Knowledge Representation

Lecture 9: Decision Problems on AFs and labelling-based Semantics

Atefeh Keshavarzi

24, November 2023

Flashback: The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Flashback: The Overall Process

Steps

- **Starting point: knowledge-base**
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

Flashback: The Overall Process

Steps

- Starting point: knowledge-base
- **Form arguments**
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

$\langle\{w, w \rightarrow \neg s\}, \neg s\rangle$

$\langle\{s, s \rightarrow \neg r\}, \neg r\rangle$ $\langle\{r, r \rightarrow \neg w\}, \neg w\rangle$

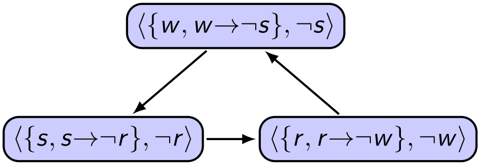
Flashback: The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- **Identify conflicts**
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

$$\Delta = \{s, r, w, s \to \neg r, r \to \neg w, w \to \neg s\}$$



Flashback: The Overall Process

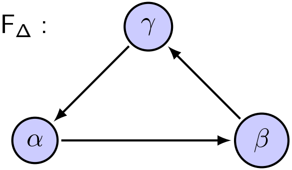
Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- **Abstract from internal structure**
- Resolve conflicts
- Draw conclusions

Example

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

$\mathrm{AF}_\Delta :$



Flashback: The Overall Process

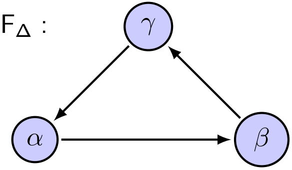
Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- **Resolve conflicts**
- Draw conclusions

Example

$$\Delta = \{s, r, w, s \to \neg r, r \to \neg w, w \to \neg s\}$$

AF_Δ :



$$\textit{pref}(\mathrm{AF}_\Delta) = \{\emptyset\}$$
$$\textit{stage}(\mathrm{AF}_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$$

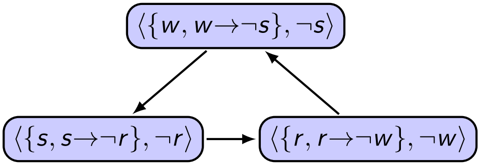
Flashback: The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- **Draw conclusions**

Example

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

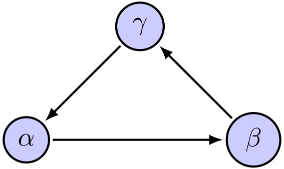


$$Cn_{pref}(\mathrm{AF}_\Delta) = Cn(\top)$$
$$Cn_{stage}(\mathrm{AF}_\Delta) = Cn(\neg r \vee \neg w \vee \neg s)$$

Dung's Abstract Argumentation Frameworks

Seminal Paper by *Phan Minh Dung*:
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.
Artif. Intell., 77(2):321–358, 1995.

Example



Remark

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)

Applications of Formalisms of Argumentation

Abstraction allows to compare several KR formalisms on a conceptual level (*calculus of conflict*)

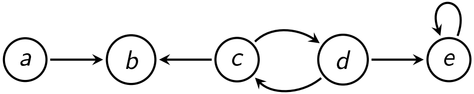
- Legal reasoning:
  - [Bench-Capon and Dunne, 2005]
  - [Collenette et al., 2020]
- Multi-agent systems
  - [McBurney et al., 2012]
  - [Amgoud et al., 2007]
- Discussion game
  - [Caminada, 2018]
  - [Keshavarzi Zafarghandi et al., 2020]
- Recommended system [Rago et al., 2018]
- Explainable AI
  - [Cocarascu et al., 2019]
  - Argumentative XAI: A Survey [Cyras et al., 2021]
  - [Chi and Liao, 2022]

Flashback: Dung's Abstract Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

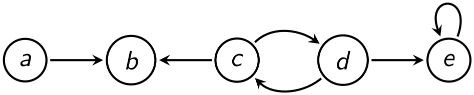


Flashback: Dung's Abstract Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")



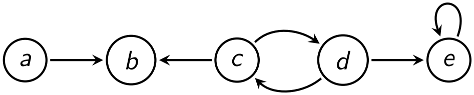
How can we assess the credibility of an argument in an AF?

An argument is believable if it can be argued successfully against the counterarguments.

- Semantics: Methods used to clarify the acceptance of arguments
  - Extension-based semantics
  - Labelling-based semantics

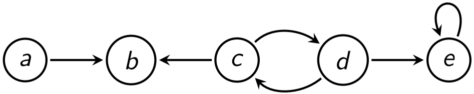
Flashback: Semantics of AFs (ctd.)

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$



Flashback: Semantics of AFs (ctd.)

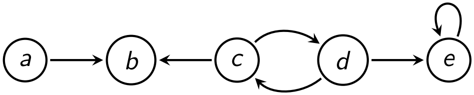
- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$



$$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, d\}\}$$

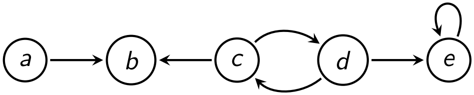
Flashback: Semantics of AFs (ctd.)

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.



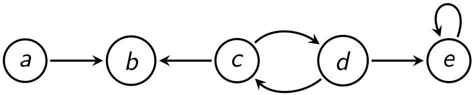
Flashback: Semantics of AFs (ctd.)

