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- Solutions to Selected Problems

Lecture 1: Introduction and Conceptual Problems

- Examples of Robots

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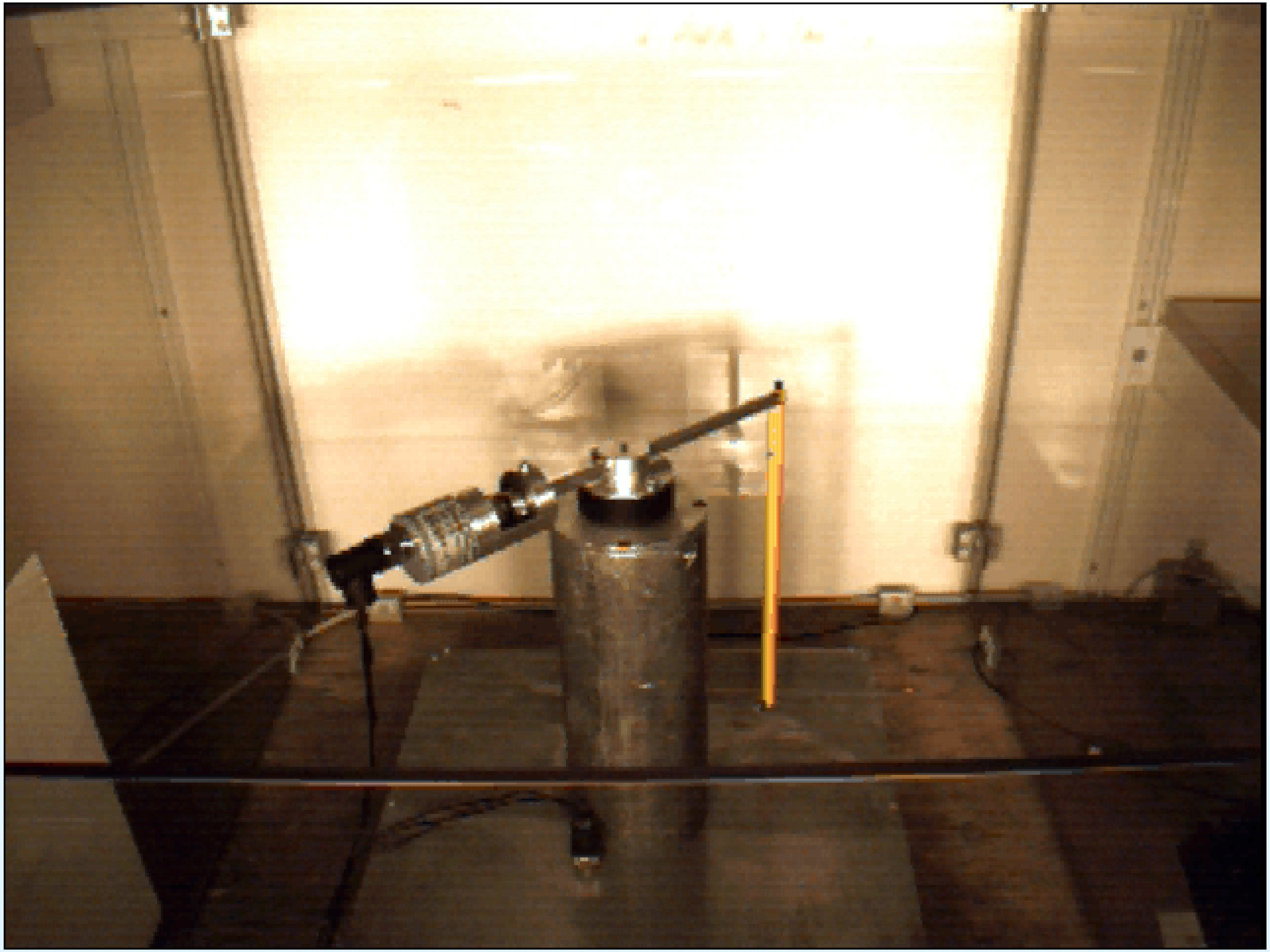
- Conceptual Problems:
  - Forward Kinematics
  - Inverse Kinematics
  - Velocity Kinematics
  - Dynamics
  - Path Planning and Trajectory Generation
  - Motion Control
  - Force Control
  - Computer Vision and Vision Based Control



