Knowledge Representation

Lecture 7: Non-monotonic Reasoning and Default Logic

Atefeh Keshavarzi

17, November 2023

Human Reasoning

Human Reasoning

- Tweety is a bird.
- Does Tweety Fly?

Human Reasoning

- Tweety is a bird.
- Does Tweety Fly?
- Tweety is a penguin.
- Does Tweety Fly?

- Birds can fly! Then, Tweety can fly.

Human Reasoning

- Tweety is a bird.
- Does Tweety Fly?
- Tweety is a penguin.
- Does Tweety Fly?

- Birds can fly! Then, Tweety can fly.
- Penguin cannot fly! Then, Tweety cannot fly.

Human Reasoning

- Tweety is a bird.
- Does Tweety Fly?
- Tweety is a penguin.
- Does Tweety Fly?
- You are not a logical person!

- Birds can fly! Then, Tweety can fly.
- Penguin cannot fly! Then, Tweety cannot fly.

Human Reasoning

- Tweety is a bird.
- Does Tweety Fly?
- Tweety is a penguin.
- Does Tweety Fly?
- You are not a logical person!

- Birds can fly! Then, Tweety can fly.
- Penguin cannot fly! Then, Tweety cannot fly.
- WHY?

Human Reasoning

- Tweety is a bird.
- Does Tweety Fly?
- Tweety is a penguin.
- Does Tweety Fly?
- You are not a logical person!
- Contradiction!

- Birds can fly! Then, Tweety can fly.
- Penguin cannot fly! Then, Tweety cannot fly.
- WHY?

Flashback: First-Order Logic

We can use first-order logic to model the parcel example a bit better:

$$\neg \mathit{AtHome}(\mathit{patrick})$$
$$\mathit{DeliversParcelTo}(\mathit{mailMan}, \mathit{patrick}, \mathit{parcel})$$
$$\mathit{Neighbour}(\mathit{patrick}, \mathit{lucia})$$
$$\forall x : \forall y : (\exists z : \mathit{DeliversParcelTo}(z, y, x) \land \mathit{AtHome}(x)) \rightarrow \mathit{ReceivesParcel}(x, y))$$
$$\vdots$$

Human Reasoning

- **Tweety is a bird. Does Tweety fly**?
- Bird(x): x is a bird.
- Fly(x): x flies.
- $\forall x : (B(x) \rightarrow F(x))$
- Bird(tweety) $\rightarrow$ Fly(tweety)

Human Reasoning

- **Tweety is a bird. Does Tweety fly**?
- Bird(x): x is a bird.
- Fly(x): x flies.
- $\forall x : (B(x) \rightarrow F(x))$
- Bird(tweety) $\rightarrow$ Fly(tweety)

- Fly(tweety)

Human Reasoning

- **Tweety is a bird. Does Tweety fly**?
- Bird(x): x is a bird.
- Fly(x): x flies.
- $\forall x : (B(x) \rightarrow F(x))$
- $\text{Bird(tweety)} \rightarrow \text{Fly(tweety)}$
- **Tweety is a penguin. Does Tweety Fly?**
- $\forall x : (\text{Penguin(x)} \rightarrow \text{Bird(x)})$
- $\forall x : (\text{Penguin(x)} \rightarrow \neg\text{Fly(x)})$

- Fly(tweety)

Human Reasoning

- **Tweety is a bird. Does Tweety fly**?
- Bird(x): x is a bird.
- Fly(x): x flies.
- $\forall x : (B(x) \rightarrow F(x))$
- Bird(tweety) $\rightarrow$ Fly(tweety)
- **Tweety is a penguin. Does Tweety Fly?**
- $\forall x$: (Penguin(x) $\rightarrow$ Bird(x))
- $\forall x$: (Penguin(x) $\rightarrow \neg$Fly(x))
- **Contradiction! Don't you believe in logic**?

- Fly(tweety)
- $\neg$Fly(tweety)

Human Reasoning

- **Tweety is a bird. Does Tweety fly**?
- Bird(x): x is a bird.
- Fly(x): x flies.
- $\forall x : (B(x) \rightarrow F(x))$
- Bird(tweety) $\rightarrow$ Fly(tweety)
- **Tweety is a penguin. Does Tweety Fly?**
- $\forall x$: (Penguin(x) $\rightarrow$ Bird(x))
- $\forall x$: (Penguin(x) $\rightarrow$ $\neg$Fly(x))
- **Contradiction! Don't you believe in logic**?

- Fly(tweety)
- $\neg$Fly(tweety)
- WHY?

Human Reasoning

What are your thoughts on the previous discussion?

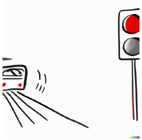
1. Mary is characterized as an irrational person.
2. Mary should not change her mind about Tweety flying, even after realizing it is a penguin.
3. Mary should delay answering Ali's first question until she is certain about Tweety's ability to fly.
4. Classical logic is not a suitable formalism for representing human reasoning.

menti
.com
2782
0921

An Automated Train System

Is propositional logic appropriate to model exception?

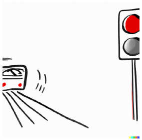
The train operates smoothly unless a train sensor detects an obstacle, in which case it must come to a stop.



An Automated Train System

Is propositional logic appropriate to model exception?

The train operates smoothly unless a train sensor detects an obstacle, in which case it must come to a stop.



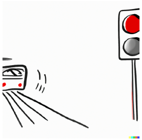
Option 1

- train
- $\text{train} \rightarrow \text{goes}$
- $\text{train} \wedge \text{obstacle} \rightarrow \text{stops}$
- obstacle
- $\text{stops} \rightarrow \neg\text{goes}$

An Automated Train System

Is propositional logic appropriate to model exception?

The train operates smoothly unless a train sensor detects an obstacle, in which case it must come to a stop.



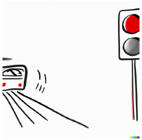
Option 1

- train
- train $\rightarrow$ goes
- train $\wedge$ obstacle $\rightarrow$ stops
- obstacle
- stops $\rightarrow$ $\neg$goes
- inconsistent!

An Automated Train System

Is propositional logic appropriate to model exception?

The train operates smoothly unless a train sensor detects an obstacle, in which case it must come to a stop.



Option 1

- train
- train $\rightarrow$ goes
- train $\wedge$ obstacle $\rightarrow$ stops
- obstacle
- stops $\rightarrow \neg$goes
- inconsistent!

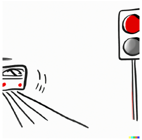
Option 2

- train
- train $\wedge \neg$obstacle $\rightarrow$ goes
- train $\wedge$ obstacle $\rightarrow$ stop

An Automated Train System

Is propositional logic appropriate to model exception?

The train operates smoothly unless a train sensor detects an obstacle, in which case it must come to a stop.



Option 1

- train
- train $\rightarrow$ goes
- train $\wedge$ obstacle $\rightarrow$ stops
- obstacle
- stops $\rightarrow$ $\neg$goes
- inconsistent!

Option 2

- train
- train $\wedge$ $\neg$obstacle $\rightarrow$ goes
- train $\wedge$ obstacle $\rightarrow$ stop
- it does not go!

Outline

Reasoning is often Defeasible

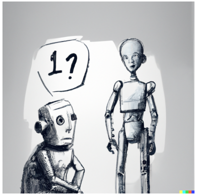
Monotonic vs. Non-monotonic Reasoning

Why is standard logic not suitable?

Default Logic

- Syntax
- Semantics

Decision-Making under Uncertainty



Knowledge is often incomplete and uncertain, so agents

- Draw tentative conclusions based on **available evidence**;
- **Revise** conclusions when new evidence **contradicts** previous assumptions.

This type of reasoning is defeasible and non-monotonic logics capture the logic of defeasible reasoning.

Defeasible Reasoning: AI System for Diagnosing COVID-19

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?

