Exam Advanced Machine Learning
27 October 2022, 18.45–21.00

This exam consists of 5 problems, each consisting of several questions. All answers should be motivated, including calculations, formulas used, etc. The use of a calculator is not allowed.

Question 1: Short questions
Please provide an argument for your answer on the following questions.

(a) Suppose that you use linear regression to model a particular dataset. To test your linear regression model, you choose at random some records to be the training set, and choose at random some of the remaining records to be the test set. Now, let us increase the size of the training set gradually. Explain what will happen to the mean training error and the mean testing error.

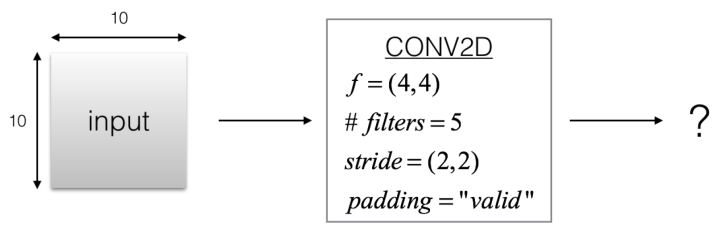
(b) Jason and Bob are discussing which structural assumptions in polynomial regression most affect the trade-off between underfitting and overfitting. Jason claims that the polynomial degree in the regression is more important for the trade-off, whereas Bob claims that the assumed variance in the Normal distribution of the error is more important. Who is right, and why?

(c) Suppose that you are given a train set X_{train}, Y_{train} and a test set X_{test}, Y_{test}. You want to normalize your data before training your model. Argue if the following statement is true or false. The test data should be normalized with its own mean and variance before being fed to the network at test time because the test distribution might be different from the train distribution.

(d) You are doing full batch gradient descent using the entire training set (not stochastic gradient descent). Is it necessary to shuffle the training data? Explain your answer.

(e) Weight sharing allows convolutional neural networks to deal with image data without using too many parameters. Does weight sharing increase the bias or the variance of a model?

(f) Consider the figure below.



The input is of shape $(n_H, n_W, n_C) = (10, 10, 1)$. There are five 4×4 convolutional filters with 'valid' padding (i.e., zero padding) and a stride of $(2, 2)$. What is the output shape after performing the convolution step? Write your answer in the following format: (n_H, n_W, n_c).

(g) You are consulting for a healthcare company. A patient may have any number of illnesses from a list of 70,000 known medical illnesses. The output of your recurrent neural network will therefore be a vector with 70,000 elements. Each element in this output vector represents the probability that the patient has the illness that maps to that particular element. Illnesses are not mutually exclusive, i.e., having one illness does not preclude you from having any other illnesses. Given this insight, what activation function would you use for your output unit? Explain your answer.

Question 2: Neural networks

The following neural network in Figure 1 has 3 units. The neural network operates as a regular neural network. Each unit takes a linear combination of the units of the previous layer, adds a bias term, and then applies an activation function g to obtain the activated units (i.e., a_1, a_2, or a_3). Additionally, this network uses the standard error function $E = \frac{1}{2}(y-t)^2$, with t the target value and $y = a_3$, the output of the neural network.

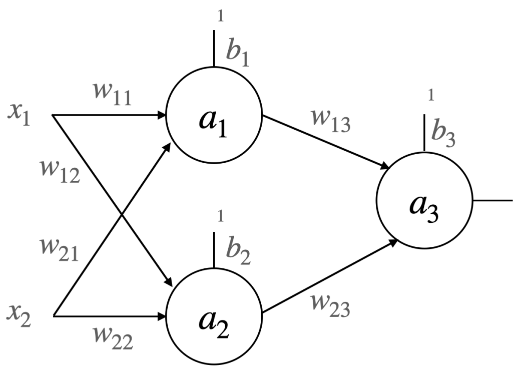


Figure 1: Neural network architecture.

