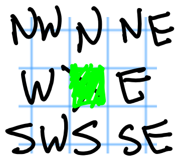
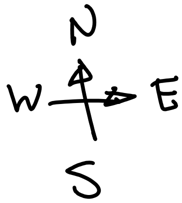
Reward Hand Calculation

March 21 Mon　　GTINE　　March 4, Fri

Driving Directions: "8- Connected Neighbors"





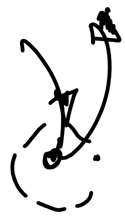
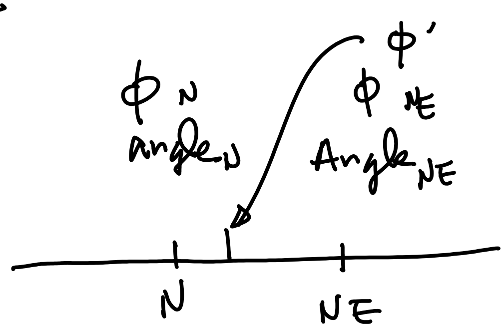


Fig.4

Find the Direction of Driving at Each Step that in the end will minize the Objective Function in Eqn (9).

Policy $\pi(\alpha_{k+1}, S_{k+1} \mid S_k) \rightarrow r_{k+1}$

Action (8 Directions)　　Reward



Action	Reward
N	$r_N = ?$
NW	$r_{NW} = ?$
W	$r_W = ?$
SW	⋮
S	
⋮	

Determine Reward Function Based ON Moving Direction of Shortest Path.

List of Possible moving Directions

1. From Fig.4. Only 5 possible Directions

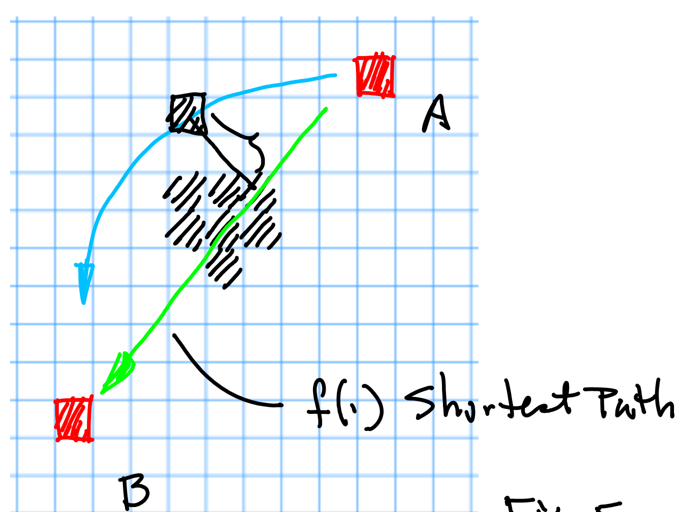


Fig.5

Example 1. 1. place 8-Direction Template (N, S, ...) on top of Point A. Use shortest path, Green Line, as a reference to Define A set of Reward function Based on the direction Matching Reward (DMR) = Policy

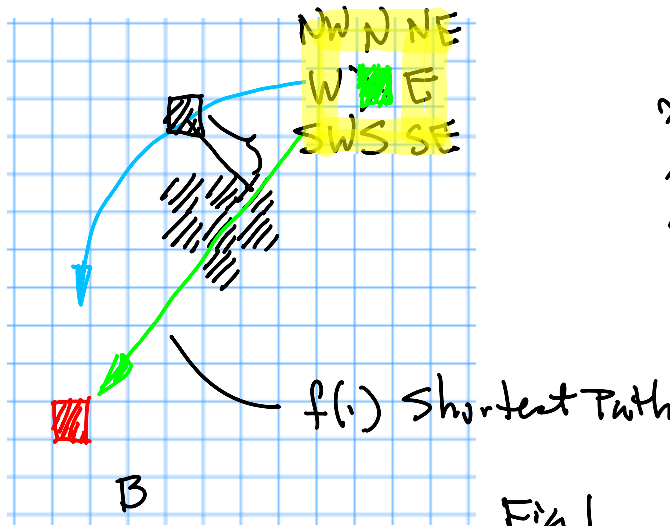


Fig. 6

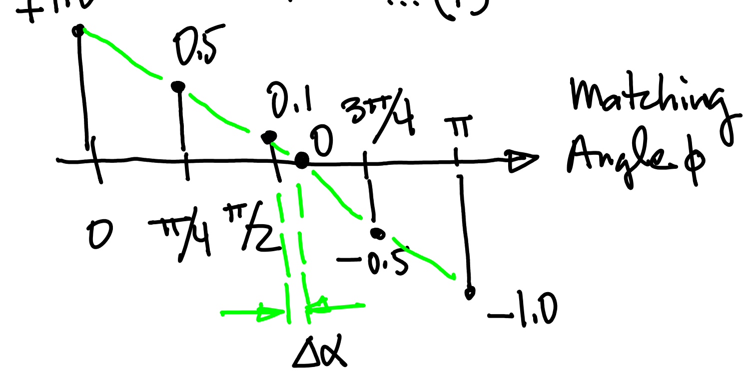
π_{DMR} : $\Delta = 0.5$

x-SW	+1.0	Best Matching	x-SW Overlap
x-W	+0.6	Next Best	x-W Angle $< \pi/2$
x-S	+0.6	" "	x-S " $< \pi/2$
x-SE	+0.1		x-SE " $< \pi/2$
x-E	−0.1	Opposite	x-E " $> \pi/2$
x-NW	−0.1		Angle $> \pi/2$
x-N	−0.6		Angle $> 3\pi/4$
x-NE	−1.0		Angle $\simeq \pi$

Algorithm: Best Matching Direction. "Highest +" Reward

Worst matching Direction "Smallest −" Reward

$$r = a\phi + b \quad \cdots (1)$$



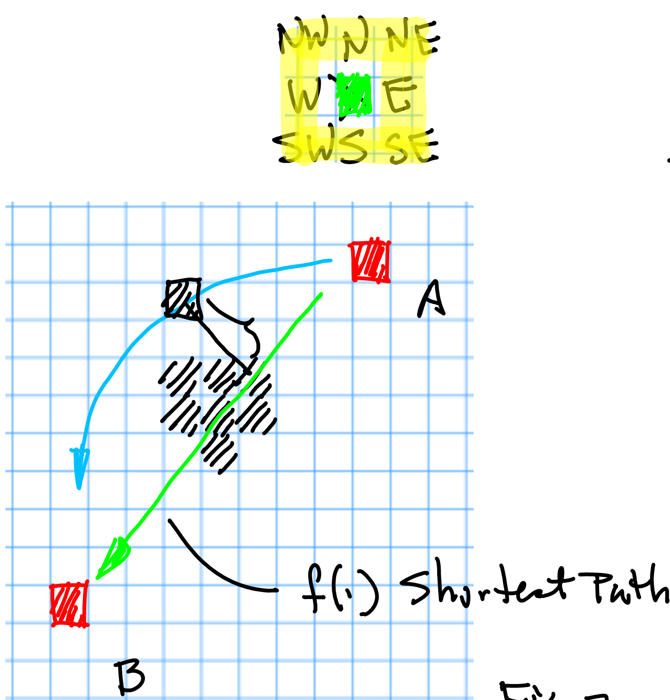


Fig. 7

Program Implementation:

1° Implement Reward Function (1).

Note: Angle ϕ is formed Between Blue line and green Line.

Compute Reward.

March 12 (Sat), 22

Step 1. Setup the System As in Fig 1., Fig. 2 is the Abstract Form for Fig. 1, So use Fig. 2.

