Knowledge Representation

Lecture 9: Decision Problems on AFs and labelling-based Semantics

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Flashback: The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

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Example

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

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$\langle \{w, w \rightarrow \neg s\}, \neg s \rangle$

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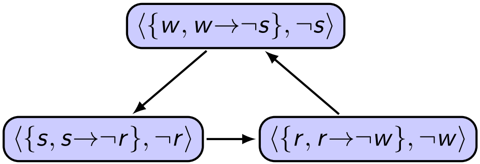

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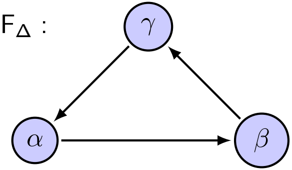
AF_Δ :



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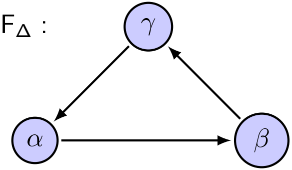

$$\mathit{pref}(\mathrm{AF}_\Delta) = \{\emptyset\}$$
$$\mathit{stage}(\mathrm{AF}_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$$

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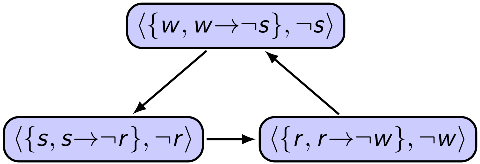
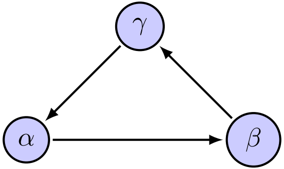


$$Cn_{pref}(\mathrm{AF}_\Delta) = Cn(\top)$$
$$Cn_{stage}(\mathrm{AF}_\Delta) = Cn(\neg r \vee \neg w \vee \neg s)$$

Dung's Abstract Argumentation Frameworks

Seminal Paper by *Phan Minh Dung*:
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.
Artif. Intell., 77(2):321–358, 1995.

Example



Remark

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)

Applications of Formalisms of Argumentation

Abstraction allows to compare several KR formalisms on a conceptual level (*calculus of conflict*)

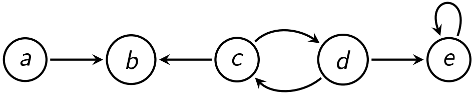
- Legal reasoning:
  - [Bench-Capon and Dunne, 2005]
  - [Collenette et al., 2020]
- Multi-agent systems
  - [McBurney et al., 2012]
  - [Amgoud et al., 2007]
- Discussion game
  - [Caminada, 2018]
  - [Keshavarzi Zafarghandi et al., 2020]
- Recommended system [Rago et al., 2018]
- Explainable AI
  - [Cocarascu et al., 2019]
  - Argumentative XAI: A Survey [Cyras et al., 2021]
  - [Chi and Liao, 2022]

Flashback: Dung's Abstract Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")



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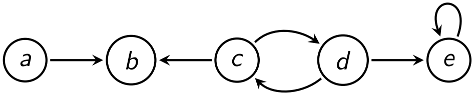


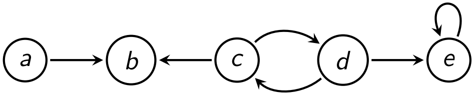
How can we assess the credibility of an argument in an AF?

An argument is believable if it can be argued successfully against the counterarguments.

- Semantics: Methods used to clarify the acceptance of arguments
  - Extension-based semantics
  - Labelling-based semantics

Flashback: Semantics of AFs (ctd.)

