VU VRIJE UNIVERSITEIT AMSTERDAM

Knowledge Representation

2023/2024

Exercise Sheet 1 – Classical and Description Logics – Solutions

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Exercise 1.1 Consider an interpretation I with

$$I(a) = \textbf{false} \qquad I(b) = \textbf{false} \qquad I(c) = \textbf{false} \qquad I(d) = \textbf{false}$$

Which of the following propositional formulas are satisfied by this interpretation:

(a) $(a \wedge b) \vee \neg c \vee \neg d$

(b) $(a \wedge b) \vee (\neg c \wedge d)$

(c) $(a \rightarrow \neg b) \vee (\neg c \rightarrow d)$

(d) $(\neg a \rightarrow b) \wedge (c \rightarrow \neg d)$

Solution:

Satisfied: (a), (c),

Not satisfied: (b),(d)

Exercise 1.2 Consider the two formulas

$$F = \neg p \wedge \neg q \qquad G = p \rightarrow q$$

(a) Which truth assignments to p and q give models of F and G respectively?

(b) Does F entail G?

(c) Does G entail F?

Solution:

intepretation	p	q	$F = \neg p \wedge \neg q$	$G = p \rightarrow q$
I_1	**false**	**false**	**true**	**true**
I_2	**false**	**true**	**false**	**true**
I_3	**true**	**false**	**false**	**false**
I_4	**true**	**true**	**false**	**true**

(a) I_1 is a model of F, I_1, I_2 and I_4 are models of G.

(b) F entails G — every model of F is also a model of G.

(c) G does not entail F: for example, I_2 is a model of G that is not a model of F.

Exercise 1.3 Assume that we use propositional logic for knowledge representation and that the reasoning problem we have to solve is the following: given a formula F, decide whether F is satisfiable. Which of the following algorithms is sound/complete/terminating?

(a) Always return "yes".

(b) Always return "no".

(c) Enter an infinite loop, never return.

(d) Go through all truth assignments for the variables in F one after the other. For each truth assignment, check whether it gives a model for F. If a satisfying truth assignment is found, return "yes". Otherwise return "no".

Solution:

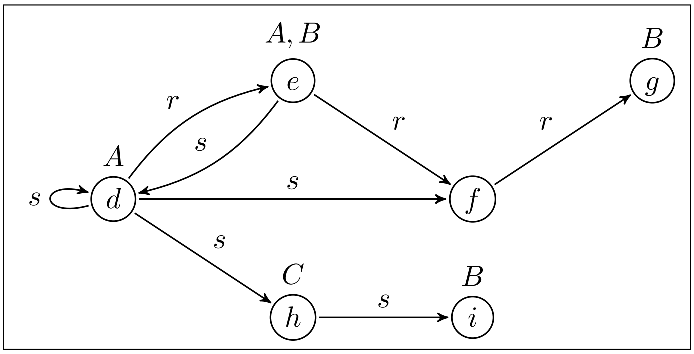
(a) complete and terminating, not sound: It never returns wrong "no"-answers, it always stops, but it may return wrong "yes" answers.

(b) sound and terminating, not complete: It never returns wrong "yes"-answers, it always stops, but it may return wrong "no" answers.

(c) sound and complete, not terminating: It never returns any wrong answers, but it also never stops.

(d) sound, complete and terminating — a decision procedure.

Exercise 1.4 Consider the interpretation $\mathcal{I}$ represented as the following graph:



For each of the following concepts D, write down the elements in their interpretation $D^{\mathcal{I}}$:

(a) $\neg A$

(b) $A \sqcap \neg B$

(c) $\exists r.(A \sqcup B)$

(d) $\forall r.(A \sqcup B)$

(e) $\exists s.\exists s.\neg A$

(f) $\exists r.\forall r.\neg A$

Solution:

(a) $(\neg A)^{\mathcal{I}} = \{f, g, h, i\}$

(b) $(A \sqcap \neg B)^{\mathcal{I}} = \{d\}$

(c) $(\exists r.(A \sqcup B))^{\mathcal{I}} = \{d, f\}$

(d) $(\forall r.(A \sqcup B))^{\mathcal{I}} = \{d, f, g, h, i\}$

(e) $(\exists s.\exists s.\neg A)^{\mathcal{I}} = \{d, e\}$

(f) $(\exists r.\forall r.\neg A)^{\mathcal{I}} = \{d, e, f\}$

Exercise 1.5

(a) Express the following phrases using $\mathcal{ALC}$ concepts:

(i) "persons that do not have a friendly neighbour,"

(ii) "persons that have a neighbour that is not friendly."

