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The adaptive speed controller for the BLDC motor using MRAC technique

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Abstract: This research presents a new adaptive speed controller for the brushless DC motor. The methodology of model reference adaptive control is applied to a novel model of BLDC motor. This novel model provides the possibility to compensate the torque ripples and load torque. The system stability is proven using Lapunov function.

Keywords: Adaptive controller, BLDC motor, speed control, MRAC,

1. INTRODUCTION

The brushless DC motor (BLDC) is widely used in industrial applications where high precision electromechanic converters are required. It can be applied as a servo application or a position tracking system. The BLDC motor has been developed based on the DC motor by applying a different commutation approach. The DC motor uses brushes to switch the current direction in windings. It enables simple control design, however its drawbacks are electric noises and low mechanic endurance. Thus, the BLDC motor's commutation is forced by an electronic converter. The most popular control mode is based on a hallotron feedback, which provides signals to a commutator. A number of methods have been proposed to design a optimal converter Krishnan (1997), Ching and Chan (2008), Song and Byun (2000) and Krishnan et al. (1996). However, a torque ripple effect, which is discussed in Yan and Wu (2006), Stumberger et al. (2006a), Kim et al. (2010) and Hanselman (1994) can not be avoided due to a trapezoidal shape of the back electromotive force (emf). Furthermore, such control approach easily enables us to design an angular speed controller or a position controller, but it is difficult to build a torque controller. Thus, a more sophisticated control system must be designed to overcome these problems.

There are wide range of methods, for instance studied in Ko et al. (2003) and Stumberger et al. (2006b), which can be applied to a Permanent Magnet Synchronous Motor (PMSM). Unfortunately, the BLDC motor has a trapezoidal back emf shape and the PMSM has a sinusoidal back emf shape. This difference is the reason, which excludes the PMSM methods for the BLDC motor.

In previous works, many solutions have been proposed to build a torque controller, which enables us to control speed in a simply way. The Sozer and Kaufman (1997) presents adaptive change of a current slope, however this controller is based on a perfect trapezoidal shape of a back emf. Another approach has been considered by Ko et al. (1999) where the BLDC controllers employ an observer. The alternative is to use a current controller presented in Taylor (1996) and Aghili (2008), where the phase currents are considered as the motor inputs. It simplifies control law, because the torque is a function of a position and current. Therefore, it is possible to calculate a current shape, which realizes a reference torque.

The alternative to the mentioned techniques is assuming a brushless DC motor as a variable structure system (VSS). This methodology is well known in control systems and it has been successfully applied to control position or speed by Lee and Youn (2004) and Lee (2006).

The adaptive controller for motors has been built by Aghili (2008), Kavanagh et al. (1991) and Shouse and Taylor (1994). However, in the previous works the control law does not adapt to all motor parameters or it assumes that a load torque model is known.

In this paper the adaptive speed controller is based on the model reference adaptive control (MRAC) discussed in Astrom and Wittenmark (1994); Ioannou (2003). The novel BLDC motor model is proposed to achieve high performance with simple structure of the adaptive controller. Thanks to the adaptive properties of this driver, the influence of the torque ripples and torque load is reduced. The stability of this system is proved using Lapunov function. The controller behavior is shown using simulations of BLDC model, which has been obtained in an identification process. The reason of new modelling technique is the alternative to feedforward controllers presented in Aghili (2008); Bernat and Stepien (2010).

2. GENERAL MODEL OF THE BLDC MOTOR

The BLDC model is based on previous work in Pillay and Krishnan (1989) and Aghili (2008). The analyzed motor has three phases and two pole pairs and each winding is defined by the equal parameters.

$$u_s(t) = Ri_s(t) + \frac{\partial \phi_s(i,\theta)}{\partial t} \tag{1}$$

where $u_s(t)$ is the phase voltage, $i_s(t)$ is the phase current, ϕ_s is the magnetic flux of the windings, R is the phase

