STAT 598Z Homework 4
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Due March 19th, 2013 **huang147@purdue.edu**

Problem 1

This problem can be solved by two different methods: either accept-reject sampling or coordinate transformation (similar to the orthogonal and polar coordinate transformation that learned in class).

- Accept-reject Sampling. This method is very straightforward. Just simply generate x and y from $uniform(-1, 1)$, then look at the range of x and y together, only the (x, y) pairs that fall into the shaded square are accepted. The efficiency of this method is about 0.5.

- Coordinate transform. Consider a square bounded by four lines:

$$x = -\sqrt{2}/2$$
$$x = \sqrt{2}/2$$
$$y = -\sqrt{2}/2$$
$$y = \sqrt{2}/2$$

If we rotate the square and its coordinate counter clockwise by $\theta = 45°$, we get the desired shaded square in this problem. We can easily sample (x, y) from the original square, and then use coordinate transformation to get (x', y') in our desired rotated square. The equation for getting an orthogonal coordinate transformation by an angle (counter clockwise) of θ is

$$\begin{cases} x' = x\cos\theta + y\sin\theta = \sqrt{2}/2 * x + \sqrt{2}/2 * y \\ y' = -x\sin\theta + y\cos\theta = -\sqrt{2}/2 * x + \sqrt{2}/2 * y \end{cases}$$

See Figure 1 for a comparison of these two methods below.

Problem 2

Since $X \sim Beta(1, \beta)$, we have the pdf of X

$$f_X(x) = \frac{1}{B(1,\beta)} x^{1-1}(1-x)^{\beta-1} = \frac{1}{\int_0^1 (1-t)^{\beta-1}dt}(1-x)^{\beta-1} = \beta(1-x)^{\beta-1} \quad (0 \le x \le 1)$$

and $Y \sim X^{\frac{1}{\gamma}}$, we have the pdf of Y from transformation of X

$$f_Y(y) = f_X(g^{-1}(y))|\frac{dx}{dy}| = \beta(1-y^\gamma)^{\beta-1}\gamma y^{\gamma-1} = \gamma\beta y^{\gamma-1}(1-y^\gamma)^{\beta-1}$$

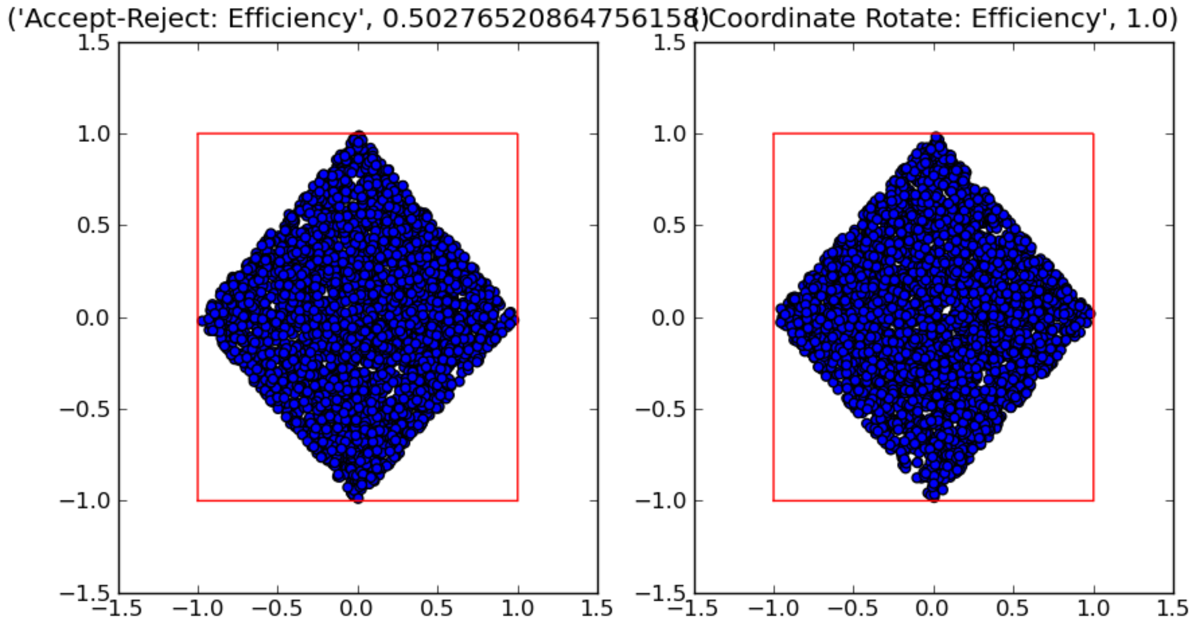

Figure 1: Comparison of Accept-Reject Sampling, and Coordinate Rotation

From integration we have the cdf of Y

$$F_Y(y) = \int_0^y f_Y(t)dt = \int_0^y \gamma\beta t^{\gamma}(1-t^{\gamma})^{\beta-1}dt = 1-(1-y^{\gamma})^{\beta}$$

