Knowledge Representation

Lecture 3: Introduction to Description Logics

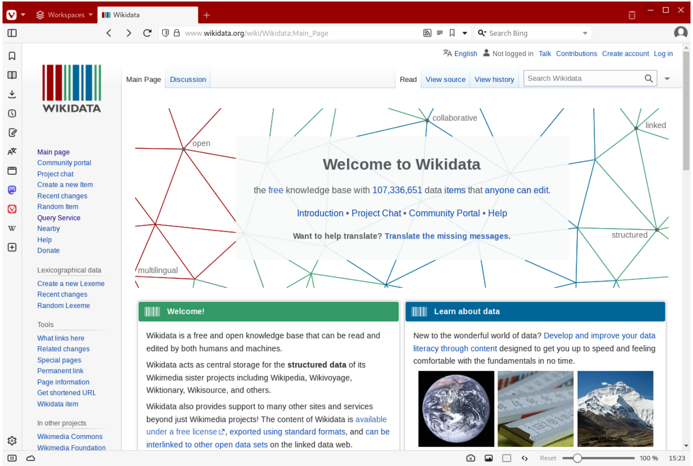
Patrick Koopmann *(using parts by Stefan Borgwardt)*

November 3, 2023

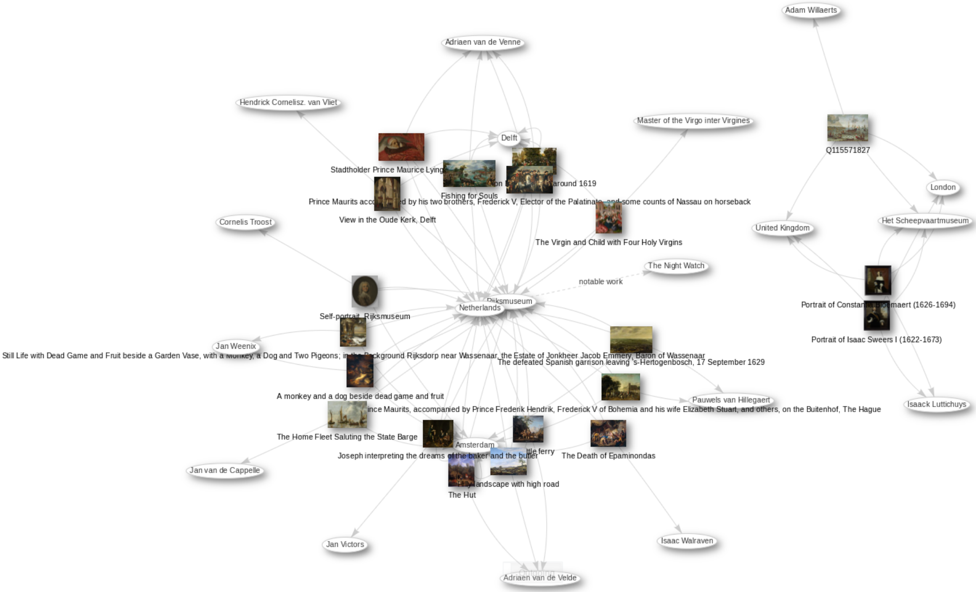
Knowledge Graphs and Ontologies

KR Formalisms You Might Know: Knowledge Graphs / Wikidata

Do you also know Wikidata?



Knowledge Graph Wikidata



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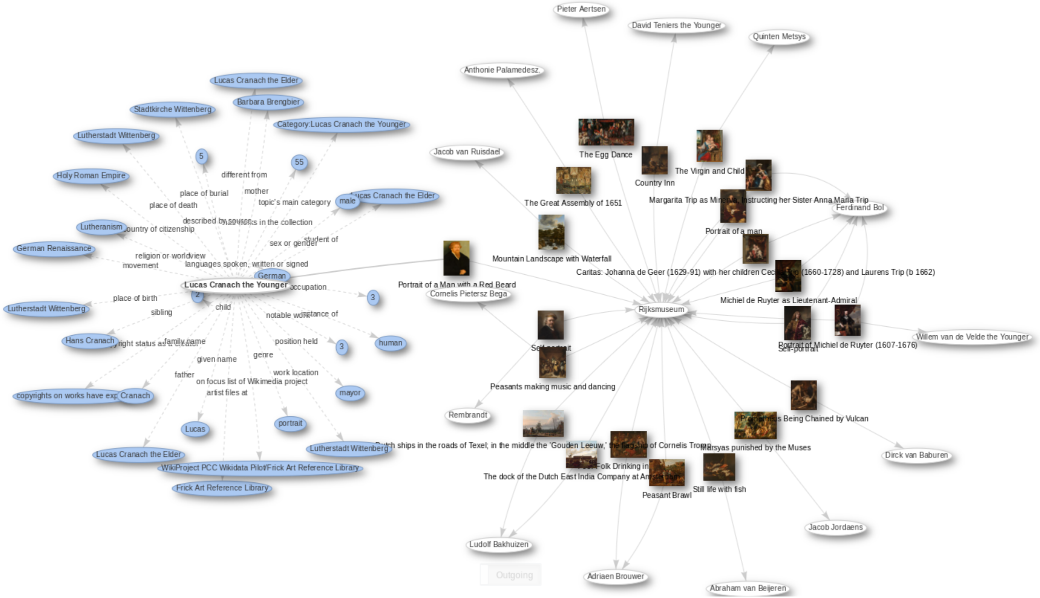
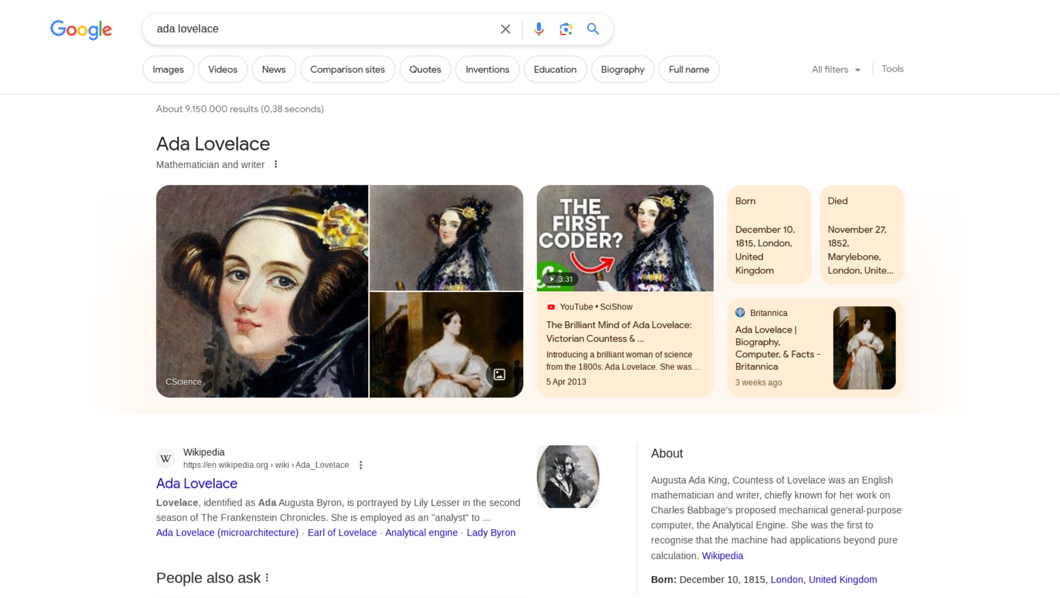


