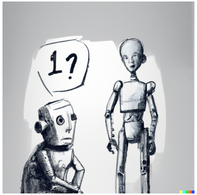
Decision-Making under Uncertainty



Knowledge is often incomplete and uncertain, so agents

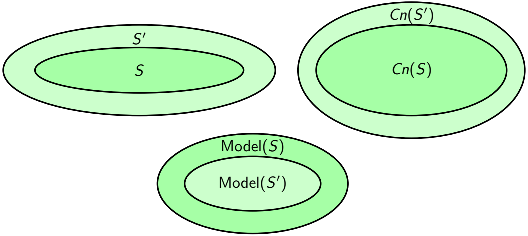
- Draw tentative conclusions based on **available evidence**;
- **Revise** conclusions when new evidence **contradicts** previous assumptions.

This type of reasoning is defeasible and non-monotonic logics capture the logic of defeasible reasoning.

Monotonic Reasoning

Definition

A logic L is monotonic iff for any sets of formula S and S' it holds that if $S \subseteq S'$ then $Cn(S) \subseteq Cn(S')$, where $Cn(S) = \{ \phi \mid S \models_L \phi \}$



Monotonic Reasoning

More information
$\implies$ More entailments
$\implies$ Less models

- Propositional logic is monotonic
- First-order logic is monotonic.

Reasoning

- Deriving logical conclusion and making predictions from available knowledge, facts, and beliefs
  - **Monotonic Reasoning**: The more you know, the more is entailed.
  - **Non-monotonic Reasoning**: Some conclusions may be invalidated if we add some more information to our knowledge base.

Example

Birds normally can fly. Tweety is a bird.

- Conclusion: Tweety can fly

Tweety is a penguin.
Exception: Penguin cannot fly!

- New information cause a revision of previous conclusions
- Conclusion: Tweety cannot fly

Advantage of Non-monotonic Reasoning

- For real-world scenarios containing exceptions, such as Robot navigation, we can use non-monotonic reasoning

Why do AI students need to know about default theory?

1. **Non-Monotonic Reasoning**: Default logic is a key formalism in non-monotonic reasoning.
2. **Knowledge Representation**: In AI, effective knowledge representation is essential for building intelligent systems that can make decisions and solve problems. Default logic provides a framework for representing and reasoning about incomplete or uncertain knowledge.
3. **Dealing with Exceptions**: In real-world scenarios, not all information is explicitly known or stated, and default reasoning allows AI systems to make plausible inferences in the absence of complete information.

Default Theories

"**Usually** birds fly, except the abnormal ones", "Tweety is a bird. All birds are animal".

- **Default theory is a pair** (W, D) **such that**:

W **presents monotonic part**: called background theory

- Tweety is a bird. All birds are animal.

$W = \{\ \text{Bird}(\text{t}), \forall x : \text{Bird}(x) \to \text{Animal}(x)\ \}$

D **presents non-monotonic part**: contains default rules

- "**Usually** birds fly except, the abnormal ones,"

$$\delta = \frac{\text{Bird}(x) : \neg\text{Ab}(x)}{\text{Fly}(x)}$$

Default Theories

Definition: Default Theory

A default theory T is a pair (W, D), s.t.,

- W : is a set of propositional or first order logic formulas (called the facts or axioms of T),
- D is a countable set of default rules

Example

$$D = \left\{ \frac{\text{train: } \neg\text{Obstacle}}{\text{Move}} \right\},\ W = \{\text{train}\}$$

$$D = \left\{ \frac{\text{Bird}(x) : \neg\text{Ab}(x)}{\text{Fly}(x)} \right\},\ W = \{\text{Bird}(T), \forall x : (\text{Bird}(x) \rightarrow \text{Animal}(x))\}$$

$$D = \left\{ \frac{\text{Robot} : \neg\text{HandsBroken}}{\text{Works}} \right\},\ W = \{\text{Robot, HandsBroken}\}$$

Default Logic

Defaults (informally): domain specific inference rules

$$\delta = \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma}$$

Intuitive reading: If α has been derived, and $\neg\beta_1, \ldots, \neg\beta_n$ are not derivable, then γ can be concluded.

Example

$$\delta = \frac{\mathsf{Bird(Tweety)} : \neg\mathsf{Ab(Tweety)}}{\mathsf{Fly(Tweety)}}$$

If it is provable that Tweety is a bird, and it is not provable that Tweety is abnormal, then we can infer that Tweety can fly.

