Knowledge Representation

Lecture 9: Decision Problems on AFs and labelling-based Semantics

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24, November 2023

Flashback: The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

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$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

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$\langle\{w, w{\rightarrow}\neg s\}, \neg s\rangle$

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$$\begin{tikzcd}
 & \langle\{w, w{\rightarrow}\neg s\}, \neg s\rangle \arrow[dl] & \\
\langle\{s, s{\rightarrow}\neg r\}, \neg r\rangle \arrow[rr] & & \langle\{r, r{\rightarrow}\neg w\}, \neg w\rangle \arrow[ul]
\end{tikzcd}$$

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AF_Δ :

$$\begin{tikzcd} & \gamma \arrow[dl] & \\ \alpha \arrow[rr] & & \beta \arrow[ul] \end{tikzcd}$$

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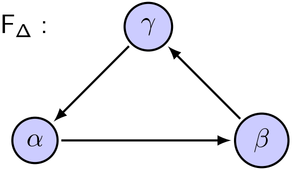

$$\mathit{pref}(\mathrm{AF}_\Delta) = \{\emptyset\}$$
$$\mathit{stage}(\mathrm{AF}_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$$

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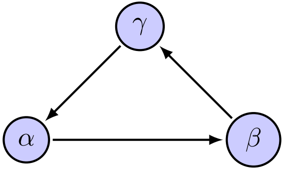
$$Cn_{pref}(\mathrm{AF}_\Delta) = Cn(\top)$$
$$Cn_{stage}(\mathrm{AF}_\Delta) = Cn(\neg r \vee \neg w \vee \neg s)$$

Dung's Abstract Argumentation Frameworks

Seminal Paper by *Phan Minh Dung*:
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.
Artif. Intell., 77(2):321–358, 1995.

Example



Remark

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)

Applications of Formalisms of Argumentation

Abstraction allows to compare several KR formalisms on a conceptual level (*calculus of conflict*)

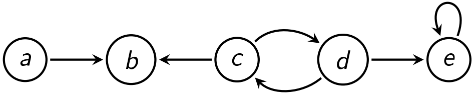
- Legal reasoning:
  - [Bench-Capon and Dunne, 2005]
  - [Collenette et al., 2020]
- Multi-agent systems
  - [McBurney et al., 2012]
  - [Amgoud et al., 2007]
- Discussion game
  - [Caminada, 2018]
  - [Keshavarzi Zafarghandi et al., 2020]
- Recommended system [Rago et al., 2018]
- Explainable AI
  - [Cocarascu et al., 2019]
  - Argumentative XAI: A Survey [Cyras et al., 2021]
  - [Chi and Liao, 2022]

Flashback: Dung's Abstract Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")



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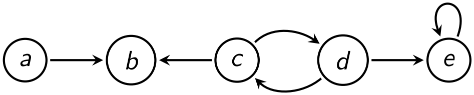


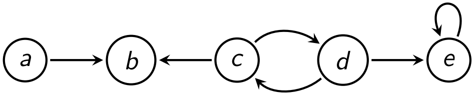
How can we assess the credibility of an argument in an AF?

An argument is believable if it can be argued successfully against the counterarguments.

- Semantics: Methods used to clarify the acceptance of arguments
  - Extension-based semantics
  - Labelling-based semantics

Flashback: Semantics of AFs (ctd.)

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$



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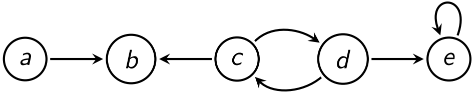

$$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, d\}\}$$

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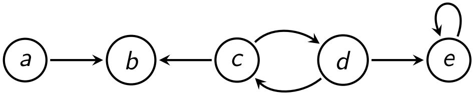

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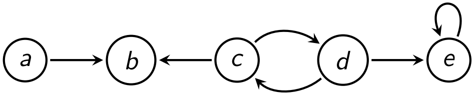

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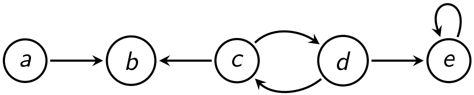

- $S_1 = \{\}$
- $S_2 = \{a\}$
- $S_3 = \{c\}$
- $S_4 = \{b\}$

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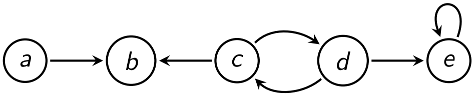

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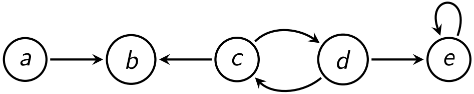

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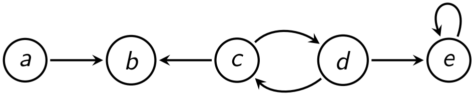

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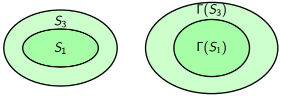
Γ_F is a monotonic function



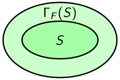
Flashback: Semantics of AFs

- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$



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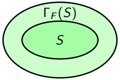

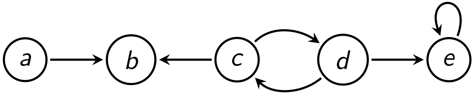

$$adm(F) = \{\{\}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \cancel{\{b, d\}}\}$$

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  That is, for each $T \subseteq A$ admissible in F, $S \not\subset T$.

