Knowledge Representation

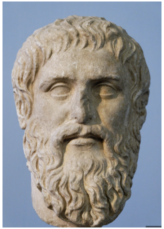
Lecture 8: Abstract Argumentation
Introduction to Formal Argumentation
*slides adapted from Stefan Woltran's lecture on Abstract Argumentation

Atefeh Keshavarzi

20, November 2023

Outline

Argumentation in History



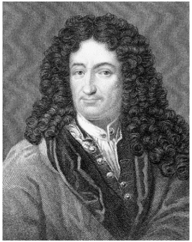
Plato's Dialectic

The dialectical method is discourse between two or more people holding different points of view about a subject, who wish to establish the truth of the matter guided by reasoned arguments.

Argumentation in History

Leibniz's Dream

The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right.



- Developing automated methods: is an old, ambitious, and ongoing research goal
- One could say that Leibniz was thinking about a machine
    1. arguing as a human
    2. reasoning automatically and finding a correct conclusion, in the presence of conflicts among arguments

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  - Purpose: Combining human and machine reasoning.
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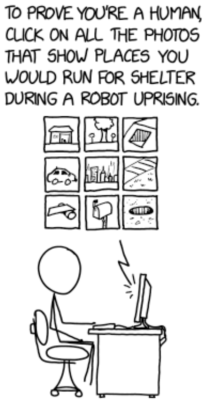

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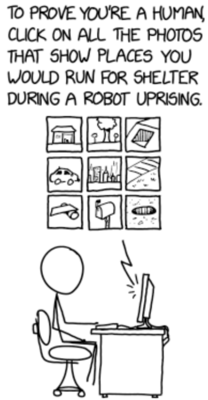
How should the process of arguing occur among automated systems?
Solid formalisms are required for modeling and evaluating argumentation



Example: Human-Human Argumentation

- a: Menzis is the best insurance

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- a: Menzis is the best insurance

- b: Univé is cheaper per month

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- *a*: Menzis is the best insurance
- *c*: You have to pay Univé from 1th Feb. You arrived 1th March. Right?!

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$$a \leftarrow b \leftarrow c$$

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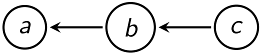

- The one who has the last word laughs best

Overview on Abstract Argumentation Frameworks

Formal Models of Argumentation are concerned with

1. representation of an argument
2. representation of the relationship between arguments
3. solving conflicts between the arguments (*acceptability*)

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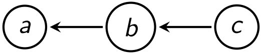

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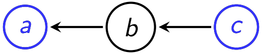

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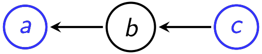

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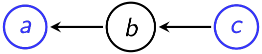

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What conclusions can be draw? Menzis is the best option for them.

The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
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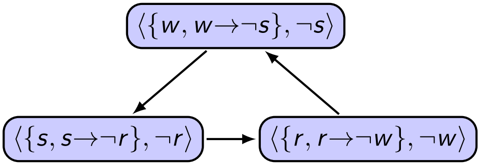

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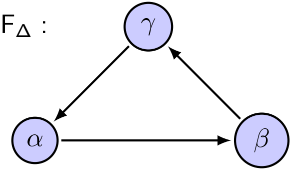

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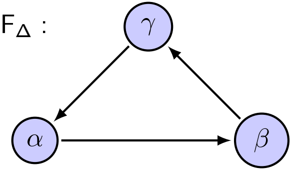

$$\mathit{pref}(\mathrm{AF}_\Delta) = \{\emptyset\}$$
$$\mathit{stage}(\mathrm{AF}_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$$

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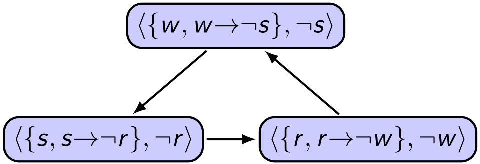

$$Cn_{pref}(\mathrm{AF}_\Delta) = Cn(\top)$$
$$Cn_{stage}(\mathrm{AF}_\Delta) = Cn(\neg r \lor \neg w \lor \neg s)$$

Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) Δ
- argument is a pair (Φ, α), such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
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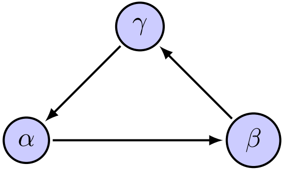
Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.

