Econometrics 1 *Applied Econometrics with R*

Lecture 10: Binary Dependent Variable

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Regression with a Binary Dependent Variable

The HMDA data

- HMDA (Home Mortgage Disclosure Act) data are data that related to mortgage applications filed in the Boston area in 1990.

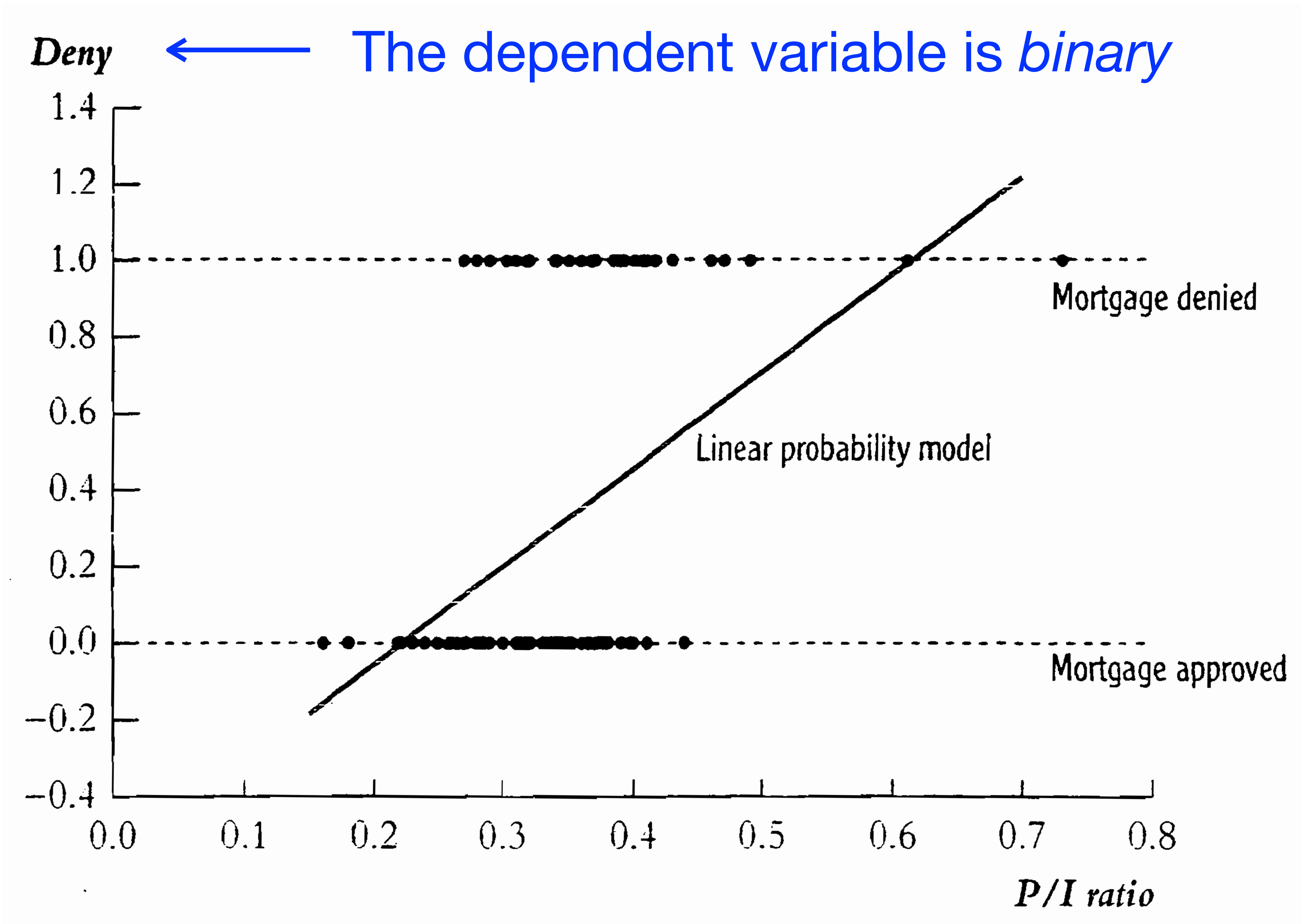
- This data is contained in the AER package.

```
> library("AER")
> data("HMDA")
```