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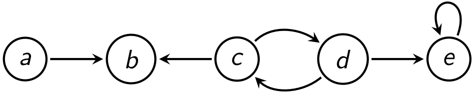

$pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

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- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

$\Gamma_F(S) = S \qquad \Gamma_F(T) = T \qquad \Rightarrow \qquad S \subseteq T$

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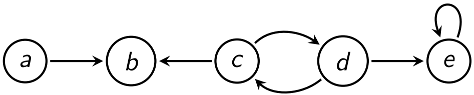

$grd(F) = \{\{a\}\}$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

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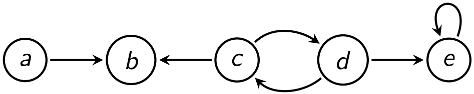

- What are the complete extensions for F?

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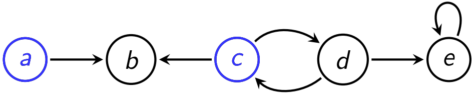

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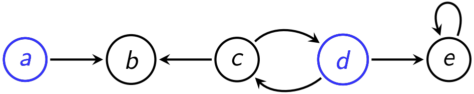

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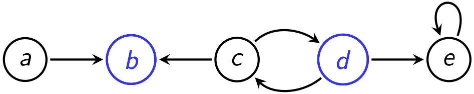

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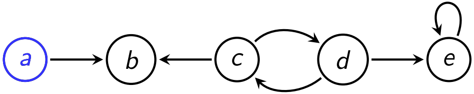

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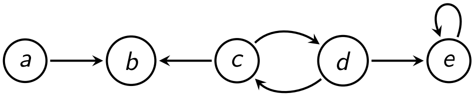

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Characterize of Semantics (ctd.)

Properties of the Extensions

Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
- F has at least one complete extension.

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Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
- F has at least one complete extension.

Remark

Since there exists exactly one grounded extension for each AF F, we often write $grd(F) = S$ instead of $grd(F) = \{S\}$.

Stable Semantics

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Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

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- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.

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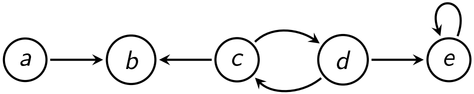

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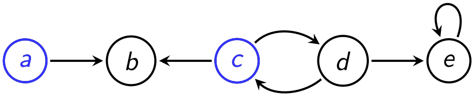

$stb(F) = \{\cancel{\{a, c\}},$

Stable Semantics

Definition

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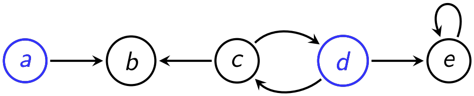

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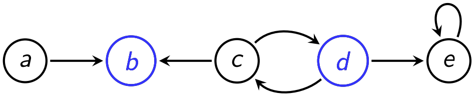

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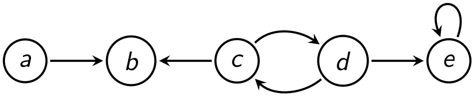


$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics (ctd.)

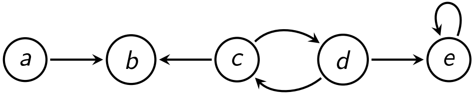
Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

- Stable semantics reflect the ‘zero-and-one’ character of classical logic in argumentation frameworks.
- An AF may not have any stable extension.

Relation between the Semantics of AFs



- $cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \{\}\}$
- $adm(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{\}\}$
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- $comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{c\}, \{d\}, \emptyset}\}$
- $grd(F) = \{\{a\}\}$

Relation between the Semantics of AFs

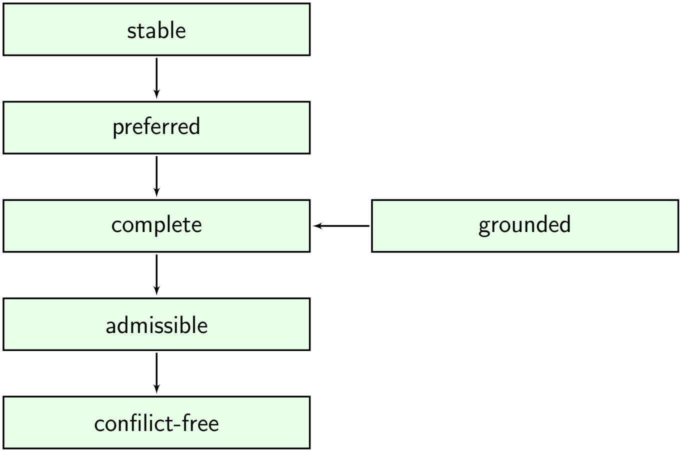

Figure: An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Well-Founded AF

Definition

Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

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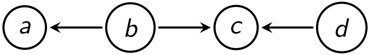
- $S \in adm(F)$ if $S \subseteq \Gamma_F(S)$

