💍 Ring Attention

Sequence Parallel Attention Across Devices

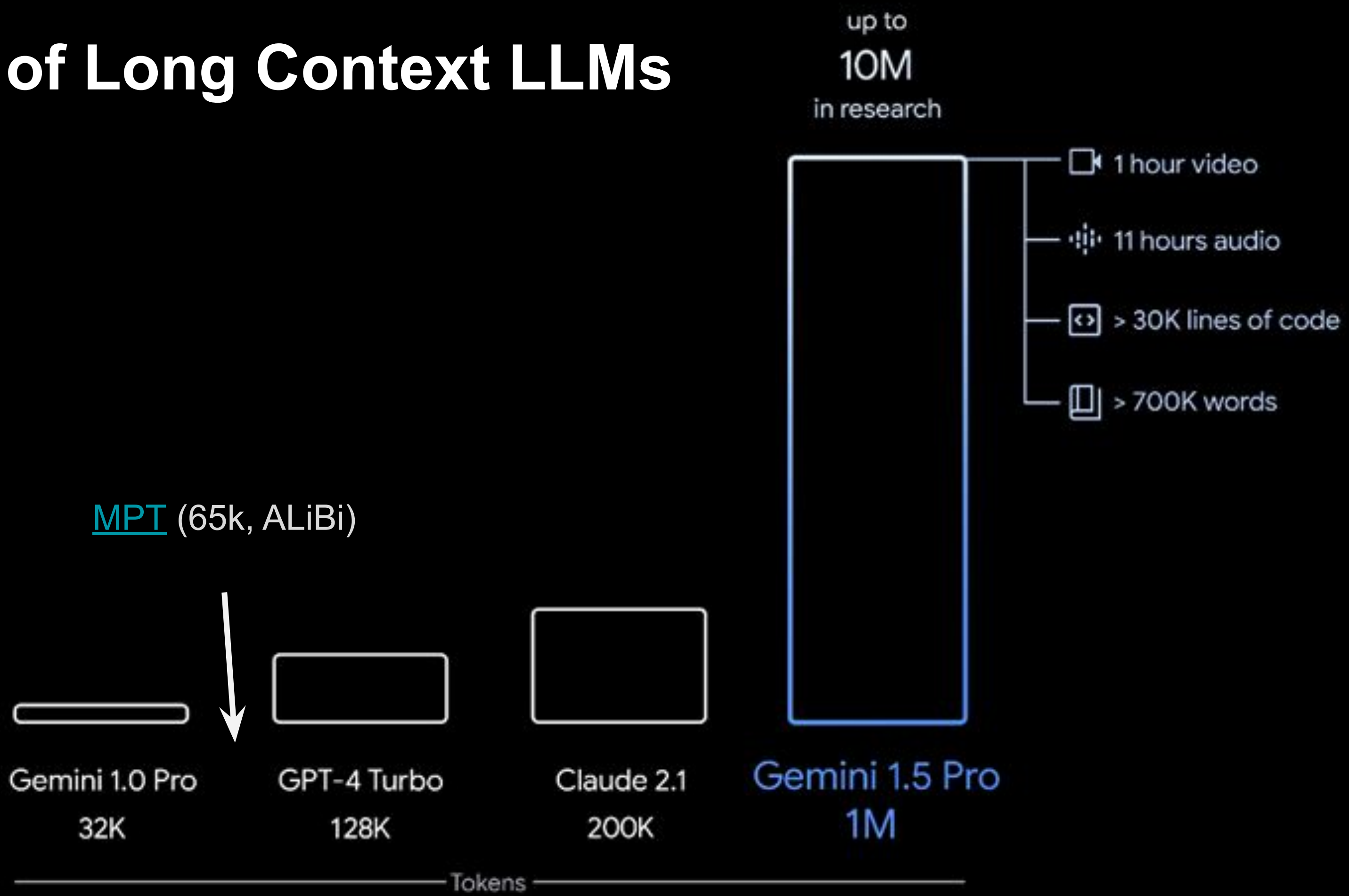
CUDA-MODE Lecture 13
Andreas Köpf
April 06, 2024

Overview

1. Motivation: Long Context Transformers & Applications
2. Recap: Vanilla Attention, Online Softmax, Log-Sum-Exp
3. Ring Attention 💍
4. Striped Attention 🦓
5. Flash Decoding ⚡

The Era of Long Context LLMs

MPT (65k, ALiBi)



GROK-1 (8K) DBRX Yarn-Mistral-128k LWM -1M

Source: Google Blog

Long-context Magic 🪄

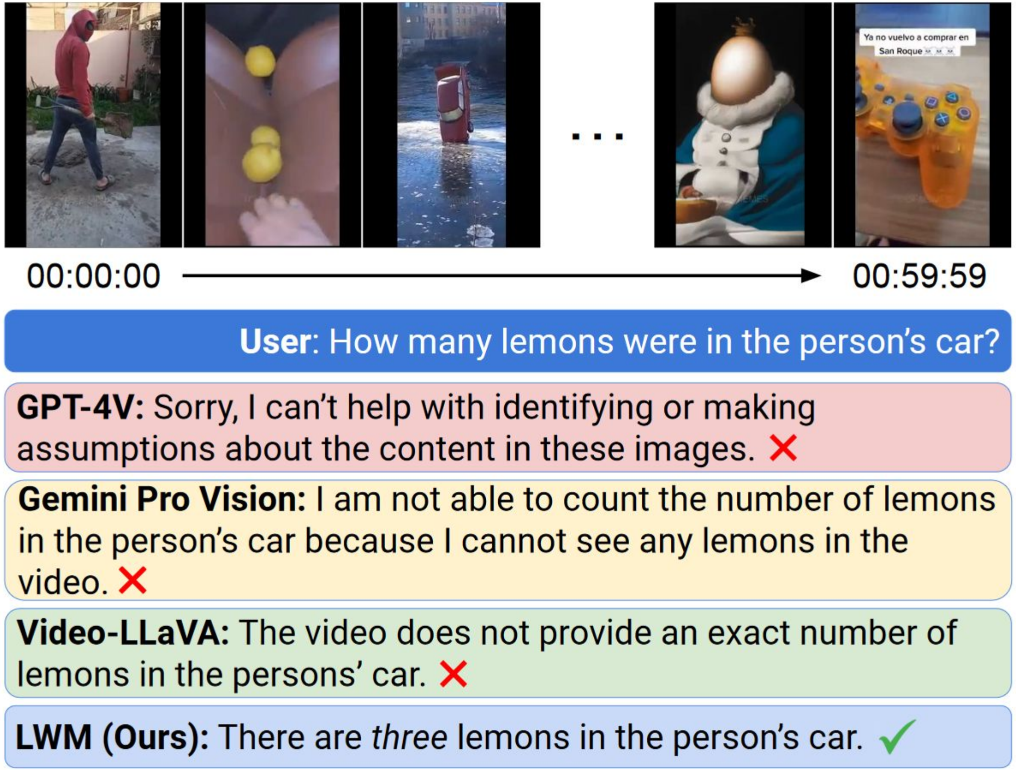


Figure 14 LWM demonstrates video understanding over 1 hour video.

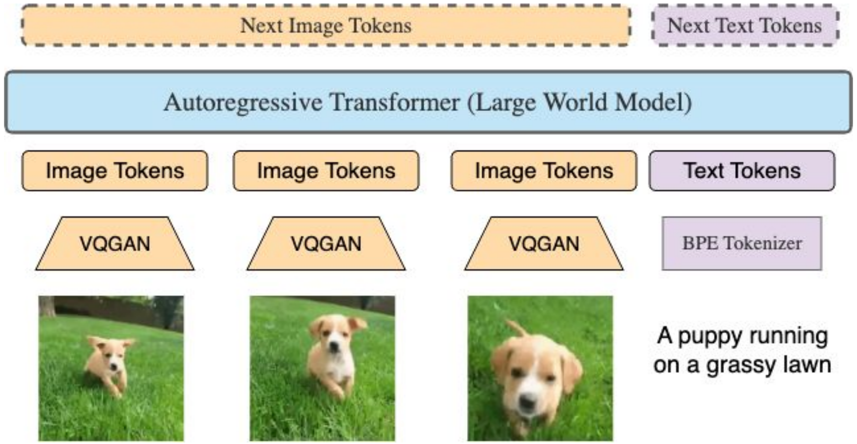
Allows to process…

- books, long documents
- web content
- chat histories
- code bases
- high-res images
- audio recordings
- videos

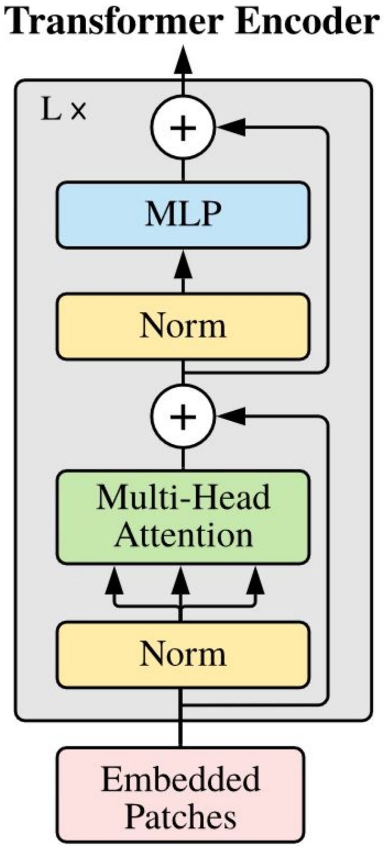
… towards multi-modal world models

More on LWM: largeworldmodel.github.io

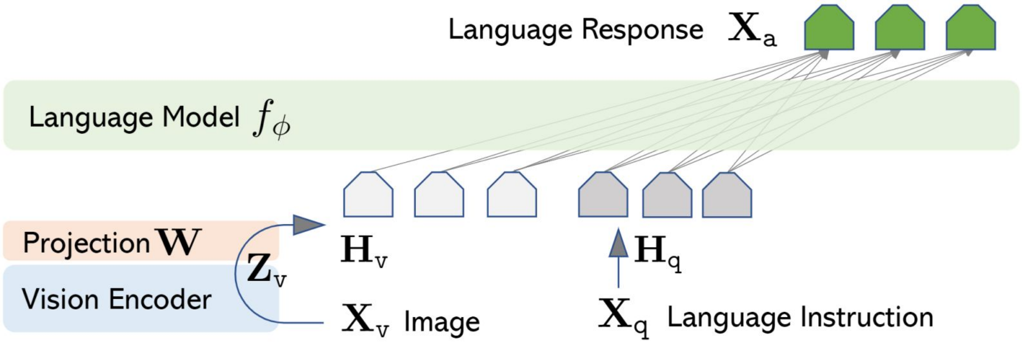
Multimodal - Any-to-Any Autoregressive Predictions



LWM: text, image, video, video-text, text-video, image-text, text-image



LLaVA: image-text



Challenge: We Run Out of Memory

"with a batch size of 1, processing 100 million tokens requires over 1000 GB of memory for a modest model with a hidden size of 1024"

Ring Attention, 2023, Hao Liu et al.

Input has to be materialized:
Memory scales linearly with Flash-Attention
- need to store input QKV + output + LSE + dout for backward

Memory of current high-end GPUs:

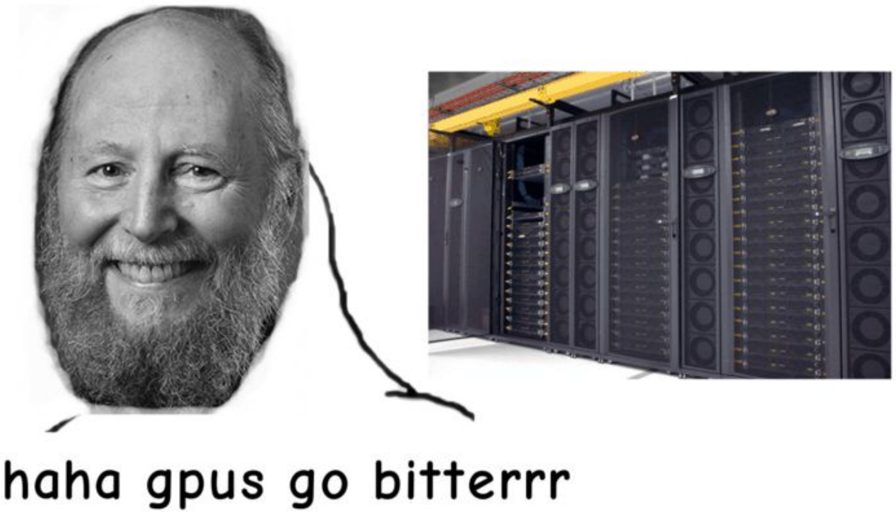
- NVIDIA H200: 141 GB
- AMD MI300X: 192 GB
- NVIDIA GB200 (Blackwell): 288 GB (available late 2024)

Approaches to Attention for Long Contex

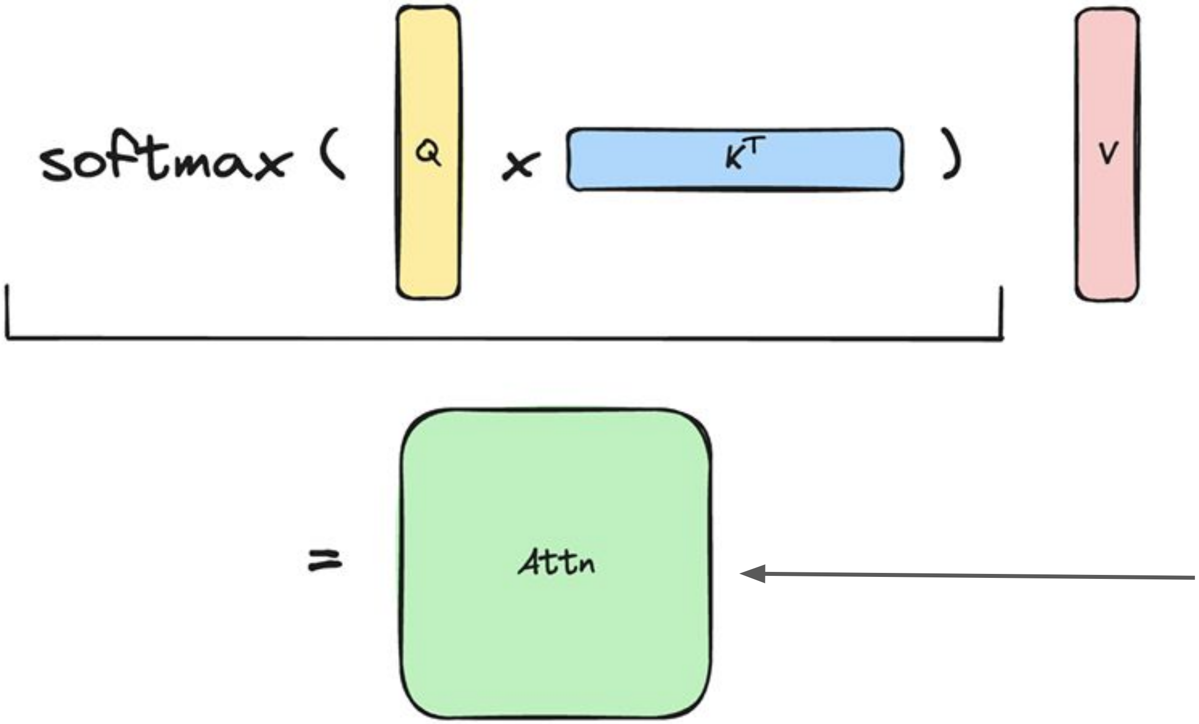
A) Approximation (e.g. Sparse, LoRA)

B) RAG / Vector-DBs (ANN search, LSH)

C) **Brute-force compute** (tiling, blockwise)



Vanilla Attention



s × s

$$\text{softmax}(QK^T)V$$

Memory complexity of naive attention is quadratic with sequence length (score matrix & softmax output).

How bad is it? FLOPS Scaling per Token

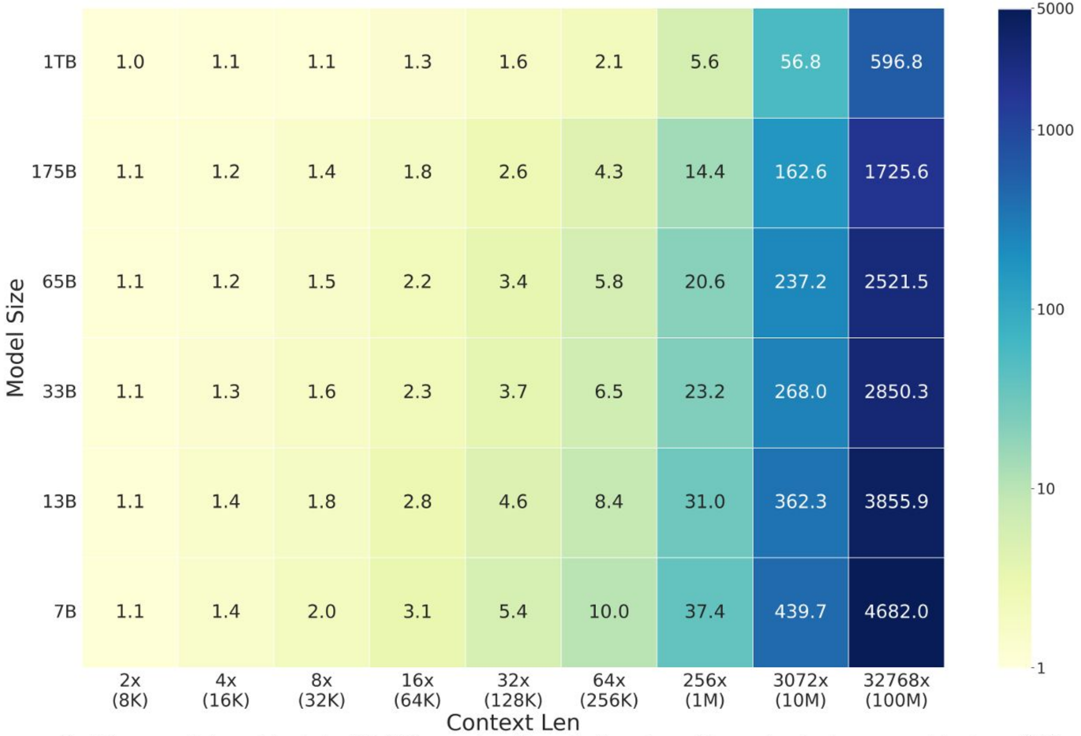


Figure 5: The per dataset trainig FLOPs cost ratio relative to a 4k context size, considering different model dimensions. On the x-axis, you'll find the context length, where, for example, 32x(128k) denotes a context length of 128k, 32x the size of the same model's 4k context length.

Surprisingly:
"as the model sizes increase, the cost ratio decreases"

FLOPS: $24sh^2 + 4s^2h$
(s=seqlen, h=hidden-dim)
given constant h: $O(s^2)$

-> sequence length will catch you - but maybe later than you thought.

Source: Ring Attention, Appendix D

The Crux of Attention: softmax

$$s\left(x_i\right) = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

Challenge: The softmax operation needs to be computed over full rows of the score matrix S = QK^T, outputs depend on the sum in the denominator.

For FlashAttention & RingAttention we need to compute the softmax part blockwise/online - i.e. with parts of this sum!

Towards Log-Sum-Exp Update - Step-by-Step

Let’s start by defining a naive softmax function …

```python
def naive_softmax(x: torch.Tensor) -> torch.Tensor:
    return x.exp() / x.exp().sum()
```

$$s(x_i) = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

… and verifying that its output matches the output of the official torch.softmax() function:

```python
x = torch.randn(10)  # generate normally distributed random numbers
a = torch.softmax(x, dim=-1) # reference output
b = naive_softmax(x) # our naive version

print("a", a)
print("b", b)
print("allclose", torch.allclose(a, b, atol=1e-6))
```

Python

```
a tensor([0.0323, 0.1455, 0.0659, 0.3275, 0.0416, 0.1432, 0.0871, 0.0258, 0.0234,
        0.1077])
b tensor([0.0323, 0.1455, 0.0659, 0.3275, 0.0416, 0.1432, 0.0871, 0.0258, 0.0234,
        0.1077])
allclose True
```

Naive & Numerical unstable

Our naive softmax function has a problem when it gets input vectors with larger elements:

```python
x = torch.randn(10)
naive_softmax(x * 100)
```

Python

```
tensor([0., 0., 0., nan, 0., 0., 0., 0., 0., 0.])
```

Before we to fix this let's first look how a block-wise computation of softmax can be realized ...

Goal: Breaking softmax() into chunks

```python
x = torch.randn(10)

x1,x2 = torch.chunk(x, 2)
s1 = naive_softmax(x1)
s2 = naive_softmax(x2)

print("We have:")
print(f"s1 = {s1}")
print(f"s2 = {s2}")

target = naive_softmax(x)
print("We want:")
print(f"target = {target}")
```

Python

```
We have:
s1 = tensor([0.1469, 0.2743, 0.1178, 0.3475, 0.1134])
s2 = tensor([0.0403, 0.4899, 0.1561, 0.2785, 0.0353])
We want:
target = tensor([0.0721, 0.1347, 0.0578, 0.1706, 0.0557, 0.0205, 0.2494, 0.0795, 0.1418,
        0.0180])
```

We generate a vector and split it into two chunks of equal size and compute softmax on each chunks individually…

But how to compute target from s1 & s2?

Undo normalization with "sum exp" $\sum_{j=1}^{n} e^{x_j}$

The softmax output had been divided by `x.exp().sum()`. If we have this value for each chuck we can "undo" the softmax normalization and combine multiple chunks.

from last slides we have:

```python
def naive_softmax(x: torch.Tensor):
    return x.exp() / x.exp().sum()
```

```python
x1,x2 = torch.chunk(x, 2)
s1 = naive_softmax(x1)
s2 = naive_softmax(x2)
```

```python
target = naive_softmax(x)
```

```python
se_x1 = x1.exp().sum()
se_x2 = x2.exp().sum()
s1_corrected = s1 * se_x1 / (se_x1 + se_x2)
s2_corrected = s2 * se_x2 / (se_x1 + se_x2)

print("After correction with help of se values:")
s_combined = torch.cat([s1_corrected, s2_corrected])
print("s_combined", s_combined)

print("allclose(s_combined, target):", torch.allclose(s_combined, target))
```

Python

```
After correction with help of lse values:
s_combined tensor([0.0721, 0.1347, 0.0578, 0.1706, 0.0557, 0.0205, 0.2494, 0.0795, 0.1418,
        0.0180])
allclose(s_combined, target): True
```

Combining blocks numerically stable

```python
x = torch.randn(20)
a = torch.softmax(x, dim=-1)
x1, x2 = x.chunk(2)
```

1. Create test input & output
2. Define stable_softmax2() function
3. Combine blockwise with help of log-sum exp.

```python
def stable_softmax2(x):
    """returns softmax result and log sum exp"""
    m = x.max()
    a = (x - m).exp()
    b = a.sum()
    lse = m + torch.log(b)
    return a / b, lse
```

```python
#c1 = b1 * torch.exp(lse1) / (torch.exp(lse1) + torch.exp(lse2))
#c2 = b2 * torch.exp(lse2) / (torch.exp(lse1) + torch.exp(lse2))
c1 = b1 / (1 + torch.exp(lse2 - lse1))
c2 = b2 / (1 + torch.exp(lse1 - lse2))
b = torch.cat([c1, c2])

print(torch.allclose(a, b))
```

```
True
```

$a/(a+b) = 1/(1+b/a)$

Trick: Do divisions as subtraction in log-scale.

Much stable!

Same trick can be used for RingAttention

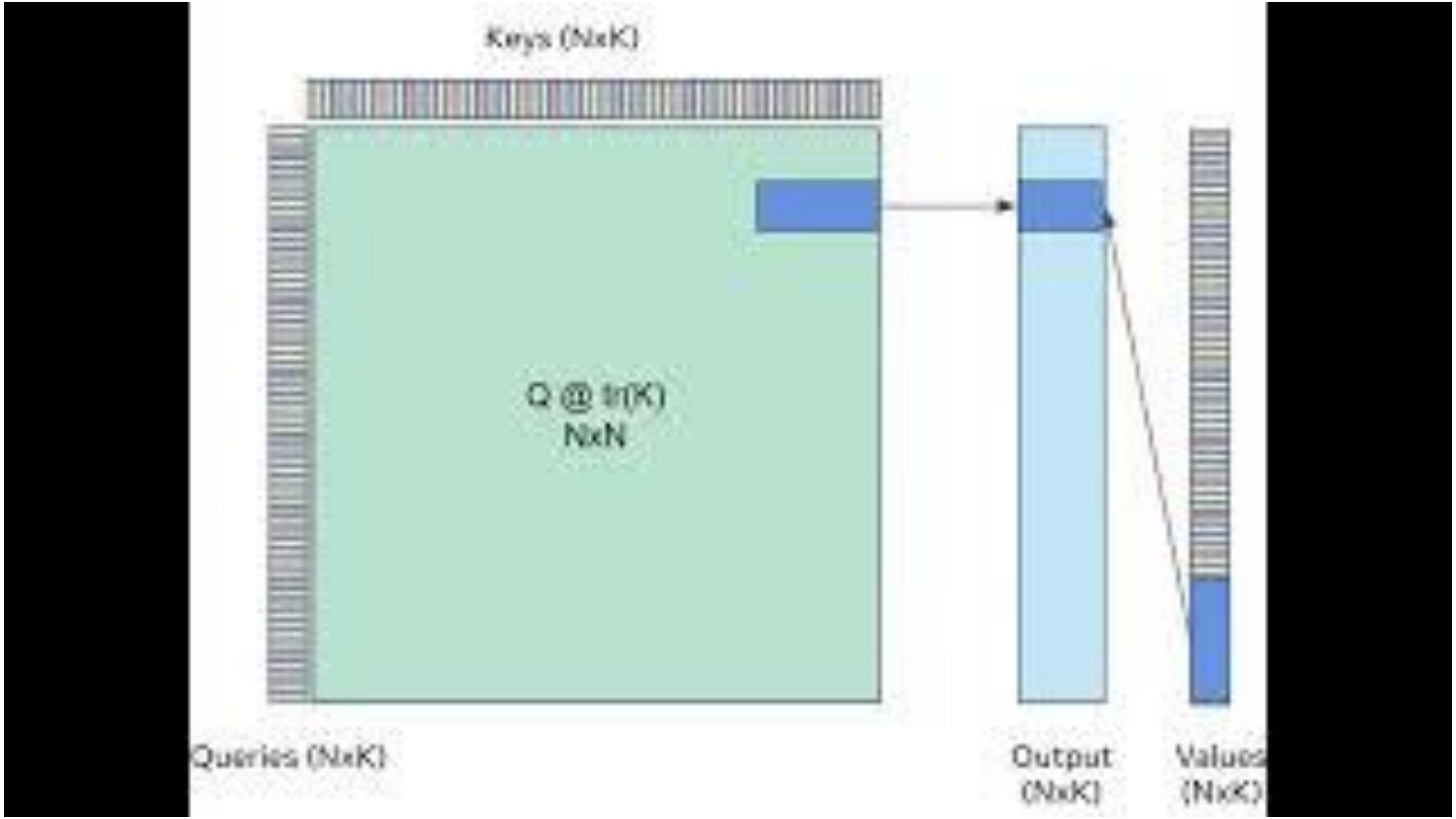
- internal flash attention functions return the log-sum-exp
- allows us to compute attention value projection blockwise/incrementally

```python
def _update_out_and_lse(
    out: torch.Tensor,
    lse: torch.Tensor,
    block_out: torch.Tensor,
    block_lse: torch.Tensor,
) -> Tuple[torch.Tensor, torch.Tensor]:
    block_out = block_out.to(torch.float32)
    block_lse = block_lse.transpose(-2, -1).unsqueeze(dim=-1)

    new_lse = lse + torch.log(1 + torch.exp(block_lse - lse))
    out = torch.exp(lse - new_lse) * out + torch.exp(block_lse - new_lse) * block_out

    lse = new_lse
    return out, lse
```

Attention value projection is linear, i.e can be corrected in same way as direct softmax block outputs.

https://github.com/zhuzilin/ring-flash-attention/blob/55ff66fd35f329dfcc24ce7a448bfdd532865966/ring_flash_attn/utils.py#L10-L24

Blockwise Output updates Animated



Source: [tweet](#) [@fvsmassa](#)

Applied in zhuzilin / ring-flash-attention

```python
comm = RingComm(process_group)
out,lse = None, None
next_k, next_v = None, None

for step in range(comm.world_size):
    if step + 1 != comm.world_size:
        next_k: torch.Tensor = comm.send_recv(k)
        next_v: torch.Tensor = comm.send_recv(v)
        comm.commit()

    if not causal or step <= comm.rank:
        block_out, _, _, _, _, block_lse, _, _ = _flash_attn_forward(
            q, k, v, dropout_p,
            softmax_scale, causal=causal and step == 0,
            window_size=window_size, alibi_slopes=alibi_slopes,
            return_softmax=True and dropout_p > 0,
        )
        out, lse = update_out_and_lse(out, lse, block_out, block_lse)

    if step + 1 != comm.world_size:
        comm.wait()
        k = next_k
        v = next_v

out = out.to(q.dtype)
lse = lse.squeeze(dim=-1).transpose(1, 2)
return out, lse
```

zhuzilin/ring-flash-attention is an excellent open-source ring attention using Tri Dao flash-attention for inner blocks.

OK, now we have already peeked into the inner core of ring-attention.

But what is RingAttention?

Sequence Parallelism

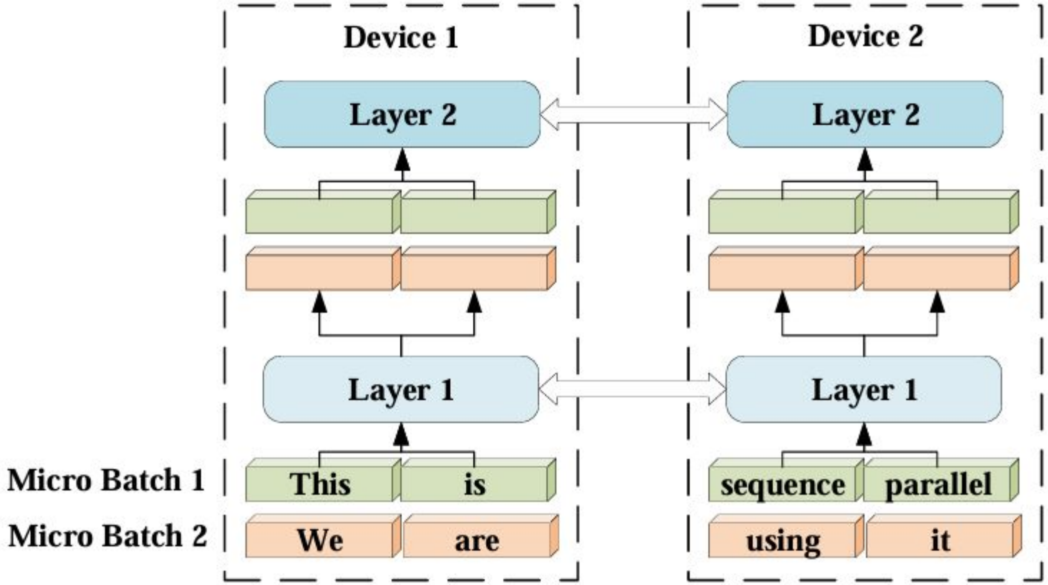
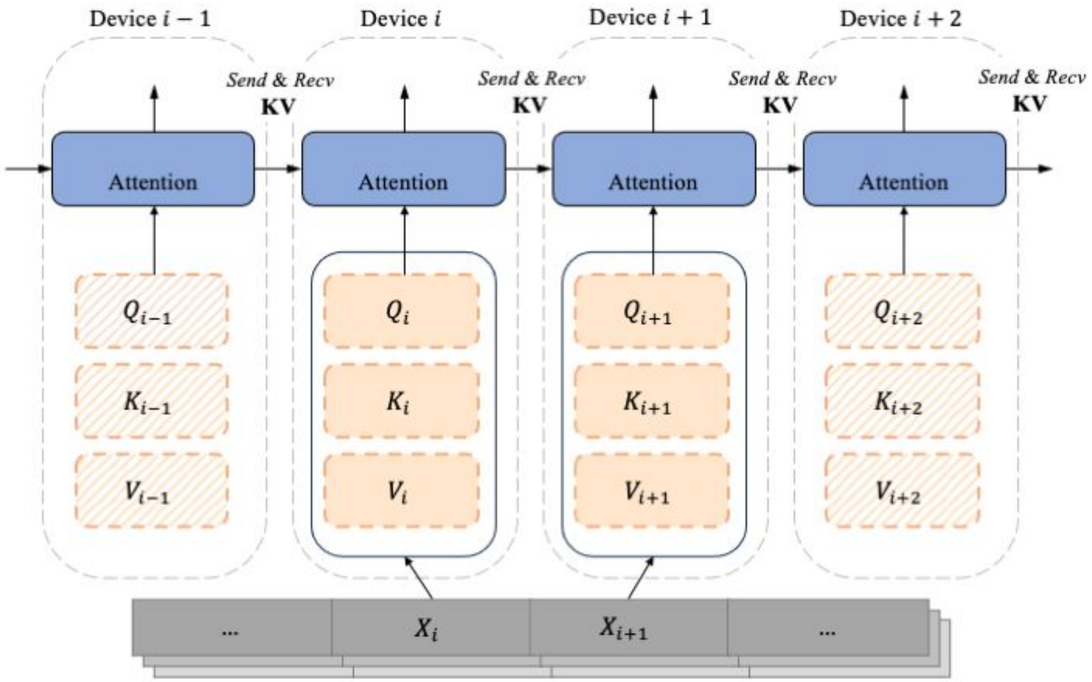


Fig: Sequence Parallelism

Batches are spread along sequence dimension evenly across devices.

Sequence Parallelism For Attention

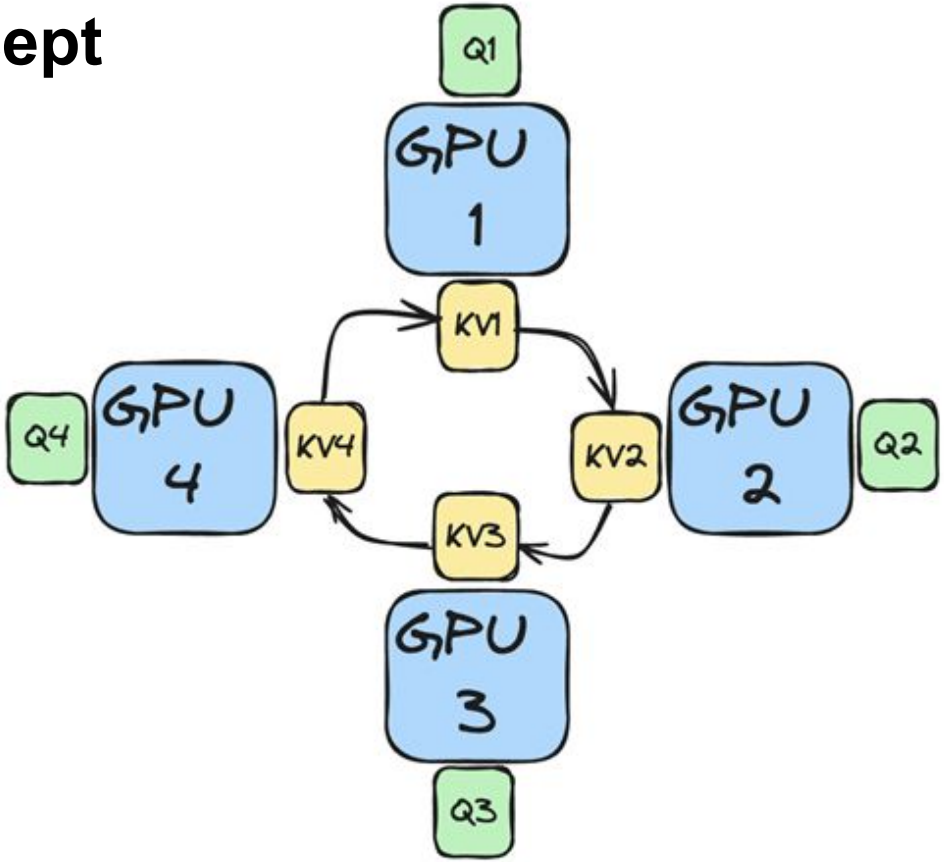


QKV split across devices

Adapted from [LASP]

Ring Attention - Main Concept

- order of block computations can be arbitrary
- split QKV sequence across N hosts
- hosts form a conceptional ring to exchange KV segments
- one pass completes when every node has seen all parts of the KV
- zero overhead for longer sequences: overlap computation and communication



Ring Attention Algo

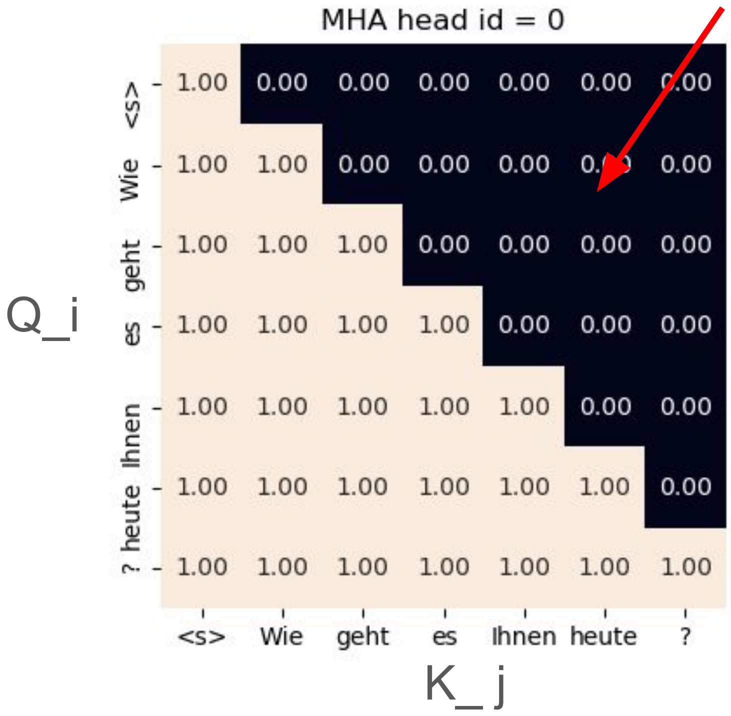
Algorithm 1 Reducing Transformers Memory Cost with Ring Attention.

Required: Input sequence x. Number of hosts N_h.
Initialize
Split input sequence into N_h blocks that each host has one input block.
Compute query, key, and value for its input block on each host.
for Each transformer layer **do**
  for $count = 1$ **to** $N_h - 1$ **do**
    for For each host concurrently. **do**
      Compute memory efficient attention incrementally using local query, key, value blocks.
      Send key and value blocks to next host and receive key and value blocks from previous host.
    end for
  end for
  for For each host concurrently. **do**
    Compute memory efficient feedforward using local attention output.
  end for
end for

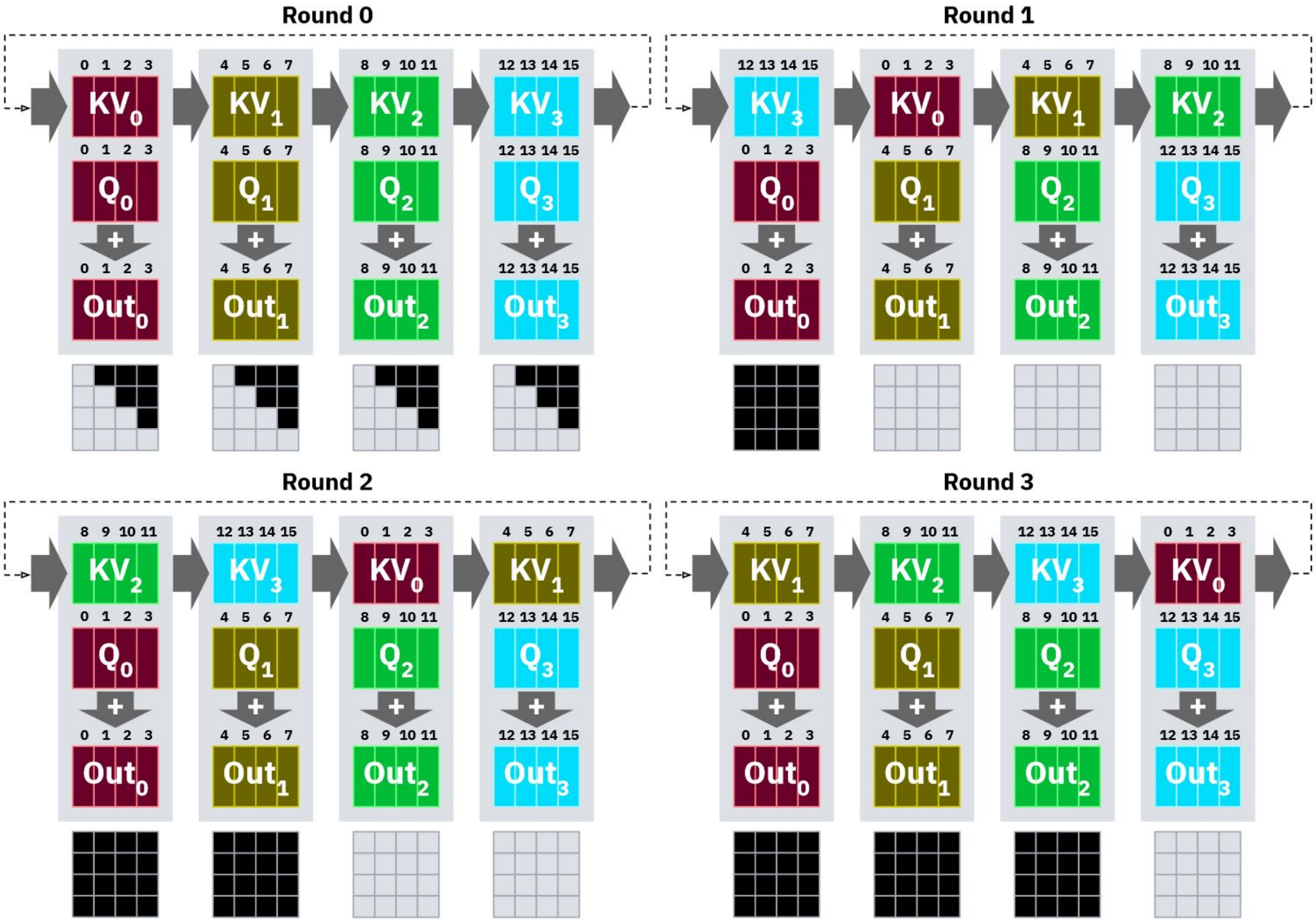
Recap: Causal Masking for Autoregressive Models

- required to support auto-regressive decoding
- outputs depend only on current and previous inputs
- attention score becomes:
  dot(Q_i, K_j) if i <= j else -inf
- no need to materialize mask: computed on the fly in kernel
- kernels like flash attention skip completely masked key blocks

mask 0 becomes -inf,
softmax output: 0



Attention Mask

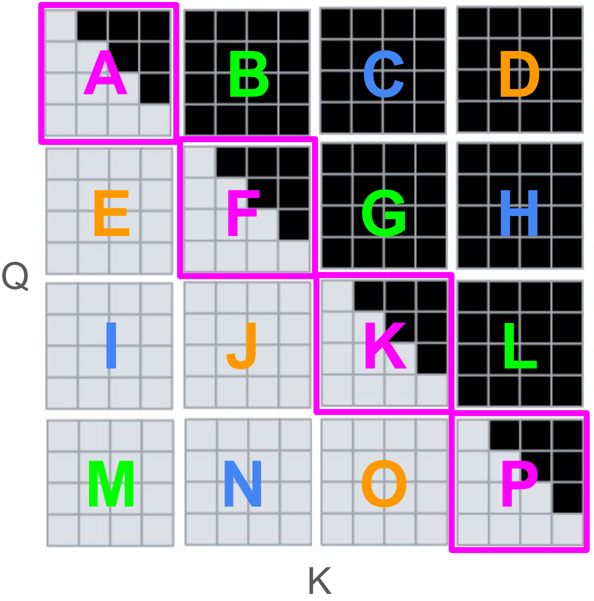


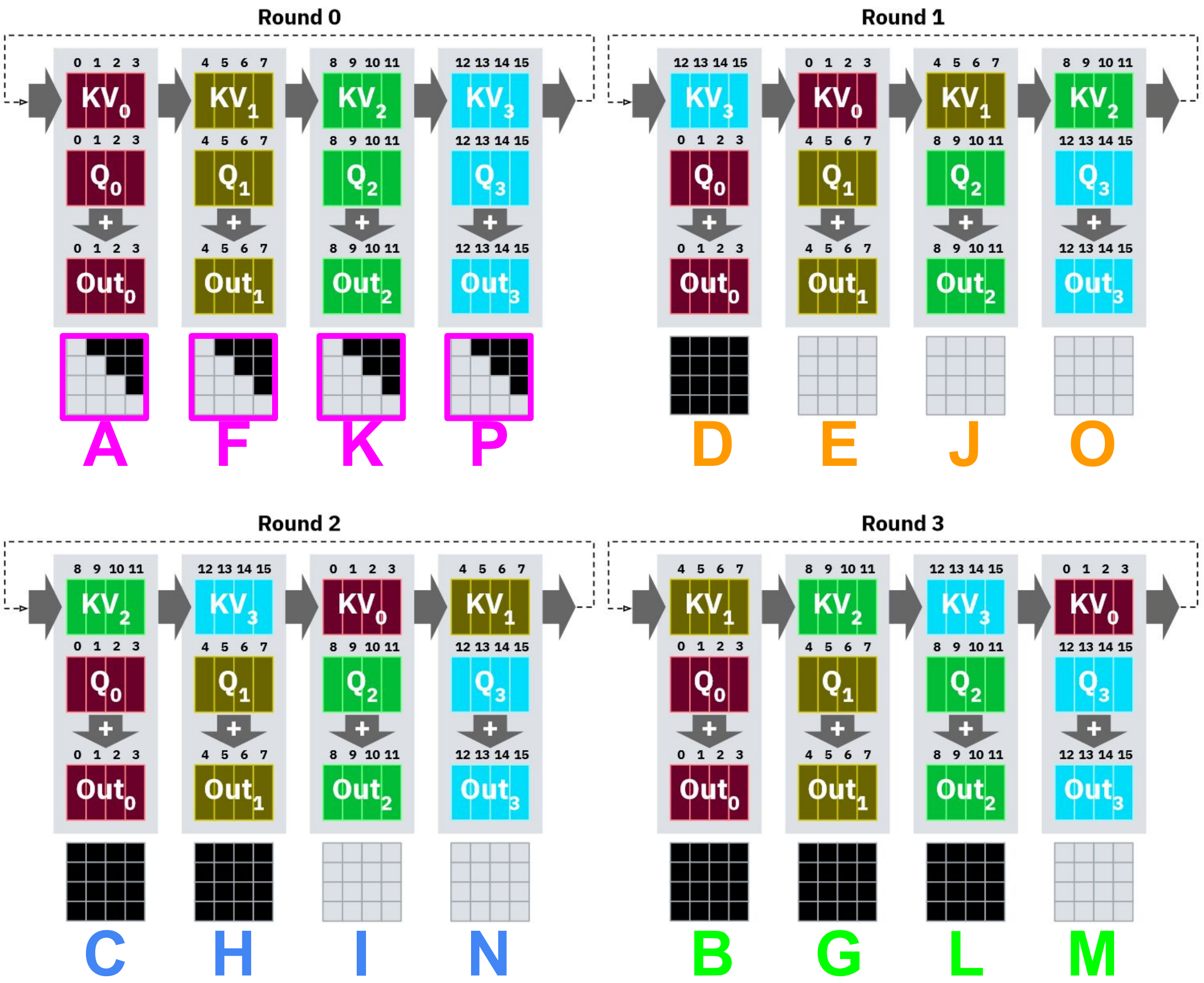
Ring Attention Problem:

Devices idle when causally masking, i.e for all auto-regressive models (very common)

output is 0 if Query_index < Key_index

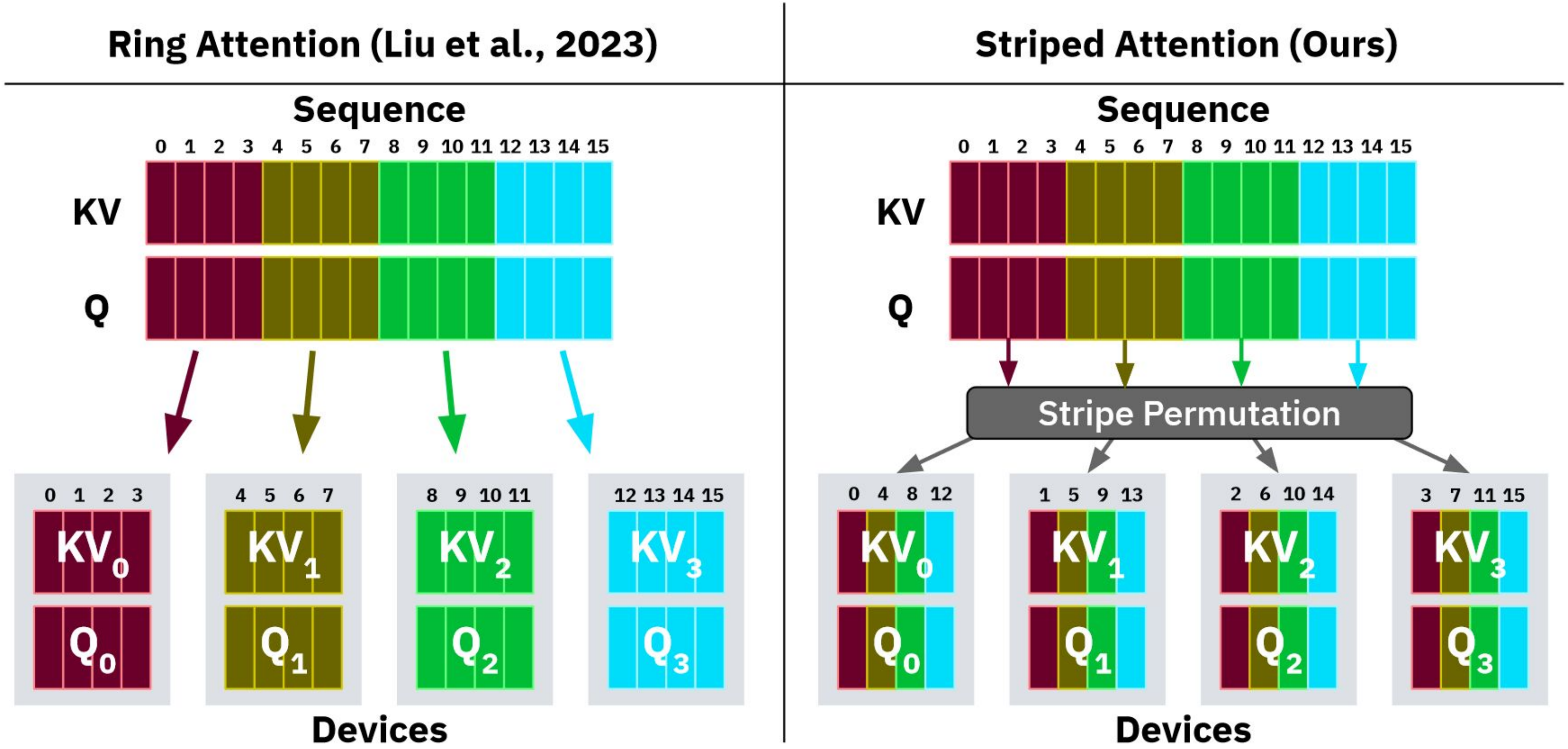
slowest ring host determines pace

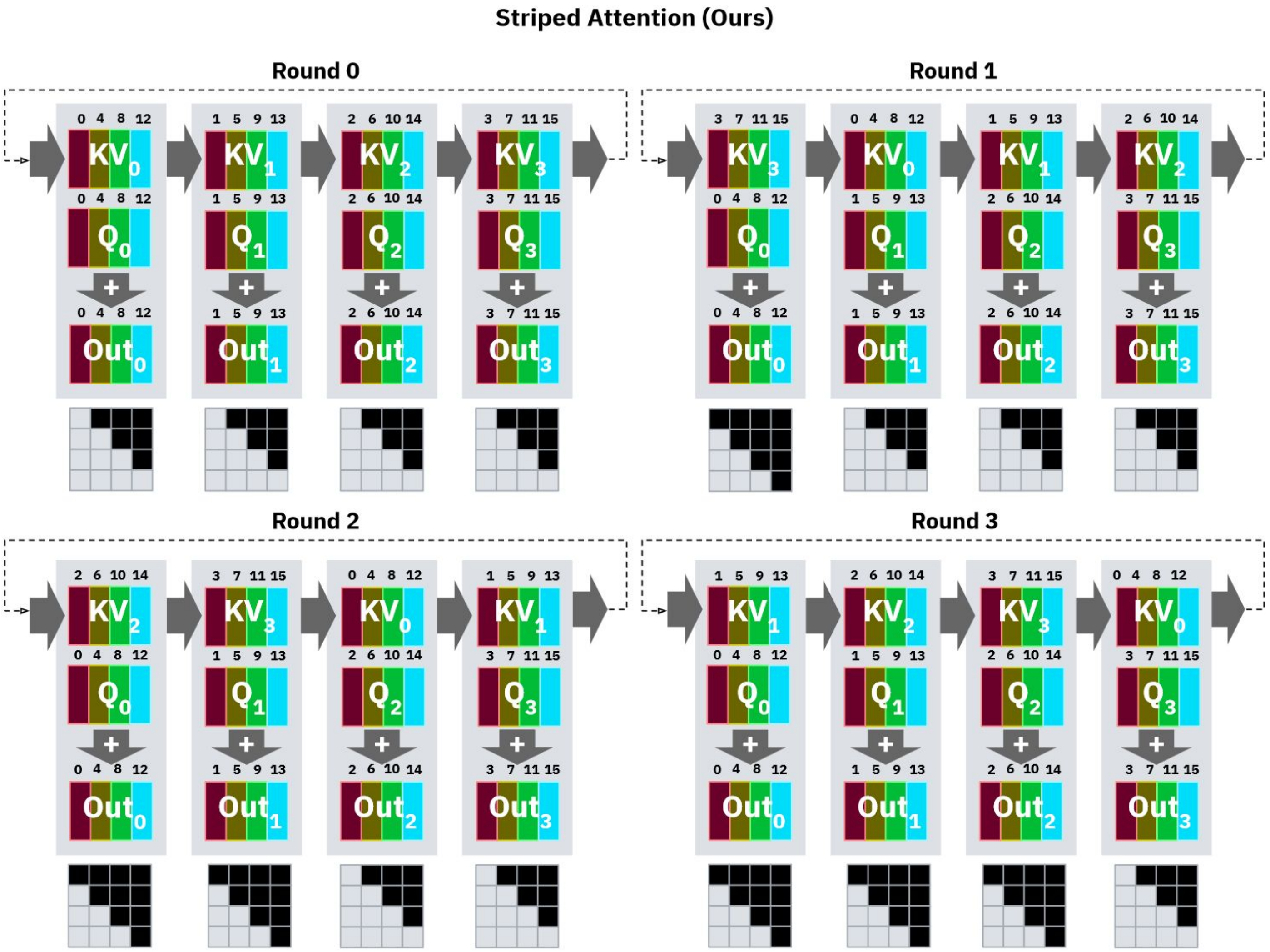




Causal mask chunks applied in different RingAttention rounds

Solution: 🦓 Striped Attention (Reorder QKV)





After reordering QKV the computations are almost perfectly distributed.

We need only to drop the first query and last key if host_id < round and can then use a standard causal flash attention kernel to compute the block.

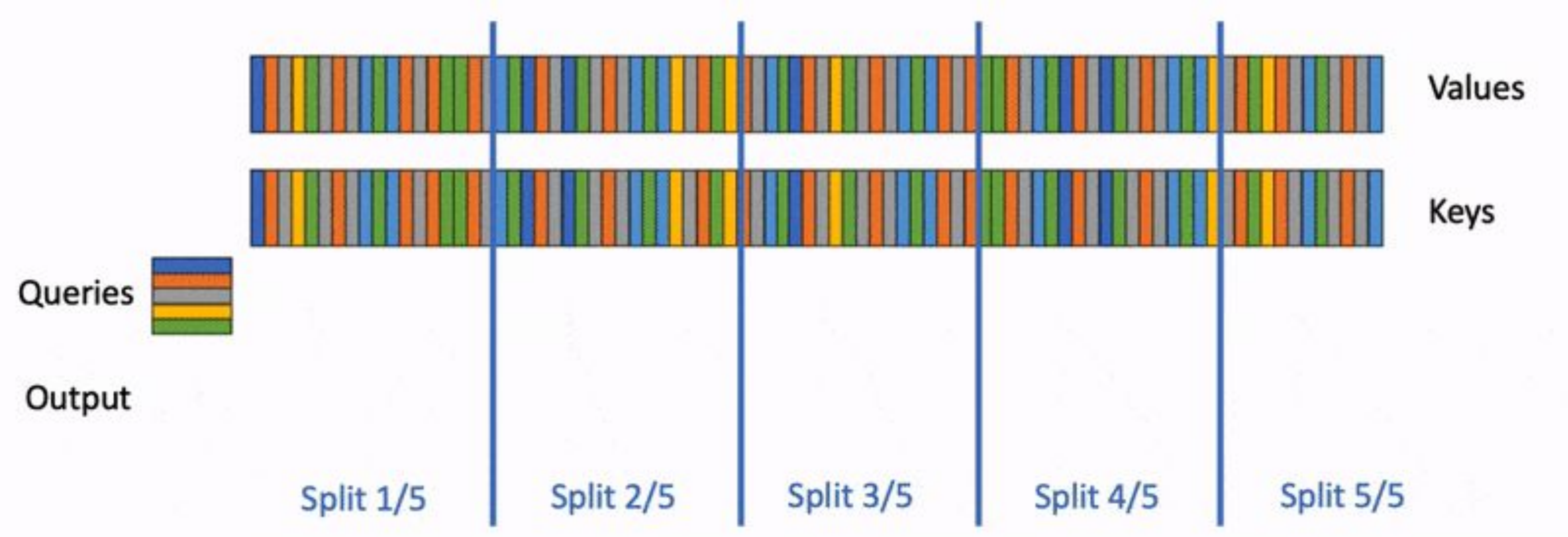
FlashAttenion: Sub-optimal for Long-Context Inference

FlashAttention parallelizes across blocks of queries and batch size only, and does not manage to occupy the entire GPU during token-by-token decoding.

Decoding with Flash Attention

Animation: Stanford CRFM Blog
Flash-Decoding

Solution: Flash-Decoding



Flash-Decoding, Dao et al.

That's all for today…

History / Papers

- May 2022, Tri Dao et al.
  FlashAttention: Fast and Memory-Efficient Exact Attention with IO-Awareness
- Aug 2023 Hao Liu et al.
  Blockwise Parallel Transformer for Large Context Models
- Nov 2023, Hao Liu et al.
  Ring Attention with Blockwise Transformers for Near-Infinite Context
- Nov 2023, Brandon et al.
  Striped Attention: Faster Ring Attention for Causal Transformers
- Feb 2024, Hao Liu et al.
  World Models on Million-Length Video and Language With RingAttention

Code:

- 🌟 zhuzilin / **ring-flash-attention**
- **cuda-mode/ring-attention: howto_log_sum_exp.ipynb**