The HMDA data

deny	Factor. Whether the mortgage was denied. Yes or no.
pirat	P/I ratio. Ratio of total monthly debt payments to total monthly income.
hirat	Ratio of monthly housing expenses to total monthly income.
lvrat	Ratio of size of loan to assessed value of property.
chist	Factor. Historical consumer credit score. 6 levels.
mhist	Factor. Historical mortgage credit score. 4 levels.
phist	Factor. Historical public bad credit record. Yes or no.
unemp	1989 Massachusetts unemployment rate in applicant's industry.
selfemp	Factor. Whether the individual is self-employed. Yes or no.
insurance	Factor. Whether the individual was denied mortgage insurance. Yes or no.
condomin	Factor. Whether the unit is a condominium. Yes or no.
afam	Factor. Whether the individual is African-American. Yes or no.
single	Factor. Whether the individual is single. Yes or no.
hschool	Factor. Whether the individual has a high-school diploma. Yes or no.

What determines whether a mortgage application is denied?



Regression with a binary dependent variable

- The population regression function of a lineal model

$$
\begin{aligned}
&\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_m x_m \\
&= \mathrm{E}(Y \mid X_{1i} = x_1, X_{2i} = x_2, \ldots, X_{mi} = x_m)
\end{aligned}
$$

- Regression with a binary variable

$$
\begin{aligned}
\mathrm{E}(Y) &= 0 \times \Pr(Y = 0) + 1 \times \Pr(Y = 1) \\
&= \Pr(Y = 1)
\end{aligned}
$$

$$
\Rightarrow \quad \mathrm{E}(Y \mid X_1, \ldots, X_m) = \Pr(Y = 1 \mid X_1, \ldots, X_m)
$$

The linear probability model

- The linear probability model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_m X_{mi} + u_i$$

$$\begin{aligned} \Rightarrow \quad & \Pr(Y = 1 \mid X_1, \ldots, X_m) \\ & = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_m X_m \end{aligned}$$

- The regression coefficient β_1 is the change in the probability $Y = 1$ associated with a unit change in X_1, holding constant the other regressors, and so forth for $\beta_2, \ldots, \beta_m$

- The regression coefficients can be estimated by OLS.

A note on the “factor” class in R

- Run the following codes

```
> a <- c(0, 1, 1, 0, 0)
> b <- as.factor(a)
```

```
> b
[1] 0 1 1 0 0
Levels: 0 1
```

Values	
a	num [1:5] 0 1 1 0 0
b	Factor w/ 2 levels "0","1": 1 2 2 1 1

A note on the “factor” class in R

- Translating a factor into a numeric vector

  A “bad” way:

```
> c <- as.numeric(b)
```

  Some “good” ways:

```
> d <- as.numeric(b) - 1
> f <- as.numeric(levels(b))[b]
> g <- as.numeric(as.character(b))
```

