

Team Control Number

**12760**

Problem Chosen

**B****2022****HiMCM****Summary Sheet**

## **Effects of CO<sub>2</sub> on Surface and Ocean Temperatures**

### **Summary**

Since the industrial society, the increase of the Earth's surface temperature and a large amount of Earth's carbon dioxide content has attracted human attention. There are three broad views on the relationship between these two factors. Based on the data given in the question, we have built the linear and quadratic regression model, the moving average model, and the time series ARIMA model to find the answer.

Then, our team proposed three models to describe the past CO<sub>2</sub> concentration and predict future CO<sub>2</sub> concentration. The three models are the linear model, the quadratic regression model, and the moving average model. The specific fitting and prediction comparison results are in Figure 4.12. According to the prediction effect of the validation set and the comparison of the R-squared and MSE, the MA model is optimal and the most accurate. The CO<sub>2</sub> concentration in 2100 is predicted to be 534.8825, 610.0136, and 572.5688 with the three models we built. The CO<sub>2</sub> concentration in 2050 is 454.1807, 488.8664, and 472.1497, respectively, none of which reaches 685 ppm predicted by the related organizations. The MA model predicts that the CO<sub>2</sub> concentration can reach 685ppm by 2125. The other two models will take longer to reach 685ppm. About the question of which model is more accurate, our team believes that the prediction in 10 years can be analogous to the validation set, i.e., the ARIMA model is the best. However, it may be difficult to evaluate which of these three models is better over a longer period of time due to many unexpected factors or no real data to verify.

For question 2, we believe there is an overall linear relationship between temperature and CO<sub>2</sub>. Our autoregressive model (5.3) has a good explanatory ability for the coefficient of determination. The model has reliable predictions for relatively long periods if the parameters are continuously updated. According to the prediction of the model (10), the change in mean land-ocean temperature reaches 1.25°C, 1.50°C, and 2°C in 2032, 2041, and 2056, respectively.

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## 1. Introduction

With the development of society and the economy, carbon dioxide (CO<sub>2</sub>) in the air has always been on the increase. Before the industrial revolution, CO<sub>2</sub> in the air had remained at around 280 ppm. In contrast, in March 2004, CO<sub>2</sub> in the air reached 337.7 ppm, the largest 10-year average CO<sub>2</sub> concentration increase to date.

According to scientists, the monthly mean CO<sub>2</sub> concentration level peaked at 421 ppm in May 2022. By 2050, the CO<sub>2</sub> level will reach 685 ppm.

In this case, it is particularly important to **study current and predict future CO<sub>2</sub> concentration levels**. Our team needs to analyze the current CO<sub>2</sub> concentration level data and build mathematical models to describe past results and predict future results. We also need to compare the predictions of our team's model with those of scientists to see if there is an agreement between data-based and empirical predictions. Most importantly, we do not limit our models to one or two fixed models and draw hasty conclusions. Instead, we propose multiple models for comparison, aiming to achieve the goal of moving away from subjective judgments and model contingency limits to obtain relatively credible predictions and comparative results.

Another important task is to **analyze the land-ocean temperature and CO<sub>2</sub> data** and to test the validity of some scientists' statements that global warming and CO<sub>2</sub> concentrations are related. We will build models to study the relationship between temperature and CO<sub>2</sub> concentration levels. Finally, the models will be used to predict future temperatures and CO<sub>2</sub> concentration levels. We will study the relationship between the two models and predict future trends.

## 2. Assumptions

### *Assumption 1:*

Assume that the CO<sub>2</sub> data and temperature difference data given in the question are annual averages for the world.

### *Justification:*

Despite the abundance of global CO<sub>2</sub> and surface temperature monitoring instruments, the observed data are localized, so we assume here that the data are global averages.

### *Assumption 2:*

Atmospheric CO<sub>2</sub> is uniformly distributed

***Justification:***

In fact, the atmospheric CO<sub>2</sub> cannot be uniformly distributed, but based on the previous data assumptions, we make this assumption to facilitate the analysis of the data.

### 3. Preparation

#### 3.1 Description of Symbols

**Table 3.1 Symbol Description**

Symbols	Meaning
$X = \{x_1, x_2, \dots, x_n\}$	X is the measured value of the annual CO <sub>2</sub> concentration vector in the air (unit/ppm) $x_i$ is the CO <sub>2</sub> concentration in the year i after the year 1959
$\hat{X} = \{\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n\}$	$\hat{X}$ is the predicted value of the vector of CO <sub>2</sub> concentration in the air in the year (unit/ppm) $\hat{x}_i$ is the predicted value of the CO <sub>2</sub> concentration in the year i
$\bar{X} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n\}$	Base difference vectors for CO <sub>2</sub> (1959-2021) *Base difference: land and ocean data compared to the temperature mean of the base period 1951-1980
$T = \{T_1, T_2, \dots, T_n\}$	T is the land-ocean temperature base difference
$\Delta_j x_i$	Average CO <sub>2</sub> concentration from the year i-j to the year i
$i$	Year i

#### 3.2 Preparation of Mathematical Models

##### 3.2.1 Linear and Quadratic Regression Models

The quadratic regression model is statistically non-linear [1].

The formula is:

$$y = \alpha x^2 + \beta x + \gamma + \varepsilon \quad (3.1)$$

where  $y$  is the dependent variable,  $x$  is the independent variable, and  $x$ ,  $y$  are random variables.  $\alpha$ ,  $\beta$  are the estimated parameters.  $\varepsilon$  is a random error and is required to follow a normal distribution.

When  $\alpha = 0$  in (1), (1) is a one-dimensional linear regression model.

### 3.2.2 Moving Average Method Model [2]

The moving average method is a commonly used time series forecasting method that has good practical value due to its simplicity.