(a) We discussed several activation functions during the lecture. We did not discuss the Exponential Linear Unit (ELU) g_1 and the SoftPlus g_2 activation functions. These activation functions are given by

$$g_1(z) = \begin{cases} \alpha(e^z - 1), & \text{for } z < 0, \\ z, & \text{for } z \geq 0, \end{cases} \qquad \text{and} \qquad g_2(z) = \log_e(1 + e^z),$$

for some $\alpha \in \mathbb{R}$. Calculate the derivative of $g_i(z)$ with respect to z for both $i = 1, 2$.

(b) Discuss the advantages and disadvantages of the ELU and SoftPlus activation functions over the sigmoid, tanh, and ReLu activation functions.

Assume that node a_1 and node a_2 are activated by g_1 and node a_3 is activated by g_2.

(c) Calculate $\partial E/\partial w_{13}$.

(d) Calculate $\partial E/\partial w_{11}$.

(e) We have discussed the linear activation function during the lecture. This was given by $z = w_0 + \sum_i w_i x_i$. Now, consider the hard threshold

$$z = \begin{cases} 1, & \text{if } w_0 + \sum_i w_i x_i \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Which of the following functions can be exactly represented by a neural network with one hidden layer which uses linear and/or hard threshold activation functions? For each case, justify your answer.

(i) polynomials of degree one
(ii) hinge loss, i.e., $h(x) = \max\{1 - x, 0\}$
(iii) polynomials of degree two
(iv) piecewise constant functions

Question 3: Graphical models
The following figure shows a graphical model over nine binary-valued variables $A, \ldots, I$. We do not know the parameters of the probability distribution associated with the graph.

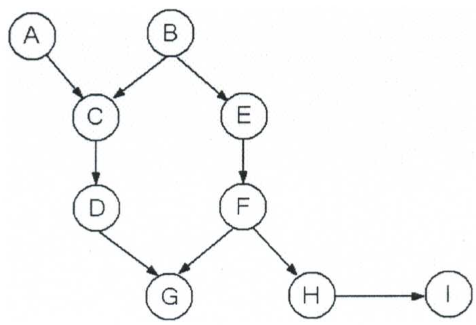


Figure 2: Graphical model.

(a) Write the expression for the joint probability $\mathbb{P}(A, B, C, D, E, F, G, H, I)$ of the network in its *reduced* factored form.

(b) Which of the following conditional independence assertions are true?

(i) $A \perp\!\!\!\perp B \,|\, G$
(ii) $A \perp\!\!\!\perp I$
(iii) $B \perp\!\!\!\perp H \,|\, E, G$
(iv) $\mathbb{P}(C \,|\, B, F) = \mathbb{P}(C \,|\, F)$

Question 4: Hidden Markov Models (HMMs)
Consider an HMM with latent states $Z_t \in \{1, 2, 3\}$, and observations $X_t \in \{A, B, C\}$. The initial distribution is given by $\pi = (\pi_1, \pi_2, \pi_3) = (1, 0, 0)$. The transition probabilities of the latent states and the emission probabilities are given by

$$A = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \text{and} \qquad \varphi = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}.$$

Thus, e.g., $\mathbb{P}(Z_t = 2 \mid Z_{t-1} = 1) = 1/4$ and $\mathbb{P}(Z_t = 2 \mid Z_{t-1} = 2) = 1/2$. Similarly, states A and B are observed with probability $1/2$ in latent state 1, A and C in latent state 2, and states B and C in latent state 3.

(a) Calculate the probability $\mathbb{P}(Z_5 = 3)$.

(b) Calculate the probability $\mathbb{P}(Z_5 = 3 \mid (X_1, \ldots, X_7) = (A, A, B, C, A, B, C))$.

(c) Write down the sequence $Z_1, \ldots, Z_7$ that maximizes the probability of observing the sequence $(X_1, \ldots, X_7) = (A, A, B, C, A, B, C)$.

(d) Suppose that you are training an HMM with a small number of latent states from a large number of observations. Explain if, in general, you can increase the training data likelihood by permitting more latent states.