Practice

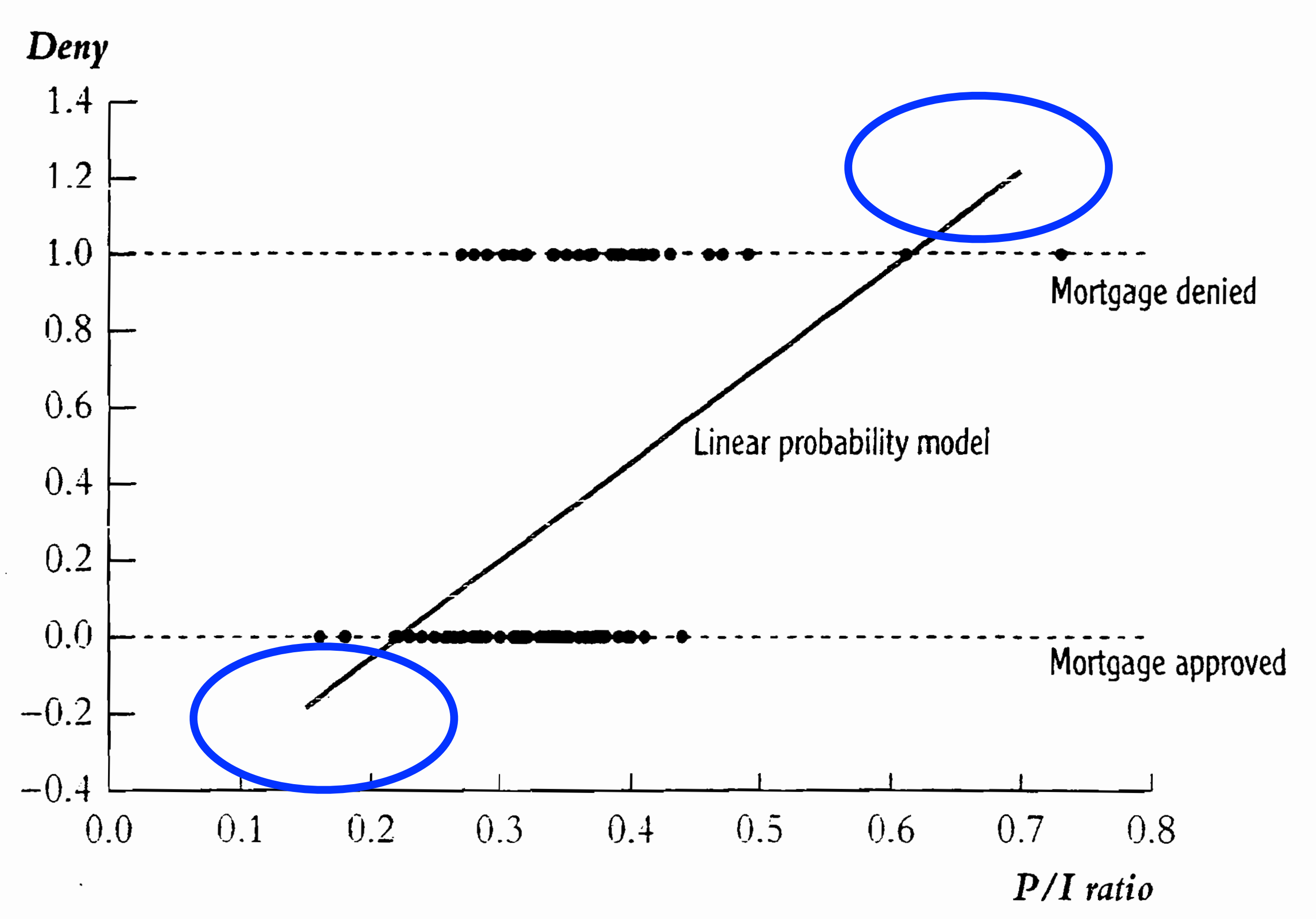
- Import the HMDA data.
- Take `deny` as the dependent variable.
- `deny` is a "factor". Translate it into a numeric vector with values 0 and 1.
- Run the following regressions with `lm()`.

$$\text{deny}_i = \beta_0 + \beta_1 \text{pirat}_i + u_i$$

$$\text{deny}_i = \beta_0 + \beta_1 \text{pirat}_i + \beta_2 \text{afam}_i + u_i$$

Shortcomings of the linear probability model

- A probability must be between 0 and 1!

