Knowledge Representation 2023/2024

Exercise Sheet 4 – Mini Exercise-Sheet on Bayesian Networks – Solutions

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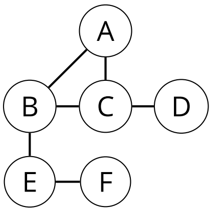
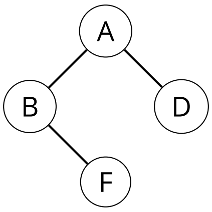
Exercise 4.1 You are given the following interaction graph:



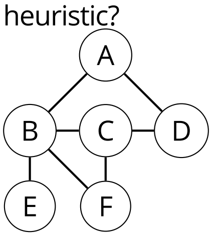
(a) Does the graph correspond to the following set of factors?

$$\{\ f(A,B,C),\ f(E,F),\ f(C,E),\ f(D,C),\ f(C,B),\ f(B,E)\ \}$$

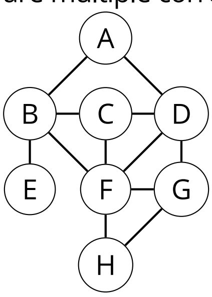
(b) Could the following graph be reached from the original graph through node elimination?



(c) Could E-A-D-F-B-C be an elimination order in the following graph according to the MinFill heuristic?



(d) Provide an elimination order on the following graph according to the MinDeg heuristic. (There are multiple correct answers.)



Solution:

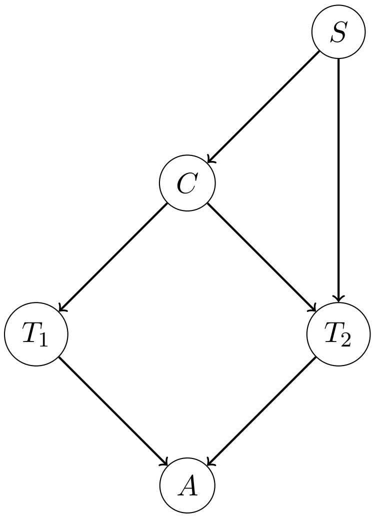
(a) No, because $f(C, E)$ does not appear in the graph.

(b) No.

(c) No. After eliminating E, the next node be eliminated according to the MinFill heuristic would be F, not A.

(d) There are different possibilities. One is E-H-G-A-B-C-D-F.

Exercise 4.2 You are given the following Bayesian Network.



S	C	T_2	$\Theta_{T_2\|C,S}$
male	yes	+ve	0.80
male	yes	-ve	0.20
male	no	+ve	0.20
male	no	-ve	0.80
female	yes	+ve	0.95
female	yes	-ve	0.05
female	no	+ve	0.05
female	no	-ve	0.95

T_1	T_2	A	$\Theta_{A\|T_1,T_2}$
+ve	+ve	yes	1
+ve	+ve	no	0
+ve	-ve	yes	0
+ve	-ve	no	1
-ve	+ve	yes	0
-ve	+ve	no	1
-ve	-ve	yes	1
-ve	-ve	no	0

S	Θ_S
male	0.55
female	0.45

S	C	$\Theta_{C\|S}$
male	yes	0.05
male	no	0.95
female	yes	0.01
female	no	0.99

C	T_1	$\Theta_{T_1\|C}$
yes	+ve	0.8
yes	-ve	0.2
no	+ve	0.2
no	-ve	0.8

(a) Write down the probability table for $\Pr(S, C, T_2)$. *(Warning: ugly numbers!)*

(b) Write down the probability table for $\Pr(T_1 \mid S = \text{male})$.

(c) Calculate MPE($S = \text{female}, C = \text{no}$).

(d) Calculate MAP($S, C \mid A = \text{yes}$).

Solution:

(a) We get as follows to the solution:

S	C	$\Pr(S, C)$
male	yes	0.0275
male	no	0.5225
female	yes	0.0045
female	no	0.4455

S	C	T_2	$\Pr(S, C, T_2)$
male	yes	+ve	0.022
male	yes	-ve	0.0055
male	no	+ve	0.1045
male	no	-ve	0.418
female	yes	+ve	0.004275
female	yes	-ve	0.000225
female	no	+ve	0.022275
female	no	-ve	0.423225

(b) We get as follows to the solution:

C	$\Pr(C \mid S = \text{male})$
yes	0.05
no	0.95

C	T_1	$\Pr(C, T_1 \mid S = \text{male})$
yes	+ve	0.04
yes	-ve	0.01
no	+ve	0.19
no	-ve	0.76

T_1	$\Pr(T_1 \mid S = \text{male})$
+ve	0.23
-ve	0.77

(c) It is $(T_1 = -ve, T_2 = -ve, A = \text{yes})$ with $P(.) = 0.76$.

(d) The solution is $(S = \text{male}, C = \text{no})$ with $P(.) = 0.493$. To get there, we calculate $\Pr(S, C, T_1, T_2)$ and then sum out S and C in the rows in which T_1 and T_2 match.