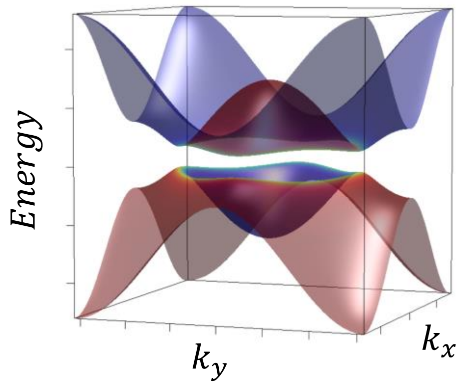
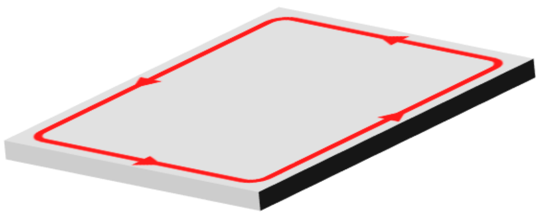
Symmetry-protected crossings

$H\psi = E\psi$

Color code: $\langle S_z \rangle$





Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H(\boldsymbol{k}) = \sum_i d_i(\boldsymbol{k})\sigma_i = d_x \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + d_y \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + d_z \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} d_z & d_x - id_y \\ d_x + id_y & -d_z \end{pmatrix}$$

$$d_x(\boldsymbol{k}) = A \sin k_x$$
$$d_y(\boldsymbol{k}) = A \sin k_y$$
$$d_z(\boldsymbol{k}) = M + 2B(2 - \cos k_x - \cos k_y)$$

$$E(\boldsymbol{k}) = \sum_i d_i{}^2$$

Inverted band structure for

$M/B > 0$

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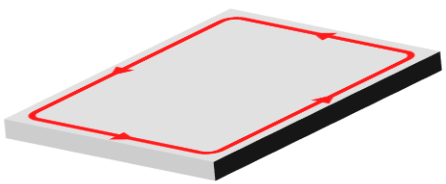
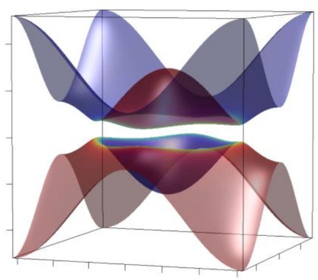
From 2D TI to 3D TI: Topological Crystalline Insulator (TCI)

2D Chern insulator





$$H(\boldsymbol{k}) = d_a(k)\sigma^a$$

Chern number (the TKNN invariant):

$$n = \frac{ie^2}{\hbar}\int \frac{d^2k}{(2\pi)^2} \sum_{n\,\epsilon\,occ} \left(\left\langle \frac{\partial u_n}{\partial k_x} \middle| \frac{\partial u_n}{\partial k_y} \right\rangle - \left\langle \frac{\partial u_n}{\partial k_y} \middle| \frac{\partial u_n}{\partial k_x} \right\rangle \right) =$$

$$= \frac{ie^2}{\hbar}\int \frac{d^2k}{(2\pi)^2} \sum_{m\neq n} \frac{\left\langle u_m \middle| \frac{\partial H(k)}{\partial k_x} \middle| u_n \right\rangle \left\langle u_n \middle| \frac{\partial H(k)}{\partial k_y} \middle| u_m \right\rangle - \left\langle u_m \middle| \frac{\partial H(k)}{\partial k_y} \middle| u_n \right\rangle \left\langle u_n \middle| \frac{\partial H(k)}{\partial k_x} \middle| u_m \right\rangle}{(\epsilon_n - \epsilon_m)^2}$$

3D TCI

Mirror symmetry can be expressed as an inversion and 180° rotation:

$$\mathcal{M} = PC_2$$

Mirror Chern number $n_{\mathcal{M}} = (n_{+i} - n_{-i})/2$