Multi-Agent Systems: Homework Assignment 1 – Group 13

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2023-11-15

1 Odd or even game

1.1 Write down the pay-off matrix for this game.

		Player B 1	Player B 2
Player A	1	$-2, 2$	$3, -3$
	2	$3, -3$	$-4, 4$

1.2 What are the regret minimisation strategies in terms of pure strategies?

The regret matrix is as follows:

		Player B 1	Player B 2
Player A	1	$5, 0$	$0, 5$
	2	$0, 7$	$7, 0$

In order to find the best strategy for player A and player B we need to find the maximum regret each of the player gets with the strategies played. For player A the maximum regret per row is 5 and 7. In order to minimize we choose the min5,7 which is 5. This corresponds to strategy 1 for player A.

We proceed doing the same for player B. The maximum regret per column is 5 and 7.In order to minimize we choose the min5,7 which is 5. This corresponds to strategy 2 for player B.

1.3 What are regret minimization strategies in terms of mixed strategies?

For player A, when $5p + (1-p) \times 0 = 0 \times p + 7(1-p)$, which simplifies to $p = \frac{7}{12}$, both strategies are equally effective, and the player can choose either strategy.
Similarly, for player B, when $q = \frac{5}{12}$, both strategies are equally effective.

		Player B 1,q	Player B 2,(1-q)	Player B L*q+R*(1-q)
Player A	1,p	5,0	0,5	7q
	2,(1-p)	0,7	7,0	5(1-q)
	T*p+B*(1-p)	5p	7(1-p)	

1.4 What are safety strategies for each player in terms of pure strategies?

If we use pure strategies, we are going to use the maximin strategy, where the first priority is the safety of each player. for Player A (row player) the minimum values we get per row are -2 and -4. In order to choose the best value we choose the maximum of those two, which corresponds to -2.

For Player B the minimal regret values are the same (-3), therefore we consider another factor, which would be the maximum gain which would be possible. Thus we would choose the strategy to play Right. As this strategy yields the biggest gain in the hope that player A plays Down.

1.5 What are the safety strategies for each player in terms of mixed strategies?

Player A: $-2pq + 3p(1-q) + 3(1-p)q - 4(1-p)(1-q)$
Player B: $2pq - 3p(1-q) - 3(1-p)q + 4(1-p)(1-q)$

2 Travelers' dilemma: Discrete version

2.1 Write down the pay-off matrix for this game.

		Player B		
		1	2	3
Player A	1	$1, 1$	$1+a, 1-a$	$1+a, 1-a$
	2	$1-a, 1+a$	$2, 2$	$2+a, 2-a$
	3	$1-a, 1+a$	$2-a, 2+a$	$3, 3$

2.2 Determine the pure Nash equilibria (PNE, there might be none, one or multiple ones).

There are in total 3 Nash equilibrium.

The first one is 1,1.

The second one is 2,2.

The third one is 3,3.

2.3 Are there mixed Nash equilibrium in which all three strategies are mixed?

For the row player we have:

First row :
$EU_2(s_1, 1) = p_1 + p_2(1+a) + p_3(1+a)$

Second row:
$EU_2(s_2, 2) = p_1(1-a) + 2p_2 + p_3(2+a)$

Third row:
$EU_2(s_3, 3) = p_1(1-a) + p_2(2-a) + 3p_3$

We equate $EU_2(s_1, 1) with EU_2(s_2, 2)$ get:

$$ap_1 - (a-1)p_2 - p_3 = 0 \tag{1}$$

and then equate $EU_2(s_2, 2) with EU_2(s_3, 3)$ get:

$$ap_2 - (a-1)p_3 = 0 \tag{2}$$

and together with:

$$p_1 + p_2 + p_3 = 1 \tag{3}$$

Solve the matrix, we get:

$$p_1 = \frac{a^2 - a + 1}{a^2 + 1}, p_2 = \frac{-a^2 + a}{a^2 + 1}, p_3 = \frac{a^2}{a^2 + 1} \tag{4}$$

And due to the symmetry of the matrix, the result of q_1, q_2, q_3 are the same.

2.4 Does this knowledge about the NE help the travelers in their decision?

In this case, the NEs are situations where both travellers choose the same value. If each traveller assumes the other is rational and also aims for an equilibrium, then choosing the same number with the highest payout (3,3) makes sense. Knowing the NE helps the traveller to know which number to choose to avoid any regret.

2.5 Write down the regret matrix. This matrix is similar to the pay-off matrix, but now specifies the regret (rather than the pay-off) for each action profile.

	1	2	3	**max regret**
1	0, 0	1-a, a	2-a, a	2-a
2	a, 1-a	0, 0	1-a, a	1-a
3	a, 2-a	a, 1-a	0, 0	a
max regret	2-a	1-a	a	

2.6 What are the (pure) regret minimisation strategies?

We can see from the previous table, both players should pick 3 to generate the least regret.

3 Cournot's Duopoly (continuous version)

3.1 What is the best response for each company given the quantity the other company will produce?

Utility function:

$$\pi_i = pq_i - c_i q_i = (p - c_i)q_i \tag{5}$$

and we have:

$$p = \alpha - \beta(q_1 + q_2) \qquad (\alpha, \beta > 0) \tag{6}$$

For given q_2,

$$\pi_1 = (\alpha - \beta(q_1 + q_2) - c_1) * q_1 = \alpha q_1 - \beta q_1 q_2 - \beta q_1^2 - c_1 q_1 \tag{7}$$

To find the maximum of the function, we set the derivative of q_2 to zero:

$$\frac{\partial \pi_2}{\partial q_2} = \alpha - \beta q_2 - c_1 - 2\beta q_1 = 0 \tag{8}$$

$$q_1 = \frac{\alpha - \beta q_2 - c_2}{2\beta} \tag{9}$$

Similarly, for given q_1:

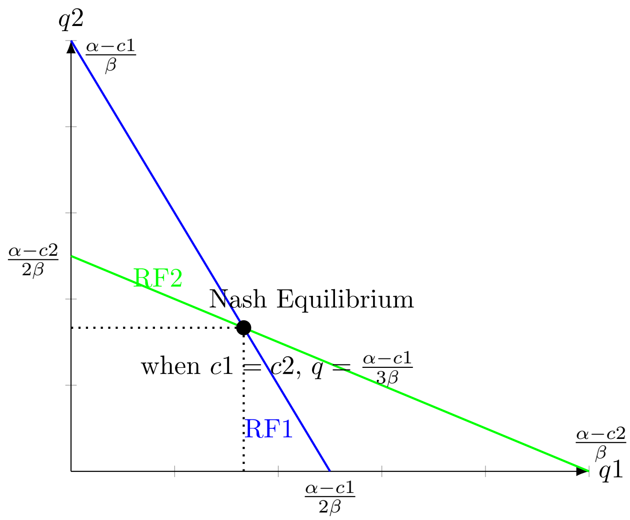
$$q_2 = \frac{\alpha - \beta q_1 - c_2}{2\beta} \tag{10}$$

3.2 Suppose the companies need not decide on their quantity at the same time, but can react to one another (an unlimited number of times). What will be the outcome? (Provide a diagram.)

Reaction functions:

$$RF_i(q_j) = -\frac{1}{2}q_j + \frac{\alpha - c_i}{2\beta} \tag{11}$$

If $q_j = 0$ then $RF_i(0) = \frac{\alpha - c_i}{2\beta}$;
If $RF_i(0) = 0$ then $q_j = \frac{\alpha - c_i}{\beta}$;



We'll get the Nash equilibrium when the two RF lines cross.

$$RF_1(q_2) = RF_2(q1) \tag{12}$$

$$-\frac{1}{2}q_2 + \frac{\alpha - c_1}{2\beta} = -\frac{1}{2}q_1 + \frac{\alpha - c_2}{2\beta} \Rightarrow q_1 - q_2 = \frac{c_2 - c_1}{\beta} \tag{13}$$

$$q_1 = q_2 + \frac{c_2 - c_1}{\beta} \tag{14}$$

If firms can react to one another, then each firm will adjust its output based on the output of the other firm. Over time, the firms will reach a point where neither has an incentive to adjust their quantity any further, namely Nash Equilibrium.

4 Ice cream time!

4.1 On a beautiful summer morning Charlize makes her way to the beach and upon arrival finds that her two competitors have already set up their stalls: Alice at location a = 0.1 and Bob at location b = 0.8. Discuss what Charlize's best response is: i.e. what location should she choose, given a = 0.1 and b = 0.8?

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4.2 Same question as above, but now assume that all we know is that a = 0.1 and $a < 6 \leq 1$.

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