Knowledge Representation

2023/2024

Exercise Sheet 3 – Argumentation and Probabilities – Solutions

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Exercise 3.1 Which of the following statements are true?

(a) $\Pr(a \vee b) = \Pr(a) + \Pr(b)$ if and only if $\Pr(a \wedge b) = \Pr(b \mid \neg b)$.

(b) $\Pr(b) = \dfrac{\Pr(a \wedge b) \cdot \Pr(a)}{\Pr(a \mid b)}$

Exercise 3.2 You are given the following table of join probability $\Pr(A, B, C)$:

A	B	C	$\Pr(.)$
true	**true**	**true**	0.2
true	**true**	**false**	0.1
true	**false**	**true**	0.2
true	**false**	**false**	0.1
false	**true**	**true**	0.0
false	**true**	**false**	0.2
false	**false**	**true**	0.1
false	**false**	**false**	0.1

(a) Write down the probability table for $\Pr(A, C)$.

(b) Write down the probability table for $\Pr(B \mid C)$.

(c) Are A and B independent?

(d) Are A and C independent?

Exercise 3.3 You are given the following directed acyclic graph:

$$\begin{tikzcd}
A \arrow[r] & B \arrow[r] \arrow[d] & C \arrow[d] & D \arrow[l] \\
& E & F & G \arrow[l]
\end{tikzcd}$$

(a) Which of the following statements are true?

(i) dsep$(\{A\}, \{E, F\}, \{G\})$

(ii) dsep$(\{E\}, \{A\}, \{D\})$

(b) We know that dsep(A, X, G), for any set X of nodes in the graph. Assume $B \notin X$. Write down all possible sets of X for which this is possible.

Exercise 3.4 Which of the following statements are true:

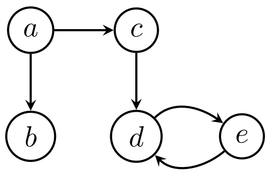
(a) $A \models B$ if and only if $\Pr(A) \leq \Pr(B)$

(b) If $A \models B$ and $B \models C$, then $\Pr(A \mid B) \geq \Pr(A \mid C)$

(c) If $A \models B$ and $B \models C$, then $\Pr(B \mid A) \geq \Pr(C)$

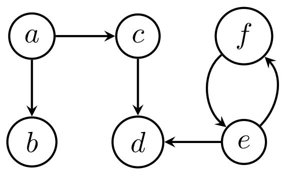
Exercise 3.5 Given an AF $F = (A, R)$. Show that any stable extension is a preferred extension.

Exercise 3.6 Given AF $F = (\{a, b, c, d, e\}, \{(a, b), (a, c), (c, d), (d, e), (e, d)\})$, depicted in the following figure. Answer the following decision problems. Either provide a witness for your answer or an explanation.



- *Skept*$_{stb}(e, F)$
- *Cred*$_{grd}(d, F)$
- *Var*$_{comp}(\{a, d\}, F)$
- *Exists*$_{adm}(F)$

Exercise 3.7 Given AF $F = (\{a, b, c, d, e, f\}, \{(a, b), (a, c), (c, d), (e, d), (e, f), (f, e)\})$, depicted in the following figure. Which of the following labelings is admissible, and which of them is complete, both, or neither? If any of the following labelings is not admissible, explain why it is not admissible. Provide the associated admissible extensions for the admissible labellings.



(a) $\{a \to \mathtt{undec}, b \to \mathtt{undec}, c \to \mathtt{undec}, d \to \mathtt{undec}, e \to \mathtt{undec}, f \to \mathtt{undec}\}$

(b) $\{a \to \mathtt{in}, b \to \mathtt{undec}, c \to \mathtt{undec}, d \to \mathtt{undec}, e \to \mathtt{undec}, f \to \mathtt{undec}\}$

(c) $\{a \to \mathtt{in}, b \to \mathtt{out}, c \to \mathtt{undec}, d \to \mathtt{undec}, e \to \mathtt{undec}, f \to \mathtt{undec}\}$

(d) $\{a \to \mathtt{in}, b \to \mathtt{out}, c \to \mathtt{out}, d \to \mathtt{in}, e \to \mathtt{undec}, f \to \mathtt{undec}\}$

(e) $\{a \to \mathtt{in}, b \to \mathtt{out}, c \to \mathtt{out}, d \to \mathtt{undec}, e \to \mathtt{undec}, f \to \mathtt{undec}\}$

(f) $\{a \to \mathtt{in}, b \to \mathtt{out}, c \to \mathtt{out}, d \to \mathtt{undec}, e \to \mathtt{in}, f \to \mathtt{out}\}$

(g) $\{a \to \texttt{in}, b \to \texttt{out}, c \to \texttt{undec}, d \to \texttt{undec}, e \to \texttt{in}, f \to \texttt{out}\}$

(h) $\{a \to \texttt{in}, b \to \texttt{out}, c \to \texttt{undec}, d \to \texttt{undec}, e \to \texttt{out}, f \to \texttt{in}\}$

(i) $\{a \to \texttt{in}, b \to \texttt{out}, c \to \texttt{out}, d \to \texttt{in}, e \to \texttt{out}, f \to \texttt{in}\}$

(j) $\{a \to \texttt{in}, b \to \texttt{out}, c \to \texttt{out}, d \to \texttt{undec}, e \to \texttt{out}, f \to \texttt{in}\}$

(k) $\{a \to \texttt{in}, b \to \texttt{out}, c \to \texttt{out}, d \to \texttt{out}, e \to \texttt{in}, f \to \texttt{out}\}$

Exercise 3.8 Given AF $F = (\{a, b, c, d, e, k\}, \{(e, d), (d, b), (b, a), (c, b), (a, k)\})$, depicted in the following figure. Proponent claims that k is labeled `in` in a preferred labeling of F. Present a preferred discussion game for this claim. Indicate whether the proponent wins the game or the opponent.