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$



Flashback: Semantics of AFs (ctd.)

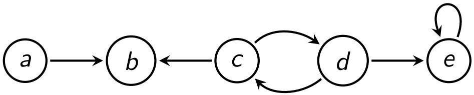
- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$



- $S_1 = \{\}$
- $S_2 = \{a\}$
- $S_3 = \{c\}$
- $S_4 = \{b\}$

Flashback: Semantics of AFs (ctd.)

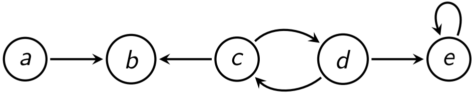
- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$



- $S_1 = \{\}$ $\qquad \Gamma_F(S_1) = \{a\}$
- $S_2 = \{a\}$
- $S_3 = \{c\}$
- $S_4 = \{b\}$

Flashback: Semantics of AFs (ctd.)

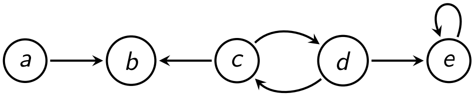
- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$



- $S_1 = \{\}$ $\qquad \Gamma_F(S_1) = \{a\}$
- $S_2 = \{a\}$ $\qquad \Gamma_F(S_2) = \{a\}$
- $S_3 = \{c\}$
- $S_4 = \{b\}$

Flashback: Semantics of AFs (ctd.)

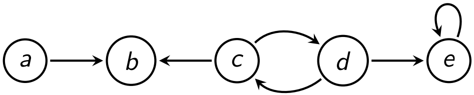
- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$



- $S_1 = \{\}$ $\qquad \Gamma_F(S_1) = \{a\}$
- $S_2 = \{a\}$ $\qquad \Gamma_F(S_2) = \{a\}$
- $S_3 = \{c\}$ $\qquad \Gamma_F(S_3) = \{a, c\}$
- $S_4 = \{b\}$

Flashback: Semantics of AFs (ctd.)

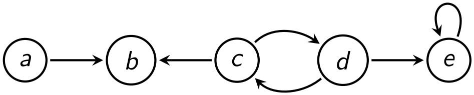
- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$



- $S_1 = \{\}$ $\quad\quad \Gamma_F(S_1) = \{a\}$
- $S_2 = \{a\}$ $\quad\quad \Gamma_F(S_2) = \{a\}$
- $S_3 = \{c\}$ $\quad\quad \Gamma_F(S_3) = \{a, c\}$
- $S_4 = \{b\}$ $\quad\quad \Gamma_F(S_4) = \{a\}$

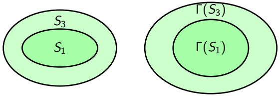
Flashback: Semantics of AFs (ctd.)

- $S \subseteq A$ is conflict-free if, for each $a, b \in S$, $(a, b) \notin R$
- An argument $a \in A$ is defended by S (or, it is *acceptable* w.r.t. S) in F, if for each $b \in A$ with $(b, a) \in R$ then there exists a $c \in S$, such that $(c, b) \in R$.
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$



- $S_1 = \{\}$ $\qquad \Gamma_F(S_1) = \{a\}$
- $S_2 = \{a\}$ $\qquad \Gamma_F(S_2) = \{a\}$
- $S_3 = \{c\}$ $\qquad \Gamma_F(S_3) = \{a, c\}$
- $S_4 = \{b\}$ $\qquad \Gamma_F(S_4) = \{a\}$

Γ_F is a monotonic function



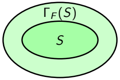
Flashback: Semantics of AFs

- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$



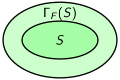
Flashback: Semantics of AFs

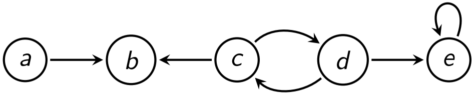
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$





$adm(F) = \{\{\}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \cancel{\{b, d\}}\}$

Flashback: Semantics of AFs

- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
  That is, for each $T \subseteq A$ admissible in F, $S \not\subset T$.

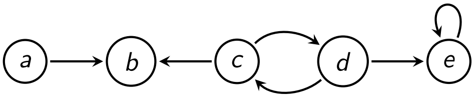
Flashback: Semantics of AFs

- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
  That is, for each $T \subseteq A$ admissible in F, $S \not\subset T$.



$pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Flashback: Semantics of AFs

- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
  That is, for each $T \subseteq A$ admissible in F, $S \not\subset T$.
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

$\Gamma_F(S) = S \quad \Gamma_F(T) = T \quad \Rightarrow \quad S \subseteq T$

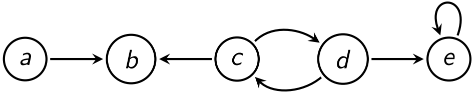
Flashback: Semantics of AFs

- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

Semantics of AFs

Given an AF $F = (A, R)$. A conflict-free set S is

- *admissible* ($S \in adm(F)$) if $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
  That is, for each $T \subseteq A$ admissible in F, $S \not\subset T$.
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$



$grd(F) = \{\{a\}\}$

Complete Semantics

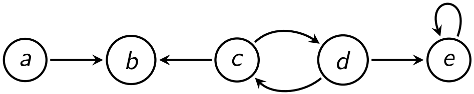
Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

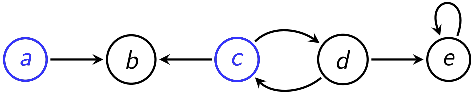


- What are the complete extensions for F?

