Image Restoration Report

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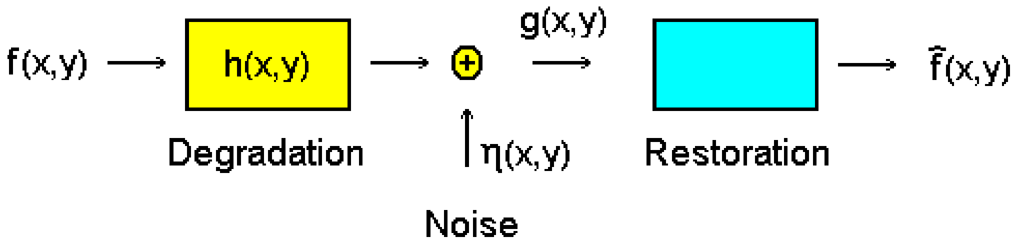
1. Introduction

What is image restoration?

A process that attempts to reconstruct or recover an image that has been degraded by using some prior knowledge of the degradation phenomenon.

Many factors can cause the degradation of the image during the image acquisition. Such as noise of sensor, not focus of camera, object movement , object illumination, light scatter.

The goal of image restoration is to improve a degrade image in some predefined sense. Schematically this process can be visualized as:



where f is the original image, g is a degraded version of the original image and $\tilde{f}$ is a restored version.

2. Degradation Model

Math Representation:

$$g(x,y) = f(x,y) * h(x,y) + h(x,y)$$

Derivation:

$$g(x,y) = H[f(x,y)] + n(x,y)$$

Where H is a linear system. So, Blurred Image Restoration's goal is to find a reverse transformation H^{-}, which satisfy

$$H^{-}[g(x,y)] = f(x,y)$$

According to the define of Impulse function:

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)d(x-a,y-b)da\,db$$

Impulse function

$$g(x,y)=H[f(x,y)]=H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)d(x-a,y-b)da\,db\right]$$

$$g(x,y)=H[f(x,y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H\left[f(a,b)d(x-a,y-b)\right]da\,db$$

$$g(x,y)=H[f(x,y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\boxed{H\left[d(x-a,y-b)\right]}da\,db$$

$$h(x,y)=H\left[d(x,y)\right] \qquad h(x,a,y,b)=H\left[d(x-a,y-b)\right]$$

If position-invariant

Impulse response (Point spread function)

$$H\left[d(x-a,y-b)\right]=h(x-a,y-b)$$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)h(x-a,y-b)da\,db$$

$$h(x,y)\neq 0$$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)h(x-a,y-b)da\,db+h(x,y)$$
$$= f(x,y)*h(x,y)+h(x,y)$$

The discrete degradation model is:

$$g(i,j)=\sum_{k=1}^{M}\sum_{l=1}^{N} h(i-k,j-l)f(k,l)+n(i,j)$$
$$=h(i,j)*f(i,j)+n(i,j)$$

$f(i,j)$: Original Image
$g(i,j)$: degraded Image
$h(i,j;k,l)$: PSF (Point Spread Function)
Dimension of Image is $M\times N$.

Image degradation can occur for many reasons, some typical degradation models are:

$$h(i)=\begin{cases}\frac{1}{L}, if -\frac{L}{2}\le i\le\frac{L}{2}\\ 0, \quad otherwise\end{cases}$$

Motion Blur: due to camera panning or subject moving quickly.

$$h(i,j)=Ke^{-\left(\frac{i^2+j^2}{2s}\right)}$$

Atmospheric Blur: long exposure

$$h(i,j)=\begin{cases}\frac{1}{L^2} & -\frac{L}{2}\le i,j\le\frac{L}{2}\\ 0 & otherwise\end{cases}$$

Uniform 2D Blur

$$h(i,j)=\begin{cases}\frac{1}{pR^2} & i^2+j^2\le R^2\\ 0 & otherwise\end{cases}$$

Out-of-Focus Blur

Motion Blur: $$h(i,j)=\begin{cases}\dfrac{1}{L^2} & -\dfrac{L}{2}\le i,j\le\dfrac{L}{2}\\ 0 & otherwise\end{cases}$$

figure1. Motion Blur, left is Original Image, right is Blurred Image.