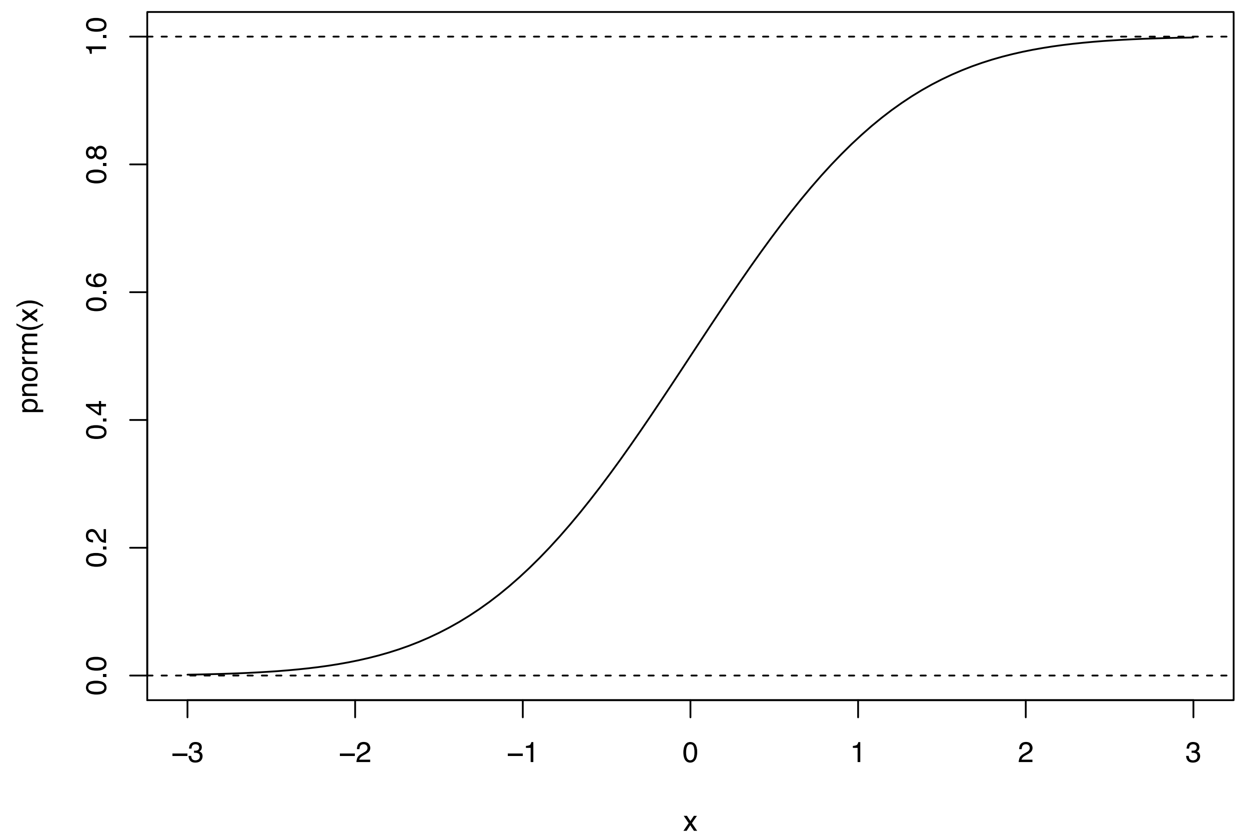


- Nonlinear models are needed.

The probit regression

- Recall the c.d.f. of the standard normal distribution

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{s^2}{2}\Big) ds$$



- Probit regression

$$\Pr(Y = 1 \mid X_1, \ldots, X_m) = \Phi(\beta_0 + \beta_1 X_1 + \cdots + \beta_m X_m)$$

The probit regression

- To predict the probability of $Y = 1$

    1. Calculate the value $z = \beta_0 + \beta_1 X_1 + \cdots + \beta_m X_m$

    2. Calculate the cumulative probability at z

- The regression coefficients can be estimated using nonlinear OLS method, or maximum likelihood method.

- The maximum likelihood estimator has a smaller variance than the nonlinear OLS estimator.

The `glm()` command in R

- GLM refers to “generalized linear model”
- The GLM is a method that allows the distribution of errors being non normal.
- `glm()` uses the “iteratively reweighed least squares” method to find the maximum likelihood estimates of a generalized linear model.
- The probit model is an example of generalized linear model.

Practice

- Take `deny` as the dependent variable and `pirat` (P/I ratio) as the independent variable.

- Use `glm()` to estimate the coefficients in

$$\Pr(\text{deny} = 1 \mid \text{pirat}) = \Phi(\beta_0 + \beta_1 \text{pirat})$$