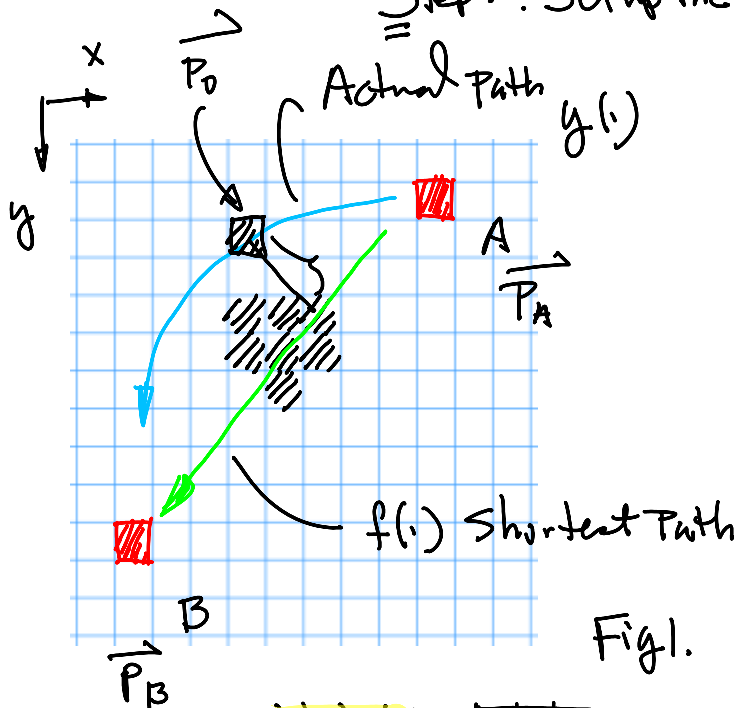


Fig 1.

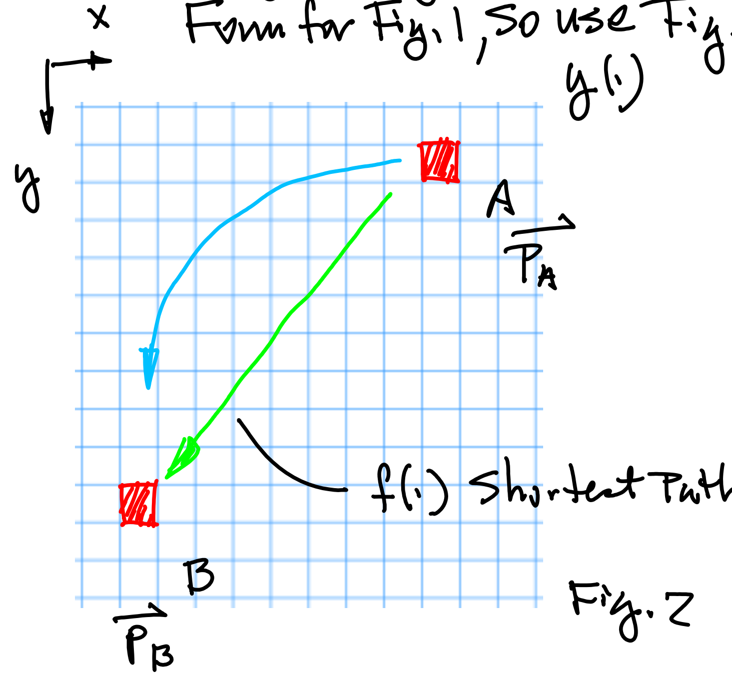


Fig. 2

Step 2.

Fig 2b. 8-Connected Neighbour.

Define Python module for the Angle Computation for 8-Connected Neighbours in Fig. 2b.

Step 3. Compute Angle Between green and Blue at location 1.

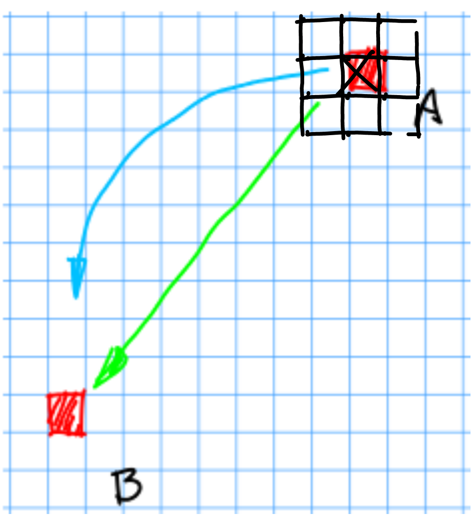


Fig. 3

Compute Angle Between green and Blue at location 2.

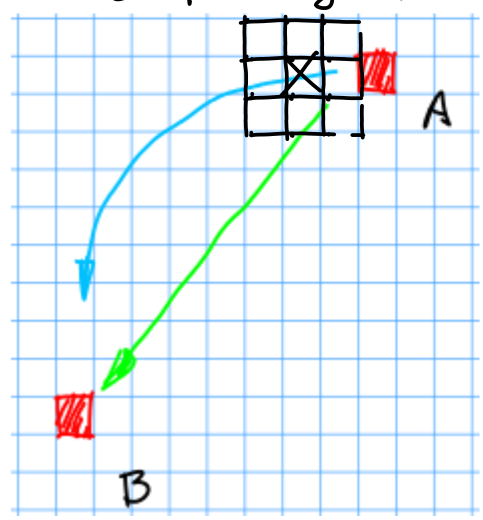


Fig. 4

March12(Sat)22

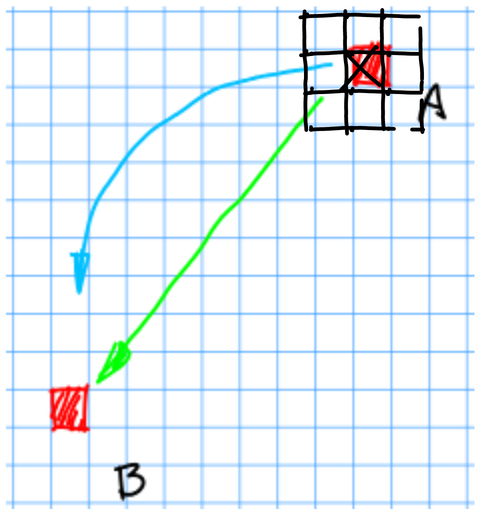
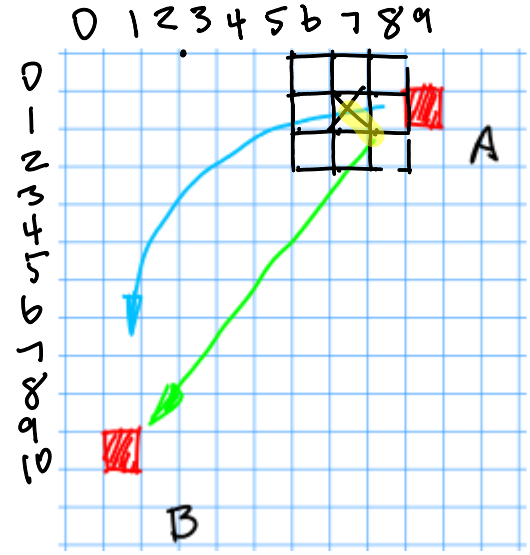


Fig.3



$i=1$
$\vec{P}(7,1)$
48.36°
dist=1.49

Fig.4

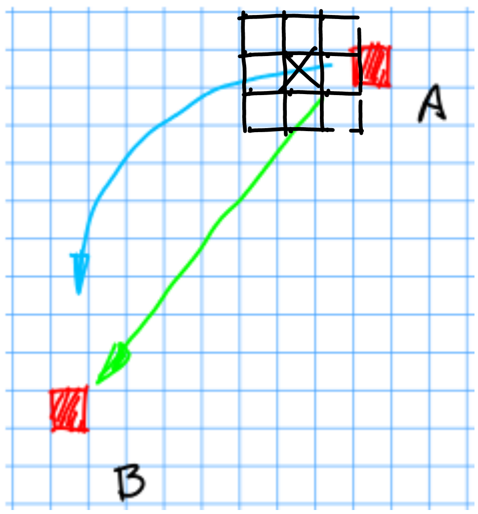
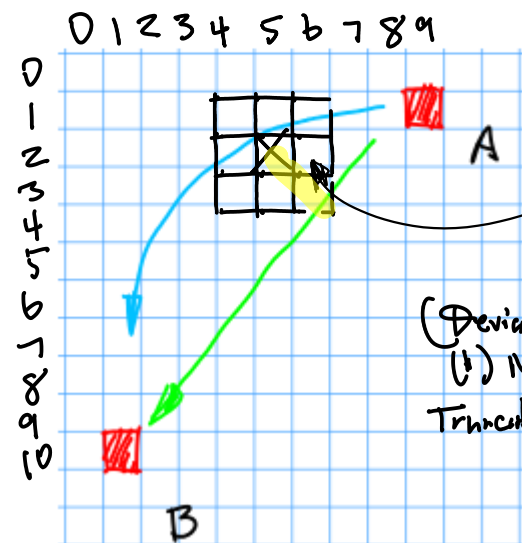


Fig5

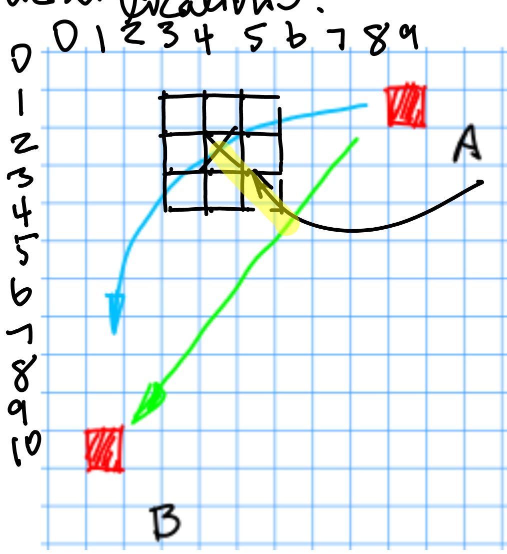
Compute Angle Between green and Blue at location3.