1. Initial knowledge: Alice has the symptoms, and no test result.

Initial Knowledge

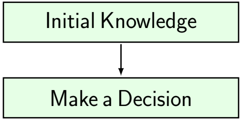
Defeasible Reasoning: AI System for Diagnosing COVID-19

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?

1. Initial knowledge: Alice has the symptoms, and no test result.
2. Apply the rule: Alice is likely to have COVID-19.



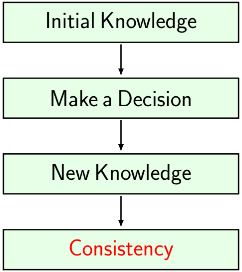
Defeasible Reasoning: AI System for Diagnosing COVID-19

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?

1. Initial knowledge: Alice has the symptoms, and no test result.
2. Apply the rule: Alice is likely to have COVID-19.
3. New knowledge: Alice gets a positive test result.

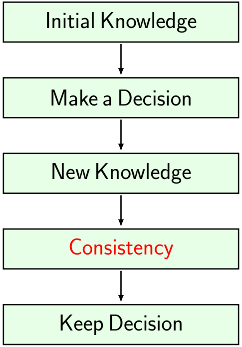


Defeasible Reasoning: AI System for Diagnosing COVID-19

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?



1. Initial knowledge: Alice has the symptoms, and no test result.
2. Apply the rule: Alice is likely to have COVID-19.
3. New knowledge: Alice gets a positive test result.
4. Keep decision: Alice has COVID-19

Defeasible Reasoning: AI System Diagnostic Reasoning

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?

1. Initial knowledge: Bob has the symptoms, and a negative test result.

Initial Knowledge

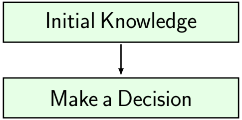
Defeasible Reasoning: AI System Diagnostic Reasoning

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?

1. Initial knowledge: Bob has the symptoms, and a negative test result.
2. Apply the exception: Bob is likely not to have COVID-19, but the flu.



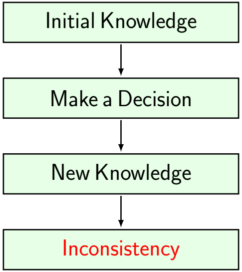
Defeasible Reasoning: AI System Diagnostic Reasoning

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?

1. Initial knowledge: Bob has the symptoms, and a negative test result.
2. Apply the exception: Bob is likely not to have COVID-19, but the flu.
3. New knowledge: Bob gets a second test result that is positive.



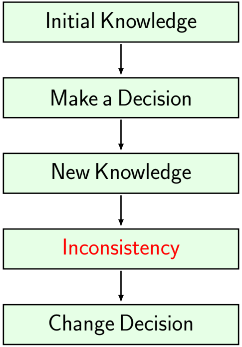
Defeasible Reasoning: AI System Diagnostic Reasoning

Example

- Rule: symptoms such as fever, cough, and difficulty breathing, likely to have COVID-19.
- Exception: a negative COVID-19 test result, the flu.

What is a system decision?

1. Initial knowledge: Bob has the symptoms, and a negative test result.
2. Apply the exception: Bob is likely not to have COVID-19, but the flu.
3. New knowledge: Bob gets a second test result that is positive.
4. Retracting the previous conclusion and applying the rule: Bob has COVID-19.



Outline

Reasoning is often Defeasible

Monotonic vs. Non-monotonic Reasoning

Why is standard logic not suitable?

Default Logic

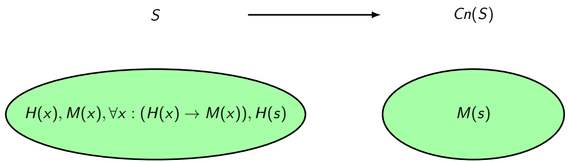
- Syntax
- Semantics

Monotonic Reasoning

Example

All humans are mortal. Socrates is a human.

- $H(x)$: x is human.
- $M(x)$: x is mortal.
- $\forall x : (H(x) \rightarrow M(x))$: All humans are mortal.
- $H(s)$: Socrates is a human.
- Entailment: $M(s)$.

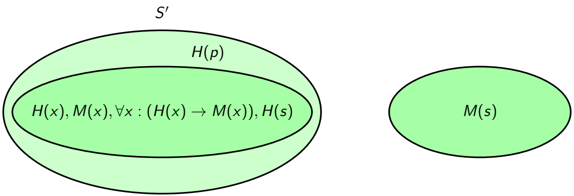


Monotonic Reasoning

Example

All humans are mortal. Socrates is a human. Plato is a human.

- $H(x)$: x is human.
- $M(x)$: x is mortal.
- $\forall x : (H(x) \to M(x))$: All humans are mortal.
- $H(s)$: Socrates is a human.
- Entailment: $M(s)$.
- $H(p)$: Plato is a human.

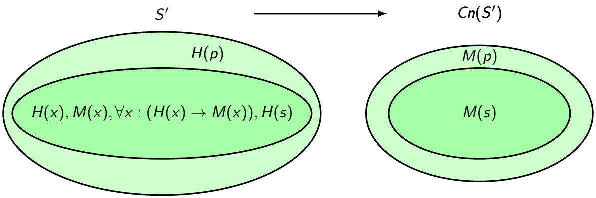


Monotonic Reasoning

Example

All humans are mortal. Socrates is a human. Plato is a human.

- $H(x)$: x is human.
- $M(x)$: x is mortal.
- $\forall x : (H(x) \rightarrow M(x))$: All humans are mortal.
- $H(s)$: Socrates is a human.
- Entailment: $M(s)$.
- $H(p)$: Plato is a human.
- Entailment: $M(s), M(p)$.



Which of the following sentences implies the notion of monotonic reasoning?

Monotonic reasoning means:

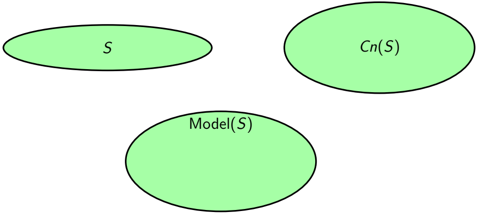
- the more information I have, the less I can derive, with the extreme case in which I cannot derive anything.
- the more information I have, the more I can derive, with the extreme case of an inconsistency in which I cannot derive anything.
- the more information I have, the more I can derive, with the extreme case of an inconsistency in which I can derive everything.

menti
.com
4434
8950

Monotonic Reasoning

Definition

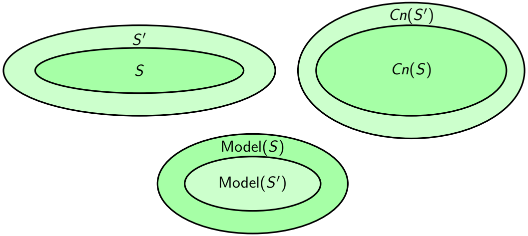
A logic L is monotonic iff for any sets of formula S and S' it holds that if $S \subseteq S'$ then $Cn(S) \subseteq Cn(S')$, where $Cn(S) = \{ \phi \mid S \models_L \phi \}$



Monotonic Reasoning

Definition

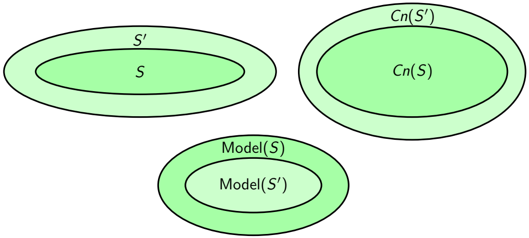
A logic L is monotonic iff for any sets of formula S and S' it holds that if $S \subseteq S'$ then $Cn(S) \subseteq Cn(S')$, where $Cn(S) = \{ \phi \mid S \models_L \phi \}$



Monotonic Reasoning

Definition

A logic L is monotonic iff for any sets of formula S and S' it holds that if $S \subseteq S'$ then $Cn(S) \subseteq Cn(S')$, where $Cn(S) = \{ \phi \mid S \models_L \phi \}$



Monotonic Reasoning

More information
$\Longrightarrow$ More entailments
$\Longrightarrow$ Less models

- Propositional logic is monotonic
- First-order logic is monotonic.