e.g.,

factor

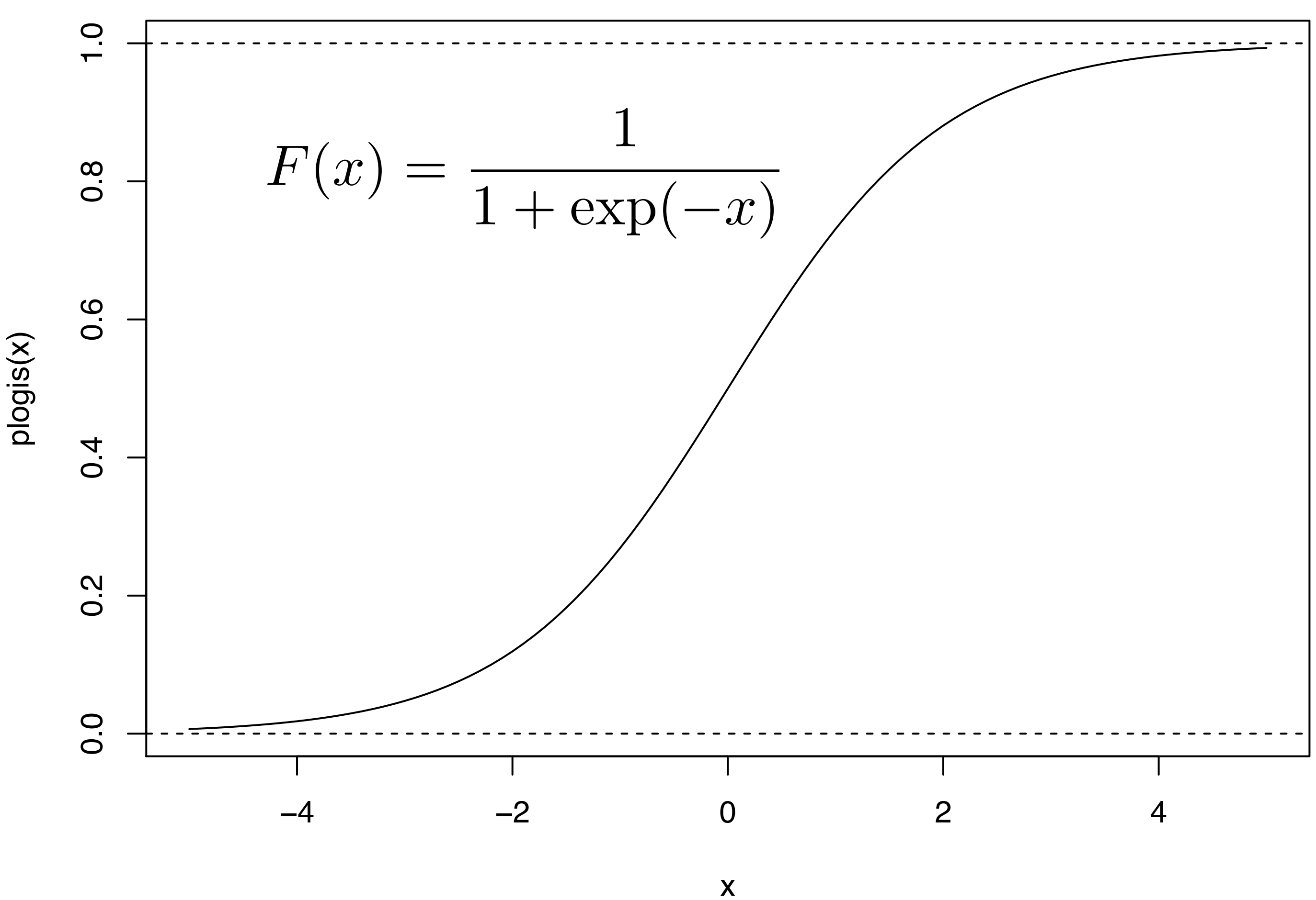
```
> glm(deny ~ pirat, family =
binomial(link = "probit"))
```

