The Final $\mathcal{EL}$-Completion Rules

$\top$-rule: Add $\top$ to any individual.

$\sqcap$-rule 1: If d has $C \sqcap D$ assigned, assign also C and D to d.

$\sqcap$-rule 2: If d has C and D assigned, assign also $C \sqcap D$ to d.

$\exists$-rule 1: If d has $\exists r.C$ assigned:

1. If there is an element e with initial concept $\underline{C}$ assigned, make e the r-successor of d.
2. Otherwise, add a new r-successor to d, and assign to it as initial concept $\underline{C}$.

$\exists$-rule 2: If d has an r-successor with C assigned, add $\exists r.C$ to d.

$\sqsubseteq$-rule: If d has C assigned and $C \sqsubseteq D \in \mathcal{T}$, then also assign D to d

The $\mathcal{EL}$-Completion Algorithm

Decide whether $\mathcal{O} \models C_0 \sqsubseteq D_0$

1. Start with initial element d_0, assign to $\underline{C_0}$ to it as initial concept
2. Set $\texttt{changed} := \textbf{true}$
3. While $\texttt{changed} = \textbf{true}$:
   - 3.1 Set $\texttt{changed} := \textbf{false}$
   - 3.2 For every element d in the current interpretation:
     - 3.2.1 Apply all the rules on d in all possible ways
       so that only concepts from the input get assigned
     - 3.2.2 If a new element was added or a new concept assigned,
       set $\texttt{changed} = \textbf{true}$
4. If D_0 was assigned to d_0, return `YES`, otherwise return `NO`

Concepts from the input: occur, possibly nested, explicitly in $\mathcal{O}$, C_0 or D_0

Example

$$\mathcal{O} = \mathcal{T} = \{ \quad B \sqsubseteq C, \qquad C \sqsubseteq \exists r.\exists t.B, \qquad A \sqcap \exists r.C \sqsubseteq \exists s.\exists t.B \quad \}$$

Example

$$\mathcal{O} = \mathcal{T} = \{ \quad B \sqsubseteq C, \qquad C \sqsubseteq \exists r.\exists t.B, \\ A \sqcap \exists r.C \sqsubseteq \exists s.\exists t.B \quad \}$$

We want to decide whether $\mathcal{O} \models A \sqcap \exists r.B \sqsubseteq \exists t.A$

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d_0

$\underline{A \sqcap \exists r.B}$

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d_0

$\underline{A \sqcap \exists r.B}$, A, $\exists r.B$

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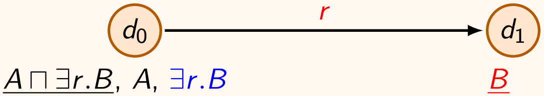

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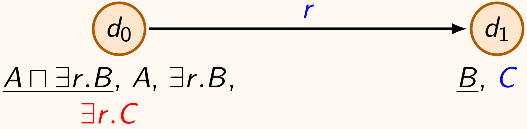

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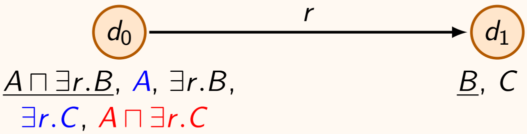

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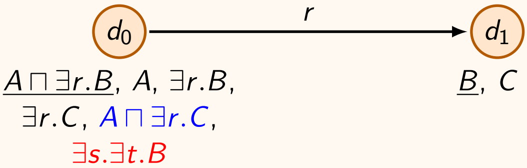

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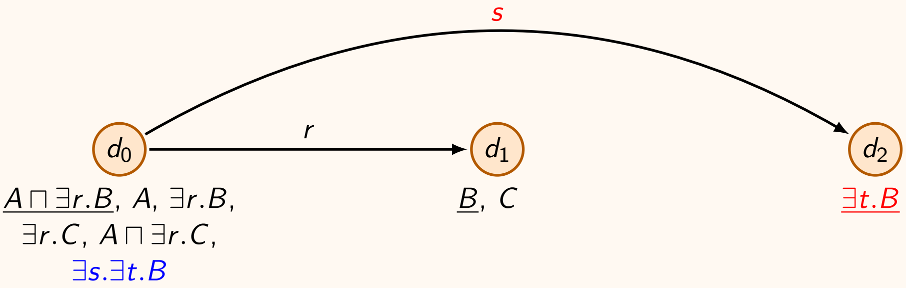