Knowledge Graphs in Google



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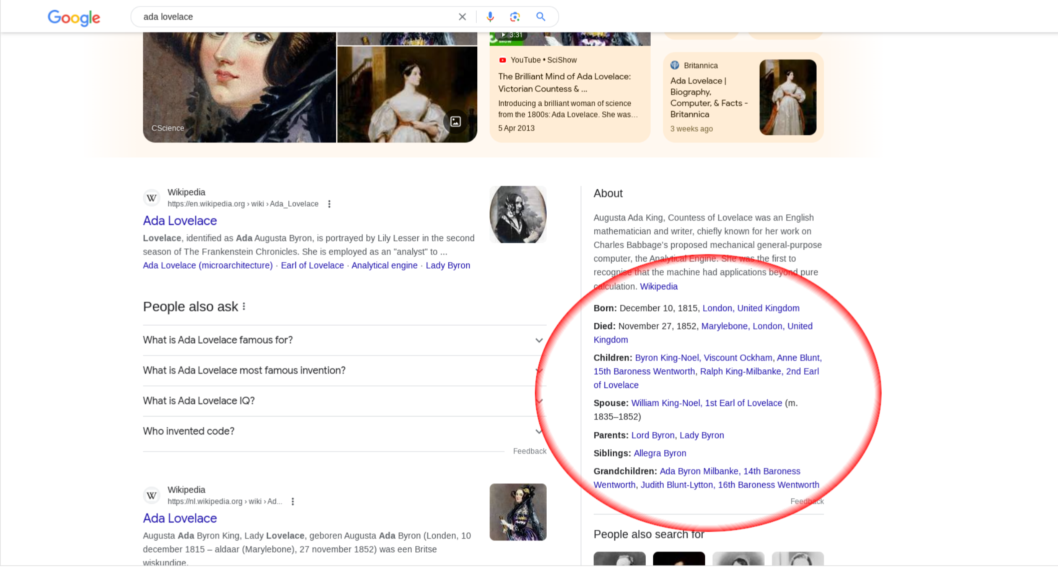

Knowledge Graphs

Knowledge graphs are used in many places:

- Wikidata and DBPedia reflect knowledge of Wikipedia
- Search Engines: Google, Bing, Yahoo, etc.
- Question answering systems: WolframAlpha, Siri, Alexa
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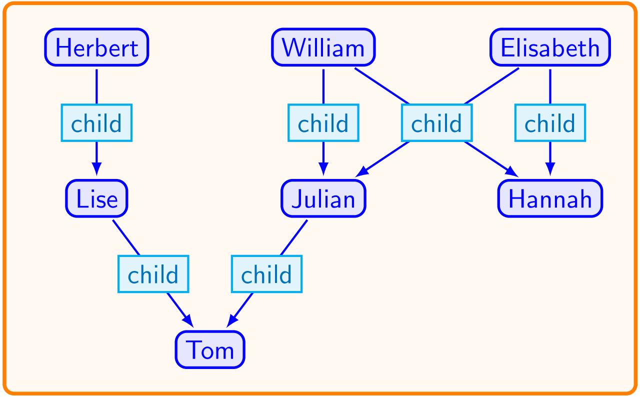
Knowledge graphs on their own are powerful, but they also have limitations:

- we only have the explicit knowledge
- graphs may be incomplete
- it is not straightforward to link different knowledge sources

Example: Family Tree



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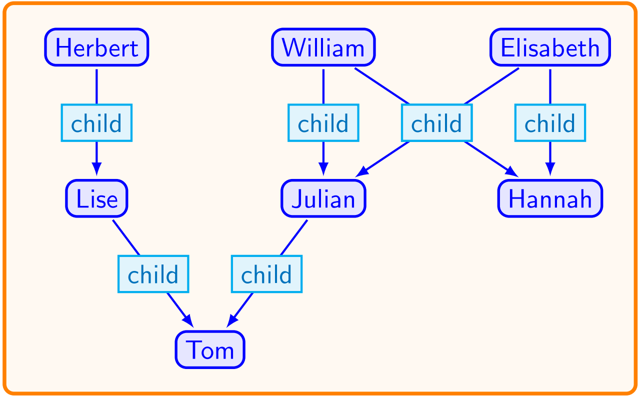

Consider the following queries:

- Who are the grandparents of Tom?
- Who is the aunt of Tom?
- Who is a parent?

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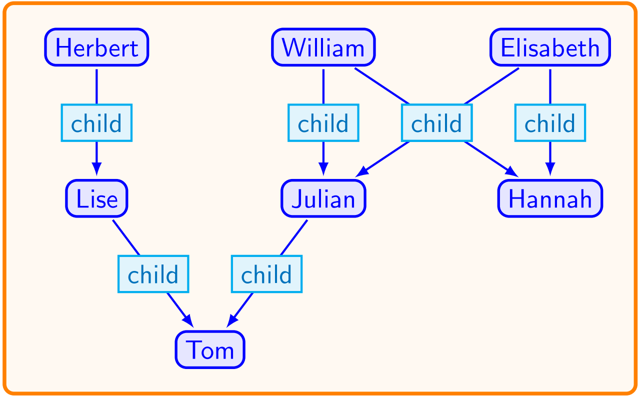

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(everyone)

Other Sources of Knowledge

Knowledge may occur in many other different forms:

- web pages
- databases
- spreadsheets
- diagrams
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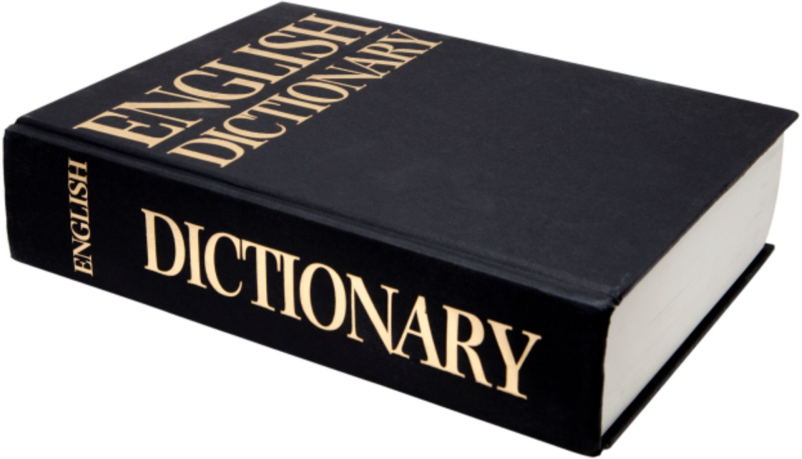
It can be useful for an AI system to "understand" these data

- ... to infer implicit information
- ... to integrate data from different sources
- ... to intelligently answer queries

Motivation for Ontologies

Intuitively, the computer would need a dictionary...