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$



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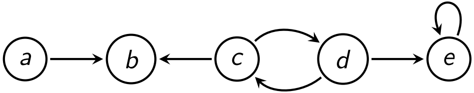

$$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, d\}\}$$

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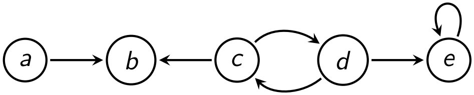

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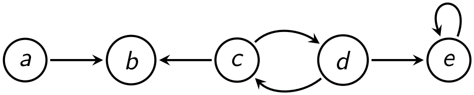

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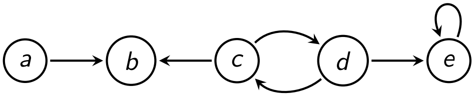

- $S_1 = \{\}$
- $S_2 = \{a\}$
- $S_3 = \{c\}$
- $S_4 = \{b\}$

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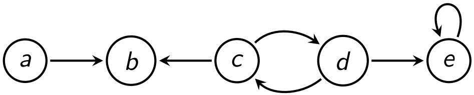

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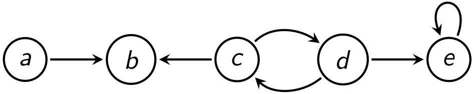

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$$
\begin{tikzcd}
a \arrow[r] & b & c \arrow[l] \arrow[r, bend left] & d \arrow[l, bend left] \arrow[r] & e \arrow[loop above]
\end{tikzcd}
$$

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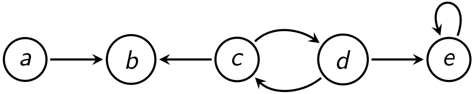

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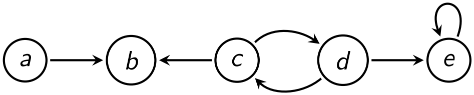

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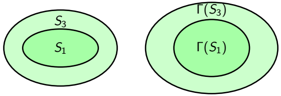
Γ_F is a monotonic function



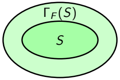
Flashback: Semantics of AFs

- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$



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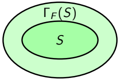

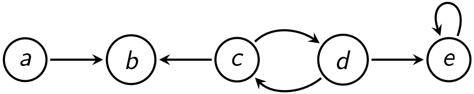

$$adm(F) = \{\{\}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \cancel{\{b, d\}}\}$$

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  That is, for each $T \subseteq A$ admissible in F, $S \not\subset T$.

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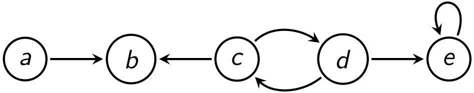

$pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

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- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

$\Gamma_F(S) = S \qquad \Gamma_F(T) = T \qquad \Rightarrow \qquad S \subseteq T$

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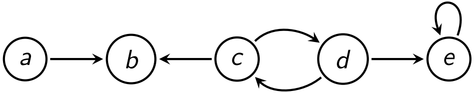

$grd(F) = \{\{a\}\}$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

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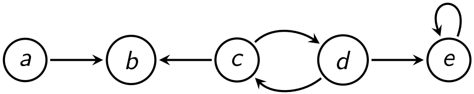

- What are the complete extensions for F?

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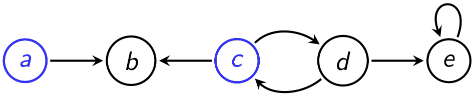

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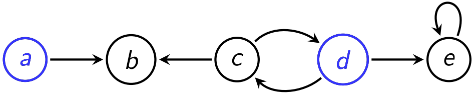

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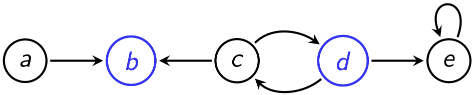

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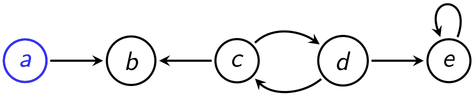

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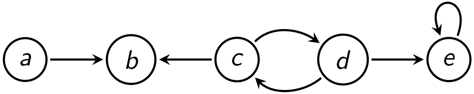

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Characterize of Semantics (ctd.)

Properties of the Extensions

Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
- F has at least one complete extension.

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Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
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Remark

Since there exists exactly one grounded extension for each AF F, we often write $grd(F) = S$ instead of $grd(F) = \{S\}$.

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.

