

# 高级计量经济学

## Assignment 2 (v.2)

1. Show that the OLS residuals of the following regression models are numerically identical.

$$(1) \mathbf{y} = \mathbf{X}_1\beta_1 + \mathbf{X}_2\beta_2 + \varepsilon,$$

$$(2) \mathbf{y}^* = \mathbf{X}_2^*\beta_2^* + \varepsilon^*,$$

where  $\mathbf{X}_2^* = \mathbf{M}_1\mathbf{X}_2$ ,  $\mathbf{y}^* = \mathbf{M}_1\mathbf{y}$ , and  $\mathbf{M}_1 = \mathbf{I} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'$ .

2. Consider random variables  $x$  and  $y$  whose joint density function is  $f(x, y)$ . Prove the following properties.

$$(1) E[xy] = E_x[xE[y|x]].$$

$$(2) \text{Cov}[x, y] = \text{Cov}_x[x, E[y|x]] = \int_x (x - E[x]) E[y|x] f_x(x) dx.$$