#### 1

# More Details for VPP-ART

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## 1 DESCRIPTIONS OF VPP-ART ALGORITHMS

Algorithm 1 presents the pseudo-code of VPP-ART; Algorithm 2 describes the pseudo-code of **InsertTCIntoVPtree** $(tc, \lambda, \varepsilon, node)$ , i.e., the Insertion/Promotion function; while Algorithm 3 gives the pseudo-code of **GetMinDistFromVPtree** $(c_j, node)$ , i.e., the search process to get the minimal distance.

#### **Algorithm 1: VPP-ART**

#### Inputs:

The candidate set size, k;

The number of dimensions in the SUT input domain, d;

The capacity of a leaf node,  $\lambda$ ;

The partitioning parameter,  $\varepsilon$ ;

#### Output:

The VP-tree with executed test cases, *T*;

- 1: Set *T* ← {}; /\* Initially, VP-tree for storing the executed test cases is empty. \*/
- 2: Set  $C \leftarrow \{\}$ ; /\* Initially, candidate test case set C, for storing k candidates in each round, is empty. \*/
- 3: Set  $min_{dist}[k] \leftarrow \infty$ ; /\* Initially, the minimum distance between each candidate test case and T is set to infinity. \*/
- 4: Randomly generate a test case tc from the input domain
- 5: Execute tc;
- 6: **while** (*No termination condition is satisfied*)
- 7: **InsertTCIntoVPtree** $(tc, \lambda, \varepsilon, node)$ ;
- 8: Randomly generate k candidate test cases  $c_1, c_2, \cdots, c_k$  from the input domain, then set  $C \leftarrow \{c_1, c_2, \cdots, c_k\}$ ;
- 9: **for** (each candidate  $c_j \in C$ , where  $j = 1, 2, \dots, k$ )
- 10: Set  $min_{dist}[j] \leftarrow \mathbf{GetMinDistFromVPtree}(c_j, node);$
- 11: end\_for
- 12: Select  $c_{best}$  from  $c_1, c_2, \dots, c_k$  such that it has the maximum distance to its nearest executed test case;
- 13: Set  $tc \leftarrow c_{best}$  and execute tc;
- 14: end\_while
- 15: return T;
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# **Algorithm 2: InsertTCIntoVPtree** $(tc, \lambda, \varepsilon, node)$

```
The executed test case need to insert into the tree, tc;
The capacity of a leaf node, \lambda;
The number of subsets partition, \varepsilon;
A node of the VP-tree, node;
Output:
Success flag, FALSE or TRUE;
1: if (node is tc-QBN (a leaf node))
       if (|tc\text{-QBN}| < \lambda)
3:
          Insert tc into tc-QBN;
          Set tc-BN \leftarrow tc-QBN;
4:
5:
          return TRUE;
 6:
       else if (|tc\text{-QBN}| = \lambda) /* Node promotion strategy */
7:
          Randomly select a test case from tc-QBN, as the vantage
          test case, vp;
8:
          for (each test case p \in tc-QBN \cup \{tc\})
             Calculate dist(p, vp);
10:
             Sort the test cases in ascending order according to the
             distance values;
11:
          end_for
12:
          Calculate the boundary distance values \mu_i according to
          Eq. (2);
13:
          Calculate \sigma of current node, using Eq. (6);
14:
          Partition the input domain into \varepsilon sub-domains;
          /* Each sub-domain contains approximately the same
          number of test cases */
15:
          Reorganize all the test cases to the children nodes of this
          new common node:
16:
          if (allocate the new tc-BN)
17:
             return TRUE;
18:
          end_if
19:
       end_if
20: else
21:
       Calculate dist(vp, tc);
       if (\mu_i < dist(vp, tc) \le \mu_{i+1} where 0 \le i \le \varepsilon - 2)
22:
23:
          InsertTCIntoVPtree(tc, \lambda, \varepsilon, node.i-th child node);
24:
25:
       if (dist(vp, tc) > \mu_{\varepsilon-1})
          InsertTCIntoVPtree(tc, node.(\varepsilon - 1)-th child node);
27:
       end if
28: end_if
```

