

# CS310 Natural Language Processing

## 自然语言处理

### Lecture 02 - Word Vectors

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# Table of Content

- **Motivation**
- Documents and Counts-based Method
- Neural Network-based Method -- word2vec
- Evaluation and Applications

# Words as one-hot vectors

- Words as discrete symbols  $\Leftrightarrow$  (equivalent to) localist representations
- **One-hot** vectors

$$\text{Vocabulary (10k)} = \begin{bmatrix} a \\ \text{about} \\ \text{all} \\ \vdots \\ \text{zoo} \end{bmatrix}$$

Apple [0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 ... 0]

Orange [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 ... 0]

I would like some **apple** juice

I would like some **orange** \_\_\_\_\_

Distance between any pair of words is **constant**:

$$\text{Euclidean distance} = \sqrt{(1 - 0)^2 + (1 - 0)^2}$$

$$\text{Cosine distance} = 0$$

One-hot vector is not helpful

# Words as real-valued vectors

	Man	Woman	King	Queen	Apple	Orange
Gender	-1	1	-0.98	0.97	0.00	-0.01
Royal	0.01	0.02	0.93	0.98	-0.01	0.00
Age	0.03	0.02	0.72	0.68	0.03	0.02
Food	0.00	0.00	0.01	0.02	0.95	0.97

main difference: gender

$$e_{Man} = \begin{bmatrix} -1 \\ 0.01 \\ 0.03 \\ 0.0 \end{bmatrix} \quad e_{Woman} = \begin{bmatrix} 1 \\ 0.02 \\ 0.02 \\ 0.0 \end{bmatrix}$$

$$e_{Man} - e_{Woman} = \begin{bmatrix} -2 \\ -0.01 \\ 0.01 \\ 0.00 \end{bmatrix}$$

main difference: gender

$$e_{King} - e_{Queen} = \begin{bmatrix} -1.95 \\ -0.05 \\ 0.04 \\ -0.01 \end{bmatrix}$$

With real-valued dense vectors, word similarity can be computed more accurately

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# Documents and Word Counts

- **Goal:** Derive word vectors from a collection of documents
- without annotation -- unsupervised/self-supervised
- **Notations:**
  - $\mathcal{x}$  is the collection of  $C$  documents
  - $x_c$  is the  $c$ th document in the corpus
  - $\ell_c$  is the length of  $x_c$  (in # of tokens)
  - $N$  is the total number of tokens,  $N = \sum_{c=1}^C \ell_c$

# Build Word-Document Matrix (term-document matrix)<sup>[1]</sup>

- Build matrix  $\mathbf{A} \in \mathbb{R}^{V \times C}$ , which contains the count of each word in each document

- Example:**

$x_1$ : 学 而 时 习 之

$x_2$ : 学 而 不 思 则 罔

$x_3$ : 思 而 不 学 则 殆

Entry  $\mathbf{A}_{v,c} = \text{count}_{x_c}(v)$ , count of word  $v$  in the  $c$ th document

		$C$		
		$x_1$	$x_2$	$x_3$
$V$	学	1	1	1
	而	1	1	1
	不	0	1	1
	思	0	1	1
	则	0	1	1
	时	1	0	0
	习	1	0	0
	之	1	0	0
	罔	0	1	0
	殆	0	0	1

[1] [https://en.wikipedia.org/wiki/Term-document\\_matrix](https://en.wikipedia.org/wiki/Term-document_matrix)

# Think: how much surprise is in each word?

- What is the expected occurrence of word  $v$  in document  $c$ ?
- Under a simple assumption, the chance of word  $v$  to occur at any position is  $\frac{\text{count}_x(v)}{N}$ , (where  $\text{count}_x(v)$  is the count of  $v$  over all documents)
- So the expected occurrence of  $v$  in a document of length  $\ell_c$  is  $\frac{\text{count}_x(v)}{N} \cdot \ell_c$
- Then we should consider the **ratio** of *observed* count of  $v$  in document  $c$ ,  $\text{count}_{x_c}(v)$ , to the expected count  $\frac{\text{count}_x(v)}{N} \cdot \ell_c$



# Intuition of surprise in word

	$x_1$	$x_2$	$x_3$
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

$$\text{count}_x(\text{学}) = 1 + 1 + 1 = 3$$

Expected count of 学 in  $x_1$  is  $\frac{\text{count}_x(\text{学})}{N} \cdot \ell_1 = \frac{3}{17} \cdot 5 \approx 0.88$

The observed count of 学 in  $x_1$  is  $\text{count}_{x_1}(\text{学}) = 1$

The **surprise** of seeing 学 in  $x_1$  is:

$$\log \frac{\text{observed}}{\text{expected}} = \log \frac{\text{count}_{x_1}(\text{学})}{\frac{\text{count}_x(\text{学})}{N} \cdot \ell_1} \approx \log \frac{1}{0.88} \approx 0.125$$

# Intuition of surprise in word

	$x_1$	$x_2$	$x_3$
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

$$\text{count}_x(\text{习}) = 1 + 0 + 0 = 1$$

Expected count of 习 in  $x_1$  is  $\frac{\text{count}_x(\text{习})}{N} \cdot \ell_1 = \frac{1}{17} \cdot 5 \approx 0.29$

The observed count of 习 in  $x_1$  is  $\text{count}_{x_1}(\text{习}) = 1$

The **surprise** of seeing 习 in  $x_1$  is:

$$\log \frac{\text{observed}}{\text{expected}} = \log \frac{\text{count}_{x_1}(\text{习})}{\frac{\text{count}_x(\text{习})}{N} \cdot \ell_1} \approx \log \frac{1}{0.29} \approx 1.223 > \text{surprise of 学}$$

# Pointwise Mutual Information

- From matrix  $\mathbf{A} \in \mathbb{R}^{V \times C}$ , derive positive **pointwise mutual information**

$$[\mathbf{A}]_{v,c} = \left[ \log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N} \cdot \ell_c} \right]_+ = \left[ \log \frac{N \cdot \text{count}_{x_c}(v)}{\text{count}_x(v) \cdot \ell_c} \right]_+ \quad \text{where } [x]_+ = \max(0, x)$$

More examples:

$$[\mathbf{A}]_{\text{学},2} = \log \frac{17 \cdot 1}{3 \cdot 6} \approx -0.057 \rightarrow 0 \quad \text{rounded to 0 because of max()}$$

$$[\mathbf{A}]_{\text{思},2} = \log \frac{17 \cdot 1}{2 \cdot 6} \approx 0.348$$

# Meaning of PMI

Pointwise mutual information for two random variables  $A$  and  $B$ :

$$\begin{aligned}\text{PMI}(a, b) &= \log \frac{p(A = a, B = b)}{p(A = a) \cdot p(B = b)} \\ &= \log \frac{p(A = a | B = b)}{p(A = a)} \\ &= \log \frac{p(B = b | A = a)}{p(B = b)}\end{aligned}$$

