Steps in Development Control Systems for Robots

- Modelling Robot Behaviors
  - Kinematics
  - Dynamics
  - Parameters’ Estimation

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- Choice of Control Architecture
  - Linear vs. Nonlinear
  - Compensation for Friction, Delays, Lack of Velocity Measurements
  - Anti-Windup Methods
  - Adaptive Mechanisms for On-line Parameters Estimation
  - Robustness . . .

Lecture 9: Independent Joint Control

- Conceptual Control Loop for One Joint

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- Controller Design
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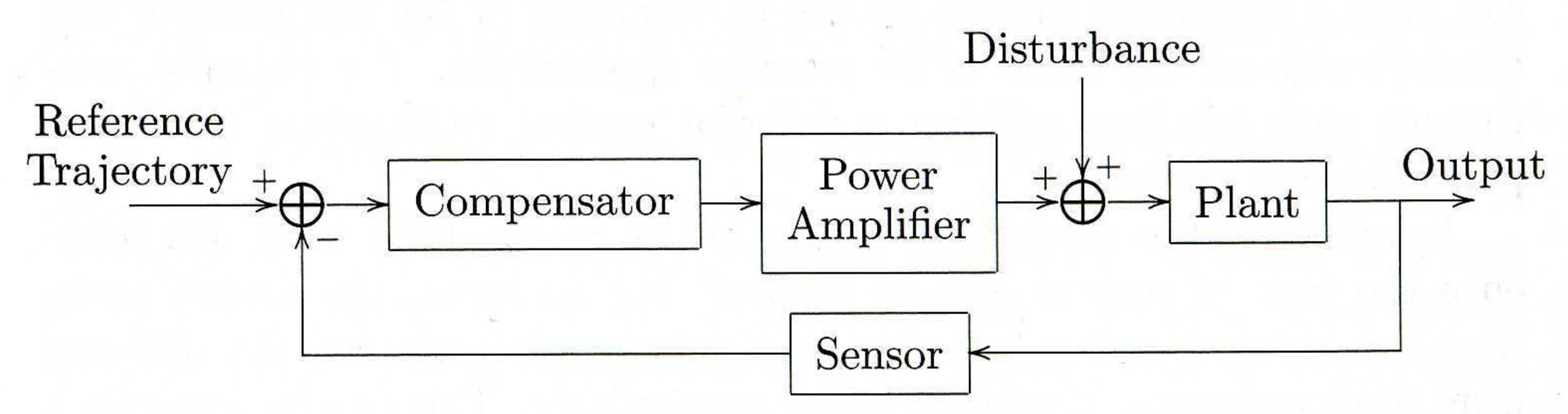

Basic structure of a feedback control system. It is often appropriate for controlling robots.

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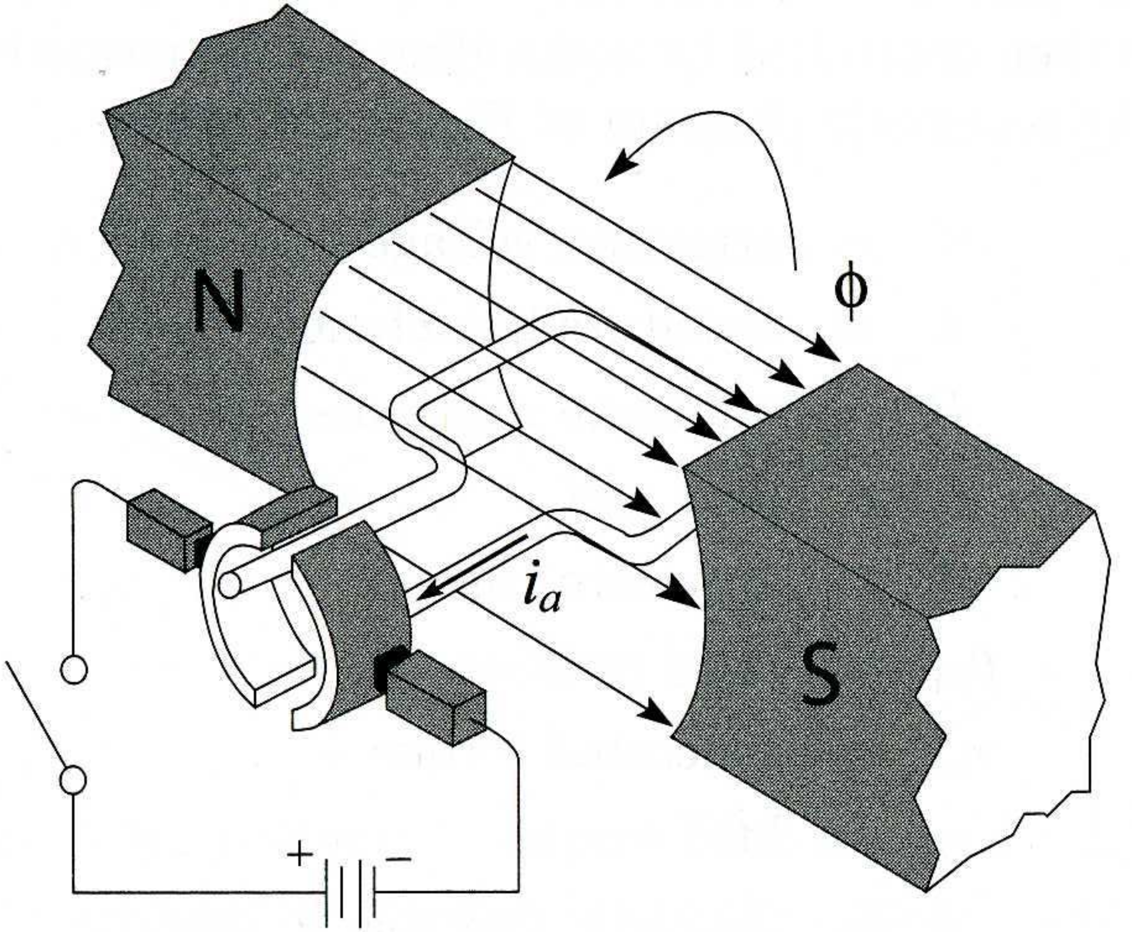
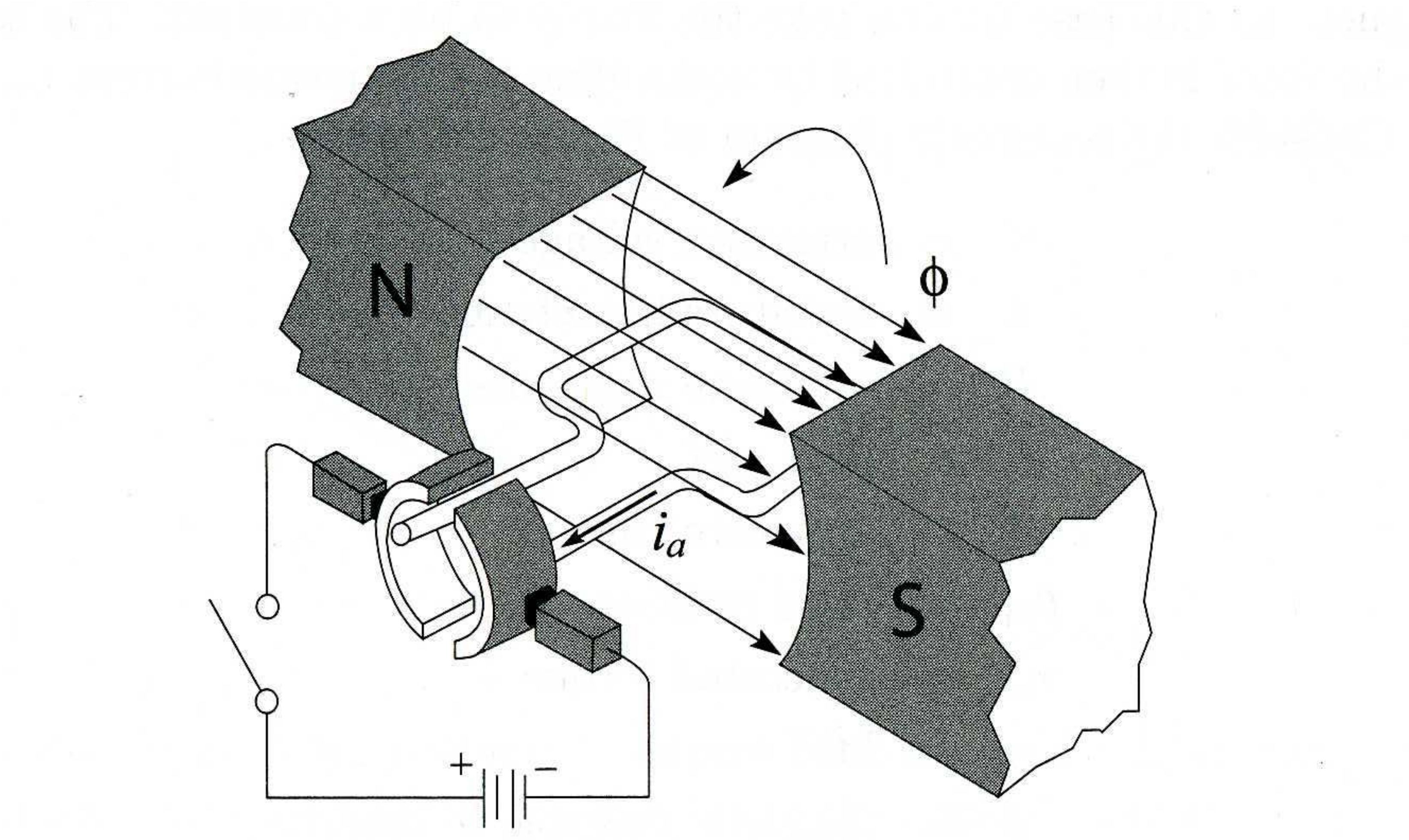