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

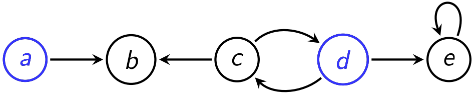


$comp(F) = \{\{a, c\},$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

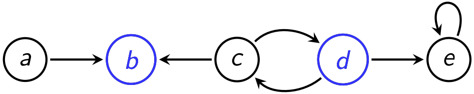


$comp(F) = \{\{a, c\}, \{a, d\},$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

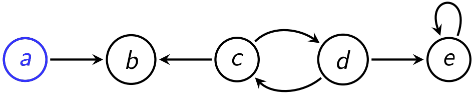


$comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}},$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.

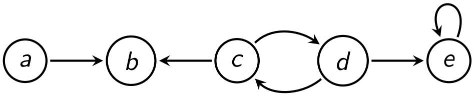


$comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\},$

Complete Semantics

Definition

Given an AF $F = (A, R)$. A conflict set $S \subseteq A$ is a *complete extension* ($S \in comp(F)$) if $S = \Gamma_F(S)$. That is, each $a \in A$ defended by S in F is contained in S.



$comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics (ctd.)

Properties of the Extensions

Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
- F has at least one complete extension.

Characterize of Semantics (ctd.)

Properties of the Extensions

Given AF $F = (A, R)$,

- F has a unique grounded extension.
- the grounded extension of F is the subset-minimal complete extension of F.
- F has at least one complete extension.

Remark

Since there exists exactly one grounded extension for each AF F, we often write $grd(F) = S$ instead of $grd(F) = \{S\}$.

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

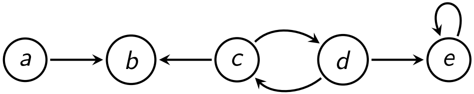
- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.

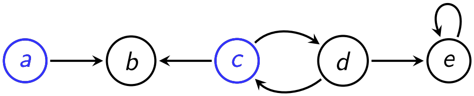


Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.



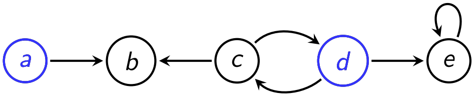
$stb(F) = \{\cancel{\{a, c\}},$

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.



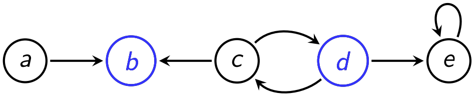
$stb(F) = \{\cancel{\{a, c\}}, \{a, d\},$

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.



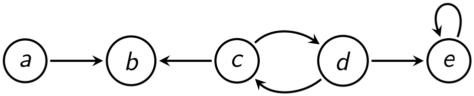
$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\},}$

Stable Semantics

Definition

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *stable extension* of F ($S \in stb(F)$) if

- S is conflict-free in F
- for each $a \in A \setminus S$: there exists a $b \in S$ such that $(b, a) \in R$.



$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Characterize of Semantics (ctd.)

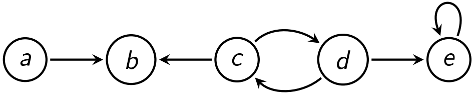
Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

- Stable semantics reflect the ‘zero-and-one’ character of classical logic in argumentation frameworks.
- An AF may not have any stable extension.

Relation between the Semantics of AFs



- $cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \{\}\}$
- $adm(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \{\}\}$
- $pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$
- $stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$
- $comp(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{c\}, \{d\}, \emptyset}\}$
- $grd(F) = \{\{a\}\}$

Relation between the Semantics of AFs

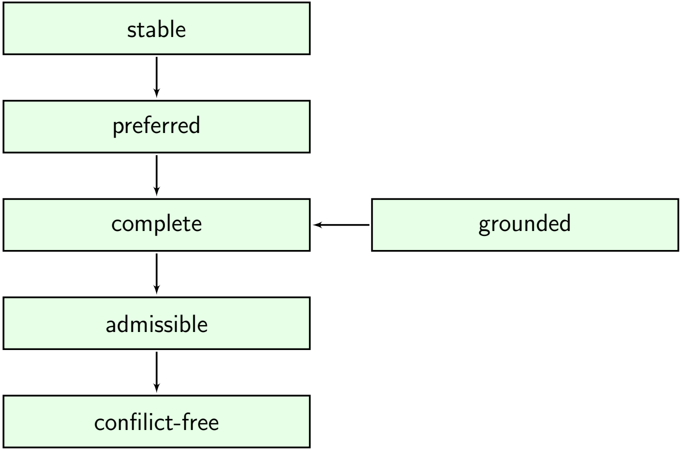


Figure: An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Well-Founded AF

Definition

Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

Well-Founded AF

Definition

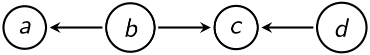
Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in adm(F)$ if $S \subseteq \Gamma_F(S)$

Example



- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$

Well-Founded AF

Definition

Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in comp(F)$ if $S = \Gamma_F(S)$

Example

$$a \longleftarrow b \longrightarrow c \longleftarrow d$$

- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$
- $comp(F) = \{\{b, d\}\}$

Well-Founded AF

Definition

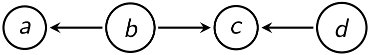
Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in grd(F)$ if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

Example



- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$
- $comp(F) = \{\{b, d\}\}$
- $grd(F) = \{\{b, d\}\}$

Well-Founded AF

Definition

Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in \mathit{pref}(F)$ if S is $\subseteq$-maximal admissible

Example

$$a \longleftarrow b \longrightarrow c \longleftarrow d$$

- $\mathit{adm}(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$
- $\mathit{comp}(F) = \{\{b, d\}\}$
- $\mathit{grd}(F) = \{\{b, d\}\}$
- $\mathit{pref}(F) = \{\{b, d\}\}$

Well-Founded AF

Definition

Given an AF $F = (A, R)$. F is well-founded iff there exists no infinite sequence $a_1, \ldots, a_i, \ldots$ s.t. $(a_{i+1}, a_i) \in R$, for each i.

Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

- $S \in stb(F)$ if $\forall a \in A$: $\exists b \in S$ s.t. $(b, a) \in R$

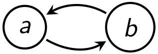
Example

$$a \longleftarrow b \longrightarrow c \longleftarrow d$$

- $adm(F) = \{\{\}, \{b\}, \{d\}, \{b, d\}\}$
- $comp(F) = \{\{b, d\}\}$
- $grd(F) = \{\{b, d\}\}$
- $pref(F) = \{\{b, d\}\}$
- $stb(F) = \{\{b, d\}\}$

Semantics of AFs (ctd.)

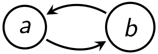
Is there always at least one argument that is skeptically accepted?



Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$

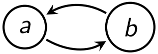


$cf(F) = \{\{\}, \{a\}, \{b\}\}$

Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* $(S \in adm(F))$ if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$

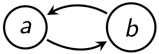


$adm(F) = \{\{\}, \{a\}, \{b\}\}$

Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible

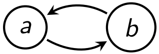


$pref(F) = \{\{a\}, \{b\}\}$

Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* $(S \in adm(F))$ if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* $(S \in pref(F))$ if S is $\subseteq$-maximal admissible
- *stable* $(S \in stb(F))$ if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$

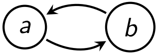


$stb(F) = \{\{a\}, \{b\}\}$

Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

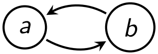


$grd(F) = \{\{\}\}$

Semantics of AFs (ctd.)

Is there always at least one argument that is skeptically accepted?

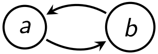
- $S \in \mathit{cf}(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in \mathit{adm}(F)$) if $S \in \mathit{cf}(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in \mathit{pref}(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in \mathit{stb}(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in \mathit{grd}(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* ($S \in \mathit{comp}(F)$) if $S = \Gamma_F(S)$



$\mathit{comp}(F) = \{\{\}, \{a\}, \{b\}\}$

Semantics of AFs (ctd.)

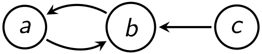
Is there always at least one argument that is skeptically accepted?



- $cf(F) = \{\{\}, \{a\}, \{b\}\}$
- $adm(F) = \{\{\}, \{a\}, \{b\}\}$
- $pref(F) = \{\{a\}, \{b\}\}$ $\quad \cap pref(F) = \{\}$
- $stb(F) = \{\{a\}, \{b\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a\}, \{b\}\}$

Semantics of AFs (ctd.)

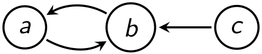
Is the existence of a loop always problematic?



Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

- $S \in \mathit{cf}(F)$ if for each $a, b \in S$, $(a, b) \notin R$

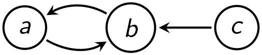


$\mathit{cf}(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, c\}\}$

Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$

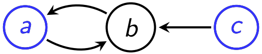


$adm(F) = \{\{\}, \{a\}, \{c\}, \{a, c\}\}$

Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible

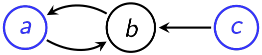


$pref(F) = \big\{\{a, c\}\big\}$

Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$

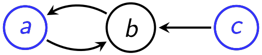


$stb(F) = \{\{a, c\}\}$

Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in grd(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

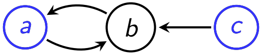


$grd(F) = \{\{a, c\}\}$

Semantics of AFs (ctd.)

Is the existence of a loop always problematic?

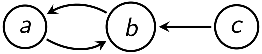
- $S \in \mathit{cf}(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in \mathit{adm}(F)$) if $S \in \mathit{cf}(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in \mathit{pref}(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in \mathit{stb}(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in \mathit{grd}(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* ($S \in \mathit{comp}(F)$) if $S = \Gamma_F(S)$



$\mathit{comp}(F) = \{\{a, c\}\}$

Semantics of AFs (ctd.)

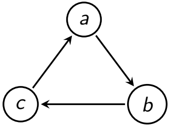
Is the existence of a loop always problematic?



- $cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, c\}\}$
- $adm(F) = \{\{\}, \{a\}, \{a, c\}\}$
- $pref(F) = \{\{a, c\}\}$
- $stb(F) = \{\{a, c\}\}$
- $grd(F) = \{\{a, c\}\}$
- $comp(F) = \{\{a, c\}\}$

Semantics of AFs (ctd.)

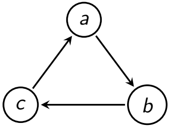
What are the effects of odd cycles on semantics?



Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$

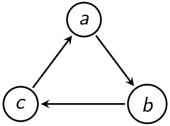


$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$

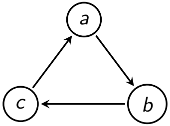


$adm(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

- $S \in \mathit{cf}(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in \mathit{adm}(F)$) if $S \in \mathit{cf}(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in \mathit{pref}(F)$) if S is $\subseteq$-maximal admissible

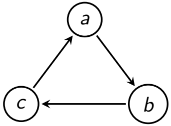


$\mathit{pref}(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

- $S \in cf(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in adm(F)$) if $S \in cf(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in pref(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in stb(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$

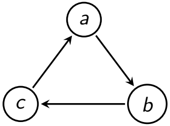


$stb(F) = \{\}$ F does not have any stable extension

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

- $S \in \mathit{cf}(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in \mathit{adm}(F)$) if $S \in \mathit{cf}(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in \mathit{pref}(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in \mathit{stb}(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in \mathit{grd}(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$

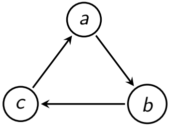


$\mathit{grd}(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?

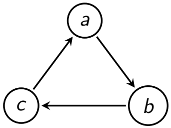
- $S \in \mathit{cf}(F)$ if for each $a, b \in S$, $(a, b) \notin R$
- *admissible* ($S \in \mathit{adm}(F)$) if $S \in \mathit{cf}(F)$ and $S \subseteq \Gamma_F(S)$
- *preferred* ($S \in \mathit{pref}(F)$) if S is $\subseteq$-maximal admissible
- *stable* ($S \in \mathit{stb}(F)$) if for each $a \in A \setminus S$: there exists $b \in S$ such that $(b, a) \in R$
- *grounded* ($S \in \mathit{grd}(F)$) if S is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- *complete* ($S \in \mathit{comp}(F)$) if $S = \Gamma_F(S)$



$\mathit{comp}(F) = \{\{\}\}$

Semantics of AFs (ctd.)

What are the effects of odd cycles on semantics?



- $cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}\}$
- $adm(F) = \{\{\}\}$
- $pref(F) = \{\{\}\}$
- $stb(F) = \{\}$ F does not have any stable extension
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}\}$

Decision problems on AFs

- Existence of extensions
- Credulous acceptance
- Skeptical acceptance
- Verifying an extension

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- Recall: Any AF has at least one *admissible/preferred/grounded/complete/conflict-free* extension.

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- Recall: Any AF has at least one *admissible/preferred/grounded/complete/conflict-free* extension.
- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, grd, comp, cf\}$, is trivially *yes*.

Decision Problem in AFs

Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- $Exists_{stb}(F)$?

Decision Problem in AFs

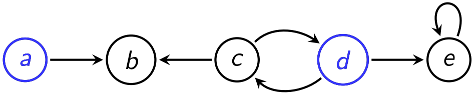
Existence of Extensions

Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

- $Exists_{stb}(F)$?



$stb(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}\}$, $\quad Exists_{stb}(F)$: Yes

Decision Problem in AFs

Existence of Extensions

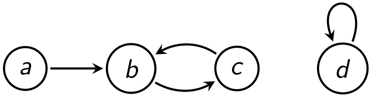
Given an AF $F = (A, R)$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Exists_\sigma(F)$: Does F has at least one σ-extension?

$$Exists_\sigma(F) = \begin{cases} \text{yes} & \text{if } F \text{ has at least one } \sigma\text{-extension} \\ \text{no} & \text{otherwise} \end{cases}$$

Answer to the existance decision problem:

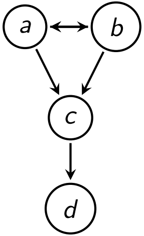
- $Exists_{stb}(F)$?

Example



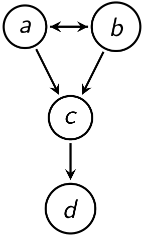
$stb(F) = \{\}, \quad Exists_{stb}(F)$: No

Decision Problem in AFs (ctd.)



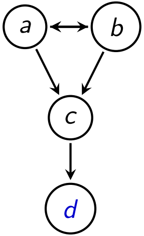
- $\mathit{Exists}_\sigma(F)$, for $\sigma \in \{\mathit{adm}, \mathit{pref}, \mathit{stb}, \mathit{grd}, \mathit{comp}, \mathit{cf}\}$ is yes.

Decision Problem in AFs (ctd.)



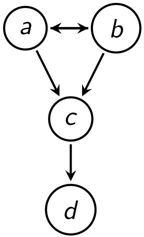
- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$

Decision Problem in AFs (ctd.)



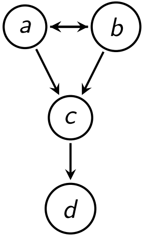
- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap pref(F) = \{d\}$, but $d \notin grd(F)$

Decision Problem in AFs (ctd.)



- $\mathit{Exists}_\sigma(F)$, for $\sigma \in \{\mathit{adm}, \mathit{pref}, \mathit{stb}, \mathit{grd}, \mathit{comp}, \mathit{cf}\}$ is yes.
- $\mathit{adm}(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $\mathit{pref}(F) = \{\{a, d\}, \{b, d\}\}$
- $\mathit{stb}(F) = \{\{a, d\}, \{b, d\}\}$
- $\mathit{grd}(F) = \{\{\}\}$
- $\mathit{comp}(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap \mathit{pref}(F) = \{d\}$, but $d \notin \mathit{grd}(F)$

Decision Problem in AFs (ctd.)