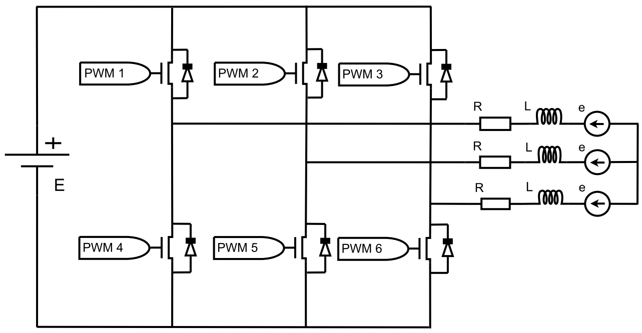

Fig. 1. The schema of BLDC motor with power stage

resistance and s is the winding index ($s = 1, 2, 3$). The flux is produced by a permanent magnet and coil windings. The permanent magnet is modelled as a virtual current, which is assumed to be constant. Thus, the permanent magnet flux is expressed as $\phi_{s,m}(\theta) = M_s(\theta)i_m(t)$, where i_m is a constant virtual magnet current and $M_s(\theta)$ describes the flux change in the motor windings due to the different rotor positions. The BLDC motor allows to assume that a self inductance is constant, because the rotor has a uniform shape. Additionally, the mutual inductance has neglected influence. Hence, the flux change is defined as $\phi_{s,ind}(i) = Li_s(t)$. In summary, the windings flux is:

$$\phi_s(\theta, i) = \phi_{s,m}(\theta) + \phi_{s,ind}(i) \tag{2}$$

Applying the flux model to circuit equation, one has:

$$u_s(t) = Ri_s(t) + L\frac{di_s(t)}{dt} + \frac{\partial M_s(\theta)}{\partial \theta}\omega i_m \tag{3}$$

The $\frac{\partial M_s(\theta)}{\partial \theta}$ causes that equation (3) is nonlinear. The $\frac{\partial M_s(\theta)}{\partial \theta}$ is related with motor geometry and for a BLDC motor has more trapezoidal shape and for a PMSM motor has more sinusoidal shape. The schema for the described circuit equation, along with the power stage, is presented in Fig. 1.

The force acting on the rotor can be calculated using the principle of virtual work, presented in Fitzgerald (2003):

$$T_e(i, \theta) = \frac{\partial W_{co}}{\partial \theta} = \frac{\partial}{\partial \theta}\int_0^t \phi^T(i, \theta)\mathbf{di} \tag{4}$$

The W_{co} is the magnetic co-energy, ϕ is the vector of the windings flux: $\phi = [\, \phi_1 \ \phi_2 \ \phi_3 \,]^T$ and $\mathbf{i}$ is the vector of windings currents: $\mathbf{i} = [\, i_1 \ i_2 \ i_3 \,]^T$. Considering the BLDC flux model, the magnetic coenergy is defined as:

$$W_{co}(\mathbf{i}, \theta) = \sum_{s=1}^{3}\left[\frac{1}{2}M_s(\theta)i_m i_s(t) + \frac{1}{2}Li_s^2(t)\right] \tag{5}$$

where M_s is the mutual inductance between virtual current i_m and phase current i_s. Hence, the rotor's torque is as follows:

$$T_e(i, \theta) = \sum_{s=1}^{3}\frac{1}{2}\frac{\partial M_s(\theta)}{\partial \theta}i_m i_s(t) \tag{6}$$

The motion model is built using the viscous friction and rotor inertia. The first order motion equation is defined as:

$$J\frac{d\omega(t)}{dt} + B\omega(t) = T_e(i, \theta) - T_{load} \tag{7}$$

where J is the rotor inertia, B is the viscous friction and T_{load} is the unknown load function.

The presented equations represent general motor model, which can be approximated to different geometries or motor structures by choosing the bemf function $\frac{\partial M_s}{\partial \theta}$. For instance, the ideal BLDC motor has trapezoidal shape of bemf $\frac{\partial M_s}{\partial \theta}$ and the ideal PMSM machine has sinusoidal shape of bemf $\frac{\partial M_s}{\partial \theta}$.

3. THE ADAPTIVE CONTROL OF MOTOR SPEED

3.1 The model approximation

The control problem in this paper is to follow the motor speed produced by a reference model. To achieve this goal the bemf function is assumed to be:

$$\frac{\partial M_s}{\partial \theta} = \hat{M}sin(2\theta + \varphi_s) \tag{8}$$

where $\hat{M}$ is the mutual inductance between virtual current i_m and phase current i_s. The sinusoidal shape of bemf causes that the motor produces the torque ripples. Therefore, the controller must assure that this known disturbance is compensated if the maximum of accuracy is wanted.

