

22/23

**THE UNIVERSITY OF HONG KONG**  
**DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE**  
**STAT8003 Time Series Forecasting**

**Assignment 1**

(Due date: October 24, 2022)

1. Are the following time series  $\{Z_t\}$  weakly stationary?
  - (a)  $Z_t = 5 + 2t + X_t$ , where  $\{X_t\}$  is a zero-mean stationary series with ACVF  $\{\gamma_k\}$ .
  - (b)  $Z_t = a_t + (a_{t-1})^{\theta_t}$ , where  $\{a_t\}$  and  $\{\theta_t\}$  are two independently and identically distributed sequences,  $a_t$  follows the standard normal distribution and  $P(\theta_t = 1) = P(\theta_t = 2) = 0.5$ .
  - (c)  $Z_t = e^{a_t} + 2a_{t-1}$ , where  $\{a_t\}$  are independent and identically distributed with the standard normal distribution.
2. Let  $Z_t = e^{a_t} - 2e^{a_{t-1}}$ , where  $\{a_t\}$  is sequence of *i.i.d.* normal random variables with mean zero and variance one.
  - (a) Is  $\{Z_t\}$  stationary?
  - (b) Calculate the ACF  $\rho_k$  with  $k = 1$  if it is stationary.
3. Find the ACF of the following MA processes.
  - (a)  $Z_t = a_t - 0.5a_{t-1}$ .
  - (b)  $Z_t = a_t - a_{t-1} + 0.5a_{t-2}$ .
  - (c)  $Z_t = a_t + 0.5a_{t-1} - a_{t-2} + 3a_{t-3}$ .
4. Consider AR(2) model,  $Z_t = 0.9 + 0.2Z_{t-1} - 0.6Z_{t-2} + a_t$ , where  $\{a_t\} \sim WN(0, \sigma_a^2)$ .
  - (a) Is it stationary?
  - (b) Find out its MA representation if possible.
  - (c) Calculate the autocovariance function  $\gamma_k$  with  $k \geq 0$ .
  - (d) Find out the variance for the sample mean  $\bar{Z}_4 = (Z_1 + Z_2 + Z_3 + Z_4)/4$ .
5. Consider an ARMA(2,2) model,

$$Z_t = 0.5Z_{t-1} + 0.25Z_{t-2} + a_t - 1.5a_{t-1} + 0.75a_{t-2},$$

where  $\{a_t\}$  is the white noise with mean zero and variance  $\sigma_a^2$ .

- (a) Calculate the expectation  $E(Z_t)$ .
- (b) Is it stationary? Is it invertible?
- (c) Find out the MA and AR representations if they exist.

6. Consider a time series model

$$Z_t = 5 + 0.3Z_{t-1} + 0.5Z_{t-2} + e^{a_t},$$

where  $\{a_t\}$  is an *i.i.d.* sequence with the standard normal distribution.

- (a) Is it an AR model? Rewrite it in the standard form if your answer is positive.
- (b) Is the model stationary? Specify the reasons for your answers.
- (c) Find out the general linear process form if it exist.