Default Formally

$$\delta = \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma}$$

- The formula α is called the prerequisite, denoted by $\mathit{pre}(\delta)$.
- $\beta_1 \ldots \beta_n$ are called justifications, denoted by $\mathit{just}(\delta)$
- γ is the consequent of δ, denoted by $\mathit{cons}(\delta)$

- (α, β_i, γ are propositional formulas)
- 'Default rule' with free variables in α, β_i, γ is a schema for all its ground instances.

$$\frac{\mathit{Bird}(x) : \neg \mathit{Ab}(x)}{\mathit{Fly}(x)}$$

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?
- Does f Fly?
- Does t abnormal?
- **What are the consequences of theory** T?
- Since ¬Ab(t) is consistent with W, we can apply the default on t and drive Fly(t).
- Since ¬Ab(f) is not consistent with W, we cannot apply the default on f.
- Consequence: we believe that Bird(t), Fly(t), Bird(f), Ab(f).

How can the logical consequences of a default theory be formally defined?

Default rule $\delta = \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then δ is applicable to T. In consequence we are led to believe that conclusion γ is true.

Informal Discussion of the Semantics

Default rule $\delta = \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then we believe that conclusion γ is true.

Example

Given default Theory $T = (W, D)$ s.t.

- $W = \{\text{Republican(Nixon), Quaker(Nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \frac{\text{Republican}(X):\neg\text{Pacifist}(X)}{\neg \mathit{Pacifist}(X)}$, $\delta_2 = \frac{\text{Quaker}(X):\text{Pacifist}(X)}{\mathit{Pacifist}(X)}$

- Is δ_1 applicable in T?
  - Yes: Republican(Nixon) $\in W$ and there is no assumption for Pacifist(Nixon).
- Is δ_2 applicable in T?
  - Yes: Quaker(Nixon) $\in W$ and there is no assumption for $\neg$Pacifist(Nixon).

Consequence: we believe that $\{\text{Pacifist(Nixon)}, \neg\text{Pacifist(Nixon)}\}$

It is inconsistent!

Default Theory: logical consequence

How can the logical consequences of a default theory be formally defined?

- The semantics of Default Logic will be given in terms of extensions.
- Recall: In classical logics, we used models to define entailment.
- In default logics, it is not so easy — extensions will play a similar role to models.
- Intuitively, extensions represent possible world views which are based on the given default theories; they seek to extend the set of known facts with “reasonable” conjectures based on the available defaults.

Given default theory $T = (W, D)$. A set of formulas of E is an Extension s.t.

- $W \subseteq E$
- $Cn(E) = E$
- E is created by applying applicable defaults
- E is maximal: formally: if $\frac{\alpha:\beta_1 \ldots \beta_n}{\gamma} \in D$, $\alpha \in E$, and $\neg\beta_1, \ldots \neg\beta_n \notin E$, then $\gamma \in E$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$
- $E_{i+1} = Cn(E_i \cup \{\gamma| \begin{array}{l} \frac{\alpha:\beta_1\ldots\beta_n}{\gamma} \in D \\ \alpha \in E_i, \\ \neg\beta_1, \ldots \neg\beta_n \notin E_i \end{array} \})$
- E is an extension if $E = \bigcup_{i=0}^{\infty} E_i$

Processes

Let $T = (W, D)$ be a default theory

- A default rule $\frac{\alpha:\beta_1,\ldots,\beta_n}{\gamma}$ is applicable to a set of formulas S iff:
  - $\alpha \in S$;
  - $\neg\beta_1, \ldots, \neg\beta_n \notin S$
- Let $\Pi = \delta_1, \ldots, \delta_n, \ldots,$ be a sequence of defaults rule from D without multiple occurrences.
  - $\Pi[k] = \delta_1, \ldots, \delta_k$: k initial segments of Π
  - $In(\Pi) = Cn(W \cup \{cons(\delta) \mid \delta \text{ occurs in } \Pi\})$
  - $Out(\Pi) = \{\neg\beta \mid \beta \in just(\delta), \text{ for some } \delta \text{ occurring in } \Pi\}$

Definition

- Π is called a **process** of T if δ_k is applicable to $In(\Pi[k])$ for every $\delta_k \in \Pi$
- Π is **successful** iff $In(\Pi) \cap Out(\Pi) = \emptyset$, otherwise it **fails**.
- Π is **closed** iff every δ that is applicable to $In(\Pi)$ already occurs in Π.

Extension

- Π is called a **process** of T if δ_k is applicable to $In(\Pi[k])$ for every $\delta_k \in \Pi$
- Π is **successful** iff $In(\Pi) \cap Out(\Pi) = \emptyset$, otherwise it **fails**.
- Π is **closed** iff every δ that is applicable to $In(\Pi)$ already occurs in Π.

