Statistics One

Lecture 16
Analysis of Variance (ANOVA)

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Two segments

- One-way ANOVA
- Post-hoc tests

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Lecture 16 ~ Segment 1

One-way ANOVA

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Analysis of Variance (ANOVA)

- Appropriate when the predictors (IVs) are all categorical and the outcome (DV) is continuous
  - Most common application is to analyze data from randomized controlled experiments

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Analysis of Variance (ANOVA)

- More specifically, randomized controlled experiments that generate more than two group means
  - If only two group means then use:
    - Independent t-test
    - Dependent t-test

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Analysis of Variance (ANOVA)

- If more than two group means then use:
  - Between groups ANOVA
  - Repeated measures ANOVA

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Example

- Working memory training
  - Four independent groups (8, 12, 17, 19)
    - IV: Number of training sessions
    - DV: IQ gain
    - Null hypothesis: All groups are equal

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Working memory training

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Analysis of Variance (ANOVA)

- ANOVA typically involves NHST
- The test statistic is the F-test (F-ratio)
  - F = (Variance between groups) / (Variance within groups)

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Analysis of Variance (ANOVA)

- Like the t-test and family of t-distributions
- The F-test has a family of F-distributions
  - The distribution to assume depends on
    - Number of subjects per group
    - Number of groups

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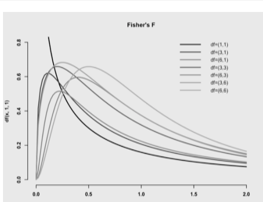
Analysis of Variance (ANOVA)



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One-way ANOVA

- F-ratio

F = between-groups variance / within-groups variance

$F = MS_{Between} / MS_{Within}$

$F = MS_{A} / MS_{S/A}$

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One-way ANOVA

- $F = MS_A / MS_{S/A}$
- $MS_A = SS_A / df_A$
- $MS_{S/A} = SS_{S/A} / df_{S/A}$

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One-way ANOVA

- $SS_A = n\ \Sigma(Y_j - Y_T)^2$
  - Y_j are the group means
  - Y_T is the grand mean

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One-way ANOVA

- $SS_{S/A} = \Sigma(Y_{ij} - Y_j)^2$
  - Y_{ij} are individual scores
  - Y_j are the group means

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One-way ANOVA

- $df_A = a - 1$
- $df_{S/A} = a(n - 1)$
- $df_{TOTAL} = N - 1$

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Summary Table

Source	SS	df	MS	F
A	$n\ \Sigma(Y_j - Y_T)^2$	a - 1	SS_A/df_A	$MS_A\ /MS_{S/A}$
S/A	$\Sigma(Y_{ij} - Y_j)^2$	a(n -1)	$SS_{S/A}/df_{S/A}$	-----
Total	$\Sigma(Y_{ij} - Y_T)^2$	N - 1	-----	-----

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Effect size

- $R^2 = \eta^2$ (eta-sqaured)
- $\eta^2 = SS_A / SS_{Total}$

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Assumptions

- DV is continuous (interval or ratio variable)
- DV is normally distributed
- Homogeneity of variance
    - Within-groups variance is equivalent for all groups
        - Levene's test

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Homogeneity of variance

- If Levene's test is significant then homogeneity of variance assumption has been violated
    - Conduct pairwise comparisons using a restricted error term

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Example

- Working memory training
  - Four independent groups (8, 12, 17, 19)
    - IV: Number of training sessions
    - DV: IQ gain
    - Null hypothesis: All groups are equal

Working memory training

Working memory training

```
> anova.WMT <- aov(WMT$IQ ~ WMT$condition)
> summary(anova.WMT)
              Df Sum Sq Mean Sq F value   Pr(>F)
WMT$condition  3  196.1   65.36   10.49 7.47e-06 ***
Residuals     76  473.4    6.23
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> # Post hoc (Tukey)
> TukeyHSD(anova.WMT)
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = WMT$IQ ~ WMT$condition)
```


Working memory training

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  Tukey multiple comparisons of means
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Fit: aov(formula = WMT$IQ ~ WMT$condition)

$`WMT$condition`
                  diff       lwr       upr     p adj
17 days-12 days  2.205  0.131797  4.278203 0.0326836
19 days-12 days  3.050  0.976797  5.123203 0.0013069
8 days-12 days  -0.790 -2.863203  1.283203 0.7492938
19 days-17 days  0.845 -1.228203  2.918203 0.7083557
8 days-17 days  -2.995 -5.068203 -0.921797 0.0016487
8 days-19 days  -3.840 -5.913203 -1.766797 0.0000354
```

Results from t-test: 12 vs. 17

```
> t.test(Days12, Days17, var.equal=T)

 Two Sample t-test

data:  Days12 and Days17
t = -2.868, df = 38, p-value = 0.006706
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.7534458 -0.6471965
sample estimates:
mean of x mean of y
 11.70287  13.90319

> cohensD(Days12, Days17, method='pooled')
[1] 0.9069322
```

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Segment summary

- ANOVA is used to compare means, typically in experimental research
  - Categorical IV
  - Continuous DV

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Segment summary

- ANOVA assumes homogeneity of variance
  - Evaluate with Levene's test

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Segment summary

- Post-hoc tests, such as Tukey's procedure, allow for multiple pairwise comparisons without an increase in the probability of a Type I error

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END SEGMENT

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Lecture 16 ~ Segment 2

Post-hoc tests

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Post-hoc tests

- Post-hoc tests, such as Tukey's procedure, allow for multiple pairwise comparisons without an increase in the probability of a Type I error

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Post-hoc tests

- Many procedures are available; the degree to which p-values are adjusted varies according to procedure
  - – Most liberal: No adjustment
  - – Most conservative: Bonferroni procedure

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NHST review

Experimenter Decision

Truth		Retain H_0	Reject H_0
	H_0 true	Correct Decision	Type I error (False alarm)
	H_0 false	Type II error (Miss)	Correct Decision

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NHST review

Experimenter Decision

Truth		Retain H_0	Reject H_0
	H_0 true	Correct Decision	Type I error $p = .05$
	H_0 false	Type II error (Miss)	Correct Decision

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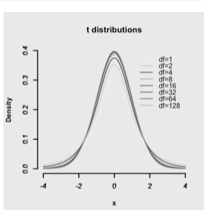
NHST review



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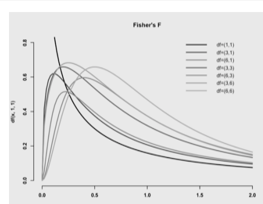
NHST review



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Working memory training

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Tukey's procedure

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[1] 0.9069322
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Bonferroni procedure

```
> p.adjust(.006706, method="bonferroni", 6)
[1] 0.040236
```

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Comparison of procedures

Procedure	p-value for 12 vs. 17
Independent t-test	0.0067
Tukey	0.0327
Bonferroni	0.0402

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Post-hoc tests

- Post-hoc tests, such as Tukey's procedure, allow for multiple pairwise comparisons without an increase in the probability of a Type I error
- Procedures vary from liberal to conservative

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END SEGMENT

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END LECTURE 16

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