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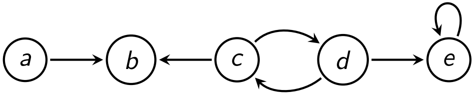

Stable Semantics

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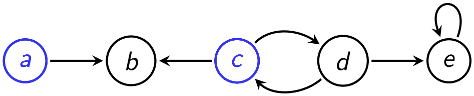

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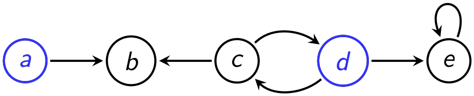

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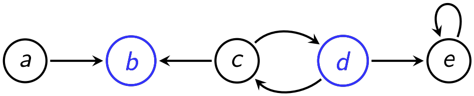

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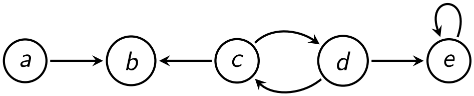


$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics (ctd.)

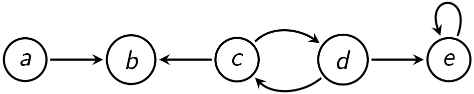
Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

- Stable semantics reflect the ‘zero-and-one’ character of classical logic in argumentation frameworks.
- An AF may not have any stable extension.

Relation between the Semantics of AFs



- $cf(F) = \{\{a,c\},\{a,d\},\{b,d\},\{a\},\{b\},\{c\},\{d\},\{\}\}$
- $adm(F) = \{\{a,c\},\{a,d\},\cancel{\{b,d\}},\{a\},\cancel{\{b\}},\{c\},\{d\},\{\}\}$
- $pref(F) = \{\{a,c\},\{a,d\},\cancel{\{a\},\{c\},\{d\},\emptyset}\}$
- $stb(F) = \{\cancel{\{a,c\}},\{a,d\},\cancel{\{b,d\}},\cancel{\{a\},\{c\},\{d\},\emptyset}\}$
- $comp(F) = \{\{a,c\},\{a,d\},\cancel{\{b,d\}},\{a\},\cancel{\{c\},\{d\},\emptyset}\}$
- $grd(F) = \{\{a\}\}$

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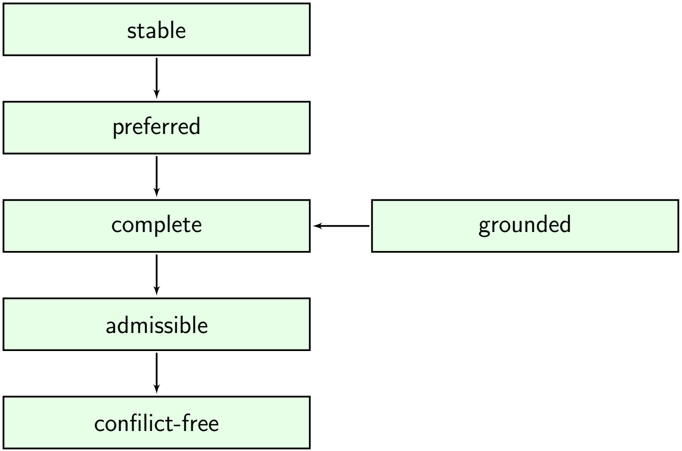

Figure: An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Well-Founded AF

Definition

Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

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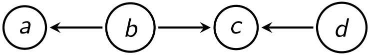
- $S \in adm(F)$ if $S \subseteq \Gamma_F(S)$

Example



- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$

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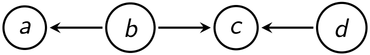
- $S \in comp(F)$ if $S = \Gamma_F(S)$

Example



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Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in grd(F)$ if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

Example

$$a \leftarrow b \rightarrow c \leftarrow d$$

- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$
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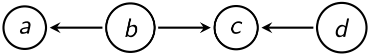
Example



- $\mathit{adm}(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$
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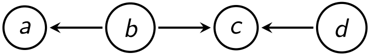
Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in stb(F)$ if $\forall a \in A$: $\exists b \in S$ s.t. $(b, a) \in R$

Example



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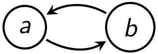
Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?