# 2 VPP-ART PARAMETER SETTINGS

The VPP-ART performance is strongly impacted by two parameters: the partitioning parameter,  $\varepsilon$ ; and the maximum test case capacity of a leaf node,  $\lambda$ . These two values play important roles in the partitioning of the input domain, and also limit the amount of executed test cases in each subdomain, which can affect the accuracy of the NN returned by VPP-ART. When VPP-ART performs the NN search in one sub-domain, the approximate NN is identified. When there are more executed test cases in this sub-domain —

#### Algorithm 3: GetMinDistFromVPtree $(c_j, node)$

```
Inputs:
A candidate test case, c_j;
A node of the VP-tree, node;
Output:
The minimum distance between c_i and E, min_{dist}[j];
 1: min_{dist}[j] \leftarrow \infty;
 2: if (node is leaf node)
       for (each test case p in node)
           if (dist(c_j, p) < min_{dist}[j])
              min_{dist}[j] \leftarrow dist(c_j, p);
 6:
           end if
 7:
        end_for
       return min_{dist}[j];
 8:
 9: else
       Calculate dist(c_j, vp);
10:
        for (0 \le i \le \varepsilon - 2)
11:
           if (\mu_i - \sigma < dist(vp, c_i) \le \mu_{i+1} + \sigma)
13:
              Get \sigma of current common node;
14:
              GetMinDistFromVPtree(c_j, node.i-th child node);
15:
           end if
16:
        end_for
17:
        if (dist(c_i, vp) > \mu_{\varepsilon-1})
           GetMinDistFromVPtree(c_j, node.(\varepsilon - 1)-th child node);
18:
19:
20: end_if
```

which is directly influenced by the static values of  $\varepsilon$  and  $\lambda$  — then VPP-ART will have a greater probability of finding a more accurate NN (of course, it may also be an approximate NN due to the construction of the modified VP-tree). If the

approximate NN returned by VPP-ART is similar to the exact NN, the failure-detection effectiveness of VPP-ART and FSCS-ART will be comparable. Therefore, this section focuses on the influence of different  $\langle \varepsilon, \lambda \rangle$  parameter pair values on VPP-ART. The specific parameter settings for the simulations were as follows:

- Dimension: d = 1, 2, 3, 4, 5, 8, 10;
- Failure rate:  $\theta = 0.0005$ ;
- Partitioning parameter,  $\varepsilon = 2, 3, 4, 5$ ;
- Maximum test case capacity of a leaf node,  $\lambda = 10, 15, 20, 25, 30, 35, 40, 45, 50$ .

Here, we briefly explain the reason for setting the failure rate  $(\theta)$  to 0.0005. The failure rate reflects the proportion of inputs that can cause software failures. Theoretically, the number of test cases used by RT to find the first failure is the reciprocal of  $\theta$ . When  $\theta=0.0005$ , RT should need (on average) 2000 test cases to find a failure. This value is appropriate for this experiment. Specifically, when the number of test cases is very small, the breadth or depth of the constructed VP tree does not reflect the advantages of the algorithm. In other words, the cost of constructing the VP tree to partition the space may be greater than the cost of finding the NN. In addition, when there are too many test cases, the verification time can be huge. These conditions are not conducive to the exploration of parameters. Therefore, we identified  $\theta=0.0005$  as an appropriate failure rate.

Table 1 presents the *ART F-ratio* simulation results of VPP-ART for the different parameter values. Based on the

TABLE 1 ART F-ratio results of VPP-ART with different  $\langle \varepsilon, \lambda \rangle$  parameter pair values