#### 1. One-time Moving Average Method

Let the observation series be  $x_1, \dots, x_T$  and take the number of terms of the moving average  $N < T$ . The formula for calculating the one-time moving average is

$$M_t^{(1)}(N) = \frac{1}{N} (x_t + x_{t-1} + \dots + x_{t-N+1}) = \frac{1}{N} \sum_{i=0}^{N-1} x_{t-i} \quad (3.2)$$

The forecast value for period  $t+1$  is  $\hat{y}_{t+1} = M_t^{(1)}(N)$ , and its forecast standard error is

$$S = \sqrt{\frac{\sum_{t=N+1}^T (\hat{x}_t - x_t)^2}{T - N}}. \quad (3.3)$$

#### 2. Secondary Moving Average Method

When the underlying trend of the predictor variables changes, the one-time moving average method cannot quickly accommodate such changes. When the change in the time series is linear, the lagged deviation of the primary moving average method makes the predicted value low and does not allow reasonable extrapolation of the trend. In this case, the quadratic moving average method is better, and its calculation formula is

$$M_t^{(2)} = \frac{1}{N} (M_t^{(1)} + \dots + M_{t-N+1}^{(1)}) = M_{t-1}^{(2)} + \frac{1}{N} (M_t^{(1)} - M_{t-N}^{(1)}) \quad (3.4)$$

In this case, when the series has both linear trend and cyclical fluctuations, the trend-moving average method can be used to build a forecasting model

$$\hat{y}_{T+m} = a_T + b_T m, \quad m = 1, 2, \dots, \quad (3.5)$$

$$\text{where } a_T = 2M_T^{(1)} - M_T^{(2)}, \quad b_T = \frac{2}{N-1} (M_T^{(1)} - M_T^{(2)}) .$$

### 3.2.3 Time Series ARIMA Model [2]

A time series model is a mathematical model that observes and measures a set of variables  $x_t$ , arranges them at the moment  $t_1, t_2, \dots, t_n$  in temporal order, and explains the interrelationships between variables. The data series of predictors over time is considered a random series, and the dependence that this set of random variables has reflects the continuity of the original data in time. The ARMA model is the most commonly used model to fit a stationary series at present. The specific equation is as follows.

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \dots + \beta_p y_{t-p} + \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2} + \dots + \alpha_q \varepsilon_{t-q} \quad (2)$$

ARIMA model, also known as the autoregressive summation moving average model, can be regarded as a stationary series when the time series itself is not stationary if its increment, also called primary difference, is stable around zero. In practical problems, most of the non-stationary series can become stationary after one or more differences, and then the corresponding models can be built.

### 3.2.4 Time Series ARMAX Model [2]

ARMAX (Autoregressive Moving Average with Extra Input): Autoregressive moving average model with extra inputs (which can be interpreted as external disturbance terms), the model structure is

$$\begin{aligned} y(t) + a_1 y(t-1) + \dots + a_{n_a} y(t-n_a) = \\ b_1 u(t-n_k) + \dots + b_{n_b} u(t-n_k-n_b+1) + \\ c_1 e(t-1) + \dots + c_{n_c} e(t-n_c) + e(t) \end{aligned} \quad (3.7)$$

where  $y(t)$  is the output value over time.  $n_a$  is the number of system poles.  $n_b$  is the number of system zeros + 1.  $n_c$  is the number of system c's.  $n_k$  is the number of input samples that occurred before the input affected the output (also called the dead time of the system).  $y(t-1)\dots y(t-n_a)$  is the previous series of outputs on which the current output depends.  $u(t-n_k)\dots u(t-n_k-n_b+1)$  is the previous and delayed series of inputs on which the current output depends.  $e(t-1)\dots e(t-n_c)$  is the white noise interference value. The parameters  $n_a$ ,  $n_b$ , and  $n_c$  are the order of the ARMAX model and  $n_k$  is the delay value.  $q$  is the delay factor.

## 4. CO2 Concentration Level Analysis and Modeling

### 4.1 Data Visualization and Pre-processing

CO2 Data Set 1 is the annual month of March averages of CO2 expressed as a mole fraction in dry air. We first analyze and process the data information to help us be able to better model it. One starts by visualizing the data (Figure 4.1), which shows a linear increase in the overall trend of CO2.

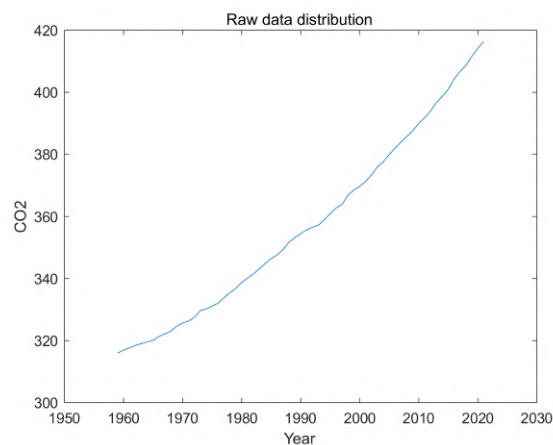


Figure 4.1

## 4.2 Analysis of Changes in Growth Trends

The analysis of the CO2 concentration data and the time series graphs give a rough idea of the slow growth trend. If we want to analyze and compare the specific trend of each growth, a time series graph of the annual CO2 concentration growth is needed.