In this paper the novel method of torque modeling is proposed. The torque ripples are defined as the periodic function, which can be expressed by a finite Fourier Series:

$$T_{ripple} = \sum_{k=1}^{N_r} h_k sin(2k\theta) + g_k cos(2k\theta) \tag{9}$$

where N_r defines the number of harmonics included in the representation, g_k and h_k are the harmonics coefficients. The presented finite Fourier Series doesn't contain the constant harmonic, because the torque ripples can't produce it. The reason of extending torque ripples is the alternative of controllers presented by Aghili (2008), Bernat and Stepien (2010). In the mentioned works the bemf functions $M_s(\theta)$ are represented as Fourier Series and the driver must be realized with feedforward. The main advantage of this work is that the gains to compensate torque ripples are calculated in the closed loop.

In summary, the motion equation along with the assumptions for the torque modeling is defined as:

$$J\frac{d\omega}{dt} + B\omega = T_e + T_{ripple} + T_{load} \tag{10}$$

The torque load T_{load} is assumed to be a constant or changing slowly. The electromagnetic torque T_e along with the equation (6) and the sinusoidal bemf function (8) is expressed as:

$$T_e(i, \theta) = \sum_{s=1}^{3}\frac{1}{2}\hat{M}sin(2\theta + \varphi_s)i_m i_s(t) \tag{11}$$

Building a controller for the presented motor is a difficult task, because its model is a nonlinear. Furthermore the torque ripples are changing for different motor speeds, therefore the driver must be adaptive.

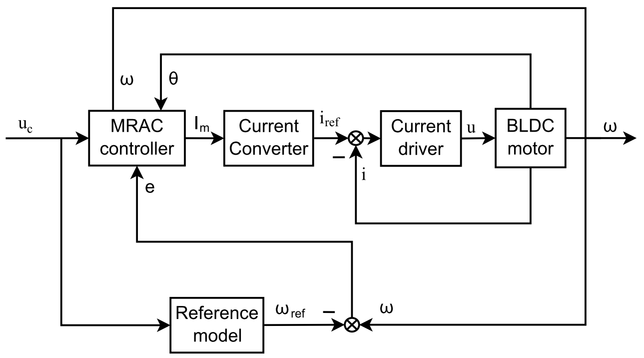

Fig. 2. The controller schema

3.2 The MRAC law

The motor is driven by the model reference adaptive control (MRAC), which is discussed in Astrom and Wittenmark (1994); Ioannou (2003). The control output is the motor speed and the input is the phase current. Therefore, the BLDC motor requires the current drivers, which is in the industrial application in common assumption. The controller schema is presented in Fig. 2 The reference model is as follows:

$$\frac{d\omega_{ref}}{dt} = -a_m\omega_{ref} + b_m u_c \tag{12}$$

where ω_{ref} is the reference motor speed, u_c is the command input and a_m, b_m are the model parameters.

The driver input is the phase current, which is defined as $i_s = \frac{4}{3}I_m sin(2\theta + \phi_s)$, where the I_m is the current amplitude. Thus the electromagnetic torque is equal to:

$$T_e = \sum_{s=1}^{3}\frac{2}{3}I_m\hat{M}i_m sin^2(2\theta+\varphi_s)) = I_m\hat{M}i_m \tag{13}$$

because $\sum_{s=1}^{3} sin^2(2\theta+\varphi_s) = \frac{3}{2}$ and now I_m is the new control input.