Lecture 1: Introduction and Conceptual Problems

- Examples of Robots
- Conceptual Problems:
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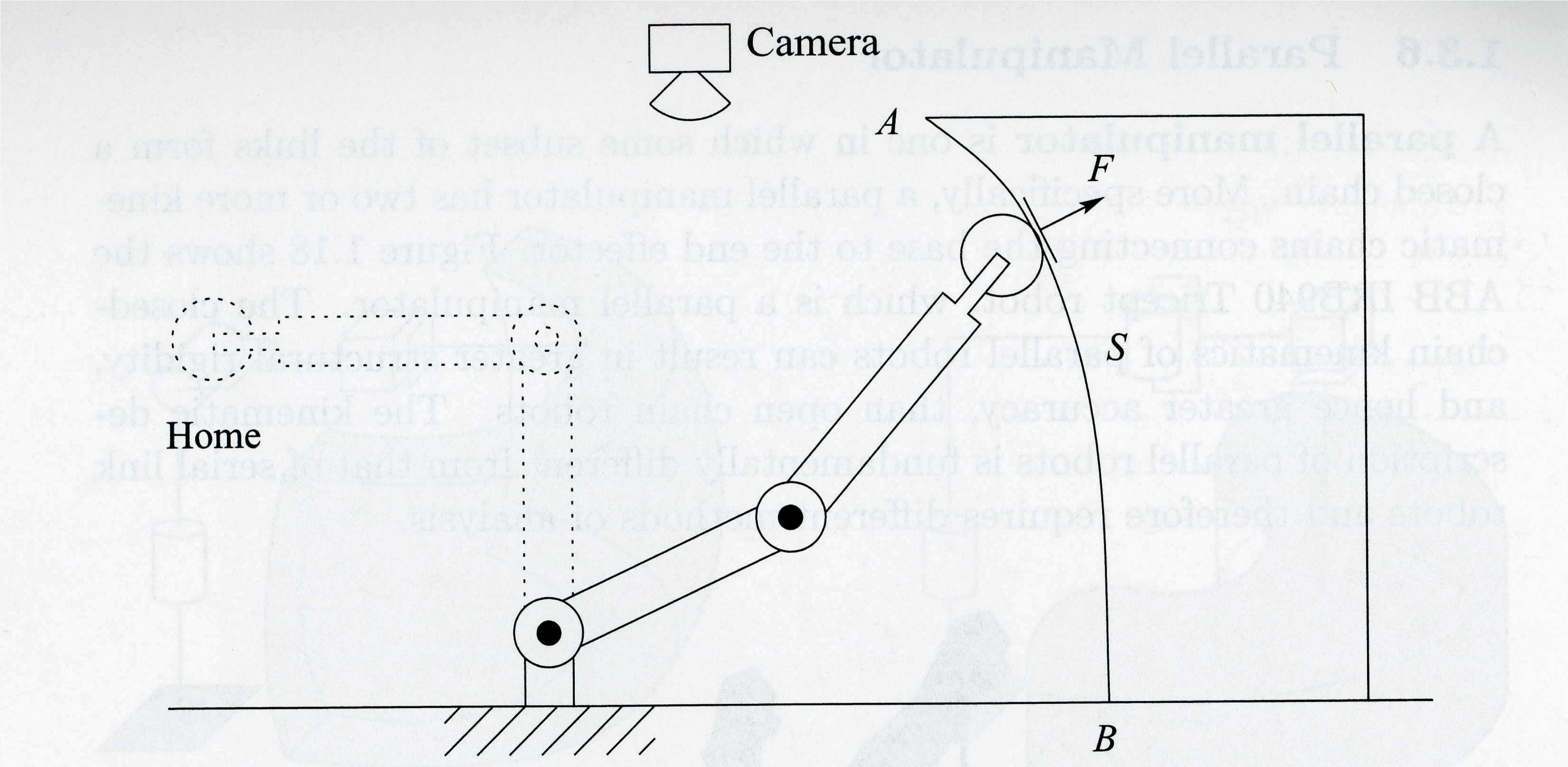
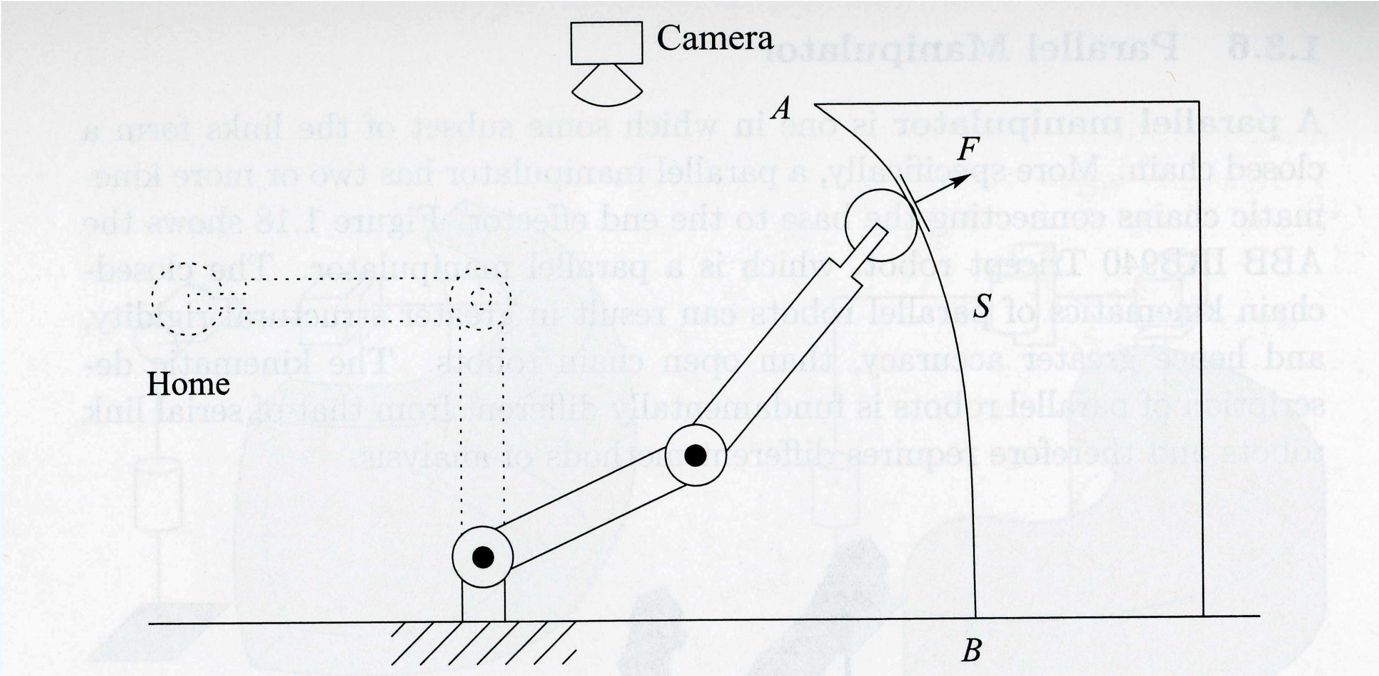
Conceptual Robotics Application:



Example: Two-link planar robot used for griding a wall

Forward and Inverse Kinematics:



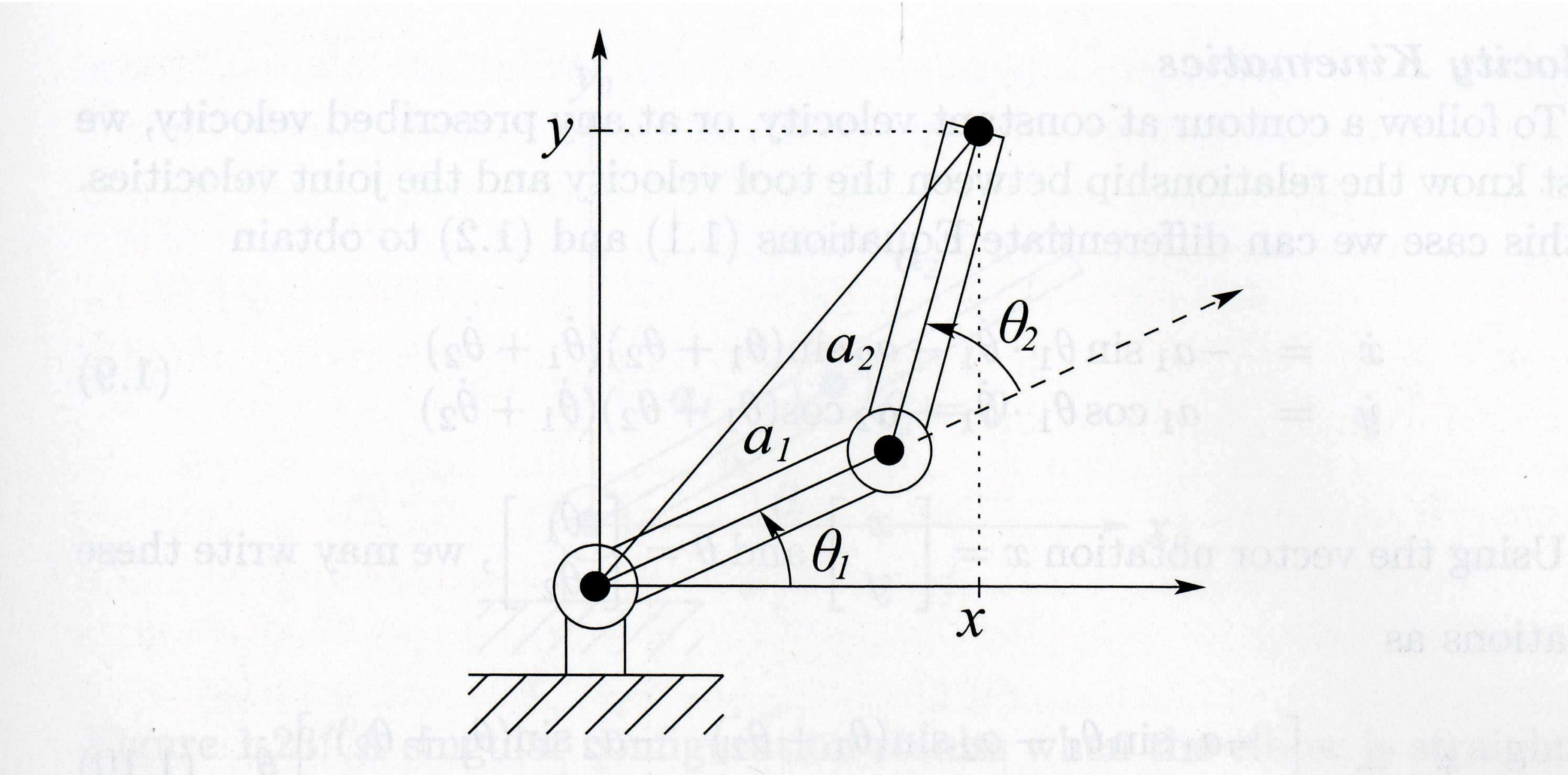
To perform the task, we need to know

- a parametrization of a wall
- configurations of the robot for which it touches the wall, i.e.

  Forward Kinematics: the position (x, y) of end-effector as a function of joint angles (θ_1, θ_2);

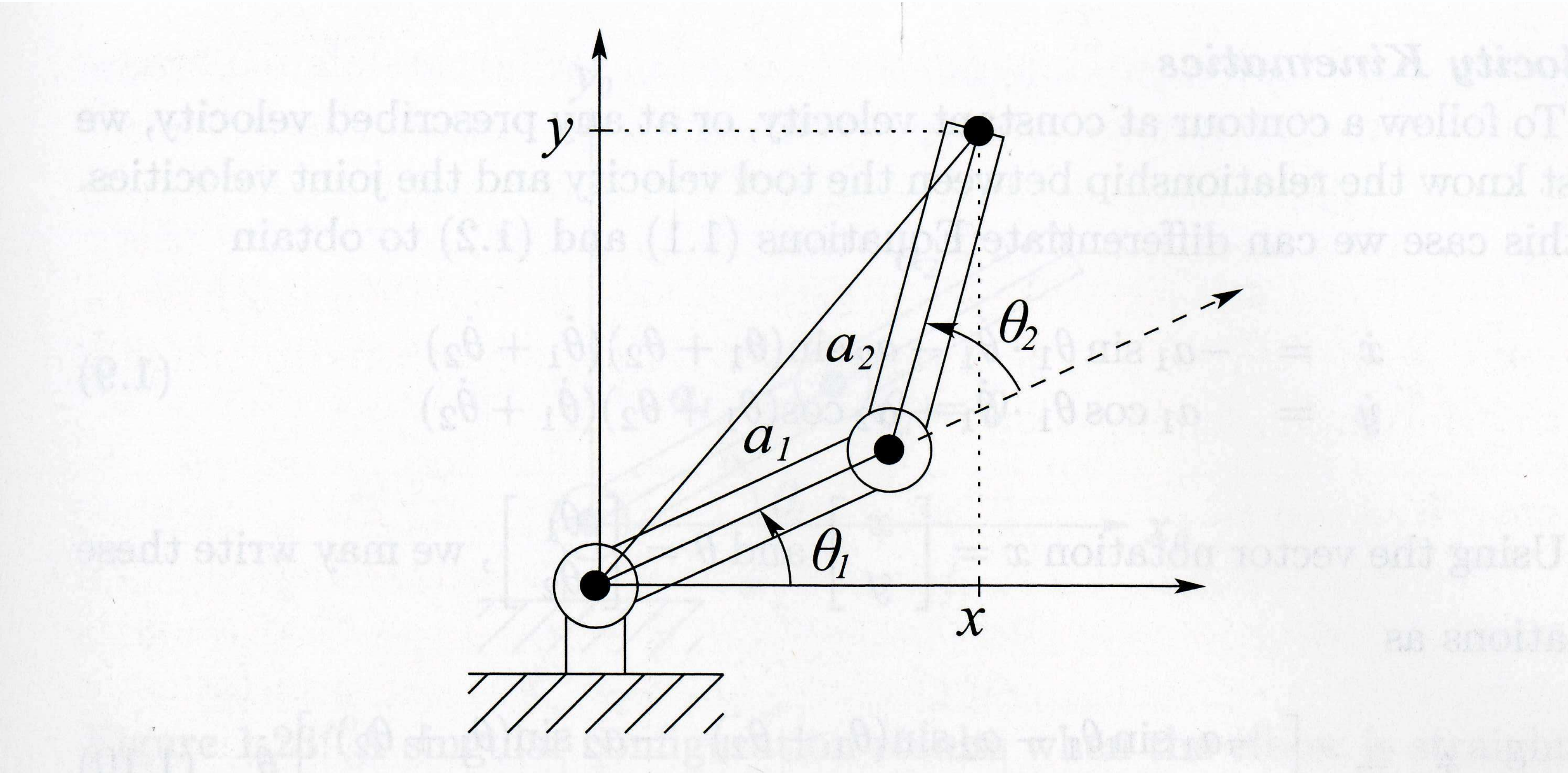
  Inverse Kinematics: angles (θ_1, θ_2) as functions of (x, y)

Forward Kinematics:



