IMPERIAL COLLEGE LONDON

MSc ASSESSMENT 2023/24

For internal students of Imperial College London

Taken by students of MSc Applied Computational Science and Engineering

Modelling and Numerical Methods Coursework 2

09:30 – 11:00 Friday 26 January 2024

+ 10 minutes upload time (11:00-11:10)

This coursework comprises three questions. Please answer ALL questions.

The marks are out of a total of 100 marks.

The coursework is run as a timed, open-book assessment, and as such we have worked hard to create a coursework that assesses synthesis of knowledge rather than factual recall. Be aware that access to the internet, notes or other sources of factual information in the time provided may not be too helpful and may well limit your time to successfully synthesise the answers required.

The use of the work of another student, past or present, constitutes plagiarism. Giving your work to another student to use may also constitute an offence. Collusion is a form of plagiarism and will be treated in a similar manner. This is an individual assessment and thus should be completed solely by you.

The College will investigate all instances where an assessment offence is reported or suspected, using plagiarism software, vivas and other tools, and apply appropriate penalties to students. In all assessments, we will analyse performance against performance on the rest of the course and against data from previous years and use an evidence-based approach to maintain a fair and robust assessment.

As with all assessments, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Please write clearly, annotate any graphs or sketches and explain your answers. Most of the marks will be awarded for answers that demonstrate a clear understanding. Only a small number of marks will be given for correct answers without an explanation or derivation.

At the end of the time, scan or take a photo of each page of your answers and combine into a single pdf to upload to GitHub classroom. Make sure you put a page number on each page of your answer script. You will have 10 minutes to upload your answers

GOOD LUCK

All questions below relate to a Newtonian fluid with the following steady-state velocity field:

$$v_x = k(x^2 - y^2)$$
$$v_y = -2kxy$$
$$v_z = 0$$

where constant k = 2 m/s, constant viscosity η is = 10 MPa s, and x, y, and z are the three Cartesian coordinate directions.

1) (*35 marks*)

(a) Plot the flow field (magnitude and arrows showing direction) for a domain $x \in [0:10]m; y \in [0:10]m\ , z = 0\ m$

(b) Describe the style of the flow plotted in (a). In your discussion include the concepts compressibility, mass conservation (considering the volume plotted) and vorticity. Use appropriate equations and calculations to make your discussion quantitative.

2) (*30 marks)*

(a) Show that the shear and normal stress in this flow field, at point (1,2,0) on a plane with normal in $\hat{\boldsymbol{e}}_y$ direction, are equal to -160 Pa and -80 Pa, respectively.

(b) Sketch the directions and relative magnitude of these two stresses on an appropriately labelled diagram.

Assume pressure equals 0 Pa.

3) (*35 marks*)

At another point in the flow, the stress tensor for the above velocity field equals:

$$\bar{\bar{\boldsymbol{\sigma}}} = 80 \begin{bmatrix} 3 & -4 & 0 \\ -4 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

(a) Transform the stress tensor in this point to a new basis with as basis vectors: $\mathbf{v}_1$ = (-4,2,0), $\mathbf{v}_2$ = (1,2,0), $\mathbf{v}_3$ = (0,0,1). Show all your steps.

(b) Is this an orthonormal basis transformation? If it is not, comment on how it could be changed to an orthonormal transformation. Clearly show the steps that motivate your answer.

(c) Describe the state of stress in this point using the tensor on the new basis $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$. The description should include a labelled sketch.

(d) Explain at least two reasons why one might want to change to a new basis for describing the state of stress.