3. Noise Model

Most noise models assume the noise is some known probability density function. The density function is chosen based on the underlining physics.

Gaussian: $$p(z)=\frac{1}{\sqrt{2p}\,s}e^{-(z-m)^2/2s^2}$$

Salt and Pepper: $$p(z)=\begin{cases}P_a & \text{for } z=a\\ P_b & \text{for } z=b\\ 0 & \text{otherwise}\end{cases}$$

In practical, a=0(black) and b=255(white).

Gaussian:

$$p(z) = \frac{1}{\sqrt{2p}\, s} e^{-(z-m)^2/2s^2}$$

figure2. Add Gaussian Noise Blur, left is Original Image, right is Degraded Image.

Salt and Pepper:

figure3. Add Salt and Pepper Noise Blur, left is Original Image, right is Degraded Image.

4. Restoration Algorithms

4.1 Inverse Filter.

Assume the image system in spatial domain:

$$g(x,y) = f(x,y) * h(x,y) + h(x,y)$$

After the FFT, In the frequency domain, we have

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

where G(u,v) and F(u,v) are the Fourier transforms of the degraded image g(x,y) and the input image f(x,y), respectively. H(u,v) is called the degradation function, and N(u,v) is a noise term.

With the estimated degradation function H(u,v):

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Unknown noise

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Estimate of original image

Problem: 0 or small values

Result of Inverse Filter:

Absolutely terrible!

Figure4. Inverse Filter, left is Original Image, middle is blurred image, right is restored Image.

4.2 Wiener Filter.

To overcome the noise sensitivity of the Inverse Filter, a collection of *least squares* filters have been developed. The Wiener Filter is a linear, spatially invariant filter which is chosen to minimize the MSE between the original and restored image.

$$MSE = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}[g(i,j) - \hat{g}(i,j)]^2}{M \times N}$$

Where $g(i,j)$, $\hat{g}(i,j)$ is gray value of position (i,j) in Original image and Restored image.

So, In frequency domain, we have:

$$F(u,v) = \frac{S_{ff}(u,v)H^*(u,v)}{S_{ff}(u,v)\left|H(u,v)\right|^2 + S_{nn}(u,v)}G(u,v)$$

$$= \frac{1}{H(u,v)} \cdot \frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + \dfrac{S_{nn}(u,v)}{S_{ff}(u,v)}}G(u,v)$$

Where $S_{ff}(u,v)$ and $S_{nn}(u,v)$ is the power Spectrum. $H^*(u,v)$ means the conjugation of $H(u,v)$.

When it has no noise, $S_{nn}(u,v)$ =0, Wiener filter becomes the Inverse filter.

In practice:

$$F(u,v) = \frac{1}{H(u,v)} \cdot \frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + K}G(u,v)$$

where K replace the $\frac{S_{nn}(u,v)}{S_{ff}(u,v)}$. Adjusting the value of K, it can obtain the satisfied result.

Result of Wiener filter:

Figure5. Wiener Filter, left is Original Image, middle is blurred image, right is restored Image.

The result is much better than Inverse Filter. Wiener Filter is used widespread, because of its good effects in restoration with less computational cost and resistance of noise.

4.3. Lucy-Richardson

Given the degraded image *H*, the point spread function S, and the requirement to find the original image W, Bayes's theorem comes readily to mind. In the notation of this problem the usual form of the Bayes's theorem' may be stated as the conditional probability of an event at W_i given an event at H_k,

$$P(W_i \mid H_k) = \frac{P(H_k \mid W_i)P(W_i)}{\sum_j P(H_k \mid W_j)P(W_j)} \qquad (1)$$

where $i = \{1,...,I\}$, $j = \{1,...,J\}$, $k = \{1,...,K\}$, H_k is for the moment an arbitrary cell of H.