$i=2$
34.33°
$\vec{P}(5,2)$
dist=2.325
(Deviation is Due to (1) No DDA, (2) Truncation During Calculati)

Fig.6

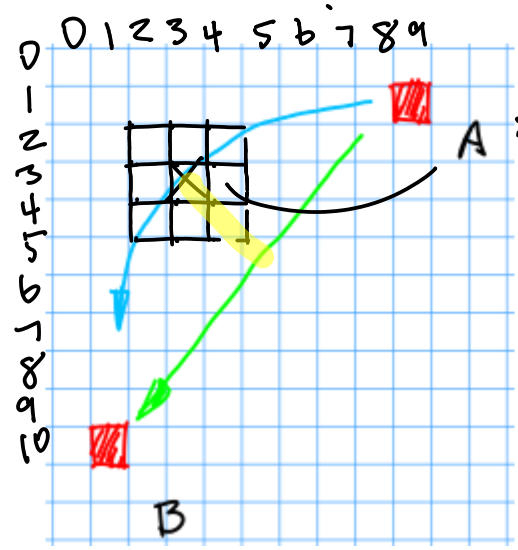
Compute Angle Between green and Blue at location4.



$i=3$
$\vec{P}(4,2)$
37.05°
dist=3.0726

Fig7

Compute Angle Between green and Blue at location5.



$i=4$
$\vec{P}(3,3)$
29.93°
dist=3.1557

Fig. 8

Compute Angle Between green and Blue at location6.

March 12 (Sat), 22

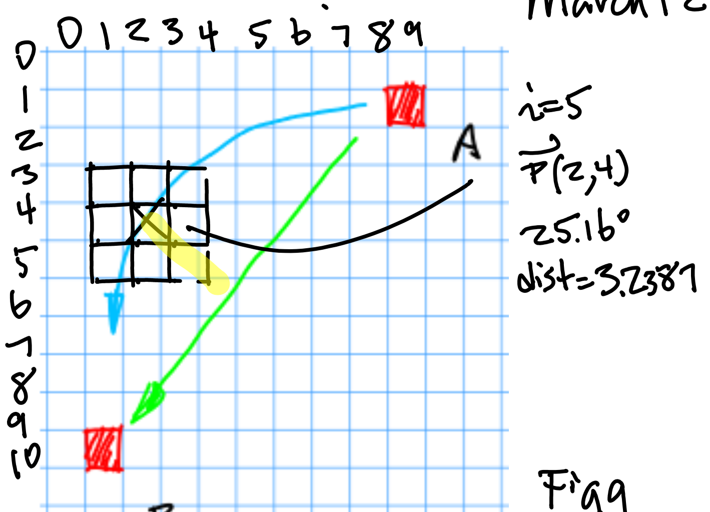


Fig 9

Compute Angle Between green and Blue at location 7.

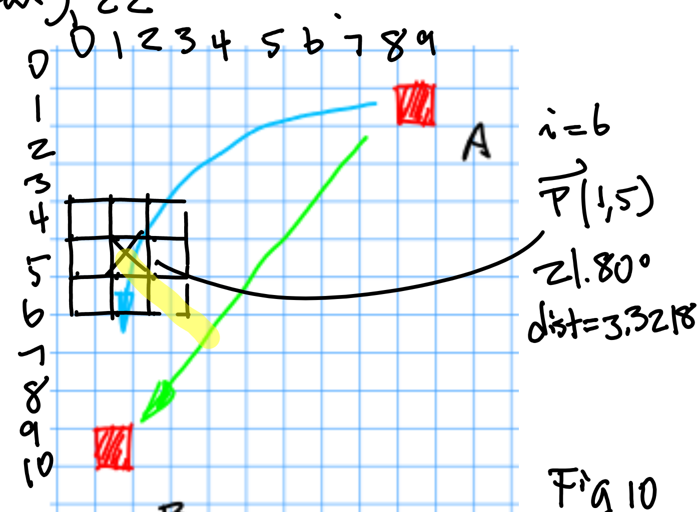


Fig. 10

Compute Angle Between green and Blue at location 8.

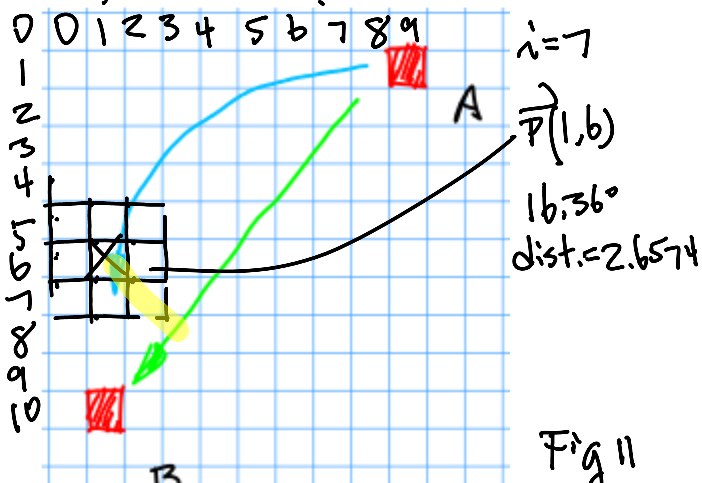


Fig 11

Compute Angle Between green and Blue at location 9.

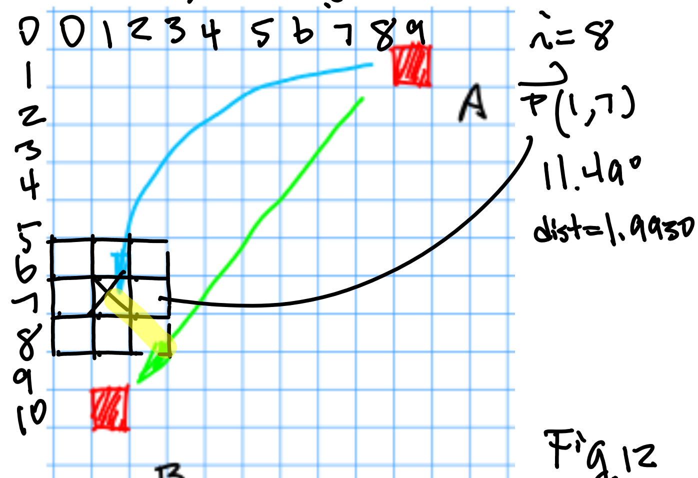


Fig. 12

Compute Angle Between green and Blue at location 10.

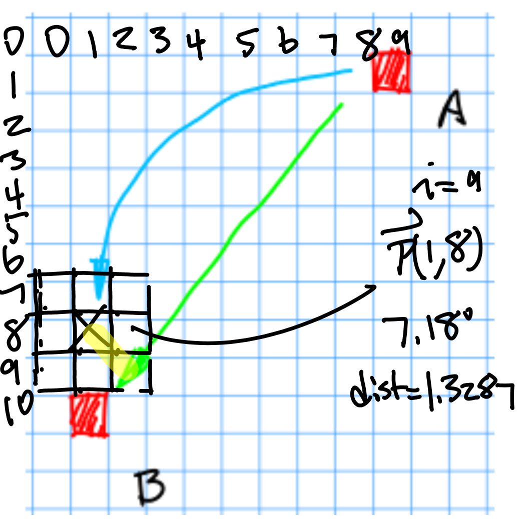


Fig 13

Compute Angle at location 11.