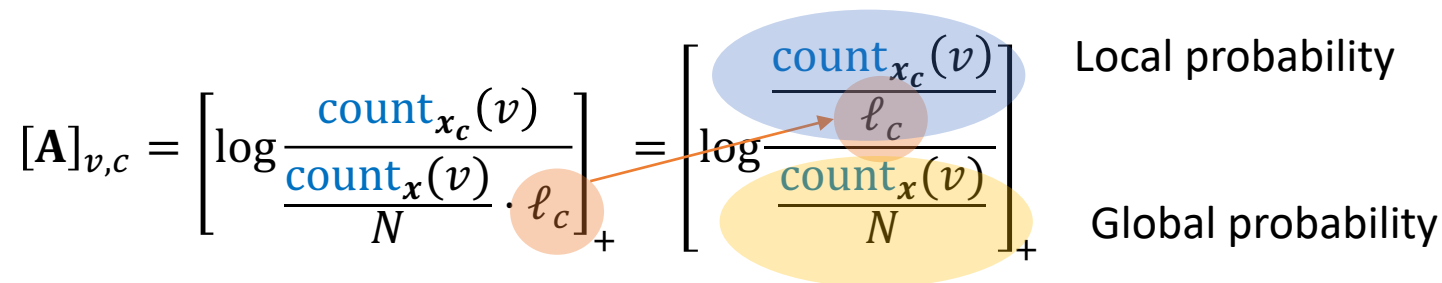
Example:

$$\log \frac{\text{count}_{x_1}(\text{习})}{\frac{\text{count}_x(\text{习})}{N} \cdot \ell_1} \approx \log \frac{1}{0.29} \approx 1.223$$

is high, which means we learn a lot about the global meaning of “习” by reading  $x_1$

## Mutual Information (MI):

The amount of information each r.v. offers about the other. I.e., how much do we know about **B** by knowing about **A**



$$[\mathbf{A}]_{v,c} = \left[ \log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N} \cdot \ell_c} \right]_+ = \left[ \log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N}} \right]_+$$

Local probability

Global probability

How much do we know about the global meaning of  $v$  by knowing about its local meaning in document  $c$

# Pointwise Mutual Information

$$PMI = [\mathbf{A}]_{v,c} = \left[ \log \frac{\text{count}_{x_c}(v)}{\frac{\text{count}_x(v)}{N} \cdot \ell_c} \right]_+$$

- If a word  $v$  has nearly same frequency in every document, then its row  $[\mathbf{A}]_{v,*}$  will be nearly all zeros
- If a word  $v$  only occurs in one document  $c$ , then its PMI will be large and positive
- Thus, PMI is sensitive to rare words; usually need to smooth the frequencies by filtering rare words

	$x_1$	$x_2$	$x_3$
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

# Reflection

- Can we directly use word-document matrix  $\mathbf{A} \in \mathbb{R}^{V \times C}$  (or smoothed PMI  $[\mathbf{A}]$ ) to represent word meanings?
- For example, can we use the row vectors as input features for a neural text classifier?
- What are the advantages/disadvantages?

# Improvement: Latent Semantic Analysis

(Deerwester et al., 1990)

- LSA seeks to find a more compact (low rank) representation of document-word matrix  $\mathbf{A}$

$$\underset{V \times C}{\mathbf{A}} \approx \underset{V \times d}{\hat{\mathbf{A}}} = \underset{V \times d}{\mathbf{M}} \times \underset{d \times d}{\text{diag}(\mathbf{s})} \times \underset{d \times C}{\mathbf{C}^T}$$

- Can be solved by applying singular value decomposition to  $\mathbf{A}$ , and then truncating to  $d$  dimensions ( $\hat{\mathbf{A}}$ )
- $\mathbf{M}$  contains left singular vectors of  $\mathbf{A}$
- $\mathbf{C}$  contains right singular vectors of  $\mathbf{A}$
- $\mathbf{s}$  are singular values of  $\mathbf{A}$

# SVD and Truncated SVD

SVD:

$$\begin{array}{c}
 \text{SVD:} \\
 \begin{array}{c}
 \boxed{\mathbf{A}} \\
 \end{array}
 =
 \begin{array}{c}
 V \times V \\
 \boxed{\mathbf{M}} \\
 \end{array}
 \begin{array}{c}
 V \times C \\
 \begin{array}{c}
 \diagdown \\
 \text{diag}(\mathbf{s}) \\
 \diagup
 \end{array}
 \end{array}
 \begin{array}{c}
 C \times C \\
 \boxed{\mathbf{C}^T}
 \end{array}
 \end{array}$$

- $\mathbf{M}$  and  $\mathbf{C}$  are unitary, i.e.,  $\mathbf{M}\mathbf{M}^T = \mathbf{I}$  and  $\mathbf{C}\mathbf{C}^T = \mathbf{I}$
- $\text{diag}(\mathbf{s})$  only has non-zero elements at diagonal
- $\mathbf{M}$  are eigenvectors of  $\mathbf{A}\mathbf{A}^T$
- $\mathbf{C}$  are eigenvectors of  $\mathbf{A}^T\mathbf{A}$
- $\mathbf{s}^2$  are eigenvalues

SVD truncated at  $d$  dimensions:

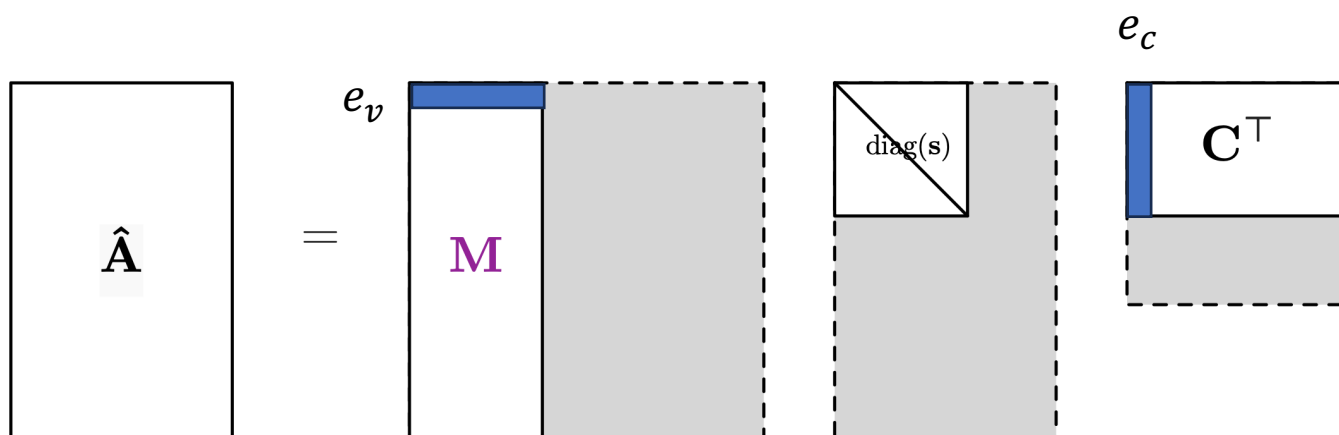
$$\begin{array}{c}
 \begin{array}{c}
 \boxed{\hat{\mathbf{A}}} \\
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{c}
 \boxed{\mathbf{M}} \\
 \text{---} \\
 V \times d
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 \diagdown \\
 \text{diag}(\mathbf{s}) \\
 \diagup
 \end{array} \\
 d \times d
 \end{array}
 \begin{array}{c}
 \begin{array}{c}
 \boxed{\mathbf{C}^T} \\
 \text{---} \\
 d \times C
 \end{array}
 \end{array}
 \end{array}$$

- Truncated: keeping only top  $d$  singular values in  $\mathbf{s}$
- corresponding  $d$  columns in  $\mathbf{M}$  and  $\mathbf{C}$