- $Exists_\sigma(F)$, for $\sigma \in \{adm, pref, stb, grd, comp, cf\}$ is yes.
- $adm(F) = \{\{\}, \{a\}, \{b\}, \{a, d\}, \{b, d\}\}$
- $pref(F) = \{\{a, d\}, \{b, d\}\}$
- $stb(F) = \{\{a, d\}, \{b, d\}\}$
- $grd(F) = \{\{\}\}$
- $comp(F) = \{\{\}, \{a, d\}, \{b, d\}\}$
- $\cap pref(F) = \{d\}$, but $d \notin grd(F)$

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

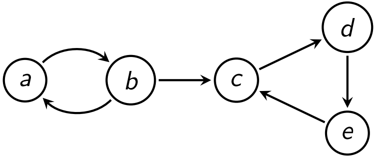
$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \ a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



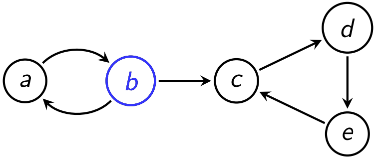
- $Cred_{cf}(b, F)$: is b contained in at least one conflict-free set of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_{cf}(a, F) = \begin{cases} \text{yes} & \text{if } (a, a) \notin R, \\ \text{no} & \text{otherwise} \end{cases}$$



$cf(F) = \{\{b\},\quad Cred_{cf}(b, F)$: Yes

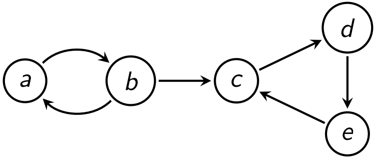
- $Cred_{cf}(b, F)$: is b contained in at least one conflict-free set of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



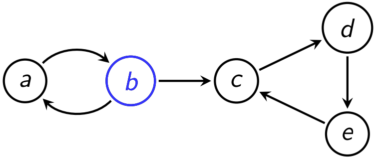
- $Cred_{adm}(b, F)$: is b contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



$adm(F) = \{\{b\}, \quad Cred_{adm}(b, F)$: Yes

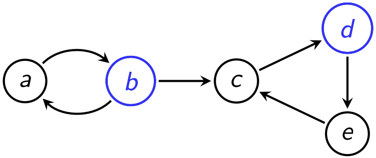
- $Cred_{adm}(b, F)$: is b contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



$pref(F) = \big\{\{b, d\}, \quad Cred_{pref}(b, F)$: Yes

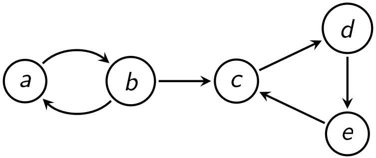
- $Cred_{pref}(b, F)$: is b contained in at least one $pref$-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



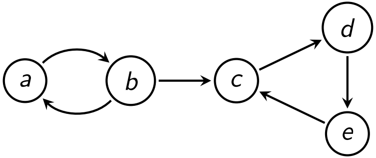
- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F?

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



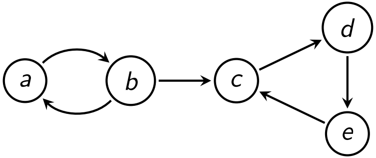
- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F? No. c in not defended against the attack from e.

Decision Problems on AFs (ctd.)

Credulous Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

$$Cred_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \exists S \in \sigma\text{-extension } F \text{ s.t. } \; a \in S, \\ \text{no} & \text{otherwise} \end{cases}$$



- $Cred_{adm}(c, F)$: is c contained in at least one adm-extension of F? No. c in not defended against the attack from e.
- $Cred_{adm}(c, F) = Cred_{pref}(c, F) = Cred_{stb}(c, F) = Cred_{comp}(c, F) = Cred_{grd}(c, F)$: No

Decision Problems on AFs (ctd.)

Characterize of Credulous Acceptance

Given an AF $F = (A, R)$:

- $Cred_{cf}(a, F)$: Check if $(a, a) \in R$
- $Cred_{adm}(a, F) = Cred_{pref}(a, F) = Cred_{comp}(a, F)$
- $Cred_{grd}(a, F)$: Evaluate the grounded extension of F
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Cred_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
- $Cred_{stb}(a, F)$: Evaluate the set of stable extensions of F

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

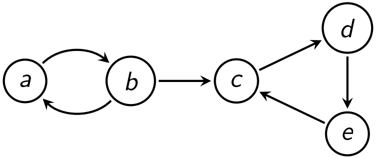
$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & \quad a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



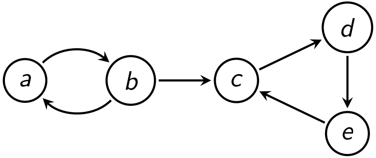
- $Skept_{pref}(b, F)$: is b contained in every $pref$-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & \quad a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



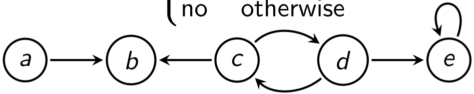
$pref(F) = \{\{a\}, \{b, d\}\}, \quad Skept_{pref}(b, F)$: No

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$



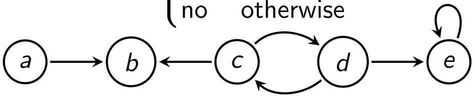
- $Skept_{pref}(a, F)$: is a contained in every *pref*-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Acceptance

Given an AF $F = (A, R)$, $a \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Skept_\sigma(a)$: is a contained in every σ-extension of F?

$$Skept_\sigma(a, F) = \begin{cases} \text{yes} & \text{if } \forall S \in \sigma\text{-extension } F \\ & a \in S \text{ holds,} \\ \text{no} & \text{otherwise} \end{cases}$$

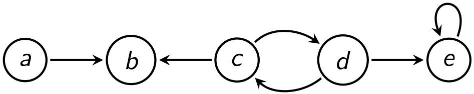


$pref(F) = \{\{a, c\}, \{a, d\}\}, \quad Skept_{pref}(a, F)$: yes

- $Skept_{pref}(a, F)$: is a contained in every $pref$-extension of F?