Reasoning

- Deriving logical conclusion and making predictions from available knowledge, facts, and beliefs
  - **Monotonic Reasoning**: The more you know, the more is entailed.
  - **Non-monotonic Reasoning**: Some conclusions may be invalidated if we add some more information to our knowledge base.

Reasoning

- Deriving logical conclusion and making predictions from available knowledge, facts, and beliefs
  - **Monotonic Reasoning**: The more you know, the more is entailed.
  - **Non-monotonic Reasoning**: Some conclusions may be invalidated if we add some more information to our knowledge base.

Example

Birds normally can fly. Tweety is a bird.

Reasoning

- Deriving logical conclusion and making predictions from available knowledge, facts, and beliefs
  - **Monotonic Reasoning**: The more you know, the more is entailed.
  - **Non-monotonic Reasoning**: Some conclusions may be invalidated if we add some more information to our knowledge base.

Example

Birds normally can fly. Tweety is a bird.

- Conclusion: Tweety can fly

Reasoning

- Deriving logical conclusion and making predictions from available knowledge, facts, and beliefs
  - **Monotonic Reasoning**: The more you know, the more is entailed.
  - **Non-monotonic Reasoning**: Some conclusions may be invalidated if we add some more information to our knowledge base.

Example

Birds normally can fly. Tweety is a bird.

- Conclusion: Tweety can fly

Tweety is a penguin.
Exception: Penguin cannot fly!

Reasoning

- Deriving logical conclusion and making predictions from available knowledge, facts, and beliefs
  - **Monotonic Reasoning**: The more you know, the more is entailed.
  - **Non-monotonic Reasoning**: Some conclusions may be invalidated if we add some more information to our knowledge base.

Example

Birds normally can fly. Tweety is a bird.

- Conclusion: Tweety can fly

Tweety is a penguin.
Exception: Penguin cannot fly!

- New information cause a revision of previous conclusions
- Conclusion: Tweety cannot fly

Advantage of Non-monotonic Reasoning

- For real-world scenarios containing exceptions, such as Robot navigation, we can use non-monotonic reasoning

Flashback: Human Reasoning

- How did you use FOL for **non-monotonic reasoning**?
- Your example was an instance of **defeasible reasoning**.

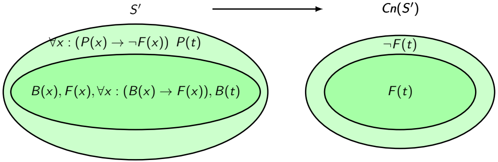
Flashback: Human Reasoning

- Here is my formalism.

- How did you use FOL for **non-monotonic reasoning**?
- Your example was an instance of **defeasible reasoning**.

Flashback: Human Reasoning

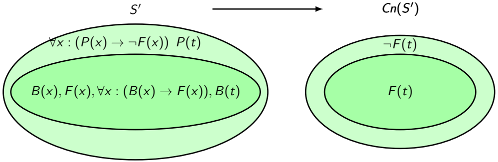
- Here is my formalism.



- How did you use FOL for **non-monotonic reasoning**?
- Your example was an instance of **defeasible reasoning**.

Flashback: Human Reasoning

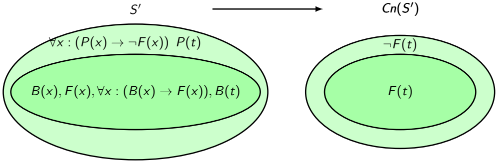
- Here is my formalism.



- How did you use FOL for **non-monotonic reasoning**?
- Your example was an instance of **defeasible reasoning**.
- You are not allowed to use FOL for non-monotonic reasoning.
- I revised conclusions when new evidence contradicts previous assumptions.

Flashback: Human Reasoning

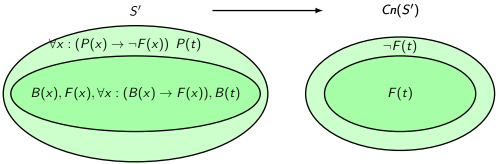
- Here is my formalism.
- Is there a formalism to represent non-monotonic reasoning?



- How did you use FOL for **non-monotonic reasoning**?
- Your example was an instance of **defeasible reasoning**.
- You are not allowed to use FOL for non-monotonic reasoning.
- I revised conclusions when new evidence contradicts previous assumptions.

Flashback: Human Reasoning

- Here is my formalism.
- Is there a formalism to represent non-monotonic reasoning?



- How did you use FOL for **non-monotonic reasoning**?
- Your example was an instance of **defeasible reasoning**.
- You are not allowed to use FOL for non-monotonic reasoning.
- I revised conclusions when new evidence contradicts previous assumptions.
- I do not know!

Different non-monotonic approaches

- Default Logic
  - By default, a robot can function, except its hand is broken.
  - By default, a train keep working, except it detect an obstacle.
- Logic Programming with Negation as Failure
- Answer Set Programming (ASP)
- Formal Argumentation

Why do AI students need to know about default theory?

1. **Non-Monotonic Reasoning**: Default logic is a key formalism in non-monotonic reasoning.
2. **Knowledge Representation**: In AI, effective knowledge representation is essential for building intelligent systems that can make decisions and solve problems. Default logic provides a framework for representing and reasoning about incomplete or uncertain knowledge.
3. **Dealing with Exceptions**: In real-world scenarios, not all information is explicitly known or stated, and default reasoning allows AI systems to make plausible inferences in the absence of complete information.

Outline

Reasoning is often Defeasible

Monotonic vs. Non-monotonic Reasoning

Why is standard logic not suitable?

Default Logic

- Syntax
- Semantics

Default Logic

Consider the knowledge base: "**Usually** birds can fly, except the abnormal ones", "Tweety is a bird. All birds are animals".

Default Logic

Consider the knowledge base: "**Usually** birds can fly, except the abnormal ones", "Tweety is a bird. All birds are animals".

Which part of our knowledge is **monotonic**?

Default Logic

Consider the knowledge base: “**Usually** birds can fly, except the abnormal ones”, “Tweety is a bird. All birds are animals”.

Which part of our knowledge is **monotonic**?

- W : monotonic part
  Tweety is a bird. All birds are animal.

Default Logic

Consider the knowledge base: "**Usually** birds can fly, except the abnormal ones", "Tweety is a bird. All birds are animals".

Which part of our knowledge is **monotonic**?

- W : monotonic part
  Tweety is a bird. All birds are animal.

Which part is **non-monotonic**?

Default Logic

Consider the knowledge base: "**Usually** birds can fly, except the abnormal ones", "Tweety is a bird. All birds are animals".

Which part of our knowledge is **monotonic**?

- W : monotonic part
  Tweety is a bird. All birds are animal.

Which part is **non-monotonic**?

- D : non-monotonic part
  Usually birds can fly, except the abnormal ones

Default Theories: Monotonic part

"Tweety is a bird. All birds are animal".

