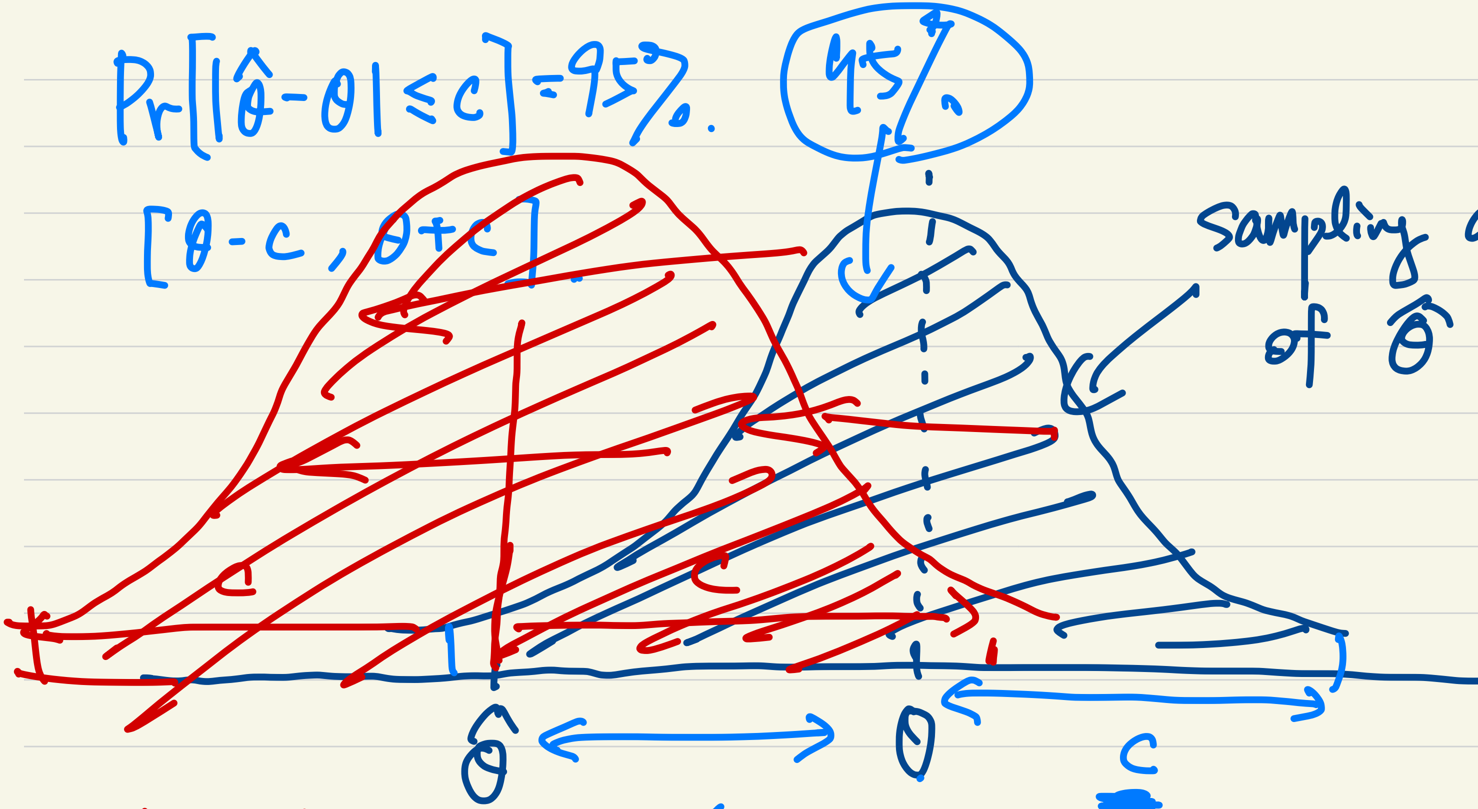
Econometrics. 2022.5.20.