The error is defined as $e = \omega - \omega_{ref}$. The error derivative with applied equations (10), (11), (13) and (12) is given by:

$$\frac{de}{dt} = -a_m e + (a_m - \frac{B}{J})\omega + \frac{\hat{M}i_m}{J}I_m - b_m u_c \tag{14}$$
$$+\frac{1}{J}T_{load} + \frac{1}{J}\sum_{k=1}^{N_r} h_k sin(2k\theta) + g_k cos(2k\theta)$$

The control law, which drives the error to 0 is defined as:

$$I_m^* = (\frac{B}{\hat{M}i_m} - a_m\frac{J}{\hat{M}i_m})\omega + b_m\frac{J}{\hat{M}i_m}u_c - \frac{1}{\hat{M}i_m}T_{load} \tag{15}$$
$$-\frac{1}{\hat{M}i_m}\sum_{k=1}^{N_r} h_k sin(2k\theta) + g_k cos(2k\theta)$$

or in the vector forms:

$$I_m^* = \lambda_0^T\varphi \tag{16}$$

where $\lambda_0 \in \mathbf{R}^{2N_r+3}$ is the vector of ideal controller parameters and $\varphi \in \mathbf{R}^{2N_r+3}$ is the vector of feedback functions. The λ_0, φ vectors are defined in the appendix (A.1) and (A.2) respectively. It is worth to notice that this control cannot be realized because h_k, g_k and T_{load} are unknown, but they exist in the vector of parameters. Therefore, the adaptive law is introduced based on the MRAC formula:

$$I_m = \lambda^T\varphi \tag{17}$$

where $\lambda \in \mathbf{R}^{2N_r+3}$ is the adapted controller coefficients. Next, the parameter adaption law is:

$$\frac{d\lambda}{dt} = -\mathbf{G}\varphi e \tag{18}$$

where $\mathbf{G} \in \mathbf{R}^{2N_r+3\times 2N_r+3}$ is the positive define matrix of gains. The stability of this control law can be proved using the following Lapunov function:

$$J = \frac{1}{2}e^2 + (\lambda-\lambda_0)^T\mathbf{G}^{-1}(\lambda-\lambda_0) > 0 \tag{19}$$

The derivative of Lapunov function J with help of formula (14) yields to:

$$\frac{dJ}{dt} = -a_m e^2 + \frac{\hat{M}i_m}{J}(\lambda-\lambda_0)^T(\varphi e + \mathbf{G}^{-1}\frac{d\lambda}{dt}) \tag{20}$$

Then, applying the parameter adaption law (18) shows that:

$$\frac{dJ}{dt} = -a_m e^2 \leq 0 \tag{21}$$

because other terms are canceled by the equation (18). This also establish that the system is stable.

4. THE ESTIMATION OF THE BLDC MOTOR MODEL

4.1 The model for simulations

The presented control in the previous section is verified using a simulation model of the BLDC motor. To improve the accuracy of the simulation model, the bemf is approximated by Fourier series. This also verifies whether the modeling of torque ripples is correct in the presented control law. To assure realism of the model, its parameters are identified by an experiment. Furthermore, the advantage of this technique is also the possibility to build a nonlinear regressor, which can be estimated by linear methods like Recursive Least Squares method, discussed in Astrom and Wittenmark (1994). Therefore, the $\frac{\partial M(\theta)}{\partial\theta}$ is expressed by the Fourier series:

$$\frac{\partial M(\theta)}{\partial\theta} = \sum_{k=1}^{N_f} A_k sin(kN_p\theta) + B_k cos(kN_p\theta) \tag{22}$$

The N_f defines the numbers of Fourier series elements, the A_k and B_k are the harmonics coefficients and N_p is the number of rotor pole pairs. To define the estimation formula, the equation (3) is written in a state space form and with help of equation (22) gives:

$$\frac{di(t)}{dt} = -\frac{R}{L}i(t) + \frac{1}{L}u(t)$$
$$-\frac{1}{L}\omega(t)i_m\sum_{k=1}^{N_f}[A_k sin(kN_p\theta) + B_k cos(kN_p\theta)] \tag{23}$$

The equation is discretized by the Euler method with the sampling time T_s:

$$
\begin{aligned}
i(n) &= (1 - T_s\frac{R}{L})i(n-1) + \frac{T_s}{L}u(n-1) \\
&-\frac{T_s}{L}\omega(n-1)i_m\sum_{k=1}^{N_f}[A_k sin(kN_p\theta(n-1))] \\
&-\frac{T_s}{L}\omega(n-1)i_m\sum_{k=1}^{N_f}[B_k cos(kN_p\theta(n-1))]
\end{aligned} \tag{24}
$$