Example



- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$

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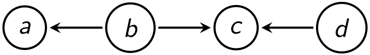
- $S \in comp(F)$ if $S = \Gamma_F(S)$

Example



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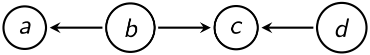
- $S \in grd(F)$ if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

Example



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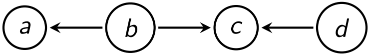
Example



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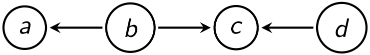
- $S \in stb(F)$ if $\forall a \in A$: $\exists b \in S$ s.t. $(b, a) \in R$

Example



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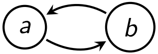
Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?



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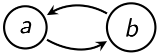

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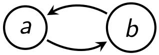

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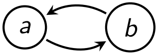

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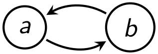

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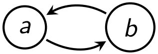

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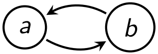

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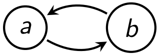

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- $stb(F) = \{\{a\}, \{b\}\}$
- $grd(F) = \{\{\}\}$
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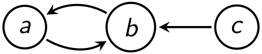
Semantics of AFs (ctd.)

Is the existence of a loop always problematic?



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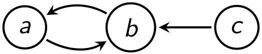

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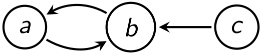

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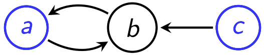

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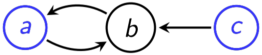

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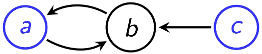

$grd(F) = \{\{a, c\}\}$

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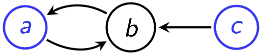

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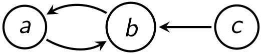

- $cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, c\}\}$
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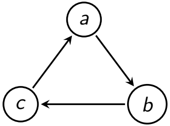
Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?



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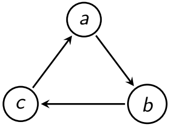

$$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}\}$$

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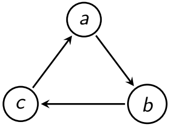

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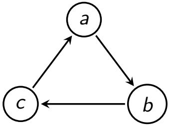

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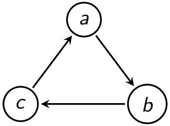

$stb(F) = \{\}$ $\quad$ F does not have any stable extension

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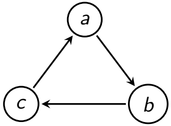
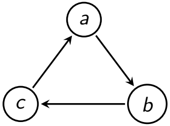


$grd(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

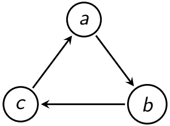
- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* ($S \in comp(F)$) if $S = \Gamma_F(S)$



$comp(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?



- $cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}\}$
- $adm(F) = \{\{\}\}$
- $pref(F) = \{\{\}\}$
- $stb(F) = \{\}$ F does not have any stable extension
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}\}$

Decision problems on AFs

- Existence of extensions
- Credulous acceptance
- Skeptical acceptance
- Verifying an extension

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problem in AFs

Existence of Extensions

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Answer to the existance decision problem:

- Recall: Any AF has at least one *admissible/preferred/grounded/complete/conflict-free* extension.

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- Recall: Any AF has at least one *admissible/preferred/grounded/complete/conflict-free* extension.
- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, grd, comp, cf\}$, is trivially *yes*.

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- $Exists_{stb}(F)$?

Decision Problem in AFs

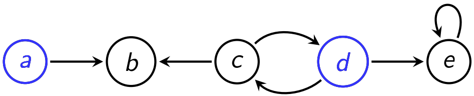
Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- $Exists_{stb}(F)$?



$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$, $Exists_{stb}(F)$: Yes

Decision Problem in AFs

Existence of Extensions

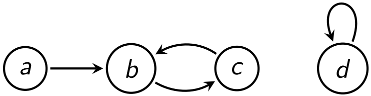
Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

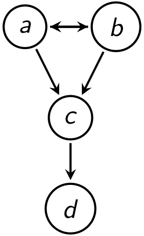
- $Exists_{stb}(F)$?

Example



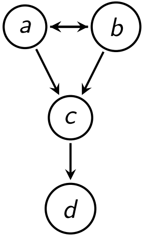
$stb(F) = \{\}$, $\quad Exists_{stb}(F)$: No

Decision Problem in AFs (ctd.)



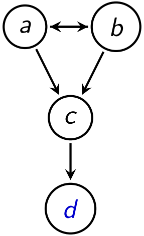
- $\mathit{Exists}_\sigma(F)$, for $\sigma \in \{\mathit{adm}, \mathit{pref}, \mathit{stb}, \mathit{grd}, \mathit{comp}, \mathit{cf}\}$ is yes.

Decision Problem in AFs (ctd.)



- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$

Decision Problem in AFs (ctd.)



- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
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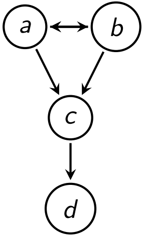
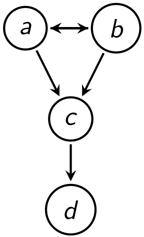
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap pref(F) = \{d\}$, but $d \notin grd(F)$

Decision Problem in AFs (ctd.)



- $\mathit{Exists}_\sigma(F)$, for $\sigma \in \{\mathit{adm}, \mathit{pref}, \mathit{stb}, \mathit{grd}, \mathit{comp}, \mathit{cf}\}$ is yes.
- $\mathit{adm}(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $\mathit{pref}(F) = \{\{a, d\}, \{b, d\}\}$
- $\mathit{stb}(F) = \{\{a, d\}, \{b, d\}\}$
- $\mathit{grd}(F) = \{\{\}\}$
- $\mathit{comp}(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap \mathit{pref}(F) = \{d\}$, but $d \notin \mathit{grd}(F)$

Decision Problem in AFs (ctd.)



- $Exists_{\sigma}(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap pref(F) = \{d\}$, but $d \notin grd(F)$

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \ a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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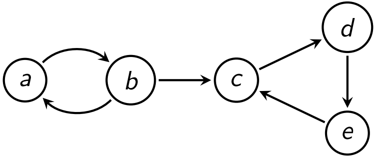


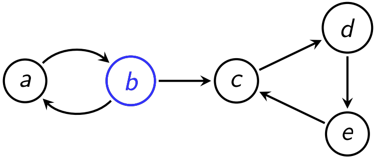
- $Cred_{cf}(b, F)$: is b contained in at least one conflict-free set of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_{cf}(a, F) = \begin{cases} \text{yes} & \text{if } (a, a) \notin R, \\ \text{no} & \text{otherwise} \end{cases}$$



$cf(F) = \{\{b\},$ $\quad Cred_{cf}(b, F)$: Yes

- $Cred_{cf}(b, F)$: is b contained in at least one conflict-free set of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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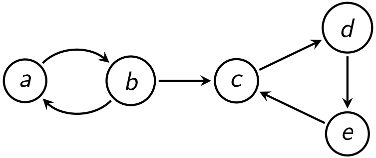

- $Cred_{adm}(b, F)$: is b contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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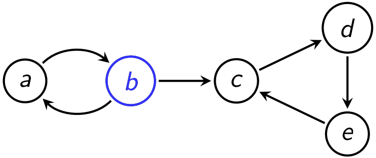

$adm(F) = \{\{b\}, \quad Cred_{adm}(b, F)$: Yes

- $Cred_{adm}(b, F)$: is b contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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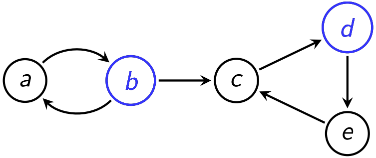
$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



$pref(F) = \{\{b, d\}, \quad Cred_{pref}(b, F)$: Yes

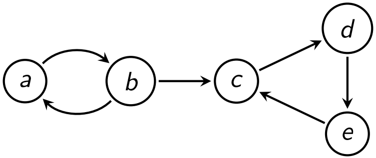
- $Cred_{pref}(b, F)$: is b contained in at least one *pref*-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

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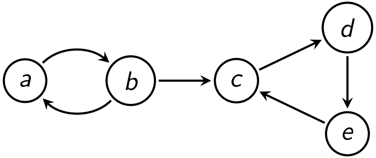

- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F? No. c in not defended against the attack from e.

Decision Problems on AFs (ctd.)

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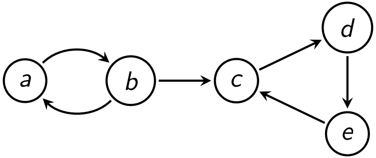

- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F? No. c in not defended against the attack from e.
- $Cred_{adm}(c, F) = Cred_{pref}(c, F) = Cred_{stb}(c, F) = Cred_{comp}(c, F) = Cred_{grd}(c, F)$: No

Decision Problems on AFs (ctd.)

Characterize of Credulous Acceptance

Given an AF $F = (A, R)$:

- $Cred_{cf}(a, F)$: Check if $(a, a) \in R$
- $Cred_{adm}(a, F) = Cred_{pref}(a, F) = Cred_{comp}(a, F)$
- $Cred_{grd}(a, F)$: Evaluate the grounded extension of F
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
- $Cred_{stb}(a, F)$: Evaluate the set of stable extensions of F

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
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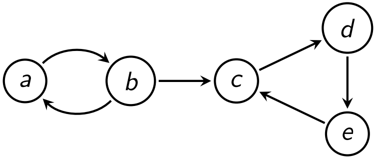


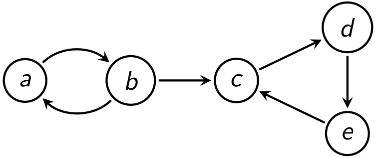
- $Skept_{pref}(b, F)$: is b contained in every *pref*-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & \quad a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