$\begin{array}{c} \textbf{Partitioning} \\ \textbf{Number} \\ (\varepsilon) \end{array}$						ART F-ra	atio				
	$\begin{array}{c} \textbf{Dimension} \\ (d) \end{array}$	TCCC ADT					VPP-ART				
	( <i>a</i> )	FSCS-ART	$\lambda = 10$	$\lambda = 15$	$\lambda = 20$	$\lambda = 25$	$\lambda = 30$	$\lambda = 35$	$\lambda = 40$	$\lambda = 45$	$\lambda = 50$
	d = 1	0.5564	0.5706	0.5652	0.5523	0.5611	0.5675	0.5747	0.5631	0.5634	0.5702
	d=2	0.6391	0.6790	0.6584	0.6752	0.6712	0.6645	0.6547	0.6641	0.6516	0.6440
	d = 3	0.7549	0.8056	0.7890	0.7809	0.7787	0.7964	0.8053	0.7957	0.8096	0.7970
$\varepsilon = 2$	d = 4	0.9033	0.9611	0.9329	0.9243	0.9391	0.9506	0.9539	0.9280	0.9368	0.9462
	d = 5	1.0462	1.0824	1.0945	1.0984	1.0852	1.0978	1.1147	1.1260	1.1200	1.1069
	d = 8	1.8607	1.7050	1.6886	1.7668	1.7673	1.8200	1.7956	1.7711	1.8460	1.8289
	d = 10	2.6138	2.4504	2.4896	2.6366	2.6495	2.6355	2.6268	2.7046	2.7357	2.7719
	d = 1	0.5564	0.5555	0.5605	0.5686	0.5584	0.5557	0.5573	0.5599	0.5514	0.5582
	d=2	0.6391	0.6510	0.6601	0.6447	0.6559	0.6377	0.6661	0.6459	0.6725	0.6523
	d = 3	0.7549	0.7553	0.7534	0.8083	0.7832	0.7753	0.7663	0.8063	0.8037	0.7940
$\varepsilon = 3$	d = 4	0.9033	0.9390	0.9448	0.9510	0.9385	0.9385	0.9345	0.9355	0.9484	0.9421
	d = 5	1.0462	1.0605	1.0890	1.0709	1.0698	1.1085	1.0876	1.0997	1.1178	1.1053
	d = 8	1.8607	1.6631	1.8007	1.7354	1.7543	1.8558	1.8406	1.8833	1.8870	1.8493
	d = 10	2.6138	2.2916	2.4821	2.4765	2.5670	2.6265	2.6319	2.7906	2.7058	2.7080
	d = 1	0.5564	0.5577	0.5636	0.5621	0.5613	0.5529	0.5726	0.5567	0.5607	0.5588
	d = 2	0.6391	0.6570	0.6587	0.6590	0.6739	0.6534	0.6296	0.6340	0.6448	0.6529
	d = 3	0.7549	0.7617	0.7680	0.7571	0.7604	0.7659	0.7519	0.7900	0.7709	0.7754
$\varepsilon = 4$	d = 4	0.9033	0.9144	0.9159	0.9272	0.9086	0.9418	0.9572	0.9427	0.9418	0.9393
	d = 5	1.0462	1.0896	1.0891	1.0734	1.0727	1.1486	1.0854	1.0964	1.0948	1.1133
	d = 8	1.8607	1.6900	1.6744	1.8339	1.8043	1.8048	1.8067	1.8578	1.9174	1.9717
	d = 10	2.6138	2.3144	2.3562	2.6492	2.6325	2.8174	2.6989	2.8207	2.8412	2.9358
	d = 1	0.5564	0.5573	0.5542	0.5507	0.5822	0.5566	0.5639	0.5558	0.5585	0.5496
	d=2	0.6391	0.6467	0.6594	0.6416	0.6360	0.6536	0.6427	0.6412	0.6433	0.6447
	d = 3	0.7549	0.7912	0.7747	0.7547	0.7745	0.7674	0.7518	0.7610	0.7671	0.7726
$\varepsilon = 5$	d = 4	0.9033	0.9276	0.9167	0.9024	0.9194	0.9582	0.9239	0.9559	0.9592	0.9391
	d = 5	1.0462	1.0932	1.0849	1.0923	1.1254	1.1063	1.1072	1.1258	1.1370	1.1280
	d = 8	1.8607	1.7753	1.7982	1.8214	1.8652	1.8581	2.0066	1.9264	1.9821	1.9547
	d = 10	2.6138	2.4743	2.5811	2.6273	2.7442	2.7095	2.8409	2.9523	2.8931	2.9514

data in the table, some observations can be summarized as follows:

(1) As the maximum test case capacity of leaf nodes  $(\lambda)$  increases, the VPP-ART *ART F-ratio* differences (when  $1 \leq d \leq 5$ ) are not significant, but are, on the whole, slightly higher than FSCS-ART; For d=8,10, the *ART F-ratio* values increase gradually, and are lower than FSCS-ART when  $\lambda$  is small. This shows that changes in  $\lambda$  have little impact on the failure-detection effectiveness of VPP-ART in low dimensional input domains, but can have a great

impact in high dimensions. Therefore, a small  $\lambda$  value can effectively ensure VPP-ART failure-detection effectiveness in low dimensions, and improve FSCS-ART performance in high dimensions.

(2) When the partition parameter ( $\varepsilon$ ) increases above 2, the VPP-ART *ART F-ratio* values are not significantly different when  $1 \le d \le 5$ . When d=8,10, the *ART F-ratio* values show a trend of first decreasing, and then increasing; an inflection point appears when  $\varepsilon=3$ . Similar to  $\lambda$ , changes in  $\varepsilon$  have little impact on VPP-ART failure-detection

TABLE 2
ART F-ratio results and statistical analysis comparisons among VPP-ART and other ART algorithms for *block patterns* 

				ART F-r	atio		Statistical Analysis									
$\begin{array}{c} \textbf{Dimension} \\ (d) \end{array}$	Failure Rate $(\theta)$	VPP-	FSCS-	Naive-	SemiBal-	LimBal-	vs. l	FSCS-ART	vs. Na	ive-KDFC	vs. Semi	Bal-KDFC	vs. Liml	Bal-KDFC		
( <i>a</i> )	(0)	ART		KDFC	KDFC			effect size	p-value	effect size	p-value	effect size	p-value	effect size		
	0.01	0.5634	0.5729	0.5664	0.5714	0.5658	0.3679	0.5067	0.7899	0.5020	0.4101	0.5061	0.5294	0.5047		
	0.005		0.5633		0.5696	0.5619	0.8994	0.4991	0.7826	0.5021	0.4768	0.5053	0.8362	0.4985		
	0.002		0.5683		0.5723	0.5605	0.5704	0.5042	0.9981	0.5000	0.5034	0.5050	0.7817	0.4979		
d = 1	0.001		0.5720		0.5690	0.5617	0.6041	0.5039	0.1890	0.4902	0.5659	0.5043	0.7915	0.4980		
	0.0005		0.5564		0.5556	0.5619	0.5919	0.5040	0.4510	0.5056	0.7454	0.5024	0.4847	0.5052		
	0.0002		0.5700		0.5662	0.5614	0.0602	0.5140	0.1353	0.5111	0.2506	0.5086	0.2506	0.5086		
	0.0001		0.5765		0.5545	0.5569	0.8939	0.4990	0.0327	0.4841	0.0559	0.4857	0.1115	0.4881		
	0.01		0.6911		0.6904	0.6953	0.1867	0.5098	0.4907	0.5051 0.4932	0.1303	0.5113	0.0805	0.5130		
	0.005	0.6750	0.6613		0.6635	0.6671	0.7798	0.4979	0.3619		0.6590	0.4967	0.8597	0.4987		
d = 2	0.002		0.6536		0.6633	0.6561	0.5933	0.4960	0.7498 0.1795	0.4976 0.4900	0.5793	0.5041	0.5865	0.4959 0.4963		
a = 2	0.001 0.0005		0.6573 <b>0.6391</b>		0.6557 0.6484	0.6595 0.6492	0.5260 0.9636	0.4953 0.4997	0.1793	0.4900	0.6460 0.4341	0.4966 0.5058	0.6193 0.3357	0.4963		
	0.0003		0.6268		0.6414	0.6388	0.3030	0.4997	0.2177	0.3092	0.4541	0.3038	0.5000	0.3072		
	0.0002		0.6248		0.6389	0.6313	0.5895	0.5040	0.0030	0.5222	0.0321	0.5129	0.4206	0.5060		
			0.8641													
	0.01 0.005		0.8314		0.8504 0.8195	<b>0.8391</b> 0.8177	0.6879 0.3398	0.5030 0.5071	0.2535 0.9302	0.4915 0.5007	0.7570 0.6204	0.4977 0.5037	0.2964 0.5875	0.4922 0.4960		
