4 Efficient Algorithms for Detecting Layered Space-Time Codes in MIMO communication system

Lu Huang

University of Sun Yat-sen

huanglu28@mail2.sysu.edu.cn

December 16, 2018

Overview

Why MIMO system is recommanded?

COMMUNICATION over multi-input multi-output(MIMO) channels have been the subject of intense research over the past several years because multiple antenna systems provide an enormous increase in capacity compared to single-input single-output(SISO) channel.

Space-Time codes are designed to exploit the high capacity of MIMO system,which use space as a second dimension of coding.

Model of MIMO System

Theorem (Equation)

$$x = H \times c + v$$

System Description

In the equation,c=(c1 c2 ... c_{nT})^T denotes the vector of transmitted symbols. x=(x1 x2 ... x_{nR})^T denotes the vector of received symbols. v=(v1 v2 ... v_{nR})^T depicts the vector of noise terms at the n_R receiving antennas. The $n_R \times n_T$ channel matrix

$$H = \begin{pmatrix} h_{1,1} & \cdots & h_{1,nT} \\ \vdots & \ddots & \vdots \\ h_{nR,1} & \cdots & h_{nR,nT} \end{pmatrix}$$

The real part and the image part of $h_{j,i}$ follow the normal distribution which change from frame to frame. The distinct fading gains are assumed to be uncorrelated and are perfectly known by the receiver.

Detecting Algorithm

- VBLAST algorithm
- Unsorted QR Decomposition
- Sorted QR Decomposition
- Geometric Mean Decomposition

VBLAST

Description

It is obvious from the equation shown before that the received signals are a linear combination of the n_T transmitted signals.

The VBLAST algorithm detects the signals at the receiver by nulling the interferer with a zero-forcing *(ZF)* nulling vector.

In every detection step,all signals but one are regarded as interferer.By applying the nulling vector to interferer cancellation,the influence of these signals is nulled out,the target signal is detected and subsequently substracted from the received signal vector.

VBLAST Algorithm

Example (MATLAB original Code)

```
function c =V_blast( H,x )
% V-blast算法检测多层时空信号
% H -- NR*NT维瑞利信道
% x -- 接收信号
% c -- 解码信号
[NR,NT,L]=size(H);
c=zeros(NT,L);
    for j=1:L
        HH=H(:,:,j);
        S=1:NR;%标记被消去的列
```

Example (MATLAB original Code)

```
        for i=1:NT
            G=pinv(HH);%G为H的二维广义逆矩阵
            [w,k]=minnorm(G);%k为G的最小行范数所在行
            y=G(k,:)*x(:,j); %生成判决统计量
            temp=S(k);
            %对y进行判决，得到解码信号
            c(temp,j)=1*(y>=0)-1*(y<0)+0;
            %从接受信号中消除c，得到修正后的信号
            x(:,j)=x(:,j)-sqrt(1/NR)*HH(:,k)*c(temp,j);
            HH(:,k)=[];
            S(k)=[];
        end
        end
    c=(c+1)/2;
end
```

USQRD

Description

The QR decomposition of the channel H derive bounds for the error probability of LST cods. Therefore,the $n_R \times n_T$ matrix is factorized into the unitary $n_R \times n_T$ matrix Q and the upper triangular $n_T \times n_T$ matrix R.

$$H = Q \cdot R$$

By multiply equation from the left with the Hermitian matrix of Q,a new equation is gotten.

$$y = Q^H \cdot x = R \cdot c + \eta$$

Because R is a upper triangular matrix the backwards substitution can be applied to solve the equation.Noting that c_{nT} need to be estimate by applying quantization operation.