Principle of operation of a permanent magnet DC motor:
A current-carrying conductor in magnetic field experience a force

$$\vec{F} = \vec{i} \times \vec{\phi}$$

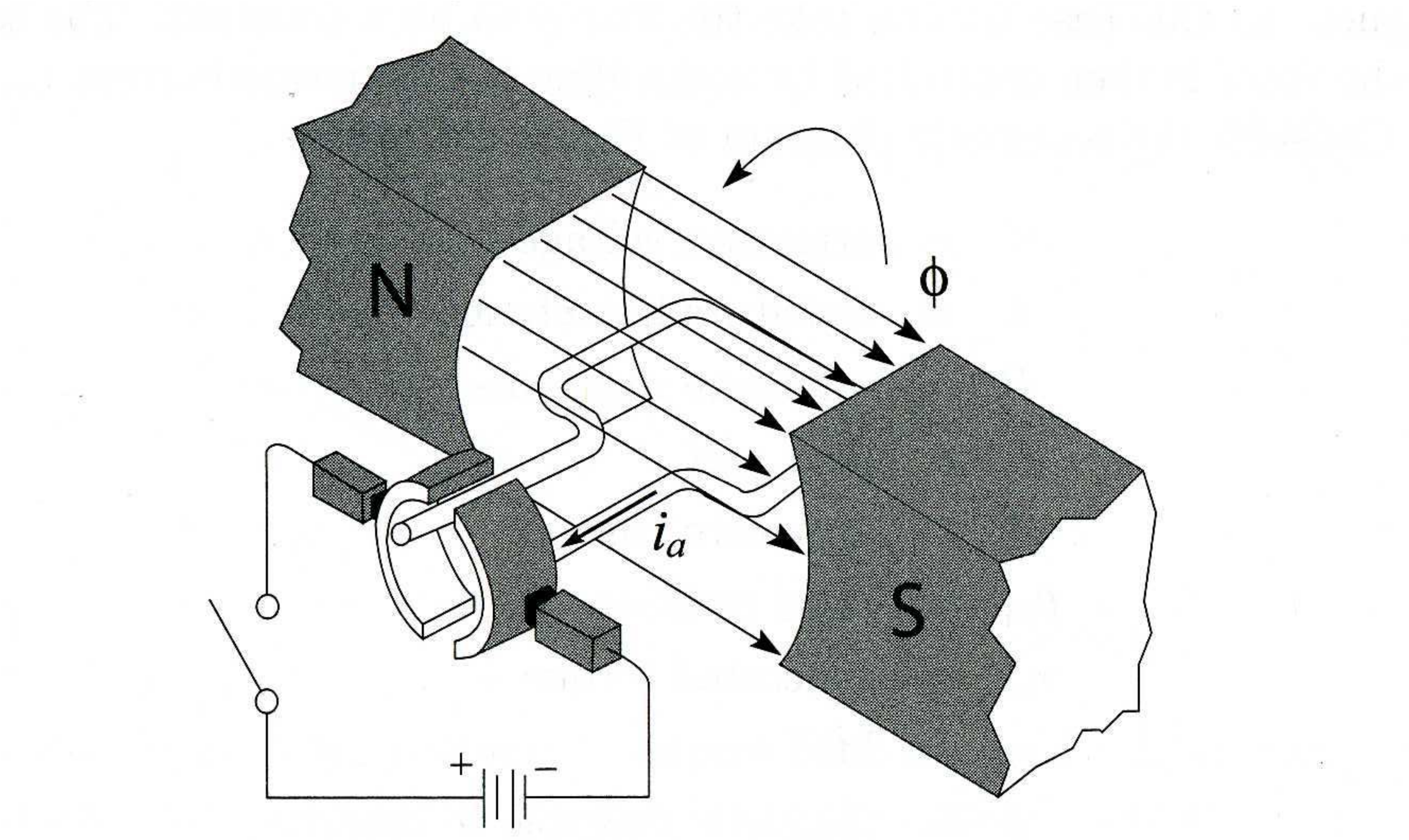
here i is is the current; ϕ is the magnetic flux



Principle of operation of a permanent magnet DC motor:
A current-carrying conductor in magnetic field experience a force

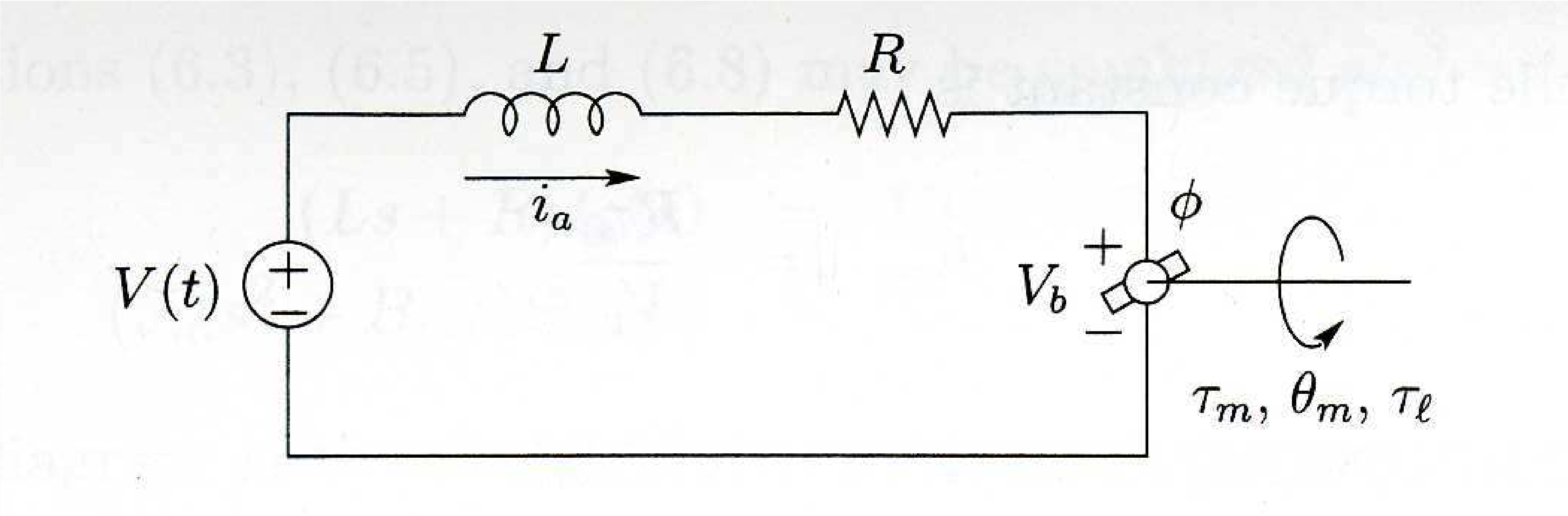
$$\vec{F} = \vec{i} \times \vec{\phi} \quad \Rightarrow \quad \tau_m = \left(K \cdot |\vec{i}| \cdot |\vec{\phi}| \right) \vec{i}$$

here i is is the current; ϕ is the magnetic flux



Principle of operation of a permanent magnet DC motor:
Whenever a conductor moves in a magnetic field, the voltage V_b is induced and it is proportional to a velocity of the conductor

$$V_b = \left(K \cdot |\vec{\phi}|\right) \frac{d}{dt}\theta_m$$



Relations between the armature current, voltage,rotor velocity and motor torque are

$$
\begin{aligned}
L\frac{d}{dt}i + Ri &= V - V_b \\
V_b &= \left(K \cdot |\vec{\phi}|\right)\frac{d}{dt}\theta_m \\
\tau_m &= \left(K_1 \cdot |\vec{i}| \cdot |\vec{\phi}|\right)\vec{i} = K_m\vec{i}
\end{aligned}
$$

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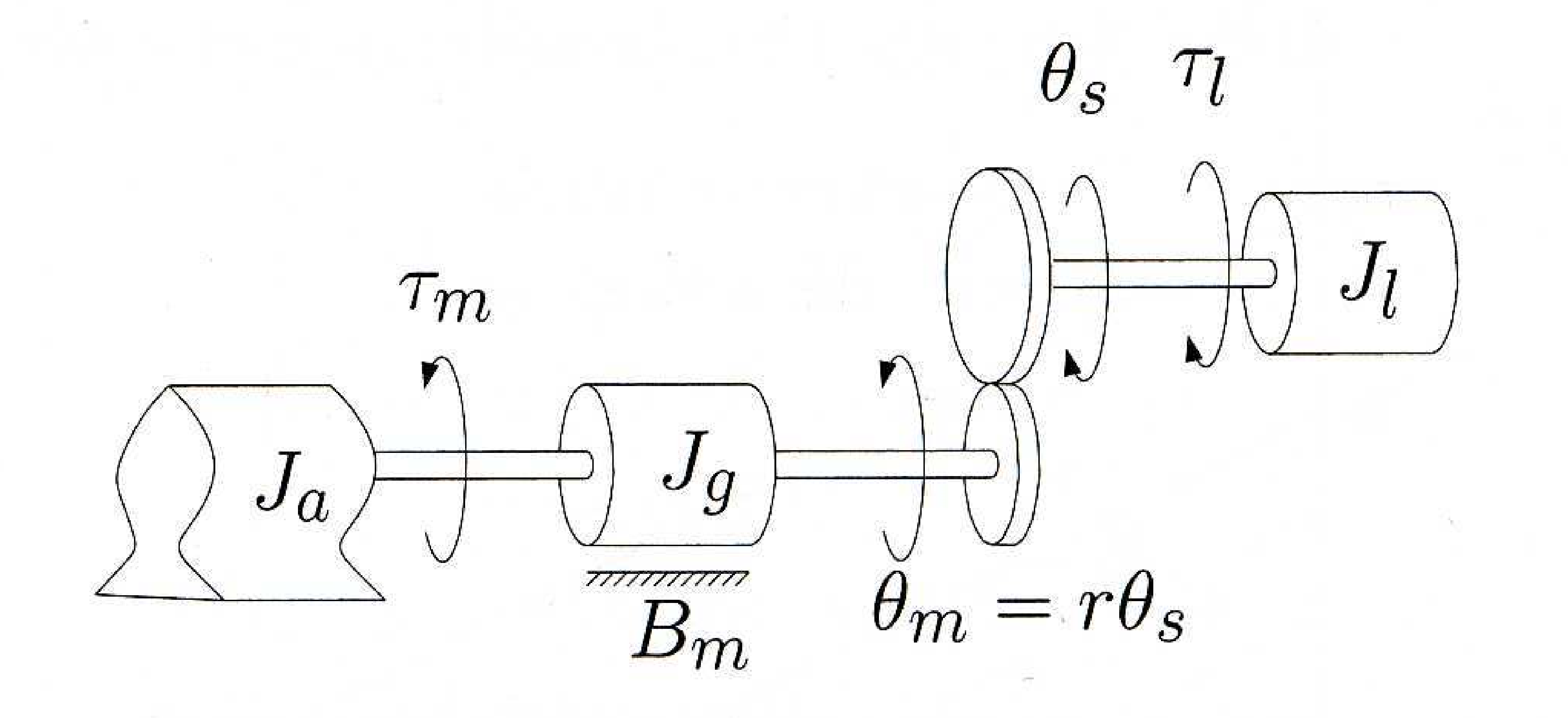
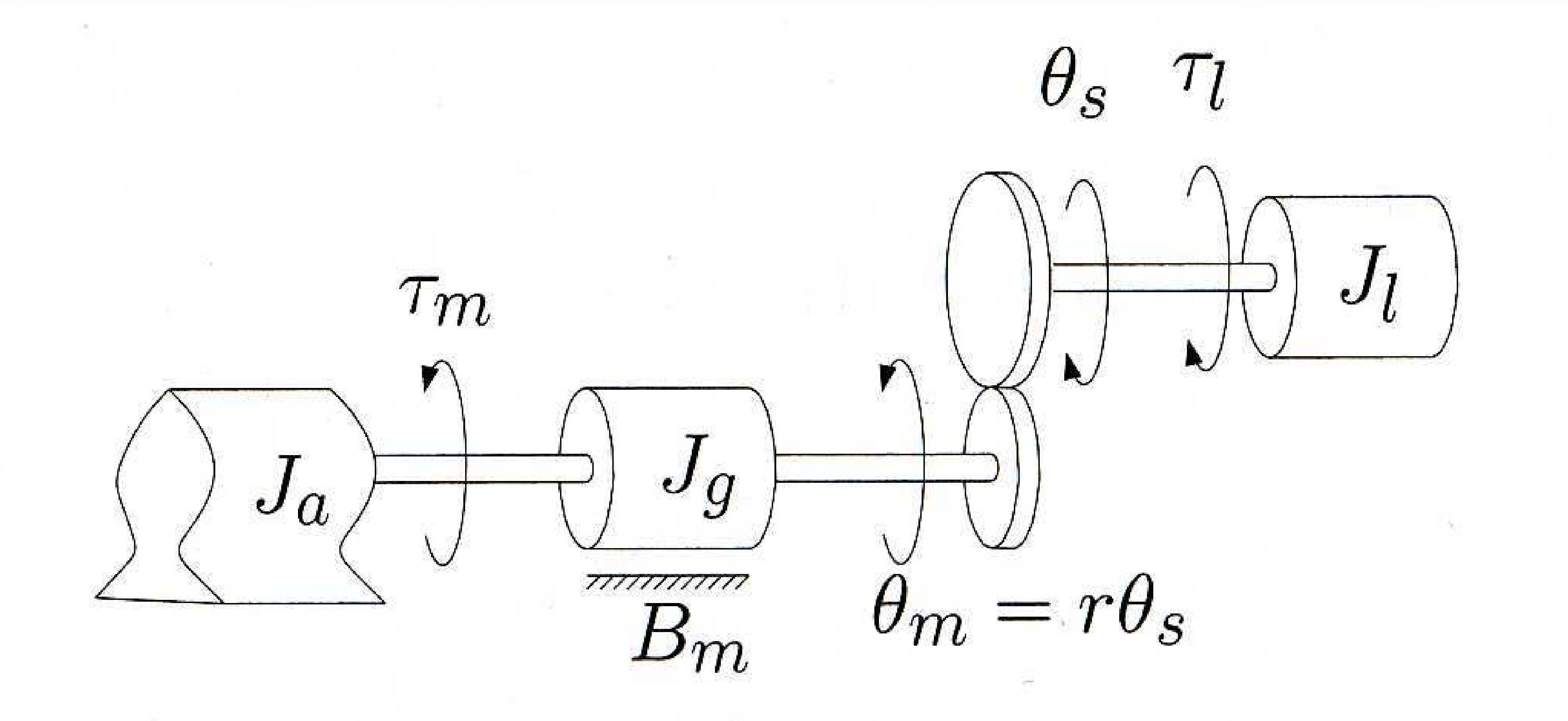
Dynamical Model of a Robot with One Joint