(b) Construct an interpretation where an element satisfies the concept for (i), but not the concept for (ii).

(c) Construct an interpretation where an element satisfies the concept for (ii), but not the concept for (i).

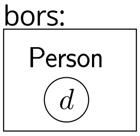
Solution:

(a) We could for instance use the following concepts:

(i) $\mathsf{Person} \sqcap \neg\exists \mathsf{neighbor}.\mathsf{Friendly}$ (or $\mathsf{Person} \sqcap \forall \mathsf{neighbor}.\neg\mathsf{Friendly}$)

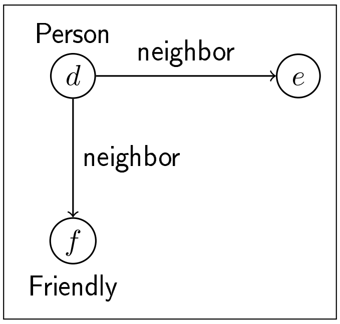
(ii) $\mathsf{Person} \sqcap \exists \mathsf{neighbor}.\neg\mathsf{Friendly}$

(b) It suffices to consider the following interpretation with just one individual d that has no neighbors:



It is true that d does not have a neighbor that is friendly, but it is not true that d has a neighbor that is not friendly.

(c) To satisfy (ii), we need at least two individuals. To also make sure that (i) is not satisfied, we need a third individual. In the following interpretation, d is an instance of the concept in (ii), but not an instance of the concept in (i):



Exercise 1.6 Which of the following $\mathcal{ALC}$ axioms corresponds to the following English statement:

- "A Mule is an animal that has a horse and a donkey as a parent."

(a) $\mathrm{Mule} \equiv \mathrm{Animal} \sqcap \exists \mathrm{hasParent}.(\mathrm{Horse} \sqcap \mathrm{Donkey})$

(b) $\mathrm{Mule} \equiv \mathrm{Animal} \sqcap \exists \mathrm{hasParent}.\mathrm{Horse} \sqcap \exists \mathrm{hasParent}.\mathrm{Donkey}$

(c) $\mathrm{Mule} \equiv \mathrm{Animal} \sqcap (\exists \mathrm{hasParent}.\mathrm{Horse} \sqcup \exists \mathrm{hasParent}.\mathrm{Donkey})$

Solution:

(b)

Exercise 1.7 Translate the following axiom into English:

$$\mathrm{KRTeacher} \equiv \exists \mathrm{teaches}.(\mathrm{Course} \sqcap \forall \mathrm{hasTopic}.(\mathrm{DL} \sqcup \mathrm{Arg} \sqcup \mathrm{PGM}))$$

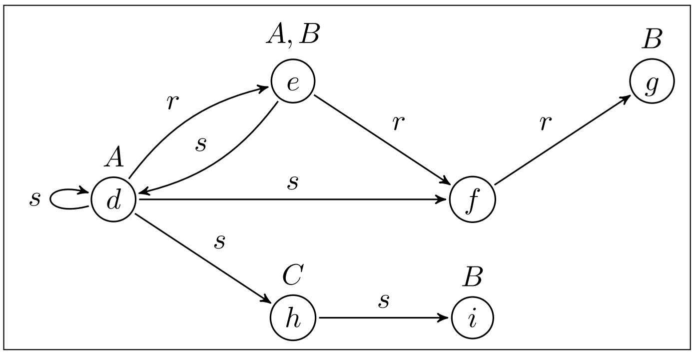
Solution:

A KR teacher is someone that teaches a course whose only topics are DL, Arg and PGM.