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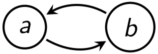

$cf(F) = \{\{\}, \{a\}, \{b\}\}$

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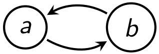

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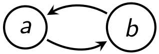

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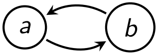

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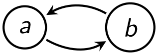

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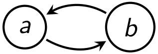

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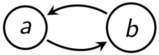

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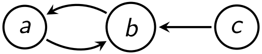
Semantics of AFs (ctd.)

Is the existence of a loop always problematic?



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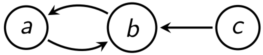

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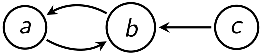

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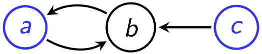

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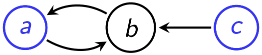

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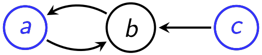

$grd(F) = \{\{a, c\}\}$

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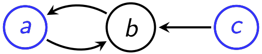
- *complete* ($S \in comp(F)$) if $S = \Gamma_F(S)$



$comp(F) = \{\{a, c\}\}$

Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

$$\begin{tikzcd} a \arrow[r, bend right] & b \arrow[l, bend right] & c \arrow[l] \end{tikzcd}$$

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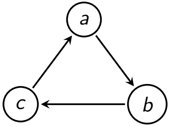
Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?



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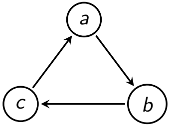

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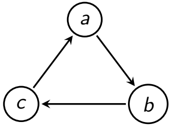

$\mathit{adm}(F) = \{\{\}\}$

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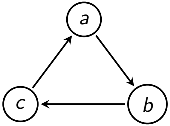

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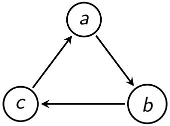

$stb(F) = \{\}$ F does not have any stable extension

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- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
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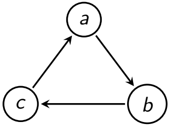
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

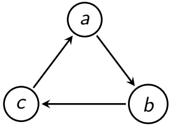


$grd(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

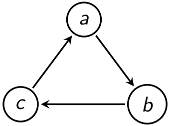
- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* $(S \in adm(F))$ if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* $(S \in pref(F))$ if S is $\subseteq$-maximal admissible
- *stable* $(S \in stb(F))$ if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* $(S \in grd(F))$ if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* $(S \in comp(F))$ if $S = \Gamma_F(S)$



$comp(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?



- $cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}\}$
- $adm(F) = \{\{\}\}$
- $pref(F) = \{\{\}\}$
- $stb(F) = \{\}$ $\quad$ F does not have any stable extension
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}\}$

Decision problems on AFs

- Existence of extensions
- Credulous acceptance
- Skeptical acceptance
- Verifying an extension

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problem in AFs

Existence of Extensions

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Answer to the existance decision problem:

- Recall: Any AF has at least one *admissible/preferred/grounded/complete/conflict-free* extension.

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- Recall: Any AF has at least one *admissible/preferred/grounded/complete/conflict-free* extension.
- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, grd, comp, cf\}$, is trivially *yes*.

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- $Exists_{stb}(F)$?

Decision Problem in AFs

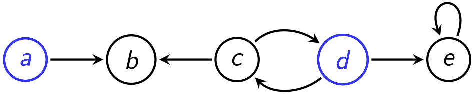
Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- $Exists_{stb}(F)$?



$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}\}$, $\quad Exists_{stb}(F)$: Yes

Decision Problem in AFs

Existence of Extensions

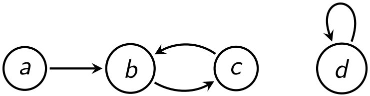
Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

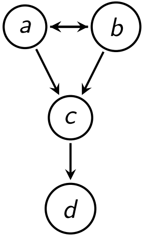
- $Exists_{stb}(F)$?

Example



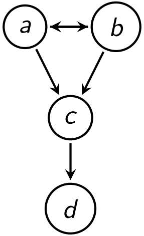
$stb(F) = \{\}, \quad Exists_{stb}(F)$: No

Decision Problem in AFs (ctd.)



