

高级计量经济学

Assignment 4

Assume OLS assumptions A1 through A6. We then have

$$\mathbf{y} \mid \mathbf{X} \sim N(\mathbf{X}\boldsymbol{\beta}, \sigma^2 \mathbf{I}_n).$$

The parameters in the regression model are the elements of vector $\boldsymbol{\beta}$ and the scalar σ^2 (here we take σ^2 as a single quantity, i.e., as $\sigma^2 = \gamma$, rather than as the square of σ). The likelihood function of $(\boldsymbol{\beta}, \sigma^2)$ is the conditional density function $f(\mathbf{y} \mid \mathbf{X})$, i.e.,

$$L(\boldsymbol{\beta}, \sigma^2) = f(\mathbf{y} \mid \mathbf{X}) = (2\pi\sigma^2)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma^2}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})\right].$$

The maximum likelihood (ML) estimator is the estimator that maximizes the (log) likelihood function. It is known that the ML estimator of $\boldsymbol{\beta}$ is the OLS estimator \mathbf{b} , and the ML estimator of σ^2 is

$$\frac{1}{n}\mathbf{e}'\mathbf{e} = \frac{\text{SSR}}{n} = \frac{n-k}{n}s^2,$$

where $s^2 = \frac{1}{n-k}\mathbf{e}'\mathbf{e}$.

Prove this result in the following two steps:

1. Find \mathbf{b} that maximizes the log likelihood function $\log L(\boldsymbol{\beta}, \sigma^2)$ for any given σ^2 .
2. Find $\hat{\sigma}^2$ that maximizes the log likelihood $\log L(\mathbf{b}, \sigma^2)$ where \mathbf{b} is obtained in step 1.