Exercise 1.8 We consider the same graph from Exercise 1.4, but with a different reading: This time, we see it as an *ABox* $\mathcal{A}$, where the nodes are individual names, i.e. $d, e, f, g, h, i \in$ **I**, and the labels correspond to concept and role assertions. In particular, it now represents the following ABox:

$$\begin{array}{lllllll} \mathcal{A} = \{ & d : A, & e : A, & e : B, & g : B, & h : C, & i : B, \\ & (d,d) : s, & (d,e) : r, & (e,d) : s, & (e,f) : r, & (d,f) : s, & (f,g) : r, \\ & (d,h) : s, & (h,i) : s & \}, & & & \end{array}$$

which is much more readable as a graph:



We also consider again the following concepts:

$$\neg A, \quad A \sqcap \neg B, \quad \exists r.(A \sqcup B), \quad \forall r.(A \sqcup B), \quad \exists s.\exists s.\neg A, \quad \exists r.\forall r.\neg A$$

(a) For each concept D from this set, which are the instances of D w.r.t. $\mathcal{A}$? In other words, for which individual name x do we have $\mathcal{A} \models x : D$?

(b) What are the instances of D w.r.t. the ontology $\mathcal{O} = \mathcal{A} \cup \mathcal{T}$, where $\mathcal{T} = \{C \sqsubseteq \neg A,\ B \sqsubseteq \forall r.C\}$?

Solution:

(a) For $\neg A$, $A \sqcap \neg B$, $\exists s.\exists s.\neg A$ we have no instances, because we can add any individual to the extension of A or B. For $\forall r.(A \sqcup B)$ and $\exists r.\forall r.\neg A$, the situation is similar: in models of $\mathcal{A}$, nothing prevents us from adding new elements as r-successors to any individual, this way making sure that the value restrictions are not satisfied for any individual name. The only concept that has instances is $\exists r.(A \sqcup B)$, which are, as in Exercise 1.4, the individual names d and f.

(b) Using the first GCI, we first observe that

$$\mathcal{O} \models h : \neg A$$

Based on the axiom $B \sqsubseteq \forall r.C$, we can determine additional individual names that have to be in the extension of $C^{\mathcal{I}}$ in any model of the ontology. In particular, this is the case for the r-successors of e, g and i. g and i do not have any r-successors, and consequently, the only affected individual name is f, which is the r-successor of e. We obtain $\mathcal{O} \models f : C$, and using the other GCI,

$$\mathcal{O} \models f : \neg A.$$

Based on this observation, we can furthermore conclude that

$$\mathcal{O} \models d : \exists s.\exists s.\neg A$$

and

$$\mathcal{O} \models e : \exists s.\exists s.\neg A,$$

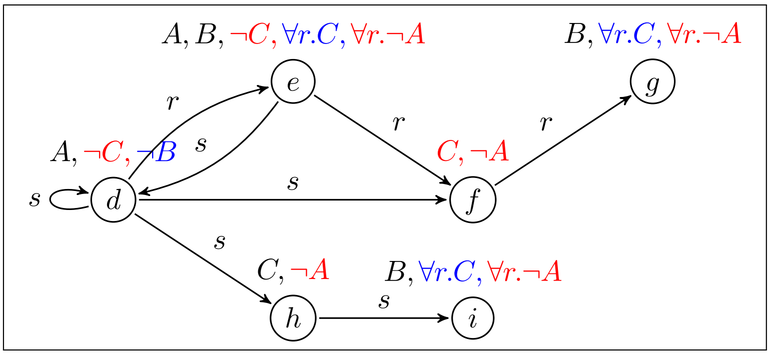
since from d and e, we can reach f by following the s-successor twice.
Finally, because $C \sqsubseteq \neg A \in \mathcal{O}$ and $B \sqsubseteq \forall r.C \in \mathcal{O}$, we can conclude that

$$\mathcal{O} \models B \sqsubseteq \forall r.\neg A,$$

which also means that

$$\mathcal{O} \models \exists r.B \sqsubseteq \exists r.\forall r.\neg A.$$

This allows us to find instances of $\exists r.\forall r.\neg A$, namely d and f, since both have an r-successor satisfying B.
We also observe that d cannot be an instance of B, because it has an r-successor that cannot be in C: this is the case for e. Consequently, we have $\mathcal{O} \models \neg B$, which gives us $\mathcal{O} \models A \sqcap \neg B$



To summarize:

(i) $\neg A$: f and h

(ii) $A \sqcap \neg B$: d

(iii) $\exists r.(A \sqcup B)$: d and f

(iv) $\forall r.(A \sqcup B)$: nothing

(v) $\exists s.\exists s.\neg A$: d and e

(vi) $\exists r.\forall r.\neg A$: d and f