- For more information, read the help of `glm` and `family`

The logit regression

- Logit regression

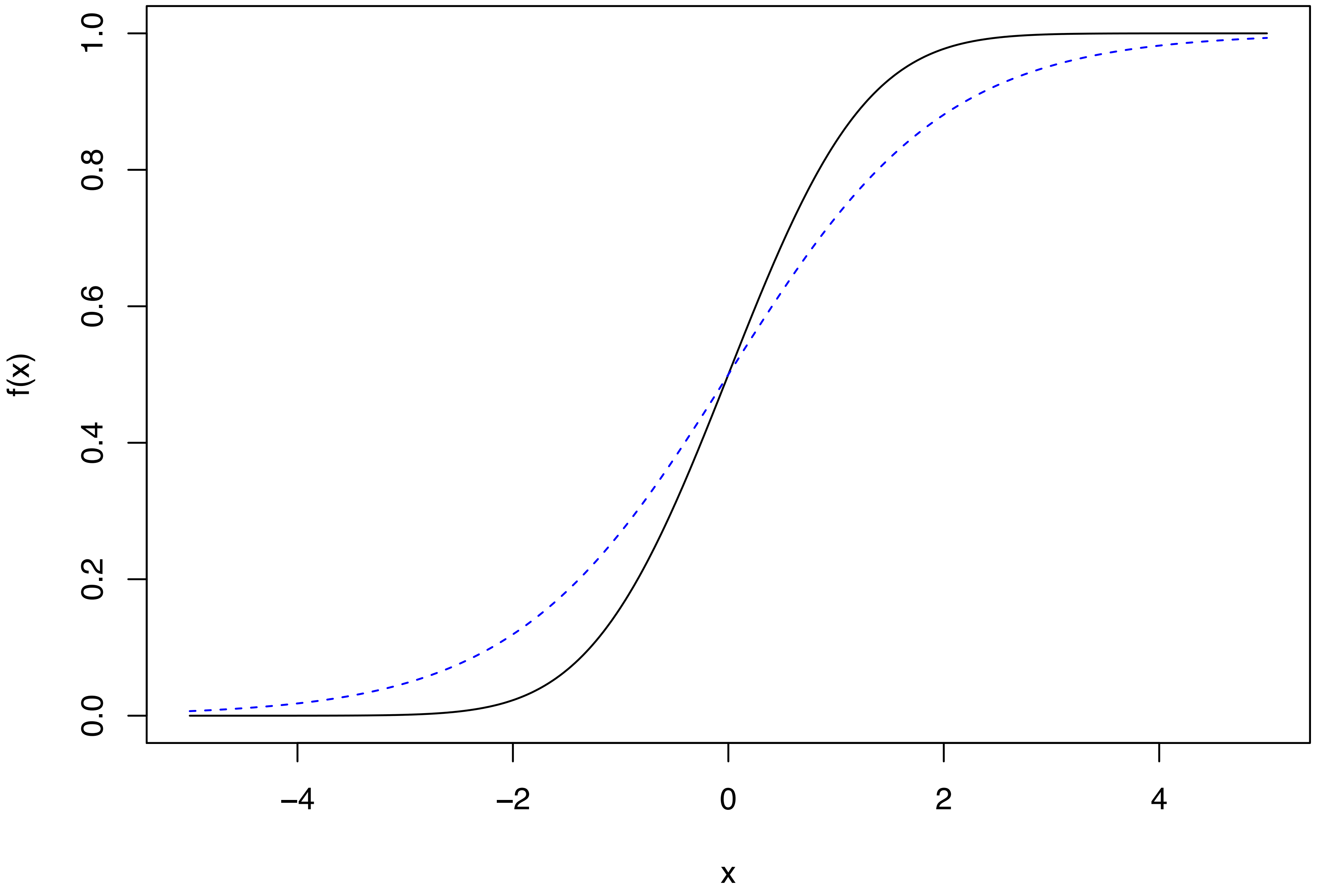
$$\Pr(Y = 1 \mid X_1, \ldots, X_m)$$
$$= \frac{1}{1 + \exp\big(-(\beta_0 + \beta_1 X_1 + \cdots + \beta_m X_m)\big)}$$



“logit” means the logistic function $F(x)$

Practice

- Draw a logistic function curve and a standard normal distribution function curve on the same plot from $x = -5$ to $x = 5$.



Practice

- Take `deny` as the dependent variable.

- Try the following regressions

```
> glm(deny ~ pirat, family =
binomial(link = "probit"))
> glm(deny ~ pirat, family =
binomial(link = “logit"))
```

- Compare the results.

