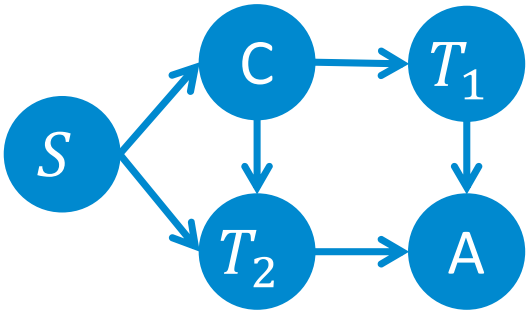
MAP Example

S	Θ_S
smoker	0.55
nonsmoker	0.45

S	C	$\Theta_{C\|S}$
smoker	yes	0.05
smoker	no	0.95
nonsmoker	yes	0.01
nonsmoker	no	0.99

C	T_1	$\Theta_{T_1\|C}$
yes	pos	0.8
yes	neg	0.2
no	pos	0.2
no	neg	0.8



Is the patient a smoker and do they have the condition, considering the tests agree?

T_1	T_2	A	$\Theta_{A\|T_1,T_2}$
pos	pos	yes	1
pos	pos	no	0
pos	neg	yes	0
pos	neg	no	1
neg	pos	yes	0
neg	pos	no	1
neg	neg	yes	1
neg	neg	no	0

S	C	T_2	$\Theta_{T_2\|C,S}$
smoker	yes	pos	0.8
smoker	yes	neg	0.2
smoker	no	pos	0.2
smoker	no	neg	0.8
nonsmoker	yes	pos	0.95
nonsmoker	yes	neg	0.05
nonsmoker	no	pos	0.05
nonsmoker	no	neg	0.95

$$MAP(Q = \{S, C\}, e = \{a\})$$
$$= argmax_{S,C} Pr(S, C, a)$$
$$= argmax_{S,C} \left(\Theta_S \Theta_{C|S} \Sigma_{T_1} \Theta_{T_1|C} \Sigma_{T_2} \Theta_{T_2|C,S} \Theta_{A|T_1,T_2}\right)^a$$
$$= argmax_S \left(\Theta_S argmax_C \Theta_{C|S} \Sigma_{T_1} \Theta_{T_1|C} \Sigma_{T_2} \Theta_{T_2|C,S} \Theta^a_{A|T_1,T_2}\right)$$

VU

MAP Example - Continued

Recall: $\Pr(S, C, a) = argmax_S\left(\Theta_S argmax_C \Theta_{C|S} \Sigma_{T_1} \Theta_{T_1|C} \Sigma_{T_2} \Theta_{T_2|C,S} \Theta^a_{A|T_1,T_2}\right)$

intermediate step: calculate $\mathrm{f}_1(\mathrm{S}, \mathrm{C}) = \Sigma_{T_1} \Theta_{T_1|C} \Sigma_{T_2} \Theta_{T_2|C,S} \Theta_{A|T_1,T_2}$

Then $\Theta_{C|S} f_1(S, C)$, $argmax_C \Theta_{C|S} f_1(S, C)$, $\Theta_S f_2(S)$, and finally $argmax_S \Theta_S f_2(S)$

S	C	$\Theta_{C\|S} f_1(S, C)$
smoker	yes	0.034
smoker	no	0.646
nonsmoker	yes	0.0076
nonsmoker	no	0.752

S	$\mathrm{f}_2(\mathrm{S}) = \mathrm{argmax}_C \Theta_{C\|S} f_1(S, C)$
smoker	0.646, $\neg c$
nonsmoker	0.752, $\neg c$

S	$\Theta_S \mathrm{f}_2(\mathrm{S})$
smoker	0.3553, $\neg c$
nonsmoker	0.3386, $\neg c$

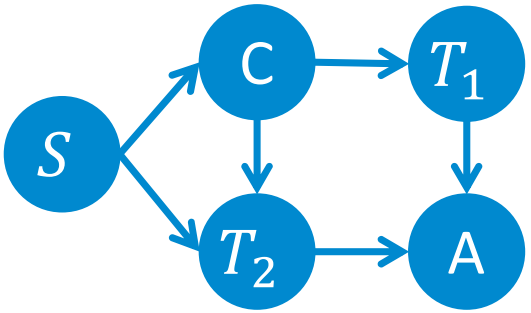
	$argmax_S \Theta_S \mathrm{f}_2(\mathrm{S})$
$\perp$	0.3553, $\neg c, s$