Definition: Extension

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Extensions: A process tree

Process tree

Given default theory $T = (W, D)$, in the process tree of the T:

- The nodes of the tree are labeled with $In(\Pi)$ (on the left) and $Out(\Pi)$ (on the right)
- The edges correspond to default applications, and are labeled with the default that is being applied.
- The paths of the process tree starting at the root correspond to processes of T.

Normal Default Theory

Normal Default Rule

A default δ is normal if $\text{just}(\delta) = \text{cons}(\delta)$.

$$\frac{\alpha : \gamma}{\gamma}$$

Example

$$\frac{\text{Bird(tweety)} : \text{Fly(tweety)}}{\text{Fly(tweety)}}$$

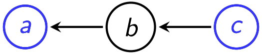
If it is provable that Tweety is a bird, and it is not provable that Tweety cannot fly, then we can infer that Tweety can fly.

Overview on Abstract Argumentation Frameworks

Formal Models of Argumentation are concerned with

1. representation of an argument
2. representation of the relationship between arguments
3. solving conflicts between the arguments (*acceptability*)

Example



1. **Form abstract arguments**: $\{a, b, c\}$
2. **Identify conflicts**: c attacks b, and b attacks a.
3. **Resolve conflicts**: a is acceptable when considered together with c.

What conclusions can be draw? Menzis is the best option for them.

The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- **Draw conclusions**

Example

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

$$\begin{tikzcd}
& \langle\{w, w{\rightarrow}\neg s\}, \neg s\rangle \arrow[dl] & \\
\langle\{s, s{\rightarrow}\neg r\}, \neg r\rangle \arrow[rr] & & \langle\{r, r{\rightarrow}\neg w\}, \neg w\rangle \arrow[ul]
\end{tikzcd}$$

$$Cn_{pref}(\mathrm{AF}_{\Delta}) = Cn(\top)$$
$$Cn_{stage}(\mathrm{AF}_{\Delta}) = Cn(\neg r \vee \neg w \vee \neg s)$$

Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) Δ
- argument is a pair (Φ, α), such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
- conflicts between arguments (Φ, α) and (Φ', α') arise if Φ and α' are contradicting.

Example

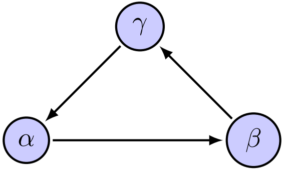
$$\langle \{s, s \to \neg r\}, \neg r \rangle \longrightarrow \langle \{r, r \to \neg w\}, \neg w \rangle$$

Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.

Dung's Abstract Argumentation Frameworks

Example



Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
  - "plethora of semantics"

Dung's Abstract Argumentation Frameworks

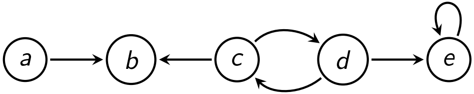
Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



The main Objection

How can we assess the credibility of an argument in an abstract argumentation framework?

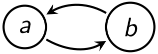
An argument is believable if it can be argued successfully against the counterarguments.

- Semantics: Methods used to clarify the acceptance of arguments
  - Extension-based semantics
  - Labelling-based semantics

How Can We Deal with Conflict in a Loop?

Example

- a: Let's go to Norway for Christmas holiday to see the northern lights.
- b: Let's go to Spain for Christmas holiday to enjoy warm weather.



- We do not accept arguments that have conflicts, do we?

Basic Properties

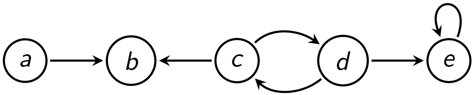
Definition

Given $F = (A, R)$.
A set $S \subseteq A$ is *conflict-free* if there is no attack/conflict within S. A set $S \subseteq A$ is *conflict-free* ($S \in cf(F)$) if, for each $a, b \in S$, $(a, b) \notin R$

- What are the conflict-free sets of F?

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \{\}\}$

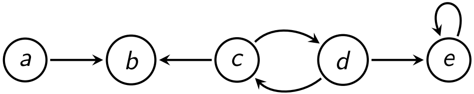
Acceptability within a Set

Definition

Given an AF $F = (A, R)$. Let $S \subseteq A$.
An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



- Is c acceptable w.r.t. any set? Is c defended by any set?
  - c is acceptable w.r.t. (defended by) $\{c\}, \{a, c\}$
- Is b acceptable w.r.t. any set? Is d defended by any set?
  - No, because $(a, b) \in R$ and a is not attacked.