Let the CO2 sequence be denoted as  $X = \{x_1, x_2, \dots, x_{63}\}$ . For this purpose, we calculate the first-order difference of Co2 as follows:

$$\Delta x_i = x_{i+1} - x_i$$

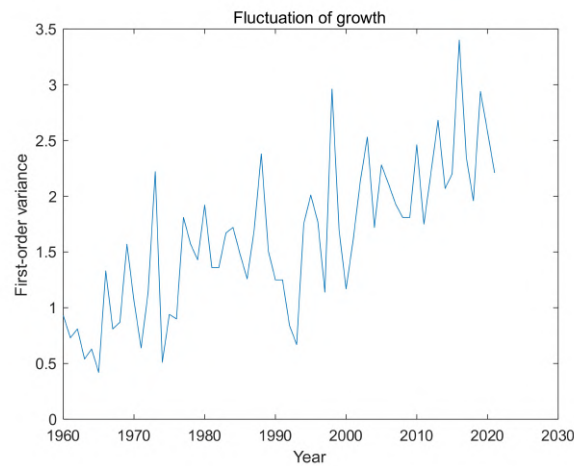


Figure 4.3 Annual increase in CO2 concentration

Figure 4.3 shows a time series graph of the annual growth of CO2 concentration, from which we can see that the CO2 growth is all positive, with large differences in fluctuations and an overall upward trend, indicating that not only CO2 is growing every year, but also its growth is rising in fluctuations. Some annual fluctuations are large, which may be due to natural and human activities and other factors, so for the influence of these factors, we use the 5-year moving average method to increase the CO2 data to eliminate the influence of fluctuations, which is calculated in Equation (3.3), where we take  $N=5$ .



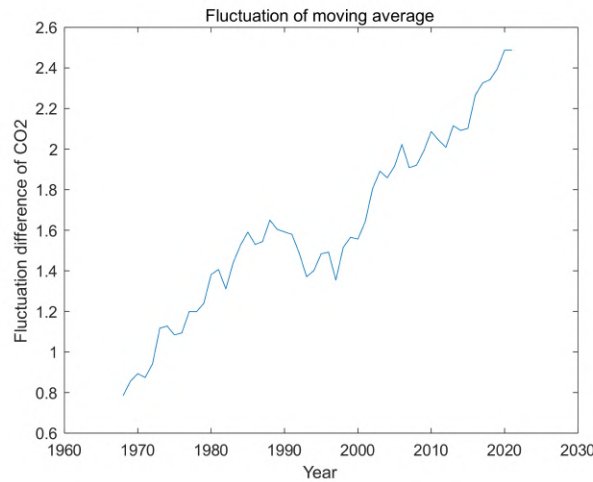


Figure 4.4: 5-year moving increase in CO2 concentration

From Figure 4.4, it can be seen that the 5-year average growth volume shows a gentle upward trend. However, in 1991-2001, the graph shows a low state, then has a sharp rise in 2001-2004. After that, it resumes a steady upward trend.

### 4.3 Comparison of The Average 10-year Growth

To address the question of whether the March 2004 increase in CO2 resulted in a larger increase than observed over any previous 10-year period? We calculated the decadal average increase in CO2 concentration from 1969-2004.

$$d(i) = \overline{\Delta x_i} = \frac{1}{9} \sum_{j=0}^8 \Delta x_{i+j} = \frac{1}{9} (x_{i+8} - x_i) (i = 1, 2, \dots, 37)$$

where  $d(i)$  denotes the average of the year-by-year growth amounts for the 10 years before 1967+i.

Table 4.1

Year	1959-1968	1960-1969	1961-1970	1962-1971	1963-1972	1964-1973
CO2	0.77	0.86	0.89	0.87	0.94	1.12
Year	1965-1974	1966-1975	1967-1976	1968-1977	1969-1978	1970-1979
CO2	1.13	1.08	1.09	1.20	1.20	1.24

Year	1971-1980	1972-1981	1973-1982	1974-1983	1975-1984	1976-1985
CO2	1.38	1.41	1.31	1.44	1.53	1.59
Year	1977-1986	1978-1987	1979-1988	1980-1989	1981-1990	1982-1991
CO2	1.53	1.54	1.65	1.60	1.59	1.58
Year	1983-1992	1984-1993	1985-1994	1986-1995	1987-1996	1988-1997
CO2	1.49	1.37	1.40	1.48	1.49	1.35
Year	1989-1988	1990-1999	1991-2000	1992-2001	1993-2002	1994-2003
CO2	1.52	1.57	1.56	1.64	1.80	1.89
Year	1995-2004					
CO2	1.86					

Table 4.1 shows that the 10-year average increase in CO<sub>2</sub> concentration in 2004 was 1.86 ppm, which is larger than the 10-year average increase over the past 1969-2002 period, but the 10-year average increase in CO<sub>2</sub> concentration in 2003 was 1.89 ppm, which is slightly larger than the value in 2004, indicating that it is not appropriate to claim that March 2004 increase of CO<sub>2</sub> resulted in a larger increase than observed over any previous 10-year period. CO<sub>2</sub> did grow significantly in 2004, but it was not enough to support a huge change or even beyond the ten-year average increase in CO<sub>2</sub> concentration in any previous decade (e.g., the increase of CO<sub>2</sub> in 2003 was more than that in 2004). Thus, our team believes that CO<sub>2</sub> did not increase much more during the 1995-2004 decade. At the same time, the 2004 increase of 1.72 ppm is also much smaller than the decadal average increase in 2003, indicating that 2004 alone is not enough to get the effect in the title.

## 4.4 Modeling of CO2 Concentration Levels

### 4.4.1 Linear Model

The data set contains data on CO2 concentrations from 1959 to 2021 (referred to as CO2 data) for a total of 63 years. We assume that the data in the data file are annual data for each year as the annual average CO2 concentration data.

First, build a linear model:

$$x_t = \beta_0 + \beta_1 t$$

where  $\beta_0, \beta_1$  is the parameter to be solved, and  $t$  means Year  $t$  (with 1959 as the base year). We intercept the first 43 CO2 data as the estimation set and the last 20 data as the validation set. From Figure 4.1, we can see that the CO2 concentration data show a linear trend, but with a little flat upward trend, so we first take the natural logarithm of the data and then use the most-squares algorithm in python to estimate  $\beta_0, \beta_1$ . The confidence intervals of  $\beta_0 = 5.741$  and  $\beta_1 = 0.003977$  are (5.738, 5.744) and (0.003868, 0.004086), respectively.