Motivation for Ontologies

... where it can look up the meaning of the used terms.

Parent *n*

1 a father or a mother

2 an ancestor or precursor

Grandparent *n*

1 a parent of a parent

Aunt *n*

1 the sister of one's father or mother.

2 the wife of one's uncle.

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  - More intelligent querying of data
  - Integrating data from different sources
  - Extending incomplete data sets

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- Ontologies are such “dictionaries for computer systems”

Ontologies

Ontology as a Discipline of Philosophy

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Branch of metaphysics

- What kinds of things/entities exist?
- What does it mean to exist?
- How are entities related?
- What are their basic categories? (eg. substances, property, relation, event,)
- Which entities are the most fundamental?

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- What kinds of things/entities exist?
- What does it mean to exist?
- How are entities related?
- What are their basic categories? (eg. substances, property, relation, event,)
- Which entities are the most fundamental?
- "Ontology" is concerned with rather general and fundamental concepts.

Ontologies in Computer Science

Computer scientists are more pragmatic:

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- We are not concerned with *Ontology*, but with *ontologies*.

Definition of Ontologies

There are many definitions of ontologies in the literature. We use the following:

> "An ontology is a formal, explicit specification of a shared conceptualization." (Gruber 1994; Staab, Studer, 2009)

Conceptualizations

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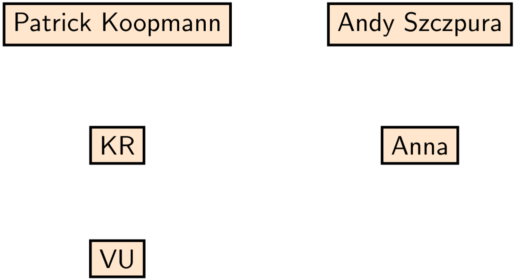
Conceptualizations (after Gruber 1994)

A conceptualization is given by:

- a domain of discourse



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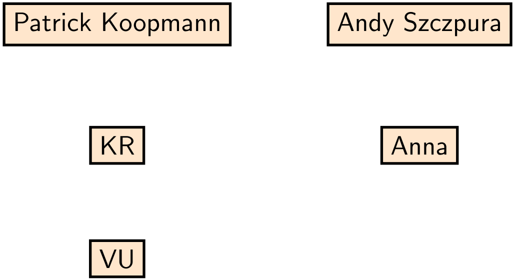

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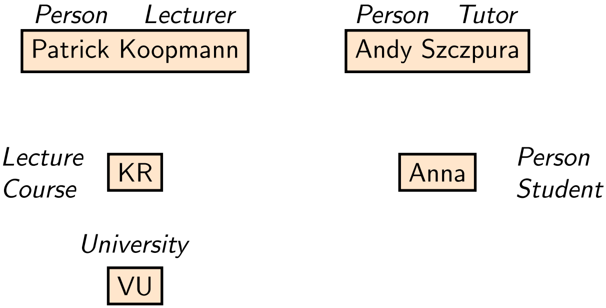

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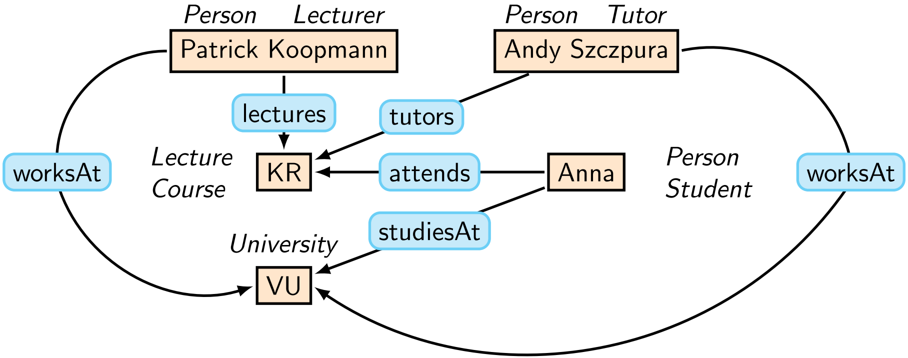
  - roles or relations (binary predicates)



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- a domain of discourse
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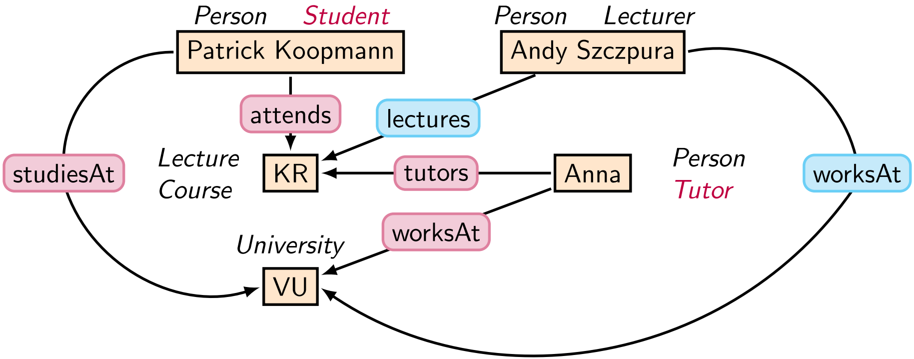
- for different states of the world, and different possible worlds



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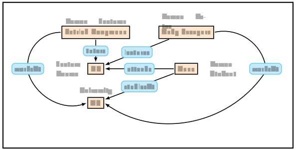

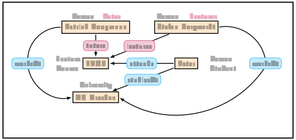

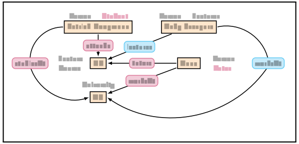

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Conceptualizations

We usually consider a simplified view based on *first-order structures*:

- possible worlds correspond to interpretations
- the domain of discourse is not fixed
- as conceptual relations we consider concepts (unary) and roles (binary)

Formal, Explicit Specifications

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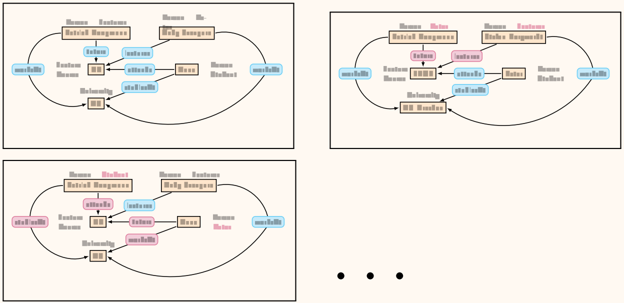
Formal, Explicit Specifications

