Knowledge Representation
2023/2024

Exercise Sheet 5 – Practice Exam – Solutions
11th December 2023

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To write formulas on your keyboard, you should use the following translation table:

$\top$	top	$\bot$	bot	$\neg$	not	$\sqcap$	and
$\sqcup$	or	$\exists$	exists	$\forall$	forall	$\geq$	min
$\leq$	max	$\sqsubseteq$	subsumedBy	$\equiv$	equivalent		
$\vee$	or	$\wedge$	and	$\rightarrow$	->	$\leftrightarrow$	<->

Exercise 5.1 Translate the following sentences from description logic into natural language or vice versa.

(a) Every *Person has* a *Hand*, which *has Finger*s.

(b) Every *Hand* has at least five *Finger*s.

(c) A *Narcicist* is defined as a *Person* who *loves* himself.

(d) $\mathrm{Car} \sqsubseteq \exists \mathrm{hasPart.Engine}$

(e) $\exists \mathrm{permitted.Driving} \sqsubseteq \exists \mathrm{owns.DrivingLicense} \sqcap \exists \mathrm{hasAge.FullAge}$

Solution:

(a) `Person subsumedBy exists has.(Hand and exists has.Finger)`

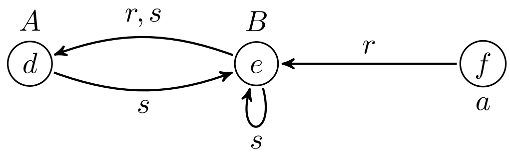
(b) `Hand subsumedBy min 5 has.Finger`

(c) `Narcicist equivalent exists loves.Self`

(d) Every car has an engine as part.

(e) If someone is permitted driving then he owns a driving license and has an age that is a full age (he is of full age).

Exercise 5.2 Consider the interpretation illustrated in the following picture:



Which elements belong to the interpretations of the following concepts:

(a) $\exists r.(A \sqcup B)$

(b) $\forall s.B$

(c) $\exists r^{-}.(\exists s.\mathsf{Self} \sqcup \{a\})$

Solution:

(a) e,f

(b) d,f

(c) e,d

Exercise 5.3 Consider the following $\mathcal{EL}$ TBox:

$$\mathcal{T} = \{ \quad B \sqcap D \sqsubseteq A, \qquad B \sqsubseteq \exists s.B,$$
$$\exists s.\top \sqsubseteq D, \qquad \exists r.A \sqsubseteq E \sqcap F \quad \}$$

Use the $\mathcal{EL}$ precompletion method to compute all subsumers of $\exists r.B$. Which concepts are assigned to the initial domain element?

Solution:

```
exists r.B
top
exists r.A
E and F
E
F
```

Exercise 5.4 Consider the following $\mathcal{ALC}$ TBox:

$$\mathcal{T} = \{ \quad \top \sqsubseteq C \sqcap \exists r.B, \quad \top \sqsubseteq \forall r.\forall r.\neg B \sqcup A\}$$

Use the tableaux method to prove the satisfiability of $\exists r.B$. To illustrate your steps, list the statements of one branch that contains a clash, and of another branch that is complete and contains no clash.

Solution:

```
Branch with a clash:

a: exists r.B
(a,b): r
b: B
a: C and exists r.B
a: C
a: exists r.B
a: forall r.forall r.not B or A
a: forall r.forall r.not B
b: forall r.not B
b: C and exists r.B
b: C
b: exists r.B
b: forall r.forall r.not B or A
b: A
(b,c): r
c: B
c: not B



Branch without a clash:
a: exists r.B
(a,b): r
b: B
a: C and exists r.B
a: C
a: exists r.B
a: forall r.forall r.not B or A
a: A
b: C and exists r.B
b: C
b: exists r.B
b: forall r.forall r.not B or A
b: A
```

```
(b,c): e
c: B
```

`c` is blocked by `b` and no more inferences are possible.

Exercise 5.5 Which of the following statements is true:

(a) Reasoning in $\mathcal{EL}$ is generally harder than reasoning in propositional logic.

(b) An Algorithm is *complete* for a decision problem if it provides a yes or a no answer to every input.

(c) Every $\mathcal{SROIQ}(D)$ axiom can be expressed in OWL.

Solution:

(a) false

(b) false

(c) true

Exercise 5.6 Consider the default theory $T = (W, D)$ with W, D defined as follows:

$$W = \{\ d, d \to e\ \}$$
$$D = \left\{\quad \delta_1 = \frac{e : f}{f}, \quad \delta_2 = \frac{d : \neg f}{\neg f}, \quad \delta_3 = \frac{f : e}{g} \quad \right\}$$

Which processes and which extensions does the process tree algorithm produce? (You may write δ_1 as "delta1" etc.).

Solution:

```
Process: delta1, delta3. Extension= Cn({d, d -> e, f, g})
Process: delta2. Extension = Cn({d, d -> e, not f})
```

Exercise 5.7 We consider the AF $F = (A, R)$ such that $A = \{a, b, c, d\}$ and

$$R = \{(a, b), (b, b), (b, c), (c, d), (d, c)\}$$

(a) Give all preferred extensions of F.