Characteristic Operator

Given $F = (A, R)$

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.

Definition

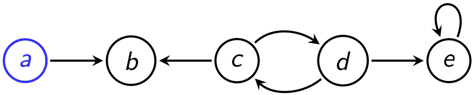
Characteristic operator $\Gamma_F(S)$ is a function that take set $S \subseteq A$ and returns the set of all arguments that are defended by S.

$$\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$$

Characteristic Operator (ctd.)

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



What are the outputs of $\Gamma_F(S)$ for any of the following sets?
Recall: $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$. $a \in A$ is defended by S , if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, s.t. $(c, b) \in R$.

- $S = \{\}$ $\Gamma_F(S) = \{a\}$
- $S = \{a\}$ $\Gamma_F(S) = \{a\}$
- $S = \{c\}$ $\Gamma_F(S) = \{a, c\}$
- $S = \{a, b\}$ $\Gamma_F(S) = \{a\}$

Properties of the Characteristic Operator

Let F be a function, and let S and S' be inputs of F :

- A function F is a monotonic function: if $S \subseteq S'$ then $F(S) \subseteq F(S')$.

Theorem

Given an AF $F = (A, R)$. Let Γ_F be the characteristic operator of F. Γ_F is a monotonic function. That is, if $S \subseteq S'$ then $\Gamma_F(S) \subseteq \Gamma_F(S')$.

proof

$$
\begin{aligned}
\Gamma_F(S') &= \{a \in A \mid a \text{ is defended by } S' \text{ in } F\} \\
&= \{a \in A \mid a \text{ is defended by } S \cup (S' \setminus S) \text{ in } F\} \\
&= \{a \in A \mid a \text{ is defended by } S \text{ in } F\} \cup \{a \in A \mid a \text{ is defended by } (S' \setminus S) \text{ in } F\} \\
&= \Gamma_F(S) \cup \{a \in A \mid a \text{ is defended by } S' \setminus S \text{ in } F\}
\end{aligned}
$$

Admissible Semantics

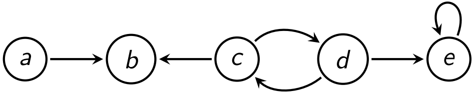
Definition

Given an AF $F = (A, R)$. A set S is *admissible* ($S \in adm(F)$) in F, if

- S is conflict-free in F
- $S \subseteq \Gamma_F(S)$
  - $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$
  - $a \in A$ is *defended* by $S \subseteq A$ in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$

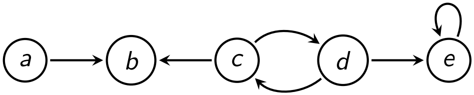


$$adm(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{\}\}$$

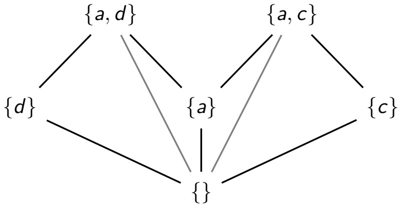
Properties of Admissible Extensions

Theorem

Every AF has at least one admissible extension.



$$adm(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \{\}\}$$



The relation between admissible extensions of F with respect to the subset relation.

Preferred Semantics

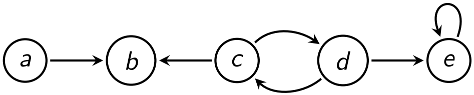
Definition

Given an AF $F = (A, R)$. A set S is a *preferred extension* ($S \in \mathit{pref}(F)$) in F if

- S is $\subseteq$-maximal admissible in F, that is, for each $T \subseteq A$ admissible in F, $S \not\subset T$. .

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



$\mathit{pref}(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics of AFs

Theorem

Any AF has at least a preferred extension.

Proof

Let F be an AF. Every AF has at least an admissible extension. For each admissible set S of AF, there exists a preferred extension E of AF such that $S \subseteq E$.

Properties of Semantics of AFs

- Any admissible extension is a conflict-free set.
- In any AF empty set is an admissible extension.
- Every AF has at least one admissible extension.
- Every AF has at least one preferred extension.
- ...

Properties of the Characteristic Operator (ctd.)

Flashback

Given an AF $F = (A, R)$, and $S \subseteq A$. The characteristic operator is $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$.

- Characteristic operator is monotonic, that is, if $S \subseteq S'$, then $\Gamma_F(S) \subseteq \Gamma_F(S')$.