The RLS method is based on the regression model:

$$
\mathbf{y_c}(n) = \varphi_{\mathbf{c}}{}^T(n)\psi_{\mathbf{c}}(n) \tag{25}
$$

where $\mathbf{y}_c(n) = [i(n)]$ is the output, φ_c is the vector of inputs, ψ_c is the system unknown parameters. The regressor φ_c and the unknown parameters vector ψ_c are defined in the appendix (B.1) and (B.2) respectively. Now, the estimation of motion parameters like friction B, rotor inertia J is performed. The motion equation is rewritten to state space form and discretized by the Euler method with the sampling time T_s:

$$
\omega(n) = (1 - T_s\frac{B}{J})\omega(n-1) + \frac{T_s}{J}T_e(n-1) \tag{26}
$$

During the identification process the load torque is equal to 0. The parameters from equation (26) can be estimated by the linear regression method expressed by equation $\mathbf{y}_m(n) = \varphi_m^T(n)\psi_m(n)$. The output is $\mathbf{y}_m = [\omega(n)]$, the regressor and the vector of unknowns is presented in the appendix (C.2) and (C.1). It is worth to notice, that the electromagnetic torque T_e is already known, because the approximation of $\frac{\partial M}{\partial \theta}$ is identified with parameters of circuit equation (24).

The identification of simulation motor has been performed on the model of BLDC motor with 3 phases built by Finite Elment Modeling technique, see in Bernat et al. (2010). The identification problem is solved iteratively Astrom and Wittenmark (1994), hence in each step the estimated vector $\widehat{\boldsymbol{\psi}}$ of unknown parameters is updated. The input voltage has a rectangular shape with the amplitude of $30.0[V]$. The motor parameters, which have been obtained in the identification process, are shown in table 1. This table includes the two main significant harmonics of bemf, which also represents the influence of permanent magnet.

Table 1. The motor parameters obtained in an identification process

Parameter	Value	Unit
R - Phase resistance	5.2	$[\Omega]$
L - Phase inductance	3.8	$[mH]$
Coeff. $\sqrt{A_1^2 + B_1^2}$	4.5	$[mH]$
Coeff. $\sqrt{A_5^2 + B_5^2}$	1.3	$[mH]$
J - Rotor inertia	$1.2 \cdot 10^{-6}$	$[Nms^s]$
B - Damping	$0.9 \cdot 10^{-5}$	$[Nms]$

5. THE SIMULATION OF ADAPTIVE LAW

The presented control is verified by simulations. Unfortunately the authors can't check the law with an experiment due to our laboratory constraints. However, the motor model has been obtained by an identification process, so it assures high reliability of results.