Dung's Abstract Argumentation Frameworks

Example



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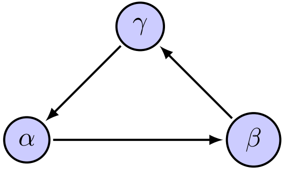

Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
  - "plethora of semantics"

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Seminal Paper by *Phan Minh Dung*:
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- Based on Google Scholar there are more than 5000 citations to [Dung, 1995]
- AFs have become a base for formal and computational argumentation [Baroni et al., 2020]
- AFs capture the essence of different non-monotonic formalisms, such as, Reiter's default logic [Reiter, 1980].
- Special issue of *Argument and Computation*, Vol. 11(1–2), 2020, dedicated to celebrate the 25 years anniversary

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An argumentation framework (AF) is a pair (A, R) where

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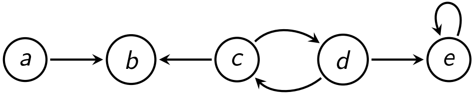

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- Semantics: Methods used to clarify the acceptance of arguments
  - Extension-based semantics
  - Labelling-based semantics

How Can We Deal with Conflict in a Loop?

Example

- a: Let's go to Norway for Christmas holiday to see the northern lights.
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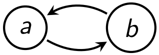

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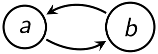

- We do not accept arguments that have conflicts, do we?

Basic Properties

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Given $F = (A, R)$.

A set $S \subseteq A$ is *conflict-free* if there is no attack/conflict within S. A set $S \subseteq A$ is *conflict-free* $(S \in cf(F))$ if, for each $a, b \in S$, $(a, b) \notin R$

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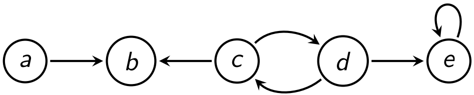

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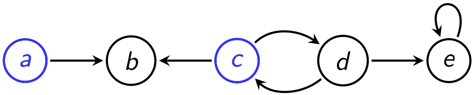

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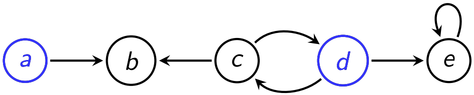

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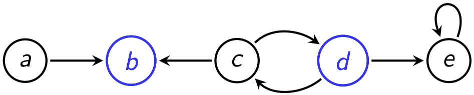

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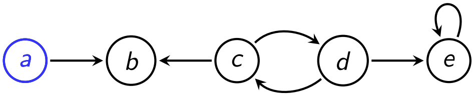

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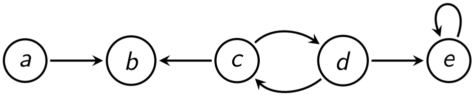

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Acceptability within a Set

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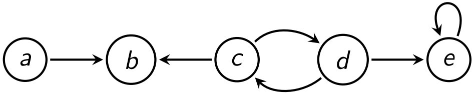

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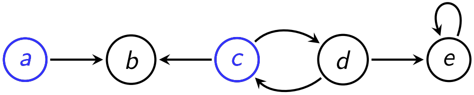

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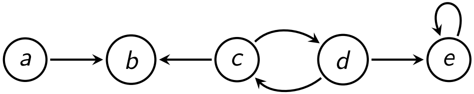

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- Is b acceptable w.r.t. any set? Is d defended by any set?

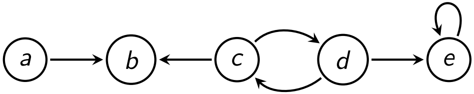
Acceptability within a Set

Definition

Given an AF $F = (A, R)$. Let $S \subseteq A$.
An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



- Is c acceptable w.r.t. any set? Is c defended by any set?
  - c is acceptable w.r.t. (defended by) $\{c\}, \{a, c\}$
- Is b acceptable w.r.t. any set? Is d defended by any set?
  - No, because $(a, b) \in R$ and a is not attacked.