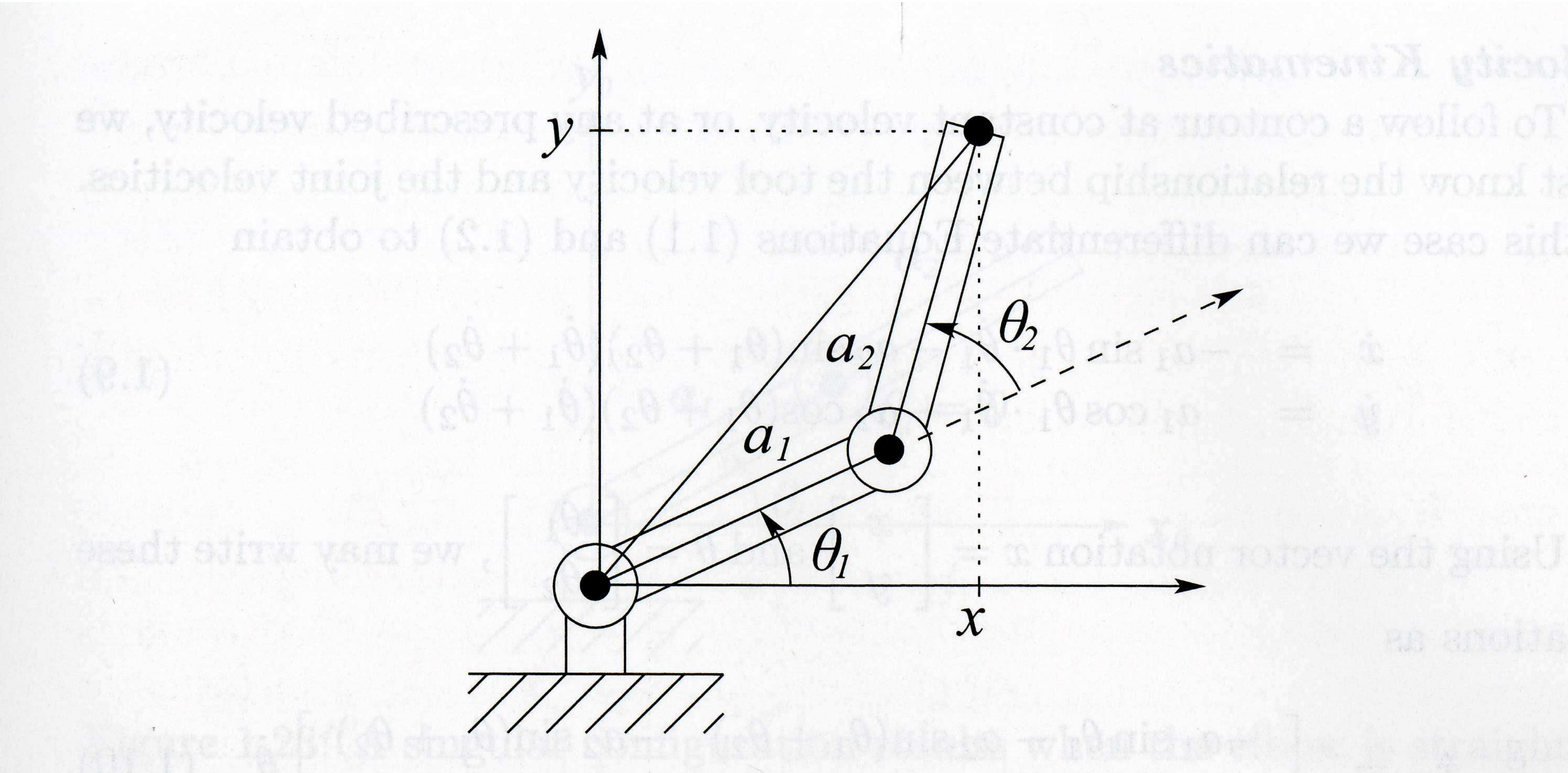
$$x \;=\; x_1 + x_2 = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$

Forward Kinematics:



$$
\begin{aligned}
x &= x_1 + x_2 = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) \\
y &= y_1 + y_2 = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)
\end{aligned}
$$

Forward Kinematics (Tool Frame Orientation):



