

Knowledge Representation

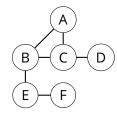
2023/2024

Exercise Sheet 4 - Mini Exercise-Sheet on Bayesian Networks

11th December 2023

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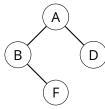
Exercise 4.1 You are given the following interaction graph:



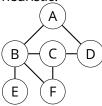
(a) Does the graph correspond to the following set of factors?

$$\{ f(A,B,C), f(E,F), f(C,E), f(D,C), f(C,B), f(B,E) \}$$

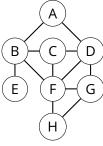
(b) Could the following graph be reached from the original graph through node elimination?



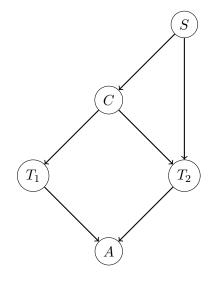
(c) Could E-A-D-F-B-C be an elimination order in the following graph according to the MinFill heuristic?



(d) Provide an elimination order on the following graph according to the MinDeg heuristic. (There are multiple correct answers.)



Exercise 4.2 You are given the following Bayesian Network.



| S | C | T_2 | $\Theta_{T_2\mid C,S}$ | T_1 | T_2 | A | $\Theta_{A T_1,T_2}$ |
|--------|-----|-------|------------------------|-------|-------|-----|----------------------|
| male | yes | +ve | 0.80 | +ve | +ve | yes | 1 |
| male | yes | -ve | 0.20 | +ve | +ve | no | 0 |
| male | no | +ve | 0.20 | +ve | -ve | yes | 0 |
| male | no | -ve | 0.80 | +ve | -ve | no | 1 |
| female | yes | +ve | 0.95 | -ve | +ve | yes | 0 |
| female | yes | -ve | 0.05 | -ve | +ve | no | 1 |
| female | no | +ve | 0.05 | -ve | -ve | yes | 1 |
| female | no | -ve | 0.95 | -ve | -ve | no | 0 |

| | | S | C | $\Theta_{C S}$ | C | T_1 | $\Theta_{T_1 C}$ |
|--------|------------|--------|-----|----------------|-----|-------|------------------|
| S | Θ_S | male | yes | 0.05 | yes | +ve | 0.8 |
| male | 0.55 | male | no | 0.95 | yes | -ve | 0.2 |
| female | 0.45 | female | yes | 0.01 | no | +ve | 0.2 |
| | | female | no | 0.99 | no | -ve | 0.8 |

- (a) Write down the probability table for $Pr(S, C, T_2)$. (Warning: ugly numbers!)
- (b) Write down the probability table for $\Pr(T_1 \mid S = \mathsf{male})$.
- (c) Calculate MPE(S = female, C = no).
- (d) Calculate $MAP(S, C \mid A = yes)$.