高级计量经济学
Assignment 2

1. Show that the OLS residuals of the following regression models are numerically identical.

   (1) $\mathbf{y} = \mathbf{X}_1\beta_1 + \mathbf{X}_2\beta_2 + \varepsilon,$

   (2) $\mathbf{y}^* = \mathbf{X}_2^*\beta_2^* + \varepsilon^*,$

   where $\mathbf{X}_2^* = \mathbf{M}_1\mathbf{X}_2$, $\mathbf{y}^* = \mathbf{M}_1\mathbf{y}$, and $\mathbf{M}_1 = \mathbf{I} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'$.

2. Consider random variables x and y whose joint density function is $f(x, y)$. Prove the following properties.

   (1) $\mathrm{E}[x\,y] = \mathrm{E}_x\big[x\mathrm{E}[y\,|\,x]\big].$

   (2) $\mathrm{Cov}[x, y] = \mathrm{Cov}_x\big[x, \mathrm{E}[x\,|\,y]\big] = \int_x \big(x - \mathrm{E}[x]\big)\,\mathrm{E}[y\,|\,x]\,f_x(x)\,dx.$