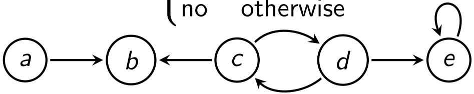
$pref(F) = \big\{\{a\}, \{b, d\}\big\}, \quad Skept_{pref}(b, F) : \text{No}$

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & \quad a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



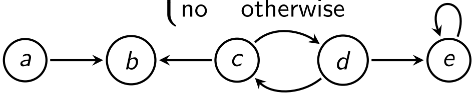
- $Skept_{pref}(a, F)$: is a contained in every $pref$-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$

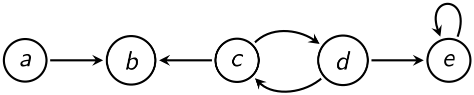


$pref(F) = \{\{a, c\}, \{a, d\}\}$, $\quad Skept_{pref}(a, F)$: yes

- $Skept_{pref}(a, F)$: is a contained in every $pref$-extension of F?

Decision Problems on AFs (ctd.)

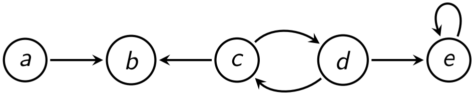
Skeptical Decision Problems under conflict-free



- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F?

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free



$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\{a\}, \{b\}, \{c\}, \{d\}, \{\}\}$ $\quad$ $Skept_{cf}(a, F)$: no.

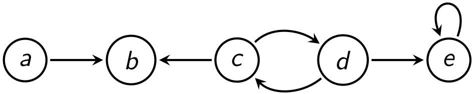
- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free

- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Skeptical Decision Problems under Admissible Semantics



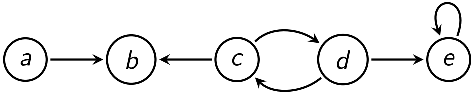
- $Skept_{adm}(a, F)$: is a contained in every adm-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free

- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Skeptical Decision Problems under Admissible Semantics

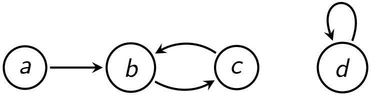


$adm(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \{\}\}$ $\quad$ $Skept_{adm}(a, F)$: no.

- $Skept_{adm}(a, F)$: is a contained in every adm-extension of F? No

Decision Problems on AFs (ctd.)

Recall: $S \subseteq A$ is an stable extension if $S \in cf(F)$ and for each $a \in A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.



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What is the answer to $Skept_{stb}(a, F)$? Why?

- $Skept_{stb}(a, F)$: yes. F has a stable extension and a is in every stable extension of F.
- $Skept_{stb}(a, F)$: no. Since F does not have any stable extension.
- $Skept_{stb}(a, F)$: yes. F does not have any stable extension. If no extension exists then all arguments are skeptically accepted.

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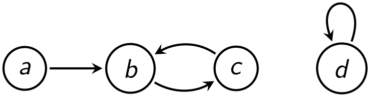

$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, c\}\}$

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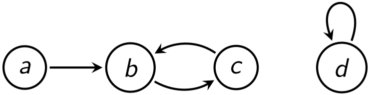

$stb(F) = \{\}, \qquad Skept_{stb}(a, F)$: yes

Decision Problems on AFs(ctd.)

Characterize of Skeptical Acceptance

- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{cf}(a, F)$: Trivially, No.
- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{adm}(a, F)$: Trivially, No.
- If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted.[1]
- $Skept_{grd}(F) = Cred_{grd}(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
  - There exists an AF F and argument a such that $Skept_{pref}(a, F)$: Yes. However, $Skept_{grd}(a, F)$: No.

[1] This is only relevant for stable semantics.

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$

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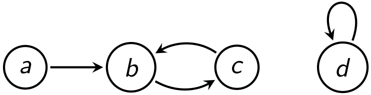

- Let $S = \{a, c\}$. $Ver_{adm}(S, F)$?

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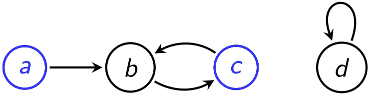

$adm(F) = \{\{a, c\},$ $\quad Ver_{adm}(S, F)$? Yes

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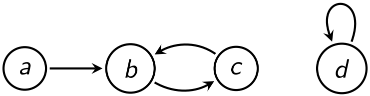

- Let $S = \{b\}$. $Ver_{adm}(S, F)$?

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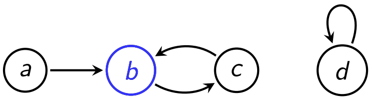

$Ver_{adm}(\{b\}, F)$: No. b is not defended

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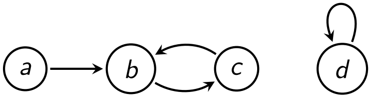

- Let $S = \{\}$. $Ver_{adm}(S, F)$?

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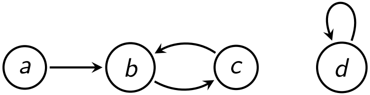

$adm(F) = \{\{\}, \quad Ver_{adm}(S, F)$? Trivially, yes.

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension?