$$x_2^0 \;=\; \left[\cos(\theta_1+\theta_2),\, \sin(\theta_1+\theta_2)\right]$$

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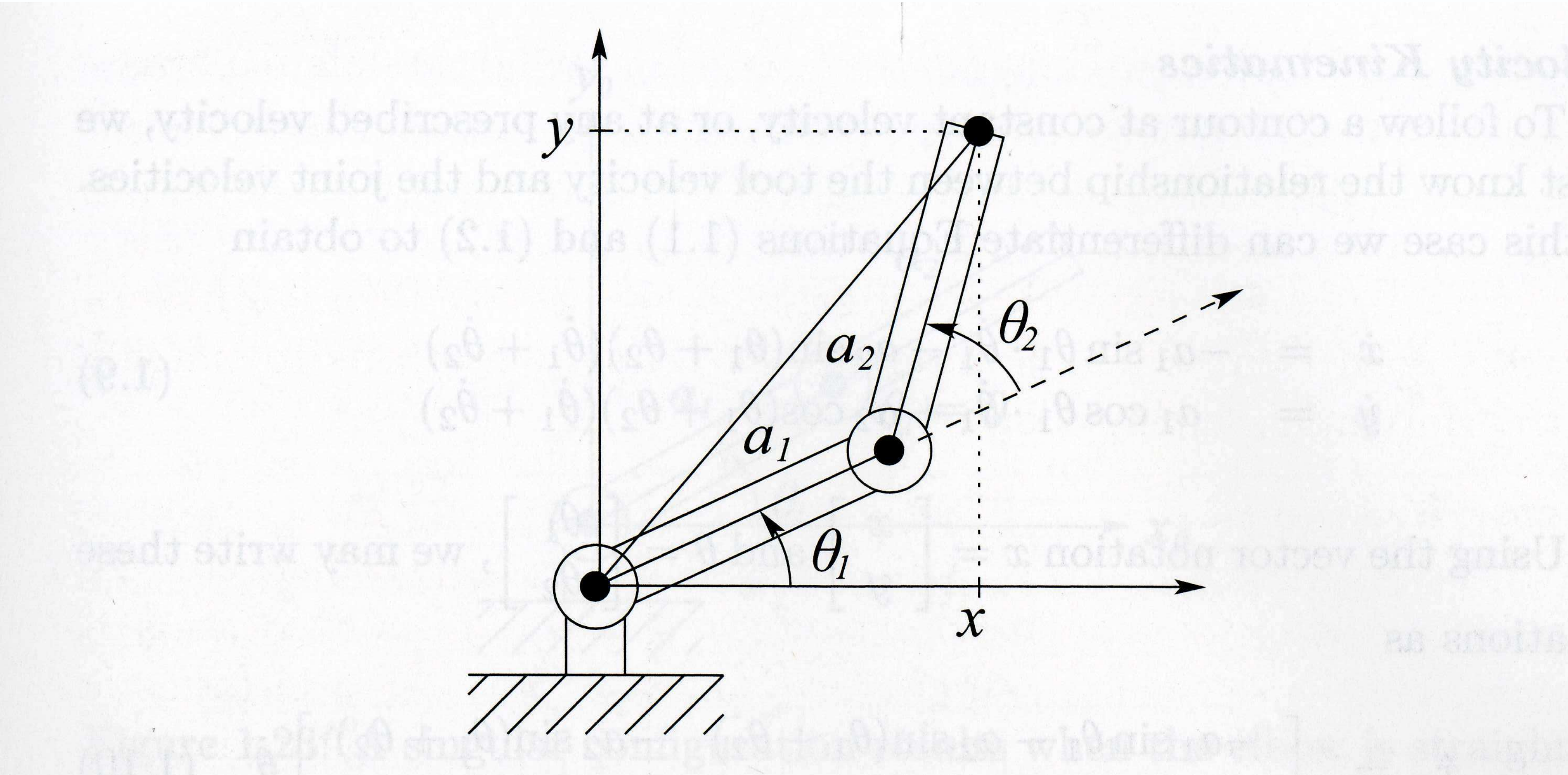
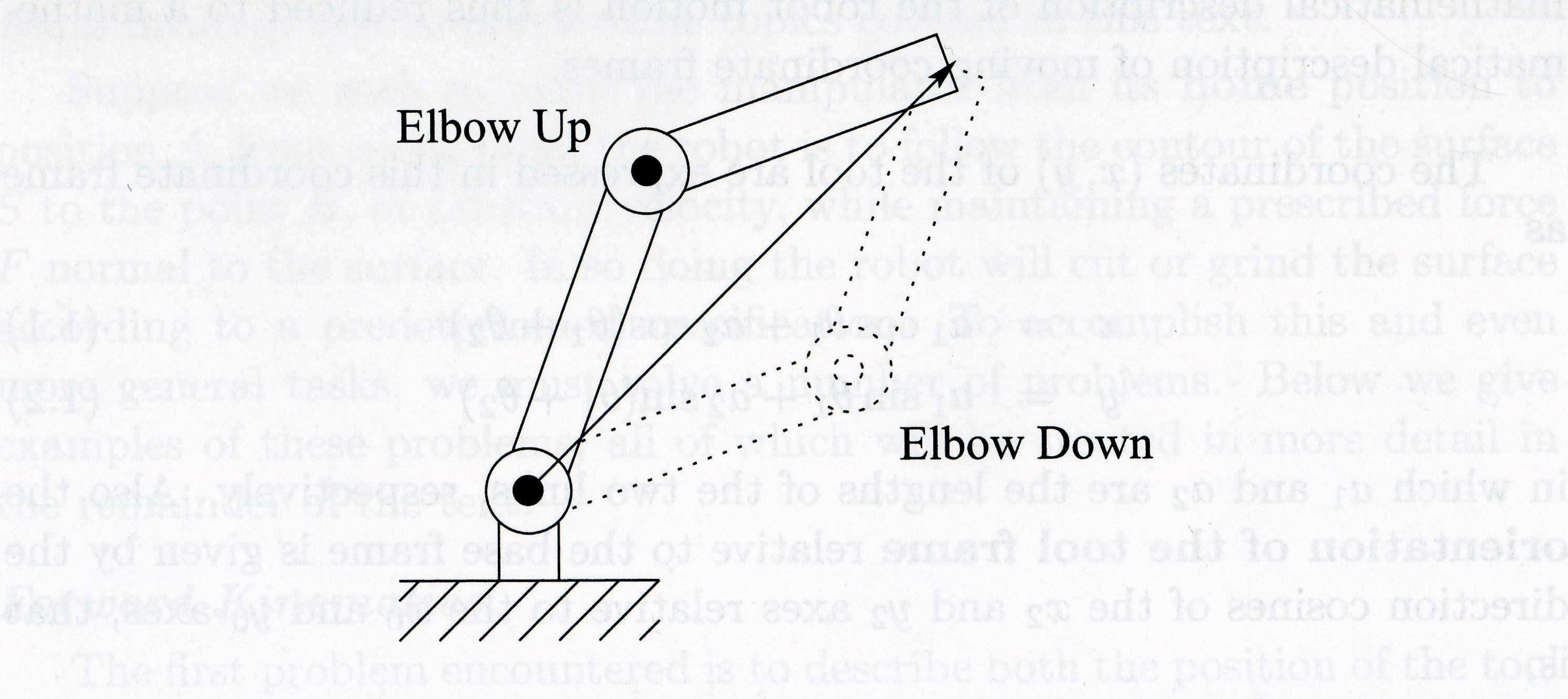