Interval Estimation.

$$\underset{\text{true parameter value.}}{\theta} = \underset{\text{estimate}}{\hat{\theta}} + \text{sampling error}$$

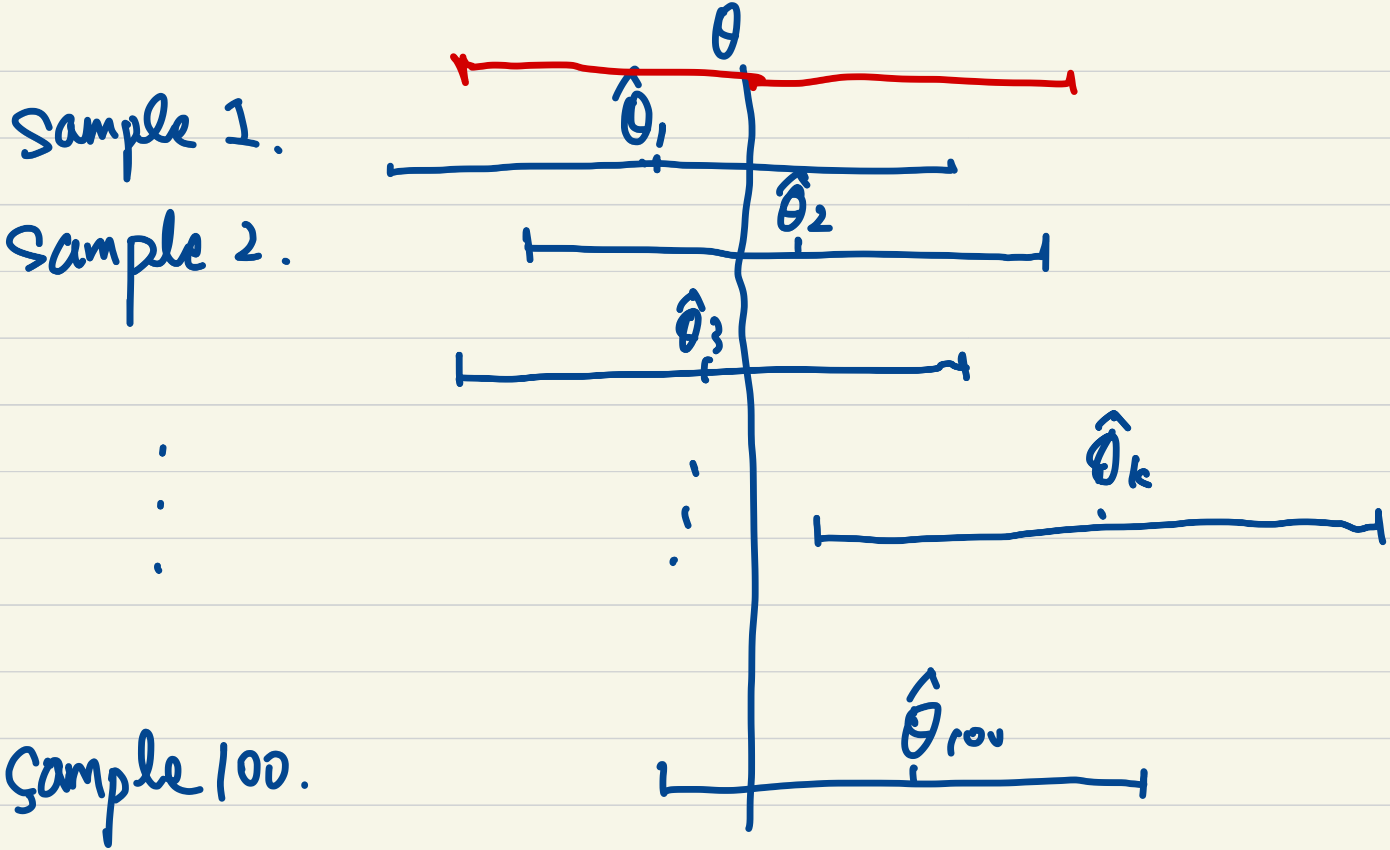
sampling distribution.

$$\Pr\left[|\hat{\theta} - \theta| \le c\right] = 95\%.$$



$$E[\hat{\theta}] = \theta$$

$[\hat{\theta} - c, \hat{\theta} + c]$ ← confidence interval.