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- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

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Complexity Results

Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ
cf	in L	trivial	in L
adm	NP-c	trivial	in L
pref	NP-c	Π2-c	co-NP-c
comp	NP-c	P-c	in P
grd	P-c	P-c	P-c
stb	NP-c	co-NP-c	in P

Table: Complexity of reasoning with AFs.

Methods and Systems

- For an overview, see:

  G. Charwat, W. Dvořák, S. Gaggl, J. Wallner and S. Woltran.
  Methods for Solving Reasoning Problems in Abstract Argumentation – A Survey. *Artificial Intelligence* 220: 28–63, 2015.

- Competition for Abstract Argumentation Solvers (ICCMA):

  `http://argumentationcompetition.org`

- ASPARTIX Web Front-End:

  `http://rull.dbai.tuwien.ac.at:8080/ASPARTIX`

- CONARG Web Front-End:

  `http://www.dmi.unipg.it/conarg/`

Labelling-based Semantics of AFs

Semantics of AFs

- Extension-based semantics
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Definition

Given an AF $F = (A, R)$. A labelling is a function $\mathbb{L} : A \to \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$

- $\mathbb{L}(a) = \mathtt{in}$, i.e., a is accepted;
- $\mathbb{L}(a) = \mathtt{out}$, i.e., a is rejected;
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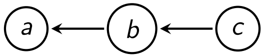
Example



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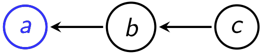

- $\mathbb{L}_1(A) = \{a \mapsto \texttt{in}, b \mapsto \texttt{undec}, c \mapsto \texttt{undec}\}$

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Example

- $\mathbb{L}_2(A) = \{a \mapsto \mathtt{out}, b \mapsto \mathtt{out}, c \mapsto \mathtt{in}\}$

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Example

- $\mathbb{L}_3(A) = \{a \mapsto \mathtt{in}, b \mapsto \mathtt{out}, c \mapsto \mathtt{in}\}$
- labelling-based argumentation semantics provides a way to select reasonable labellings among all the possible ones, according to some criterion.

Labelling-based Semantics of AFs (ctd.)

Each argument is labelled `in`, `out` or `undec`

an argument is `in` $\Leftrightarrow$
all its attackers are `out`

an argument is `out` $\Leftrightarrow$
it has an attacker that is `in`

an argument is `undec` $\Leftrightarrow$
not all its attackers are `out` and it does not have an attacker that is `in`

Applying Argument Labellings

$\texttt{in} \Leftrightarrow$ all attackers are $\texttt{out}$

$\texttt{out} \Leftrightarrow$ there is an attacker that is $\texttt{in}$

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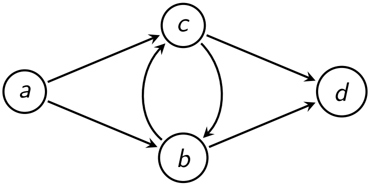

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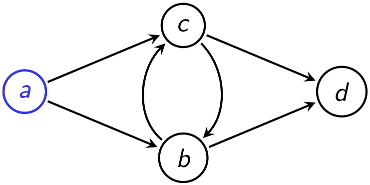

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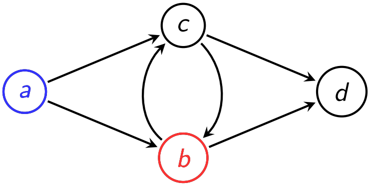

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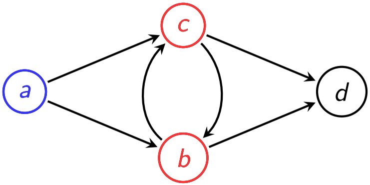

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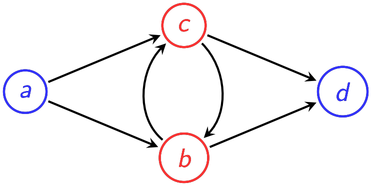

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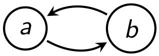

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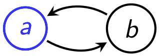

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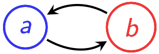

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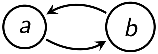

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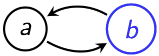

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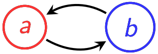

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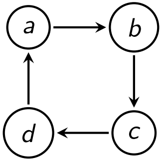

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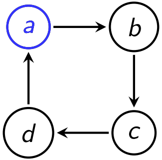

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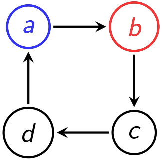

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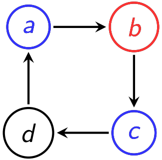

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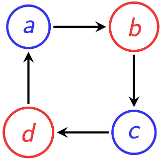

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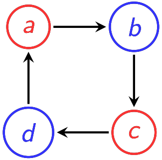

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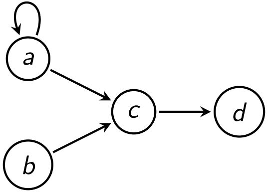

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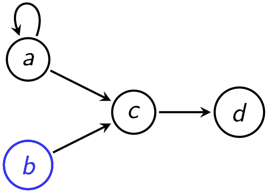