$$x_2^0 = \left[\cos(\theta_1 + \theta_2), \sin(\theta_1 + \theta_2)\right]$$

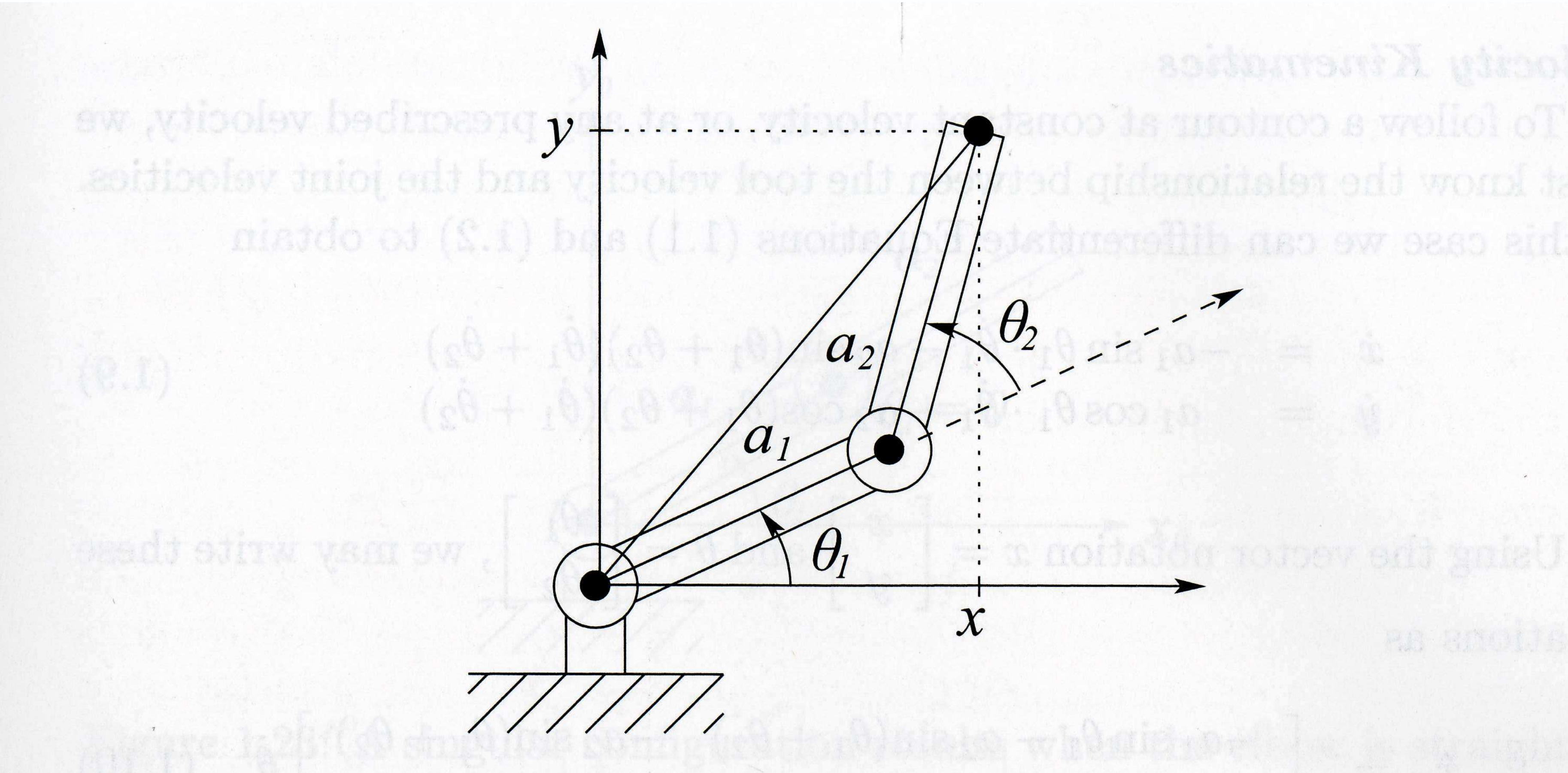
$$y_2^0 = \left[-\sin(\theta_1 + \theta_2), \cos(\theta_1 + \theta_2)\right]$$

Inverse Kinematics:



If the tool position (x, y) is given, but the orientation is not defined, then, except particular cases, there are two configurations: Elbow Up and Elbow Down.

Inverse Kinematics:



$$\cos\theta_2 = \frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2} := D, \quad \left\{\Rightarrow \sin\theta_2 = \pm\sqrt{1-D^2}\right\}$$

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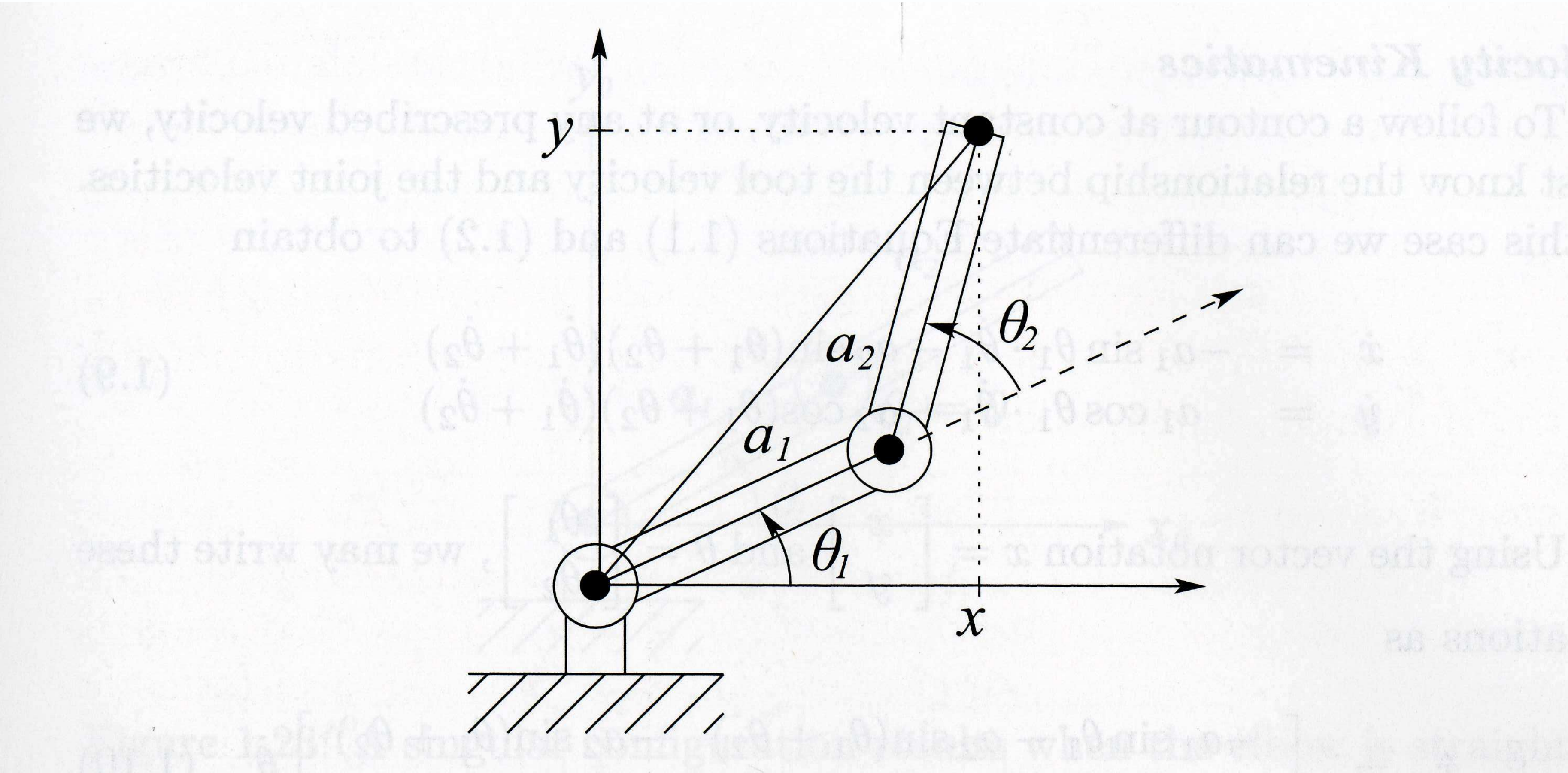