	0.003		0.7847		0.8193	0.8052	0.3398	0.3071	0.9302	0.3007	0.5126	0.3037	0.3310	0.4980		
d = 3	0.002		0.7735		0.7735	0.7772	0.2033	0.4917	0.0981	0.4877	0.3120	0.4894	0.0703	0.4865		
a = 3	0.0005		0.7549		0.7618	0.7615	0.2115	0.5093	0.3457	0.5070	0.1302	0.5081	0.2614	0.5084		
	0.0003		0.7499		0.7441	0.7464	0.5469	0.4955	0.5871	0.4960	0.1881	0.4902	0.2464	0.4914		
	0.0001		0.7358		0.7518	0.7387	0.6934	0.4971	0.1312	0.4887	0.7021	0.4971	0.2108	0.4907		
-	0.01		1.0786		1.0711	1.0666	0.9147	0.5008	0.6676	0.5032	0.9517	0.4995	0.9651	0.5003		
	0.005		1.0272		1.0200	1.0202	0.5394	0.4954	0.8604	0.5013	0.4641	0.4945	0.5352	0.4954		
	0.002		0.9606		0.9711	0.9754	0.8243	0.4983	0.5681	0.4957	0.9008	0.5009	0.9820	0.4998		
d = 4	0.001		0.9155		0.9190	0.9366	0.7930	0.5020	0.4660	0.4946	0.8987	0.4991	0.8987	0.5009		
	0.0005	0.9390	0.9033	0.8908	0.8904	0.9067	0.1379	0.4889	0.1067	0.4880	0.0464	0.4852	0.2141	0.4907		
	0.0002	0.8965	0.8522	0.8494	0.8651	0.8708	0.0963	0.4876	0.2810	0.4920	0.1513	0.4893	0.6204	0.4963		
	0.0001	0.8857	0.8357	0.8234	0.8687	0.8491	0.3925	0.4936	0.3509	0.4930	0.4141	0.5061	0.8148	0.5017		
	0.01	1.3417	1.3346	1.3416	1.3268	1.3209	0.6070	0.5038	0.9107	0.5008	0.8170	0.4983	0.5754	0.5042		
	0.005		1.2694		1.2632	1.2550	0.7747	0.4979	0.8919	0.4990	0.6168	0.4963	0.7876	0.5020		
	0.002		1.1661		1.1685	1.1550	0.3047	0.5077	0.1059	0.5121	0.5585	0.5044	0.5848	0.5041		
d = 5	0.001		1.1097		1.1317	1.0850	0.9833	0.5002	0.6000	0.5039	0.2411	0.5087	0.6566	0.4967		
	0.0005		1.0462		1.0584	1.0217	0.7815	0.5021	0.7631	0.5022	0.8088	0.5018	0.4692	0.4946		
	0.0002 0.0001		1.0156 0.9935		1.0215 <b>0.9810</b>	1.0054 0.9867	0.8649 0.8739	0.4987 0.5012	0.5564 0.7783	0.4956 0.4979	0.5235 0.3574	0.5048 0.4931	0.5712 0.4456	0.4958 0.4943		
	0.01		2.6802		2.6390	2.5701	0.0000	0.5319	0.0030	0.5221	0.0046	0.5211	0.0026	0.5225		
	0.005		2.4032		2.3672	2.2685	0.0081	0.5197	0.0010	0.5246	0.0079	0.5198	0.1401	0.5110		
1 0	0.002		2.1176		2.0986	2.0333	0.0672	0.5136	0.0511	0.5145	0.1144	0.5118	0.1073	0.5120		
d = 8	0.001 0.0005		1.9526 1.8607		1.9525 1.8163	1.8306 1.7219	0.0004	0.5263 0.5352	0.0027 0.0000	0.5224 0.5303	0.0068 0.0015	0.5202 0.5237	0.1988 0.1321	0.5096 0.5112		
				1.7099	1.6956	1.7219		0.5332		0.5314		0.5237		0.5112		
	0.0002 0.0001		1.6325		1.6110	1.5027	0.0000	0.5510	0.0000 $0.0000$	0.5452	0.0000	0.5502	0.1752 <b>0.0003</b>	0.5267		
-	0.01		3.9718		3.9454	3.8735	0.0000	0.5539	0.0000	0.5552	0.0000	0.5567	0.0000	0.5583		
	0.005		3.5995		3.5591	3.4597	0.0000	0.5359	0.0000	0.5332	0.0000	0.5447	0.0000	0.5333		
	0.003		3.1565		3.1184	2.9093	0.0000	0.5372	0.0000	0.5342	0.0000	0.5306	0.0000	0.5168		
d = 10	0.001		2.8741		2.9142	2.6868	0.0004	0.5263	0.0000	0.5243	0.0005	0.5259	0.1794	0.5100		
. 10	0.0005		2.7049		2.7002	2.4310	0.0000	0.5432	0.0000	0.5466	0.0000	0.5443	0.0728	0.5134		
	0.0002		2.4118		2.4727	2.1770	0.0000	0.5536	0.0000	0.5528	0.0000	0.5614	0.0007	0.5252		
	0.0001		2.2568		2.3156	2.0289	0.0000	0.5481	0.0000	0.5466	0.0000	0.5549	0.0150	0.5181		