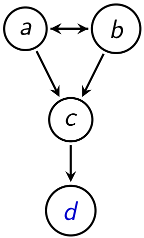
- $\mathit{Exists}_{\sigma}(F)$, for $\sigma \in \{\mathit{adm}, \mathit{pref}, \mathit{stb}, \mathit{grd}, \mathit{comp}, \mathit{cf}\}$ is yes.

Decision Problem in AFs (ctd.)



- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$

Decision Problem in AFs (ctd.)



- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
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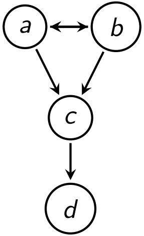
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap pref(F) = \{d\}$, but $d \notin grd(F)$

Decision Problem in AFs (ctd.)



- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
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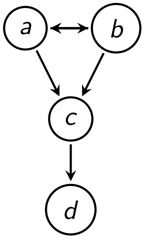
Decision Problem in AFs (ctd.)



- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap pref(F) = \{d\}$, but $d \notin grd(F)$

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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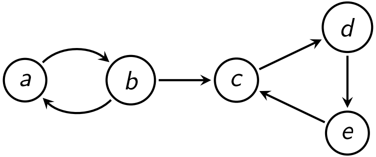


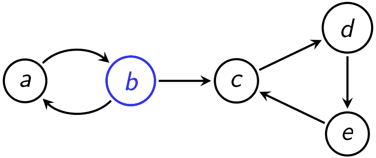
- $Cred_{cf}(b, F)$: is b contained in at least one conflict-free set of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_{cf}(a, F) = \begin{cases} \text{yes} & \text{if } (a, a) \notin R, \\ \text{no} & \text{otherwise} \end{cases}$$



$cf(F) = \{\{b\},\quad Cred_{cf}(b, F)$: Yes

- $Cred_{cf}(b, F)$: is b contained in at least one conflict-free set of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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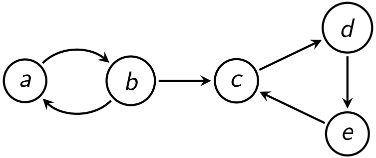


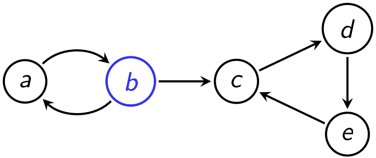
- $Cred_{adm}(b, F)$: is b contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



$adm(F) = \{\{b\}, \quad Cred_{adm}(b, F)$: Yes

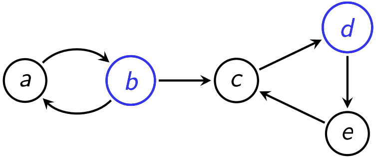
- $Cred_{adm}(b, F)$: is b contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \ a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



$pref(F) = \{\{b, d\}, \quad Cred_{pref}(b, F)$: Yes

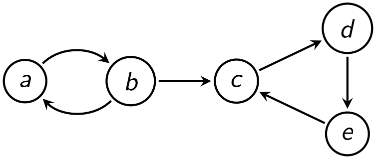
- $Cred_{pref}(b, F)$: is b contained in at least one *pref*-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

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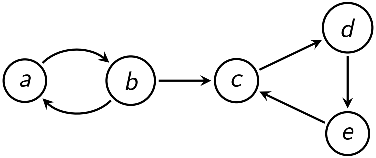

- $Cred_{adm}(c, F)$: is c contained in at least one *adm*-extension of F? No. c in not defended against the attack from e.

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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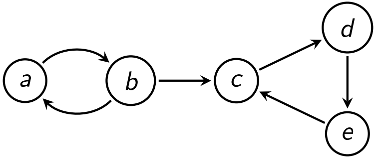


- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F? No. c in not defended against the attack from e.
- $Cred_{adm}(c, F) = Cred_{pref}(c, F) = Cred_{stb}(c, F) = Cred_{comp}(c, F) = Cred_{grd}(c, F)$: No

Decision Problems on AFs (ctd.)