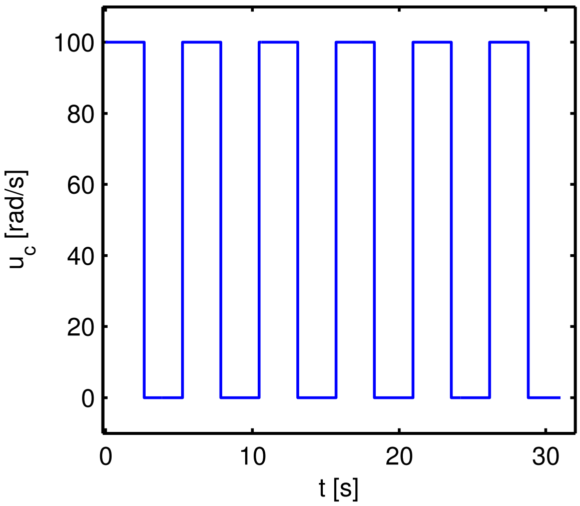

Fig. 3. The command signal applied to system

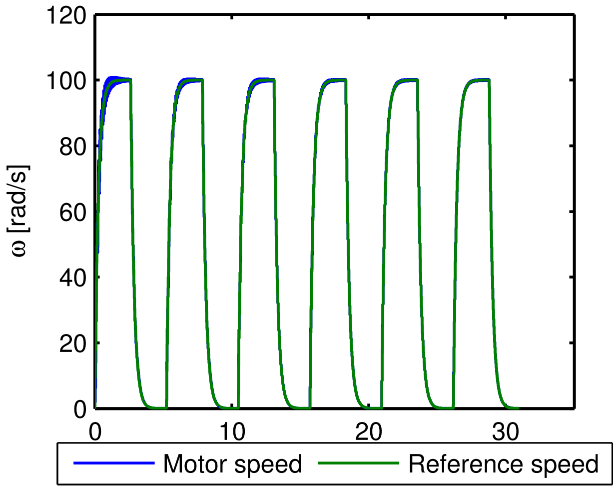

Fig. 4. The comparison of motor speed with reference signal

The control law requires the reference model, which has two design parameters a_m and b_m. Both of them can be chosen freely, nevertheless it is worth to set $a_m = b_m$ to get a unit gain of reference dynamics. In this experiment $a_m = b_m$ is equal to 4.0, which gives a time constant equal to $T_{a_m} = \frac{1}{a_m} = 0.25[s]$. The gain matrix is chosen as diagonal with gains equal to $\mathbf{G} = [g_{kk}] = 0.0001, k = 1, \ldots, 2N_r + 3$ and the number of torque ripple harmonics N_r is set to 10. The command signal has a rectangular shape with amplitude 100 $\left[\frac{rad}{s}\right]$ as it is shown in Fig. 3.

The adaptive controller follows the reference speed as it is shown in Fig. 4. Nevertheless the first few motor starts have low performance, which is related to the adaption of controller parameters, the target dynamics is successfully achieved. The process of adaption of driver parameters, presented in Fig. 5 is very dynamic at the beginning and it stabilizes with time. Fig. 5 shows only three of the most

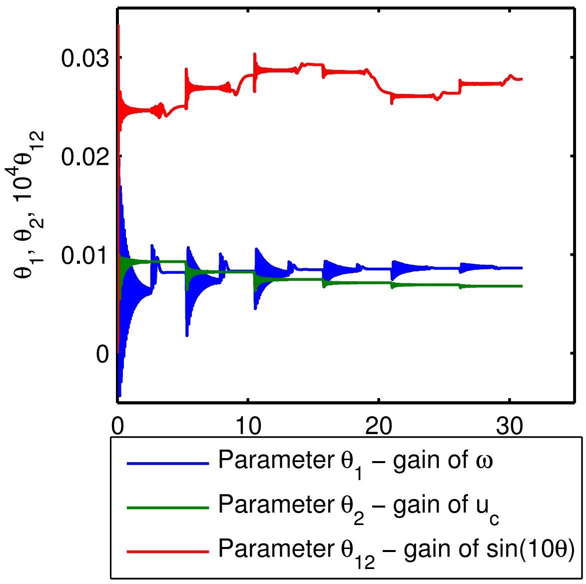

Fig. 5. The transients of controller parameters

significant controller coefficients, because the other values are close to 0. The output motor speed doesn't contain

Table 2. The motor torque coefficients in steady state

Coefficient	MRAC control	BLDC motor	Unit
T_{avg}	$1.0 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$	$[Nm]$
T_{max}	$1.08 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$[Nm]$
T_{min}	$0.92 \cdot 10^{-3}$	$0.7 \cdot 10^{-3}$	$[Nm]$
T_{pulse}	16	60	[%]

higher harmonics, because the controller successfully compensates torque ripples. Table 2 shows the comparison of the torque coefficients in the motor controlled by presented adaptive system and in the BLDC motor with feedback from hallotrons. The coefficients presented in the table describes the torques in a steady state and means: T_{avg} is the average torque, T_{max} is the maximal torque, T_{min} is the minimal and T_{pulse} is the torque pulsation defined as: $T_{pulse} = \frac{T_{max}-T_{min}}{T_{avg}}100\%$. The avarage torque is the same for both motors, however the torque pulsation, which describes a level of torque ripples, is significantly lower for MRAC controller than for hallotron driver. The 16% of torque ripples in presented control system is caused by the current driver imperfections. Nevertheless this results enables us to control succefully the motor speed.

6. CONCLUSIONS

This research presents the adaptive speed control method of the brushless DC motor. The controller uses a novel model of brushless DC motor, which has as input the current and as output the speed. This model provides also the possibility to compensate the torque ripples and load torque. The control law stability has been proved using a Lapunov function. The theoretical results has been verified in the simulations.