MPE Example

S	Θ_S
smoker	0.55
nonsmoker	0.45

S	C	$\Theta_{C\|S}$
smoker	yes	0.05
smoker	no	0.95
nonsmoker	yes	0.01
nonsmoker	no	0.99

C	T_1	$\Theta_{T_1\|C}$
yes	pos	0.8
yes	neg	0.2
no	pos	0.2
no	neg	0.8



What are the most likely values, considering the tests agree?

T_1	T_2	A	$\Theta_{A\|T_1,T_2}$
pos	pos	yes	1
pos	pos	no	0
pos	neg	yes	0
pos	neg	no	1
neg	pos	yes	0
neg	pos	no	1
neg	neg	yes	1
neg	neg	no	0

S	C	T_2	$\Theta_{T_2\|C,S}$
smoker	yes	pos	0.8
smoker	yes	neg	0.2
smoker	no	pos	0.2
smoker	no	neg	0.8
nonsmoker	yes	pos	0.95
nonsmoker	yes	neg	0.05
nonsmoker	no	pos	0.05
nonsmoker	no	neg	0.95

$$MPE(e = \{a\})$$
$$= argmax_{V \setminus \mathrm{A}} Pr(S, C, T_1, T_2, a)$$
$$= argmax_{V \setminus \mathrm{A}} \left(\Theta_S \Theta_{C|S} \Theta_{T_1|C} \Theta_{T_2|C,S} \Theta_{A|T_1,T_2}\right)^a$$
$$= argmax_S \left(\Theta_S argmax_C \left(\Theta_{C|S} argmax_{T_1} \left(\Theta_{T_1|C} argmax_{T_2} \left(\Theta_{T_2|C,S} \Theta^a_{A|T_1,T_2}\right)\right)\right)\right)$$

VU

MPE Example - Continued

$$argmax_S\left(\Theta_S argmax_C\left(\Theta_{C|S} argmax_{T_1}\left(\Theta_{T_1|C} argmax_{T_2}\left(\Theta_{T_2|C,S}\Theta^a_{A|T_1,T_2}\right)\right)\right)\right)$$

intermediate step: calculate $\mathrm{f}_1(\mathrm{S},\mathrm{C},\mathrm{T}_1) = argmax_{T_2}\Theta_{T_2|C,S}\Theta^a_{A|T_1,T_2}$

Then $\mathrm{f}_2(\mathrm{S},\mathrm{C}) = argmax_{T_1}\Theta_{T_1|C} f_1(S,C,T_1)$, $f_3(S) = argmax_C\Theta_{C|S} f_2(S,C)$ and finally $argmax_S\Theta_S f_3(S)$

S	C	T_2	$f_1(S,C,T_1)$
smoker	yes	pos	0.8, t_2
smoker	yes	neg	0.2, $\neg t_2$
smoker	no	pos	0.2, t_2
smoker	no	neg	0.8, $\neg t_2$
nonsmoker	yes	pos	0.95, t_2
nonsmoker	yes	neg	0.05, $\neg t_2$
nonsmoker	no	pos	0.05, t_2
nonsmoker	no	neg	0.95, $\neg t_2$

S	C	$f_2(S,C)$
smoker	yes	0.64, t_2, t_1
smoker	no	0.16, $t_2, \neg t_1$
nonsmoker	yes	0.19, t_2, t_1
nonsmoker	no	0.76, $\neg t_2, \neg t_1$

S	$f_3(S)$
smoker	0.4096, t_2, t_1, c
nonsmoker	0.7524, $\neg t_2, \neg t_1, \neg c$

Either all true or all false!

$argmax_S\Theta_S f_3(S)$	
$\perp$	0.339, $\neg t_2, \neg t_1, \neg c, \neg s$

Disagrees with previous MAP query