Multi-Agent Systems

Introduction to Reinforcement Learning

Part 4: Actor-Critic Algorithm

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Outline

Advantage Actor-Critic (A2C)

Actor-Critic combines valued-based and policy-based learning.

Actor-Critic algorithms therefore have two components that are learned jointly:

- Actor learns a parametrised policy
- Critic learns value function to evaluate state-action pairs;

Advantage function: Select action based on how it performs relative to other actions in that state:

$$a_\pi(s, a) := q_\pi(s, a) - v_\pi(s)$$

Quick Recap

Policy gradient along trajectory τ

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} R_t(\tau)\nabla_\theta \log \pi_\theta(a_t \mid s_t)\right]$$

Restricting the trajectory to the part starting at s_t:

$$R_t(\tau) = R(s_t, a_t, \ldots, s_T) \quad \Longrightarrow \quad \mathbb{E}_{\tau\sim\pi_\theta}[R_t(\tau)] = q_{\pi_\theta}(s_t, a_t);$$

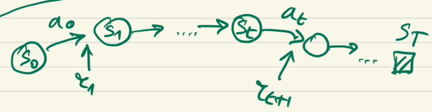
$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} q_{\pi_\theta}(s_t, a_t)\nabla_\theta \log \pi_\theta(a_t \mid s_t)\right]$$

Policy Gradient via **Monte Carlo Sampling**

$$\nabla_\theta J(\theta) = E_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} R_t(\tau)\, \nabla_\theta \log \pi_\theta(a_t|s_t)\right]$$

$\downarrow$ single sample = single trajectory τ

$$\nabla_\theta J(\theta) \simeq \sum_{t=0}^{T} R_t(\tau)\, \nabla_\theta \log \pi_\theta(a_t|s_t)$$



$$R_t(\tau) = r_{t+1} + r_{t+2} + \dots + r_T \;\hat{=}\; q_{\pi_\theta}(s_t, a_t)$$

generated

1-Sample.

$$s_t \xrightarrow{a_t} s_{t+1}$$

Policy gradient: Quick Recap

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} R_t(\tau)\nabla_\theta \log \pi_\theta(a_t \mid s_t)\right] \quad \text{(MC)}$$

$$= \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} q_{\pi_\theta}(s_t, a_t)\nabla_\theta \log \pi_\theta(a_t \mid s_t)\right] \quad \text{(value fct)}$$

For N sampled paths $\tau_i = \{s_{i,0}, a_{i,0}, s_{i,1}, r_{i,1}, \ldots\}$:

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=0}^{T} R_t(\tau_i)\nabla_\theta \log \pi_\theta(a_{i,t} \mid s_{i,t})\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=0}^{T} q_{\pi_\theta}(s_{i,t}, a_{i,t})\nabla_\theta \log \pi_\theta(a_{i,t} \mid s_{i,t})\right]$$

Actor-Critic Methods

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \underbrace{\pi_\theta(a_t \mid s_t)}_{ACTOR} \underbrace{q_{\pi_\theta}(s_t, a_t)}_{CRITIC}\right]$$

- **Critic** estimates the value function (could be action-value q or state-value v function).
- **Actor** updates the policy distribution in the direction suggested by the critic, i.e.:
  - Changes the policy to increase the likelihood of actions that get high values from the critic. the critic, more l

Gradient Policy Theorem: Interpretation

$$\frac{dJ}{d\theta} = E_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T} q_{\pi_\theta}(s_t,a_t)\,\frac{d}{d\theta}\log\pi_\theta(a_t|s_t)\right]$$

Interpretation:

* If $q_{\pi_\theta}(s_t,a_t)$ "high", changing θ such that a_t becomes more likely (i.e.: $(\log)\,\pi_\theta(a_t|s_t)\uparrow$) increases $J(\theta)$

* Hence:

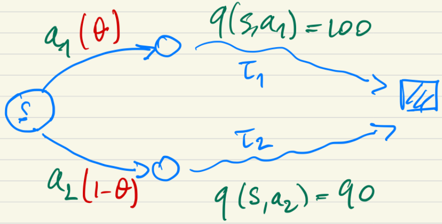
if $\frac{d}{d\theta}\log\pi_\theta(a_t|s_t) > 0 \Rightarrow \frac{dJ}{d\theta} > 0$

increase θ to increase $J(\theta)$

if $\frac{d}{d\theta}\log\pi_\theta(a_t|s_t) < 0 \Rightarrow \frac{dJ}{d\theta} < 0$

decrease θ to increase $J(\theta)$.

