Appendix A: Detailed Comparison Results of S_{ratio}

TABLE A.1
$\mathcal{S}_{ratio}$ results of three SB algorithms with the shape being a rectangle or hypercube and $\theta = 0.005$

d	N	δ	Method	γ						
				$\gamma = 0°$	$\gamma = 30°$	$\gamma = 60°$	$\gamma = 90°$	$\gamma = 120°$	$\gamma = 150°$	$\gamma = 180°$
$d = 2$	$N = 100$	$\delta = 1$	FSB-1	1.0000	0.7882	0.7913	1.0000	0.7900	0.7882	1.0000
			FSB-2	1.0000	0.9842	0.9844	1.0000	0.9846	0.9839	1.0000
			DSB	0.9960	0.9959	0.9959	0.9958	0.9959	0.9958	0.9958
		$\delta = 10$	FSB-1	1.0000	0.5168	0.5143	1.0000	0.5556	0.5574	1.0000
			FSB-2	1.0000	0.6641	0.6561	1.0000	0.7112	0.7187	1.0000
			DSB	0.9241	0.9284	0.9306	0.9246	0.9288	0.9290	0.9227
		$\delta = 100$	FSB-1	1.0000	0.0625	0.0623	1.0000	0.0676	0.0679	1.0000
			FSB-2	1.0000	0.0858	0.0827	1.0000	0.0931	0.0935	1.0000
			DSB	0.4464	0.3789	0.3802	0.4450	0.3869	0.3810	0.4431
	$N = 1000$	$\delta = 1$	FSB-1	1.0000	0.8339	0.8282	1.0000	0.8306	0.8266	1.0000
			FSB-2	1.0000	0.9983	0.9984	1.0000	0.9983	0.9984	1.0000
			DSB	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		$\delta = 10$	FSB-1	1.0000	0.7965	0.7959	1.0000	0.7884	0.7957	1.0000
			FSB-2	1.0000	0.9279	0.9361	1.0000	0.9308	0.9499	1.0000
			DSB	0.9982	0.9984	0.9984	0.9982	0.9983	0.9984	0.9982
		$\delta = 100$	FSB-1	1.0000	0.1121	0.1116	1.0000	0.1120	0.1122	1.0000
			FSB-2	1.0000	0.1448	0.1503	1.0000	0.1495	0.1572	1.0000
			DSB	0.9230	0.9289	0.9313	0.9228	0.9313	0.9288	0.9262
$d = 3$	$N = 100$	$\delta = 1$	FSB-1	1.0000	0.7016	0.6995	0.9948	0.7025	0.7039	0.9879
			FSB-2	0.9285	0.8678	0.8679	0.9283	0.8683	0.8686	0.9281
			DSB	0.8354	0.8355	0.8361	0.8347	0.8357	0.8344	0.8365
		$\delta = 10$	FSB-1	1.0000	0.3323	0.3308	0.9987	0.3269	0.3351	0.9768
			FSB-2	0.8523	0.4633	0.4587	0.8522	0.4265	0.4259	0.8518
			DSB	0.4479	0.3952	0.3948	0.4397	0.4004	0.3996	0.4433
		$\delta = 100$	FSB-1	1.0000	0.0380	0.0380	0.9999	0.0380	0.0380	0.9565
			FSB-2	0.8355	0.0541	0.0539	0.8355	0.0499	0.0498	0.8354
			DSB	0.0762	0.0471	0.0464	0.0818	0.0468	0.0474	0.0769
	$N = 1000$	$\delta = 1$	FSB-1	1.0000	0.7922	0.7956	1.0000	0.7949	0.7936	1.0000
			FSB-2	1.0000	0.9851	0.9852	1.0000	0.9853	0.9854	1.0000
			DSB	0.9887	0.9887	0.9887	0.9887	0.9886	0.9886	0.9885
		$\delta = 10$	FSB-1	1.0000	0.5544	0.5468	1.0000	0.5456	0.5427	1.0000
			FSB-2	1.0000	0.7072	0.7464	1.0000	0.7533	0.7067	1.0000
			DSB	0.9017	0.9009	0.9013	0.9023	0.8989	0.9016	0.9023
		$\delta = 100$	FSB-1	1.0000	0.0668	0.0671	1.0000	0.0672	0.0670	1.0000
			FSB-2	1.0000	0.0950	0.0998	1.0000	0.1001	0.0949	1.0000
			DSB	0.4682	0.3872	0.3857	0.4697	0.3819	0.3831	0.4635
$d = 4$	$N = 100$	$\delta = 1$	FSB-1	0.6454	0.4715	0.4689	0.6455	0.4711	0.4719	0.6388
			FSB-2	0.5815	0.5332	0.5317	0.5740	0.5292	0.5334	0.5688
			DSB	0.3489	0.3534	0.3521	0.3479	0.3523	0.3527	0.3510
		$\delta = 10$	FSB-1	0.6439	0.1758	0.1764	0.6479	0.1758	0.1761	0.6332
			FSB-2	0.3827	0.2151	0.2508	0.4200	0.2497	0.2152	0.3665
			DSB	0.0941	0.0758	0.0784	0.0917	0.0752	0.0770	0.0945
		$\delta = 100$	FSB-1	0.6472	0.0195	0.0195	0.6439	0.0195	0.0195	0.6236
			FSB-2	0.3756	0.0209	0.0258	0.3797	0.0257	0.0209	0.3503
			DSB	0.0080	0.0038	0.0039	0.0076	0.0039	0.0037	0.0078
	$N = 1000$	$\delta = 1$	FSB-1	0.9791	0.7319	0.7359	0.9786	0.7302	0.7335	0.9777
			FSB-2	0.9305	0.8920	0.8920	0.9307	0.8918	0.8923	0.9283
			DSB	0.8647	0.8661	0.8663	0.8648	0.8653	0.8661	0.8649
		$\delta = 10$	FSB-1	0.9795	0.3904	0.3898	0.9791	0.3866	0.3871	0.9747
			FSB-2	0.7227	0.4620	0.5147	0.7287	0.5095	0.4590	0.7024
			DSB	0.5511	0.5343	0.5287	0.5548	0.5327	0.5304	0.5490
		$\delta = 100$	FSB-1	0.9791	0.0454	0.0453	0.9794	0.0456	0.0456	0.9731
			FSB-2	0.6730	0.0495	0.0549	0.6719	0.0548	0.0493	0.6449
			DSB	0.1135	0.0846	0.0832	0.1121	0.0820	0.0828	0.1135

TABLE A.2
$\mathcal{S}_{ratio}$ results of three SB algorithms with the shape being a rectangle or hypercube and $\theta = 0.001$

d	N	δ	Method	γ						
				$\gamma = 0°$	$\gamma = 30°$	$\gamma = 60°$	$\gamma = 90°$	$\gamma = 120°$	$\gamma = 150°$	$\gamma = 180°$
$d = 2$	$N = 100$	$\delta = 1$	FSB-1	1.0000	0.7877	0.7869	1.0000	0.7901	0.7880	1.0000
			FSB-2	1.0000	0.9844	0.9839	1.0000	0.9843	0.9837	1.0000
			DSB	0.9959	0.9959	0.9958	0.9959	0.9958	0.9958	0.9958
		$\delta = 10$	FSB-1	1.0000	0.5218	0.5148	1.0000	0.5510	0.5567	1.0000
			FSB-2	1.0000	0.6711	0.6559	1.0000	0.7110	0.7141	1.0000
			DSB	0.9229	0.9277	0.9276	0.9240	0.9265	0.9283	0.9233
		$\delta = 100$	FSB-1	1.0000	0.0624	0.0622	1.0000	0.0680	0.0679	1.0000
			FSB-2	1.0000	0.0859	0.0825	1.0000	0.0930	0.0933	1.0000
			DSB	0.4466	0.3772	0.3776	0.4421	0.3835	0.3815	0.4420
	$N = 1000$	$\delta = 1$	FSB-1	1.0000	0.8304	0.8329	1.0000	0.8291	0.8306	1.0000
			FSB-2	1.0000	0.9984	0.9984	1.0000	0.9983	0.9984	1.0000
			DSB	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		$\delta = 10$	FSB-1	1.0000	0.7961	0.7952	1.0000	0.7949	0.7961	1.0000
			FSB-2	1.0000	0.9242	0.9391	1.0000	0.9302	0.9500	1.0000
			DSB	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9982
		$\delta = 100$	FSB-1	1.0000	0.1117	0.1120	1.0000	0.1117	0.1118	1.0000
			FSB-2	1.0000	0.1449	0.1502	1.0000	0.1496	0.1569	1.0000
			DSB	0.9238	0.9305	0.9300	0.9240	0.9318	0.9300	0.9236
$d = 3$	$N = 100$	$\delta = 1$	FSB-1	1.0000	0.7025	0.6981	0.9965	0.7007	0.7033	0.9924
			FSB-2	0.9288	0.8660	0.8675	0.9283	0.8676	0.8685	0.9288
			DSB	0.8355	0.8376	0.8373	0.8352	0.8358	0.8357	0.8349
		$\delta = 10$	FSB-1	1.0000	0.3331	0.3285	0.9992	0.3292	0.3338	0.9865
			FSB-2	0.8524	0.4565	0.4597	0.8521	0.4286	0.4286	0.8520
			DSB	0.4412	0.4034	0.3991	0.4443	0.3972	0.3931	0.4449
		$\delta = 100$	FSB-1	1.0000	0.0380	0.0380	0.9998	0.0379	0.0380	0.9718
			FSB-2	0.8355	0.0541	0.0545	0.8354	0.0498	0.0498	0.8354
			DSB	0.0779	0.0479	0.0466	0.0764	0.0458	0.0481	0.0778
	$N = 1000$	$\delta = 1$	FSB-1	1.0000	0.7959	0.7899	1.0000	0.7952	0.7898	1.0000
			FSB-2	1.0000	0.9853	0.9851	1.0000	0.9855	0.9850	1.0000
			DSB	0.9886	0.9886	0.9886	0.9884	0.9886	0.9887	0.9883
		$\delta = 10$	FSB-1	1.0000	0.5444	0.5464	1.0000	0.5481	0.5454	1.0000
			FSB-2	1.0000	0.7077	0.7512	1.0000	0.7519	0.7083	1.0000
			DSB	0.9014	0.9014	0.9005	0.9018	0.9014	0.8988	0.9031
		$\delta = 100$	FSB-1	1.0000	0.0671	0.0671	1.0000	0.0668	0.0672	1.0000
			FSB-2	1.0000	0.0951	0.1004	1.0000	0.1001	0.0948	1.0000
			DSB	0.4631	0.3894	0.3917	0.4671	0.3820	0.3846	0.4598
$d = 4$	$N = 100$	$\delta = 1$	FSB-1	0.6465	0.4708	0.4672	0.6421	0.4687	0.4724	0.6418
			FSB-2	0.5807	0.5315	0.5330	0.5775	0.5330	0.5343	0.5747
			DSB	0.3454	0.3498	0.3521	0.3495	0.3486	0.3475	0.3479
		$\delta = 10$	FSB-1	0.6500	0.1765	0.1759	0.6462	0.1769	0.1769	0.6354
			FSB-2	0.3826	0.2149	0.2509	0.4204	0.2504	0.2175	0.3708
			DSB	0.0945	0.0789	0.0765	0.0923	0.0755	0.0776	0.0908
		$\delta = 100$	FSB-1	0.6453	0.0196	0.0196	0.6466	0.0195	0.0195	0.6293
			FSB-2	0.3756	0.0209	0.0257	0.3798	0.0257	0.0209	0.3551
			DSB	0.0076	0.0040	0.0038	0.0070	0.0037	0.0037	0.0090
	$N = 1000$	$\delta = 1$	FSB-1	0.9791	0.7308	0.7307	0.9783	0.7320	0.7345	0.9780
			FSB-2	0.9320	0.8912	0.8920	0.9304	0.8900	0.8933	0.9285
			DSB	0.8640	0.8661	0.8675	0.8642	0.8663	0.8668	0.8648
		$\delta = 10$	FSB-1	0.9791	0.3875	0.3857	0.9790	0.3881	0.3887	0.9761
			FSB-2	0.7223	0.4612	0.5073	0.7263	0.5069	0.4623	0.7094
			DSB	0.5521	0.5312	0.5320	0.5513	0.5322	0.5314	0.5483
		$\delta = 100$	FSB-1	0.9793	0.0455	0.0454	0.9791	0.0454	0.0454	0.9746
			FSB-2	0.6732	0.0495	0.0547	0.6707	0.0548	0.0494	0.6572
			DSB	0.1113	0.0860	0.0827	0.1115	0.0836	0.0855	0.1149

TABLE A.3
$\mathcal{S}_{ratio}$ results of three SB algorithms with the shape being an ellipse or hyperellipsoid and $\theta = 0.005$

d	N	δ	Method	γ						
				$\gamma = 0°$	$\gamma = 30°$	$\gamma = 60°$	$\gamma = 90°$	$\gamma = 120°$	$\gamma = 150°$	$\gamma = 180°$
$d = 2$	$N = 100$	$\delta = 1$	FSB-1	0.7525	-	-	-	-	-	-
			FSB-2	0.9715	-	-	-	-	-	-
			DSB	0.9981	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.7550	0.5851	0.5890	0.7545	0.5442	0.5471	0.7538
			FSB-2	0.8893	0.7289	0.7388	0.8814	0.7001	0.6843	0.8892
			DSB	0.9356	0.9427	0.9425	0.9375	0.9418	0.9420	0.9378
		$\delta = 100$	FSB-1	0.7558	0.0663	0.0657	0.7558	0.0602	0.0608	0.7554
			FSB-2	0.7683	0.0911	0.0910	0.7705	0.0844	0.0796	0.7746
			DSB	0.4513	0.3833	0.3864	0.4534	0.3874	0.3808	0.4543
	$N = 1000$	$\delta = 1$	FSB-1	0.7520	-	-	-	-	-	-
			FSB-2	0.9722	-	-	-	-	-	-
			DSB	1.0000	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.7561	0.8253	0.8233	0.7586	0.8275	0.8262	0.7564
			FSB-2	0.9624	0.9364	0.9513	0.9619	0.9311	0.9409	0.9634
			DSB	0.9992	0.9993	0.9993	0.9992	0.9993	0.9993	0.9992
		$\delta = 100$	FSB-1	0.7543	0.1112	0.1099	0.7599	0.1093	0.1097	0.7551
			FSB-2	0.7919	0.1477	0.1556	0.7893	0.1421	0.1493	0.7926
			DSB	0.9376	0.9440	0.9447	0.9392	0.9427	0.9433	0.9387
$d = 3$	$N = 100$	$\delta = 1$	FSB-1	0.4750	-	-	-	-	-	-
			FSB-2	0.7626	-	-	-	-	-	-
			DSB	0.8460	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.4676	0.2338	0.2332	0.4730	0.2312	0.2333	0.4734
			FSB-2	0.5385	0.3902	0.3881	0.5376	0.3950	0.4123	0.5355
			DSB	0.4672	0.4201	0.4193	0.4696	0.4201	0.4147	0.4685
		$\delta = 100$	FSB-1	0.4774	0.0266	0.0267	0.4758	0.0255	0.0256	0.4768
			FSB-2	0.4537	0.0385	0.0383	0.4530	0.0375	0.0412	0.4526
			DSB	0.0870	0.0500	0.0509	0.0843	0.0491	0.0519	0.0841
	$N = 1000$	$\delta = 1$	FSB-1	0.5065	-	-	-	-	-	-
			FSB-2	0.9174	-	-	-	-	-	-
			DSB	0.9853	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.5088	0.3771	0.3771	0.5082	0.3794	0.3781	0.5062
			FSB-2	0.7328	0.7202	0.6870	0.7099	0.6906	0.7208	0.7322
			DSB	0.8988	0.8961	0.8952	0.8993	0.8965	0.8958	0.8991
		$\delta = 100$	FSB-1	0.5159	0.0448	0.0450	0.5089	0.0429	0.0435	0.5147
			FSB-2	0.5324	0.0778	0.0730	0.5270	0.0698	0.0756	0.5359
			DSB	0.4902	0.4093	0.4005	0.4924	0.4089	0.4052	0.4885
$d = 4$	$N = 100$	$\delta = 1$	FSB-1	0.2017	-	-	-	-	-	-
			FSB-2	0.3779	-	-	-	-	-	-
			DSB	0.3756	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.2034	0.0776	0.0784	0.2065	0.0783	0.0776	0.2032
			FSB-2	0.1578	0.1575	0.1333	0.1319	0.1344	0.1565	0.1570
			DSB	0.1091	0.0877	0.0913	0.1109	0.0886	0.0892	0.1072
		$\delta = 100$	FSB-1	0.2120	0.0084	0.0082	0.2101	0.0084	0.0081	0.2089
			FSB-2	0.1092	0.0109	0.0090	0.1084	0.0089	0.0110	0.1121
			DSB	0.0106	0.0049	0.0044	0.0095	0.0045	0.0042	0.0096
	$N = 1000$	$\delta = 1$	FSB-1	0.3033	-	-	-	-	-	-
			FSB-2	0.7038	-	-	-	-	-	-
			DSB	0.8416	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.3042	0.1677	0.1706	0.3032	0.1703	0.1683	0.3003
			FSB-2	0.3482	0.4083	0.3730	0.3289	0.3698	0.4164	0.3454
			DSB	0.5593	0.5372	0.5386	0.5615	0.5428	0.5386	0.5614
		$\delta = 100$	FSB-1	0.3108	0.0186	0.0189	0.3129	0.0188	0.01865	0.3125
			FSB-2	0.2214	0.0254	0.0239	0.2267	0.0239	0.0260	0.2272
			DSB	0.1354	0.0952	0.0988	0.1330	0.0972	0.1002	0.1353

TABLE A.4
$\mathcal{S}_{ratio}$ results of three SB algorithms with the shape being an ellipse or hyperellipsoid and $\theta = 0.001$

d	N	δ	Method	γ						
				$\gamma = 0°$	$\gamma = 30°$	$\gamma = 60°$	$\gamma = 90°$	$\gamma = 120°$	$\gamma = 150°$	$\gamma = 180°$
$d = 2$	$N = 100$	$\delta = 1$	FSB-1	0.7627	-	-	-	-	-	-
			FSB-2	0.9726	-	-	-	-	-	-
			DSB	0.9981	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.7529	0.5763	0.5823	0.7560	0.5468	0.5463	0.7539
			FSB-2	0.8898	0.7382	0.7392	0.8837	0.6997	0.6814	0.8905
			DSB	0.9373	0.9424	0.9426	0.9369	0.9419	0.9426	0.9360
		$\delta = 100$	FSB-1	0.7545	0.0657	0.0656	0.7581	0.0606	0.0606	0.7547
			FSB-2	0.7720	0.0916	0.0910	0.7714	0.0844	0.0807	0.7758
			DSB	0.4584	0.3854	0.3840	0.4526	0.3815	0.3794	0.4531
	$N = 1000$	$\delta = 1$	FSB-1	0.7551	-	-	-	-	-	-
			FSB-2	0.9715	-	-	-	-	-	-
			DSB	1.0000	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.7524	0.8257	0.8247	0.7591	0.8289	0.8318	0.7549
			FSB-2	0.9609	0.9356	0.9527	0.9603	0.9321	0.9411	0.9627
			DSB	0.9992	0.9993	0.9993	0.9992	0.9993	0.9993	0.9992
		$\delta = 100$	FSB-1	0.7581	0.1095	0.1094	0.7571	0.1102	0.1096	0.7539
			FSB-2	0.7978	0.1477	0.1561	0.7925	0.1429	0.1487	0.7926
			DSB	0.9393	0.9427	0.9450	0.9374	0.9439	0.9431	0.9389
$d = 3$	$N = 100$	$\delta = 1$	FSB-1	0.4663	-	-	-	-	-	-
			FSB-2	0.7599	-	-	-	-	-	-
			DSB	0.8446	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.4725	0.2304	0.2268	0.4757	0.2267	0.2285	0.4698
			FSB-2	0.5382	0.3821	0.3817	0.5333	0.3905	0.4058	0.5356
			DSB	0.4714	0.4209	0.4176	0.4708	0.4177	0.4184	0.4685
		$\delta = 100$	FSB-1	0.4665	0.0250	0.0248	0.4756	0.0246	0.0246	0.4765
			FSB-2	0.4456	0.0366	0.0359	0.4441	0.0362	0.0387	0.4504
			DSB	0.0852	0.0492	0.0511	0.0840	0.0498	0.0499	0.0850
	$N = 1000$	$\delta = 1$	FSB-1	0.5105	-	-	-	-	-	-
			FSB-2	0.9157	-	-	-	-	-	-
			DSB	0.9853	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.5059	0.3723	0.3710	0.5068	0.3734	0.3689	0.5060
			FSB-2	0.7296	0.7154	0.6875	0.7165	0.6878	0.7166	0.7320
			DSB	0.8999	0.8949	0.8948	0.8994	0.8948	0.8950	0.8999
		$\delta = 100$	FSB-1	0.5072	0.0430	0.0420	0.5143	0.0421	0.0421	0.5129
			FSB-2	0.5240	0.0725	0.0682	0.5275	0.0680	0.0713	0.5296
			DSB	0.4937	0.4087	0.4011	0.4938	0.4062	0.4051	0.4897
$d = 4$	$N = 100$	$\delta = 1$	FSB-1	0.2051	-	-	-	-	-	-
			FSB-2	0.3766	-	-	-	-	-	-
			DSB	0.3746	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.2033	0.0776	0.0771	0.2056	0.0763	0.0789	0.2040
			FSB-2	0.1551	0.1575	0.1346	0.1306	0.1342	0.1577	0.1567
			DSB	0.1091	0.0906	0.0908	0.1109	0.0901	0.0902	0.1103
		$\delta = 100$	FSB-1	0.2005	0.0081	0.0081	0.2010	0.0080	0.0080	0.2037
			FSB-2	0.1070	0.0107	0.0088	0.1053	0.0088	0.0109	0.1085
			DSB	0.0099	0.0051	0.0044	0.0102	0.0047	0.0046	0.0091
	$N = 1000$	$\delta = 1$	FSB-1	0.3002	-	-	-	-	-	-
			FSB-2	0.7039	-	-	-	-	-	-
			DSB	0.8424	-	-	-	-	-	-
		$\delta = 10$	FSB-1	0.3015	0.1702	0.1668	0.2980	0.1707	0.1651	0.3052
			FSB-2	0.3495	0.4102	0.3662	0.3314	0.3706	0.4080	0.3464
			DSB	0.5583	0.5428	0.5385	0.5589	0.5415	0.5383	0.5594
		$\delta = 100$	FSB-1	0.3031	0.0182	0.0184	0.3062	0.0183	0.0184	0.3014
			FSB-2	0.2188	0.0253	0.0237	0.2202	0.0233	0.0249	0.2186
			DSB	0.1301	0.0968	0.0976	0.1325	0.0958	0.0989	0.1318