Lumped model of a single link with actuator/gear transmission.

Dynamical Model of a Robot with One Joint



Lumped model of a single link with actuator/gear transmission.

In terms of the motor angle θ_m the equation of motion is

$$J_m\left(\frac{d^2}{dt^2}\theta_m\right) = \tau_m - \frac{1}{r}\tau_l - B_m\left(\frac{d}{dt}\theta_m\right) = K_m \cdot i - \frac{1}{r}\tau_l - B_m\left(\frac{d}{dt}\theta_m\right)$$

Dynamical Model of a Robot with One Joint

Augmenting the mechanical and electrical models, we obtain:

$$
\begin{aligned}
L\left(\frac{d}{dt}i\right) + R\cdot i &= V - K_b\left(\frac{d}{dt}\theta_m\right) \\
J_m\left(\frac{d^2}{dt^2}\theta_m\right) + B_m\left(\frac{d}{dt}\theta_m\right) &= K_m\cdot i - \frac{1}{r}\tau_l
\end{aligned}
$$

Dynamical Model of a Robot with One Joint

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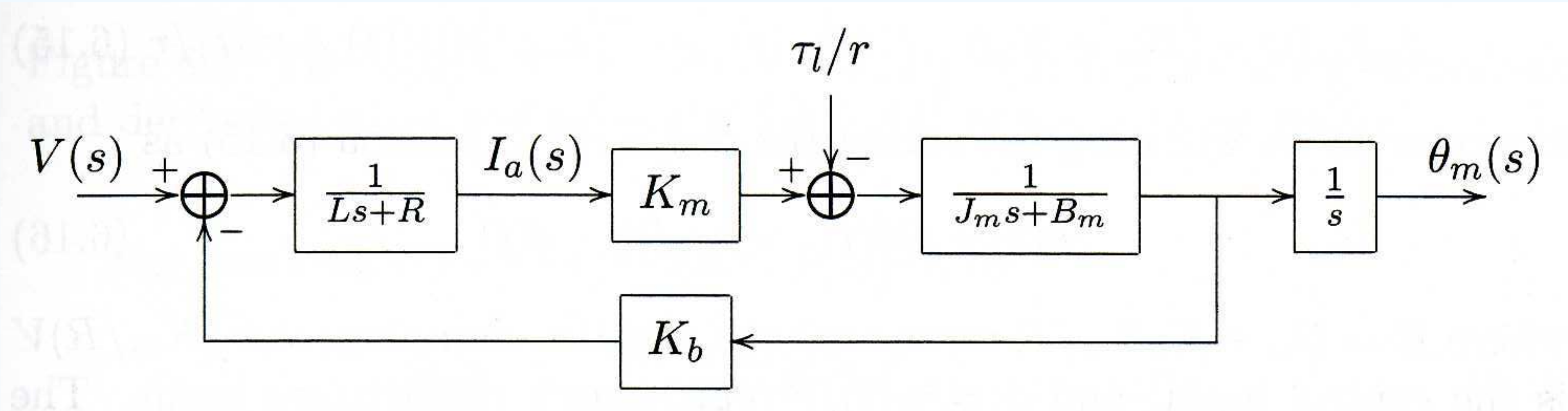

System is linear! We can use classical methods for controlling it!

Dynamical Model of a Robot with One Joint

The transfer function from $V(s)$ to $\Theta_m(s)$ is

$$G_{\{v \to \theta\}}(s) = \frac{\Theta_m(s)}{V(s)} = \frac{K_m}{\left[(Ls + R)(J_m s + B_m) + K_m K_b\right] s}$$

Dynamical Model of a Robot with One Joint

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The transfer function from $\tau_l(s)$ to $\Theta_m(s)$ is

$$G_{\{\tau_l \to \theta\}}(s) = \frac{\Theta_m(s)}{\tau_l(s)} = \frac{1}{r} \frac{-(Ls + R)}{\left[(Ls + R)(J_m s + B_m) + K_m K_b\right] s}$$

Dynamical Model of a Robot with One Joint

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If $\dfrac{L}{R} \approx 0 \quad \Rightarrow \quad G_{\{v\to\theta\}}(s) \approx \boxed{\dfrac{K_m}{\left[R(J_m s + B_m) + K_m K_b\right] s}}$

Dynamical Model of a Robot with One Joint

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Correctness of reduction of the model based on size of the parameter can be justified via e.g. balanced model reduction

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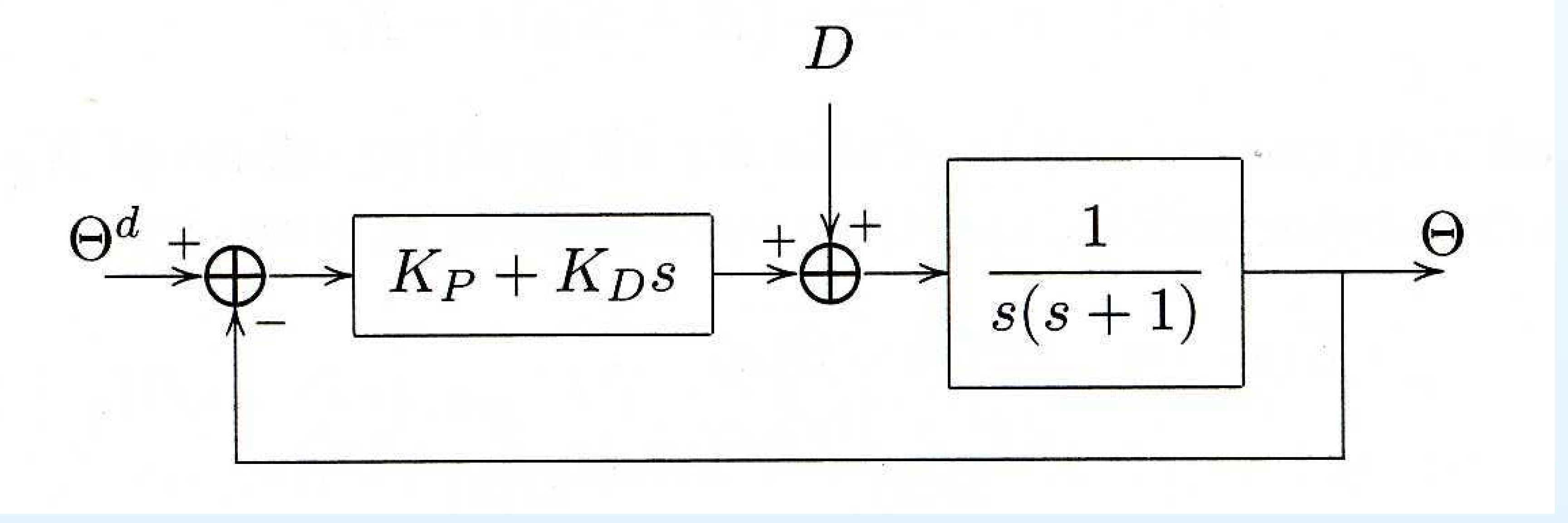
Example:

Suppose that $K_b = 1$ and

$$G_{\{v\to\theta\}}(s) \approx \boxed{\frac{K_m}{\left[R(J_m s + B_m) + K_m K_b\right] s}} = \frac{1}{(s+1)\, s}$$

Design a PD-controller that closed loop poles are at -3, -1

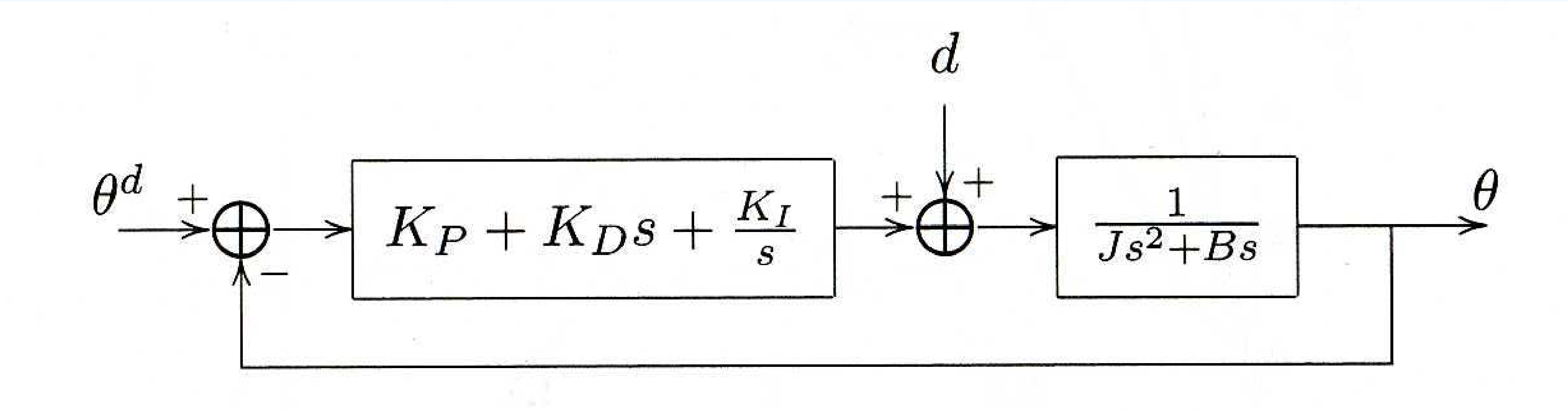


Example:

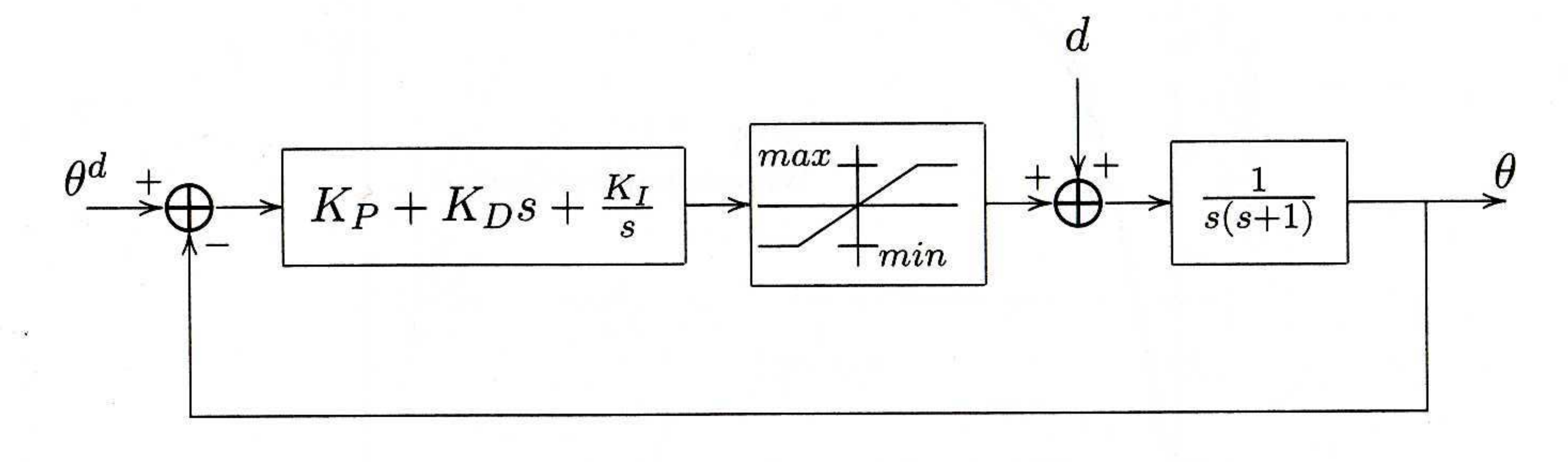
Suppose that $K_b = 1$ and

$$G_{\{v \to \theta\}}(s) \approx \boxed{\frac{K_m}{\left[R(J_m s + B_m) + K_m K_b\right] s}} = \frac{1}{J s^2 + B s}$$

Find a PID-controller gains such that that closed loop is stable



Dealing with Input Saturation



The closed loop system can be drastically different if we take into account signals limits in the loop!

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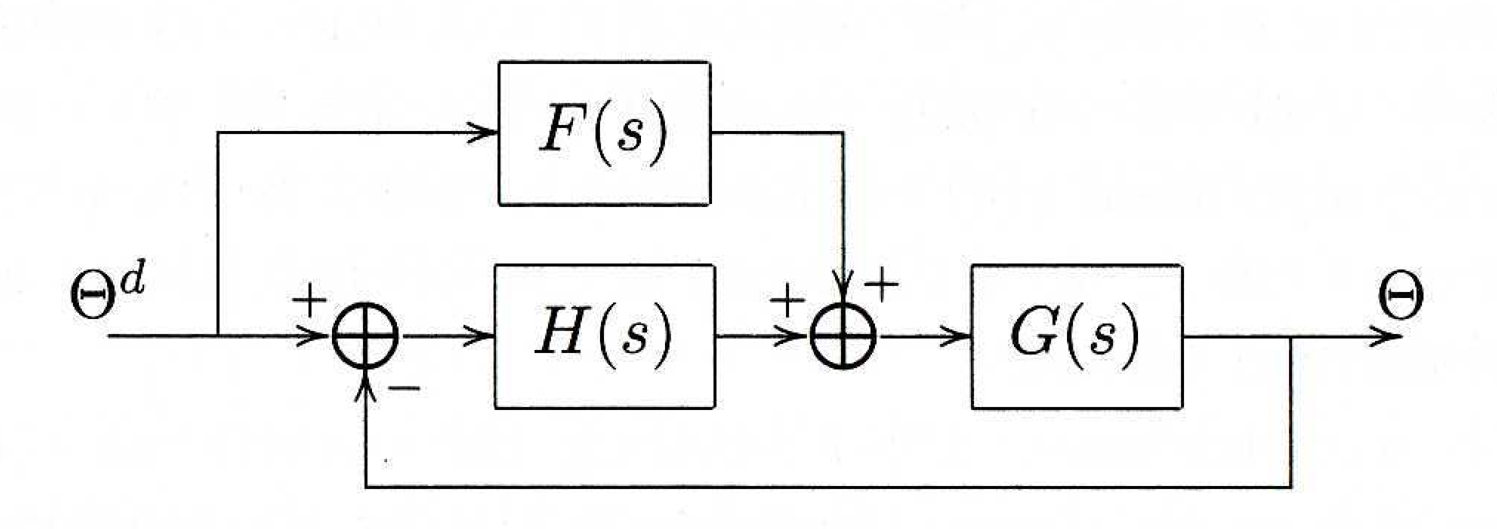
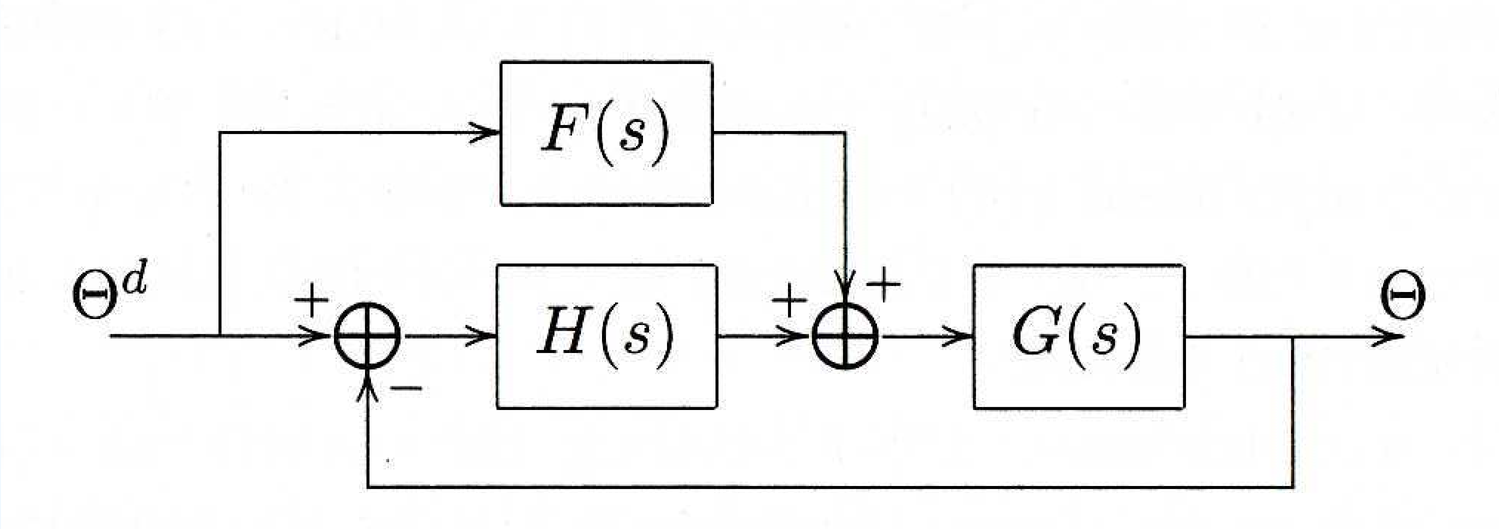
Idea of Feedforward Control



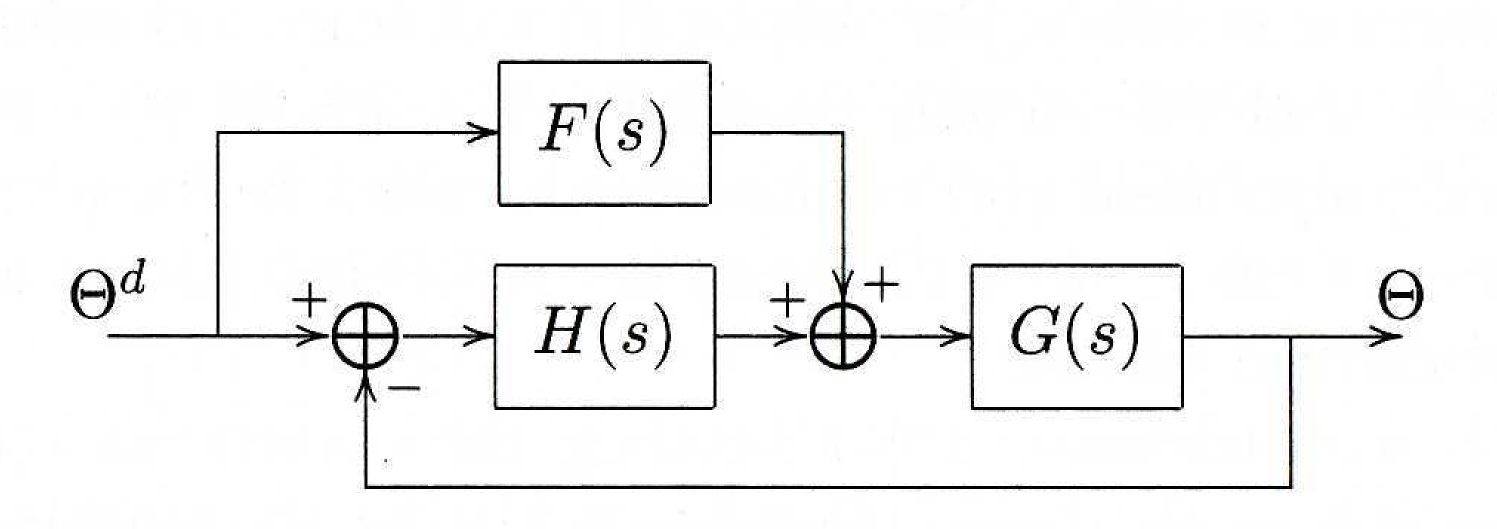
- Transfer function $F(s)$ is new parameter for design