Characterize of Credulous Acceptance

Given an AF $F = (A, R)$:

- $Cred_{cf}(a, F)$: Check if $(a, a) \in R$
- $Cred_{adm}(a, F) = Cred_{pref}(a, F) = Cred_{comp}(a, F)$
- $Cred_{grd}(a, F)$: Evaluate the grounded extension of F
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
- $Cred_{stb}(a, F)$: Evaluate the set of stable extensions of F

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

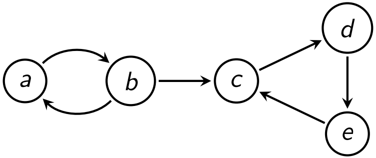
$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & \quad a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



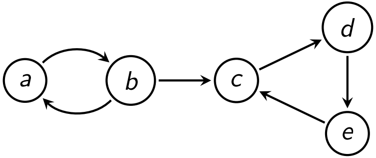
- $Skept_{pref}(b, F)$: is b contained in every $pref$-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



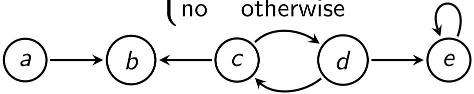
$pref(F) = \{\{a\}, \{b, d\}\}$, $\quad Skept_{pref}(b, F)$: No

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



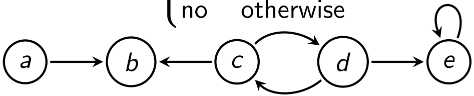
- $Skept_{pref}(a, F)$: is a contained in every *pref*-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$

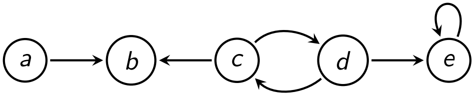


$pref(F) = \{\{a, c\}, \{a, d\}\}, \quad Skept_{pref}(a, F)$: yes

- $Skept_{pref}(a, F)$: is a contained in every $pref$-extension of F?

Decision Problems on AFs (ctd.)

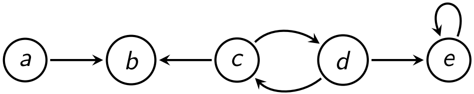
Skeptical Decision Problems under conflict-free



- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F?

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free



$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\{a\}, \{b\}, \{c\}, \{d\}, \{\}\}$ $\qquad$ $Skept_{cf}(a, F)$: no.

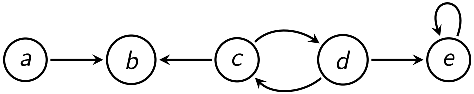
- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free

- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Skeptical Decision Problems under Admissible Semantics



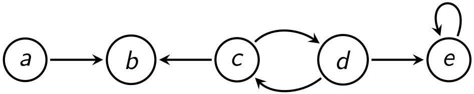
- $Skept_{adm}(a, F)$: is a contained in every *adm*-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free

- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Skeptical Decision Problems under Admissible Semantics

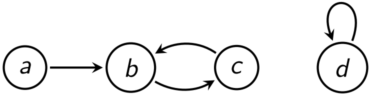


$adm(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \{\}\}$ $Skept_{adm}(a, F)$: no.

- $Skept_{adm}(a, F)$: is a contained in every adm-extension of F? No

Decision Problems on AFs (ctd.)

Recall: $S \subseteq A$ is an stable extension if $S \in cf(F)$ and for each $a \in A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.



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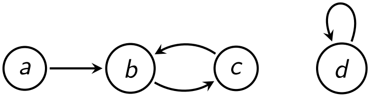
What is the answer to $Skept_{stb}(a, F)$? Why?

- $Skept_{stb}(a, F)$: yes. F has a stable extension and a is in every stable extension of F.
- $Skept_{stb}(a, F)$: no. Since F does not have any stable extension.
- $Skept_{stb}(a, F)$: yes. F does not have any stable extension. If no extension exists then all arguments are skeptically accepted.

Decision Problems on AFs (ctd.)

Recall: $S \subseteq A$ is an stable extension if $S \in cf(F)$ and for each $a \in A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.



$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, c\}\}$

Decision Problems on AFs (ctd.)