- **Monotonic part**: We use first order logic formula
- "Tweety is a bird."
  - Bird(t)
- All birds are animal.
  - $\forall x : (\text{Bird}(x) \rightarrow \text{Animal}(x))$
- $W = \{ \text{ Bird}(t), \forall x : \text{Bird}(x) \rightarrow \text{Animal}(x) \ \}$

Default Theories: Non-Monotonic part

- "**Usually** birds can fly, except the abnormal ones"

- **Non-monotonic part**: default rules of inference
- "**Usually** birds can fly, except the abnormal ones"
  - $\mathrm{Bird}(x)$: x is a bird
  - $\mathrm{Ab}(x)$: x is an abnormal bird
  - $\mathrm{Fly}(x)$: x can fly
- "**Usually** birds fly, except the abnormal ones"

$$\delta = \frac{\mathrm{Bird}(x) : \neg \mathrm{AB}(x)}{\mathrm{Fly}(x)}$$

Default Theories

"**Usually** birds fly, except the abnormal ones", "Tweety is a bird. All birds are animal".

- **Default theory is a pair (W, D) such that**:

W **presents monotonic part**: called background theory

- Tweety is a bird. All birds are animal.

$W = \{ \text{Bird}(\text{t}), \forall x : \text{Bird}(x) \rightarrow \text{Animal}(x) \}$

D **presents non-monotonic part**: contains default rules

- "**Usually** birds fly except, the abnormal ones,"

$$\delta = \frac{\text{Bird}(x) : \neg\text{Ab}(x)}{\text{Fly}(x)}$$

Default Theories

Definition: Default Theory

A default theory T is a pair (W, D), s.t.,

- W : is a set of propositional or first order logic formulas (called the facts or axioms of T),
- D is a countable set of default rules

Example

$$D = \left\{ \frac{\text{train}: \neg\text{Obstacle}}{\text{Move}} \right\},\ W = \{\text{train}\}$$

Default Theories

Definition: Default Theory

A default theory T is a pair (W, D), s.t.,

- W : is a set of propositional or first order logic formulas (called the facts or axioms of T),
- D is a countable set of default rules

Example

$$D = \left\{ \frac{\text{train: } \neg\text{Obstacle}}{\text{Move}} \right\},\ W = \{\text{train}\}$$

$$D = \left\{ \frac{\text{Bird}(x) : \neg\text{Ab}(x)}{\text{Fly}(x)} \right\},\ W = \{\text{Bird}(T), \forall x : (\text{Bird}(x) \rightarrow \text{Animal}(x))\}$$

Default Theories

Definition: Default Theory

A default theory T is a pair (W, D), s.t.,

- W : is a set of propositional or first order logic formulas (called the facts or axioms of T),
- D is a countable set of default rules

Example

$$D = \left\{ \frac{\text{train: } \neg\text{Obstacle}}{\text{Move}} \right\}, \; W = \{\text{train}\}$$

$$D = \left\{ \frac{\text{Bird}(x) : \neg\text{Ab}(x)}{\text{Fly}(x)} \right\}, \; W = \{\text{Bird}(T), \forall x : (\text{Bird}(x) \rightarrow \text{Animal}(x))\}$$

$$D = \left\{ \frac{\text{Robot} : \neg\text{HandsBroken}}{\text{Works}} \right\}, \; W = \{\text{Robot, HandsBroken}\}$$

Default Logic

Defaults (informally): domain specific inference rules

$$\delta = \frac{\alpha : \beta_1, \dots, \beta_n}{\gamma}$$

Intuitive reading: If α has been derived, and $\neg\beta_1, \dots, \neg\beta_n$ are not derivable, then γ can be concluded.

Example

$$\delta = \frac{\mathsf{Bird(Tweety)} : \neg\mathsf{Ab(Tweety)}}{\mathsf{Fly(Tweety)}}$$

If it is provable that Tweety is a bird, and it is not provable that Tweety is abnormal, then we can infer that Tweety can fly.

Default Formally

$$\delta = \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma}$$

- The formula α is called the prerequisite, denoted by $pre(\delta)$.
- $\beta_1 \ldots \beta_n$ are called justifications, denoted by $just(\delta)$
- γ is the consequent of δ, denoted by $cons(\delta)$

Default Formally

$$\delta = \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma}$$

- The formula α is called the prerequisite, denoted by $pre(\delta)$.
- $\beta_1 \ldots \beta_n$ are called justifications, denoted by $just(\delta)$
- γ is the consequent of δ, denoted by $cons(\delta)$

- (α, β_i, γ are propositional formulas)
- 'Default rule' with free variables in α, β_i, γ is a schema for all its ground instances.

$$\frac{Bird(x) : \neg Ab(x)}{Fly(x)}$$

Flashback: Human Reasoning

- What's up?!

Flashback: Human Reasoning

- I found my mistakes.

- What's up?!

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about: $\frac{\text{Bird}(x) : \neg\text{Penguin}(x)}{\text{Fly}(x)}$?

- What's up?!

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about: $\frac{\text{Bird}(x) : \neg\text{Penguin}(x)}{\text{Fly}(x)}$?

- What's up?!
- It works!

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about:
  $\frac{\text{Bird}(x) : \neg\text{Penguin}(x)}{\text{Fly}(x)}$?

- What's up?!
- It works!
- It means if it is a bird and there is no assumption that it is a penguin, then it can fly.

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about:
  $\frac{\text{Bird}(x) : \neg\text{Penguin}(x)}{\text{Fly}(x)}$?
- First, I gave you $W = \{\ \text{Bird}(\text{tweety})\ \}$. Your answer was correct, because there was no assumption that Tweety is a penguin.

- What's up?!
- It works!
- It means if it is a bird and there is no assumption that it is a penguin, then it can fly.

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about: $\dfrac{\text{Bird(x)} : \neg\text{Penguin(x)}}{\text{Fly(x)}}$?
- First, I gave you $W = \{ \text{Bird(tweety)} \}$. Your answer was correct, because there was no assumption that Tweety is a penguin.
- Next, more information more set of facts!

- What's up?!
- It works!
- It means if it is a bird and there is no assumption that it is a penguin, then it can fly.

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about: $\dfrac{\text{Bird(x)} : \neg\text{Penguin(x)}}{\text{Fly(x)}}$?
- First, I gave you $W = \{$ Bird(tweety) $\}$. Your answer was correct, because there was no assumption that Tweety is a penguin.
- Next, more information more set of facts!

- What's up?!
- It works!
- It means if it is a bird and there is no assumption that it is a penguin, then it can fly.
- $W' = \{\text{Bird(tweety)}, \text{Penguin(tweety)}, \forall x : (\text{Penguin(x)} \rightarrow \neg\text{Fly(x)})\}$

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about:
  $\dfrac{\text{Bird}(x) : \neg\text{Penguin}(x)}{\text{Fly}(x)}$?
- First, I gave you $W = \{\ \text{Bird}(\text{tweety})\ \}$. Your answer was correct, because there was no assumption that Tweety is a penguin.
- Next, more information more set of facts!

- What's up?!
- It works!
- It means if it is a bird and there is no assumption that it is a penguin, then it can fly.
- $W' = \{\text{Bird}(\text{tweety}), \text{Penguin}(\text{tweety}), \forall x : (\text{Penguin}(x) \rightarrow \neg\text{Fly}(x))\}$
- I revised my conclusion with $\neg\text{Fly}(\text{tweety})$

Flashback: Human Reasoning

- usually birds can fly, except penguin!
- What is your idea about:
  $\dfrac{\text{Bird(x)} : \neg\text{Penguin(x)}}{\text{Fly(x)}}$?
- First, I gave you $W = \{$ Bird(tweety) $\}$. Your answer was correct, because there was no assumption that Tweety is a penguin.
- Next, more information more set of facts!
- How did you get this new consequence?

- What's up?!
- It works!
- It means if it is a bird and there is no assumption that it is a penguin, then it can fly.
- $W' = \{\text{Bird(tweety)}, \text{Penguin(tweety)}, \forall x : (\text{Penguin(x)} \rightarrow \neg\text{Fly(x)})\}$
- I revised my conclusion with $\neg$Fly(tweety)

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?
- Does f Fly?

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?
- Does f Fly?
- Does t abnormal?

How can the logical consequences of a default theory be formally defined?