(b) Give the grounded extension of the AF.

(c) Answer the following decision problems. Either provide a witness for your answer or an explanation:

  (i) $\text{Skept}_{\text{adm}}(a, F)$

  (ii) $\text{Exists}_{\text{stb}}(F)$

  (iii) $\text{Cred}_{\text{com}}(d, F)$

Solution:

(a) $\text{prf}(F) = \{\{a, c\}, \{a, d\}\}$

(b) $\text{grd}(F) = \{a\}$

(c) (i) No, the empty set $\emptyset$ is an admissible extension.

  (ii) Yes: $\text{stb}(F) = \{\{a, c\}, \{a, d\}\}$.

  (iii) Yes. Because $\{a, d\}$ is a complete extension.

Exercise 5.8 Consider the AF $F = (\{a, b, c, d\}, \{(a, b), (b, a), (a, c), (b, c), (c, d), (d, c)\})$. Which of the following labelings is admissible, and which of them is complete, both, or neither? If any of the following labelings is not admissible, explain why it is not admissible.

(a) $\{a \to \texttt{in}, b \to \texttt{out}, c \to \texttt{out}, d \to \texttt{undec}\}$

(b) $\{a \to \texttt{out}, b \to \texttt{out}, c \to \texttt{in}, d \to \texttt{out}\}$

(c) $\{a \to \texttt{in}, b \to \texttt{out}, c \to \texttt{undec}, d \to \texttt{undec}\}$

Solution:

(a) is admissible and not complete, (b) is not admissible and also not complete, and (c) is admissible and not complete. (b) is not admissible because a and b are out and both of them are not attacked by any node labeled `in`.

Exercise 5.9 Consider the AF $F = (A, R)$ with $A = \{a, b, c, d\}$ and $R = \{(a, b), (b, d), (a, c), (a, d)\}$. The proponent claims that d is labeled with `in`. Present a preferred discussion game for this claim. Indicate whether the proponent wins the game or the opponent.

Solution:

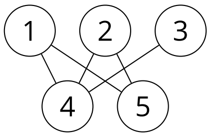
```
P: d is 'in'
O: Then a is 'out'. Why?
P is out of moves and O wins.
Then O wins the game because P is out of moves.
That is, there is no preferred labelling in which d is labelled 'in'
```

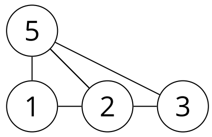
Exercise 5.10 Which of the following statements is true?

(a) If $\alpha \models \beta$ and $\Pr(\alpha) = 0$, then $\Pr(\beta) = 0$

(b) If $\alpha \models \beta$ and $\beta \models \gamma$, then $\Pr(\alpha \mid \beta) \geq \Pr(\alpha \mid \gamma)$

(c) In the interaction graph G below, if we eliminate 4, we get G':

G: 

G': 

(d) Assume G' as above. One possible elimination order according to the "min-degree heuristic" is 1, 5, 3, 2.

Solution:

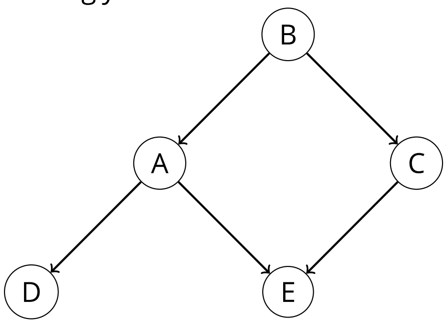
(a) false

(b) true

(c) false

(d) true

Exercise 5.11 Consider the following Bayesian network in the figure, and answer the questions accordingly.



B	Θ_B
true	0.7
false	0.3

B	A	$\Theta_{A\|B}$
true	true	0.1
true	false	0.9
false	true	0.4
false	false	0.6

B	C	$\Theta_{C\|B}$
true	true	0.8
true	false	0.2
false	true	0.6
false	false	0.4

A	D	$\Theta_{D\|A}$
true	true	0.2
true	false	0.8
false	true	0.5
false	false	0.5

A	C	E	$\Theta_{E\|AC}$
true	true	true	0.1
true	true	false	0.9
true	false	true	0.05
true	false	false	0.95
false	true	true	0.7
false	true	false	0.3
false	false	true	0.4
false	false	false	0.6

(a) Which of the following statements are true?

(i) $\{D\} \perp\!\!\!\perp \{C\} \mid \{B\}$

(ii) $\{E\} \perp\!\!\!\perp \{B\} \mid \{A\}$

(iii) $\{B, D\} \perp\!\!\!\perp \{E\} \mid \{A, C\}$

(iv) $\{C\} \perp\!\!\!\perp \{A\} \mid \{B, E\}$

(b) Calculate the probability $\Pr(A = \text{true}, B = \text{false}, E = \text{true})$.

(c) Calculate the probability $\Pr(A = \text{false}, B = \text{true} \mid E = \text{true})$.

Solution:

(a) (i) true

(ii) true

(iii) false

(iv) true

(b) 0.096

(c) 0.681