STAT 598Z: Homework 4
Due: 20th March 2012

1. This homework will contribute 10 points towards your final score.

2. Attempt as many problems as possible.

3. Only neatly handwritten solutions will be accepted. Alternatively you may use LaTeX to typeset your solutions.

4. Hand in your HW (including print outs of your source code) at the beginning of the class on 20th March 2012. Additionally source code (if any) should be emailed to `stat598z@gmail.com` **before** the assignments are submitted in the class. No late submissions will be accepted!

5. Program files should be named after the problem (e.g. solution to problem 1 should be problem1.py etc).

6. Remember to seed your random number generators!

Problem 1 (2 pt) Derive a scheme for drawing samples uniformly from the unit disk. Generate 3,000 samples using your scheme and plot them.

Problem 1a (1 pt) Let $(X_1, Y_1), \ldots, (X_{3000}, Y_{3000})$ denote samples generated from **Problem 1**, where X_i and Y_i denote the x-coordinate and y-coordinate of the i-th sample respectively. Define $Z_i = X_i / Y_i$. Plot the histogram of Z_i's.

Problem 1b (2 pt) Show that Z_i's follow a standard Cauchy distribution. **Hint:** Recall that the ratio of two normal distributions is Cauchy.

Problem 2 (5 pt) Let $X \sim Beta(1, \beta)$, and let $Y \sim X^{\frac{1}{\gamma}}$. Recall that the Beta distribution is given by:

$$Beta(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where $B(\cdot, \cdot)$ is the beta function.

- Derive the density function of Y.
- Derive the cumulative distribution function of Y.
- Generate 3,000 independent samples from the distribution of Y using $\gamma = 5, \beta = 5$, and plot their histogram.