Characteristic Operator

Given $F = (A, R)$

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.

Definition

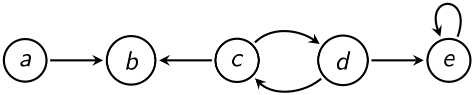
Characteristic operator $\Gamma_F(S)$ is a function that take set $S \subseteq A$ and returns the set of all arguments that are defended by S.

$$\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$$

Characteristic Operator (ctd.)

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



What are the outputs of $\Gamma_F(S)$ for any of the following sets?
Recall: $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$. $a \in A$ is defended by S , if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, s.t. $(c, b) \in R$.

- $S = \{\}$
- $S = \{a\}$
- $S = \{c\}$
- $S = \{a, b\}$

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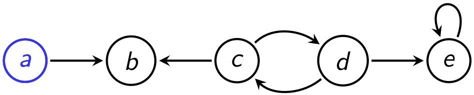
Characteristic Operator (ctd.)

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



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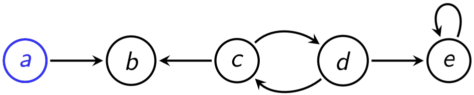

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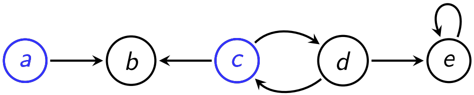

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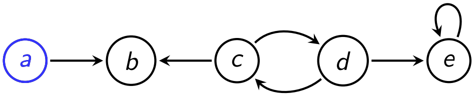

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Properties of the Characteristic Operator

Let F be a function, and let S and S' be inputs of F :

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proof

$$\begin{aligned}
\Gamma_F(S') &= \{a \in A \mid a \text{ is defended by } S' \text{ in } F\} \\
&= \{a \in A \mid a \text{ is defended by } S \cup (S' \setminus S) \text{ in } F\} \\
&= \{a \in A \mid a \text{ is defended by } S \text{ in } F\} \cup \{a \in A \mid a \text{ is defended by } (S' \setminus S) \text{ in } F\} \\
&= \Gamma_F(S) \cup \{a \in A \mid a \text{ is defended by } S' \setminus S \text{ in } F\}
\end{aligned}$$

Admissible Semantics

Definition

Given an AF $F = (A, R)$. A set S is *admissible* ($S \in adm(F)$) in F, if

- S is conflict-free in F
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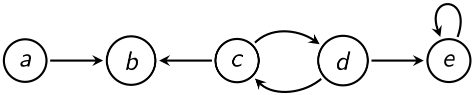

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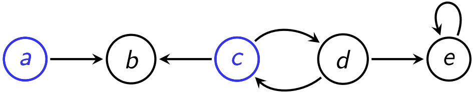

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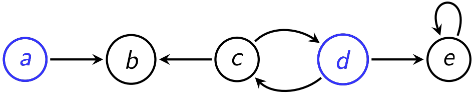

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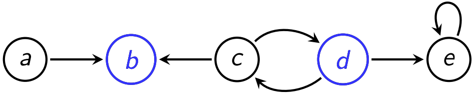

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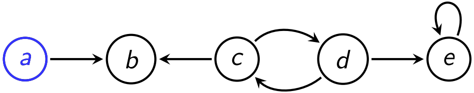

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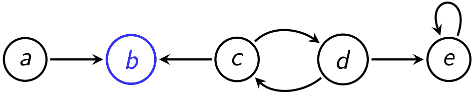

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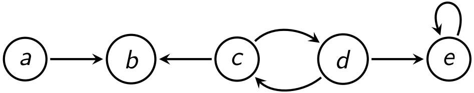

$adm(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{\}\}$

Properties of Admissible Extensions

Theorem

Every AF has at least one admissible extension.

Proof

In any AF empty set is an admissible extension.