Definition

S is termed the least fixed point of Γ_F if:

- $S = \Gamma_F(S)$,
- for each $S' \subseteq A$, if $\Gamma_F(S') = S'$, then $S \subseteq S'$.

Theorem

Given an AF $F = (A, R)$. Γ_F has the least fixed point.

Proof.

Let $S = \emptyset$, and let $S' = \Gamma_F(\emptyset)$. clearly, $S \subseteq S'$
Since Γ_F is a monotonic function, $\Gamma_F^n(S) \subseteq \Gamma_F^{n+1}(S')$, where . $\Gamma_F^{n+1} = \Gamma_F(\Gamma_F^n)$. Since A is countable, there exists m s.t. $\Gamma_F^m(S) = \Gamma_F^{m+1}(S')$ $\square$

Grounded Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is the *grounded extension* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$.

Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of F is defined as the outcome S of the following "algorithm":

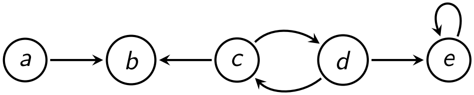
1. put each argument $a \in A$ which is not attacked in F into S; if no such argument exists, return S;
2. remove from F all (new) arguments in S and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.

Grounded Semantics (ctd.)

Recall: S is the grounded extension of F if it is the $\subseteq$-least fixed point of $\Gamma_F(S)$.

Example

Given $F = (A, R)$ s.t. $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$



What is the grounded extension of F?

1. $grd(F) = \{\{\}\}$
2. $grd(F) = \{\{a\}\}$
3. $grd(F) = \{\{a, c\}\}$
4. $grd(F) = \{\{a, d\}\}$
5. $grd(F) = \{\{a, c\}, \{a, d\}\}$

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Semantics in Summery

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$ $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

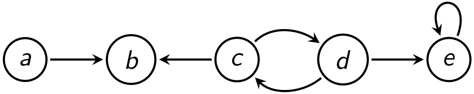
Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.



$comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics (ctd.)

Properties of the Extensions

Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
- F has at least one complete extension.

Remark

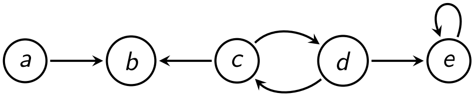
Since there exists exactly one grounded extension for each AF F, we often write $grd(F) = S$ instead of $grd(F) = \{S\}$.

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.



$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics (ctd.)

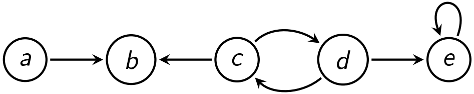
Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

- Stable semantics reflect the 'zero-and-one' character of classical logic in argumentation frameworks.
- An AF may not have any stable extension.

Relation between the Semantics of AFs



- $cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \{\}\}$
- $adm(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{\}\}$
- $pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$
- $stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$
- $comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{c\}, \{d\}, \emptyset}\}$
- $grd(F) = \{\{a\}\}$

Relation between the Semantics of AFs

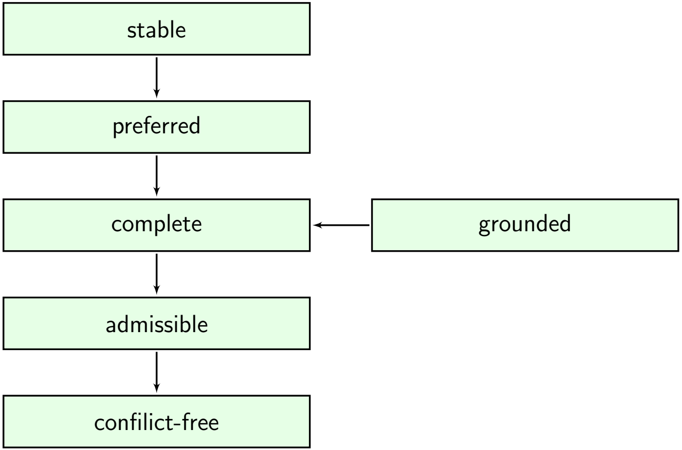


Figure: An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Well-Founded AF

Definition

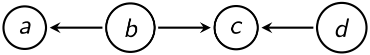
Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in stb(F)$ if $\forall a \in A$: $\exists b \in S$ s.t. $(b, a) \in R$

Example

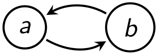


- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$
- $comp(F) = \{\{b, d\}\}$
- $grd(F) = \{\{b, d\}\}$
- $pref(F) = \{\{b, d\}\}$
- $stb(F) = \{\{b, d\}\}$

Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* ($S \in comp(F)$) if $S = \Gamma_F(S)$

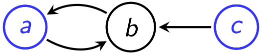


$comp(F) = \{\{\}, \{a\}, \{b\}\}$

Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* ($S \in comp(F)$) if $S = \Gamma_F(S)$

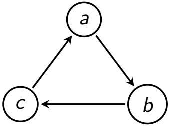


$comp(F) = \{\{a, c\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* $(S \in adm(F))$ if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* $(S \in pref(F))$ if S is $\subseteq$-maximal admissible
- *stable* $(S \in stb(F))$ if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* $(S \in grd(F))$ if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* $(S \in comp(F))$ if $S = \Gamma_F(S)$



$comp(F) = \{\{\}\}$

Decision Problem in AFs

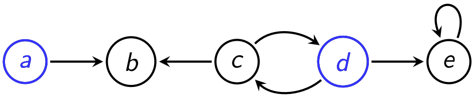
Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

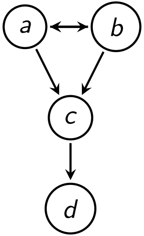
Answer to the existance decision problem:

- $Exists_{stb}(F)$?



$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$, $\quad Exists_{stb}(F)$: Yes

Decision Problem in AFs (ctd.)



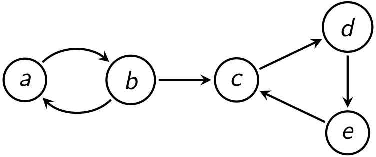
- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap pref(F) = \{d\}$, but $d \notin grd(F)$

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



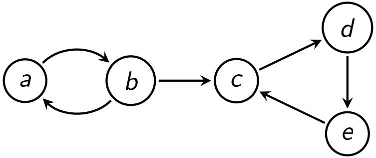
- $Cred_{cf}(b, F)$: is b contained in at least one conflict-free set of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F? No. c in not defended against the attack from e.
- $Cred_{adm}(c, F) = Cred_{pref}(c, F) = Cred_{stb}(c, F) = Cred_{comp}(c, F) = Cred_{grd}(c, F)$: No

Decision Problems on AFs (ctd.)

Characterize of Credulous Acceptance

Given an AF $F = (A, R)$:

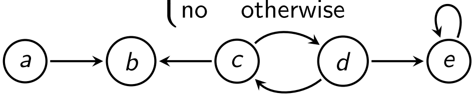
- $Cred_{cf}(a, F)$: Check if $(a, a) \in R$
- $Cred_{adm}(a, F) = Cred_{pref}(a, F) = Cred_{comp}(a, F)$
- $Cred_{grd}(a, F)$: Evaluate the grounded extension of F
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
- $Cred_{stb}(a, F)$: Evaluate the set of stable extensions of F

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & \quad a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



$pref(F) = \{\{a, c\}, \{a, d\}\}$, $\quad Skept_{pref}(a, F)$: yes

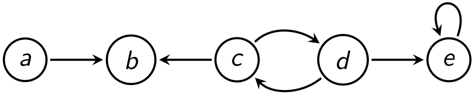
- $Skept_{pref}(a, F)$: is a contained in every $pref$-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free

- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Skeptical Decision Problems under Admissible Semantics



$adm(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \{\}\}$ $\qquad$ $Skept_{adm}(a, F)$: no.

- $Skept_{adm}(a, F)$: is a contained in every adm-extension of F? No

Decision Problems on AFs(ctd.)

Characterize of Skeptical Acceptance

- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{cf}(a, F)$: Trivially, No.
- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{adm}(a, F)$: Trivially, No.
- If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted.[1]
- $Skept_{grd}(F) = Cred_{grd}(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
  - There exists an AF F and argument a such that $Skept_{pref}(a, F)$: Yes. However, $Skept_{grd}(a, F)$: No.

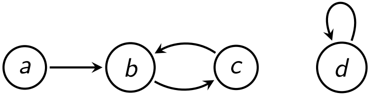
[1] This is only relevant for stable semantics.

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



$adm(F) = \{\{\}, \quad Ver_{adm}(S, F)$? Trivially, yes.

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension? NO
- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F? NO
- $Skept_\sigma(a)$: is a contained in every σ-extension of F? If it is not trivial, yes
- $Ver_\sigma(S, F)$: is S σ-extension of F? NO

Complexity Results

Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ
cf	in L	trivial	in L
adm	NP-c	trivial	in L
pref	NP-c	Π2-c	co-NP-c
comp	NP-c	P-c	in P
grd	P-c	P-c	P-c
stb	NP-c	co-NP-c	in P

Table: Complexity of reasoning with AFs.