From inverse transformation, we have

$$F_Y^{-1}(u) = [1-(1-u)^{\frac{1}{\beta}}]^{\frac{1}{\gamma}}$$

Which means we only need to draw samples from $u \sim uniform(0,1)$, and then transform it into $F_Y^{-1}(u)$ to get the sample of Y. See my code in the attached pages. Figure 2 is showing the plot of histogram versus the function itself.

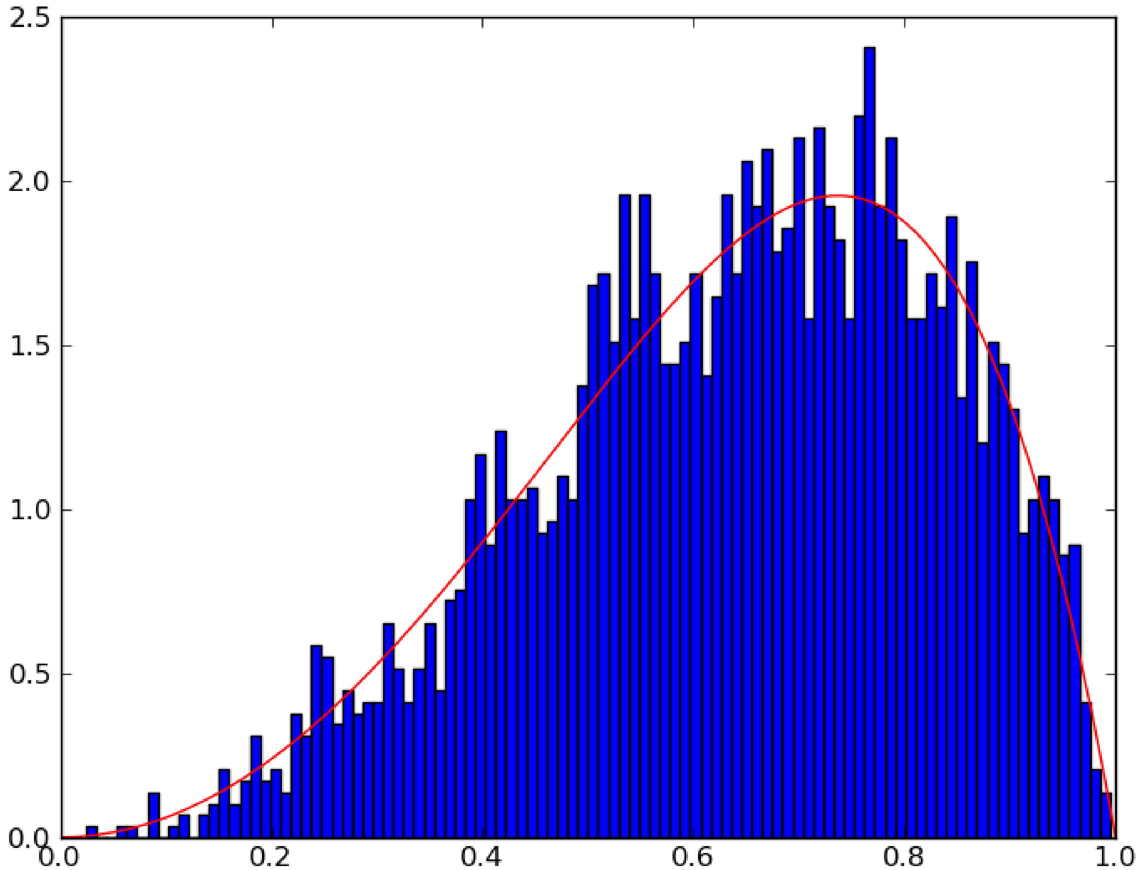

Figure 2: Plot of histogram and function of Y