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Appendix A. THE ADAPTIVE CONTROLLER PARAMETERS

The vector of controller parameters λ_0 is as follows:

$$\lambda_0 = \begin{bmatrix} \frac{B}{\hat{M}i_m} - a_m \frac{J}{\hat{M}i_m} \\ b_m \frac{J}{\hat{M}i_m} \\ -\frac{T_{load}}{\hat{M}i_m} \\ -\frac{h_1}{\hat{M}i_m} \\ -\frac{g_1}{\hat{M}i_m} \\ \vdots \\ -\frac{h_{N_r}}{\hat{M}i_m} \\ -\frac{g_{N_r}}{\hat{M}i_m} \end{bmatrix} \tag{A.1}$$

The vector of feedback functions φ is given by:

$$\varphi = \begin{bmatrix} \omega \\ u_c \\ 1 \\ sin(2\theta) \\ cos(2\theta) \\ \vdots \\ sin(2N_r\theta) \\ cos(2N_r\theta) \end{bmatrix} \tag{A.2}$$

Appendix B. THE REGRESSOR FOR ESTIMATION OF CIRCUIT PARAMETERS

The nonlinear regressor for electric circuit (24) is defined as:

$$\begin{aligned} \varphi_{c,1}(n) &= i(n-1) \\ \varphi_{c,2}(n) &= u(n-1) \\ \varphi_{c,3}(n) &= \omega(n-1)sin(N_p\theta(n-1)) \\ \varphi_{c,4}(n) &= \omega(n-1)cos(N_p\theta(n-1)) \\ &\vdots \\ \varphi_{c,p}(n) &= \omega(n-1)sin(kN_p\theta(n-1)) \\ \varphi_{c,p+1}(n) &= \omega(n-1)cos(kN_p\theta(n-1)) \\ &\vdots \\ \varphi_{c,2N_f+1}(n) &= \omega(n-1)sin(N_fN_p\theta(n-1)) \\ \varphi_{c,2N_f+2}(n) &= \omega(n-1)cos(N_fN_p\theta(n-1)) \end{aligned} \tag{B.1}$$

and the unknown parameters vector elements are following:

$$\begin{aligned} \psi_{c,1}(n) = 1 - T_s\frac{R}{L},\ \psi_{c,2}(n) &= \frac{T_s}{L} \\ \psi_{c,3}(n) = \frac{T_s}{L}i_mA_1,\ \psi_{c,4}(n) &= \frac{T_s}{L}i_mB_1 \\ &\vdots \\ \psi_{c,p}(n) = \frac{T_s}{L}i_mA_k,\ \psi_{c,p+1}(n) &= \frac{T_s}{L}i_mB_k \\ &\vdots \\ \psi_{c,2N_f+1}(n) = \frac{T_s}{L}i_mA_{N_f},\ \psi_{c,2N_f+2}(n) &= \frac{T_s}{L}i_mB_{N_f} \end{aligned} \tag{B.2}$$

Without loss of generality the virtual current i_m can be employed to 1. Therefore the BLDC parameters are equal to:

$$\begin{aligned} R = \frac{1-\psi_{c,1}}{\psi_{c,2}},\ L &= \frac{T_s}{\psi_{c,2}} \\ A_1 = \frac{\psi_{c,2}}{\psi_{c,3}},\ B_1 &= \frac{\psi_{c,2}}{\psi_{c,4}} \\ &\vdots \\ A_k = \frac{\psi_{c,2}}{\psi_{c,p}},\ B_k &= \frac{\psi_{c,2}}{\psi_{c,p+1}} \\ &\vdots \\ A_{N_f} = \frac{\psi_{c,2}}{\psi_{c,2N_f+2}},\ B_{N_f} &= \frac{\psi_{c,2}}{\psi_{c,2N_f+2}} \end{aligned} \tag{B.3}$$

Appendix C. THE REGRESSOR FOR ESTIMATION OF MOTION PARAMETERS

The vector of parameters for motion circuit (7) is defined as:

$$\psi_{m,1}(n) = 1 - T_s\frac{B}{J},\ \psi_{m,2}(n) = \frac{T_s}{J} \tag{C.1}$$

and the linear regressor is given by:

$$\varphi_{m,1}(n) = \omega(n-1),\ \varphi_{m,2}(n) = T_e(n-1) \tag{C.2}$$