Properties of Admissible Extensions

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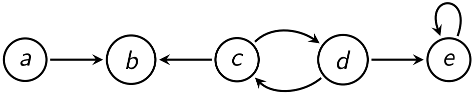

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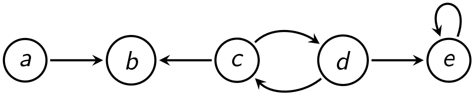

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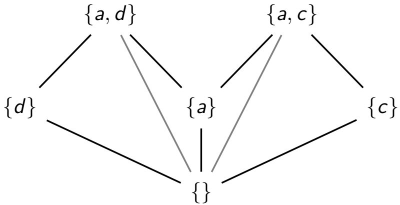

The relation between admissible extensions of F with respect to the subset relation.

Preferred Semantics

Definition

Given an AF $F = (A, R)$. A set S is a *preferred extension* ($S \in \mathit{pref}(F)$) in F if

- S is $\subseteq$-maximal admissible in F, that is, for each $T \subseteq A$ admissible in F, $S \not\subset T$. .

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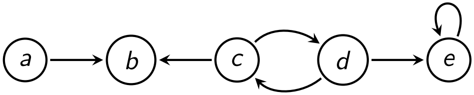

- What are the preferred extensions for F?

Preferred Semantics

Definition

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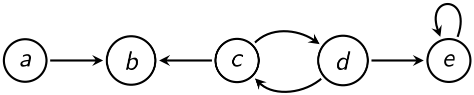

$pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics of AFs

Theorem

Any AF has at least a preferred extension.

Proof

Let F be an AF. Every AF has at least an admissible extension. For each admissible set S of AF, there exists a preferred extension E of AF such that $S \subseteq E$.

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Properties of Semantics of AFs

- Any admissible extension is a conflict-free set.
- In any AF empty set is an admissible extension.
- Every AF has at least one admissible extension.
- Every AF has at least one preferred extension.
- ...

Properties of the Characteristic Operator (ctd.)

Flashback

Given an AF $F = (A, R)$, and $S \subseteq A$. The characteristic operator is $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$.

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Definition

S is termed the least fixed point of Γ_F if:

- $S = \Gamma_F(S)$,
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Theorem

Given an AF $F = (A, R)$. Γ_F has the least fixed point.

Proof.

Let $S = \emptyset$, and let $S' = \Gamma_F(\emptyset)$. clearly, $S \subseteq S'$
Since Γ_F is a monotonic function, $\Gamma_F^n(S) \subseteq \Gamma_F^{n+1}(S')$, where . $\Gamma_F^{n+1} = \Gamma_F(\Gamma_F^n)$. Since A is countable, there exists m s.t. $\Gamma_F^m(S) = \Gamma_F^{m+1}(S')$ $\square$

Grounded Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is the *grounded extension* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$.

Grounded Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is the *grounded extension* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$.

Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of F is defined as the outcome S of the following "algorithm":

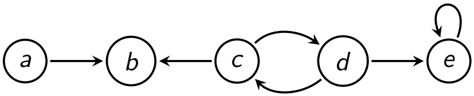
1. put each argument $a \in A$ which is not attacked in F into S; if no such argument exists, return S;
2. remove from F all (new) arguments in S and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.

Grounded Semantics (ctd.)

Recall: S is the grounded extension of F if it is the $\subseteq$-least fixed point of $\Gamma_F(S)$.

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



What is the grounded extension of F?

1. $grd(F) = \{\{\}\}$
2. $grd(F) = \{\{a\}\}$
3. $grd(F) = \{\{a, c\}\}$
4. $grd(F) = \{\{a, d\}\}$
5. $grd(F) = \{\{a, c\}, \{a, d\}\}$

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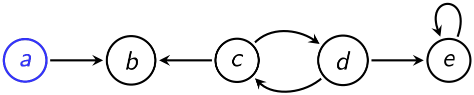
Grounded Semantics (ctd.)

Recall: S is the grounded extension of F if it is the $\subseteq$-least fixed point of $\Gamma_F(S)$.

