


Question 1

Let the matrix be as $\begin{bmatrix} A_{so} & b_{so} \\ 0 & 1 \end{bmatrix}$ where $b_{so} \in \mathbb{R}^d$, $A_{so} \in GLC(3)$

$$A_{so} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{and} \quad b_{so} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Plug in OP_1 to OP_4 and SP_1 to SP_4

$$\begin{bmatrix} A_{so} & b_{so} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} OP \\ 1 \end{bmatrix} = T_{so}(OP_1) = \begin{bmatrix} SP_1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} \textcircled{1} \quad 2a_{11} + 3a_{12} - 3a_{13} + b_1 &= 1.8641 & \textcircled{2} \quad -3a_{13} + b_1 &= -0.096 & \textcircled{3} \quad -a_{11} - 2a_{12} + 2a_{13} + b_1 &= -0.0827 \\ \textcircled{4} \quad -a_{11} + 2a_{13} + b_1 &= -0.0827 & \text{solve for } a_{11}, a_{12}, a_{13} \text{ \& } b_1 \end{aligned}$$

$$\textcircled{4} - \textcircled{3} : a_{12} = 0 \quad \textcircled{1} - \textcircled{2} : 2a_{11} + 3a_{12} = 1.9601 \Rightarrow a_{11} = 0.98005$$

$$\textcircled{4} \Rightarrow 2a_{13} + b_1 = 0.89735 \quad \Rightarrow -\textcircled{2} \Rightarrow 5a_{13} = 0.99335 \quad a_{13} = 0.19867$$

$$\text{plug in } \textcircled{2} : b_1 = 0.50001$$

$$\text{so } \begin{cases} a_{11} = 0.98005 \\ a_{12} = 0 \\ a_{13} = 0.19867 \\ b_1 = 0.50001 \end{cases}$$

Similarly solve for the other variables and we can find:

$$T_{so} = \left\{ \begin{bmatrix} A_{so} & b_{so} \\ 0 & 1 \end{bmatrix} \right\} = \begin{bmatrix} 0.98005 & 0 & 0.19867 & 0.50001 \\ 0.0198 & 0.995 & 0.01786 & 0.29998 \\ -0.19765 & 0.0992 & 0.9757 & 2.50001 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2

I will assume red is X, blue is Y,

green is Z

rotation along axis c-clockwise is as follows:

$$R_x(a) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & -\sin a \\ 0 & \sin a & \cos a \end{bmatrix}$$

$$R_y(a) = \begin{bmatrix} \cos a & 0 & \sin a \\ 0 & 1 & 0 \\ -\sin a & 0 & \cos a \end{bmatrix}$$

$$R_z(a) = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so: (a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.86 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0.5 & 0.866 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$(e) \begin{bmatrix} 0 & 0 & 1 & 0.2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(f) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(g) \begin{bmatrix} 0.707 & 0 & 0.707 & 0 \\ 0 & 1 & 0 & 1 \\ -0.707 & 0 & 0.707 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(h) \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0.707 & 0 & -0.707 & -2 \\ 0.707 & 0 & 0.707 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Some of Above rotation matrix involves multiply:
 $R_x(a) \times R_y(b)$ etc. The results are final results.