Decision Problems on AFs (ctd.)

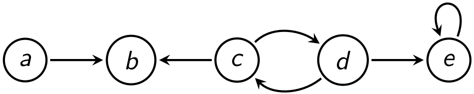
Skeptical Decision Problems under conflict-free



- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F?

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free



$cf(F) = \{\{a,c\},\{a,d\},\{b,d\}\{a\},\{b\},\{c\},\{d\},\{\}\}$ $\quad$ $Skept_{cf}(a,F)$: no.

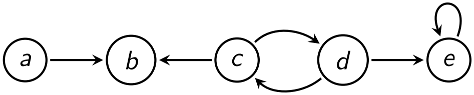
- $Skept_{cf}(a,F)$: is a contained in every conflict-free set of F? No

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free

- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Skeptical Decision Problems under Admissible Semantics



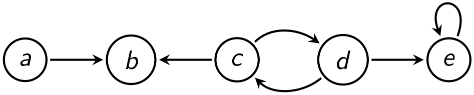
- $Skept_{adm}(a, F)$: is a contained in every *adm*-extension of F?

Decision Problems on AFs (ctd.)

Skeptical Decision Problems under conflict-free

- $Skept_{cf}(a, F)$: is a contained in every conflict-free set of F? No

Skeptical Decision Problems under Admissible Semantics

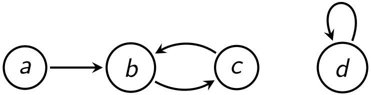


$adm(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \{\}\}$ $\quad Skept_{adm}(a, F)$: no.

- $Skept_{adm}(a, F)$: is a contained in every adm-extension of F? No

Decision Problems on AFs (ctd.)

Recall: $S \subseteq A$ is an stable extension if $S \in cf(F)$ and for each $a \in A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.



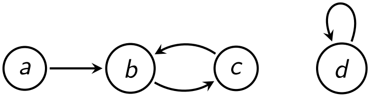
menti
.com
5221
5761

What is the answer to $Skept_{stb}(a, F)$? Why?

- $Skept_{stb}(a, F)$: yes. F has a stable extension and a is in every stable extension of F.
- $Skept_{stb}(a, F)$: no. Since F does not have any stable extension.
- $Skept_{stb}(a, F)$: yes. F does not have any stable extension. If no extension exists then all arguments are skeptically accepted.

Decision Problems on AFs (ctd.)

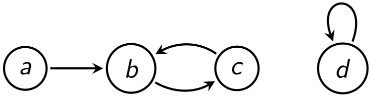
Recall: $S \subseteq A$ is an stable extension if $S \in cf(F)$ and for each $a \in A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.



$cf(F) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, c\}\}$

Decision Problems on AFs (ctd.)

Recall: $S \subseteq A$ is an stable extension if $S \in cf(F)$ and for each $a \in A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.



$stb(F) = \{\}$, $\quad$ $Skept_{stb}(a, F)$: yes

Decision Problems on AFs(ctd.)

Characterize of Skeptical Acceptance

- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{cf}(a, F)$: Trivially, No.
- For every AF $F = (A, R)$ and for every argument $a \in A$: $Skept_{adm}(a, F)$: Trivially, No.
- If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted.[1]
- $Skept_{grd}(F) = Cred_{grd}(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap comp(F)$
  - If $Skept_{grd}(a, F)$: Yes, then $a \in \cap pref(F)$
  - Note that it is possible to have a such that $a \in \cap pref(F)$, but $a \notin grd(F)$
  - There exists an AF F and argument a such that $Skept_{pref}(a, F)$: Yes. However, $Skept_{grd}(a, F)$: No.

[1] This is only relevant for stable semantics.

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

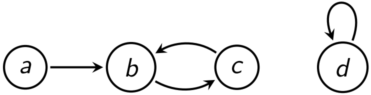
$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



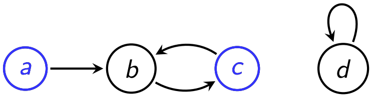
- Let $S = \{a, c\}$. $Ver_{adm}(S, F)$?

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



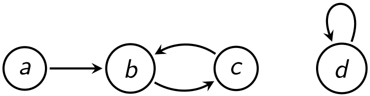
$adm(F) = \{\{a, c\},\quad Ver_{adm}(S, F)$? Yes

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



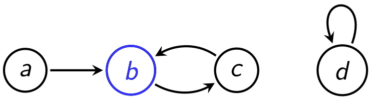
- Let $S = \{b\}$. $Ver_{adm}(S, F)$?