# Truncated SVD => word vectors

$$\mathbf{A} \approx \hat{\mathbf{A}} = \mathbf{M} \times \text{diag}(\mathbf{s}) \times \mathbf{C}^\top$$

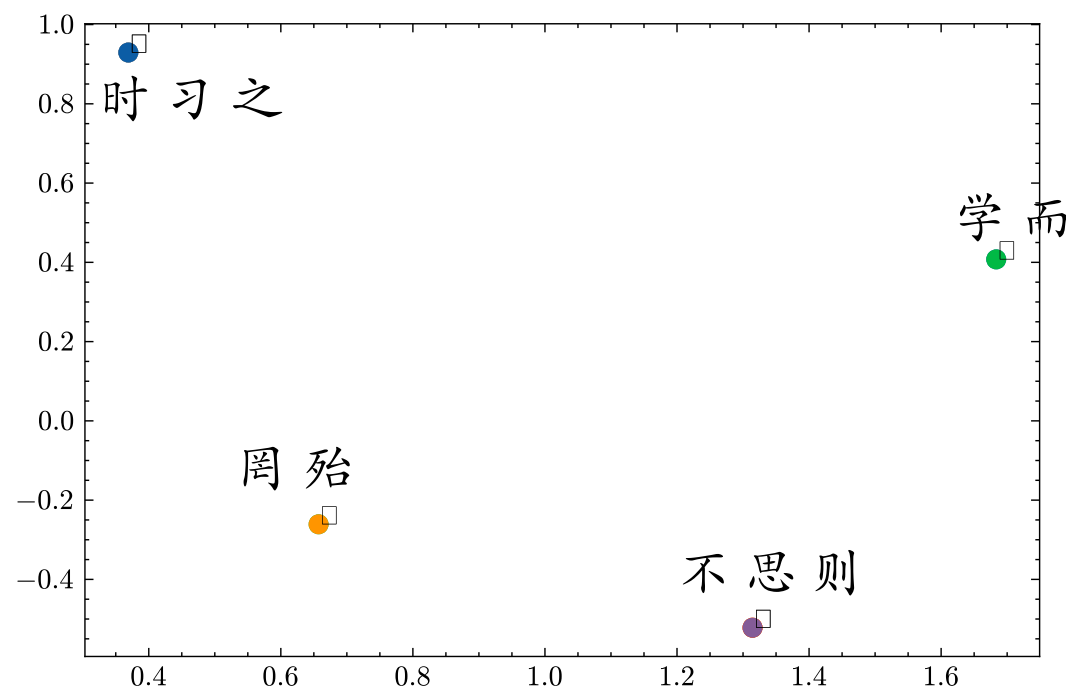


- $v$ th column in  $\mathbf{M}$  is the embedding vector for word  $v$
- $c$ th column in  $\mathbf{C}$  is the embedding vector for document  $c$

- $\mathbf{M}$  contains useful word vectors (“embeddings”) of  $d$  dimensions
- $\mathbf{C}$  contains document vectors

# LSA Example $d = 2$

- Word vectors  $\mathbf{M}$  plotted
- Note that some words are in the same spot. Why?

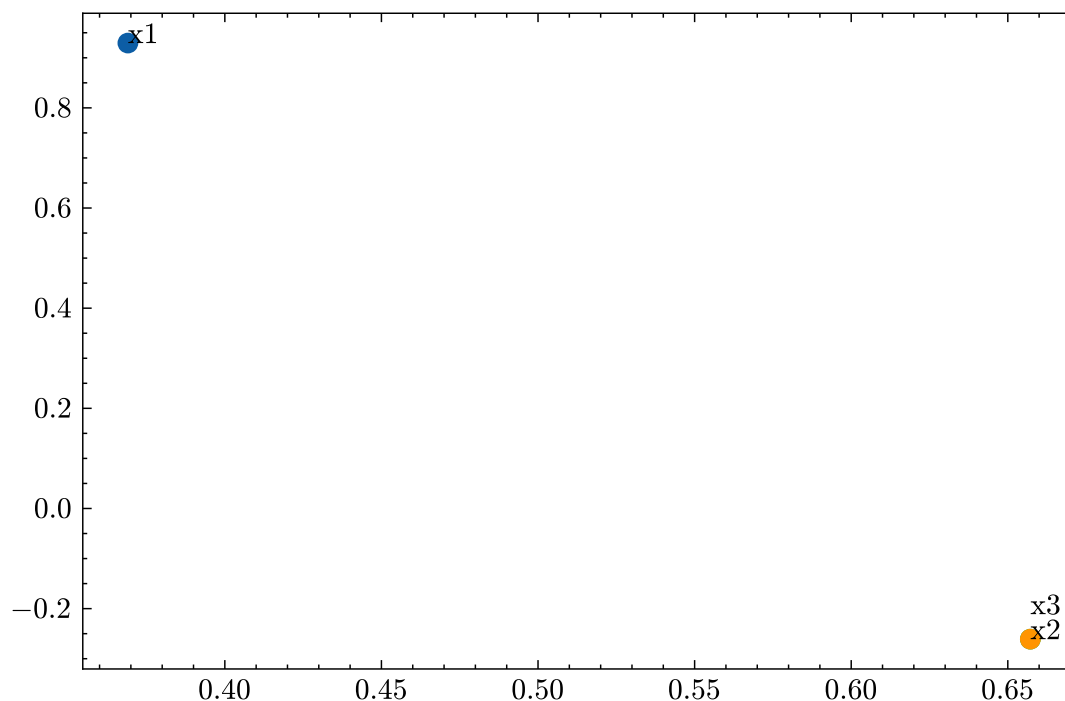


$\mathbf{A} =$

	$x_1$	$x_2$	$x_3$
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

# LSA Example $d = 2$

- Document vectors  $\mathbf{C}$  plotted
- Note that documents  $x_2$  and  $x_3$  are in the same spot. Why?



$\mathbf{A} =$

	$x_1$	$x_2$	$x_3$
学	1	1	1
而	1	1	1
不	0	1	1
思	0	1	1
则	0	1	1
时	1	0	0
习	1	0	0
之	1	0	0
罔	0	1	0
殆	0	0	1

# LSA Summarized

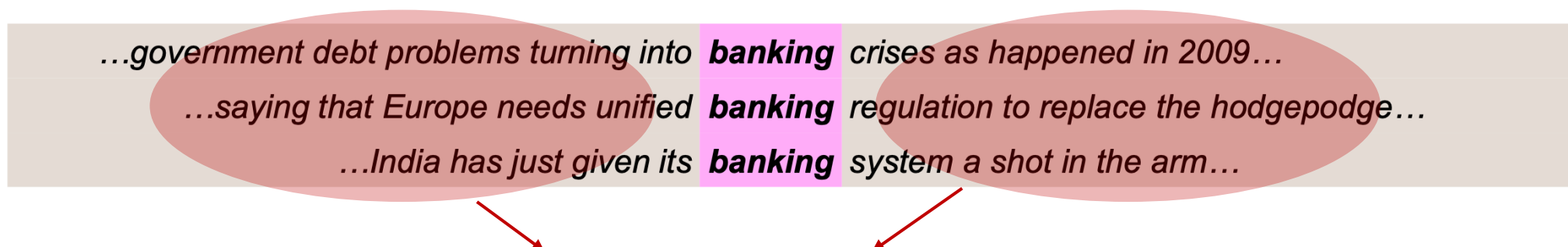
- It creates a mapping of words and documents into the same low-dimensional space.
- Bag-of-words assumption (Salton et al., 1975):
  - A document is nothing more than the distribution of words it contains.
- Distributional hypothesis (Harris, 1954; J.R. Firth, 1957):
  - Words' meanings are nothing more than the distribution of *contexts* (here, documents) they occur in.
  - Words that occur in similar contexts have similar meanings.
- Word-document matrix  $\mathbf{A}$  is sparse and noisy; LSA “fills in” the zeroes and tries to eliminate the noise.
- It finds the best rank- $d$  approximation to  $\mathbf{A}$ .