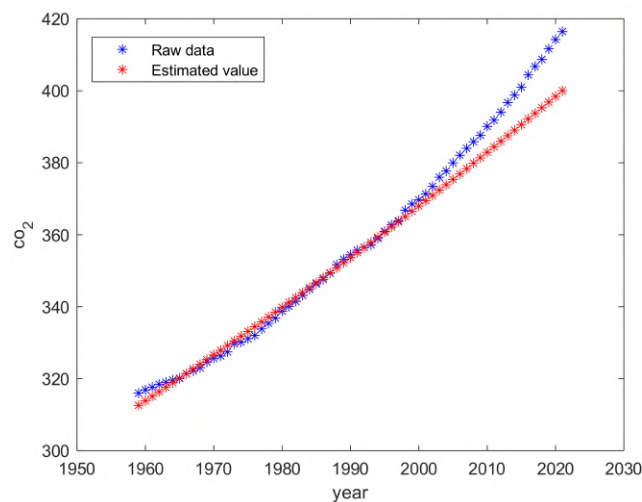


Figure 4.5: Linear model fitting and prediction results

It can be seen from the figure that the linear model roughly fits the rising trend of CO2 concentration data, but the fit is still unsatisfactory from the validation set point of view. , where R-square=0.9893, it can be seen that the model is reliable on the estimation set, but the residual is 9.7154, which indicates that the linear model cannot predict some dramatic changes in the data and can only predict the trend direction.

### 4.4.2 Secondary Regression Model

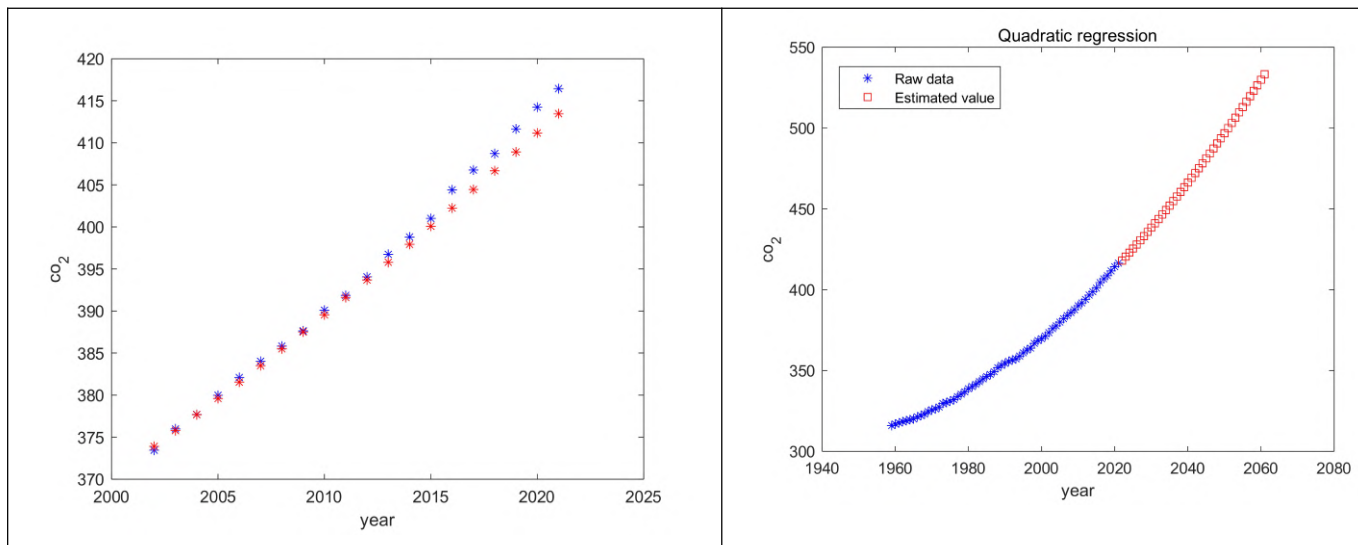
The CO<sub>2</sub> concentration data is a time-based series, and the linear model is built by treating the time difference as the dependent variable, and then we consider a quadratic regression model to fit the CO<sub>2</sub> concentration.

$$y = \beta_0 + \beta_1 x + \beta_2 x^2$$

where,  $\beta_2$  is the parameter to be solved, and t means year (with 1959 as the base year). We intercept the first 43 of CO<sub>2</sub> data as the estimation set and the last 20 data as the validation set. We use progress in MATLAB to find

$$\beta_0 = 314.73, \beta_1 = 0.7796, \beta_2 = 0.0130$$

The confidence intervals of  $\beta_0, \beta_1, \beta_2$  were  
(314.1528, 315.3049), (0.738096, 0.821158), (0.012409, 0.013666).



From the figure, it can be seen that the quadratic regression model fits the rising trend of CO<sub>2</sub> concentration data, and the fitting effect is relatively good, where R-square=0.9975, which shows that the model is reliable in the estimation set. The residual of the prediction for the validation set is 1.4845, which indicates that the quadratic regression model is more reliable.

### 4.4.3 Time Series Moving Average (MA) Model

Both of the above methods are based on statistical methods, and for time series prediction, we also consider using the model where the moving time is 3. The specific data set is constructed in the way shown in Table 4.6.

Therefore, the results of the model parameters are not given here, and we are more interested in the fitting and prediction results of the model to the data. The results are shown in Figure 4.8 From the figure, we can see that the MA method has a smoother fitting to the data and can have a better prediction effect in the short term, and can also predict the general trend in the long term.

