给定B，对于指定的C_i，有

$$\frac{r_i}{\sin\angle OAC_i} = \frac{R}{\sin\angle AC_iO} \tag{1}$$
$$\frac{r_i}{\sin\angle OBC_i} = \frac{R}{\sin\angle BC_iO} \tag{2}$$

（1） $\tilde{C}_i \in \overset{\frown}{AB}$时

插图

$$\begin{aligned}
\sin\angle OAC_i &= \sin(\pi - (\angle AC_iO + \theta_i)) \\
&= \sin(\angle AC_iO + \tilde{\theta}_i + \delta_i) \\
&= \sin(\angle AC_iO + i\Theta + \delta_i) \\
&= \sin(\angle AC_iO + i\Theta)\cos\delta_i + \cos(\angle AC_iO + i\Theta)\sin(\delta_i) \\
&\approx \sin(\angle AC_iO + i\Theta)(1 - \frac{1}{2}x^2) + \cos(\angle AC_iO + i\Theta)x
\end{aligned}$$

$$\begin{aligned}
\sin\angle OBC_i &= \sin(\pi - (\angle BC_iO + \angle AOB - \theta_i)) \\
&= \sin(\angle BC_iO + k\Theta - (\tilde{\theta}_i + \delta_i)) \\
&= \sin(\angle BC_iO + k\Theta - i\Theta - \delta_i)) \\
&= \sin(\angle BC_iO + (k-i)\Theta)\cos\delta_i - \cos(\angle BC_iO + \Theta(k-i))\sin\delta_i \\
&\approx \sin(\angle BC_iO + (k-i)\Theta)(1 - \frac{1}{2}x^2) - \cos(\angle BC_iO + (k-i)\Theta)x
\end{aligned}$$

记

$$\begin{aligned}
\sin(\angle AC_iO + i\Theta) &= \sin(\angle AC_iO + i\Theta) \\
b_1 &= \cos(\angle AC_iO + i\Theta) \\
c_1 &= \sin\angle AC_iO \\
a_2 &= \sin(\angle BC_iO + (k-i)\Theta) \\
b_2 &= \cos(\angle BC_iO + (k-i)\Theta) \\
c_2 &= \sin\angle BC_iO
\end{aligned}$$

$$
\begin{aligned}
0 &= (a_1c_2 - a_2c_1)x^2 - 2(b_1c_2 + b_2c_1)x - 2(a_1c_2 - a_2c_1) \\
a &= (a_1c_2 - a_2c_1) \\
&= \sin(\angle AC_iO + i\Theta)\sin\angle BC_iO - \sin(\angle BC_iO + (k-i)\Theta)\sin\angle AC_iO \\
b &= 2(b_1c_2 + b_2c_1) \\
&= 2(\cos(\angle AC_iO + i\Theta)\sin\angle BC_iO + \cos(\angle BC_iO + (k-i)\Theta)\sin\angle AC_iO) \\
c &= -2(a_1c_2 - a_2c_1) \\
&= -2a
\end{aligned}
$$

于是方程 (1) 和 (2) 联立，代换后可以得到