Default rule $\delta = \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then δ is applicable to T. In consequence we are led to believe that conclusion γ is true.

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?
- Does f Fly?
- Does t abnormal?
- **What are the consequences of theory T?**

How can the logical consequences of a default theory be formally defined?

Default rule $\delta = \frac{\alpha:\beta_1\ldots\beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \le i \le n$, is consistent with our current beliefs, then δ is applicable to T. In consequence we are led to believe that conclusion γ is true.

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?
- Does f Fly?
- Does t abnormal?
- **What are the consequences of theory** T?
- Since ¬Ab(t) is consistent with W, we can apply the default on t and drive Fly(t).

How can the logical consequences of a default theory be formally defined?

Default rule $\delta = \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then δ is applicable to T. In consequence we are led to believe that conclusion γ is true.

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?
- Does f Fly?
- Does t abnormal?
- **What are the consequences of theory** T?
- Since ¬Ab(t) is consistent with W, we can apply the default on t and drive Fly(t).
- Since ¬Ab(f) is not consistent with W, we cannot apply the default on f.

How can the logical consequences of a default theory be formally defined?

Default rule $\delta = \frac{\alpha:\beta_1 \dots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then δ is applicable to T. In consequence we are led to believe that conclusion γ is true.

Informal Discussion of the Semantics

Example

Given $T = (W, D)$ s.t., $D = \left\{ \frac{Bird(x):\neg Ab(x)}{Fly(x)} \right\}$ and $W = \{Bird(t), Bird(f), Ab(f)\}$

- Does t Fly?
- Does f Fly?
- Does t abnormal?
- **What are the consequences of theory T?**
- Since ¬Ab(t) is consistent with W, we can apply the default on t and drive Fly(t).
- Since ¬Ab(f) is not consistent with W, we cannot apply the default on f.
- Consequence: we believe that Bird(t), Fly(t), Bird(f), Ab(f).

How can the logical consequences of a default theory be formally defined?

Default rule $\delta = \frac{\alpha:\beta_1\ldots\beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then δ is applicable to T. In consequence we are led to believe that conclusion γ is true.

Informal Discussion of the Semantics

Default rule $\delta = \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then we believe that conclusion γ is true.

Example

Given default Theory $T = (W, D)$ s.t.

- $W = \{\text{Republican(Nixon), Quaker(Nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \frac{\text{Republican}(X):\neg\text{Pacifist}(X)}{\neg \mathit{Pacifist}(X)}$, $\delta_2 = \frac{\text{Quaker}(X):\text{Pacifist}(X)}{\mathit{Pacifist}(X)}$

- Is δ_1 applicable in T?
  - Yes: Republican(Nixon) $\in W$ and there is no assumption for Pacifist(Nixon).

Informal Discussion of the Semantics

Default rule $\delta = \frac{\alpha:\beta_1\ldots\beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then we believe that conclusion γ is true.

Example

Given default Theory $T = (W, D)$ s.t.

- $W = \{\text{Republican(Nixon), Quaker(Nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \frac{\text{Republican}(X):\neg\text{Pacifist}(X)}{\neg \mathit{Pacifist}(X)}$, $\delta_2 = \frac{\text{Quaker}(X):\text{Pacifist}(X)}{\mathit{Pacifist}(X)}$

- Is δ_1 applicable in T?
  - Yes: Republican(Nixon) $\in W$ and there is no assumption for Pacifist(Nixon).
- Is δ_2 applicable in T?
  - Yes: Quaker(Nixon) $\in W$ and there is no assumption for $\neg$Pacifist(Nixon).

Informal Discussion of the Semantics

Default rule $\delta = \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then we believe that conclusion γ is true.

Example

Given default Theory $T = (W, D)$ s.t.

- $W = \{\text{Republican(Nixon)}, \text{Quaker(Nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \frac{\text{Republican}(X):\neg\text{Pacifist}(X)}{\neg \mathit{Pacifist}(X)}$, $\delta_2 = \frac{\text{Quaker}(X):\text{Pacifist}(X)}{\mathit{Pacifist}(X)}$

- Is δ_1 applicable in T?
  - Yes: Republican(Nixon) $\in W$ and there is no assumption for Pacifist(Nixon).
- Is δ_2 applicable in T?
  - Yes: Quaker(Nixon) $\in W$ and there is no assumption for $\neg$Pacifist(Nixon).

Consequence: we believe that $\{\text{Pacifist(Nixon)}, \neg\text{Pacifist(Nixon)}\}$

Informal Discussion of the Semantics

Default rule $\delta = \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma}$ is applicable in default theory T if the prerequisite α is true, and each β_i, for $1 \leq i \leq n$, is consistent with our current beliefs, then we believe that conclusion γ is true.

Example

Given default Theory $T = (W, D)$ s.t.

- $W = \{\text{Republican(Nixon), Quaker(Nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \frac{\text{Republican}(X):\neg\text{Pacifist}(X)}{\neg \mathit{Pacifist}(X)}$, $\delta_2 = \frac{\text{Quaker}(X):\text{Pacifist}(X)}{\mathit{Pacifist}(X)}$

- Is δ_1 applicable in T?
  - Yes: Republican(Nixon) $\in W$ and there is no assumption for Pacifist(Nixon).
- Is δ_2 applicable in T?
  - Yes: Quaker(Nixon) $\in W$ and there is no assumption for $\neg$Pacifist(Nixon).

Consequence: we believe that $\{\text{Pacifist(Nixon)}, \neg\text{Pacifist(Nixon)}\}$

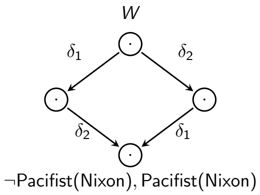
It is inconsistent!

Informal Discussion of the Semantics

Example

Given default Theory $T = (W, D)$ s.t.

- $W = \{\text{Republican(Nixon), Quaker(Nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \dfrac{\text{Republican}(X) : \neg\text{Pacifist}(X)}{\neg Pacifist(X)}$, $\delta_2 = \dfrac{\text{Quaker}(X) : \text{Pacifist}(X)}{Pacifist(X)}$



$$E = \{\text{Pacifist(Nixon)}, \neg Pacifist(Nixon)\}$$

E is inconsistent!

- **The current method has to be modified!**

Default Theory: logical consequence

How can the logical consequences of a default theory be formally defined?

- The semantics of Default Logic will be given in terms of extensions.
- Recall: In classical logics, we used models to define entailment.
- In default logics, it is not so easy — extensions will play a similar role to models.
- Intuitively, extensions represent possible world views which are based on the given default theories; they seek to extend the set of known facts with “reasonable” conjectures based on the available defaults.

Default Theory: logical consequence

How can the logical consequences of a default theory be formally defined?

- The semantics of Default Logic will be given in terms of extensions.
- Recall: In classical logics, we used models to define entailment.
- In default logics, it is not so easy — extensions will play a similar role to models.
- Intuitively, extensions represent possible world views which are based on the given default theories; they seek to extend the set of known facts with "reasonable" conjectures based on the available defaults.

Given default theory $T = (W, D)$. A set of formulas of E is an Extension s.t.