effectiveness in low dimensions, but show a certain change trend in the high dimensions. Therefore, a smaller  $\varepsilon$  value can enhance the VPP-ART performance.

Based on the above, the parameter pair  $\langle \varepsilon, \lambda \rangle$  were assigned  $\langle 3, 10 \rangle$  for the simulations and experiments.

## 3 DETAILED EXPERIMENTAL DATA

In the tables in this section, the *blue bold* typeface indicates the minimum value of *ART F-ratio*, *F-measure* or *F-time* across the several ART algorithms; and the *red bold* means that

the p-value of the comparison between VPP-ART and the corresponding ART algorithm is less than 0.05, indicating significance.

#### 3.1 Comparisons of Failure-Detection Effectiveness

## 3.1.1 Answer to RQ1 - Part 1: Results of Simulations

Tables 2 to 4 provide the detailed ART F-ratio simulation results and the statistical comparisons of VPP-ART against other ART algorithms, according to the block, strip, and point patterns, respectively.

TABLE 3
ART F-ratio results and statistical analysis comparisons among VPP-ART and other ART algorithms for *strip patterns* 

	F" D'			ART F-1	atio		Statistical Analysis									
Dimension (d)	Failure Rate $(\theta)$	VPP-	FSCS-	Naive-	SemiBal-	LimBal-	vs. l	SCS-ART	vs. Na	ive-KDFC	vs. Semi	Bal-KDFC	vs. Lim	Bal-KDFC		
( <i>u</i> )	(0)	ART		KDFC	KDFC			effect size	p-value	effect size	p-value	effect size	p-value	effect size		
	0.01		0.5729		0.5714	0.5658	0.3679	0.5067	0.7899	0.5020	0.4101	0.5061	0.5294	0.5047		
	0.005		0.5633		0.5696	0.5619	0.8994	0.4991	0.7826	0.5021	0.4768	0.5053	0.8362	0.4985		
	0.002		0.5683		0.5723	0.5605	0.5704	0.5042	0.9981	0.5000	0.5034	0.5050	0.7817	0.4979		
d = 1	0.001		0.5720		0.5690	0.5617	0.6041	0.5039	0.1890	0.4902	0.5659	0.5043	0.7915	0.4980		
	0.0005		0.5564		0.5556	0.5619	0.5919	0.5040	0.4510	0.5056	0.7454	0.5024	0.4847	0.5052		
	0.0002		0.5700		0.5662	0.5614	0.0602	0.5140	0.1353	0.5111	0.2506	0.5086	0.2506	0.5086		
	0.0001		0.5765		0.5545	0.5569	0.8939	0.4990	0.0327	0.4841	0.0559	0.4857	0.1115	0.4881		
	0.01 0.005		0.9816		0.9490	<b>0.9276</b> 0.9456	<b>0.0415</b> 0.2303	0.5152 0.5089	0.3121 0.7102	0.5075 0.5028	0.2007 0.9005	0.5095 0.5009	0.4732 0.4696	0.5053		
