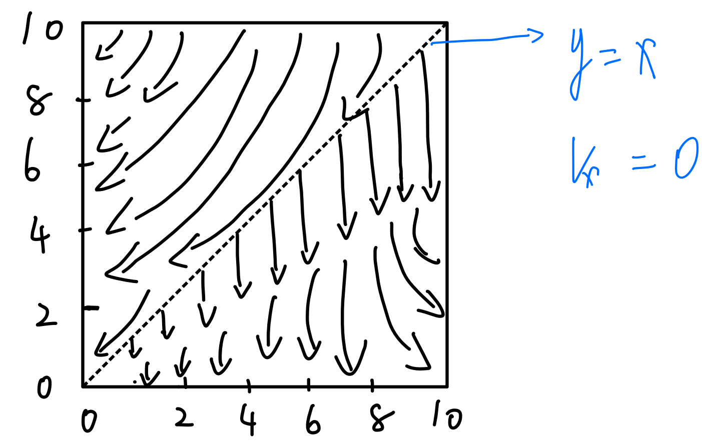
(1)

(a)



(b) $V_x = k(x^2 - y^2)$

When the value of "x" and "y" are similar, the value of V_x is close to 0.

$\vec{w} = \nabla \times \vec{v}$

For 2-D flow, $\vec{w}$ only have "z" component, which can be written as $W_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}$.

"$\nabla \cdot \vec{v}$" is a measure of the divergence of a vector field, which is "0" for incompressible fluids.

$\nabla \cdot \vec{v} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 2kx - 2kx + 0 = 0$

For this question, "$\nabla \cdot \vec{v} = 0$", it shows, this fluid is incompressible.

(2) (a)

$\tau_{ij} = \eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$, $\sigma = -P + 2\eta\frac{\partial u}{\partial x}$, We assume $P = 0$.

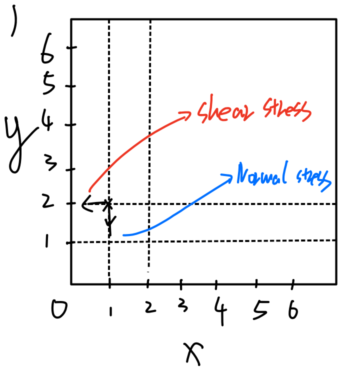
Since, $V_x = k(x^2 - y^2)$, $V_y = -2kxy$.

So, $\frac{\partial V_x}{\partial y} = -2ky$, $\frac{\partial V_y}{\partial x} = -2ky$, $\frac{\partial V_x}{\partial x} = 2kx$, $\frac{\partial V_y}{\partial y} = -2k$

when $x = 1$, $y = 2$, $\eta = 10$ MPa·s, $\tau_{xy} = \eta\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right) = \eta(-2ky - 2ky) = -4 \times 2 \times 2\eta = -160$ MPa.

$\tau_{yy} = \eta\left(2\frac{\partial V_y}{\partial y}\right) = \eta(2 \times (-2k)) = -8\eta = -80$ MPa

(b)



Magnitude of shear stress is twice the magnitude of normal stress.

3) (a)

$$\bar{\bar{\sigma}} = 80\begin{bmatrix} 3 & -4 & 0 \\ -4 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa},$$

$$V_1 = (-4, 2, 0),\ V_2 = (1, 2, 0),\ V_3 = (0, 0, 1)$$

$$\bar{\bar{\sigma}}' = P^T \bar{\bar{\sigma}} P = \begin{bmatrix} 400 & 0 & 0 \\ 0 & -400 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(b) To check is this an orthonormal basis trasformation.

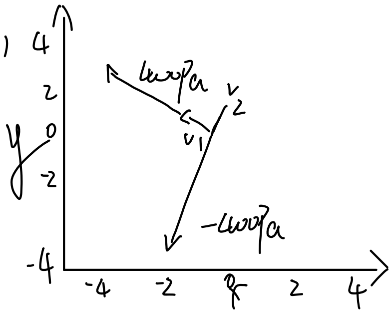
We need to check, $V_i \cdot V_j = 0$ $(i \neq j)$, and $|V_i| = 1$.

$V_1 \cdot V_2 \neq 0$, So this is not an orthonormal basis transformation.

$\sigma_{v_1} = \frac{V_1}{|V_1|} = (-0.894, 0.447, 0)$, $\sigma_{v_2} = \frac{V_2}{|V_2|} = (0.447, 0.894, 0)$, $\sigma_{v_3} = (0, 0, 1)$

We need to normalise each vector so that the changed matrices are orthogonal matrices.

(c)



There is tension in the V_1 direction, and compression in the V_2 direction, with no shear stress component.

(d) Reasons for replacing the basis may include, "simplifying the calculations", such as diagonalising the stress tensor to find the normal stress and direction.

"Physical or geometrical structure alignment", simplify the resolution of a problem or provide a more intuitive analysis.