Gradient Policy Theorem: Interpretation



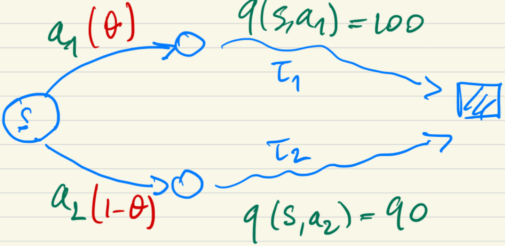
"Mixed message":

① $q(s,a_1)=100 \longrightarrow$ make a_1 more likely $\Rightarrow \theta\uparrow$

② $q(s,a_2)=90 \longrightarrow$ make a_2 more likely $\Rightarrow \theta\downarrow$

Gradient Policy Theorem: Interpretation

$$\frac{dJ}{d\theta} \approx \sum_{t=0}^{T} q_\theta(s_t, a_t) \frac{d}{d\theta}\left(\log \pi_\theta(a_t | s_t)\right)$$

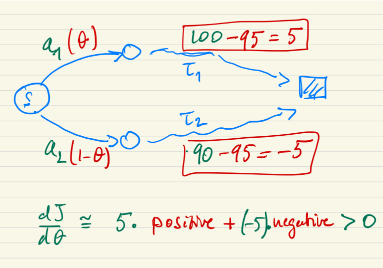


$$\frac{dJ}{d\theta} \cong 100 \cdot \text{positive} + 90 \cdot \text{negative} \approx 0$$

Gradient Policy Theorem: Introducing baselines

- "Raw" q-values don't give clear feed-back signal
- *Baseline*: some average wrt which q-values can be compared:
- Comparing q-values to baseline results in clearer feedback!
- E.g.: baseline for state s is 95 ($b(s) = 95$):
  - $q(s, a_1) - b(s) = 100 - 95 = 5 > 0$
  - $q(s, a_2) - b(s) = 90 - 95 = -5 < 0$
- Recall: increasing θ makes a_1 (a_2) more (less) likely; hence:

$$\frac{d}{d\theta} \log \pi_\theta(a_1 \mid s) > 0 \quad \text{(positive)} \qquad \frac{d}{d\theta} \log \pi_\theta(a_2 \mid s) < 0 \quad \text{(negative}$$



Introducing baselines: A2C

- Policy gradient theorem:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t \mid s_t) q(s_t, a_t)\right]$$

- Problem: value of $q(s, a)$ is not very informative;
- We need a reference point or baseline: natural choice $= v(s)$
- Advantage: Relative value of an action as compared to other actions in that state:

$$A(s, a) := q(s, a) - v(s)$$

- **Advantage actor-critic (A2C)**

$$\nabla_\theta J(\theta) \propto \mathbb{E}_{\tau}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t \mid s_t)(q_{\pi_\theta}(s_t, a_t) - v_{\pi_\theta}(s_t))\right]$$

Estimating Advantage

- Typically two neural networks to estimate
  1. policy $\to \pi_\theta$
  2. value functions and advantage: $\to v_w, q_w$
  3. Network weights: θ and w
- Computational strategy:
  - Estimate $v(s)$
  - Estimate $q(s, a)$ using Bellman eqs:

$$q(s_t, a_t) = \mathbb{E}\left[r_{t+1} + \gamma v(s_{t+1})\right]$$

- Along sampled trajectory:

$$\hat{q}(s_t, a_t) = r_{t+1} + \gamma \hat{v}(s_{t+1})$$

$$\hat{A}(s_t, a_t) = r_{t+1} + \gamma \hat{v}(s_{t+1}) - \hat{v}(s_t)$$

Estimating Advantage (2)