Extensional specification:

- explicitly state the elements of the conceptual relations



Formal, Explicit Specifications

~~Extensional specification:~~

- ~~explicitly state the elements of the conceptual relations~~

Intensional specification:

- constrain concepts and roles in the different possible worlds
- *axiomatize*

> Every lecture is a course.
>
> A TA is someone who teaches a course.
>
> Every TA works at a university.
>
> ⋮

Formal, Explicit Specifications

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- constrain concepts and roles in the different possible worlds
- *axiomatize*
- formal languages make these specifications *precise*

$$\forall x.\,(\mathit{Lecture}(x) \rightarrow \mathit{Course}(x))$$
$$\forall x.\,(\mathit{TA}(x) \leftrightarrow \exists y.\,(\mathit{teaches}(x, y) \wedge \mathit{Course}(y)))$$
$$\forall x.\,(\mathit{TA}(x) \rightarrow \exists y.\,(\mathit{worksAt}(x, y) \wedge \mathit{University}(y)))$$
$$\vdots$$

Formal, Explicit Specifications

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Intensional specification:

- constrain concepts and roles in the different possible worlds
- *axiomatize*
- formal languages make these specifications *precise*
- dedicated languages make them human readable

```
Lecture SubClassOf Course
TA EquivalentTo teaches some Course
TA SubClassOf worksAt some University
...
```

Formal, Explicit Specifications

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$$\begin{aligned}&\mathit{Lecture} \sqsubseteq \mathit{Course}\\&\mathit{TA} \equiv \exists \mathit{teaches}.\mathit{Course}\\&\mathit{TA} \sqsubseteq \exists \mathit{worksAt}.\mathit{University}\\&\vdots\end{aligned}$$

Formal, Explicit Specifications

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Ontologies allow to share a fixed conceptualization with different parties

- AI system and user
- different user groups/companies
- different systems (communication via the web)

Ontologies in Practice

Ontologies in Practice

- Many knowledge graphs are used without or with very simple ontologies
  - in Wikidata, a lot of relevant knowledge is already there
  - terms are often not used coherently enough to allow for logical reasoning
- Existing ontologies vary a lot in size and expressivity
  - some ontologies are just taxonomies
  - others allow for complex logical inferencing
- In the context of this course, we are interested in the more expressive ontology formalisms

Search and Semantic Web

Schema.org

- Launched in 2011 by Bing, Google and Yahoo!
- Selection of schemas for metadata of web content
- Influences for example search results by Google's Knowledge Graph
- Relatively small:
  - 797 Concepts, 1,453 Roles

```
<div itemscope itemtype="http://schema.org/Movie">
   <h1 itemprop="name">Inglourious Basterds</h1>
   <div itemprop="director" itemscope itemtype="http://schema.org/Person">
     Director:  <span itemprop="name">Quentin Tarantino</span>
     (born <time itemprop="birthDate" datetime="1963-03-27">March 27, 1963</time>)
  </div>
  <span itemprop="genre">War Film</span>
  <a href="../movies/ingbast-theatrical-trailer.html" itemprop="trailer">Trailer</a>
</div>
```

SNOMED CT

- Clinical healthcare terminology
- 361,042 concepts and 242 roles
- Concepts for:
  - clinical findings,
  - symptoms
  - diagnoses
  - procedures
  - body structures
  - organisms and other etiologies,
  - substances,
  - pharmaceuticals,
  - devices,
  - specimens.
- Applications:
  - electronic health records
  - catalogues of clinical services
  - clinical decision support systems
  - ...

SNOMED CT

7 new concepts were added to SNOMED CT on March 9, 2020.

ConceptID	Description
840534001	COVID-19 vaccination
840544004	Suspected COVID-19
840535000	Antibody to SARS-CoV-2
840536004	Antigen of SARS-CoV-2
840539006	COVID-19
840546002	Exposure to SARS-CoV-2
840533007	SARS-CoV-2

These were used by doctors around the world (US, UK, Germany, Argentina, India, Israel, . . .) to record their diagnoses.

Gene Ontology (GO)

- Developed by the Gene Ontology consortium since 1998
- Biological processes and their interactions
- 63.000 concepts and 300 roles
- Main application: biological research

$$\mathit{DNAMetabolicProcess} \equiv \mathit{MetabolicProcess} \sqcap \exists \mathit{hasParticipant}.\mathit{DNA}$$
$$\mathit{MAPKCascade} \sqsubseteq \mathit{MetabolicProcess} \sqcap \exists \mathit{partOf}.\mathit{CellCommunication}$$

Biology and Medicine

NCBO BioPortal

- Source of ontologies on biology and medicine
- `https://bioportal.bioontology.org/`
- 975 ontologies (over 100 more than last year)
- 13,794,030 concepts
- 36,286 Roles
- Some examples:
  - National Cancer Institute Thesaurus (NCIT)
  - SNOMED Clinical Terms (SNOMED CT)
  - Gene Ontology (GO)
  - Semantic Web for Earth and Environment Technology Ontology (SWEET)
  - COVID-19 Ontology (COVID-19)

Common Core Ontologies

- Developed by non-profit R&D company CUBRC, since 2010
- Formalize generic notions found in many applications

Time Ontology:

"A day is a temporal interval. An hour occurs during a day. The relation 'during' is transitive."

$Day \sqsubseteq OneDimensionalTemporalRegion$

$Hour \sqsubseteq \exists intervalDuring.Day$

$intervalDuring \circ intervalDuring \sqsubseteq intervalDuring$

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Agent Ontology:

"An agent is an organization or person that acts in some process".
"A group of agents contains at least one agent and consists only of agents."

$$Agent \equiv (Organization \sqcup Person) \sqcap \exists agentIn.Process$$

$$GroupOfAgents \sqsubseteq \exists hasPart.Agent \sqcap \forall hasPart.Agent$$

More Examples

- GeoNames
  - ontology to specify geographical information (countries, cities, rivers, borders, etc)
  - used for GIS (Geo Information Systems)
- LKIF Legal Core Ontology
  - collection of ontologies for the legal domain
- General Ontology for Linguistic Description (GOLD)
  - ontology about human language
- Process Specification Language (PSL)
  - used to model manufactoring processes
- FoodOn
  - Ontology about food
- SWARMS
  - Ontology about autonomous underwater vehicles (submarine robots)

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- OWL
  - "**W**eb **O**ntology **L**anguage"
  - Most expressive decidable formalism in this list
  - Based on description logics

The Description Logic $\mathcal{ALC}$

Description Logics

Description logics (DLs)

- are decidable fragments of first-order logic
- are restricted to unary and binary predicates
- use a special syntax for formulas
- have the specification of ontologies as main use case

Vocabulary

Concepts / Classes / Categories:

> Person, Student, Teacher, Room, Building, University, ...