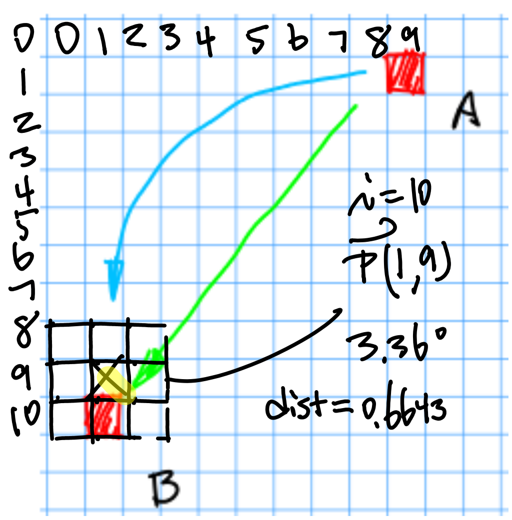


Fig. 14

Compute Angle at location 12.

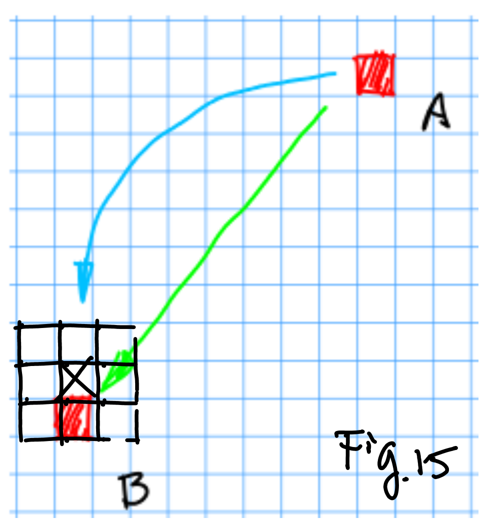


Fig. 15

Compute Angle at location 13.

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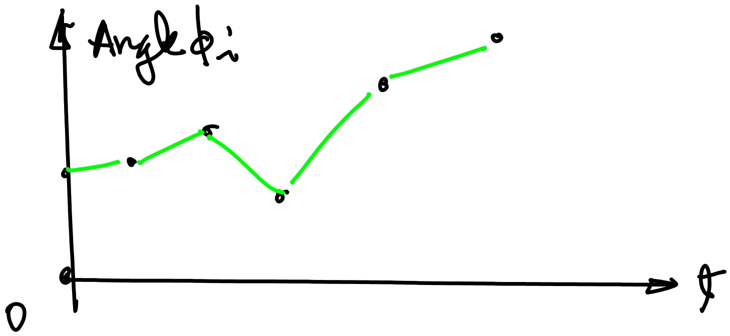
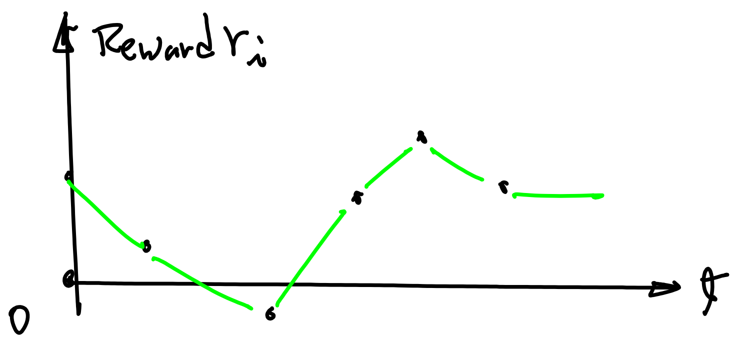
Step 4

Plot All the Angles $\phi_1, \phi_2, \cdots, \phi_i \cdots$ in the plot below, plot All Reward Function values $r_1, r_2, \cdots, r_i \cdots$ in the plot below. Then find Sum of all Rewards.



$$R_\Sigma = \sum_{i=1}^{N-1} r_i \quad \cdots (1)$$



March 13th (Sun) with B.P.

(End)

$$\vec{x} \times \vec{y} = \vec{z}$$

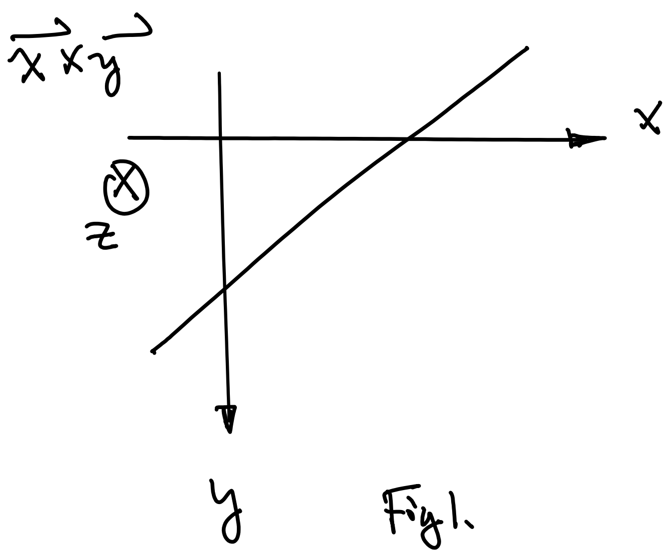
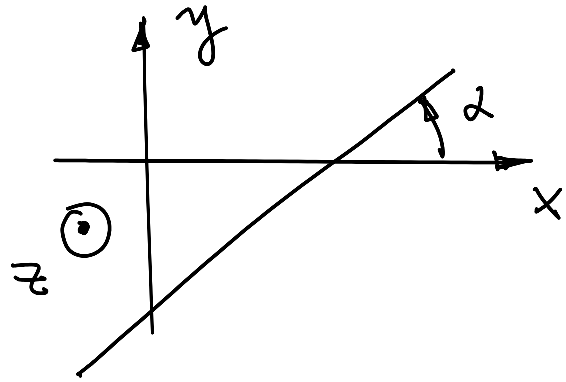


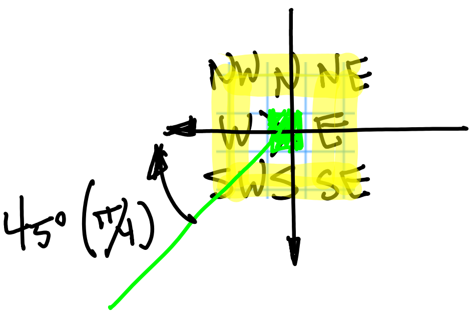
Fig 1.



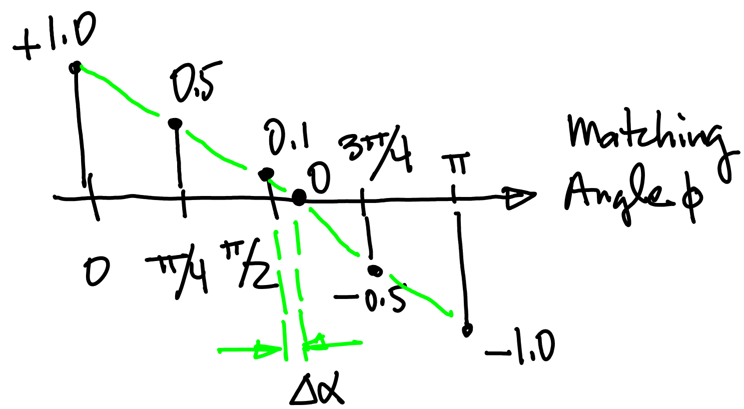
Guideline

Fig. 2

No "World coordinate System" yet



W.r.t GreenLineDirection



π_{DMR} : $\Delta = 0.5$

α-SW	+1.0	Best Matching	α-SW	Overlap
α-W	+0.6	Next Best	α-W Angle	< π/2
α-S	+0.6	" "	α-S "	< π/2
α-SE	+0.1		α-SE "	< π/2
α-E	−0.1	Opposite	α-E "	> π/2
α-NW	−0.1		Angle	> π/2
α-N	−0.6		Angle	> 3π/4
α-NE	−1.0		Angle	≃ π

March 14 (Monday), With. YY, BP.

robotics-open_abb / aiv200 / 190g-deep-reinforcement-learning / **190g-3-6DoF-Action-State-Reward-SS-2021-03-17.pdf**

6 DoF Robot Unity

[How to train your Robot Arm?. Training a 6 axis robot arm using Unity… | by Raju K | XRPractices | Medium]

[rkandas/RobotArmMLAgentUnity: Training 6 axis robot arm Inverse kinematics using Unity ML Agents (github.com)]

1. Actions: An array of actions – each action in the array represents the degree of rotation. We have 5 types of actions in total: 1 Rotate and 4 Bends.