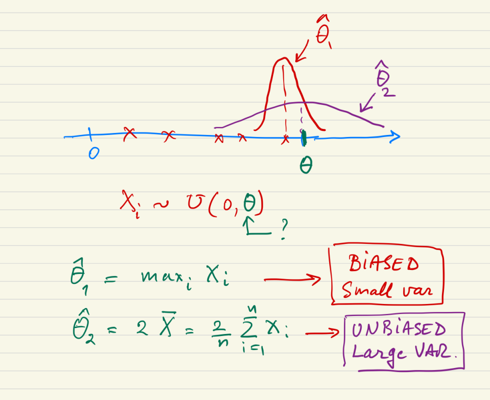
- *n*-step returns along sampled trajectory:

$$\hat{q}(s_t, a_t) = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t_3} + \ldots + \gamma^{n-1} r_{t+n} + \gamma^n \hat{v}(s_{t+n})$$

- Combination of biased and unbiased estimate:
  - Actual returns: unbiased but high variance
    (sample paths can be very different!)
  - Bias due to inclusion of estimate $\hat{v}$, but lower variance
    (average over all actions);

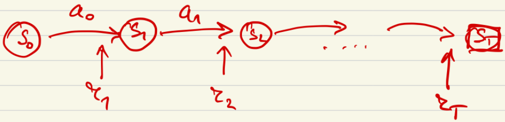
Mathematical aside (1): Bias vs. Variance

- Consider sample $X_1, X_2, \ldots, X_n \sim U(0, \theta)$ where θ is unknown and needs to be estimated;
- There are two natural estimators $\hat{\theta}$ for θ:
  1. $\hat{\theta}_1 = \max_i X_i$: biased but low variance
  2. $\hat{\theta}_2 = 2\overline{X} = 2/n \sum_i X_i$: unbiased but high variance



Mathematical aside (2a): Discount factor

Discount factor. $\boxed{\gamma = \text{Prob of Continuing}}$



$$ET = \sum_{k=1}^{\infty} k \, \mathbb{P}(T=k)$$

$$= \sum_{k=1}^{\infty} k \cdot \gamma^k (1-\gamma)$$

$$= (1-\gamma)\gamma \underbrace{\sum_{k=1}^{\infty} k \gamma^{k-1}}$$

Mathematical aside (2b): Discount factor

$$\sum_{k=1}^{\infty} k\gamma^{k-1} = \frac{d}{d\gamma}\left(\sum_{k=1}^{\infty}\gamma^k\right)$$

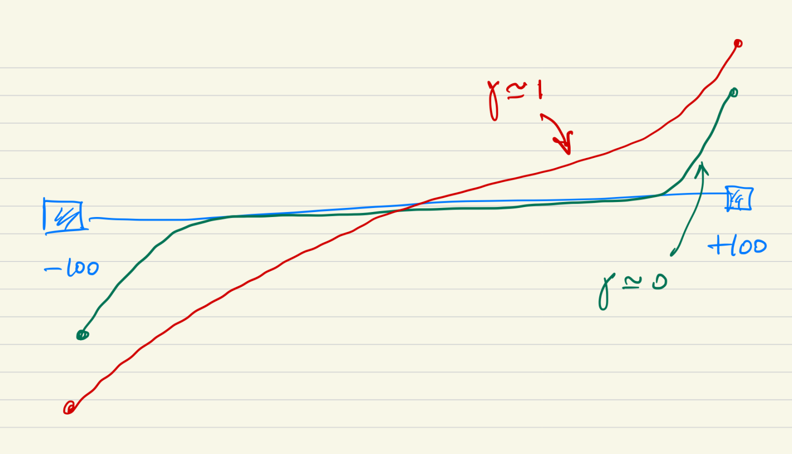
$$= \frac{d}{d\gamma}\left(\frac{\gamma}{1-\gamma}\right)$$

$$= \frac{(1-\gamma)-\gamma(-1)}{(1-\gamma)^2} = \frac{1}{(1-\gamma)^2}$$

$$\boxed{ET = (1-\gamma)\gamma\frac{1}{(1-\gamma)^2} = \frac{\gamma}{1-\gamma}}$$

Eg:
$\gamma = 0.9$
$ET = 9.$

Mathematical aside (2c): Discount factor



Further reading

https://lilianweng.github.io/lil-log/2018/04/08/policy-gradient-algorithms.html