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



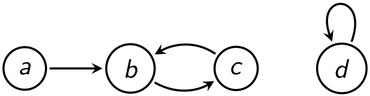
$Ver_{adm}(\{b\}, F)$: No. b is not defended

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



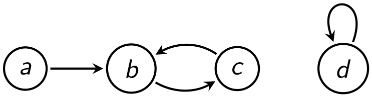
- Let $S = \{\}$. $Ver_{adm}(S, F)$?

Decision Problems on AFs (ctd.)

Verifying an extension

Given an AF $F = (A, R)$, $S \in A$, and $\sigma \in \{adm, pref, stb, grd, comp, cf\}$.
$Ver_\sigma(S, F)$: is S σ-extension of F?

$$Ver_\sigma(S, F) = \begin{cases} \text{yes} & \text{if } S \text{ is a } \sigma\text{-extension of } F, \\ \text{no} & \text{otherwise} \end{cases}$$



$adm(F) = \{\{\},\quad Ver_{adm}(S, F)$? Trivially, yes.

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_{\sigma}(F)$: Does F has a σ-extension?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension? NO
- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension? NO
- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F? NO
- $Skept_\sigma(a)$: is a contained in every σ-extension of F?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension? NO
- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F? NO
- $Skept_\sigma(a)$: is a contained in every σ-extension of F? If it is not trivial, yes
- $Ver_\sigma(S, F)$: is S σ-extension of F?

Decision Problems on AFs (ctd.)

Given an AF $F = (A, R)$, $a \in A$, and $S \in A$.

Do we need to construct the set of all σ extensions of F to answer any of the following decision problems?

- $Exists_\sigma(F)$: Does F has a σ-extension? NO
- $Cred_\sigma(a, F)$: is a contained in at least one σ-extension of F? NO
- $Skept_\sigma(a)$: is a contained in every σ-extension of F? If it is not trivial, yes
- $Ver_\sigma(S, F)$: is S σ-extension of F? NO

Complexity Results

Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ
cf	in L	trivial	in L
adm	NP-c	trivial	in L
pref	NP-c	Π_2-c	co-NP-c
comp	NP-c	P-c	in P
grd	P-c	P-c	P-c
stb	NP-c	co-NP-c	in P

Table: Complexity of reasoning with AFs.

Methods and Systems

- For an overview, see:

  G. Charwat, W. Dvořák, S. Gaggl, J. Wallner and S. Woltran.
  Methods for Solving Reasoning Problems in Abstract Argumentation – A Survey. *Artificial Intelligence* 220: 28–63, 2015.

- Competition for Abstract Argumentation Solvers (ICCMA):

  `http://argumentationcompetition.org`

- ASPARTIX Web Front-End:

  `http://rull.dbai.tuwien.ac.at:8080/ASPARTIX`

- CONARG Web Front-End:

  `http://www.dmi.unipg.it/conarg/`

Summery

We have seen

- Abstract Argumentation Frameworks
- Conflict-free sets
- Admissible semantics
- Preferred semantics
- Complete semantics
- Grounded semantics
- Stable semantics

Next

- Labelling-based semantics
- Discussion games

Amgoud, L., Dimopoulos, Y., and Moraitis, P. (2007).
A unified and general framework for argumentation-based negotiation.
In Proceedings of the 6th international joint conference on Autonomous agents and multiagent systems, pages 1–8.

Bench-Capon, T. J. and Dunne, P. E. (2005).
Argumentation in ai and law: Editors' introduction.
Artificial Intelligence and Law, 13(1):1–8.

Caminada, M. (2018).
Argumentation semantics as formal discussion.
In Baroni, P., Gabbay, D., Giacomin, M., and van der Torre, L., editors, Handbook of Formal Argumentation, pages 487–518.

Chi, H. and Liao, B. (2022).
A quantitative argumentation-based automated explainable decision system for fake news detection on social media.
Knowl. Based Syst., 242:108378.

Cocarascu, O., Rago, A., and Toni, F. (2019).

Extracting dialogical explanations for review aggregations with argumentative dialogical agents.
In AAMAS, pages 1261–1269. International Foundation for Autonomous Agents and Multiagent Systems.

Collenette, J., Atkinson, K., and Bench-Capon, T. J. M. (2020).
An explainable approach to deducing outcomes in european court of human rights cases using adfs.
In Proceedings of Computational Models of Argument COMMA 2020, volume 326 of Frontiers in Artificial Intelligence and Applications, pages 21–32. IOS Press.

Cyras, K., Rago, A., Albini, E., Baroni, P., and Toni, F. (2021).
Argumentative XAI: A survey.
In IJCAI, pages 4392–4399. ijcai.org.

Keshavarzi Zafarghandi, A., Verbrugge, R., and Verheij, B. (2020).
A discussion game for the grounded semantics of abstract dialectical frameworks.
In Proceedings of Computational Models of Argument COMMA 2020, volume 326 of Frontiers in Artificial Intelligence and Applications, pages 431–442. IOS Press.

McBurney, P., Parsons, S., and Rahwan, I., editors (2012).

Argumentation in Multi-Agent Systems - 8th International Workshop, ArgMAS 2011, Taipei, Taiwan, May 3, 2011, Revised Selected Papers, volume 7543 of Lecture Notes in Computer Science. Springer.

Rago, A., Cocarascu, O., and Toni, F. (2018).
Argumentation-based recommendations: Fantastic explanations and how to find them.
In IJCAI, pages 1949–1955. ijcai.org.