Regression results

- The probit model

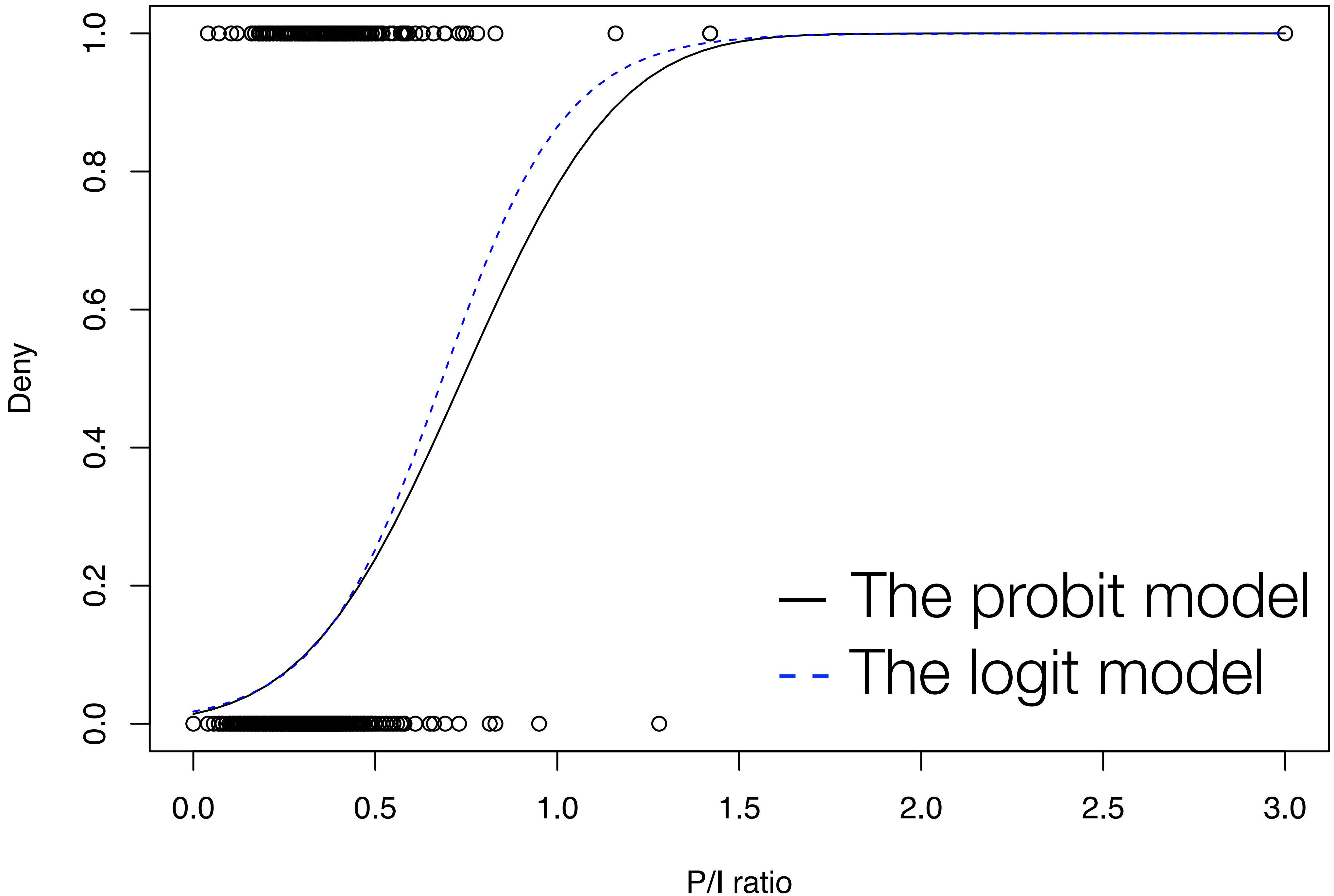
$$\beta_0 = \underset{(0.14)}{-2.19}, \quad \beta_1 = \underset{(0.39)}{2.97}$$

- The logit model

$$\beta_0 = \underset{(0.27)}{-4.03}, \quad \beta_1 = \underset{(0.73)}{5.88}$$

Practice

- Reproduce the following figure.



Practice

- Take the subset of the HMDA data such that the P/I ratio (`pirat`) is between 0 and 1.

- Randomly choose 200 observations from this subset, and run the probit and logit regressions with a single regressor `pirat`. Compare the results by plotting the data and the population regression lines.

- Repeat the above 10 times. Can you tell the difference between the probit and the logit models?

Goodness of fit

- McFadden's pseudo-R^2

$$\text{pseudo-}R^2 = 1 - \frac{\ell(\hat{\beta})}{\ell(\bar{y})}$$

  where $\ell(\hat{\beta})$ is the log-likelihood function of the fitted model, and $\ell(\bar{y})$ is the log-likelihood function of the model containing only a constant term.

- Use `logLik()` to obtain the log-likelihood function of an `glm` object.

Practice

- Example

```
> fprobit1 <- glm(deny ~ pirat, family =
binomial(link = "probit"))

> fprobit0 <- glm(deny ~ 1, family =
binomial(link = “probit"))

> pR2probit1 <- 1 - as.numeric(
     logLik(fprobit1) / logLik(fprobit0))
```

Practice

- Use `pirat` and `afam` as independent variables, i.e.

$$\Pr(\text{deny} = 1 \mid \text{pirat}, \text{afam}) = \Phi(\beta_0 + \beta_1 \text{pirat} + \beta_2 \text{afam})$$

- Run this probit regression. Compare the results with the probit regression with single regressor `pirat`. Does the variable `afam` affects deny probability? Does this model have a better fit than the single regressor model?

- Try the same with logit regression.

References

1. Stock, J. H. and Watson, M. M., *Introduction to Econometrics*, 3rd Edition, Pearson, 2012.

2. Kleiber, C. and Zeileis, A., *Applied Econometrics with R*, Springer, 2008.