θ
Sample 1.
θ̂₁
Sample 2.
θ̂₂
θ̂₃
θ̂ₖ
Sample 100.
θ̂₁₀₀

Some Important Probability Distributions. (B.4).

· Normal Distribution.

$$x \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right].$$

$$E[x] = \mu, \quad Var[x] = \sigma^2$$

$$\Rightarrow z = \frac{x-\mu}{\sigma} \sim N(0, 1)$$

standard normal dist.

· χ^2 - distribution (Chi - Squared)

If $x_1, \cdots, x_n$ are independent, normally distributed variables with $\mu = 0$, and $\sigma^2 = 1$, then

$$z = \sum_{i=1}^{n} x_i^2 \sim \chi^2[n] \text{ or } \chi^2_n$$

Chi-squared distribution with degrees of freedom n.

$E[z] = n$, $Var[z] = 2n$.

$\Rightarrow$ If $z_1 \sim \chi^2_n$ and $z_2 \sim \chi^2_m$, z_1 and z_2 are independent, then $z_1 + z_2 \sim \chi^2_{n+m}$.

Degree of freedom.

$$\begin{cases} \underline{x_1 + x_2 = 2} \\ x_1 - x_2 = 1 \end{cases} \Rightarrow \text{Let } x_2 = c,\ x_1 = 2-c$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2-c \\ \underline{c} \end{pmatrix}.$$

· The F-distribution.

If $y_1 \sim \chi^2_{n_1}$, $y_2 \sim \chi^2_{n_2}$, y_1 and y_2 are independent,

then

$$z = \frac{y_1/n_1}{y_2/n_2} \sim F_{n_1, n_2} \text{ or } F[n_1, n_2].$$

- The t-distribution

If $x \sim N(0,1)$, $y \sim \chi^2_n$, x and y are independent, then

$$Z = \frac{x}{\sqrt{y/n}} \sim t_n \text{ or } t[n].$$

$\Rightarrow$ If $Z \sim t_n$, then $Z^2 \sim F_{1,n}$.

· Multivariate normal distribution.

$$\underset{\text{mean}}{x \sim N(\mathbf{0},} \underset{\text{covariance matrix.}}{\Sigma)}$$

$\rightarrow y = x + \mu \Rightarrow y \sim N(\mu, \Sigma)$.

$\Rightarrow a'y \sim N(a'\mu, a'\Sigma a)$.

$\rightarrow z \sim N(\mathbf{0}, I) \Leftrightarrow z_1, \cdots, z_n \sim IN(0, 1)$.

→ The m-vector $x \sim N(0, \Sigma)$, then

$$x'\Sigma^{-1}x \sim \chi^2_m.$$

→ If A is ~~$(n \times n)$~~ and z is n-vector.

$z \sim N(0, I)$. then

$$z'Az \sim \chi^2_r.$$

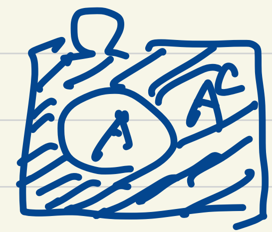
$\boxed{\text{Correction}}$

$(n \times n)$ and $\operatorname{rank}(A) = r$.

Hypothesis Testing.

- Hypothesis : a statement about a population parameter.
- H_0 (null hypothesis)

  H_1 (alternative ") $\quad H_1 = H_0^c$



- Hypothesis test :
  - For which sample values (statistic.) to reject H_0 and accept H_1. ← rejection region
  - For which " to accept H_0. (not to reject). ← acceptance region.

Error probabilities.

		Decision: accept H_0	Decision: accept H_1
Truth	H_0	correct	type I error
	H_1	type II error	correct.

- The level of a test: $\alpha = Pr(\text{type I error})$.

(the smaller the better) $= Pr(\text{reject } H_0 \mid H_0 \text{ is true})$

- $Pr(\text{type II error}) = Pr(\text{accept } H_0 \mid H_1 \text{ is true}) = \beta$.
- The power of a test: $1-\beta = Pr(\text{reject } H_0 \mid H_1 \text{ is true})$

(the greater the better).

A good test : a test with low α and β.

$\Rightarrow$ Control α and minimize β.