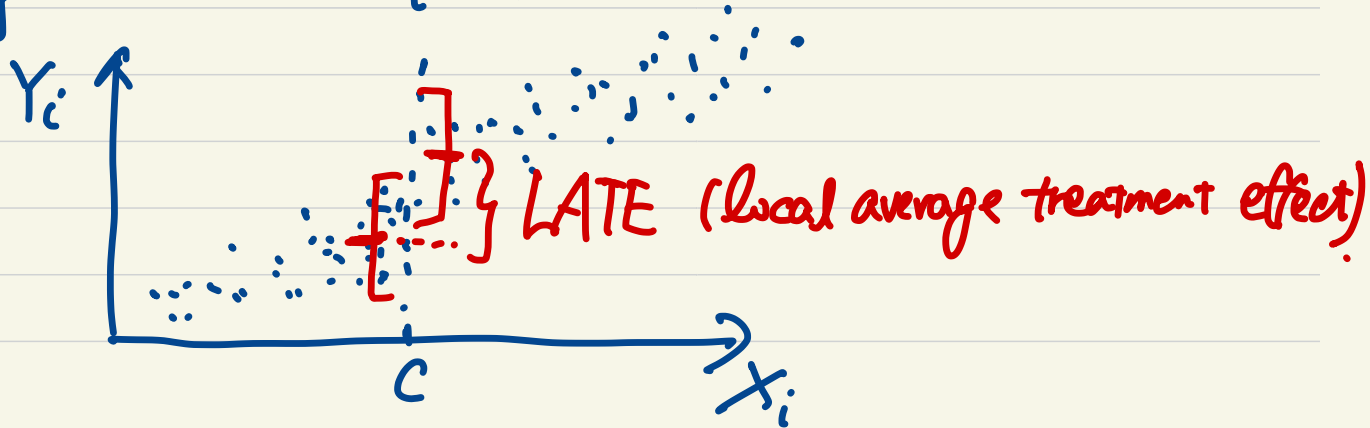# Regression Discontinuity Design (RDD).

Outcome : Y_i

Treatment : A_i

Running variable : X_i
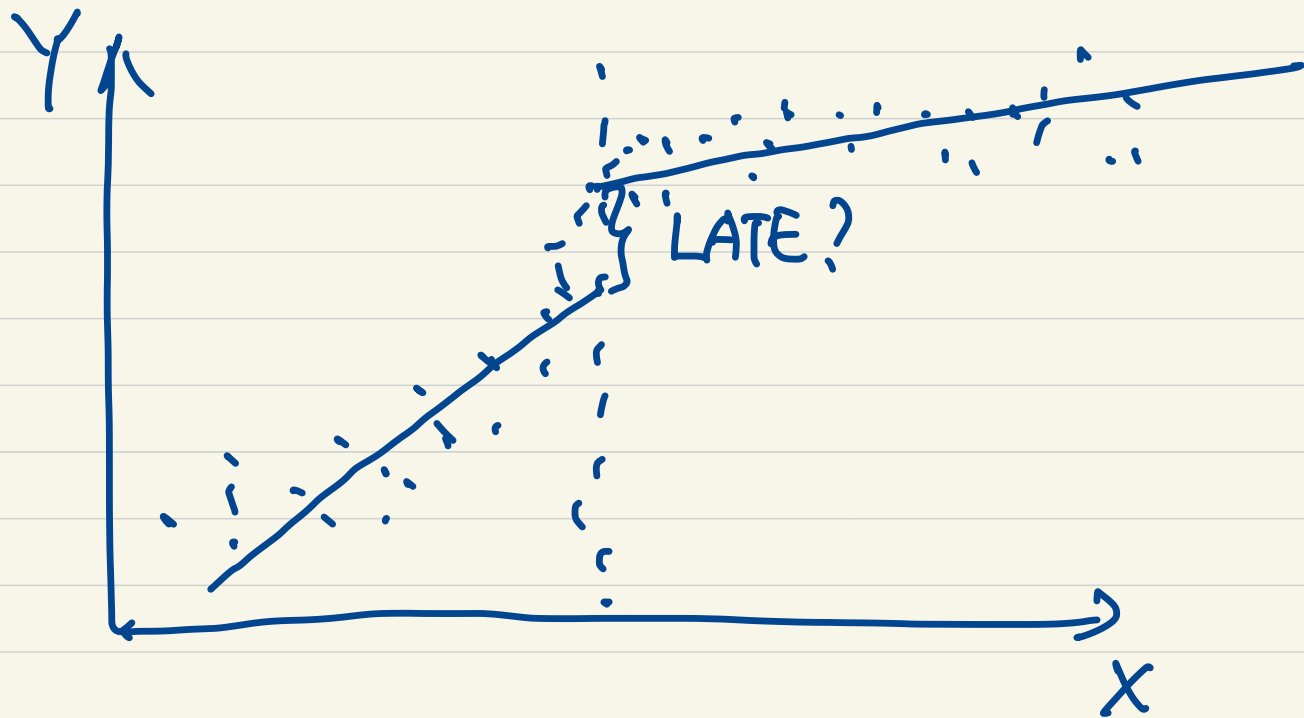
$$\Rightarrow \quad A_i = \begin{cases} 1 & \text{if } X_i \geq C \\ 0 & \text{if } X_i < C. \end{cases}$$

LATE (local average treatment effect)

$$LATE = E[Y_i' \mid X_i \to C^+] - E[Y_i^0 \mid X_i \to C^-]$$

$\longrightarrow$ use regression

LATE ?

- Synthetic Control.

  → Comparative case study.

  Units : ① 2 3 $\cdots$ J+1

  ① ↑ treatment

  2 3 ... J+1 → potential controls (donor pool).

Outcome : $Y_{it} = A_{it} Y_{it}^1 + (1 - A_{it}) Y_{it}^0$

Time of treatment : T_0.

Treatment status : $A_{it} = \begin{cases} 1 & \text{if } i=1 \text{ and } t = T_0+1, \cdots T. \\ 0 & \text{otherwise}. \end{cases}$

Causal effect : $\{ Y_{1t} - Y_{1t}^0 : t = T_0+1, \cdots, T \}$

$\underset{\underset{Y_{1t}^1}{\shortparallel}}{\underbrace{\qquad}}$ $\underset{\underset{?}{\shortparallel}}{\underline{\underline{\qquad}}}$

Covariates : $Z_{i1}, \cdots, Z_{ir}$.

Weights : $w_2, \cdots, w_{J+1}$

Minimize the diferences between

$$
\begin{array}{lll}
Y_{11} & \text{and} & \sum_j w_j Y_{j1} \\
\vdots & \vdots & \vdots \\
Y_{1T_0} & \text{and} & \sum_j w_j Y_{jT_0} \\
Z_{11} & \text{and} & \sum_j w_j Z_{j1} \\
\vdots & & \vdots \\
Z_{1r} & \text{and} & \sum_j w_j Z_{jr}
\end{array}
\Bigg\} \Rightarrow w_2^*, \cdots, w_{J+1}^*.
$$

$$\Rightarrow \left\{ \sum_{j=1}^{J+1} w_j^* Y_{jt} : t = 1, \cdots, T \right\} \text{ is the synthetic}$$

control of Y_{1t}.

$$\Rightarrow Y_{1t}^0 \text{ is estimated by } \left\{ \sum_{j=2}^{J+1} w_j^* Y_{jt}^0 : t = T_0+1, \cdots, T \right\}$$

can be observed .

Y

Synthetic
California

California

year