Example (MATLAB original Code)

```
function c = USQR( H,x )
% unsorted QR算法检测多层时空信号
% H -- NR*NT维瑞利信道
% x -- 接收信号
% c -- 解码信号
[NR,NT,L]=size(H);
c=zeros(NT,L);
for j=1:L
    R=zeros(NT,NT);
    Q=H(:,:,j);
    for i=1:NT
        %R的对角线元素等于qi的1范数
        R(i,i)=norm(Q(:,i),1);
        Q(:,i)=Q(:,i)/R(i,i);
        for l=i+1:NT
```

USQRD Algorithm

Example (MATLAB original Code)

```
    y=Q'*x(:,j);
    c(NT,j)=y(NT)/R(NT,NT);
    c(NT,j)=(c(NT,j)>=0)-(c(NT,j)<0)+0;
    for k=NT-1:-1:1
        d=0;
        for i=k+1:NT
            d=d+sqrt(1/NT)*R(k,i)*c(i,j);
        end
        z=y(k)-d;
        z=z/R(k,k);
        c(k,j)=(z>=0)-(z<0)+0;
    end
end
c=(c+1)/2;
end
```

SQRD

Description

The sorted QR decomposition bases on the unsorted QR decomposition but the sorted Gram-Schmidt process*(SQRD)*proposed here search for the detection sequence **S** that acheive small SNR_k in the upper layers by finding the column with the minimal norm of **Q**.

The column chosen from the vectors $q_i,\cdots,q_{nT}$ and denoted as q_{k_i}.The corresponding h_{k_i} has the smallest component orthogonal to the space span by $q_1,\cdots,q_{i-1}$, which leads to the smallest $r_{i,i}$ of the possible permutations in the step i and thereby the smallest SNR_k.

SQRD Algorithm

Example (MATLAB original Code)

```
function c= SQRD( H,x )
% SQRD算法检测多层时空信号
% H -- NR*NT维瑞利信道
% x -- 接收信号
% c -- 解码信号
[NR,NT,L]=size(H);
c=zeros(NT,L);
for j=1:L
    R=zeros(NT,NT);
    Q=H(:,:,j);
    S=1:NT;
    for i=1:NT
    %找出Q的最小范数列和该列的范数
    [min_norm,k]=minnormc(Q,i,NT);
```

SQRD Algorithm

Example (MATLAB original Code)

```
    if(i~=k)
    %交换Q、R、S的第i列和第k列
        Q=swapc(Q,i,k);
        R=swapc(R,i,k);
        S=swapc(S,i,k);
    end

    %R的对角线元素等于最小范数列1范数
    R(i,i)=min_norm;
    Q(:,i)=Q(:,i)/R(i,i);
    for l=i+1:NT
        R(i,l)=Q(:,i)'*Q(:,l);
        Q(:,l)=Q(:,l)-R(i,l)*Q(:,i);
    end
end
```

SQRD Algorithm

Example (MATLAB original Code)

```
y=Q'*x(:,j);
c_u=zeros(NT,1);  %未置换的c矩阵
%计算第NT个信号的大小
c_u(NT)=y(NT)/(R(NT,NT)*sqrt(1/2));
c_u(NT)=(c_u(NT)>=0)-(c_u(NT)<0)+0;

for k=NT-1:-1:1
    d=0;
    for i=k+1:NT
        d=d+R(k,i)*c_u(i)*sqrt(1/2);
    end
    z=y(k)-d;
    z=z/R(k,k);
    c_u(k)=(z>=0)-(z<0)+0;
end
```

GMD

Description

The geometric mean decomposition combines either the conventional VBLAST decoder or the more recent zero-forcing dirty paper precoder*(ZFDP)*.

The scheme decomposes a MIMO channel into multiple identical parallel subchannels which is proved to be asymptotically optimal for(moderately) high SNR in both channel throughput and BER performance.

Description

The GMD algorithm begin with decomposing the $n_R \times n_T$ matrix H by Singular Value Decomposition(SVD).

$$H = QRP_*$$

Then,the [2] prove that the upper triangular matrix R can be decomposed with SVD

$$R = U_R \sigma V_R^*$$

with the ith diagonal element $r_{ii} = \overline{\lambda}_H$.

Description

Combining above two equation yields $H = UU_R^* RV_R V^*$. Next,by replacing x with **Ps**,the equivalent data model is

$$y = QRs + z$$

The next step is nothing but Backwards Substitution.