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



$grd(F) = \{\{a\}\}$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

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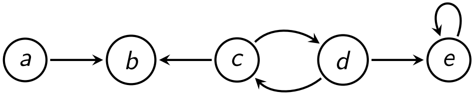

- What are the complete extensions for F?

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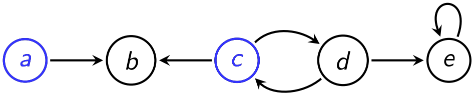

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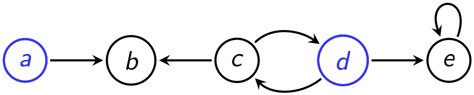

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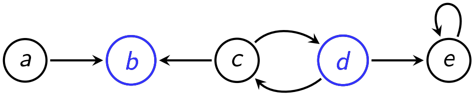

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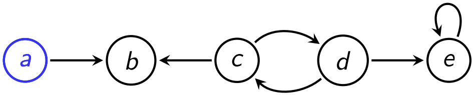

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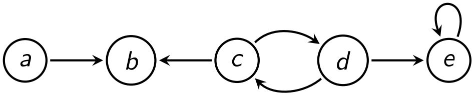

$comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics (ctd.)

Properties of the Extensions

Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
- F has at least one complete extension.

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Remark

Since there exists exactly one grounded extension for each AF F, we often write $grd(F) = S$ instead of $grd(F) = \{S\}$.

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
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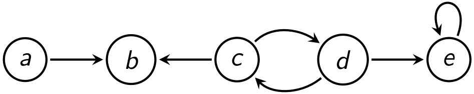

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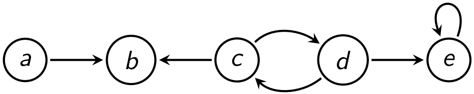

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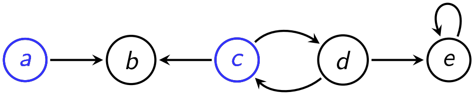

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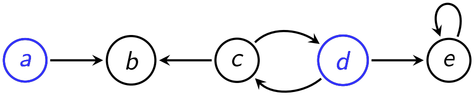

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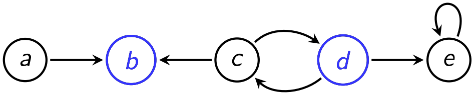

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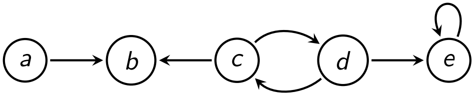

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Characterize of Semantics (ctd.)

Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

- Stable semantics reflect the 'zero-and-one' character of classical logic in argumentation frameworks.
- An AF may not have any stable extension.

Relation between the Semantics of AFs

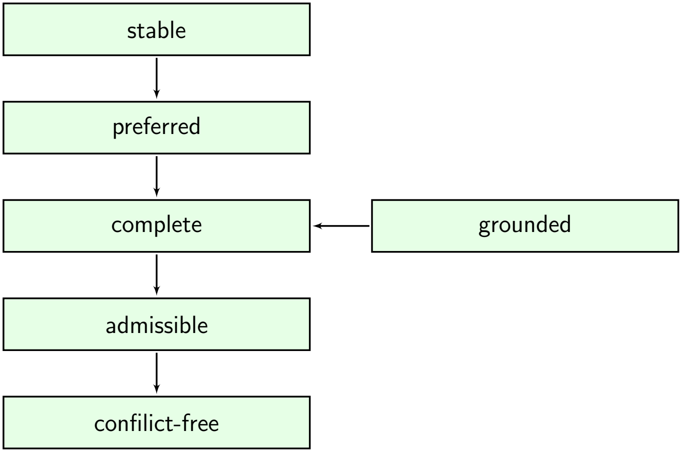

Figure: An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Semantics in Summery

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$ $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* ($S \in comp(F)$) if $S = \Gamma_F(S)$

Summery

We have seen

- Abstract Argumentation Frameworks
- Conflict-free sets
- Admissible semantics
- Preferred semantics
- Complete semantics
- Grounded semantics
- Stable semantics

Next

- Decision problems in AFs
- Labelling-based argumentation

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