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# Motivation: Distributional semantics

- Distributional semantics: A word's meaning is given by the words that frequently appear close-by
- “You shall know a word by the company it keeps” (J. R. Firth 1957: 11)
- When a word  $w$  appears in a text, its local **context** is the set of words that co-occur within a fixed-size window



*...government debt problems turning into **banking** crises as happened in 2009...*  
*...saying that Europe needs unified **banking** regulation to replace the hodgepodge...*  
*...India has just given its **banking** system a shot in the arm...*

The meaning of “banking” is represented by these context words

# In What Form of Representation?

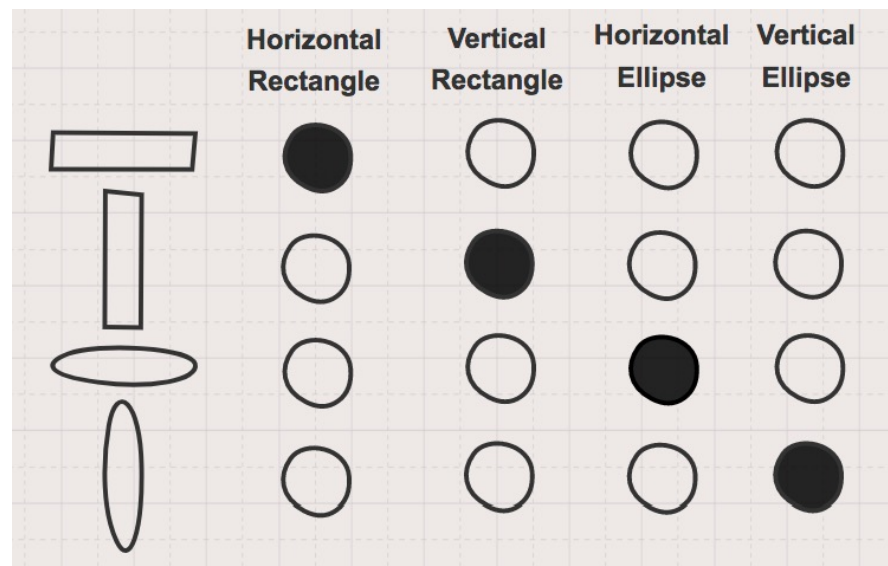
- **Goal:** Obtain a dense vector for each word, so that word sense similarity can be computed via vector distance, such as dot product

$$e_{apple} = \begin{bmatrix} 0.00 \\ -0.01 \\ 0.03 \\ 0.95 \\ \vdots \\ 0.21 \end{bmatrix} \quad e_{orange} = \begin{bmatrix} -0.01 \\ 0.00 \\ 0.02 \\ 0.97 \\ \vdots \\ 0.22 \end{bmatrix} \quad \left. \vphantom{\begin{bmatrix} 0.00 \\ -0.01 \\ 0.03 \\ 0.95 \\ \vdots \\ 0.21 \end{bmatrix}} \right\} \text{Common dimension size: } 100\text{-d}, 200\text{-d}, 300\text{-d}, \dots$$

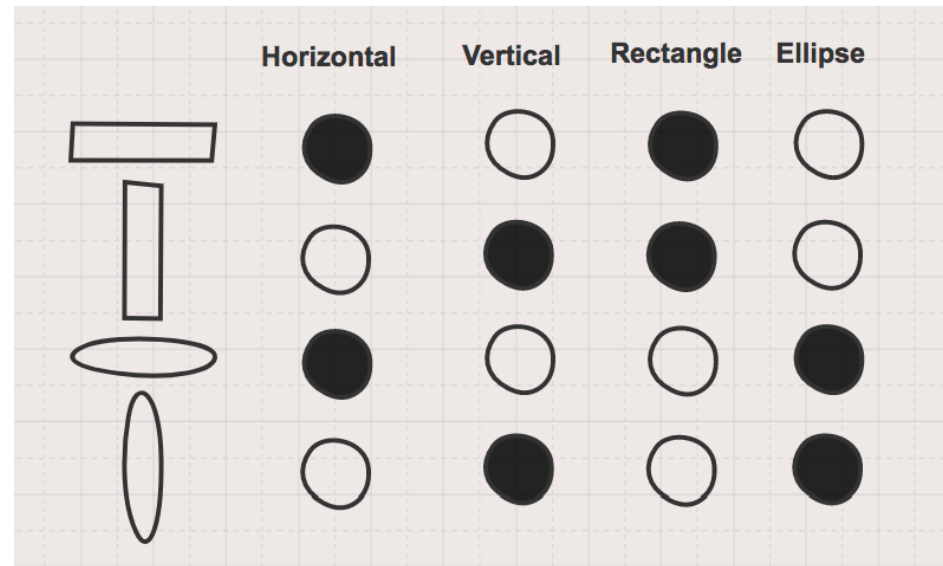
These dense word vectors are also called word **embeddings** (嵌入)  
(which implies the idea of placing or mapping words into some continuous vector space)

# Intuition: One-hot vs. Distributed repr.

One-hot representation



Distributed representation



The individual dimensions of a word embedding do not have concrete “meanings”

$$E_{Orange} = \begin{bmatrix} -0.01 \\ 0.00 \\ 0.02 \\ 0.97 \\ \dots \\ 0.22 \end{bmatrix}$$

For instance,  $e_{orange}$  It does NOT mean  
 1<sup>st</sup> dimension -0.01 is for “animalness”  
 4<sup>th</sup> dimension 0.97 is for “fruitness”  
 They are only meaningful when compared to other words



# Question: How to obtain word embeddings?

- An effective and efficient method: Word2vec (Mikolov et al. 2013 a&b)
- **Basic Idea:**
- Given a corpus as a list of words
- Go through each position  $t$  in the text, which has a center word  $c$  and context (“outside”) words  $o$
- Use the **similarity of word vectors** between  $c$  and  $o$  to **compute the probability** of  $o$  given  $c$ , i.e., conditional probability  $P(o|c)$  (or vice versa)
- **Maximize this probability** by keep adjusting the word vectors