- $W \subseteq E$
- $Cn(E) = E$
- E is created by applying applicable defaults
- E is maximal: formally: if $\frac{\alpha:\beta_1 \ldots \beta_n}{\gamma} \in D$, $\alpha \in E$, and $\neg\beta_1, \ldots \neg\beta_n \notin E$, then $\gamma \in E$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$
- $E_{i+1} = Cn(E_i \cup \{\gamma \mid$

  $\frac{\alpha:\beta_1 \ldots \beta_n}{\gamma} \in D$

  $\alpha \in E_i,$

  $\neg\beta_1, \ldots \neg\beta_n \notin E_i \;\})$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$
- $E_{i+1} = Cn(E_i \cup \{\gamma|$
  $\frac{\alpha:\beta_1 \ldots \beta_n}{\gamma} \in D$
  $\alpha \in E_i,$
  $\neg\beta_1, \ldots \neg\beta_n \notin E_i \})$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$
- $E_{i+1} = Cn(E_i \cup \{\gamma | \frac{\alpha:\beta_1 \ldots \beta_n}{\gamma} \in D, \alpha \in E_i, \neg\beta_1, \ldots \neg\beta_n \notin E_i\})$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$
- $E_{i+1} = Cn(E_i \cup \{\gamma |$
  $\frac{\alpha : \beta_1 \ldots \beta_n}{\gamma} \in D$
  $\alpha \in E_i,$
  $\neg\beta_1, \ldots \neg\beta_n \notin E_i \})$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$
- $E_{i+1} = Cn(E_i \cup \{\gamma |$
  $\frac{\alpha:\beta_1 \ldots \beta_n}{\gamma} \in D$
  $\alpha \in E_i,$
  $\neg\beta_1, \ldots \neg\beta_n \notin E_i \})$

Extensions

Naive definition of extension

Let $T = (W, D)$ be a default theory. Define a sequence $E_0, E_1, \ldots$ as follows:

- $E_0 = Cn(W)$
- $E_{i+1} = Cn(E_i \cup \{\gamma| \begin{array}{l} \frac{\alpha:\beta_1\ldots\beta_n}{\gamma} \in D \\ \alpha \in E_i, \\ \neg\beta_1, \ldots \neg\beta_n \notin E_i \end{array} \})$
- E is an extension if $E = \bigcup_{i=0}^{\infty} E_i$

Naive Extension: example

Given $T = (W, D)$ s.t. $W = \{\text{Bird(t)}\}$, $D = \left\{ \frac{\text{Bird(x)}:\neg\text{Ab(x)}}{\text{Fly(x)}}, \frac{\text{Fly(x)}:\neg\text{Exception(x)}}{\text{Ascend(x)}} \right\}$

- $E_0 = Cn(\{\text{Bird(t)}\})$;

Naive Extension: example

Given $T = (W, D)$ s.t. $W = \{\text{Bird(t)}\}$, $D = \left\{ \frac{\text{Bird(x)}:\neg\text{Ab(x)}}{\text{Fly(x)}}, \frac{\text{Fly(x)}:\neg\text{Exception(x)}}{\text{Ascend(x)}} \right\}$

- $E_0 = Cn(\{\text{Bird(t)}\})$;
- $E_1 = Cn(E_0 \cup \{\text{Fly(t)}\})$;

Naive Extension: example

Given $T = (W, D)$ s.t. $W = \{\text{Bird(t)}\}$, $D = \left\{ \frac{\text{Bird(x)}:\neg\text{Ab(x)}}{\text{Fly(x)}}, \frac{\text{Fly(x)}:\neg\text{Exception(x)}}{\text{Ascend(x)}} \right\}$

- $E_0 = Cn(\{\text{Bird(t)}\})$;
- $E_1 = Cn(E_0 \cup \{\text{Fly(t)}\})$;
- $E_2 = Cn(E_1 \cup \{\text{Ascend(t)}\})$;

Naive Extension: example

Given $T = (W, D)$ s.t. $W = \{\text{Bird(t)}\}$, $D = \left\{ \frac{\text{Bird(x)}:\neg\text{Ab(x)}}{\text{Fly(x)}}, \frac{\text{Fly(x)}:\neg\text{Exception(x)}}{\text{Ascend(x)}} \right\}$

- $E_0 = Cn(\{\text{Bird(t)}\})$;
- $E_1 = Cn(E_0 \cup \{\text{Fly(t)}\})$;
- $E_2 = Cn(E_1 \cup \{\text{Ascend(t)}\})$;
- $E_3 = Cn(E_2) = E_2 = \{\text{Bird(t)}, \text{Fly(t)}, \text{Ascend(t)}\}$

Naive Extension Problem

Given default theory $T = (W, D)$ s.t $W = \{\text{Dutch(bart), Logician(bart)}\}$,
$\delta_1 = \frac{\text{Dutch(x):Sporty(x)}}{\text{IceScater(x)}}$, $\delta_2 : \frac{\text{Logician(x) : Philosopher(x)}}{\text{Philosopher(x)}}$ $\delta_3 : \frac{\text{Philosopher(x) : } \neg\text{Sporty(x)}}{\neg\text{Sporty(x)}}$

- $E_0 = Cn(W)$;

Naive Extension Problem

Given default theory $T = (W, D)$ s.t $W = \{\text{Dutch(bart), Logician(bart)}\}$,

$\delta_1 = \frac{\text{Dutch(x):Sporty(x)}}{\text{IceScater(x)}}$, $\delta_2 : \frac{\text{Logician(x) : Philosopher(x)}}{\text{Philosopher(x)}}$ $\delta_3 : \frac{\text{Philosopher(x) : }\neg\text{Sporty(x)}}{\neg\text{Sporty(x)}}$

- $E_0 = Cn(W)$;
- $E_1 = Cn(E_0 \cup \{\text{IceScater(bart)}\})$;

Naive Extension Problem

Given default theory $T = (W, D)$ s.t $W = \{\text{Dutch(bart), Logician(bart)}\}$,
$\delta_1 = \frac{\text{Dutch(x):Sporty(x)}}{\text{IceScater(x)}}$, $\delta_2 : \frac{\text{Logician(x) : Philosopher(x)}}{\text{Philosopher(x)}}$ $\delta_3 : \frac{\text{Philosopher(x) : }\neg\text{Sporty(x)}}{\neg\text{Sporty(x)}}$

- $E_0 = Cn(W)$;
- $E_1 = Cn(E_0 \cup \{\text{IceScater(bart)}\})$;
- $E_2 = Cn(E_1 \cup \{\text{Philosopher(bart)}\})$;

Naive Extension Problem

Given default theory $T = (W, D)$ s.t $W = \{\text{Dutch(bart), Logician(bart)}\}$,
$\delta_1 = \dfrac{\text{Dutch(x):Sporty(x)}}{\text{IceScater(x)}}$, $\delta_2 : \dfrac{\text{Logician(x) : Philosopher(x)}}{\text{Philosopher(x)}}$ $\delta_3 : \dfrac{\text{Philosopher(x) : }\neg\text{Sporty(x)}}{\neg\text{Sporty(x)}}$

- $E_0 = Cn(W)$;
- $E_1 = Cn(E_0 \cup \{\text{IceScater(bart)}\})$;
- $E_2 = Cn(E_1 \cup \{\text{Philosopher(bart)}\})$;
- $E_3 = Cn(E_2 \cup \{\neg\text{Sporty(bart)}\})$

Naive Extension Problem

Given default theory $T = (W, D)$ s.t $W = \{\text{Dutch(bart), Logician(bart)}\}$,
$\delta_1 = \frac{\text{Dutch(x):Sporty(x)}}{\text{IceScater(x)}}$, $\delta_2 : \frac{\text{Logician(x) : Philosopher(x)}}{\text{Philosopher(x)}}$ $\delta_3 : \frac{\text{Philosopher(x) : }\neg\text{Sporty(x)}}{\neg\text{Sporty(x)}}$

- $E_0 = Cn(W)$;
- $E_1 = Cn(E_0 \cup \{\text{IceScater(bart)}\})$;
- $E_2 = Cn(E_1 \cup \{\text{Philosopher(bart)}\})$;
- $E_3 = Cn(E_2 \cup \{\neg\text{Sporty(bart)}\})$

$E = \quad Cn(\{\text{ Dutch(bart), Logician(bart), IceScater(bart), Philosopher(bart), }\neg\text{Sporty(bart) }\})$

Naive Extension

Is there any problem in extension
$E = Cn(\{\text{ Dutch(bart), Logician(bart), IceScater(bart)}, \text{Philosopher(bart)}, \neg\text{Sporty(bart)}\})$?

