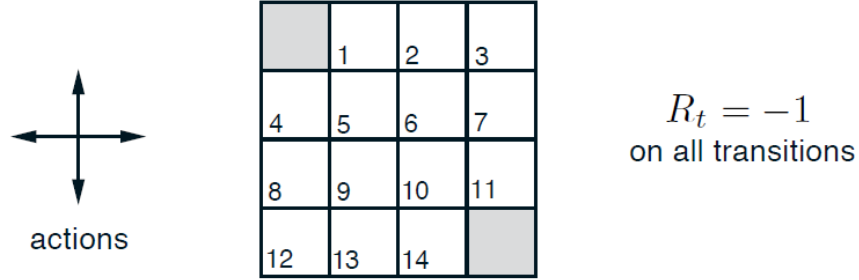


Value Function Evaluation and Policy Update Example

Xiuwen Liu, Department of Computer Science, Florida State University

Here we use the following example from the book, where there are 14 nonterminal states and one terminal state. The value for the terminal state is fixed at 0. For simplicity, we will state 0 to represent the terminal state.



As shown here, all the transitions have an immediate reward of -1. Here we use the iterative evaluation algorithm where the initial values are 0 for the nonterminal states. The policy is the equiprobable random policy; in other words, the probability of taking each of the four actions is 0.25. The reward in the example is undiscounted; in other words, $\gamma = 1$.

Input π , the policy to be evaluated

Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation

Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\text{terminal}) = 0$

Loop:

$\Delta \leftarrow 0$

Loop for each $s \in \mathcal{S}$:

$v \leftarrow V(s)$

$V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a) [r + \gamma V(s')]$

$\Delta \leftarrow \max(\Delta, |v - V(s)|)$

until $\Delta < \theta$

For simplicity, we update all the state values using the values from the previous iteration. In other words, the updated values are not available until all the values are updated, known as synchronous evaluation. According to the algorithm, at each iteration, we need to update the value for each state.

All the values are initialized to be zero. For the next iteration ($k=1$), for state 1, we have

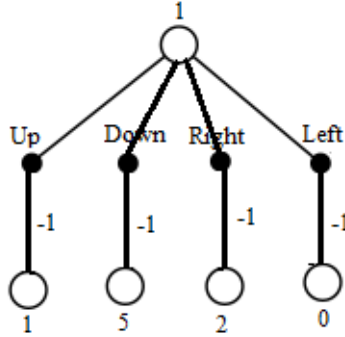
$$V(1) = 0.25 \times (1 \times [-1 + \gamma \times V(1)]) + 0.25 \times (1 \times [-1 + \gamma \times V(5)]) + 0.25 \times (1 \times [-1 + \gamma \times V(2)]) + 0.25 \times (1 \times [-1 + \gamma \times V(0)]) = -1 + \gamma \frac{V(1)+V(5)+V(2)+V(0)}{4} = -1.$$

Similarly, for all the states, the updated values are -1.

For the next iteration ($k=2$), again for state 1, we have

$$V(1) = 0.25 \times (1 \times [-1 + \gamma \times V(1)]) + 0.25 \times (1 \times [-1 + \gamma \times V(5)]) + 0.25 \times (1 \times [-1 + \gamma \times V(2)]) + 0.25 \times (1 \times [-1 + \gamma \times V(0)]) = -1 + \gamma \frac{V(1)+V(5)+V(2)+V(0)}{4} = -1.75 \approx -1.7.$$

The corresponding backup diagram is given as



Similarly, for state 2, we have

$$V(2) = 0.25 \times (1 \times [-1 + \gamma \times V(2)]) + 0.25 \times (1 \times [-1 + \gamma \times V(6)]) + 0.25 \times (1 \times [-1 + \gamma \times V(3)]) + 0.25 \times (1 \times [-1 + \gamma \times V(1)]) = -1 + \gamma \frac{V(2)+V(6)+V(3)+V(1)}{4} = -2.$$

For the next iteration (k=3), again for state 1, we have

$$V(1) = 0.25 \times (1 \times [-1 + \gamma \times V(1)]) + 0.25 \times (1 \times [-1 + \gamma \times V(5)]) + 0.25 \times (1 \times [-1 + \gamma \times V(2)]) + 0.25 \times (1 \times [-1 + \gamma \times V(0)]) = -1 + \gamma \frac{V(1)+V(5)+V(2)+V(0)}{4} = -1 + 1.0 \times \frac{-1.7-2.0-2.0-0}{4} = -2.425 \approx -2.4.$$

For this problem, if we continue to update the state values, the values will converge. Again for state 1, we have

$$V(1) = 0.25 \times (1 \times [-1 + \gamma \times V(1)]) + 0.25 \times (1 \times [-1 + \gamma \times V(5)]) + 0.25 \times (1 \times [-1 + \gamma \times V(2)]) + 0.25 \times (1 \times [-1 + \gamma \times V(0)]) = -1 + \gamma \frac{V(1)+V(5)+V(2)+V(0)}{4} = -1 + 1.0 \times \frac{-14-20-18-0}{4} = -14. \text{ In other words, the value will remain the same.}$$

Please make sure that you know how to update the value for any state.

Given a state-value function, to improve the policy, we will take the action that we will give us the maximal reward. For example, if we use the state value function for k=2, for state 1, the resulting states and the rewards are 1, -2.7 (up), 5, -3.0 (down), 2, -3.0 (right), and 0, -1 (left) respectively. Therefore, we can improve the current policy by going left always if we are in state 1. The equation for policy improvement is

$$\begin{aligned} \pi'(s) &= \arg \max_a Q^\pi(s, a) \\ &= \arg \max_a E \{ r_{t+1} + \gamma V^\pi(s_{t+1}) \mid s_t = s, a_t = a \} \\ &= \arg \max_a \sum_{s'} \mathcal{P}_{ss'}^a \left[\mathcal{R}_{ss'}^a + \gamma V^\pi(s') \right], \end{aligned}$$