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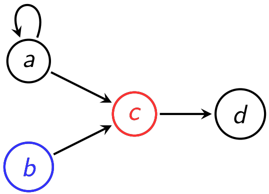

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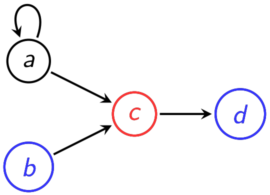

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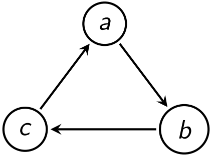

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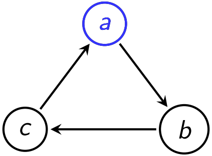

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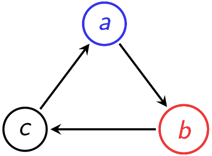

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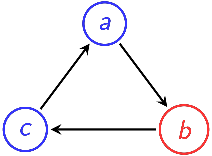

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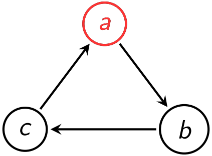

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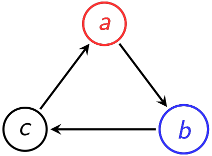

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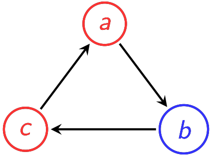

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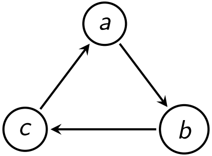
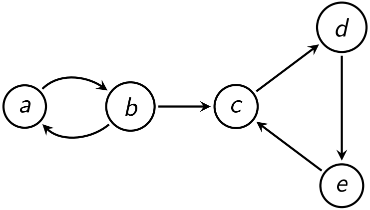


Applying Argument Labellings, Exercise

Give the three labellings of this argumentation framework.



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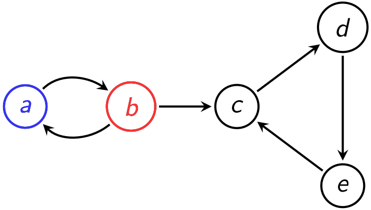

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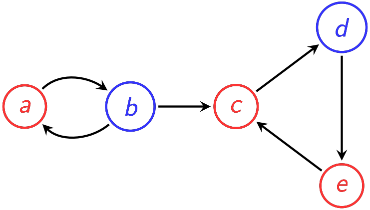
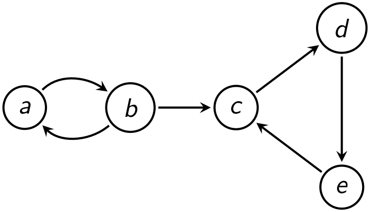


Maximisation/Minimisation

maximal: there is no other that has the same plus something

minimal: there is no other that has the same minus something



a, b, c, d, e min `in`, min `out`, max `undec`

a, b, c, d, e max `in`, max `out`, min `undec`

a,b,c,d,e max `in`, max `out`

Admissible Labelling

Definition: Admissible labelling

Given an AF $F = (A, R)$. Let $\mathbb{L}$ be a labelling function on A. $\mathbb{L}$ is an admissible labelling iff for each argument $a \in A$ it holds that:

- if $\mathbb{L}(a) = \texttt{in}$ then for each b, such that $(b, a) \in R$ then $\mathbb{L}(b) = \texttt{out}$;
- if $\mathbb{L}(a) = \texttt{out}$ then there exists $b \in A$, such that $(b, a) \in A$ and $\mathbb{L}(b) = \texttt{in}$.

Admissible Labelling

Admissible labeling:

`in` $\Rightarrow$ all attackers are `out`

`out` $\Rightarrow$ there is an attacker that is `in`

Example

$$a \leftarrow b \leftarrow c$$

Admissible Labelling

Admissible labeling:

> `in` $\Rightarrow$ all attackers are `out`
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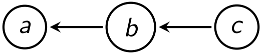
Example



- $adm(F) = \{a \mapsto \texttt{undec}, b \mapsto \texttt{undec}, c \mapsto \texttt{undec}\}$

Admissible Labelling

Admissible labeling:

$\texttt{in} \Rightarrow$ all attackers are $\texttt{out}$

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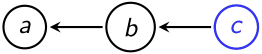

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Example

$$a \longleftarrow b \longleftarrow c$$

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Preferred Labelling

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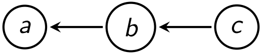
Given an AF $F = (A, R)$. Let $\mathbb{L}$ be a labelling function on A. $\mathbb{L}$ is a preferred labelling if

- $\mathbb{L}$ is an admissible labelling,
- $\{a \mid \mathbb{L}(a) = \mathtt{in}\} \cup \{a \mid \mathbb{L}(a) = \mathtt{out}\}$ is $\subseteq$-maximal among all admissible labellings.

