Econometrics. 2022-5-27

$H_0 : \theta = \theta_0 \ (=2)$

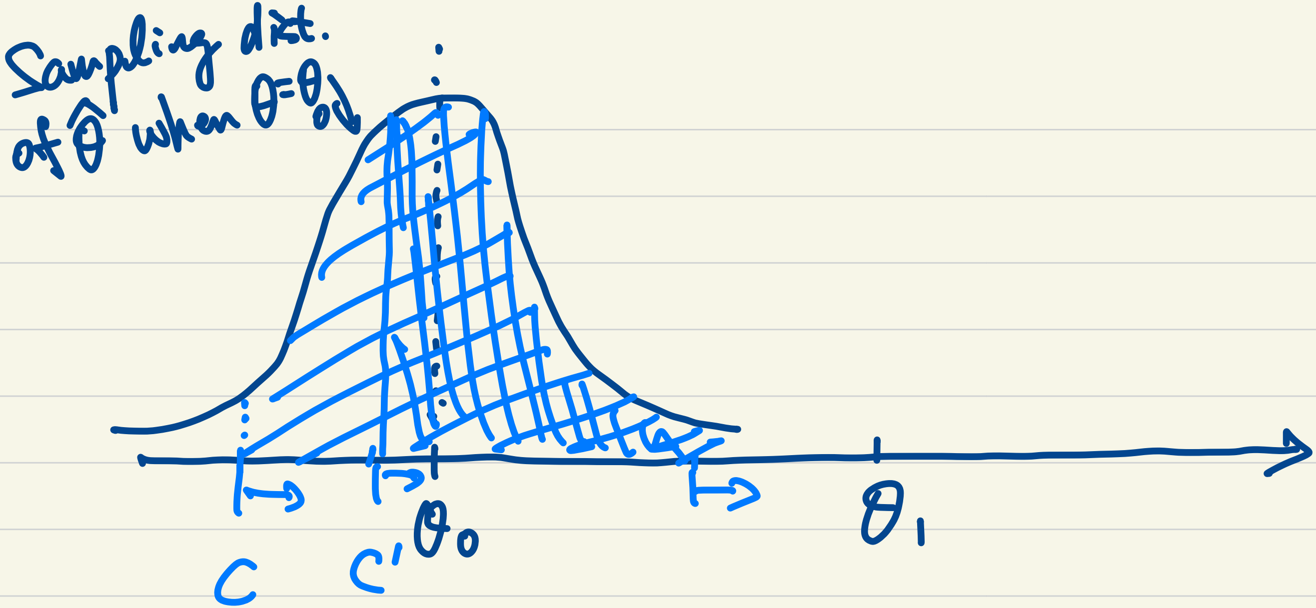
$H_1 : \theta = \theta_1 \ (=5)$ | assume $\theta_1 > \theta_0$

Test statistic : $\hat{\theta}$

Rejection region : $\hat{\theta} > \underline{\underline{c}}$ ← What if $\hat{\theta} < c$

$\Rightarrow$ Acceptance region : $\hat{\theta} \leq c$

Sampling dist. of $\hat{\theta}$.

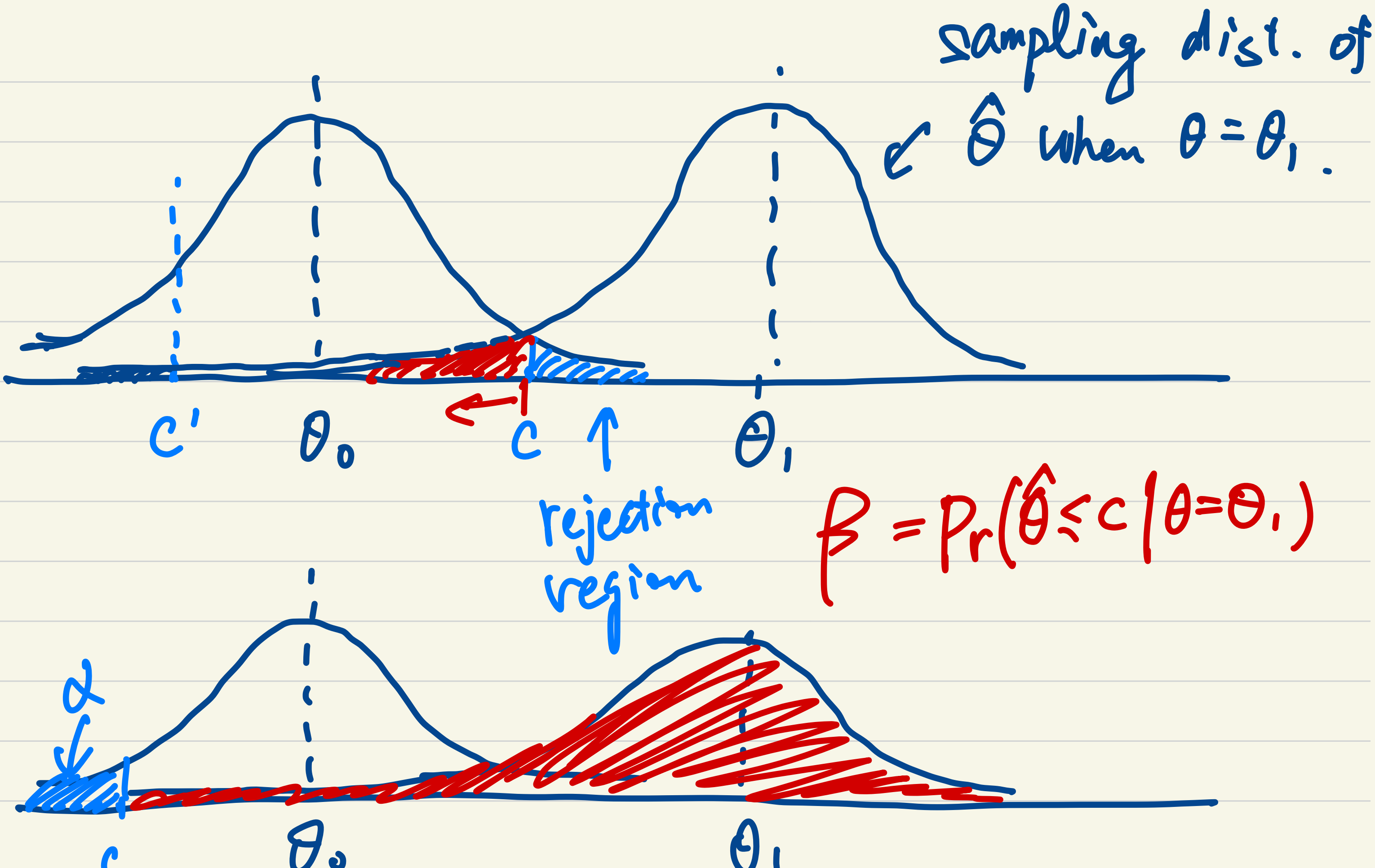


$$\alpha = \Pr(\hat{\theta} > c \mid \theta = \theta_0)$$

$$\vee$$

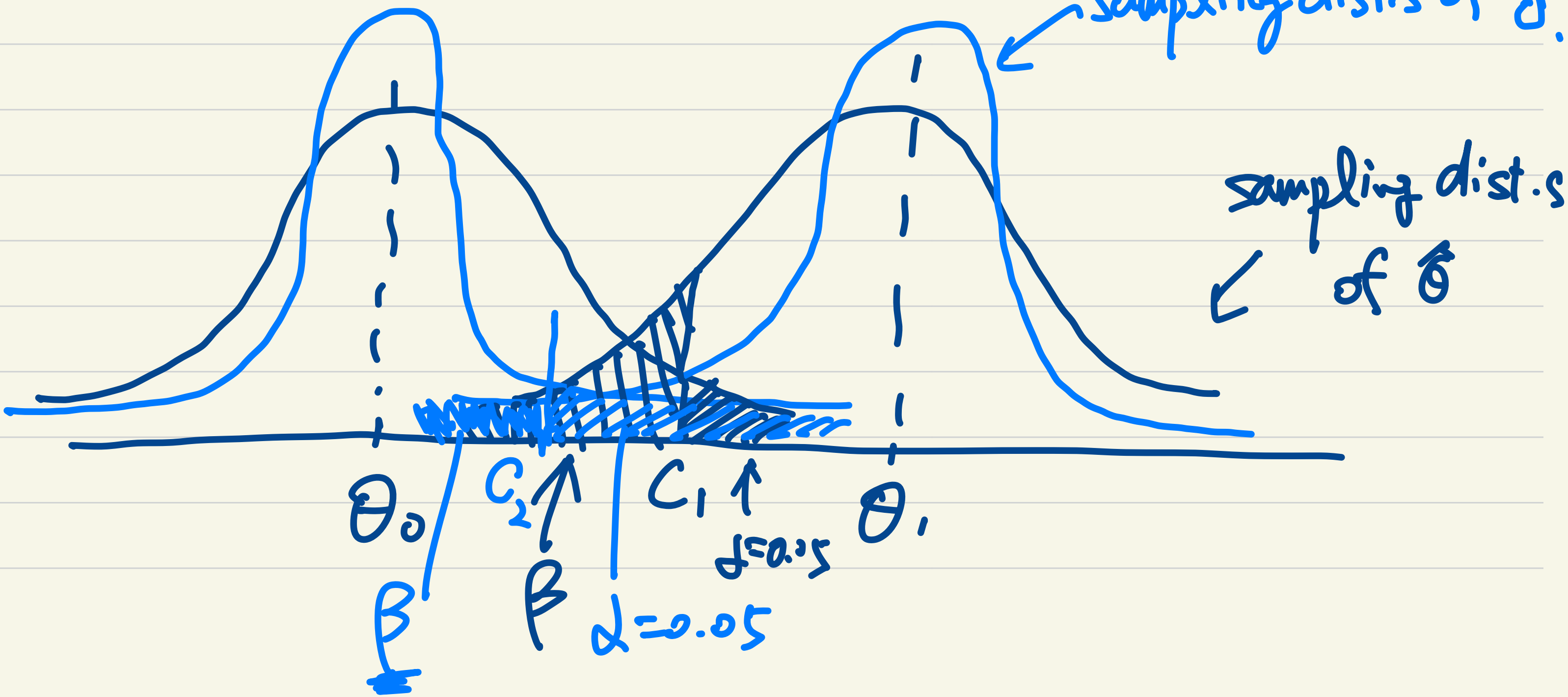
$$\alpha' = \Pr(\hat{\theta} > c' \mid \theta = \theta_0)$$

$$\alpha = 0.05,\ 0.01.\ 0.1$$


sampling dist. of $\hat{\theta}$ when $\theta = \theta_1$.
c'
θ_0
c
θ_1
rejection region
$\beta = \Pr(\hat{\theta} \leq c \mid \theta = \theta_1)$
α
c
θ_0
θ_1

Test statistics $\hat{\theta}$ vs. $\hat{\delta}$

Rejection region : $\hat{\theta} > c_1$, $\hat{\delta} > c_2$



$H_0 : \theta = \theta_0 \quad , \quad H_1 : \theta = \theta_1$ ← simple hypothesis

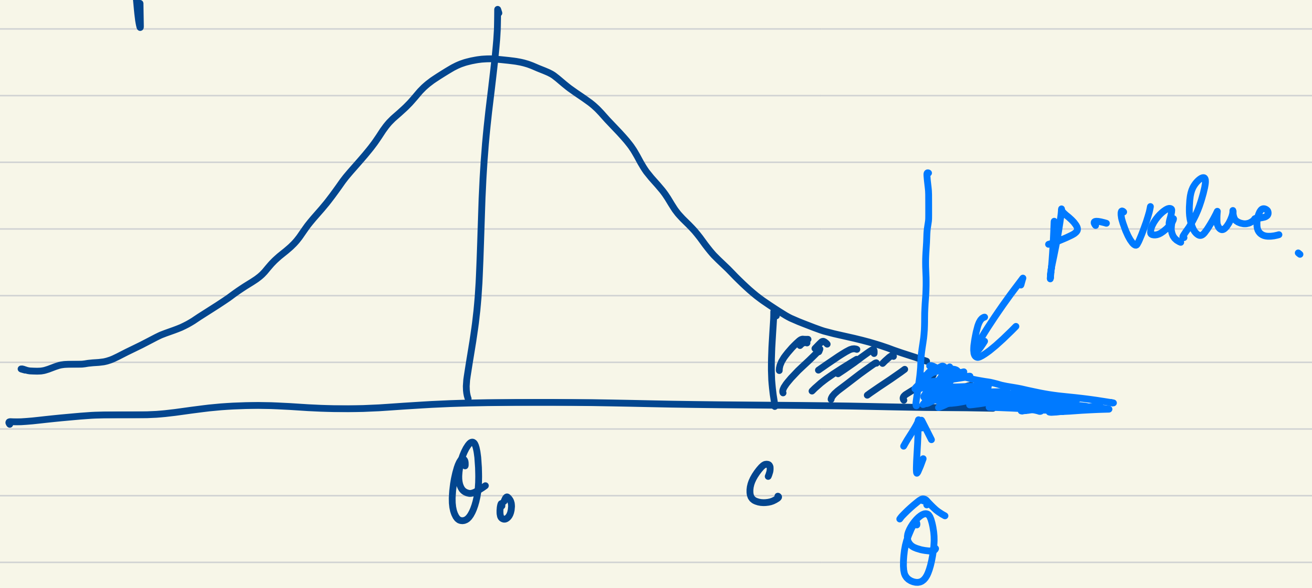
$H_0 : \theta = \theta_0 \quad , \quad H_1 : \theta \neq \theta_0$ ← Composite hypothesis.

Uniformly most powerful test. (UMP test).

→ Neyman-Pearson Lemma.

→ Likelihood ratio test is UMP.

The p-value.



p-value $\downarrow$ sample size. $\uparrow$

Three widely used tests (for large samples)

- The likelihood ratio test (LR)
- The Wald test (W)
- The Lagrangian multiplier test (LM).

$\rightarrow$ Chapter 14.

Linear hypotheses :

$H_0: R\beta = q$, $\underset{(J\times k)}{R}$ $\; J\Big\{\underbrace{\begin{bmatrix} & R & \end{bmatrix}}_{k} \overset{\beta}{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}\Big\}k = \overset{q}{\begin{bmatrix} \\ \end{bmatrix}}\Big\}J.$

$H_1: R\beta \neq q$

E.g. $\beta_2 = 0$: $R = [0\ 1\ 0 \cdots 0]\begin{bmatrix}\beta_1\\ \beta_2\\ \vdots \\ \beta_K\end{bmatrix} = 0$

$\uparrow$ for β_2

$\beta_3 + \beta_4 = 1$ $\rightarrow$ $[0\ 0\ 1\ 1\ 0 \cdots 0][\beta] = 1$

regression under H_0 : restricted regression $\to$ LM, LR

original regression : unrestricted regression $\to$ LR, W

The Wald test.

If $H_0: R\beta = q$ is true, using the unrestricted estimator $\mathbb{b}$, $R\mathbb{b} - q$ must be small.

Let $\mathbb{v} = R\mathbb{b} - q$, Under H_0.

$$\mathbb{v} = R\mathbb{b} - q = R\mathbb{b} - R\beta = R(\mathbb{b} - \beta).$$

$$E[\mathbb{b}|X]=\beta, \quad Var[\mathbb{b}|X]=\sigma^2(X'X)^{-1}.$$

Under A.6, $\mathbb{b}-\beta|X \sim N(\mathbb{0}, \sigma^2(X'X)^{-1})$.

$$\Rightarrow R(\mathbb{b}-\beta)|X \sim N(\mathbb{0}, \sigma^2 R(X'X)^{-1}R').$$

$$\Rightarrow \underset{\cdot}{W} = \underbrace{[R(\mathbb{b}-\beta)]'}_{x'} \cdot \underbrace{[\sigma^2 R(X'X)^{-1}R']^{-1}}_{\Sigma^{-1}} \cdot \underbrace{[R(\mathbb{b}-\beta)]}_{x}.$$

$$\sim \chi^2_J$$

$\Rightarrow$ Wald test: reject H_0 when W is large.

- The F-test.

Let σ be unknown. $s^2 = \frac{e'e}{n-k}$ is an unbiased estimator of σ^2.

Let $r = \frac{e'e}{\sigma^2}$, then it can be shown that

$$r \mid X \sim \chi^2_{(n-k)}.$$

w and r are independent, conditional on X.

$\Rightarrow$ Let $F = \frac{w/J}{r/(n-k)} \sim F(J, n-k)$.

$\Rightarrow$ reject H_0 if F is large.