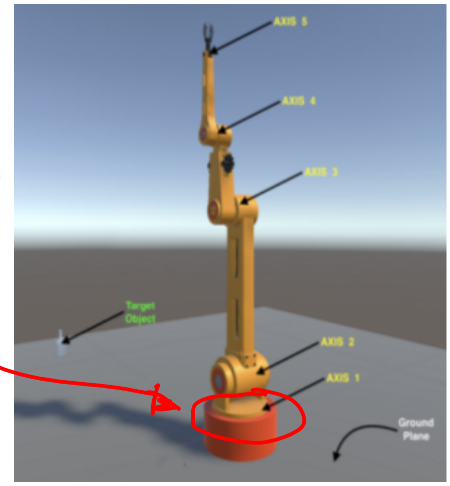
   1.1. Axis 1: is the bottom-most axis and can rotate 0 to 360 degrees [Rotate]

```
armAxes[0].transform.localRotation =
Quaternion.AngleAxis(angles[0] * 180f, armAxes[0].GetComponent<Axis>().rotationAxis);
```

CTI ONE march 14, 22

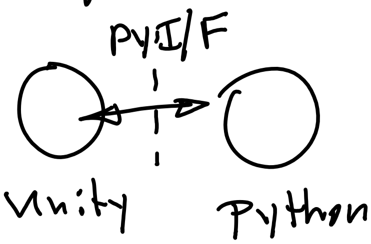
1.1. Axis 1: is the bottom-most axis and can rotate 0 to 360 degrees [Rotate]

```
armAxes[0].transform.localRotation =
Quaternion.AngleAxis(angles[0] * 180f, armAxes[0].GetComponent<Axis>().rotationAxis);
```



C# { a. Physical model (Dimension)
b. Physics of the model. { Rotation Direction
Angle
c. Graphical model

{ Step 1. a–c : Move OpenCV model to Unity from CTI ONE model, And the implementation code is from CTI One Team, especially from Mr. Yusuke Yakuwa.
Step 2. C# ML Interface



OpenCV { 1. Angle
2. Index
3. Reward (I)

March 16 (Wed)

1. Verification of YY's Implementation

2. Provide Hand Calculation.

1. from start position to the end position.

	Position	Angle	Distance	Reward
$\vec{P}_A$	(9,1)	ϕ_i	~~d_i~~ L_i	$r(\phi_i, L_i)$
⋮	⋮			
$\vec{P}_i$	(4,2)			
⋮	⋮			
$\vec{P}_B$	(1,10)			

2. Record Heuristic Motion, Path.txt

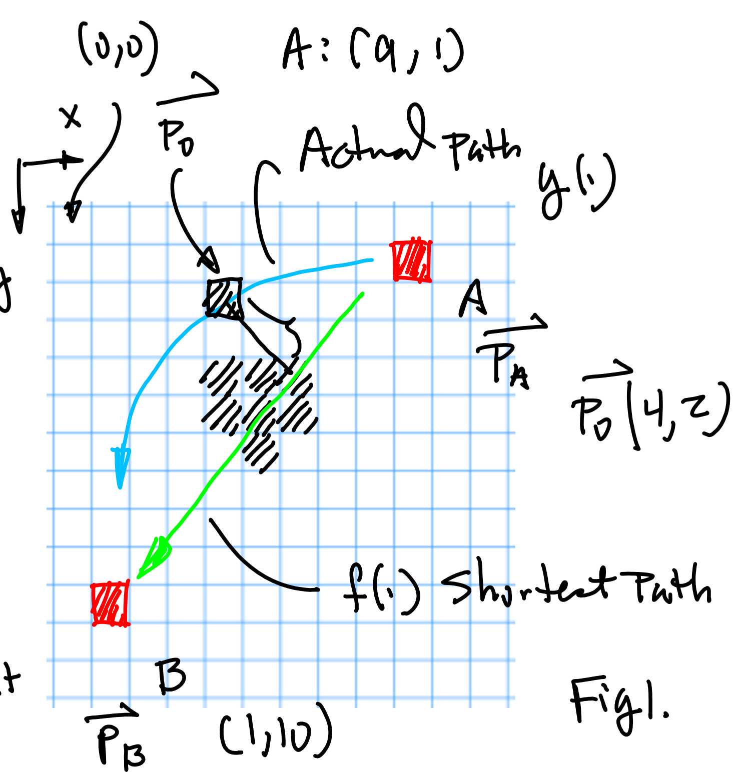


Fig 1.

③ Run GTI One Version 0.1 Code,
Make Comparison for Verification.

March 17 (Thursday) Hand Calculation

Step 1. Given initial condition
$A=(9,1)$, $B=(1,0)$
x-y Coordinate System Setup
as shown in the figure below.

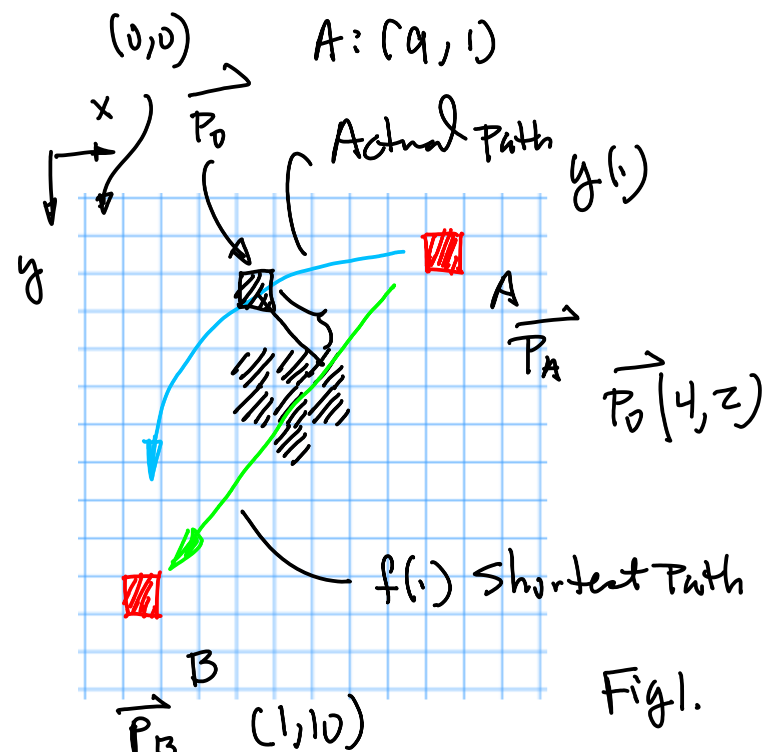


Fig 1.

Step 2. The Angle calculation
Formula:

$$\vec{P_A}-\vec{P_i}=(x_a-x_i,\ y_a-y_i) \quad \ldots (1) \text{ (Blue Line)}$$

$$\vec{P_A}-\vec{P_B}=(x_a-x_b,\ y_a-y_b) \quad \ldots (2) \text{ (Green Line)}$$

$$(\vec{P_A}-\vec{P_i})\cdot(\vec{P_A}-\vec{P_B}) = \|\vec{P_A}-\vec{P_i}\|\,\|\vec{P_A}-\vec{P_B}\|\cos\alpha \quad \ldots (3)$$

Therefore, for Eqn (3), we have Eqn (4) below,

$$\cos\alpha = \frac{(\vec{P_A}-\vec{P_i})\cdot(\vec{P_A}-\vec{P_B})}{\|\vec{P_A}-\vec{P_i}\|\,\|\vec{P_A}-\vec{P_B}\|} = \frac{(x_a-x_i,\ y_a-y_i)\cdot(x_a-x_b,\ y_a-y_b)}{\sqrt{(x_a-x_i)^2+(y_a-y_i)^2}\sqrt{(x_a-x_b)^2+(y_a-y_b)^2}} =$$

$$\frac{(x_a-x_i)(x_a-x_b)+(y_a-y_i)(y_a-y_b)}{\sqrt{(x_a-x_i)^2+(y_a-y_i)^2}\sqrt{(x_a-x_b)^2+(y_a-y_b)^2}} \quad \ldots (4)$$