Idea of Feedforward Control



- Transfer function $F(s)$ is new parameter for design
- Not any $F(s)$ can be used:
  - $F(s)$ should be stable;
  - $F(s)$ should be proper

Idea of Feedforward Control



- Transfer function $F(s)$ is new parameter for design
- Not any $F(s)$ can be used:
  - $F(s)$ should be stable;
  - $F(s)$ should be proper
- Reasonable choice is $F(s) \approx 1/G(s)$

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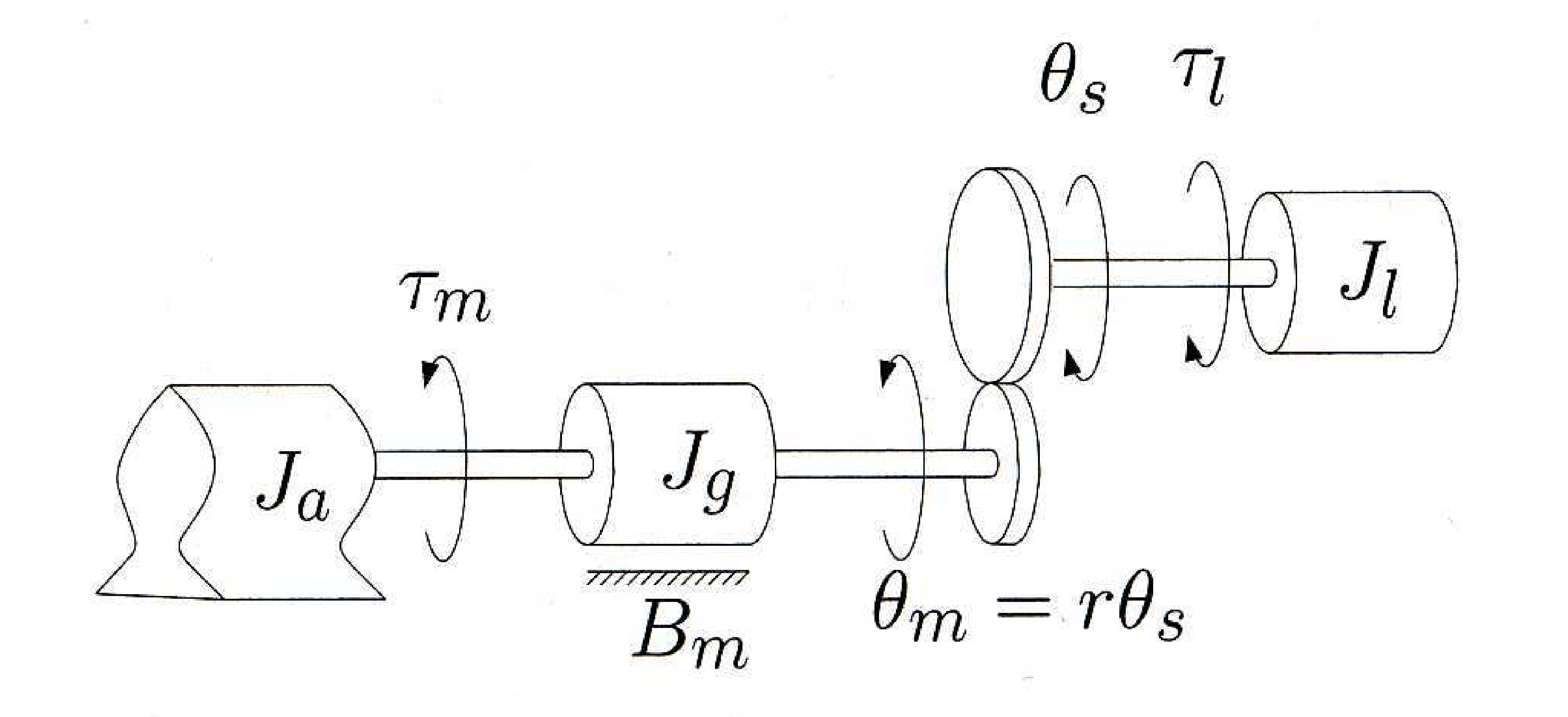
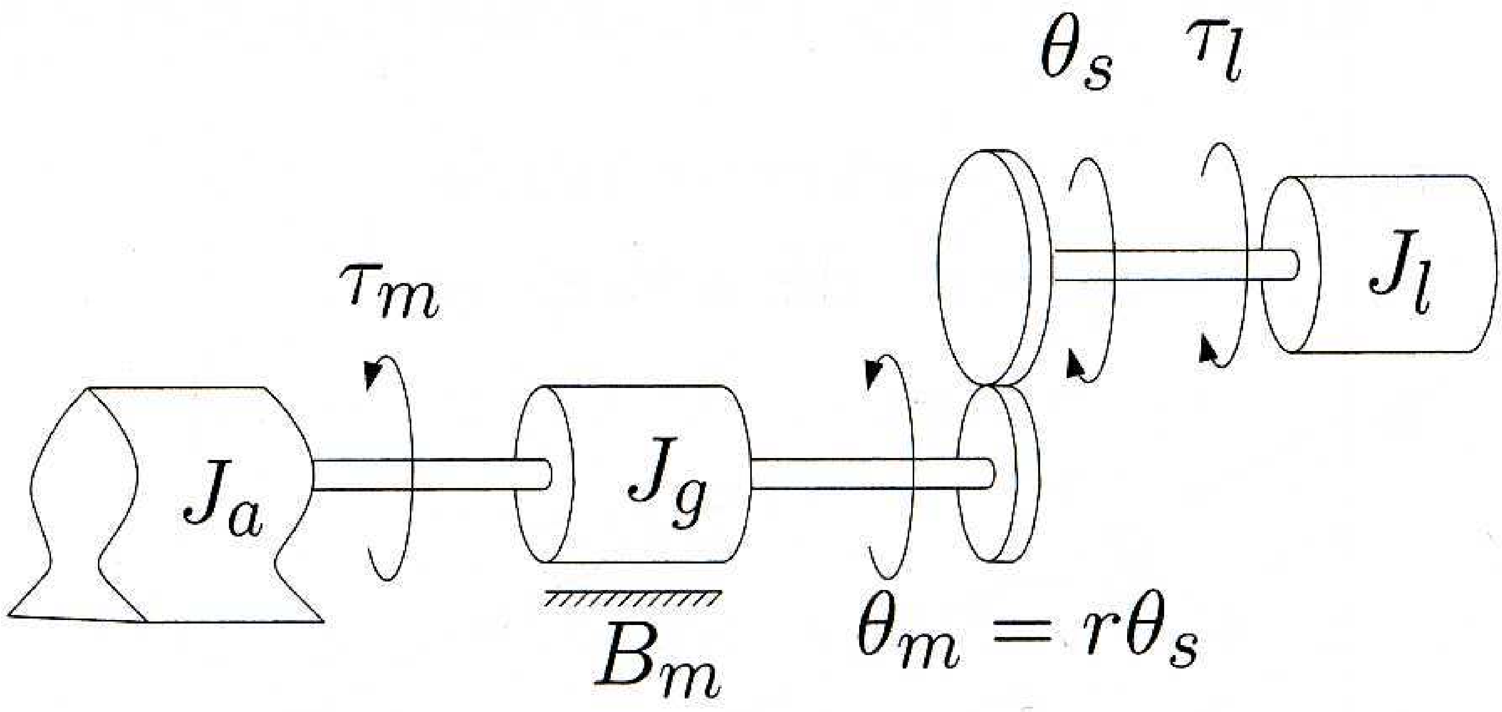
Gear-Boxes



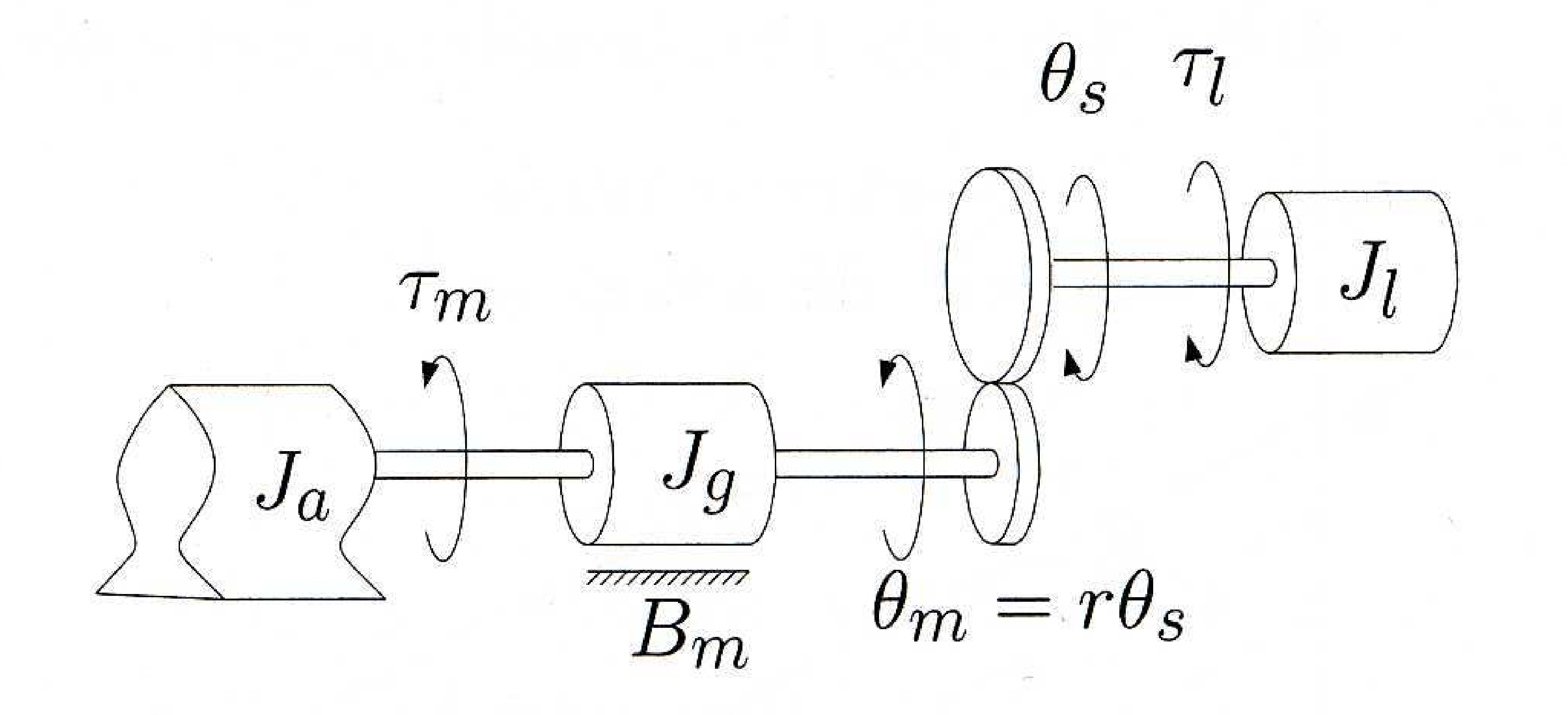
- Lossless transmission means that $\tau_m \dot{\theta}_m = \tau_s \dot{\theta}_s$;