	0.003		0.9716 0.9961		0.9279	0.9436	0.2303		0.7102	0.5028	0.3378	0.5009	0.4696	0.5054 0.5072		
<i>1</i> _ 0					0.9611			0.5138				0.5071				
d=2	0.001 0.0005		0.9561 0.9784		0.9775 <b>0.9641</b>	<b>0.9547</b> 0.9808	0.2154 0.4873	0.4908 0.4948	0.9536 0.3322	0.5004 0.4928	0.9391 0.0716	0.4994	0.9948 0.7090	0.5000 0.4972		
	0.0003		0.9764		0.9915	0.9811	0.4073	0.4948	0.3322	0.4928	0.5323	0.4953	0.7090	0.4972		
	0.0002		1.0130		0.9513 0.9534	0.9760	0.2020	0.5091	0.1222	0.5052	0.3523	0.4933	0.5044	0.5050		
	0.01		0.9639		0.9850	0.9491	0.6975	0.5029	0.8514	0.5014	0.2837	0.5080	0.6560	0.4967		
	0.005		0.9404		0.9803	0.9809	0.5083	0.4951	0.6035	0.5039	0.4249	0.5059	0.3965	0.5063		
1 _ 2	0.002		0.9853		0.9918	0.9653	0.3922	0.4936	0.0275	0.4836	0.9863	0.4999 0.4984	0.1312	0.4887		
d = 3	0.001		0.9514		0.9757	1.0010 0.9832	0.1059	0.4879	0.5522	0.5044	0.8288		0.7370	0.5025		
	0.0005 0.0002		0.9978		0.9510	0.9832	0.7362 0.6592	0.4975	0.4889 0.7665	0.4948	0.1556	0.4894	0.2067	0.4906 0.5024		
	0.0002		0.9734 <b>0.9945</b>		<b>0.9730</b> 1.0572	1.0066	0.6392	0.4967 0.4998	0.4034	0.4978 0.5062	0.5442 <b>0.0476</b>	0.4955 0.5148	0.7425 0.6856	0.3024		
	0.01		0.9733		0.9895	0.9723	0.7517	0.4976	0.5908	0.5040	0.9209	0.5007	0.4972	0.5051		
	0.005		0.9830		0.9604	0.9602	0.9922	0.4999	0.7150	0.5027	0.6021	0.4961	0.3329	0.4928		
7 4	0.002		1.0274		0.9749	0.9919	0.2591	0.5084	0.6052	0.5039	0.8414	0.4985	0.7604	0.5023		
d = 4	0.001		0.9982		0.9767	0.9807	0.9649	0.5003	0.9953	0.5000	0.4106	0.4939	0.9186	0.4992		
	0.0005		1.0038		0.9968	0.9792	0.8139	0.5018	0.2896	0.5079	0.8391	0.4985	0.4397	0.4942		
	0.0002 0.0001		1.0081 1.0268		1.0117 1.0038	1.0206 0.9911	0.1872 <b>0.0456</b>	0.5098 0.5149	0.5527 0.5598	0.5044 0.5043	0.0689 0.5370	0.5136 0.5046	0.2540 0.3980	0.5085 0.5063		
-	0.01		1.0162		0.9806	1.0228	0.1548	0.5106	0.3827	0.5065	0.4749	0.5053	0.0056	0.5206		
	0.005		1.0102		1.0002	0.9613	0.3629	0.5068	0.3627	0.5104	0.5924	0.5040	0.6791	0.4969		
	0.002		1.0108		0.9871	1.0363	0.6384	0.5035	0.4244	0.4940	0.5290	0.4953	0.3940	0.5064		
d = 5	0.001		0.9791		1.0275	1.0236	0.4323	0.4941	0.8983	0.4990	0.5204	0.5048	0.5039	0.5050		