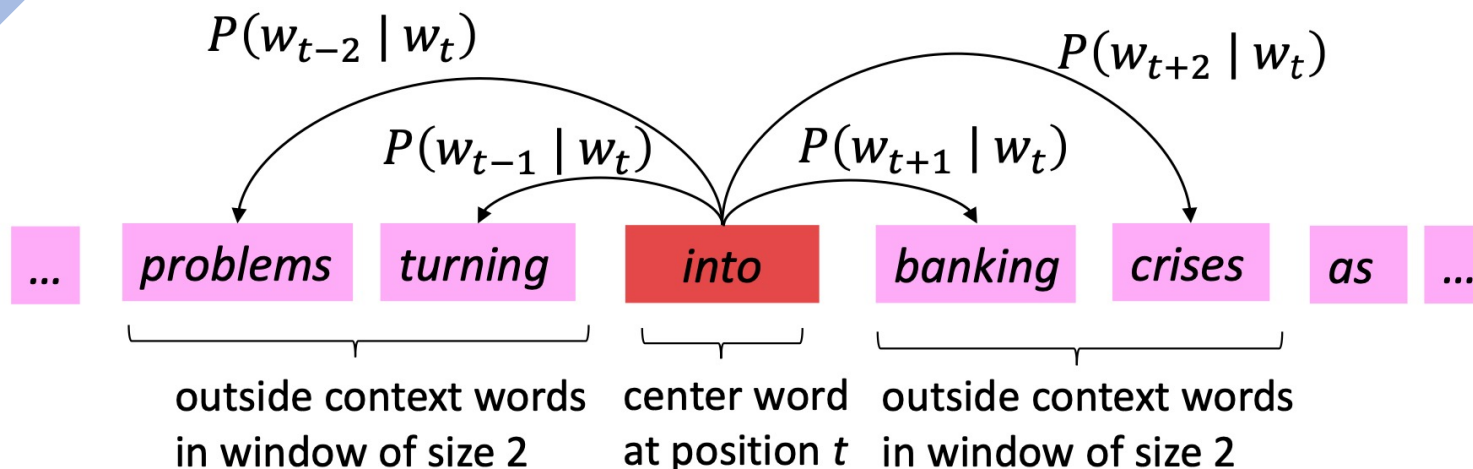
# Two architectures of Word2vec

Word2vec { **Skip-gram**: Maximize  $P(o|c)$

- “o” for outside (context) words
- “c” for center word

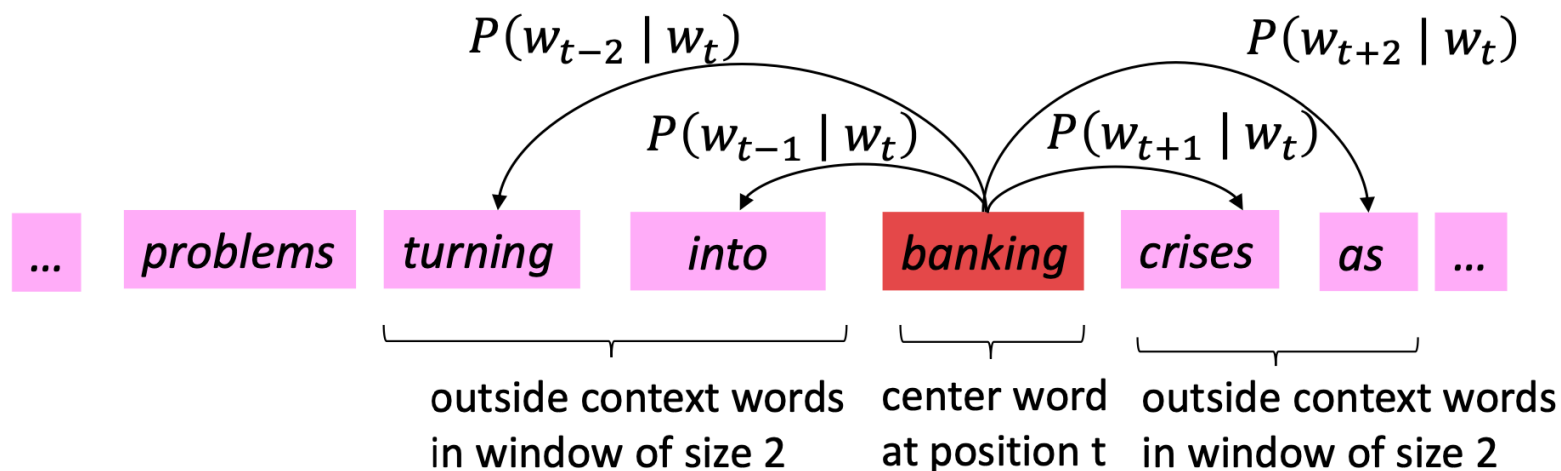
Continuous Bag-of-words (**CBOW**): Maximize  $P(c|o)$

Compute probability  
 $P(w_{t+j}|w_t)$ ,  
for  $j \in \{-2, -1, 1, 2\}$   
when window size is 2



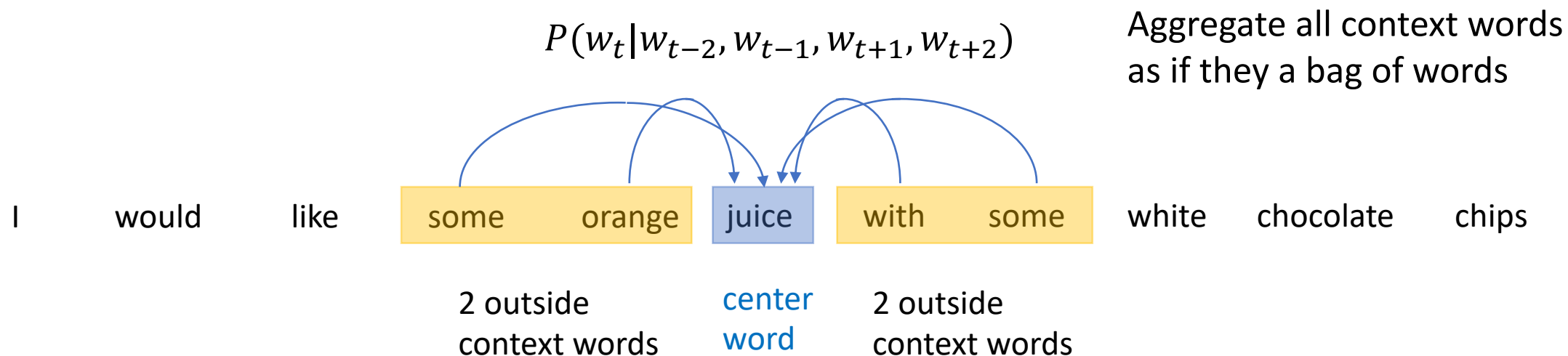
# Use A Moving Window $t \leftarrow t + 1$

Skip-gram: Compute probability  $P(w_{t+j}|w_t)$ , for  $j \in \{-2, -1, 1, 2\}$  when window size is 2



Example from: <https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/>

# Continuous Bag-of-Words (CBOW)



Compute only one probability at position  $t$ :

$P(w_t | w_{t-2}, w_{t-1}, w_{t+1}, w_{t+2})$ , for window size 2

# Word2vec Objective Function (Skip-gram as example)

- Given a data set of  $T$  tokens, for each position  $t = 1, \dots, T$ , we compute the conditional probability  $P(w_{t+j}|w_t)$ , for  $j \in \{-m, \dots, m\}$ , with window size  $m$
- Then the *likelihood* of data is:

$$\mathcal{L}(\theta) = \prod_{t=1}^T \prod_{\substack{-m \leq j \leq m \\ j \neq 0}} P(w_{t+j}|w_t; \theta)$$

$\theta$  denotes model parameters, that is, all the word **embeddings** to be learned!