Considering all the H_k and their dependence on all W_i in accordance with S, we can say:

$$P(W_i) = \sum_k P(W_i \cdot H_k) = \sum_k P(W_i \mid H_k)P(H_k) \qquad (2)$$

Since $P(W_i \mid H_k) = P(W_i \cdot H_k) / P(H_k)$. Substituting Eq.(1) in Eq.(2) gives

$$P(W_i) = \sum_k \frac{P(H_k \mid W_i)P(W_i)P(H_k)}{\sum_j P(H_k \mid W_j)P(W_j)} \qquad (3)$$

In the right side of this equation, the term $P(W_i)$ is also the desired solution. But in many applications of Bayes's theorem, when this $P(W_i)$ term is not known, an accepted practice is to make the best of a bad situation and use an estimate of the $P(W_i)$ function to obtain, from Eq. (1), an approximation of $P(W_i \mid H_k)$**.** When this practice is applied here, Eq. (3) becomes

$$P_{r+1}(W_i) = P_r(W_i) \sum_k \frac{P(H_k \mid W_i) P(H_k)}{\sum_j P(H_k \mid W_j) P_r(W_j)} \tag{4}$$

where $r = \{0, 1, ...\}$

This results in an iterative procedure where the initial $P_0(W_i)$ is estimated. An estimation often used is Bayes's postulate (also known as the equidistribution of ignorance), which assumes a uniform distribution so that $P_0(W_i) = 1/I$ or $W_{i,0} = W/I$.

Equation (4) can be reduced to a more easily workable form by $P(W_i) = W_i / W$ and $P(H_k) = H_k / H = H_k / W$, since the restoration is a conservative process and $W = H$, and also $P(H_k \mid W_i) = P(S_{i,k}) = S_{i,k} / S$,

$$S = \sum_j S_j, \quad j = \{1, ..., J\}$$

Then Eq.(4) becomes:

$$W_{i,r+1} / W = (W_{i,r} / W) \sum_k \frac{(S_{i,k} / S) \cdot (H_k / W)}{\sum_j (S_{j,k} / S)(W_{j,r} / W)}$$

or $$W_{i,r+1} = W_{i,r} \sum_k \frac{S_{i,k} \cdot H_k}{\sum_j S_{j,k} W_{j,r}} \tag{5}$$

Lucy-Richardson

Result of Lucy-Richardson

Figure6. Wiener Filter, left is Original Image, middle is blurred image, right is restored Image-50 iterations of Lucy-Richardson..

5. Result

This section, I compared two methods about Wiener and Lucy-richardson's result. See figure 7-9.

Compared about restore motion blur:

Figure7. Restored motion blur image. Top left is Original Image, Top middle is blurred image, Top right is restored by Wiener. Bottom left is L-R 50 iterations, Bottom right is L-R 200 iterations.

Compared about restore motion blur and noise degradation:

Figure8. Restored motion blur and Salt pepper noise image. Top left is Original Image, Top middle is blurred image, Top right is restored by Wiener. Bottom left is L-R 50 iterations, Bottom right is L-R 200 iterations.

Figure9. Restored motion blur and Gaussian noise image. Top left is Original Image, Top middle is blurred image, Top right is restored by Wiener. Bottom left is L-R 50

iterations, Bottom right is L-R 200 iterations.

So, Wiener filter is good result for non-noise image with little cost, but the drawback is little bad handle for noise image. Lucy-Richardson is good result for any image, however, its cost is more higher than Wiener.

6. Conclusion

Experience for this research, I have some useful gain. first, this research make me know the Image restoration’s concept, Math model of degradation, Several of motion blur methods and noise method.

Second, for this research, I found the Inverse filter is worse for use. Wiener and Lucy-Richardson is much better.

Third, since this research depend on the know Point Spread Function, in the future research, I will find a method to estimate this function, is also called Blind Deconvolution.

This research is make by single..