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$$\Rightarrow \quad \theta_2 = \tan^{-1}\left(\frac{\sin\theta_2}{\cos\theta_2}\right) = \tan^{-1}\left(\frac{\pm\sqrt{1 - D^2}}{D}\right)$$

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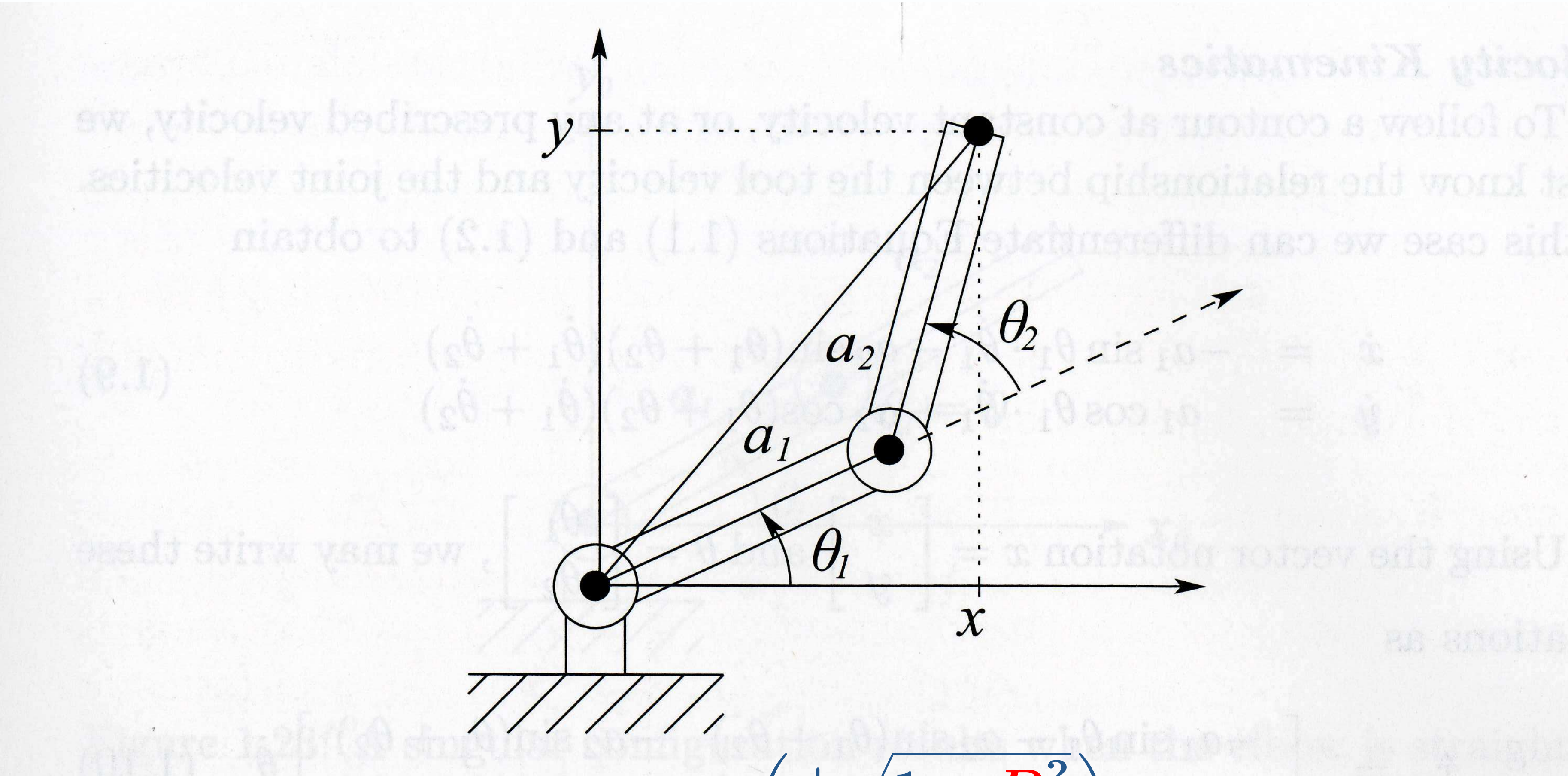

$$\theta_2 = \tan^{-1}\left(\frac{\pm\sqrt{1 - D^2}}{D}\right)$$

$$\Rightarrow \quad \theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{a_2 \sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$$

Velocity Kinematics:

Geometrical relations between (x, y) and (θ_1, θ_2)

$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2), \quad y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$

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imply the relations between velocities

$$\begin{aligned} \frac{d}{dt} x(t) &= -a_1 \sin\theta_1 \frac{d}{dt}\theta_1 - a_2 \sin(\theta_1 + \theta_2) \left(\frac{d}{dt}\theta_1 + \frac{d}{dt}\theta_2 \right) \\ \frac{d}{dt} y(t) &= a_1 \cos\theta_1 \frac{d}{dt}\theta_1 + a_2 \cos(\theta_1 + \theta_2) \left(\frac{d}{dt}\theta_1 + \frac{d}{dt}\theta_2 \right) \end{aligned}$$

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In compact form it is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} -a_1 \sin\theta_1 - a_2 \sin(\theta_1 + \theta_2) & -a_2 \sin(\theta_1 + \theta_2) \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) & a_2 \cos(\theta_1 + \theta_2) \end{array}\right]}_{=:\, J(\theta_1, \theta_2)} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

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The matrix $J(\cdot)$ is called the Jacobian of the manipulator.

Velocity Kinematics:

The relation between the joint velocities and the tool velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J(\theta_1, \theta_2) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

allows to compute the joint velocities $\dot{\theta}_1(t)$, $\dot{\theta}_2(t)$ to achieve the particular velocity of the tool!