- The objective function (cost/loss) is the negative log-likelihood

$$J(\theta) = -\frac{1}{T} \log \mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^T \sum_{\substack{-m \leq j \leq m \\ j \neq 0}} \log P(w_{t+j}|w_t; \theta)$$

# Question: How to compute $P(w_{t+j}|w_t; \theta)$ ?

- **Solution:** Use *two* vectors per word  $w$
- When  $w$  is a center word, its vector is  $v_w$
- When  $w$  is a context (outside) word, its vector is  $u_w$
- Then the conditional probability of context word  $o$  given center word  $c$  can be computed using **softmax** function:

$$P(o|c) = \frac{\exp(u_o^\top v_c)}{\sum_{w \in V} \exp(u_w^\top v_c)}$$

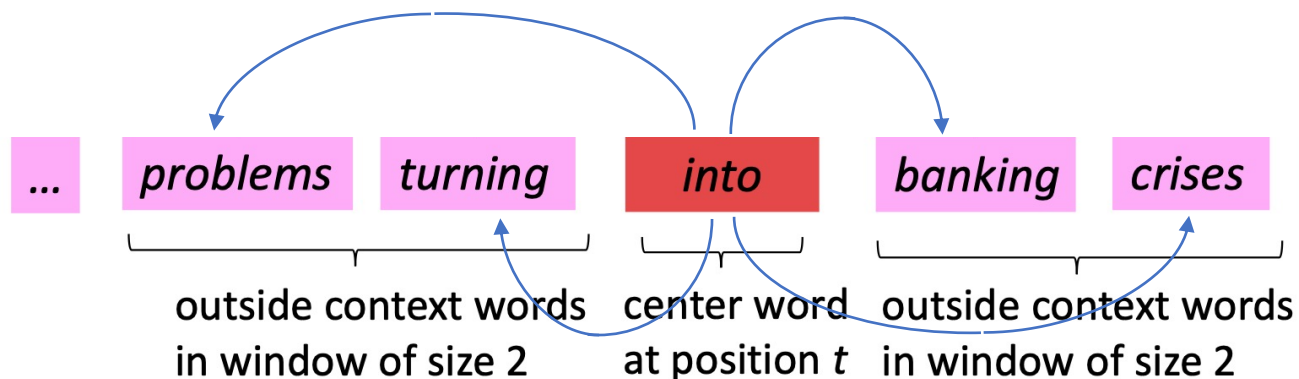
Dot product measures the similarity between  $o$  and  $c$

Normalized over the entire vocabulary

# Compute probabilities using softmax

$$P(\text{problems}|\text{into}) = \frac{\exp(u_{\text{problems}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$

$$P(\text{banking}|\text{into}) = \frac{\exp(u_{\text{banking}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$



$$P(\text{turning}|\text{into}) = \frac{\exp(u_{\text{turning}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$

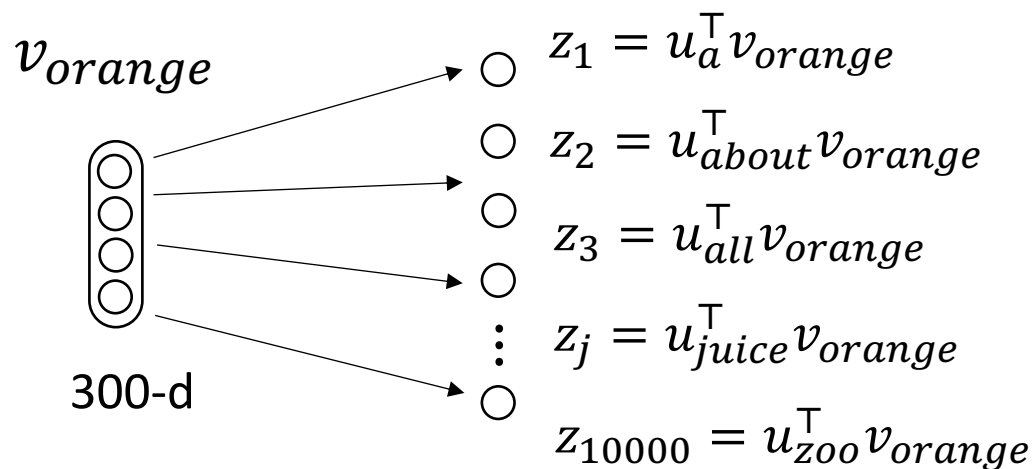
$$P(\text{crises}|\text{into}) = \frac{\exp(u_{\text{crises}}^T v_{\text{into}})}{\sum_{w \in V} \exp(u_w^T v_{\text{into}})}$$

Example from: <https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/>

# Problem with Softmax

center    context  
*I        would        like        some        orange        juice        with        some        white        chocolate        chips*

$$P(\text{juice}|\text{orange}) = \frac{\exp(u_{\text{juice}}^{\top} v_{\text{orange}})}{\sum_{w \in V} \exp(u_w^{\top} v_{\text{orange}})}$$



For a vocabulary of 10,000 words

Needs 10,000 times of dot product to compute the denominator



# Number of Parameters

- Because *two* vectors are used per word  $w$ :  $v_w$  and  $u_w$
- => Two parameter tables, or, embedding matrices