Now, Calculation. Denote Robot Position as $\vec{P_i}(x_i, y_i)$,

for Postion $i=1$, $\vec{P_1}(7,1)$ illustrated in Fig.4

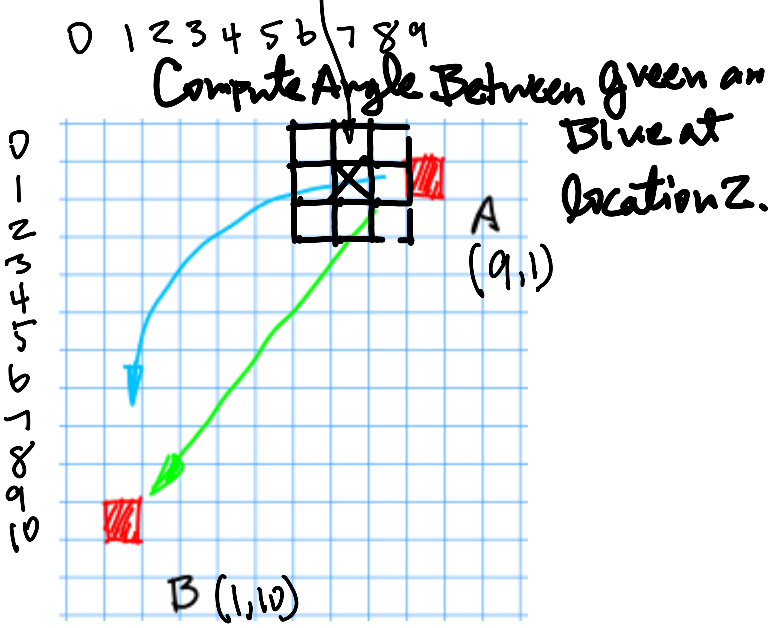


Fig.4

From Eqn (4), we have

$$\cos\alpha = \frac{(x_a - x_i)(x_a - x_b) + (y_a - y_i)(y_a - y_b)}{\sqrt{(x_a - x_i)^2 + (y_a - y_i)^2}\sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}}$$

see my Spreadsheet for the hand Calculation.

Now, Compute Reward Function, Based on the Formula on PP.

March 17 (Th), 22

From pp.4.

π_{DMR} : $\Delta = 0.5$

x-SW	+1.0	Best Matching	x-SW	Overlap
x-W	+0.6	Next Best	x-W	Angle < $\pi/2$
x-S	+0.6	" "	x-S	" < $\pi/2$
x-SE	+0.1		x-SE	" < $\pi/2$
x-E	-0.1	Opposite	x-E	" > $\pi/2$
x-NW	-0.1			Angle > $\pi/2$
x-N	-0.6			Angle > $3\pi/4$
x-NE	-1.0			Angle $\simeq \pi$

Note: March 21 (Mon).

$$R(\phi) = \begin{cases} -1 & \text{if collision} \\ +1 & \text{if Reach Target B} \\ a\phi + b & \text{if } \phi > 0 \\ a'\phi + b' & \text{if } \phi < 0 \end{cases} \quad \cdots (1)$$

$\phi > 0$ from $\vec{P_A}$ to $\vec{P_B}$, Clockwise

$$\frac{x - x_2}{x_2 - x_1} = \frac{y - y_2}{y_2 - y_1} \quad \cdots (2)$$

(See PP. 15)

Algorithm: Best Matching Direction.
Highest "+" Reward
Worst matching Direction
Smallest "−" Reward

$$r = a\phi + b \quad \cdots (1)$$

March 18, Fri

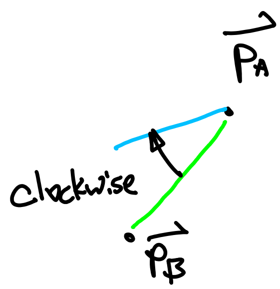


Fig. 1

Reward Function On Positive Angle (Clockwise)

Fig. 1

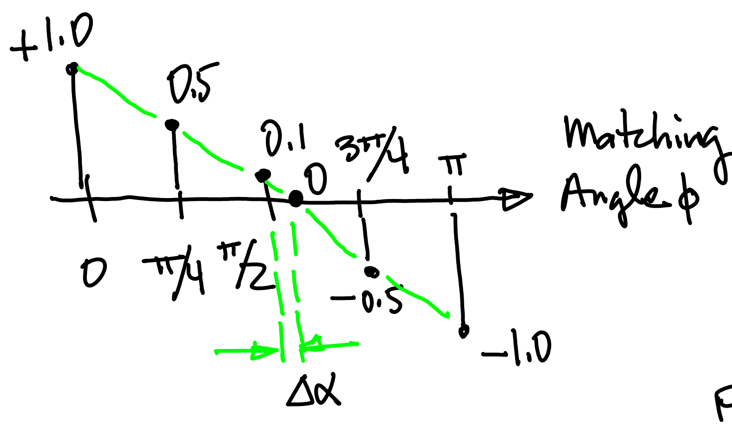
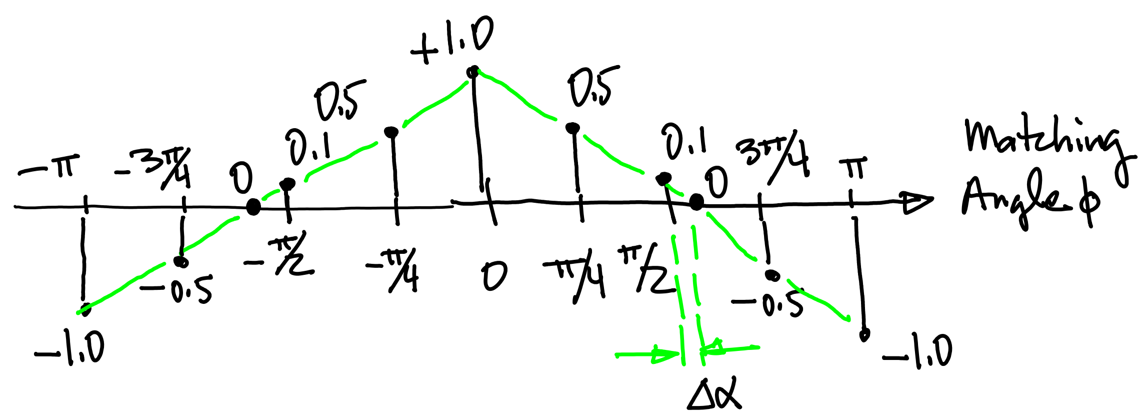


Fig. 2



Perpendicular Line Calculation.

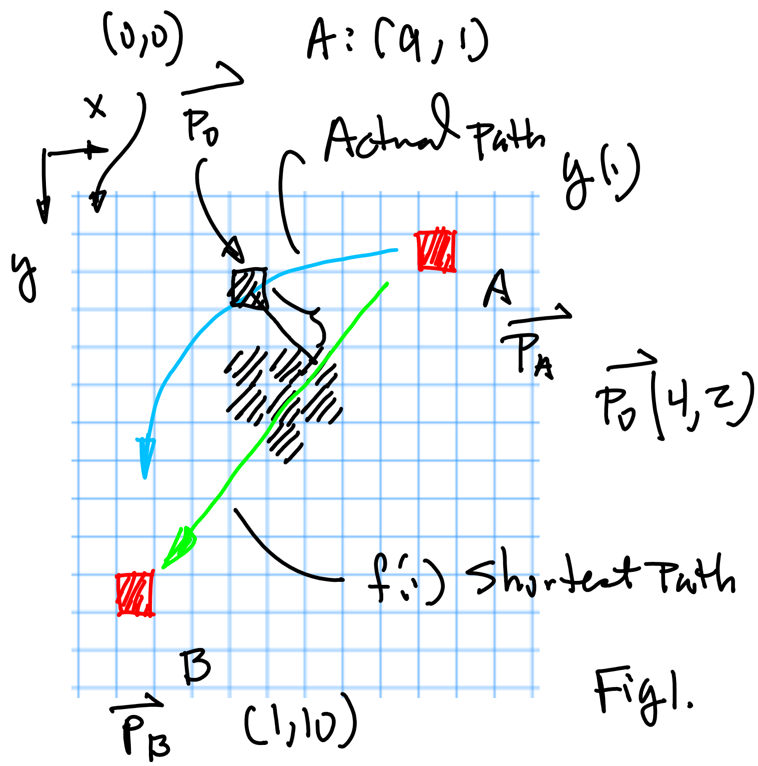


Fig 1.