Recall: $S \subseteq A$ is an stable extension if $S \in cf(F)$ and for each $a \in A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.

$$\begin{tikzcd} a \arrow[r] & b \arrow[r, bend right] & c \arrow[l, bend right] & d \arrow[loop above] \end{tikzcd}$$

$stb(F) = \{\}$, $\qquad$ $Skept_{stb}(a, F)$: yes

Decision Problems on AFs(ctd.)

Characterize of Skeptical Acceptance

- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{cf}(a, F)$: Trivially, No.
- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{adm}(a, F)$: Trivially, No.
- If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted.[1]
- $Skept_{grd}(F) = Cred_{grd}(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
  - There exists an AF F and argument a such that $Skept_{pref}(a, F)$: Yes. However, $Skept_{grd}(a, F)$: No.

[1] This is only relevant for stable semantics.

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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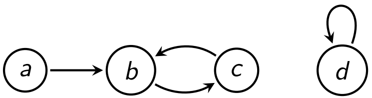

- Let $S = \{a, c\}$. $Ver_{adm}(S, F)$?

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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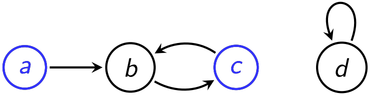

$adm(F) = \{\{a, c\},\quad Ver_{adm}(S, F)$? Yes

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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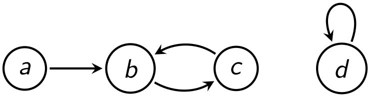


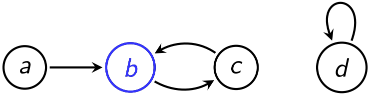
- Let $S = \{b\}$. $Ver_{adm}(S, F)$?

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



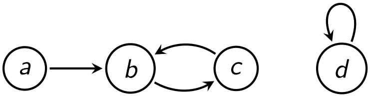
$Ver_{adm}(\{b\}, F)$: No. b is not defended

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



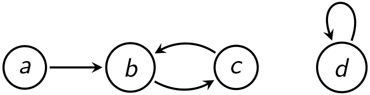
- Let $S = \{\}$. $Ver_{adm}(S, F)$?

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



$adm(F) = \{\{\},\quad Ver_{adm}(S, F)$? Trivially, yes.

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension? NO
- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $\mathit{Exists}_{\sigma}(F)$: Does F has a σ-extension? NO
- $\mathit{Cred}_{\sigma}(a, F)$: is a contained in at least one σ-extension of F? NO
- $\mathit{Skept}_{\sigma}(a)$: is a contained in every σ-extension of F?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension? NO
- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F? NO
- $Skept_\sigma(a)$: is a contained in every σ-extension of F? If it is not trivial, yes
- $Ver_\sigma(S, F)$: is S σ-extension of F?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

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- $Skept_\sigma(a)$: is a contained in every σ-extension of F? If it is not trivial, yes
- $Ver_\sigma(S, F)$: is S σ-extension of F? NO

Complexity Results

Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ
cf	in L	trivial	in L
adm	NP-c	trivial	in L
pref	NP-c	Π_2-c	co-NP-c
comp	NP-c	P-c	in P
grd	P-c	P-c	P-c
stb	NP-c	co-NP-c	in P

Table: Complexity of reasoning with AFs.

Methods and Systems

- For an overview, see:

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  Methods for Solving Reasoning Problems in Abstract Argumentation – A Survey. *Artificial Intelligence* 220: 28–63, 2015.

- Competition for Abstract Argumentation Solvers (ICCMA):

  `http://argumentationcompetition.org`

- ASPARTIX Web Front-End:

  `http://rull.dbai.tuwien.ac.at:8080/ASPARTIX`

- CONARG Web Front-End:

  `http://www.dmi.unipg.it/conarg/`

Summery

We have seen

- Abstract Argumentation Frameworks
- Conflict-free sets
- Admissible semantics
- Preferred semantics
- Complete semantics
- Grounded semantics
- Stable semantics

Next

- Labelling-based semantics
- Discussion games

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In IJCAI, pages 1949–1955. ijcai.org.