Usually we keep the target table **V** as the trained embeddings

Total # of params:  
 **$2 * [\text{vocab-size}] * [\text{emb-size}]$**

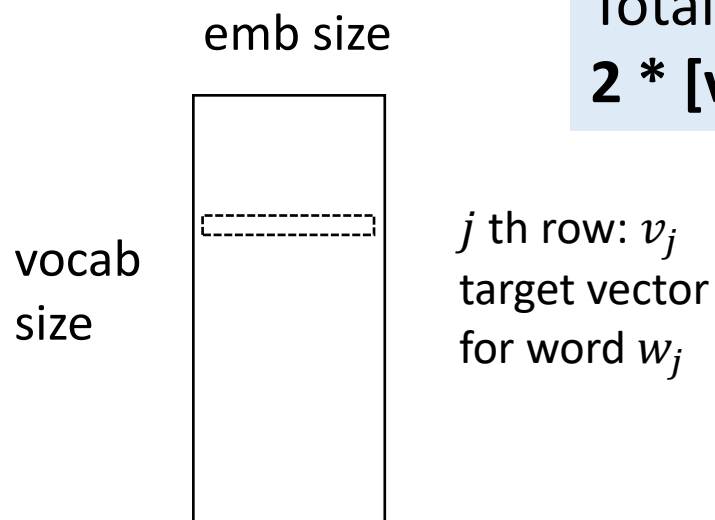


Table **V** contains all parameters for **center** vectors

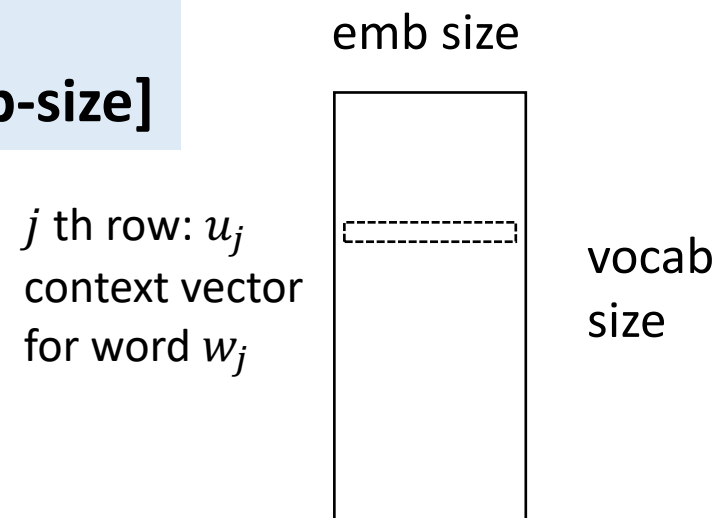


Table **U** contains all parameters for **context** vectors

# To Overcome Softmax

## Solutions

- 1. Hierarchical softmax
- 2. **Negative sampling**

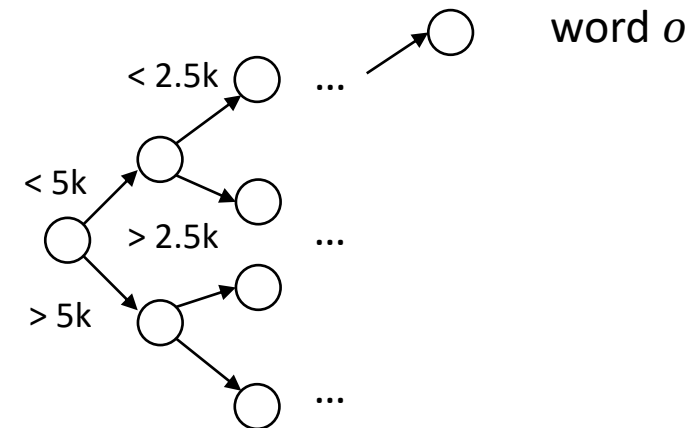


Make binary predictions instead:

$$P\left(o < \frac{|V|}{2} \mid c\right)$$

The probability of word  $o$  belongs to the 1<sup>st</sup> half of vocabulary

For vocabulary size  $|V| = 10k$



Multiple steps of binary predictions until word  $o$  is found

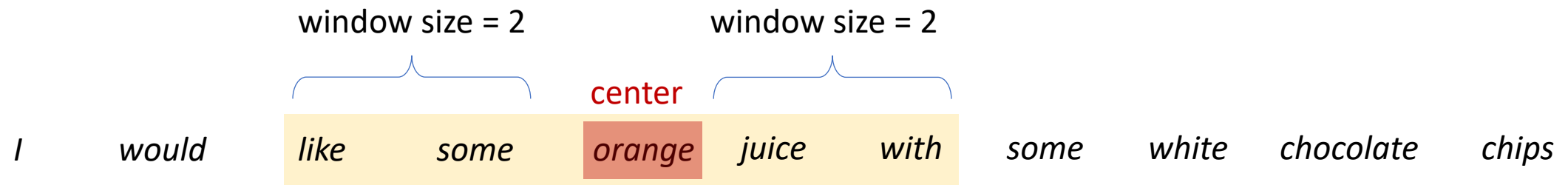
$$\text{Then } P(o|c) = P(o < 5k|c) \cdot P(o < 2.5k|c) \cdot P(o < 1.25k|c) \dots$$

product of probabilities along the path

Time complexity  $O(\log(|V|))$

Reference: <http://ruder.io/word-embeddings-softmax/>

# Solution 2: Negative sampling



**Goal:** Given a center word, predict if a *randomly sampled* word is its context or not (within a fixed window)

<b>Center</b>	<b>Target Word</b>	<b>Label</b>
<i>orange</i>	<i>juice</i>	<i>1</i>
<i>orange</i>	<i>king</i>	<i>0</i>
<i>orange</i>	<i>the</i>	<i>0</i>
<i>orange</i>	<i>of</i>	<i>0</i>
<i>orange</i>	<i>book</i>	<i>0</i>

Diagram illustrating the data structure for negative sampling:

The table shows the Center word (*orange*), the Target Word, and the Label (*1* for positive, *0* for negative).

The Center word is labeled  $x$  and the Target Word is labeled  $y$ .

Step 1: Pick a context word within the window

**Positive sample**

Step 2: Randomly pick  $k$  words from the entire vocabulary that do not appear in the window

**Negative samples**

# Negative Sampling: Objective Function

- For token at position  $t$ , maximize the log-likelihood:

Word  $o$  is the positive sample

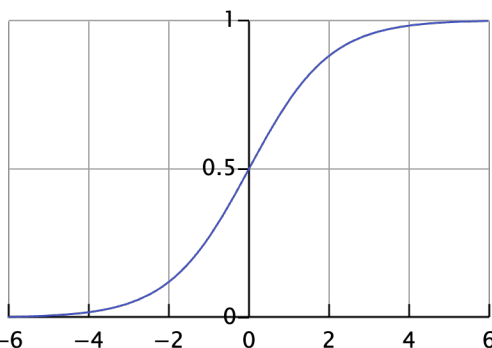
$$J_t(\theta) = \log \sigma(u_o^\top v_c) + \sum_{i=1}^k \mathbb{E}_{w_i \sim P(w)} [\log \sigma(-u_{w_i}^\top v_c)]$$

The  $k$  words  $w_i$  ( $i = 1 \dots k$ ) are the negative samples

- Sigmoid function  $\sigma(u_o^\top v_c)$  outputs the probability of  $o$  in the context window of  $c$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

a monotone increasing function



Maximizing this term will push the dot product  $u_o^\top v_c$  to **larger** values, i.e., making  $o$  and  $c$  closer in semantic space

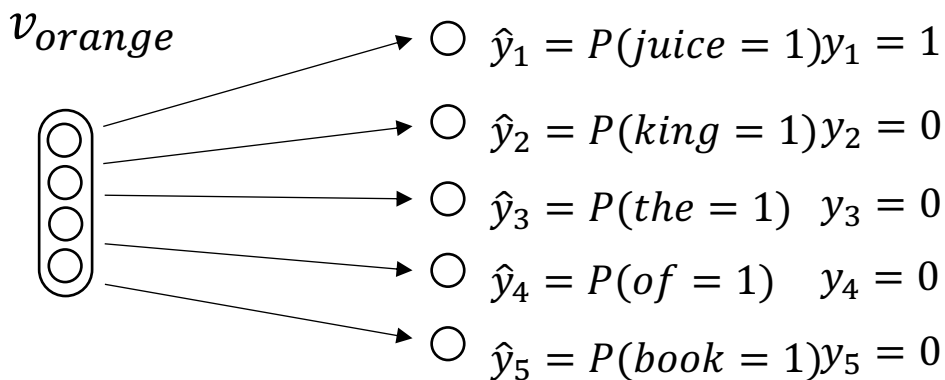
Maximizing this term will push the dot product  $u_{w_i}^\top v_c$  to **smaller** values, i.e., making  $w_i$  and  $c$  farther apart in semantic space

# Negative sampling: Example

$c$	$t$	$y$
Center	Target	Context or not
<i>orange</i>	<i>juice</i>	1
<i>orange</i>	<i>king</i>	0
<i>orange</i>	<i>the</i>	0
<i>orange</i>	<i>of</i>	0
<i>orange</i>	<i>book</i>	0

Instead of using softmax:  $P(t|c) = \frac{\exp(u_t \cdot v_c)}{\sum_{j=1 \dots |V|} \exp(u_j \cdot v_c)} = \hat{y}_t$

Use logistic regression:  $P(y = 1|c, t) = \sigma(u_t \cdot v_c)$



$k+1$  times of logistic regression

For each center word,  
the  $k$  negative examples  
are different

$k = 5 \sim 20$  for small dataset

$k = 2 \sim 5$  for large dataset

# Negative Sampling: More Details

- Maximize probability that *real* outside word appears;
- Minimize probability that *random* words appear around center word
- Sample from the distribution  $P(w) = \frac{U(w)^{\frac{3}{4}}}{Z}$ , the unigram frequency distribution  $U(w)$  raised to the  $\frac{3}{4}$  power ( $Z$  is normalization term)
- The power makes less frequent words be sampled more often
- $0.9^{3/4} \approx 0.924 \Rightarrow$  a 2.7% increase in chance being sampled
- $0.1^{3/4} \approx 0.178 \Rightarrow$  a 77.8% increase in chance being sampled