Labelling-based Semantics of AFs

Definition

Given an AF $F = (A, R)$. A labelling is a function $\mathbb{L} : A \to \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$

- $\mathbb{L}(a) = \mathtt{in}$, i.e., a is accepted;
- $\mathbb{L}(a) = \mathtt{out}$, i.e., a is rejected;
- $\mathbb{L}(a) = \mathtt{undec}$, i.e., a is undecided/unknown.

- $\mathbb{L}_3(A) = \{a \mapsto \mathtt{in}, b \mapsto \mathtt{out}, c \mapsto \mathtt{in}\}$
- labelling-based argumentation semantics provides a way to select reasonable labellings among all the possible ones, according to some criterion.

Labelling-based Semantics of AFs (ctd.)

Each argument is labelled `in`, `out` or `undec`

an argument is `in` $\Leftrightarrow$
all its attackers are `out`

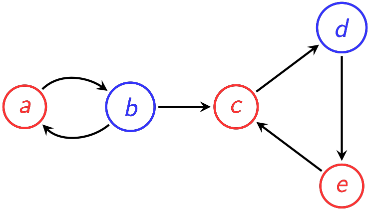
an argument is `out` $\Leftrightarrow$
it has an attacker that is `in`

an argument is `undec` $\Leftrightarrow$
not all its attackers are `out` and it does not have an attacker that is `in`

Maximisation/Minimisation

maximal: there is no other that has the same plus something

minimal: there is no other that has the same minus something



a, b, c, d, e max `in`, max `out`, min `undec`

Extension-based Semantics vs. Labelling-based Semantics

`in` $\Leftrightarrow$ all attackers are `out`

`out` $\leftrightarrow$ there is an attacker that is `in`

`undec` $\Leftrightarrow$ not all attackers are `out`, and no attacker is `in`

restriction on labelling	Extension-based semantics
maximal `in`	preferred semantics
maximal `out`	preferred semantics
maximal `undec`	grounded semantics
minimal `in`	grounded semantics
minimal `out`	grounded semantics
empty `undec`	Stable semantics

An extension is the in-labelled part of a labelling

Admissible Labelling

Definition: Admissible labelling

Given an AF $F = (A, R)$. Let $\mathbb{L}$ be a labelling function on A. $\mathbb{L}$ is an admissible labelling iff for each argument $a \in A$ it holds that:

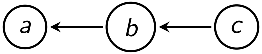
- if $\mathbb{L}(a) = \texttt{in}$ then for each b, such that $(b, a) \in R$ then $\mathbb{L}(b) = \texttt{out}$;
- if $\mathbb{L}(a) = \texttt{out}$ then there exists $b \in A$, such that $(b, a) \in A$ and $\mathbb{L}(b) = \texttt{in}$.

Admissible Labelling

Admissible labeling:

$\texttt{in} \Rightarrow$ all attackers are $\texttt{out}$

$\texttt{out} \Rightarrow$ there is an attacker that is $\texttt{in}$



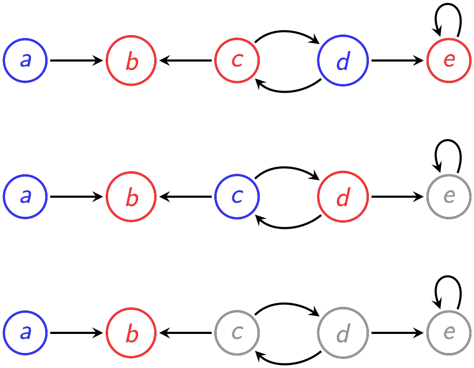
- $adm_{\mathbb{L}_1}(F) = \{a \mapsto \texttt{undec}, b \mapsto \texttt{undec}, c \mapsto \texttt{undec}\}$ $adm_1(F) = \{\}$
- $adm_{\mathbb{L}_2}(F) = \{a \mapsto \texttt{undec}, b \mapsto \texttt{undec}, c \mapsto \texttt{in}\}$ $adm_2(F) = \{c\}$
- $adm_{\mathbb{L}_3}(F) = \{a \mapsto \texttt{undec}, b \mapsto \texttt{out}, c \mapsto \texttt{in}\}$ $adm_3(F) = \{c\}$
- $adm_{\mathbb{L}_4}(F) = \{a \mapsto \texttt{in}, b \mapsto \texttt{out}, c \mapsto \texttt{in}\}$ $adm_4(F) = \{a, c\}$