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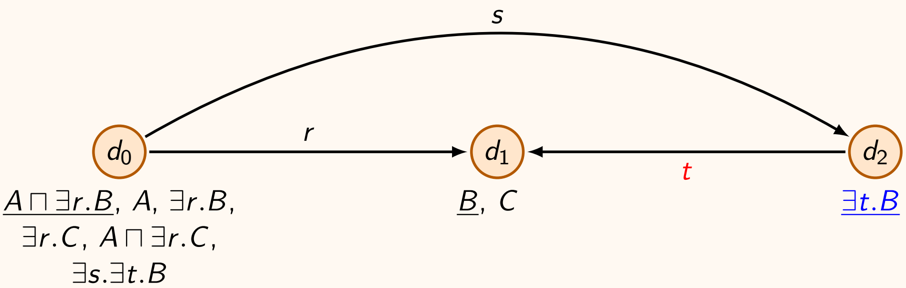

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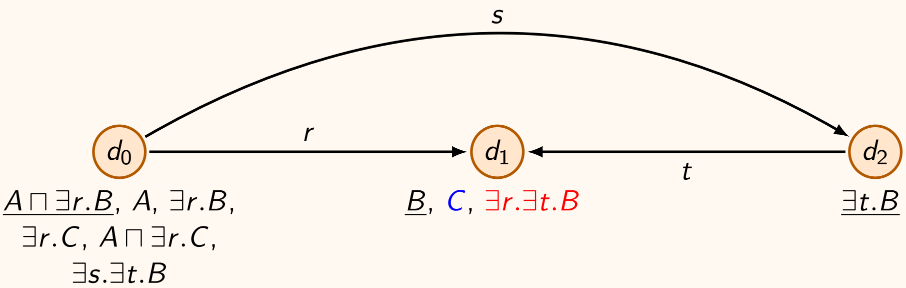

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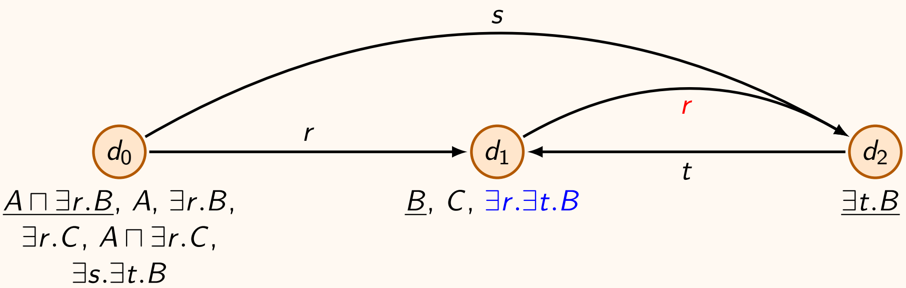

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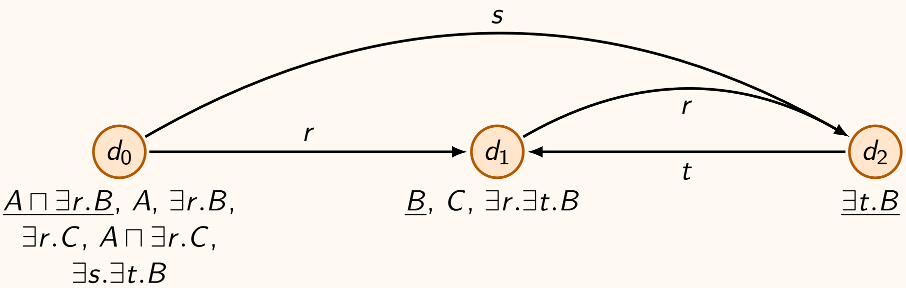

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We want to decide whether $\mathcal{O} \not\models A \sqcap \exists r.B \sqsubseteq \exists t.A$ NO