# Table of Content

- Motivation
- Documents and Counts-based Method
- Neural Network-based Method -- word2vec
- **Evaluation and Applications**

# General Evaluation in NLP

- Intrinsic (内在的) vs. Extrinsic (外在的)
- Intrinsic:
  - Evaluation on a specific/intermediate subtask
  - Fast to compute
  - Not clear if really helpful unless correlation to real task is found
- Extrinsic:
  - Evaluation on a real task
  - Can take a long time to compute accuracy
  - Unclear if the subsystem is the problem or its interaction with other subsystems

Adapted from: <https://web.stanford.edu/class/archive/cs/cs224n/cs224n.1224/>



# Evaluate Word Vectors (Embeddings)

- Intrinsic task: Word semantic similarity task

$$d_1 = \cos(e_{book}, e_{library}), \text{ cosine similarity}$$

Word1	Word2	Human score	Cosine distance
book	library	7.46	d1
bank	money	8.12	d2
wood	forest	7.73	d3
professor	cucumber	0.31	d4
...	...	...	...

Spearman's correlation between the two columns are used to evaluate the quality of word embeddings

# Evaluate Word Vectors (Embeddings)

- Intrinsic task: Word analogy task

Question: What is to “King” as “woman” to “man”?

$$e_{Man} - e_{Woman} \approx e_{King} - e_w \quad w = ?$$

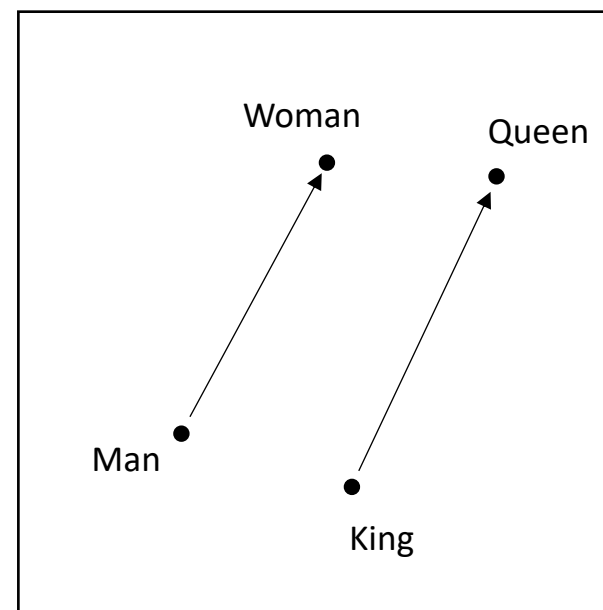
Find the word  $w$  so that:

$$\arg \max_w \text{sim}(e_w, e_{King} - e_{Man} + e_{Woman})$$

Here,  $\text{sim}()$  is a similarity function,  
for example, cosine similarity

$$\text{sim}(u, v) = \frac{u^T v}{\|u\| \|v\|}$$

Finding the most similar vector  $e_w$  will hopefully pick up the word  $w = \text{Queen}$



# Word Analogy Task (as an interesting application)

Capital-common-countries:

Athens Greece Baghdad **Iraq**  
Athens Greece Bangkok **Thailand**  
Athens Greece Beijing **China**  
Athens Greece Berlin **Germany**  
...

Family:

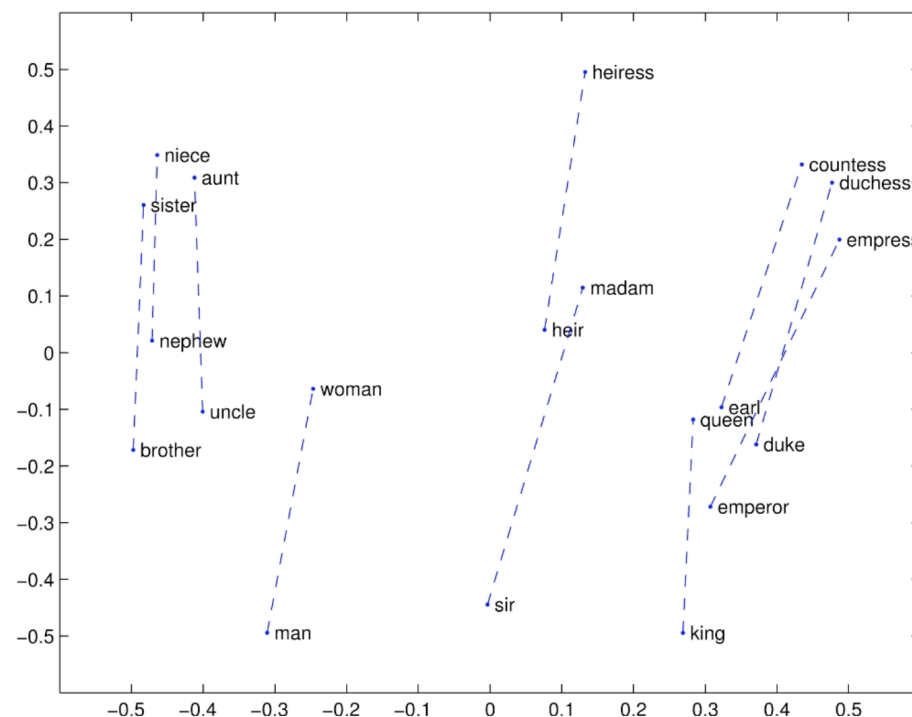
boy girl brother **sister**  
boy girl brothers **sisters**  
boy girl dad **mom**  
boy girl father **mother**  
...

Comparative:

bad worse big bigger  
bad worse bright brighter  
bad worse cheap cheaper  
bad worse cold colder  
...

City-in-state:

Chicago Illinois Houston **Texas**  
Chicago Illinois Philadelphia **Pennsylvania**  
Chicago Illinois Phoenix **Arizona**  
Chicago Illinois Dallas **Texas**  
...



70 - 80 % accuracy reported in Mikolov et al., 2013

Figure from: Pennington et al. (2014). Glove: Global vectors for word representation.

# Extension: GloVe

Pennington, J., Socher, R., & Manning, C. (2014). Glove: Global vectors for word representation.

A Different way from Word2vec

How the paper gets to this function step by step is deep and enlightening (reading recommended)

Cost function: 
$$J = \sum_{i,j=1}^V f(X_{ij})(e'_i e_j + b_i + b'_j - \log(X_{ij}))^2$$

Dot product of two embeddings

Frequency counts of word i and j co-occur (within a fixed window)

Basic idea: words that appear together more often (larger  $X_{ij}$ ) should have closer meanings (larger dot product  $e'_i e_j$ )

**Advantages:** Fast training; scalable to huge corpora

# Fun Application: Emoji2vec

Eisner, B., Rocktäschel, T., Augenstein, I., Bošnjak, M., & Riedel, S. (2016). emoji2vec: Learning emoji representations from their description. *arXiv preprint arXiv:1609.08359*.



# To-Do

- Attend Lab 3
- Continue working on A2
- Read Chapter 9 - RNNs and LSTMs

# References

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