Complete Labelling

`in` ⇔ all attackers are `out`

`out` ⇔ there is an attacker that is `in`

`undec` ⇔ not all attackers are `out`, and no attacker is `in`



Labelling-based Semantics

Admissible Labelling

in $\Leftrightarrow$ all attackers are out

out $\Leftrightarrow$ there is an attacker that is in

Complete labelling

in $\Leftrightarrow$ all attackers are out

out $\Leftrightarrow$ there is an attacker that is in

undec $\leftrightarrow$ not all attackers are out, and no attacker is in

Grounded labelling: Complete with min in/ min out / max undec

Preferred labelling: Complete with max in/ max out

Stable labelling: complete with no undec

because versus *but-then*

$$\top \ldots \xRightarrow[\xleftarrow{\quad}]{} \xRightarrow[\xleftarrow{\quad}]{} \xRightarrow[\xleftarrow{\quad}]{} \Phi$$
because because because

reasoning goes backward

proponent constructs path

originates from true

both parties become committed

$$\Phi \xRightarrow[\xrightarrow{\quad}]{} \xRightarrow[\xrightarrow{\quad}]{} \xRightarrow[\xrightarrow{\quad}]{} \ldots \bot$$
but-then but-then but-then

reasoning goes forward

opponent constructs path

leads to false

only proponent becomes committed

Preferred Semantics as Socratic Discussion

Recall: Definition

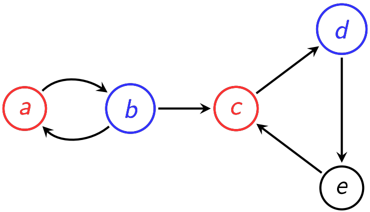
Admissible labeling:

> if argument is `in` then all attackers are `out`
>
> if argument is `out` then it has an attacker that is `in`

Recall: Proposition

An argument is in a preferred extension
  iff it is in a complete extension
  iff it is in an admissible set
  iff it is labelled `in` by an admissible labelling

Preferred Semantics as Socratic Discussion



- P: in(d): I have an admissible labelling in which d is labelled in
- O: out(c): But then in your labelling it must also be the case that d's attacker c is labelled out. Based on which grounds?
- P: in(b): c is labelled out because b is labelled in.
- O: out(a): But then in your labelling it must also be the case that b's attacker a is labelled out. Based on which grounds?
- P: in(b): a is labelled out because b is labelled in.
  If O cannot make a move any more, P wins the discussion.

Preferred Semantics as Socratic Discussion

- P: $\texttt{in}(d)$: I have an admissible labelling in which d is labelled $\texttt{in}$
- O: $\texttt{out}(c)$: But then in your labelling it must also be the case that d's attacker c is labelled $\texttt{out}$. Based on which grounds?
- P: $\texttt{in}(b)$: c is labelled $\texttt{out}$ because b is labelled $\texttt{in}$.
- O: $\texttt{out}(a)$: But then in your labelling it must also be the case that b's attacker a is labelled $\texttt{out}$. Based on which grounds?
- P: $\texttt{in}(b)$: a is labelled $\texttt{out}$ because b is labelled $\texttt{in}$.

1. Each move of P (except the first) contains an attacker of the directly preceding move of O.
2. Each move of O contains an attacker of <u>some</u> previous move of P.
3. O is not allowed to repeat his moves.
4. P is allowed to repeat his moves.

Preferred Semantics as Socratic Discussion

- P: in(e): I have an admissible labelling in which e is labelled in
- O: out(d): But then in your labelling it must also be the case that e's attacker d is labelled out. Based on which grounds?
- P: in(c): d is labelled out because c is labelled in.
- O: out(e): But then in your labelling it must also be the case that c's attacker e is labelled out.
  This **contradicts** with your earlier claim that e is labelled in
- **If O uses an argument previously used by P, then O wins the discussion.**

Preferred Semantics as Socratic Discussion

Winning rules

1. If O uses an argument previously used by P, then O **wins the discussion**.
2. If P uses an argument previously used by O, then O **wins the discussion**.
3. If P cannot make a move any more, O **wins the discussion**.
4. If O cannot make a move any more, P **wins the discussion**.

Preferred Semantics as Socratic Discussion

Theorem

Argument a is labelled `in` by at least one admissible labelling iff M can win the Socratic discussion game (for a).

Theorem

Argument a is labelled `in` by at least one preferred labelling iff M can win the Socratic discussion game (for a).