1. Everything is quite fine with this extension.
2. It does not make sense to conclude that Bart is an ice scater and he is not sporty.
3. δ_1 is applicable if there is no assumption that Bart is not sporty.
4. δ_3 is applicable if there is no assumption that Bart is sporty.
5. We cannot apply both δ_1 and δ_3, but it is not covered in the naive extension definition.

menti
.com
3872
9471

Processes

Let $T = (W, D)$ be a default theory

- A default rule $\frac{\alpha:\beta_1,\ldots,\beta_n}{\gamma}$ is applicable to a set of formulas S iff:
  - $\alpha \in S$;
  - $\neg\beta_1, \ldots, \neg\beta_n \notin S$

Processes

Let $T = (W, D)$ be a default theory

- A default rule $\frac{\alpha:\beta_1,\ldots,\beta_n}{\gamma}$ is applicable to a set of formulas S iff:
  - $\alpha \in S$;
  - $\neg\beta_1, \ldots, \neg\beta_n \notin S$
- Let $\Pi = \delta_1, \ldots, \delta_n, \ldots,$ be a sequence of defaults rule from D without multiple occurrences.
  - $\Pi[k] = \delta_1, \ldots, \delta_k$: k initial segments of Π
  - $In(\Pi) = Cn(W \cup \{cons(\delta) \mid \delta \text{ occurs in } \Pi\})$
  - $Out(\Pi) = \left\{\neg\beta \mid \beta \in just(\delta), \text{ for some } \delta \text{ occurring in } \Pi\right\}$

Processes

Let $T = (W, D)$ be a default theory

- A default rule $\frac{\alpha:\beta_1,\dots,\beta_n}{\gamma}$ is applicable to a set of formulas S iff:
  - $\alpha \in S$;
  - $\neg\beta_1, \dots, \neg\beta_n \notin S$
- Let $\Pi = \delta_1, \dots, \delta_n, \dots,$ be a sequence of defaults rule from D without multiple occurrences.
  - $\Pi[k] = \delta_1, \dots, \delta_k$: k initial segments of Π
  - $In(\Pi) = Cn(W \cup \{cons(\delta) \mid \delta \text{ occurs in } \Pi\})$
  - $Out(\Pi) = \{\neg\beta \mid \beta \in just(\delta), \text{ for some } \delta \text{ occurring in } \Pi\}$

Definition

- Π is called a **process** of T if δ_k is applicable to $In(\Pi[k])$ for every $\delta_k \in \Pi$
- Π is **successful** iff $In(\Pi) \cap Out(\Pi) = \emptyset$, otherwise it **fails**.
- Π is **closed** iff every δ that is applicable to $In(\Pi)$ already occurs in Π.

Extension

- Π is called a **process** of T if δ_k is applicable to $In(\Pi[k])$ for every $\delta_k \in \Pi$
- Π is **successful** iff $In(\Pi) \cap Out(\Pi) = \emptyset$, otherwise it **fails**.
- Π is **closed** iff every δ that is applicable to $In(\Pi)$ already occurs in Π.

Definition: Extension

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Extension

Let $T = (W, D)$ be a default theory, s.t., s.t. $W = \{\mathrm{Bird}(\mathrm{tweety})\}$ and
$D = \{\delta = \frac{\mathrm{Birds}(x):\neg\mathrm{Ab}(x)}{\mathit{Fly}(x)}\}$

$$\begin{tikzcd}
\mathit{Cn}(\mathrm{Bird}(\mathrm{tweety})) & \odot \arrow[d, "\delta"'] & \emptyset \\
\mathit{Cn}(\mathrm{Bird}(\mathrm{tweety}), \mathrm{Fly}(\mathrm{tweety})) & \odot & \{\mathrm{Ab}(\mathrm{tweety})\}
\end{tikzcd}$$

- $\Pi = (\delta)$ is successful and closed
- $E = \mathit{Cn}(\mathrm{Bird}(\mathrm{tweety}), \mathrm{Fly}(\mathrm{tweety}))$

Extensions: A process tree

Process tree

Given default theory $T = (W, D)$, in the process tree of the T:

- The nodes of the tree are labeled with $In(\Pi)$ (on the left) and $Out(\Pi)$ (on the right)
- The edges correspond to default applications, and are labeled with the default that is being applied.
- The paths of the process tree starting at the root correspond to processes of T.

Example: A Process Tree

Given default theory $T = (W, D)$ with $W = \{a\}$ and $D = \{\delta_1, \delta_2\}$:

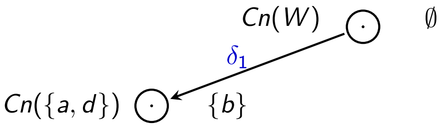
$$\delta_1 = \frac{a : \neg b}{d}, \quad \delta_2 = \frac{true : c}{b}$$

$$Cn(W) \quad \odot \quad \emptyset$$

Example: A Process Tree

Given default theory $T = (W, D)$ with $W = \{a\}$ and $D = \{\delta_1, \delta_2\}$:

$$\delta_1 = \frac{a : \neg b}{d}, \quad \delta_2 = \frac{true : c}{b}$$

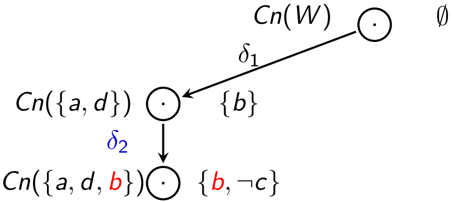


- For $\Pi_1 = (\delta_1)$ we have $In(\Pi_1) = Cn(\{a, b\})$, and $Out(\Pi_1) = \{b\}$. $In(\Pi_1)$ successful but not closed since δ_2 can be applied to $In(\Pi_1)$.

Example: A Process Tree

Given default theory $T = (W, D)$ with $W = \{a\}$ and $D = \{\delta_1, \delta_2\}$:

$$\delta_1 = \frac{a : \neg b}{d}, \quad \delta_2 = \frac{true : c}{b}$$



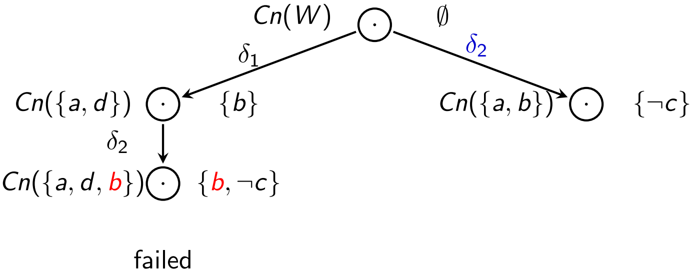
failed

- Let $\Pi_2 = (\delta_1, \delta_2)$. It is failed because $In(\Pi_2) \cap Out(\Pi_2) = \{b\}$.

Example: A Process Tree

Given default theory $T = (W, D)$ with $W = \{a\}$ and $D = \{\delta_1, \delta_2\}$:

$$\delta_1 = \frac{a : \neg b}{d}, \quad \delta_2 = \frac{true : c}{b}$$

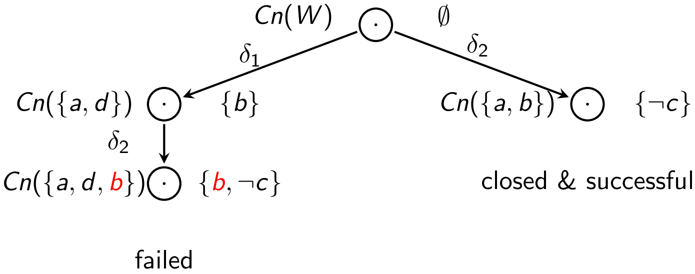


- Let $\Pi_3 = (\delta_2)$.