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Indeed, $\dot{\theta}_1(t)$, $\dot{\theta}_2(t)$ are found by

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J(\theta_1, \theta_2)^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

with

$$J^{-1} = \frac{1}{a_1 a_2 \sin\theta_2} \left[\begin{array}{c|c} a_2 \cos(\theta_1 + \theta_2) & a_2 \sin(\theta_1 + \theta_2) \\ -a_1 \cos\theta_1 - a_2 \cos(\theta_1 + \theta_2) & -a_1 \sin\theta_1 - a_2 \sin(\theta_1 + \theta_2) \end{array} \right]$$

Velocity Kinematics:

It is clear that the inverse of the Jacobian

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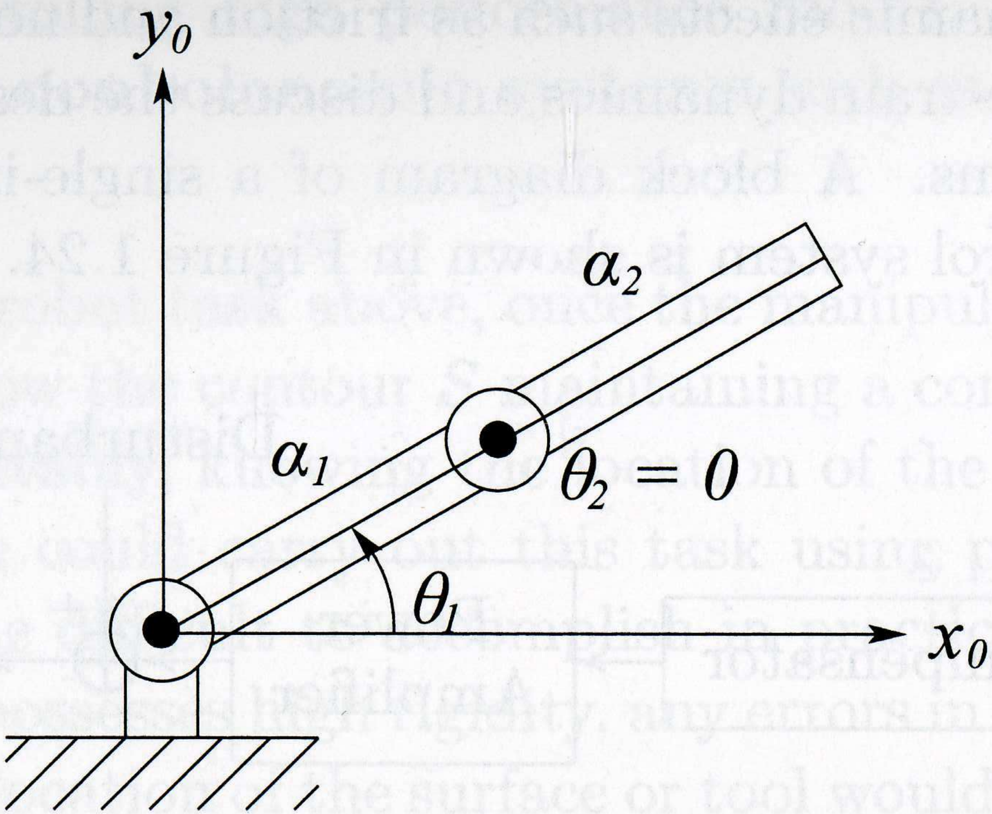
What does this mean?

Singular Configurations



If $\theta_2 = 0$, then the Jacobian $J(\cdot)$ looses the rank and cannot be inverted. It means that the tool velocity cannot be any, but should belong to particular line irrespective of the joint velocities.

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- Dynamics are equations, which allow to introduce effect of control action and define variables as functions of time;
- For instance, for the double pendulum the dynamics are

$$\begin{bmatrix} p_1 + p_2 + 2p_3 \cos\theta_2 & p_2 + p_3 \cos\theta_2 \\ p_2 + p_3 \cos\theta_2 & p_2 \end{bmatrix} \begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix} + p_3 \sin\theta_2 \begin{bmatrix} -\dot\theta_2 & -\dot\theta_2 - \dot\theta_1 \\ \dot\theta_1 & 0 \end{bmatrix} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \\ + \begin{bmatrix} p_4 g \cos\theta_1 + p_5 g \cos(\theta_1 + \theta_2) \\ p_5 g \cos(\theta_1 + \theta_2) \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

where

- p_1-p_5 are constants defined by physical parameters;
- u_1, u_2 are control torques.

Path Planning and Trajectory Generation:

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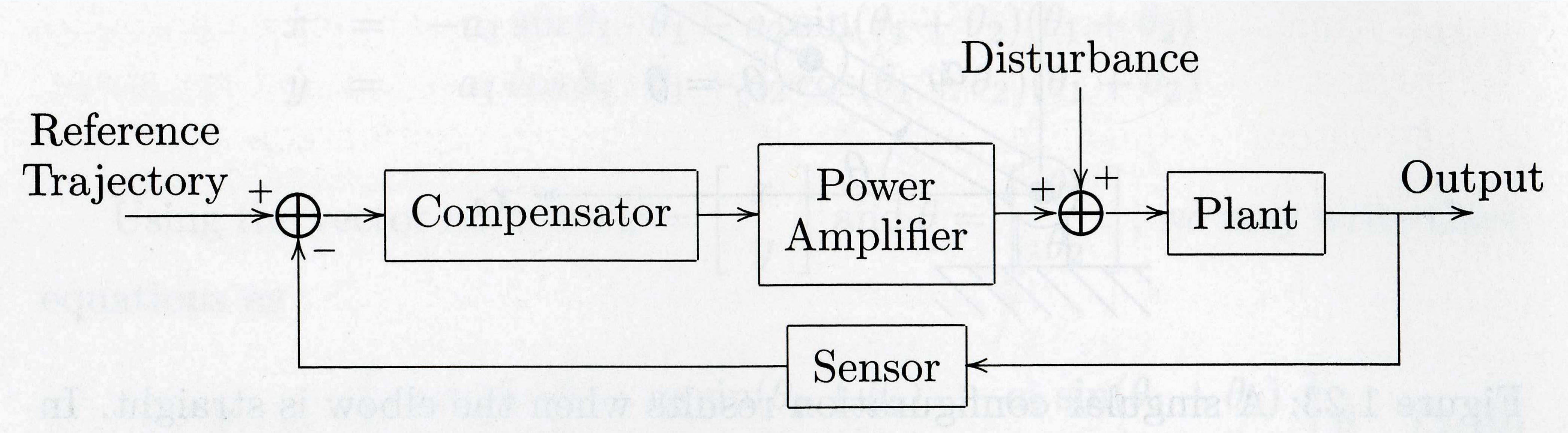
- Trajectory is the path augmented with the information on velocities how the system links will travel along the path;
- Planning trajectory might include also specifications on accelerations

Motion Control:



Basic structure of a feedback control system.