Gear-Boxes



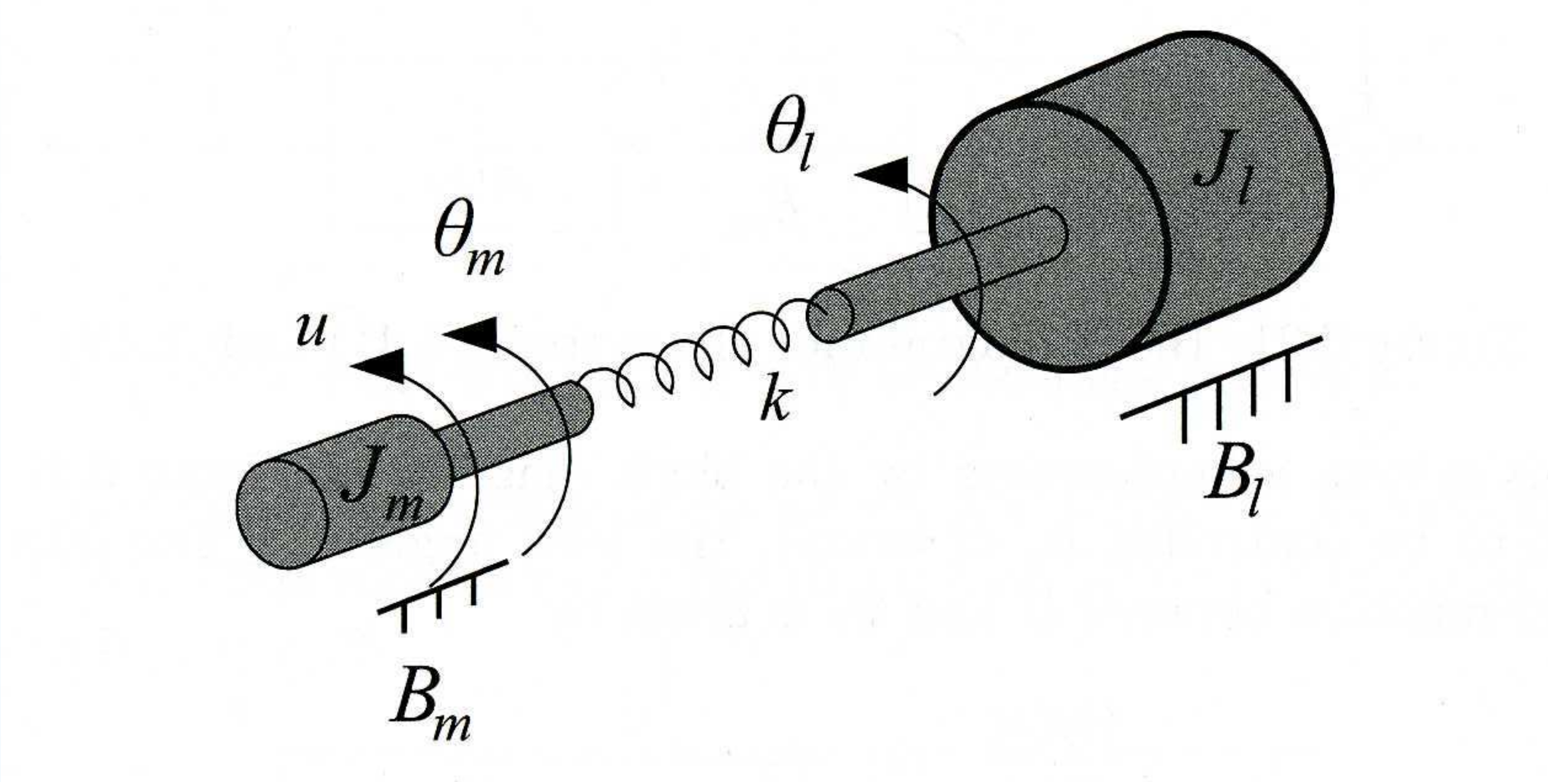
- Lossless transmission means that $\tau_m \dot{\theta}_m = \tau_s \dot{\theta}_s$;
- Gear-box always introduces additional friction and delay;

Gear-Boxes



- Lossless transmission means that $\tau_m \dot{\theta}_m = \tau_s \dot{\theta}_s$;
- Gear-box always introduces additional friction and delay;
- Gear-box can complicate the dynamics and limit the performance.

Flexibility of in Harmonic Drive Transmission

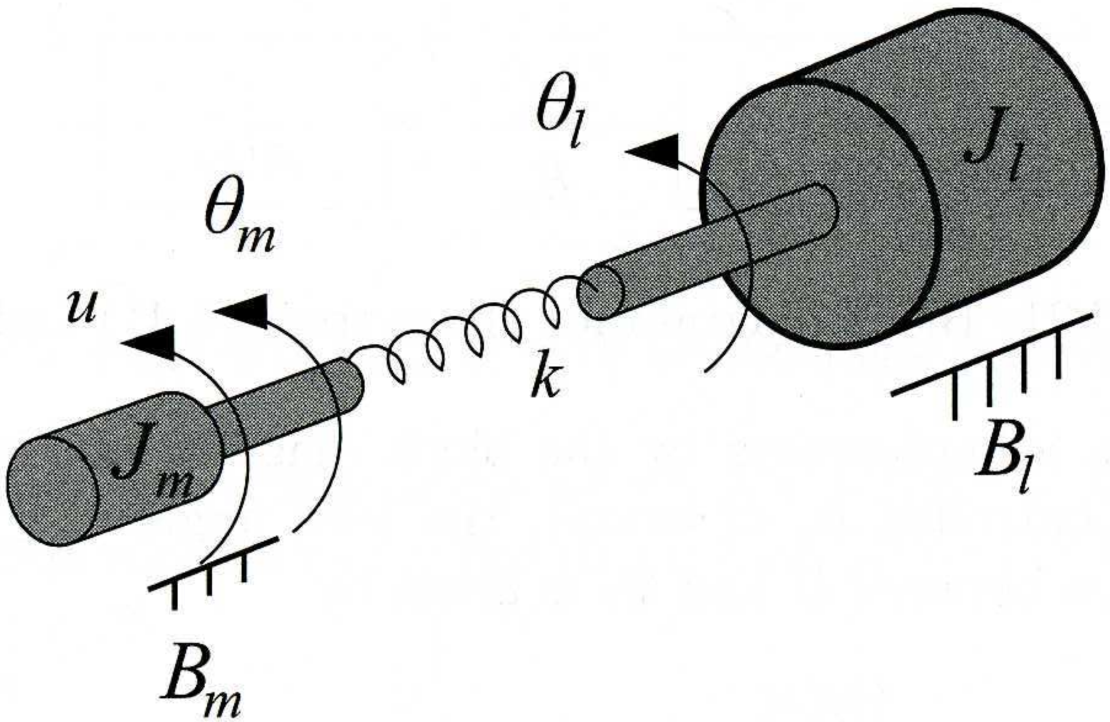


The dynamics are

$$\begin{aligned} J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k\left(\theta_l - \theta_m\right) &= 0 \\ J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k\left(\theta_l - \theta_m\right) &= u \end{aligned}$$

where u is a control torque.

Flexibility of in Harmonic Drive Transmission

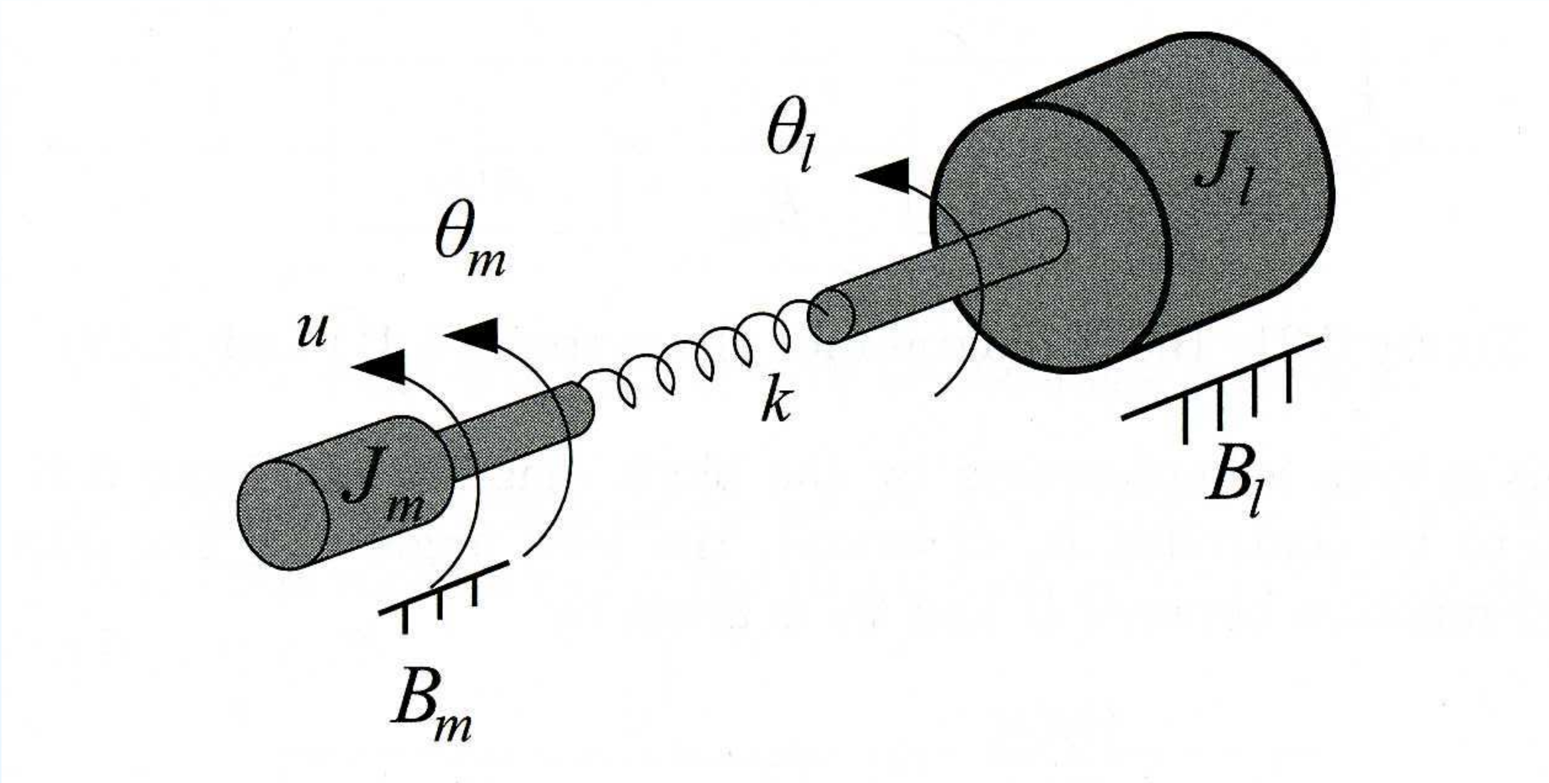


The dynamics are

$$
\begin{aligned}
J_l \ddot{\theta}_l + B_l \dot{\theta}_l + k\theta_l &= k\theta_m \\
J_m \ddot{\theta}_m + B_m \dot{\theta}_m + k\theta_m &= k\theta_l + u
\end{aligned}
$$

where u is a control torque.