Example



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Example

- $\mathit{pref}(F) = \{a \mapsto \mathtt{in}, b \mapsto \mathtt{out}, c \mapsto \mathtt{in}\}$

Complete Labelling

in $\Leftrightarrow$ all attackers are out

out $\Leftrightarrow$ there is an attacker that is in

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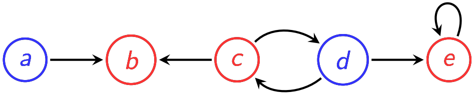

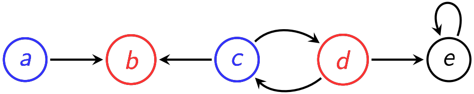

Extension-based Semantics vs. Labelling-based Semantics

`in` $\Leftrightarrow$ all attackers are `out`

`out` $\leftrightarrow$ there is an attacker that is `in`

`undec` $\Leftrightarrow$ not all attackers are `out`, and no attacker is `in`

restriction on labelling

maximal `in`

maximal `out`

maximal `undec`

minimal `in`

minimal `out`

empty `undec`

Extension-based Semantics vs. Labelling-based Semantics

in $\Leftrightarrow$ all attackers are out

out $\leftrightarrow$ there is an attacker that is in

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restriction on labelling	Extension-based semantics
maximal in	preferred semantics
maximal out	preferred semantics
maximal undec	grounded semantics
minimal in	grounded semantics
minimal out	grounded semantics
empty undec	Stable semantics

An extension is the in-labelled part of a labelling

Labelling-based Semantics

Admissible Labelling

in $\Leftrightarrow$ all attackers are out

out $\Leftrightarrow$ there is an attacker that is in

Complete labelling

in $\Leftrightarrow$ all attackers are out

out $\Leftrightarrow$ there is an attacker that is in

undec $\leftrightarrow$ not all attackers are out, and no attacker is in

Grounded labelling: Complete with min in/ min out / max undec

Preferred labelling: Complete with max in/ max out

Stable labelling: complete with no undec

Properties of Labelling-based Semantics

Given an AF $F = (A, R)$. Let $\mathbb{L}$ be a labelling.

- If $\mathbb{L}$ is an admissible labelling of F, then $\{a \mid \mathbb{L}(a) = \texttt{in}\}$ is an admissible extension of F.

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Given an AF $F = (A, R)$. Let $\mathbb{L}$ be a labelling.

- If $\mathbb{L}$ is an admissible labelling of F, then $\{a \mid \mathbb{L}(a) = \mathtt{in}\}$ is an admissible extension of F.
- If $S \subseteq A$ is an admissible extension of F, then there exists a labelling function $\mathbb{L}$ on A, such that $S = \{a \mid \mathbb{L}(a) = \mathtt{in}\}$.

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- For each a, if $\mathbb{L}(a) = \mathtt{in}$ in a preferred labelling then there exists an admissible labelling in which $\mathbb{L}(a) = \mathtt{in}$.

Amgoud, L., Dimopoulos, Y., and Moraitis, P. (2007).
A unified and general framework for argumentation-based negotiation.
In Proceedings of the 6th international joint conference on Autonomous agents and multiagent systems, pages 1–8.

Bench-Capon, T. J. and Dunne, P. E. (2005).
Argumentation in ai and law: Editors' introduction.
Artificial Intelligence and Law, 13(1):1–8.

Caminada, M. (2018).
Argumentation semantics as formal discussion.
In Baroni, P., Gabbay, D., Giacomin, M., and van der Torre, L., editors, Handbook of Formal Argumentation, pages 487–518.

Chi, H. and Liao, B. (2022).
A quantitative argumentation-based automated explainable decision system for fake news detection on social media.
Knowl. Based Syst., 242:108378.

Cocarascu, O., Rago, A., and Toni, F. (2019).

Extracting dialogical explanations for review aggregations with argumentative dialogical agents.
In AAMAS, pages 1261–1269. International Foundation for Autonomous Agents and Multiagent Systems.

Collenette, J., Atkinson, K., and Bench-Capon, T. J. M. (2020).
An explainable approach to deducing outcomes in european court of human rights cases using adfs.
In Proceedings of Computational Models of Argument COMMA 2020, volume 326 of Frontiers in Artificial Intelligence and Applications, pages 21–32. IOS Press.

Cyras, K., Rago, A., Albini, E., Baroni, P., and Toni, F. (2021).
Argumentative XAI: A survey.
In IJCAI, pages 4392–4399. ijcai.org.

Keshavarzi Zafarghandi, A., Verbrugge, R., and Verheij, B. (2020).
A discussion game for the grounded semantics of abstract dialectical frameworks.
In Proceedings of Computational Models of Argument COMMA 2020, volume 326 of Frontiers in Artificial Intelligence and Applications, pages 431–442. IOS Press.

McBurney, P., Parsons, S., and Rahwan, I., editors (2012).

Argumentation in Multi-Agent Systems - 8th International Workshop, ArgMAS 2011, Taipei, Taiwan, May 3, 2011, Revised Selected Papers, volume 7543 of Lecture Notes in Computer Science. Springer.

Rago, A., Cocarascu, O., and Toni, F. (2018).
Argumentation-based recommendations: Fantastic explanations and how to find them.
In IJCAI, pages 1949–1955. ijcai.org.