Given:

$\vec{P}_A - \vec{P}_i = (x_a - x_i, y_a - y_i)$ ...(1) (Blue Line)

$\vec{P}_A - \vec{P}_B = (x_a - x_b, y_a - y_b)$ ...(2) (Green Line)

direction Vectors:

$\vec{d}_B = (x_a - x_i, y_a - y_i) = (x_B, y_B)$ ...(3)

$\vec{d}_G = (x_a - x_b, y_a - y_b) = (x_G, y_G)$ ...(4)

Parametric Equation for the Lines in

$ay = bx + c$ Form,

$f_B(x, y) = a_B y - (b_B x + c)$ ...(5)

$f_G(x, y) = a_G y - (b_G x + c)$ ...(6)

where

$a_B = x_a - x_i,\ b_B = y_a - y_i$ ...(7)

$a_G = x_a - x_b,\ b_G = y_a - y_b$ ...(8)

And $\vec{d}_G = (a_G, b_G)$ ... (8b)

The intersection Point of the Perpendicular line from $\vec{P}_i$ on Blue to Green Line

$\vec{P}(x, y) = \vec{P}_i + \lambda \vec{d}$ ... (9)

where

$\vec{d} \cdot \vec{d}_G = 0,\ (x_d, y_d) \cdot (x_G, y_G) = x_d x_G + y_d y_G = 0$ ...(10)

$(x_d, y_d) = (-y_G, x_G)$ ...(11)

So, Perpendicular line to Green

$\vec{P}(x, y) = \vec{P}_i + \lambda \vec{d} = \vec{P}_i + \lambda (x_d, y_d) = \vec{P}_i + \lambda (-y_G, x_G)$ ...(11b)

The distance (Perpendicular Line from Blue Line $\vec{P}_i$ to Green Line):

$$\lambda = -\frac{f_G(x_i, y_i)}{a_G^2 + b_G^2} \quad \text{...(12)}$$

The distance (Perpendicular Line from Blue Line $\vec{P_i}$ to Green Line):

$$\vec{P}(x,y) = \vec{P_i} + \lambda\vec{d} = \vec{P_i} + \lambda(x_d, y_d) = \vec{P_i} + \lambda(-y_G, x_G)$$

$$= (x_i, y_i) + \lambda(-y_G, x_G) \quad \dots (11b)$$

where

$$\lambda = -\frac{f_G(x_i, y_i)}{a_G^2 + b_G^2} \quad \dots (12)$$

$$f_G(x,y) = a_G y - (b_G x + c_G) \quad \dots (6)$$

$$a_G = x_a - x_b,\ b_G = y_a - y_b \quad \dots (8)$$

$$\vec{d_G} = (a_G, b_G) = (x_G, y_G) \quad \dots (8b)$$

Then, $\mathrm{dist}(\vec{P_i}, \text{Green}) = \sqrt{(x_i - x)^2 + (y_i - y)^2} \quad \dots (13)$

Testing of the Above Equations By a Special case (for Simplicity) Below

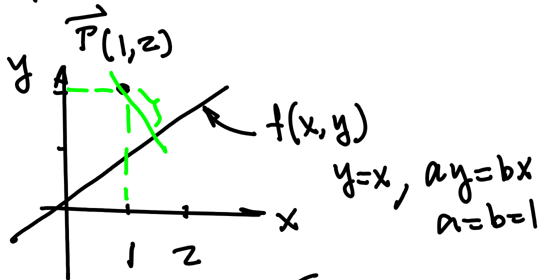


Fig 2

Step 1.

For line f (Green Line Equivalent)

$$\vec{d_G} = (x_G, y_G) = (a_G, b_G) = (1,1)$$

from Eqn (12)

$$\lambda = -\frac{f_G(x_i, y_i)}{a_G^2 + b_G^2} = -\frac{a_G y_i - (b_G x_i + c_G)}{a_G^2 + b_G^2} = -\frac{y_i - x_i}{1+1}\Bigg|_{\substack{x_i=1\\ y_i=2}}$$

$$= -\frac{2-1}{2} = -\frac{1}{2}$$

Step 2. Find Intersection Point of the Perpendicular line on Green

From Eqn (11b)

$$\vec{P}(x,y) = (x_i, y_i) + \lambda(-y_G, x_G)$$

$$= (1,2) + \lambda(-b_G, a_G)\Big|_{a_G = b_G = 1}$$

$$= (1,2) + \lambda(-1,1)\Big|_{\lambda = -\frac{1}{2}}$$

$$= (1,2) - \frac{1}{2}(-1,1) = (1+\tfrac{1}{2}, 2-\tfrac{1}{2})$$

$$= \left(\tfrac{3}{2}, \tfrac{3}{2}\right) = (1.5, 1.5)$$

From Eqn (13),

$$\mathrm{dist}(\vec{P_i}, \text{Green}) = \sqrt{(x_i - x)^2 + (y_i - y)^2}$$

$$= \sqrt{(1-1.5)^2 + (2-1.5)^2}$$

$$= \sqrt{0.5^2 + 0.5^2} = 0.5\sqrt{2} = \frac{\sqrt{2}}{2}$$

$$= 0.707$$

Note: c_G Calculation

$$f_G(x,y) = a_G y - (b_G x + c_G)\Big|_{\substack{a_G = x_G = x_a - x_b\\ b_G = y_G = y_a - y_b}}$$

Take $(x,y) = (x_a, y_a)$

Substitute into the above equation.

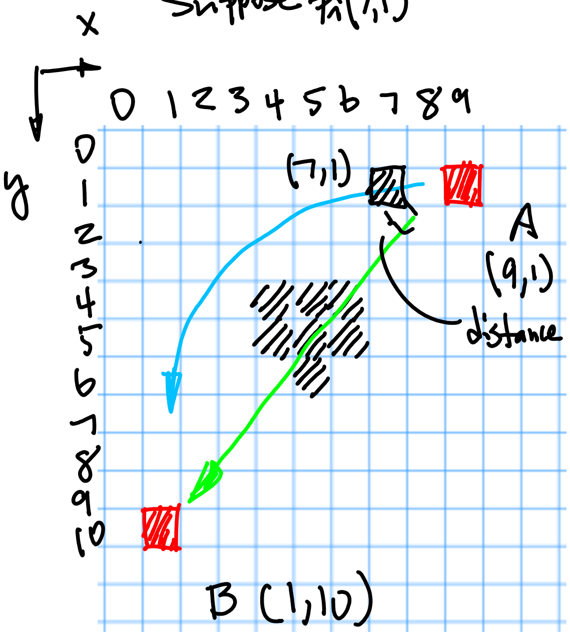
$$0 = a_G y_a - (b_G x_a + c_G),\quad c_G = a_G y_a - b_G x_a$$

$$= (x_a - x_b) y_a - (y_a - y_b) x_a$$

$$\therefore c_G = (x_a - x_b) y_a - (y_a - y_b) x_a \quad \dots (13)$$

Example: Hand Calculation of Distance

Suppose $\vec{P_i}(7,1)$



Step 1. Find direction vector of Green Line

$\vec{d}_G = (x_a - x_b, y_a - y_b) = (x_G, y_G)$ ...(4)

$= (9-1, 1-10) = (8, -9)$

Step 2. Find Green Line Equation in Parametric Form.

$f_G(x,y) = a_G y - (b_G x + c_G)$.

From

$a_G = x_a - x_b,\ b_G = y_a - y_b$ ...(8)

$\vec{d}_G = (a_G, b_G) = (x_G, y_G)$ ... (8b)

$a_G = 8,\ b_G = -9$, hence $= (9-1, 1-10) = (8, -9)$

$f_G(x,y) = 8y - (-9x + c_G)$

From Eqn (13)

$c_G = (x_a - x_b)\, y_a - (y_a - y_b)\, x_a$

$= a_G y_a - b_G x_a$

$= 8 \cdot y_a - (-9) x_a = 8 \cdot 1 - (-9) 9$

$= 8 + 81 = 89$

Step 3, Find f_G.