Roles / Relations / Properties / Attributes:

> attends, teaches, is part of, is a, belongs to, is employed by, ...

Objects / Individuals:

> Patrick Koopmann, KR Lecture, Vrije Universiteit Amsterdam, Netherlands, ...

Vocabulary

Formally the syntax of DLs is based on the following infinite, disjoint sets:

concept names	role names	individual names
$\mathbf{C} = \{A, B, \dots\}$	$\mathbf{R} = \{r, s, \dots\}$	$\mathbf{I} = \{a, b, \dots\}$

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- Individual names describe individual objects.

  vrijeUniversiteit denotes this university.

Semantics

- Semantics is used to specify the meaning of DL expressions
- As for classical logics, we use interpretations for this
- Recall propositional logic:
  - Vocabulary consists of propositional variables
  - Interpretations assign **true** or **false** to those
- For the DL vocabulary, we need to talk about individual objects and their relations

Interpretations

The semantics of DLs is based on (first-order) interpretations.

A DL interpretation is a tuple $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where

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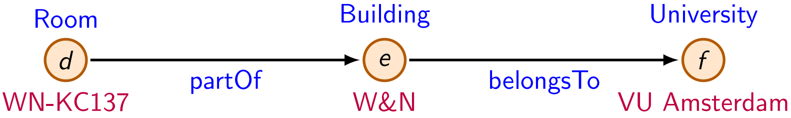
Interpretations can be represented as labeled graphs:



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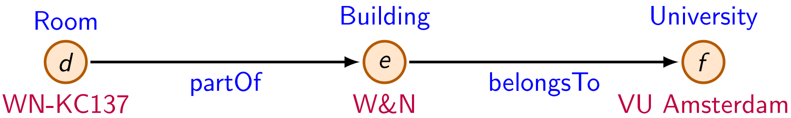

$\mathbf{C} = \{\mathit{Room},\ \mathit{Building},\ \mathit{University},\ \mathit{Lecture},\ \ldots\}$

$\mathbf{R} = \{\mathit{partOf},\ \mathit{belongsTo},\ \ldots\}$

$\mathbf{I} = \{\mathit{WN\text{-}KC137},\ \mathit{W\&N},\ \mathit{VUAmsterdam},\ \ldots\}$

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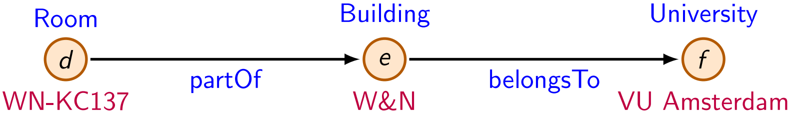

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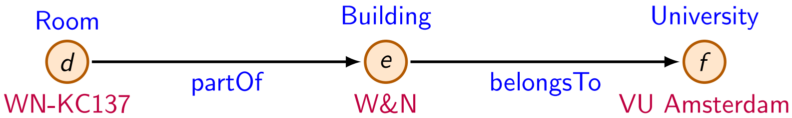
$\Delta^{\mathcal{I}} = \{d, e, f\}$

Example: An Interpretation



$\mathbf{C} = \{Room,\ Building,\ University,\ Lecture,\ \ldots\}$
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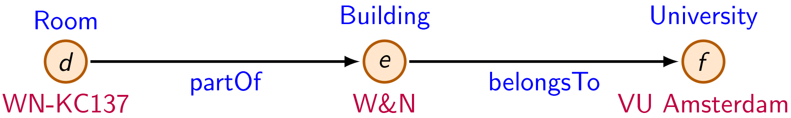

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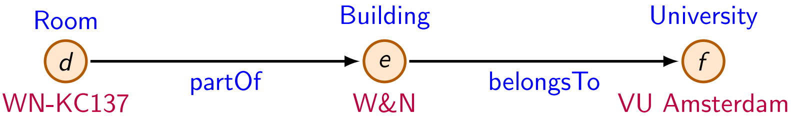

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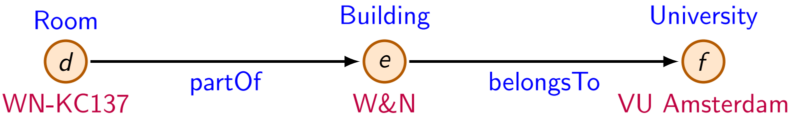
Important Facts about Interpretations



- The domain $\Delta^{\mathcal{I}}$ can be infinite.

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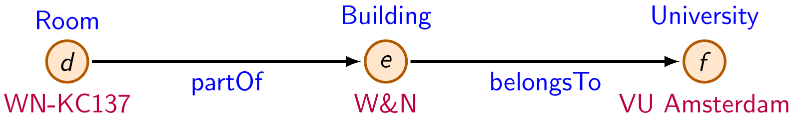

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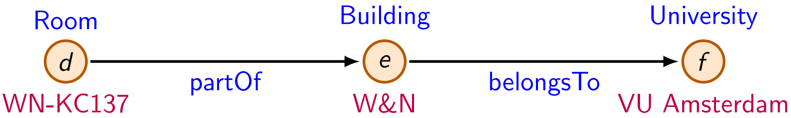

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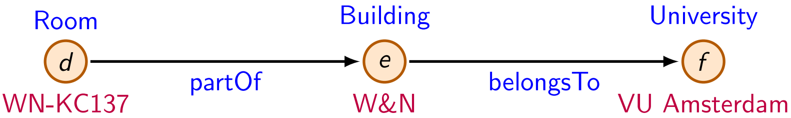

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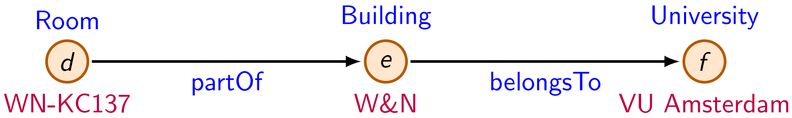

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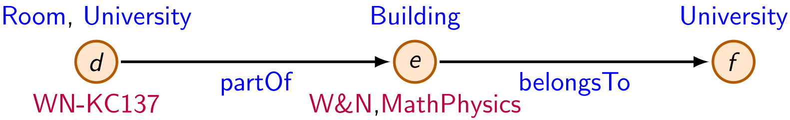
- f is called belongsTo-successor of e.
- e is called belongsTo-predecessor of f.

