6.4 $E(\hat{\theta}) = E\left(\frac{\sum(x_i-\bar{x})^2}{n}\right)$

$\sum(x_i-\bar{x})^2 = \sum(x_i^2 - 2x_i\bar{x} + \bar{x}^2)$

$= \sum x_i^2 - \sum 2x_i\bar{x} + \sum\bar{x}^2$

$= \sum x_i^2 - 2\sum n\bar{x}\bar{x} + \sum x_i^2 - n\bar{x}^2$

$= \sum x_i^2 - n\bar{x}^2$

$E(\hat{\theta}) = E\left(\frac{\sum(x_i-\bar{x})^2}{n}\right) = \frac{1}{n}E(\sum x_i^2 - n\bar{x}^2)$

$E(\sum x_i^2) = \sum E(x_i^2)$

$V(X) = E(X^2) - E^2(X) \quad E(x_i^2) = V(x_i) + E^2(x_i)$

因此 $\sum E(x_i^2) = \sum[V(x_i) + E^2(x_i)]$

$\sum E(x_i^2) = n\sigma^2 + n\mu^2$

$E(n\bar{x}^2) = nE(\bar{x}^2)$

$V(X) = E(X^2) - E^2(X) \quad E(\bar{x}^2) = V(\bar{x}) + E^2(\bar{x})$

$E(\bar{x}^2) = \frac{\sigma^2}{n} + \mu^2 \quad nE(\bar{x}^2) = \sigma^2 + n\mu^2$

$E(\sum x_i^2 - n\bar{x}^2) = \sum E(x_i^2) - nE(\bar{x}^2)$

$= (n\sigma^2 + n\mu^2) - (\sigma^2 + n\mu^2)$

$E(\hat{\theta}) = \frac{n-1}{n}\sigma^2.$