Description of the IFT method for low sidelobe pattern synthesis

Will P.M.N. Keizer

ArraySoft, Netherlands

willkeizer@ieee.org

INTRODUCTION

The iterative Fourier technique (IFT) uses the property that for an array with uniform spacing of the elements, an inverse Fourier transform relationship exists between the array factor (AF) and the element excitations. Because of this relationship, a direct Fourier transform performed on AF will yield the element excitations. The underlying approach relies on the repeatedly use of both types of Fourier transforms. At each iteration, the newly calculated AF is adapted to the sidelobe requirements, which then is used to derive a new set of excitation coefficients. Only those excitation coefficients constituting the array are used to calculate a new AF.
A key characteristic of this iterative synthesis method is that the algorithm itself is very simple, highly robust, and easy to implement in software requiring only a few lines of code when programmed in MATLAB. The computational speed is very high because the core calculations are based on direct and inverse fast Fourier transforms (FFTs).
The presented results are related to linear arrays consisting of 100 elements located in periodic grid with half wavelength inter-element spacing.

FORMULATION OF THE IFT METHOD

The far-field *F(u)* of a linear array with *M* elements arranged along a periodic grid at distance *d* apart, can be written as the product of the embedded element pattern *EF* and the array factor AF

$$F(u) = EF(u)AF(u) \tag{1}$$

$$AF(u) = \sum_{m=0}^{M-1} A_m e^{jkmdu} \tag{2}$$

where A_m is the complex excitation of the *m*th element, *k* is wavenumber ($2\pi/\lambda$), λ is the wavelength, u = sinθ and θ angular coordinate measured between far-field direction and the array normal.
Equation (2) forms a finite Fourier series that relates the element excitation coefficients A_m of the array to its AF through a discrete inverse Fourier transform. AF is periodic in u-dimension over the interval d/λ. Since AF is related to the to the element excitations through a discrete inverse Fourier transform, a discrete direct Fourier transform applied on AF over the period λ/d will yield the element excitations A_m. These Fourier transform relationships are used in an iterative way to synthesize low sidelobe pattern for arrays with a periodic element arrangement.
The synthesis procedure starts with the calculation of AF using an initial set for the *M* element excitation coefficients. The calculation of AF is carried out with a *K*-point inverse FFT with *K*>*M* and using zero padding. This is followed by an adaptation of the sidelobe region of AF

to the sidelobe requirements. Only the sidelobe samples of AF that exceed the sidelobe thresholds are corrected, the other samples of AF are left unchanged. After this correction, a direct K-point FFT is performed on the adapted AF to get an updated set of excitation coefficients. From those K excitation coefficients only the M samples belonging to the array are retained. When an amplitude-only synthesis has to be performed, the phases of the retained M excitation coefficients are made equal to the phases of the initial element excitations at the start of the synthesis after which a new updated AF will be calculated. This process is repeated until the new updated AF will satisfy the sidelobe requirements. Constraints for the element excitations can be quite easily incorporated in the IFT synthesis method. Dynamic range constraints for the array illumination are included by setting a lower limit to the amplitude values of the updated element excitation coefficients at each iteration. The occurrence of defective elements across the aperture can be included by setting the associated excitation coefficients to zero. Both element coefficient constraints are executed near the end of each iteration just before the next iteration of a new updated AF is performed.

Implementation of the IFT algorithm for the synthesis of low sidelobe patterns for linear arrays using amplitude-only element weighting proceeds as follows.

1. *Start the synthesis using a uniform excitation for M elements in case of the sum pattern.*
2. *Compute AF from* $\{A_m\}$ *using a K-point inverse FFT with K>M.*
3. *Adapt AF to the prescribed sidelobe constraints.*
4. *Compute* $\{A_m\}$ *for the adapted AF using a K-point direct FFT.*
5. *Truncate* $\{A_m\}$ *from K samples to M samples by making zero all samples outside the array.*
6. *Make the phase of the M samples of* $\{A_m\}$ *equal to the phase of initial excitation at Step 1.*
7. *Set the magnitude of the excitations violating the amplitude dynamic range constraint to the lowest permissible value.*
8. *Enforce the optional defective element constraint. Take element failures into account by setting their excitation values to zero.*
9. *Repeat Steps 2-9 until the prescribed sidelobe requirements for AF are satisfied or the allowed number of iterations is reached.*

The above algorithm refers to the amplitude-only low sidelobe synthesis. In order to apply phase-only synthesis the present Step 6 has to be replaced by: "*Make the amplitude of the M samples of* $\{A_m\}$ *equal to one*". When Step 6 is deleted, the synthesis is of the complex weight type.
Any low sidelobe AF consisting of K samples can be realized with K element excitations. The objective of the IFT low sidelobe synthesis method is that the contribution of the excitations of the (K-M) virtual elements located outside the array to AF, is transferred to the excitations of M elements located inside the array.
The IFT method is implemented in the *MATLAB* program S*idelobeSynthesis.* This program is able to synthesize a low sidelobe taper for any linear array with the elements positioned in a periodic grid. S*idelobeSynthesis* determines the element excitations producing an array factor that matches the user defined peak sidelobe requirements.

DESCRIPTION THE PROGRAM *SidelobeSynthesis*

The input data to the program *SidelobeSynthesis* are contained by the statements of Lines 48-54.

Lines 48-49 define the array configuration in terms of the number of array elements and the element spacing.
Line 50 defines the user defined maximum peak SLL requirement. This requirement applies to the whole sidelobe region of AF.
Line 51 defines the dynamic range of the amplitude of element excitations, defined as the ratio of its maximum value to its minimum value $|A_{max}|/|A_{min}|$.

Using the program a low-sidelobe pattern is synthesised that features a maximum peak SLL of – 40 dB for the whole sidelobe region. This result is obtained for a linear array consisting of 100 elements spaced at 0.5 wavelength apart. During the synthesis the dynamic range of the amplitude of element excitations was not allowed to exceed 19 dB. The consequence of 19 dB dynamic range is that all peaks of the sidelobes are below -40 dB as required but that their maximum levels are not uniform. A uniform maximum peak level of -40 dB can be obtained for all sidelobes by raising $|A_{\mathrm{max}}|/|A_{\mathrm{min}}|$ to 21 dB followed by a new synthesis.

Pattern recovery in case of element failures, can be activated by uncommenting Line 153. In that case the array is corrupted by eight defective elements. For these defective elements the program is able to synthesize a new taper that provides again – 40 dB sidelobes. The penalty is a reduction in taper efficiency.

When a phase-only synthesis has to be performed the statement of Line 74 has to be replaced by the following two statements:

```
Phase  = exp(1i*0.3*pi*randn(1,noEl));
IllumS = IllumS.*Phase;
```

The same action is required for a complex weighted synthesis.
As is indicated in the formulation of the IFT method, page 2, also Step 6 has to be modified for both the phase-only synthesis as well as for the complex weighted synthesis.

DESCRIPTION OF THE PLOTTED RESULTS

Figure (a) depicts the synthesized AF as a function of u over the interval $-1 \leq u \leq 1$.
Figure (b) shows how the maximum peak SLL decreases during the iteration process.
Figure (c) displays the synthesized taper and Fig (d) illustrates how the number of far-field directions of the sidelobe region exceeding the SLL requirement of -40 dB, decreases with increasing iteration number. This figure shows also the widening of the main beam due to the drop in maximum peak SLL as the synthesis progresses. The number of far-field directions contained in the main lobe region, Figure (d), is a direct measure of its width.