Example: A Process Tree

Given default theory $T = (W, D)$ with $W = \{a\}$ and $D = \{\delta_1, \delta_2\}$:

$$\delta_1 = \frac{a : \neg b}{d}, \quad \delta_2 = \frac{true : c}{b}$$

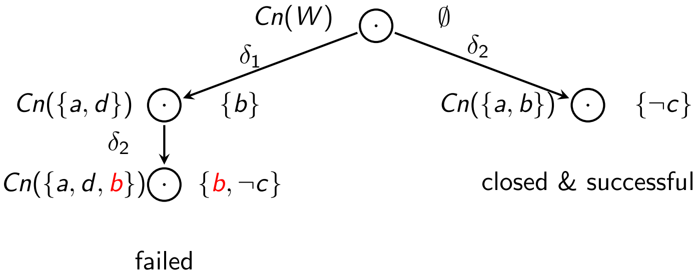


- Let $\Pi_3 = (\delta_2)$. It is closed and successful. $E = Cn(\{a, b\})$.

Example: A Process Tree

Given default theory $T = (W, D)$ with $W = \{a\}$ and $D = \{\delta_1, \delta_2\}$:

$$\delta_1 = \frac{a : \neg b}{d}, \quad \delta_2 = \frac{true : c}{b}$$



- T has one extension: $E = Cn(\{a, b\})$.

Extensions: A process tree

Definition

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Example

- $W = \{\text{Republican}(\text{nixon}), \text{Quaker}(\text{nixon})\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \dfrac{\text{Republican}(x) : \neg\text{Pacifist}(x)}{\neg \mathit{Pacifist}(x)}$, $\delta_2 = \dfrac{\text{Quaker}(x) : \text{Pacifist}(x)}{\text{Pacifist}(x)}$

$Cn(W)$ $\odot$ $\emptyset$

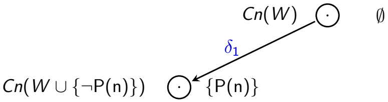
Extensions: A process tree

Definition

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Example

- $W = \{\text{Republican}(\text{nixon}), \text{Quaker}(\text{nixon})\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \dfrac{\text{Republican}(x) : \neg\text{Pacifist}(x)}{\neg \mathit{Pacifist}(x)}$, $\delta_2 = \dfrac{\text{Quaker}(x) : \text{Pacifist}(x)}{\text{Pacifist}(x)}$



- Let $\Pi_1 = (\delta_1)$.

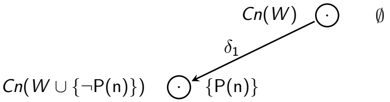
Extensions: A process tree

Definition

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Example

- $W = \{\text{Republican(nixon)}, \text{Quaker(nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \dfrac{\text{Republican}(x) : \neg\text{Pacifist}(x)}{\neg \mathit{Pacifist}(x)}$, $\delta_2 = \dfrac{\text{Quaker}(x) : \text{Pacifist}(x)}{\text{Pacifist}(x)}$



closed & successful

- Let $\Pi_1 = (\delta_1)$. It is closed and successful. $E_1 = Cn(W \cup \{\neg P(n)\})$

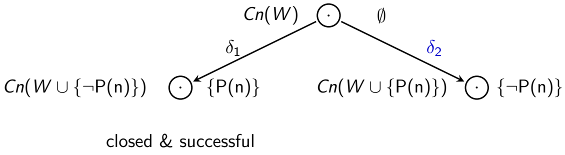
Extensions: A process tree

Definition

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Example

- $W = \{\text{Republican(nixon)}, \text{Quaker(nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \dfrac{\text{Republican}(x) : \neg\text{Pacifist}(x)}{\neg \mathit{Pacifist}(x)}$, $\delta_2 = \dfrac{\text{Quaker}(x) : \text{Pacifist}(x)}{\text{Pacifist}(x)}$



- Let $\Pi_2 = (\delta_2)$.

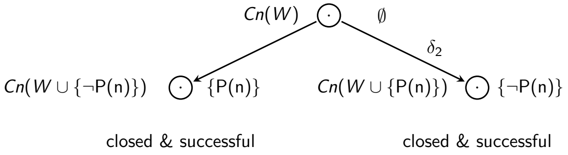
Extensions: A process tree

Definition

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Example

- $W = \{\text{Republican}(\text{nixon}), \text{Quaker}(\text{nixon})\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \dfrac{\text{Republican}(x) : \neg\text{Pacifist}(x)}{\neg \mathit{Pacifist}(x)}$, $\delta_2 = \dfrac{\text{Quaker}(x) : \text{Pacifist}(x)}{\text{Pacifist}(x)}$



- Let $\Pi_2 = (\delta_2)$. It is closed and successful. $E_2 = Cn(W \cup \{P(n)\})$

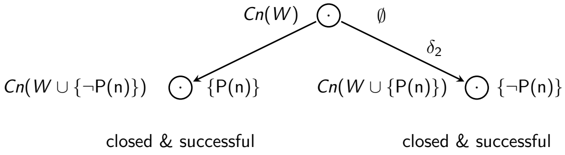
Extensions: A process tree

Definition

A set of formulae E is an extension of the default theory T iff there is some closed and successful process Π of T such that $E = In(\Pi)$

Example

- $W = \{\text{Republican(nixon)}, \text{Quaker(nixon)}\}$
- $D = \{\delta_1, \delta_2\}$ s.t. $\delta_1 = \dfrac{\text{Republican}(x) : \neg\text{Pacifist}(x)}{\neg \mathit{Pacifist}(x)}$, $\delta_2 = \dfrac{\text{Quaker}(x) : \text{Pacifist}(x)}{\text{Pacifist}(x)}$



- This default theory has two extensions: $E_1 = Cn(W \cup \{\neg P(n)\})$ and $E_2 = Cn(W \cup \{P(n)\})$.

Is there always an extension?

Is there always an extension?

Let $T = (W, D)$ with $W = \{\}$ and $D = \{\frac{true:a}{\neg a}\}$

Is there always an extension?

Let $T = (W, D)$ with $W = \{\}$ and $D = \{\frac{true:a}{\neg a}\}$

$$\begin{tikzcd} Cn(\emptyset) & \odot \arrow[d, "\delta"'] & \emptyset \\ Cn(\{\neg a\}) & \odot & \{\neg a\} \end{tikzcd}$$

failed

- A default logic may not have an extension

Is there always an extension?

Let $T = (W, D)$ with $W = \{b\}$ and $D = \{\frac{true:a}{\neg a}\}$

Is there always an extension?

Let $T = (W, D)$ with $W = \{b\}$ and $D = \{\frac{true:a}{\neg a}\}$

$$\begin{tikzcd}
Cn(b) & \odot \arrow[d] & \emptyset \\
Cn(\{b, \neg a\}) & \odot & \{\neg a\}
\end{tikzcd}$$

failed

- It does not have any extension!

Normal Default Theory

Normal Default Rule

A default δ is normal if $\text{just}(\delta) = \text{cons}(\delta)$.

$$\frac{\alpha : \gamma}{\gamma}$$

Example

$$\frac{\text{Bird(tweety)} : \text{Fly(tweety)}}{\text{Fly(tweety)}}$$

If it is provable that Tweety is a bird, and it is not provable that Tweety cannot fly, then we can infer that Tweety can fly.

Normal Default Theory

Theorem

Normal default theories always have extensions.

Exercise

- Does the default theory (W, D) where, $W = \{a, d\}$, $\delta_1 = \frac{a : b}{b}$, $\delta_2 = \frac{b : c}{c}$, $\delta_3 = \frac{d : \neg c}{\neg c}$ has an extension?
- Draw the process tree of this default theory.

This Lecture

- Defeasible reasoning
- Monotonic vs. non-monotonic reasoning
- Default Theory

Next lecture

- Abstract argumentation frameworks