$f_G(x_i, y_i) = a_G y_i - (b_G x_i + c_G)$ ...(b)

$= a_G y_i - b_G x_i - c_G = 8y_i - (-9)x_i - 89 \Big|_{x_i = 7,\ y_i = 1}$

$= 8 \cdot 1 + 9 \cdot 7 - 89 = 71 - 89 = -18$

Step 4. Find λ

$\lambda = -\dfrac{f_G(x_i, y_i)}{a_G^2 + b_G^2} \Big|_{x_i = 7,\ y_i = 1}$ ...(12)

$= -\dfrac{-18}{a_G^2 + b_G^2} = \dfrac{18}{8^2 + (-9)^2} = \dfrac{18}{64 + 81}$

$= \dfrac{18}{145} = 0.12$

Step 5 Find intersection point $P(x,y)$ on the Green Line, Eqn (11b)

$\vec{P}(x,y) = \vec{P_i}(x_i, y_i) + \lambda \vec{d}$

$= (x_i, y_i) + \lambda(-y_G, x_G)$

$x = x_i + \lambda(-y_G) \Big|_{x_i = 7,\ \lambda = 0.12} = 7 + 0.12(-(-9)) = 7 + 0.12 \times 9$

$= 7 + 1.08$

$= 8.08$

$y = y_i + \lambda x_G = 1 + \lambda(x_G) = 1 + 0.12 \times 8 = 1 + 0.96$

$= 1.96$

Step 6. Find distance, from Eqn (12)

$dist(\vec{P_i}, green) = \sqrt{(x_i - x)^2 + (y_i - y)^2}$

$= \sqrt{(7 - 8.08)^2 + (1 - 1.96)^2}$

$= \sqrt{1.08^2 + 0.96^2} = \sqrt{1.1664 + 0.9216}$

$= 1.445$

The Spreadsheet Implementation:

Hand Calculation of Reward Function

Define Piece-wise Linear Reward function as Type-I Reward:

$$R_I(\phi)=\begin{cases}-1 & \text{if collision}\\ +1 & \text{if Reach Target } B\\ R_{I\text{-}1}(\phi)=a\phi+b & \text{if } \phi>0 \quad \ldots(1a)\\ R_{I\text{-}2}(\phi)=a'\phi+b' & \text{if } \phi<0 \quad \ldots(1b)\end{cases} \quad \ldots(1)$$

OR

$$R_I(\phi)=\begin{cases}-1 & \text{if collision}\\ +1 & \text{if Reach The target}\\ R_{I\text{-}1}(\phi) & \text{if } \phi>0 \quad \ldots(1a)\\ R_{I\text{-}2}(\phi) & \text{if } \phi<0 \quad \ldots(1b)\end{cases}$$

$\phi>0$ from $\vec{P_A}$ to $\vec{P_B}$, Clockwise

$$\frac{x-x_2}{x_2-x_1}=\frac{y-y_2}{y_2-y_1} \quad \ldots(2)$$

Note: Angle Definition

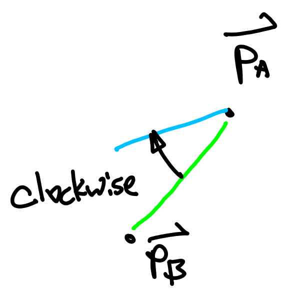


Fig.1

To find Eqn (1a), using Eqn (2), where

$(x_1, y_1)=(0,1)$, $(x_2, y_2)=(180,-1)$

So Eqn (2):

$$\frac{x-180}{180-0}=\frac{y-(-1)}{-1-1}, \quad \frac{x-180}{180}=\frac{y+1}{-2}$$

Reward Function On Positive Angle (Clockwise) Fig.1

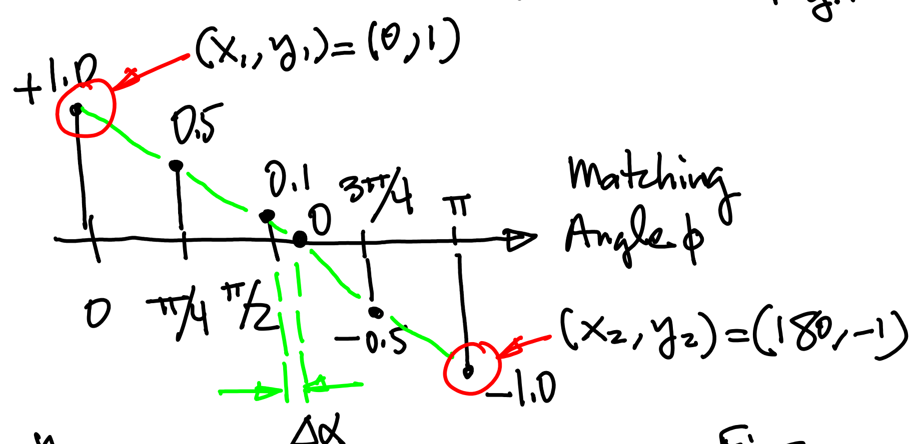


Fig.2

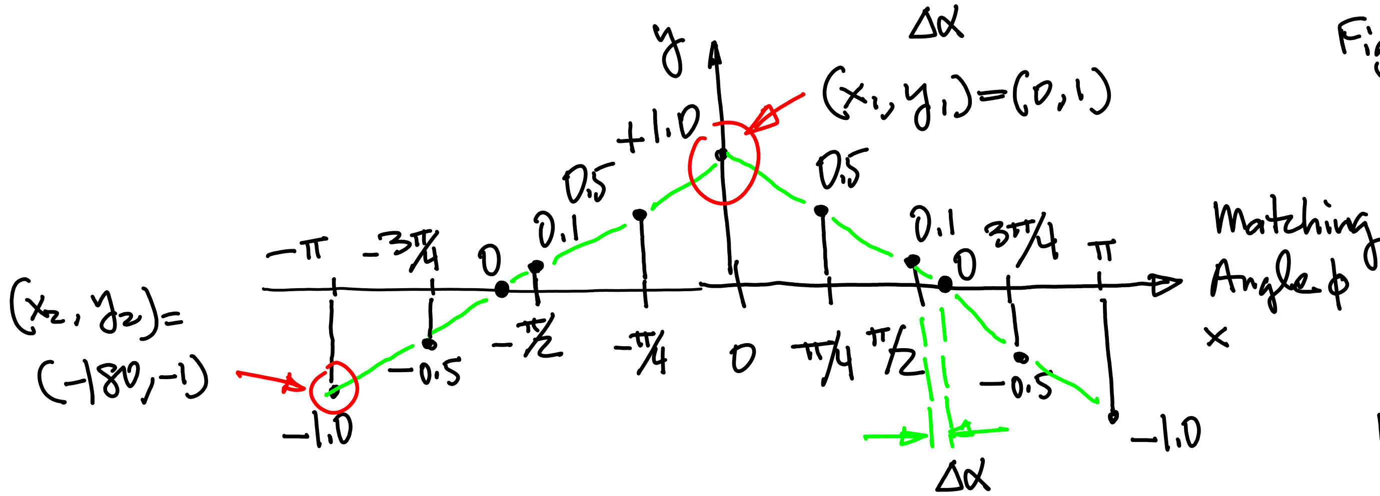


Fig.3

$$\frac{x-180}{180-0} = \frac{y-(-1)}{-1-1}, \quad \frac{x-180}{180} = \frac{y+1}{-2}$$

$$-2 \cdot \frac{x-180}{180} = y+1, \quad y = -\frac{1}{90}(x-180)-1$$

$$y = -\frac{1}{90}x + \frac{180}{90} - 1 = -\frac{1}{90}x + 1$$

$$\therefore y = -\frac{1}{90}x + 1, \quad a = -\frac{1}{90}, \quad b = 1. \quad \ldots (3)$$

$R_{I\text{-}1}(\phi) = a\phi + b$ if $\phi > 0$

For $R_{I\text{-}2}(\phi) = a'\phi + b'$ if $\phi < 0$.
(See Fig3, pp 15), from Eq.(2)

$$\frac{x-x_2}{x_2-x_1} = \frac{y-y_2}{y_2-y_1}$$

And

$(x_1, y_1) = (0, 1)$
$(x_2, y_2) = (-180, -1)$

So, $$\frac{x-(-180)}{-180-0} = \frac{y-(-1)}{-1-1}$$

$$\frac{x+180}{-180} = -\frac{y+1}{2}, \quad 2 \cdot \frac{x+180}{180} = y+1$$

$$\frac{x+180}{90} = y+1, \quad y = \frac{1}{90}x + 2 - 1$$

$$y = \frac{1}{90}x + 1$$

$$a' = \frac{1}{90}, \quad b' = 1 \quad \ldots (4)$$