Example: Another Interpretation

Interpretations can be completely arbitrary:



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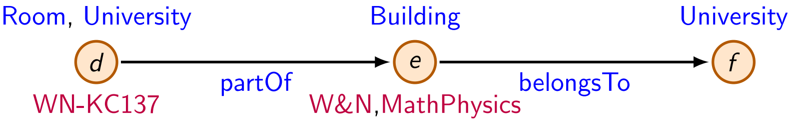

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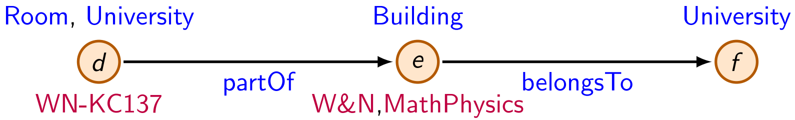

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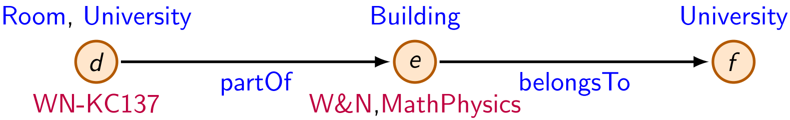

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- There can be unnamed individuals, e.g. f, which do not have a corresponding individual name.
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- The axioms in an ontology restrict the set of interpretations, e.g. by stating that rooms cannot be lectures.

Concepts

Before we introduce axioms, we introduce complex concepts

- also: *concept descriptions*, *compound concepts*, or just *concepts*
- describe sets of objects in an interpretation
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- also: *concept descriptions*, *compound concepts*, or just *concepts*
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- central elements in ontologies and ontology axioms
- the semantics of concepts is captured by the interpretation function:
  - $\cdot^{\mathcal{I}}$ is extended to map concepts C to subsets $C^{\mathcal{I}}$ of the domain $\Delta^{\mathcal{I}}$

Concepts

We define $\mathcal{ALC}$ concepts C and their semantics $C^{\mathcal{I}}$ inductively.

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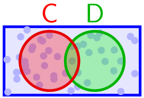
Name:
Syntax:
Semantics:



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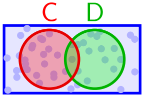
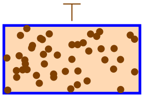
Name:	top
Syntax:	$\top$
Semantics:	$\Delta^{\mathcal{I}}$





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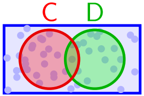
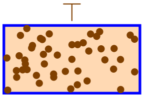
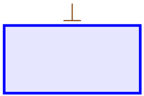
Name:	top	bottom
Syntax:	$\top$	$\bot$
Semantics:	$\Delta^\mathcal{I}$	$\emptyset$







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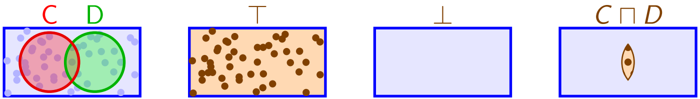
Name:	top	bottom	conjunction
Syntax:	$\top$	$\bot$	$C \sqcap D$
Semantics:	$\Delta^{\mathcal{I}}$	$\emptyset$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$



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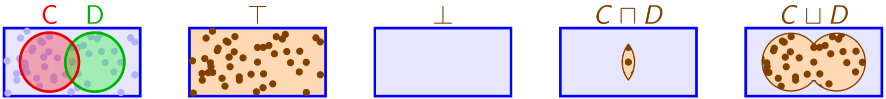
Syntax:	$\top$	$\bot$	$C \sqcap D$	$C \sqcup D$
Semantics:	$\Delta^{\mathcal{I}}$	$\emptyset$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$



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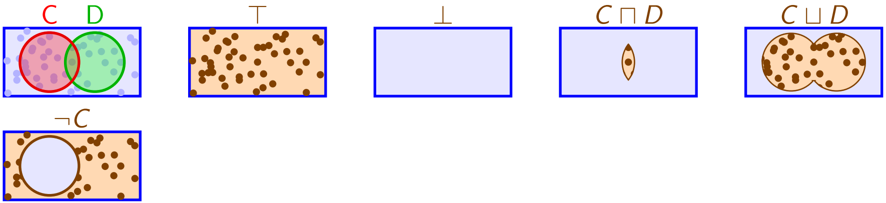
Name:	top	bottom	conjunction	disjunction	complement
Syntax:	$\top$	$\bot$	$C \sqcap D$	$C \sqcup D$	$\neg C$
Semantics:	$\Delta^{\mathcal{I}}$	$\emptyset$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$



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- These correspond to the operators in propositional logic.

Examples for complex concepts:

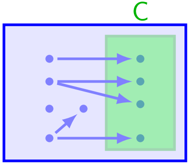
$\mathit{Room} \sqcap \mathit{Lecture}$ $\qquad$ $\neg \mathit{Building}$

Concepts using Roles

Additionally, we can describe outgoing role connections by specifying the concepts of the role successors.

- If C is a concept and r is a role, then the following are also concepts:

Name:
Syntax:
Semantics:



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- If C is a concept and r is a role, then the following are also concepts:

Name:	existential restriction
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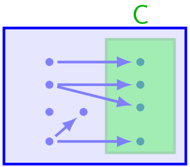
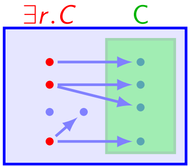
Semantics:	$\{d \mid \exists e.(d,e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\}$



Some r-successor in C: $\exists r.C$



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Name:	existential restriction	value restriction
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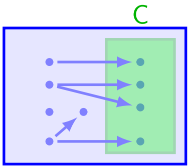

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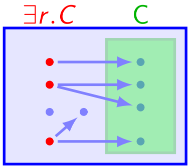
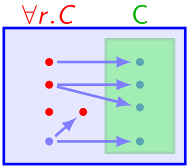


All r-successors in C: $\forall r.C$



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Examples:

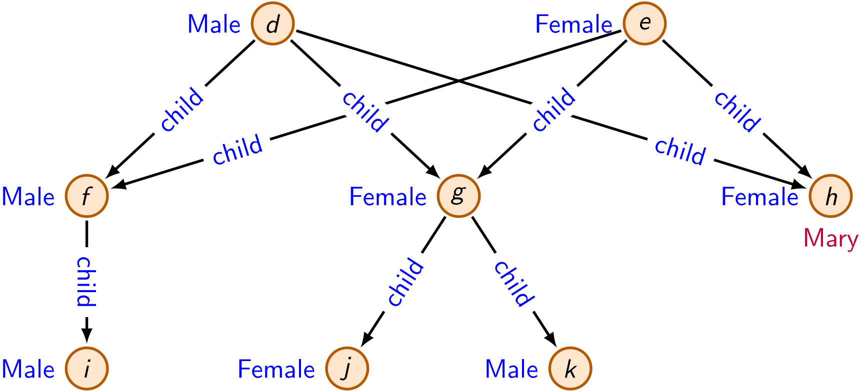
$\exists \mathit{hasChild}.\mathit{Girl}$ $\quad$ $\exists \mathit{hasChild}.\exists \mathit{hasChild}.\top$ $\quad$ $\forall \mathit{eats}.\mathit{PlantBased}$ $\quad$ $\exists \mathit{belongsTo}.\top$

Syntax of $\mathcal{ALC}$ Concepts

$\mathcal{ALC}$ is the description logic that allows the concept constructors $\top$, $\bot$, $\sqcap$, $\sqcup$, $\neg$, $\exists$, and $\forall$ to build concepts.