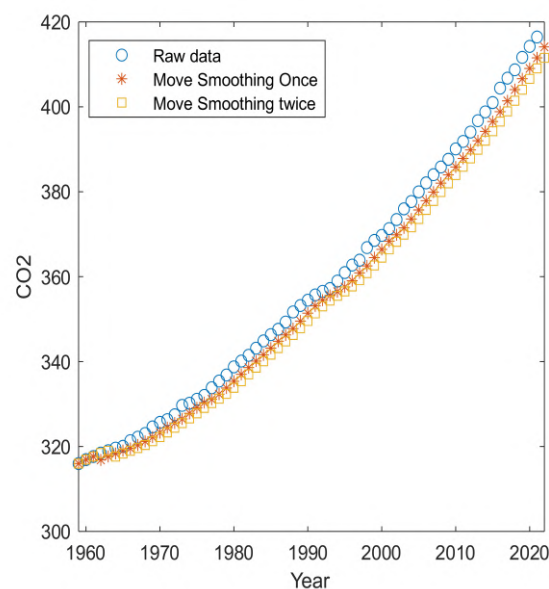


Figure 4.8 Fitting and prediction results of the MA model

## 4.4 Model Predictions

The three models established are now projected for the long term, and the results are shown in Figure 4.9. The three models have different growth trends, with the quadratic regression model having the fastest growth rate, the Linear model having the slowest predicted growth rate, and the MA model having a growth rate between these two.

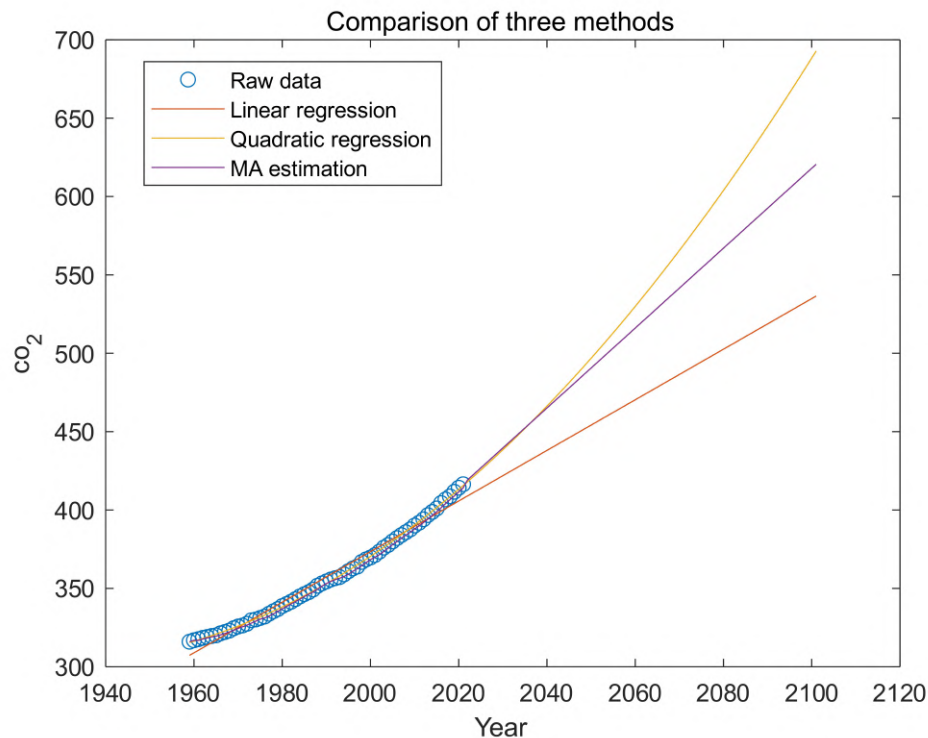


Figure 4.9 Long-term prediction results of the three models

#### 4.4. Answer to Question 1(c)

Table 4.8 shows some of the prediction results of the three models, from the table, the linear regression, quadratic regression, and MA models predict the atmospheric CO<sub>2</sub> concentration in 2100 to be 534.8825, 496.8051, and 495.6811, respectively. The quadratic regression model predicts that CO<sub>2</sub> will reach 685 ppm in 2099, and the MA model predicts that CO<sub>2</sub> will reach 685 ppm by 2125.

Table 4.2 Selected prediction results of the three models

Year	Predicted results (CO <sub>2</sub> /ppm)		
	Linear	Quadratic fitting	MA
2022	408.9877	421.2362	415.4198
2050	454.1807	496.8051	495.6811
2100	534.8825	688.3268	623.1256
2125	580.0756		686.8478

#### 4.5 Model Comparison and Answer to Question 1(d)

In order to compare the effects of the three types of models, the coefficient of

determination ( $R^2$ ), and mean squared error (MSE), which are two types of measures, are used for comparison. Where the closer the R-square is to 1, the better the model is; the smaller the MSE is, the better the model is, as calculated by the following equation, where  $\bar{Y}$  is the mean value.

$$R^2 = 1 - \frac{\sum (\hat{Y} - Y)^2}{\sum (\bar{Y} - Y)^2}$$

$$MSE = \frac{1}{n} \sum (Y - \hat{Y})^2$$

The results of the comparison are shown in Table 4.3. From the two indicators, we can see that the ARIMA model has the best effect, which means that the ARIMA model can best fit the data for prediction and can summarize and extract the information from the data, which is the best model among these three models. From Figure 4.11, we can also see the fitting effect on the training set and the prediction effect on the validation set. The ARIMA model and the LSTM model are both better than the linear model, and the ARIMA model can predict the trend better and fit more smoothly.

Table 4.3 Comparison of the three models

Models	$R^2$	$MSE$
Linear	0.5074	124.9219
Quadratic fitting	0.9462	4.4581
MA	0.9505	4.0983

The results of the comparison are shown in Table 4.3. From the two indicators, it can be seen that the MA model has the best results, which means that the MA model is the best model among these three models because it can fit the data together for prediction and can summarize and extract the information from the data, and the MA model can predict the trend better.

About the question of which model is more accurate, we believe that the forecast within 10 years can be analogous to the forecast of the validation set, that is, the MA model is optimal, and the specific comparison results are shown in Table 4.3. However, due to the existence of many unexpected factors or because there is no real data for validation, it may not be easy to assess which of the three models is better for a longer period of time.

## 5. Modeling and Analysis of The Relationship Between Temperature and CO2

### 5.1 Relationship Between Temperature and CO2

We know that water vapor and clouds are the main contributors to the Earth's greenhouse effect and that carbon dioxide, a component of the Earth's atmosphere, is one of several heat-trapping gases near the Earth's surface, and these gases are known as greenhouse gases. Some scientists have shown in a new atmosphere-ocean climate simulation study [3] that the temperature of the Earth ultimately depends on the level of CO2 in the atmosphere, and that a large increase in CO2 abundance will lead to an increase in the Earth's surface temperature, i.e., CO2 is the main cause of the Earth's surface temperature, with an overall linear correlation. It has also been pointed out that there is a correlation between CO2 changes and changes in surface temperature, but not a linear relationship [4]. Also, some researchers have shown that CO2 changes do not have any direct relationship with changes in surface temperature [5].

What are the facts? It seems that there is far from a consensus. We are only making a preliminary analysis here based on the data given in the title.

### 5.2 Data Preparation and Analysis

The annual temperature data given in this question is the difference from the average temperature of the base period, which takes the value interval  $[-0.2, 1.02]$ . In order to investigate the relationship between CO2 and temperature difference, we first treated the CO2 data similarly. After calculation, we took the average value of CO2 data from 1959 to 1980 (325.7359 ppm) as the base CO2 level. The difference between the original CO2 data at  $X = \{x_1, x_2, \dots, x_{63}\}$  and the base CO2 level at is given by  $\bar{X} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n\}$ . Therefore,  $\bar{X} = X - 325.7359$ , and the range of values for  $\bar{X}$  is given by  $[-9.76, 90.71]$ .



### 5.3 Predictive Model (answer to question 2(a))

The fluctuating growth trend of temperature indicates that temperature is a non-stationary time series with an embedded curvilinear trend, and we can treat it similarly to Problem 1, where the non-stationary deterministic information in the original series can be sufficiently extracted with enough times of difference operation, but the excessive difference will cause the waste of useful information [2]. However, the difference operation has a powerful ability to extract deterministic information, so the stationary series obtained after the difference operation on the data can be fitted with the ARMA model first. This is the ARIMA model.

We used the ARIMA model for correlation analysis of CO2 and temperature data. Above all, the primary difference between the original data and the autocorrelation plot of the difference data is shown in Figure 5.1. Then, we take some values and propose a comparative analysis with the ARIMA(2,1,1) model and the ARIMA(1,2,1) model according to the indicators such as AIC and BIC. Using Python software, the obtained ARIMA(2,1,1) fitted residuals' values, and distribution are shown in Figure 5.2 (the relevant graph of ARIMA(1,2,1) is omitted).

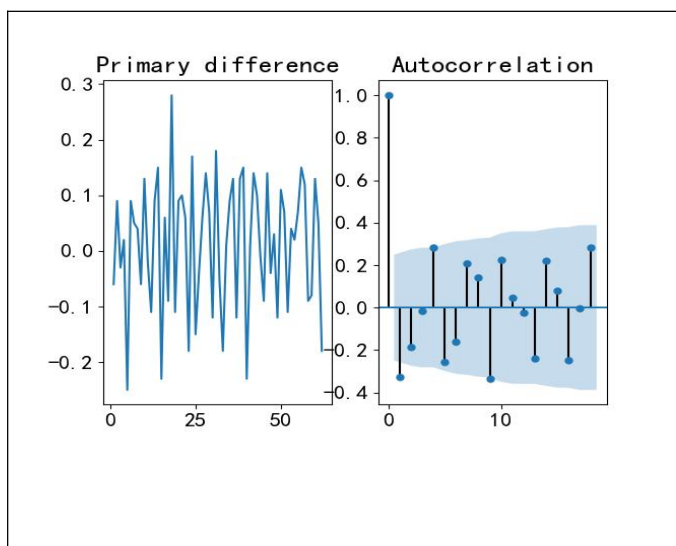


Figure 5.1 Primary differential and differential autocorrelation

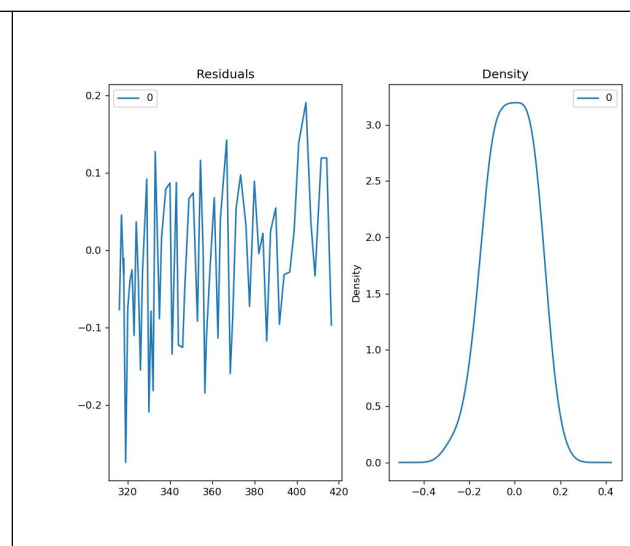
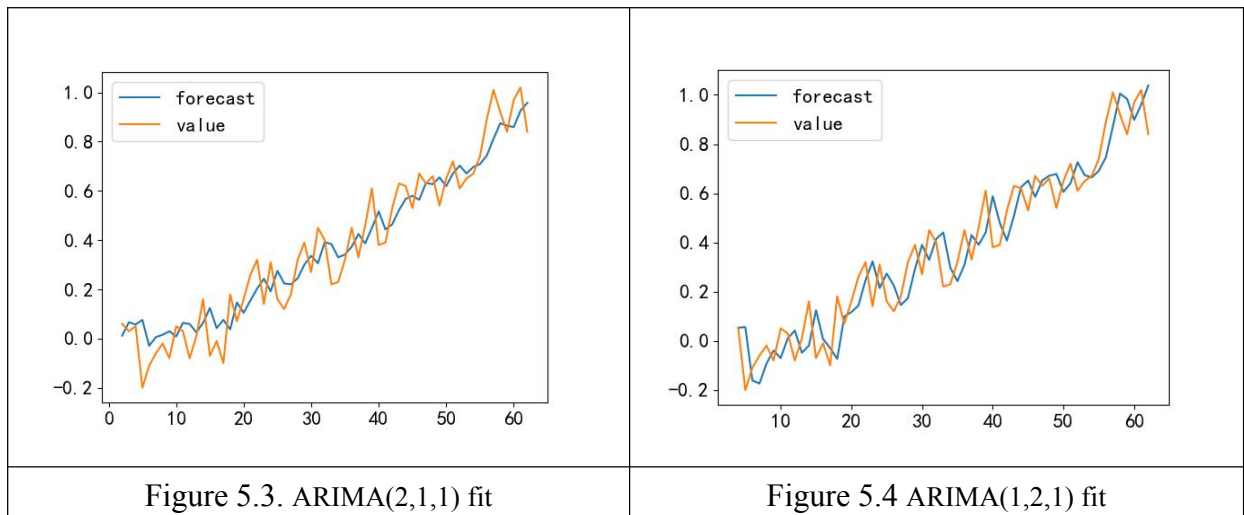


Figure 5.2 Residual values and distribution



Then, we use for ARIMA(2,1,1), respectively, to get  $AIC=-99.337$ ,  $BIC=-88.702$ ,  $\text{LogLikelihood}=54.669$ ,  $\text{stderr}=0.00$ . For ARIMA(1,2,1) model, we get  $AIC=-81.663$ ,  $BIC=-73.219$ ,  $\text{Log Likelihood}=44.831$ ,  $\text{stderr}=0.001$ .

As can be seen in Figures 5.3 and 5.4, the effect of the quadratic fit with a primary shift containing a random term is essentially the same as the linear fit with a quadratic shift containing a random term. Therefore, the fluctuations in temperature are largely eliminated after the quadratic shift, and therefore it can be assumed that the temperature change exhibits an apparent linear increase over the last 100 years, but the two growth processes are fluctuating due to many other factors.

We choose the ARIMA(1,2,1) model as the prediction model. The prediction graph is shown in Figure 5.5.

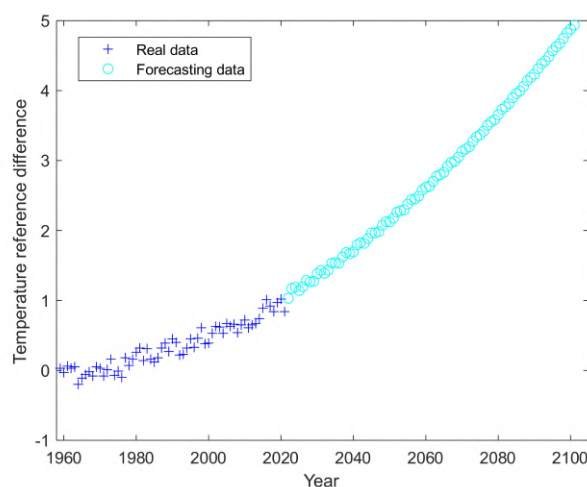


Figure 5.5 ARMIA Forecast Chart (2022-2115)

As can be seen from the prediction figure 5.5, the predicted trend of the ARIMA model is slightly upward compared to the trend of the original data but is generally consistent.

According to the model projections, the change in mean land-ocean temperature will reach 1.25°C, 1.50°C, and 2°C by 2027, 2034, and 2049, respectively.

## 5.4 Answer to Question 2(b) .

From the discussion in Section 5.1, it is clear that there is no uniform understanding of this relationship, but based on a large amount of historical data, CO2 and temperature are approximately linear [4]. In the following, we apply the processed CO2 data with temperature data to give a graph of CO2 versus temperature from 1959-2021 See Figure 5.6.

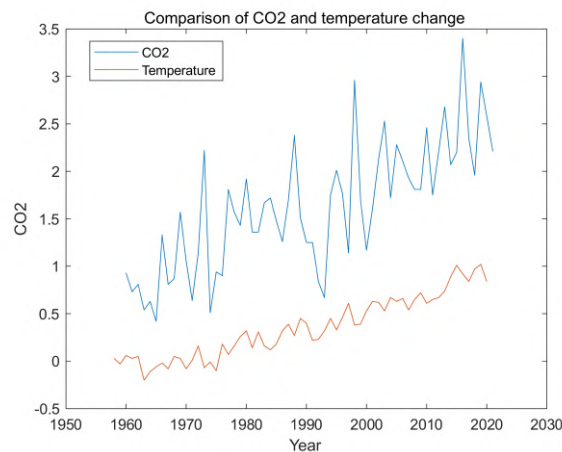


Figure 5.6 CO2 vs. temperature trend (1959-2021)

From Fig. 5.6, it can be seen that both of them show roughly low growth quadratic curve shape, but the temperature fluctuation is more, which also indicates that the CO2 growth is roughly linear with the change of temperature.

The following is based on the regression model (3.1), which fits the temperature variation with CO2. Using the curve fitting package in MATLAB, the results are solved as

Coefficients (with 95% confidence bounds):  $p_1 = 0.01048$  (0.009708, 0.01125),  $p_2 = -3.393$  (-3.668, -3.117), Goodness of fit.  $sse = 0.4977$ ,  $R\text{-square} = 0.9241$ , Adjusted  $R\text{-square} = 0.9229$ ,  $RMSE = 0.09033$

The fitted model is :

$$f(x) = 0.01048x - 3.393 \quad (5).$$

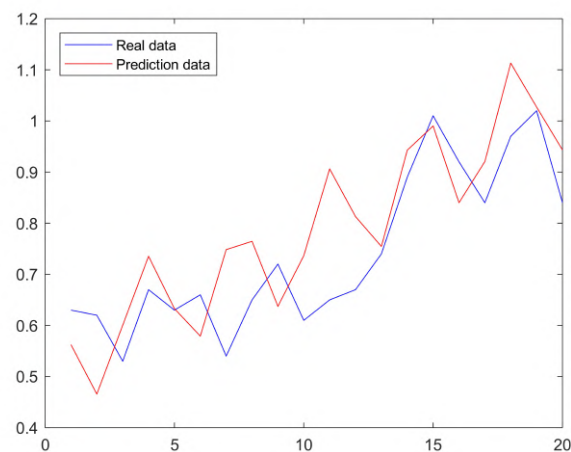


Figure 5.7 Fitted graph

From Figure 5.7, it can be seen that the fluctuation trend of CO<sub>2</sub> is generally consistent with the fluctuation trend of temperature. Therefore, we believe that the changes in CO<sub>2</sub> and temperature show a linear correlation at least for these 70 years.

### 5.5 Answer to Question 2(c) .

We apply the regression model above to calculate the temperature change for the next 80 years based on the results of the first question. As can be seen from Figure 5.8, the temperature change after 2021 is generally consistent with the previous growth trend, but somewhat elevated.

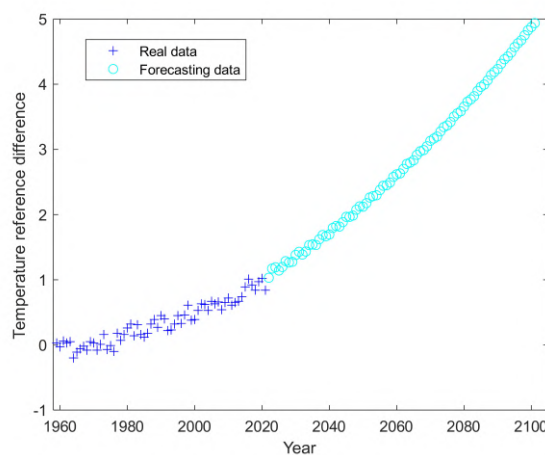


Figure 5.8 Prediction results

## 6. Proposal

### Effects of CO<sub>2</sub> on Temperatures

Global warming is the most outstanding problem in recent years, and the increasing concentration of carbon dioxide is one of the reasons. The concentration of CO<sub>2</sub> in the air is continually growing. To figure out the relationship between temperature and CO<sub>2</sub>, the scientific world has proposed several assumptions.

Three different attitudes are provided by scientists regard to the relationship between temperature and CO<sub>2</sub>, scientific world has three different opinions, a linear relationship, a non-linear relationship, and no relationship. After days of research, our team members strongly believe there is a linear relationship which means the rising temperature leads to an increasing concentration of carbon dioxide. Also, considering the research and data in the past decades, it is more convincing to believe that it is a linear relationship. Given that the high concentration of carbon dioxide is currently the main factor contributing to the global temperature rise, the solution to alleviate global warming is to control the concentration of carbon dioxide. In this regard, our group has proposed the following solutions:

#### 1. Countermeasure program for forest protection

Tropical rainforests have suffered varying degrees of destruction. Over the past 20 years, the Amazon rainforest has lost more than 8% of its tree area. The result is a reduction in carbon dioxide absorption. Therefore, reducing the destruction of forests and promoting forest regeneration through extensive reforestation can help reduce the CO<sub>2</sub> concentration in the atmosphere.

#### 2. Use renewable energy and reduce the use of CFCs

As it is impossible to diminish the discharge of carbon dioxide, the better solution is to replace some fossil energy sources with renewable energy sources like solar energy, tidal energy, and wind energy. Also, using new energy vehicles and optimizing the combustion efficiency controls the use of CFCs.

In conclusion, controlling the discharge of carbon dioxide is a crucial task for everyone. All of us should make an effort to protect the environment of Earth so that we can live a healthy life in this world.

## 7. Evaluation of Models

The Earth system is a highly complex ecosystem, and the various factors have very complex relationships with each other. Therefore, the long-term relationship between CO<sub>2</sub> and temperature change may be quite different from the short-term relationship. The change in surface temperature is the result of many factors, but its growth trend is linear and fluctuating. In this paper, we present a preliminary analysis of only the average data for the last 70 years, using either linear or quadratic prediction models, and the resulting predictions are generally consistent with temperature changes. We believe that, despite the simplicity of the model we used, it also reflects the basic relationship between the two quantities.

We suggest that the effect of CO<sub>2</sub> or other geochemical components on temperature change can be studied and evaluated in an integrated manner by applying multiple models combining long-term and short-term data in the study of surface temperature change, with a view to obtaining more credible conclusions

## 8. References

- [1].Samprit Chatterjee, Ali S, Hadi, Regression Analysis by Example (In Chinese), 2013, China Machine Press.
- [2].Jonathan D. Cryer, Kung-Sik Chan, Time Series Analysis with Applications in R (in Chinese), 2011, China Machine Press.
- [3].<https://lederniercarbone.org/en/relation-co2-temperature>
- [4].<https://iowaclimate.org/2017/04/10/the-relationship-between-co2-and-temperature-simply-isnt-linear/>
- [5].<https://skepticalscience.com/co2-temperature-correlation.htm>