高级计量经济学
Assignment 1

Consider the regression model

$$y_i = \beta_1 x_{1,i} + \beta_2 x_{2,i} + \varepsilon_i$$

where $x_{k,i}$ denotes the ith observation of the kth variable, $k \in \{1,2\}$. We assume that $\mathbf{x}_1 \neq \mathbf{0}$, $\mathbf{x}_2 \neq \mathbf{0}$, and $\mathbf{x}_2 \neq c\,\mathbf{x}_1$ for any real number c. Let the residual e_i be $e_i = y_i - (b_1 x_{1,i} + b_2 x_{2,i})$. Find the least squares fitted values (b_1, b_2) of parameters (β_1, β_2) following the steps below.

1. Let $S = \sum e_i^2$. Express the least squares problem as a minimization problem of S.
2. Find $\dfrac{\partial S}{\partial b_1}$, $\dfrac{\partial S}{\partial b_2}$, $\dfrac{\partial^2 S}{\partial b_1^2}$, $\dfrac{\partial^2 S}{\partial b_2^2}$, and $\dfrac{\partial^2 S}{\partial b_1 \partial b_2}$.
3. Derive the first and second order conditions.
4. Show that the second order condition is satisfied. You can use the Cauchy-Schwarz inequality if necessary.
5. Find the least squares solution by solving the first order condition.
6. Calculate $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ for this model, and confirm that the result coincides with the one you obtained in the previous step.