Other DLs may use different constructors.

Example: Interpreting Concepts



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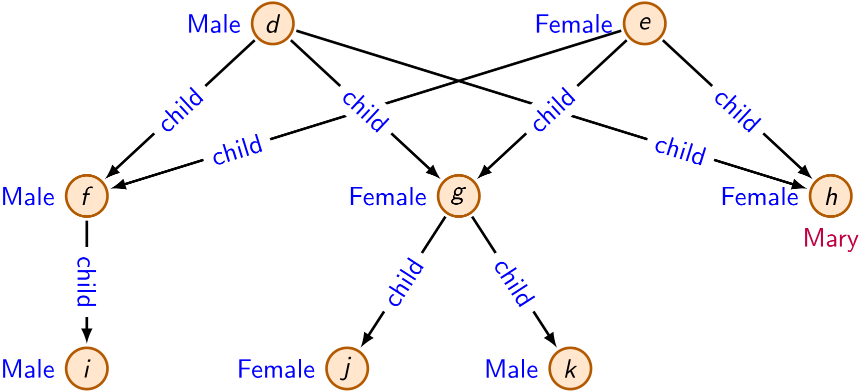

$(\exists child.\top)^{\mathcal{I}} =$

$(Female \sqcap \exists child.\top)^{\mathcal{I}} =$

$(\neg\exists child.\top)^{\mathcal{I}} =$

$(\exists child.Female)^{\mathcal{I}} =$

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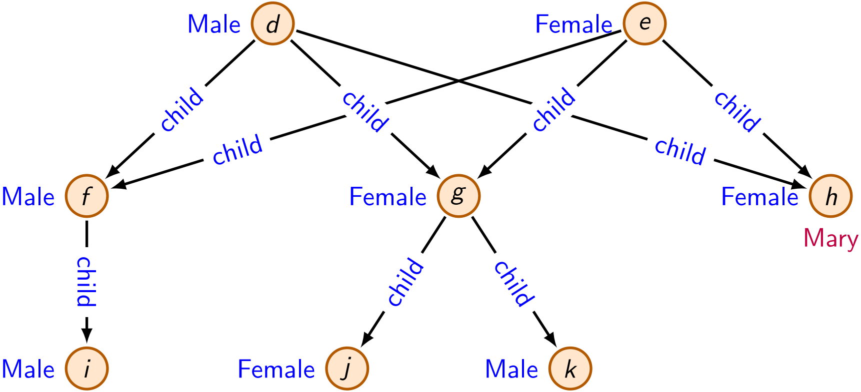

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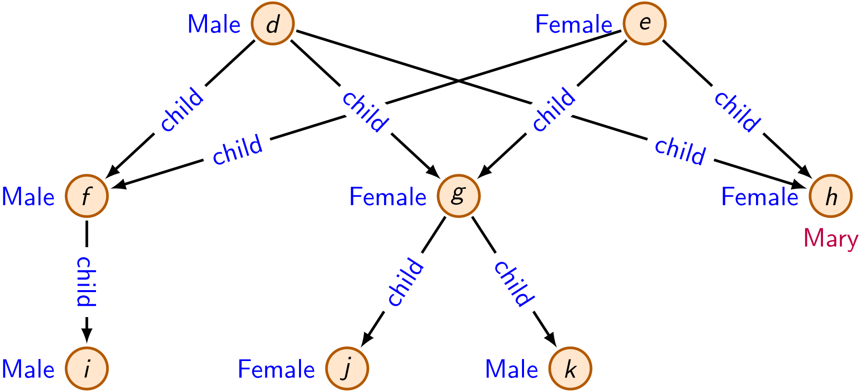

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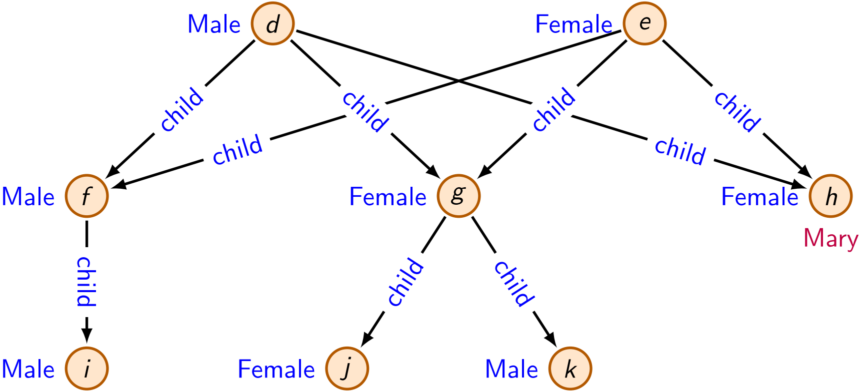

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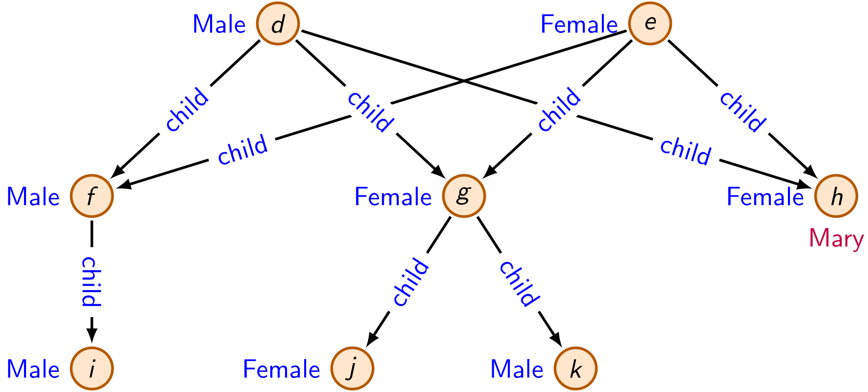

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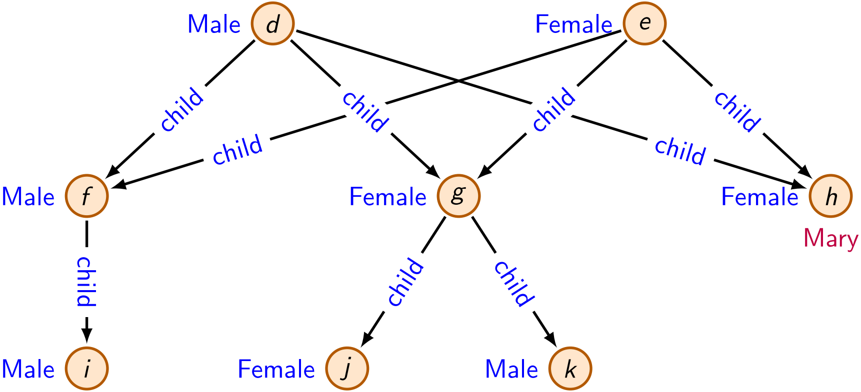