Question 5: Machine learning for blackjack

Armed with the power of Q-learning, you go to Holland Casino. You play a simplified version of blackjack where the deck is infinite and the dealer always has a fixed count of 15. The deck contains cards 2 through 10, J, Q, K, and A, each of which is equally likely to appear when a card is drawn. Each number card is worth the number of points shown on it, the cards J, Q, and K are worth 10 points, and A is worth 11.

At each turn, you may either *hit* or *stay*. If you choose to *hit*, you receive no immediate reward and are dealt an additional card. If you *stay*, you receive a reward of 0 if your current point total is exactly 15, $+10$ if it is higher than 15 but not higher than 21, and -10 otherwise (i.e., lower than 15 or larger than 21). After taking the *stay* action, the game enters a terminal state *end* and ends. A total of 22 or higher is refered to as a *bust*; from a bust, you can only choose the action *stay*.

As your state space, you take the set $\{0, 2, \ldots, 21, \text{bust}, \text{end}\}$ indicating point totals, "bust" if your point total exceeds 21, and "end" for the end of the game.

(a) Suppose you have performed k iterations of value iteration. Compute $V_{k+1}(12)$ given the partial table below for $V_k(s)$. Give your answer in terms of the discount γ as a variable. Note: do not worry about whether the listed V_k values could actually result from this MDP!

s	$V_k(s)$
13	2
14	10
15	10
16	10
17	10
18	10
19	10
20	10
21	10
bust	-10
end	0

(b) You suspect that the cards do not actually appear with equal probability and decide to use Q-learning instead of value iteration. Given the partial table of initial Q-values below, update the partial table of Q-values after the following episode occurred. Assume a learning rate of 0.5 and a discount factor of $\gamma = 1$. The initial portion of the episode has been omitted.

s	a	$Q(s,a)$
19	hit	-2
19	stay	5
20	hit	-4
20	stay	7
21	hit	-6
21	stay	8
bust	stay	-8

Episode

s	a	r	s	a	r	s	a	r
19	hit	0	21	hit	0	bust	stay	-10

(c) Unhappy with your experience with basic Q-learning, you decide to featurize your Q-values, representing them in the form $\sum_i w_i f_i(s,a)$ for some feature functions $f_i(s,a)$. Consider the two feature functions

$$f_1(s,a) = \begin{cases} 0, & \text{if } a = \text{stay}, \\ +1, & \text{if } a = \text{hit and } s \geq 15, \\ -1, & \text{if } a = \text{hit and } s < 15. \end{cases} \quad \text{and} \quad f_2(s,a) = \begin{cases} 0, & \text{if } a = \text{stay}, \\ +1, & \text{if } a = \text{hit and } s \geq 18, \\ -1, & \text{if } a = \text{hit and } s < 18. \end{cases}$$

For which of the following partial policy tables (i)–(v) is it possible to represent Q-values in the form $w_1 f_1(s,a) + w_2 f_2(s,a)$ that imply that policy unambiguously (i.e., without having to break ties)?

(i)

s	$\pi(s)$
14	hit
15	hit
16	hit
17	hit
18	hit
19	hit

(ii)

s	$\pi(s)$
14	stay
15	hit
16	hit
17	hit
18	stay
19	stay

(iii)

s	$\pi(s)$
14	hit
15	hit
16	hit
17	hit
18	stay
19	stay

(iv)

s	$\pi(s)$
14	hit
15	hit
16	hit
17	hit
18	hit
19	stay

(v)

s	$\pi(s)$
14	hit
15	hit
16	hit
17	stay
18	hit
19	stay

partial grade	1	2	3	4	5
(a)	1	2	1	2	1
(b)	1	2	4	2	2
(c)	1	2		2	3
(d)	1	2		1	
(e)	1	3			
(f)	1				
(g)	1				

Final grade is: (sum of partial grades) / 4.0 + 1.0