PERFORMANCE ANALYSIS

Performance Analysis

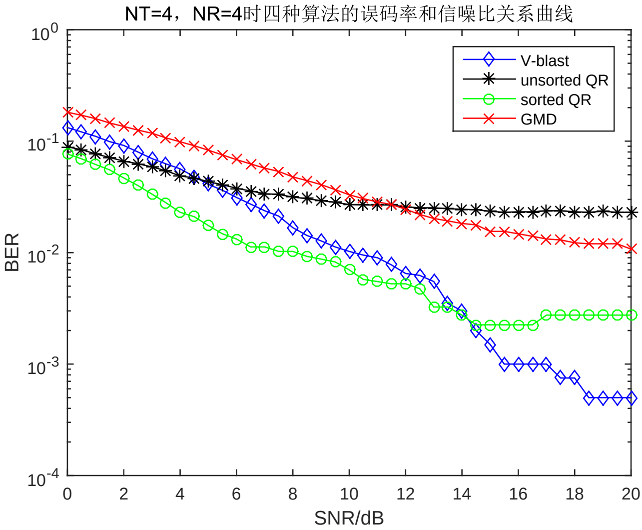

Figure: Simulation with $n_T = 4$ and $n_R = 4$ antennas over 1000 Monte Carlo

Performance Analysis

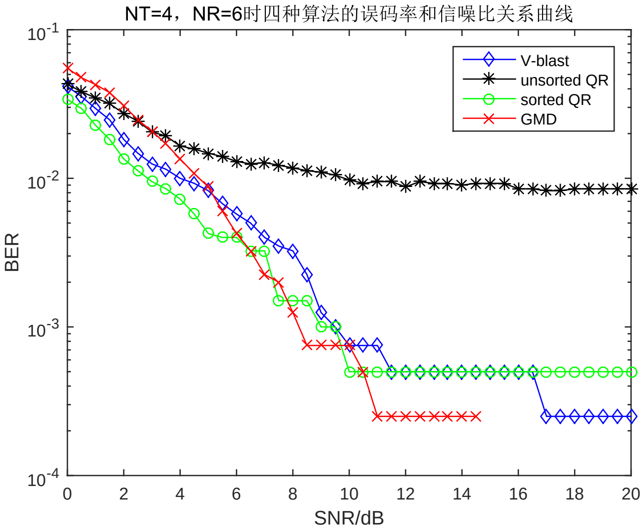

Figure: Simulation with $n_T = 4$ and $n_R = 6$ antennas over 1000 Monte Carlo

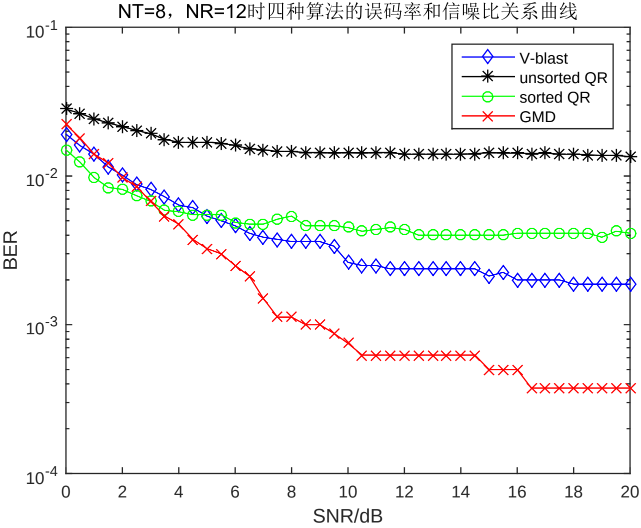

Figure: Simulation with $n_T = 8$ and $n_R = 12$ antennas over 1000 Monte Carlo

Performance Analysis

Figure 1

Figure 1 shows the BER of the different detection algorithms for an uncoded system with $n_T = 4$ and $n_R = 4$ antennas,and the frame length is defined as 1000. We see that the VBLAST scheme has more than 1-dB improvement over the other three schemes.

Figure 2

In *Figure 2*,GMD ,SQRD and VBLAST tends to zero while the GMD performs better than other three.

Figure 3

According to *Figure 3*,the BER of unsorted QR decomposition are obviously high than any other three algorithms.The GMD algorithm has more than 1-dB improvement than the VBLAST.

Conclusion

The performance analyse suggest that the GMD may be preferred over the others if the channel has a large dimensionality.

References

Dirk Wubben,Volker Kueh,Efficient Alogorithm for Detecting Layered Space–Time Codes

Yi Jiang,Jian Li,William W.Hager,Joint Transceiver Design for MIMO Comunications Using Geometric Mean Decompostion

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