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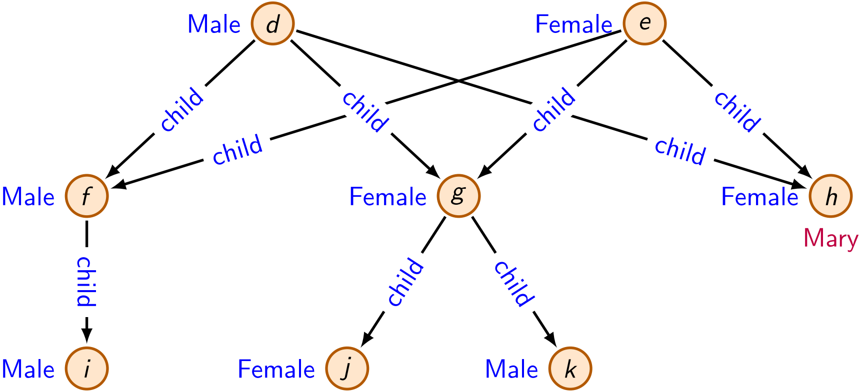

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- Axioms
  - express constraints on interpretations
  - relate concepts to other concepts
  - relate concepts to individuals

From Concepts to Ontologies

- Interpretations are arbitrary
  - We usually do not know which interpretation is the correct one
- Only some will be models of our conceptualization
- To specify what is a model, we use axioms
- Axioms
  - express constraints on interpretations
  - relate concepts to other concepts
  - relate concepts to individuals
- An ontology is then a collection of axioms.

From Concepts to Ontologies

- DL ontologies are sets of axioms
- Axioms put constraints on interpretations
- We distinguish two types:
  - terminological axioms put concepts in relation
  - assertions relate individual names with concepts and roles
- They respectively form the TBox and the ABox of an ontology
- An interpretation satisfying such an axiom/ontology is a *model*
  - $\mathcal{I}$ *satisfies* α: $\mathcal{I} \models \alpha$

Terminological Axioms

If C and D are concepts, then the following is a (terminological) axiom:

Name: general concept inclusion (GCI)
Syntax: $C \sqsubseteq D$
Semantics: $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$

$\mathit{Lecture} \sqsubseteq \mathit{Course}$ $\quad$ $\mathit{Room} \sqsubseteq \neg\mathit{Lecture}$

$\mathit{Room} \sqsubseteq \mathit{Structure} \sqcap \exists \mathit{partOf}.\mathit{Building}$

Terminological Axioms

If C and D are concepts, then the following is a (terminological) axiom:

Name:	general concept inclusion (GCI)	equivalence axiom
Syntax:	$C \sqsubseteq D$	$C \equiv D$
Semantics:	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$	$C^{\mathcal{I}} \equiv D^{\mathcal{I}}$

$Lecture \sqsubseteq Course$ $\quad$ $Room \sqsubseteq \neg Lecture$

$Room \sqsubseteq Structure \sqcap \exists partOf.Building$

$TA \equiv Person \sqcap \exists teaches.Course$

$Lecturer \equiv Person \sqcap \exists teaches.Lecture$

Assertions

Assertions (also called facts) are axioms about named individuals.

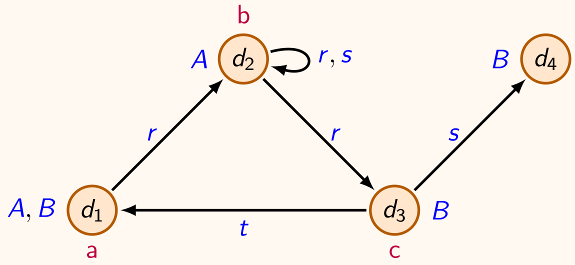
Given $a, b \in \mathbf{I}$, a concept C, and $r \in \mathbf{R}$, the following are assertions:

Name:	concept assertion	role assertion
Syntax:	$a : C$	$(a, b) : r$
Semantics:	$a^{\mathcal{I}} \in C^{\mathcal{I}}$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$

$\mathit{WN\text{-}KC137} : \mathit{Room}$ $\quad$ $(\mathit{W\&N}, \mathit{VUAmsterdam}) : \mathit{belongsTo}$

Exercise: Axioms

Let's look at the following interpretation $\mathcal{I}$:



Which of the following axioms does it satisfy:

1. $(a, b) : r$
2. $c : A$
3. $c : \exists s.B$
4. $A \sqsubseteq \exists r.A$
5. $A \sqsubseteq \forall r.(A \sqcup B)$
6. $A \equiv \forall r.(A \sqcup B)$
7. $\exists r.\top \sqsubseteq A$
8. $\exists r.\bot \sqsubseteq B$
9. $\exists r.A \sqsubseteq \forall s.A$

Ontologies

An ontology is a set $\mathcal{O} = \mathcal{A} \cup \mathcal{T}$, where

- $\mathcal{A}$ is an ABox, a finite set of assertions,
- $\mathcal{T}$ is a TBox, a finite set of GCIs and equivalence axioms,

An interpretation is a model of $\mathcal{O}$ (written $\mathcal{I} \models \mathcal{O}$) if it is a model of all axioms in $\mathcal{O}$.

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An interpretation is a model of $\mathcal{O}$ (written $\mathcal{I} \models \mathcal{O}$) if it is a model of all axioms in $\mathcal{O}$.

- The ABox contains facts about named individuals (data), the TBox contains (terminological) knowledge that applies to all individuals.

Example: An Ontology

$\mathcal{O} = \mathcal{A} \cup \mathcal{T}$ with

$\mathcal{A} = \{\mathit{WN\text{-}KC137} : \mathit{Room}, (\mathit{WN\text{-}KC137}, \mathit{W\&N}) : \mathit{partOf}\}$

$\mathcal{T} = \{\mathit{Room} \sqsubseteq \neg\mathit{University}, \mathit{Room} \sqsubseteq \exists\mathit{partOf}.\mathit{Building},$
$\mathit{Building} \sqsubseteq \neg\mathit{University}, \mathit{Building} \sqsubseteq \neg\mathit{Room}\}$

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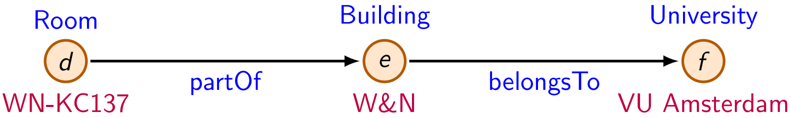
This ontology has many models, for example the following:



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$\quad \mathit{Building} \sqsubseteq \neg\mathit{University},\ \mathit{Building} \sqsubseteq \neg\mathit{Room}\}$

The following interpretation is not a model of the ontology: