高级计量经济学 2022-3-18

Friday, March 18, 2022 13:56

$$y = X\beta + \varepsilon$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = {}_n\underbrace{\begin{bmatrix} & x_i' & \\ & & \end{bmatrix}}_{k} \begin{bmatrix} \\ \\ \end{bmatrix}_{k \times 1} + \begin{bmatrix} \\ \\ \end{bmatrix}_{n \times 1}$$

k-th variable

y constant

No.	income	edu	age	sex	nationality	...	age^2
1						...	
2						...	
⋮							⋮
n							

$$X$$

$$y_i = \alpha + \beta x_i + \varepsilon_i, \quad i = 1, \dots, n$$

$$\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha + \beta x_1 \\ \alpha + \beta x_2 \\ \vdots \\ \alpha + \beta x_n \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & & & \\ 1 & \ddots & & \\ 1 & & \ddots & \\ \vdots & & & \ddots \\ 1 & & & \end{bmatrix}$$

Probability

(x, y)

$$f_X(x) = \int_y f(x, y)\, dy$$

$$E[X] = \int_x x f_X(x)\, dx = \int_x x \int_y f(x, y)\, dy\, dx$$

$$= \int_x \int_y x f(x, y)\, dy\, dx$$

$$Var[X] = \sigma_x^2 \Rightarrow \sigma_x$$

$$Cov[x,y] = E[xy] - E[x]E[y] = \sigma_{xy}.$$

$$r[x,y] = \rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}.$$

Independence : $f(x,y) = f_x(x) \cdot f_x(y)$.

$P[X=x] = 0$.

$\rho_{xy} = 0 \quad \Leftarrow \;\not\Rightarrow \quad$ x, y are independent.

uncorrelated.

$$f(-1,1) = f(0,0) = f(1,1) = \frac{1}{3}.$$

$E[x] = 0$, $E[y] = \frac{2}{3}$. $\sigma_{xy} = 0 - 0 \cdot \frac{2}{3} = 0$.

$\rho_{xy} = 0$. $f_x(1) = \frac{1}{3}$. $f_y(1) = \frac{2}{3}$

$$f(x,y) = \frac{1}{3}$$

$$(X,Y) \sim N(\mu, \Sigma)$$

Conditional distribution.

$$f(y|x) = \frac{f(x,y)}{f_x(x)} \underset{\text{independent}}{=} f_y(y) \qquad \frac{P[X,Y]}{P[X=x]}$$

Y=y X=x

$$f(x|y) = \frac{f(x,y)}{f_y(y)} = f_x(x)$$

$$E[y|x] = \int y \cdot f(y|x)\, dy$$

$$E[y|x] = \int_y y \, f(y|x) \, dy.$$

$$y = E[y|x] - E[y|x] + y$$
$$= E[y|x] + \underbrace{(y - E[y|x])}_{\varepsilon}$$

$$Var[y|x] = \int_y (y - E[y|x])^2 f(y|x) \, dy$$
$$= E[y^2|x] - (E[y|x])^2$$

Law of Iterated (Total) Expectation

$$E_x[E[y|x]] = E[y]$$

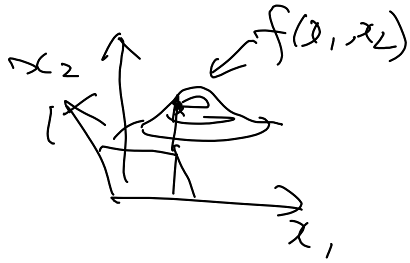
variable

Proof. $E[y] = \int_x \int_y y \, f(x,y) \, dy \, dx$

$$= \int_x \int_y y \, f(y|x) \cdot f_x(x) \, dy \, dx$$
$$= \int_x \left(\int_y y \cdot f(y|x) \, dy \right) f_x(x) dx$$
$$= \int_x \underbrace{E[y|x]}_{f(x)} f_x(x) dx$$
$$= E_x[E[y|x]]$$

Multivariate distribution

$$x = \begin{bmatrix} x_1 \\ \vdots \end{bmatrix} \leftarrow f(x)$$



random vector $\begin{bmatrix} x_k \end{bmatrix}$

· $E[\mathbf{x}] = \begin{bmatrix} E[x_1] \\ E[x_2] \\ \vdots \\ E[x_k] \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_k \end{bmatrix} = \mu$

· Covariance matrix

$$Var[\mathbf{x}] = \Sigma = E[(\mathbf{x}-\mu)(\mathbf{x}-\mu)']$$

σ_1^2

$$= \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1k} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{k1} & \sigma_{k2} & \cdots & \sigma_{kk} \end{bmatrix}$$

$$= E[\mathbf{x}\mathbf{x}'] - \mu\mu'$$

· Correlation matrix.

$$R = \begin{bmatrix} \frac{\sigma_{11}}{\sigma_1\sigma_1} & \frac{\sigma_{12}}{\sigma_1\sigma_2} & \cdots & \frac{\sigma_{1k}}{\sigma_1\sigma_k} \\ \vdots & & & \\ \frac{\sigma_{k1}}{\sigma_k\sigma_1} & \cdots & \cdots & \frac{\sigma_{kk}}{\sigma_k\sigma_k} \end{bmatrix} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1k} \\ \rho_{21} & 1 & \cdots & \rho_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{k1} & \rho_{k2} & \cdots & 1 \end{bmatrix}$$

Assumptions of linear regression model

A1. Linearity. $y = X\beta + \varepsilon$.

$X \qquad \beta \qquad\qquad X \qquad \beta$

$$\begin{bmatrix} (& (& (& \cdots & (\end{bmatrix}\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} = [x_1\ x_2 \cdots x_k] \cdot \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}$$

$x_1 \ x_2 \ \cdots \ x_k$

$\beta_1 \varepsilon \ \beta_2 \varepsilon^2$

$= x_1 \cdot \beta_1 + x_2 \cdot \beta_2 + \cdots + x_k \cdot \beta_k$

$$y = A L^{\alpha} K^{1-\alpha} e^{\varepsilon}$$

$$\ln y = \ln A + \underset{\beta_1}{\alpha} \cdot \ln L + \underset{\beta_2}{(1-\alpha)} \cdot \ln K + \varepsilon$$

$$\ln y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

A2. Full rank of X

满秩.

$$x_1 = \alpha_2 x_2 + \cdots + \alpha_k x_k$$

$$\begin{array}{cc} n & \left[\ () \ () \ () \ () \ () \ () \ \right] \\ & k \end{array}$$

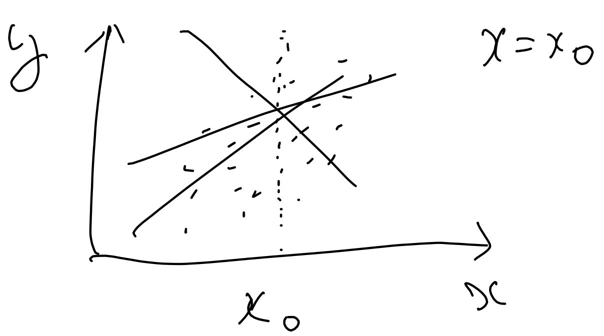
$n > k$, $\quad rank(X) = k$

$n \le k$, $\quad rank(X) = n \le k$

$n < k$, $\quad rank(X) < k$

$$X b = 0$$

$$y = \beta_1 + \beta_2 x + \varepsilon.$$



A3. Exogeneity $E[\varepsilon_i \mid X] = 0$ for all i

x_i

$E[\varepsilon_i \mid x_i] = 0$

- $E_X\left[E[\varepsilon_i \mid X]\right] = E[\varepsilon_i] = E[0] = 0$.

- $Cov[\varepsilon_i, X] = Cov_X\left[E[\varepsilon_i \mid X], X\right]$

Theorem B.2

$= Cov[0, X]$

$= 0$

- $E[y \mid X] = X\beta$ $\hat{\beta}$

$y = X\beta + \varepsilon$

$E[y \mid X] = E[X\beta + \varepsilon \mid X]$

$= E[X\beta \mid X] + E[\varepsilon \mid X]$

$= X\beta$