Flexibility of in Harmonic Drive Transmission

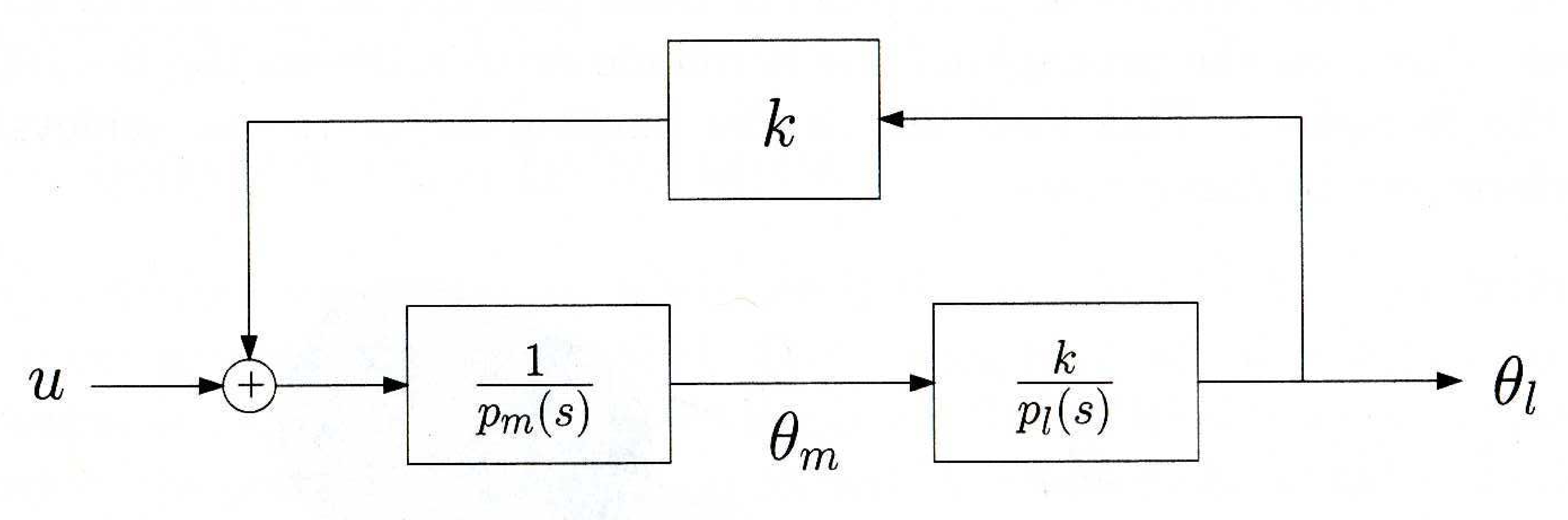


The dynamics are

$$\Theta_l(s) = \frac{k}{p_l(s)}\Theta_m(s) = \frac{k}{J_l s^2 + B_l s + k}\Theta_m(s)$$

$$\Theta_m(s) = \frac{1}{p_m(s)}\left(k\Theta_l(s) + U(s)\right) = \frac{k\Theta_l(s) + U(s)}{J_m s^2 + B_m s + k}$$

Flexibility of in Harmonic Drive Transmission



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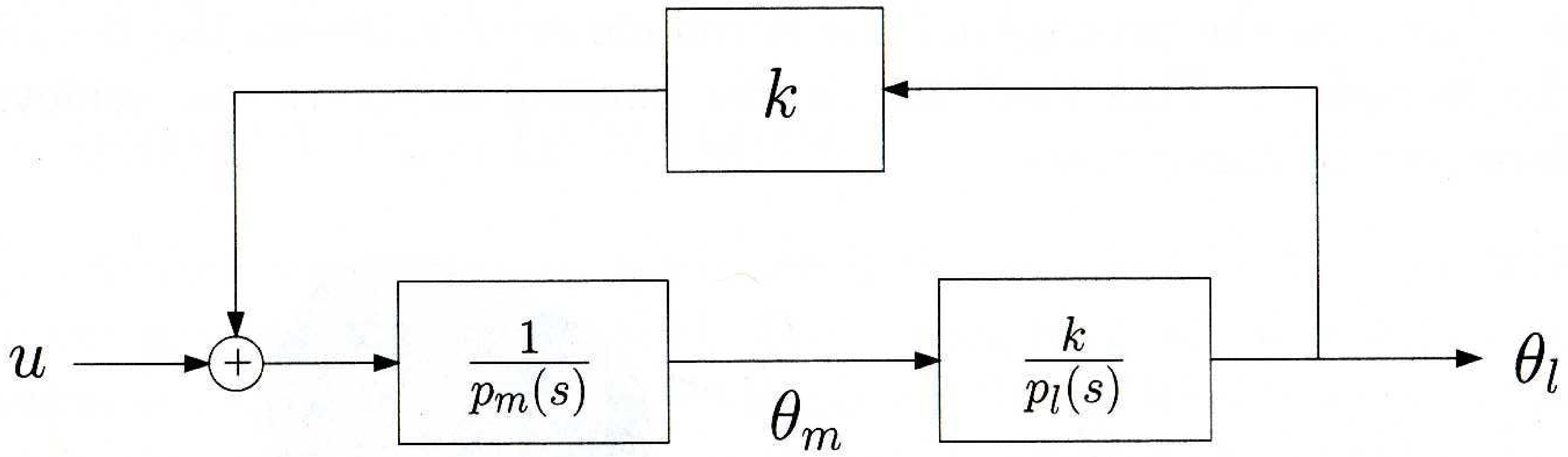
Flexibility of in Harmonic Drive Transmission

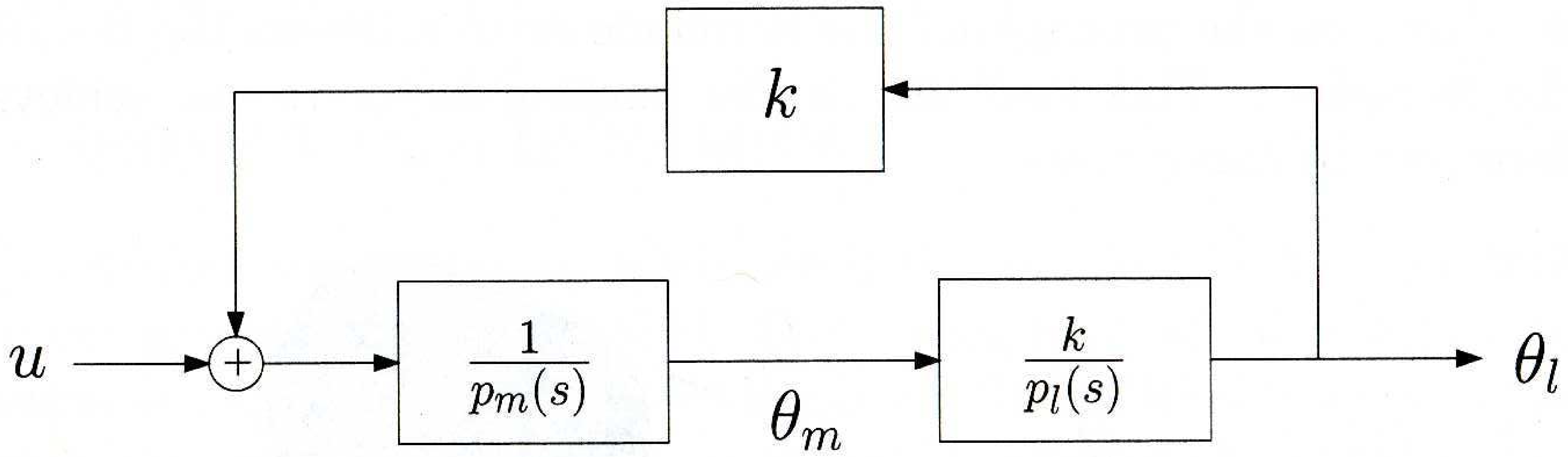


The dynamics are

$$\Theta_l(s) = \frac{k}{p_l(s)p_m(s) - k^2} U(s)$$

$$= \frac{k}{J_m J_l s^4 + (J_l B_m + J_m B_l)s^3 + (k(J_m + J_l) + B_m B_l)s^2 + k(B_m + B_l)s} U(s)$$

Flexibility of in Harmonic Drive Transmission



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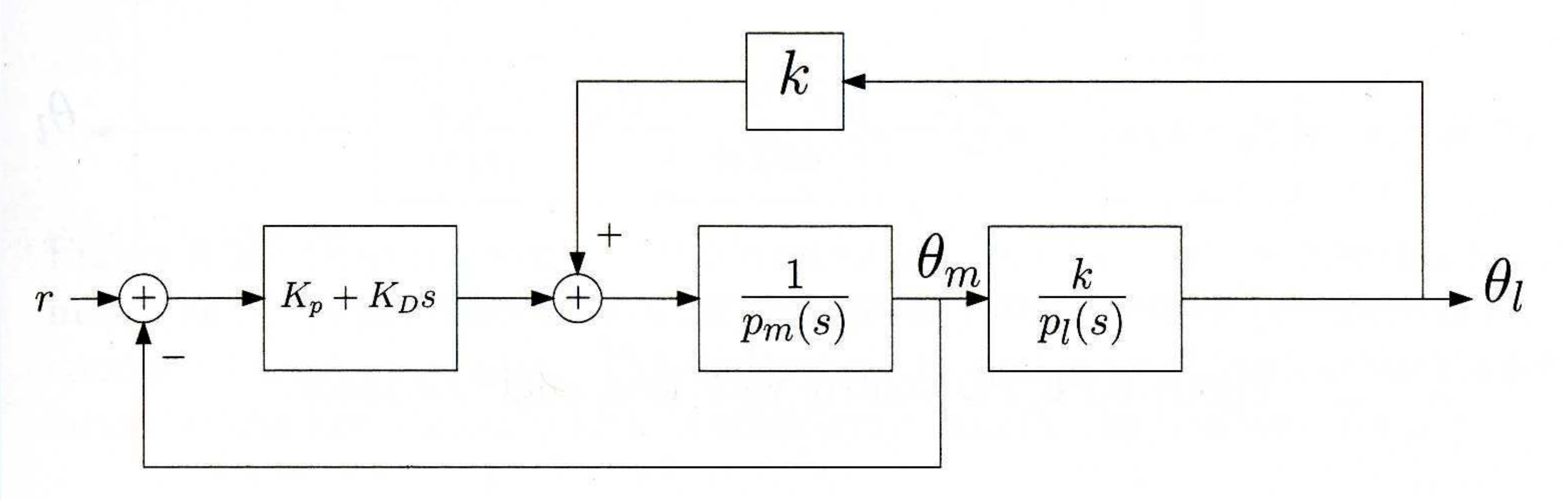
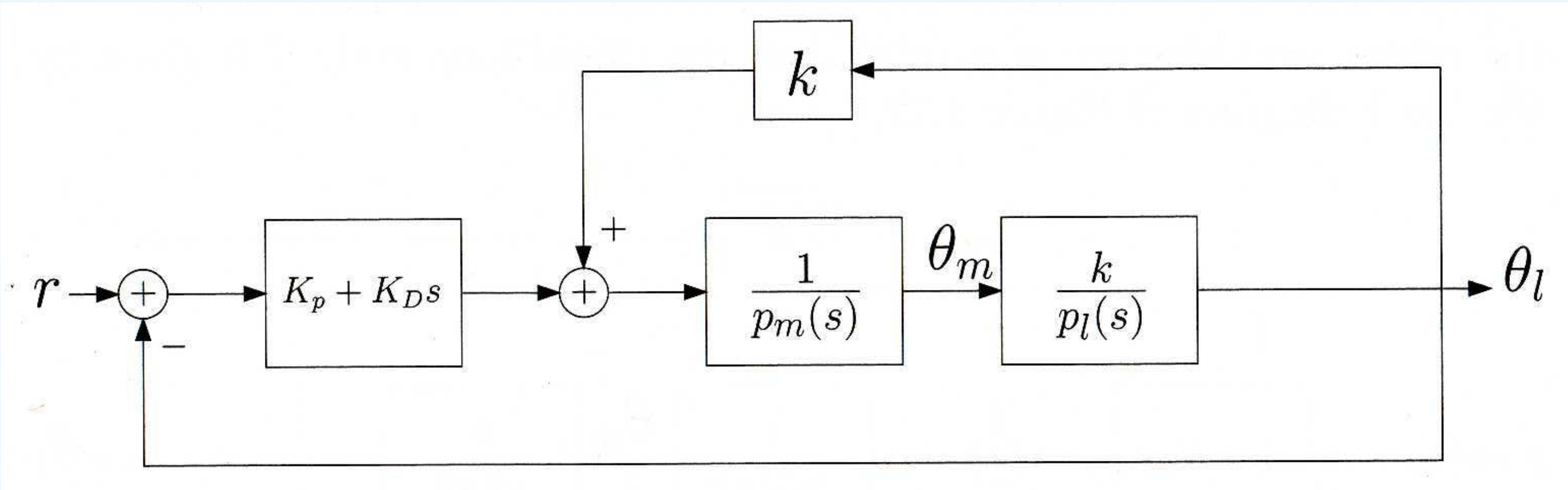
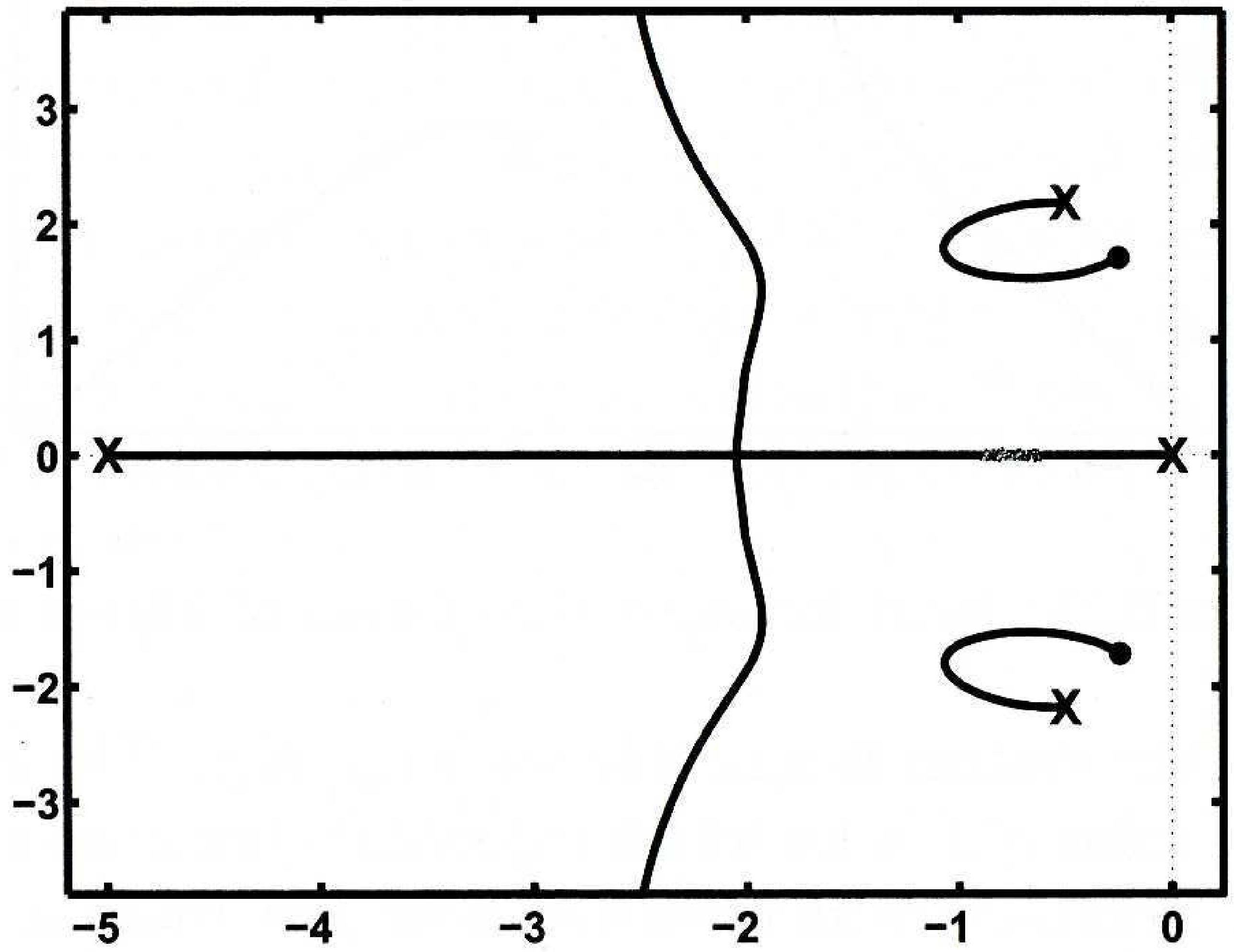
$$= \frac{k}{J_m J_l s^4 + (J_l B_m + J_m B_l)s^3 + (k(J_m + J_l) + B_m B_l)s^2 + k(B_m + B_l)s} U(s)$$

$$\approx \frac{k}{J_m J_l s^4 + k(J_m + J_l)s^2} U(s)$$

PD controller with θ_m or θ_l feedbacks

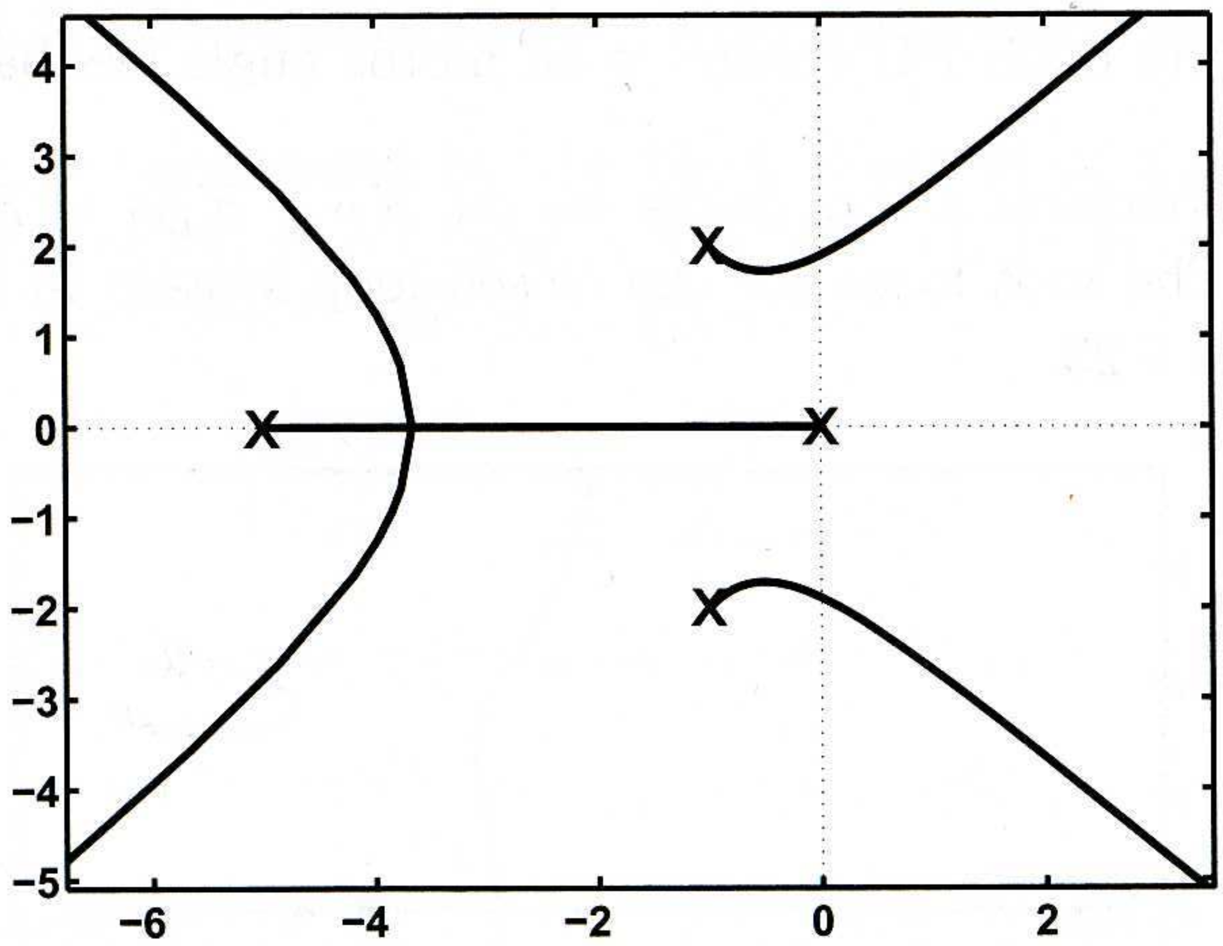






The root-locus of the closed loop system with θ_m-feedback and

$$PD = K_P + K_D s = K(a + s)$$



The root-locus of the closed loop system with θ_l-feedback and

$$PD = K_P + K_D s = K(a + s)$$