a = 0	0.0005		1.0236		0.9708	1.0223	0.3717	0.5067	0.2420	0.5087	0.7043	0.4972	0.4610	0.5055		
	0.0002		0.9751		1.0208	0.9881	0.5492	0.4955	0.9281	0.5007	0.3255	0.5073	0.6977	0.5029		
	0.0001		1.0039		0.9953	0.9832	0.7696	0.4978	0.4521	0.4944	0.6092	0.4962	0.4384	0.4942		
	0.01	0.9446	0.9907	0.9836	0.9847	1.0045	0.0642	0.5138	0.3624	0.5068	0.2449	0.5087	0.2238	0.5091		
	0.005		1.0145		1.0094	0.9781	0.3021	0.4923	0.0229	0.4830	0.6840	0.4970	0.3628	0.4932		
	0.002		0.9905		1.0024	1.0316	0.3255	0.5073	0.1617	0.5104	0.6831	0.5030	0.3427	0.5071		
d = 8	0.001	1.0189	1.0107	0.9999	1.0069	1.0411	0.4169	0.4939	0.7538	0.4977	0.6408	0.4965	0.9866	0.5001		
	0.0005		1.0123		0.9830	0.9866	0.1190	0.4884	0.1358	0.4889	0.0036	0.4783	0.0290	0.4837		
	0.0002			0.9967		0.9942	0.5020	0.5050	0.8309	0.5016	0.5052	0.4950	0.6009	0.5039		
	0.0001	0.9932	1.0207	1.0074	0.9979	0.9935	0.3650	0.5068	0.3201	0.5074	0.4655	0.5054	0.5424	0.5045		
	0.01		1.0068		0.9771	0.9967	0.8753	0.5012	0.1357	0.5111	0.3455	0.4930	0.9348	0.4994		
	0.005		1.0265		1.0067	1.0089	0.7691	0.5022	0.5492	0.5045	0.8914	0.5010	0.5979	0.4961		
	0.002		0.9933		0.9882	0.9974	0.5222	0.5048	0.8152	0.4983	0.8856	0.5011	0.7253	0.5026		
d = 10	0.001		1.0083		0.9946	0.9834	0.6407	0.5035	0.7539	0.5023	0.9126	0.5008	0.7832	0.4979		
	0.0005		1.0054		1.0388	1.0265	0.6510	0.4966	0.4760	0.4947	0.2862	0.5079	0.4809	0.5053		
	0.0002		1.0073	1.0096 1.0097	<b>0.9823</b> 0.9832	1.0246 1.0149	0.5084 0.2287	0.5049 0.5090	0.9860 0.1881	0.5001 0.5098	0.6003 0.9243	0.4961 0.4993	0.8356 0.3204	0.5015 0.5074		
	0.0001															

TABLE 4
ART F-ratio results and statistical analysis comparisons among VPP-ART and other ART algorithms for *point patterns* 

-				ART F-r	atio		Statistical Analysis										
Dimension $(d)$	Failure Rate $(\theta)$	VPP-	FSCS-	Naive-	SemiBal-	LimBal-	vs. I	SCS-ART	vs. Na	ive-KDFC	vs. Sem	iBal-KDFC	vs. Liml	Bal-KDFC			
	(*)	ART		KDFC	KDFC			effect size	p-value	effect size	p-value	effect size	p-value	effect size			
	0.01		0.9607		0.9621	0.9763	0.6231	0.5037	0.4279	0.5059	0.3374	0.5072	0.0750	0.5133			
	0.005		0.9543		0.9576	0.9320	0.0654	0.5137	0.0993	0.5123	0.1321	0.5112	0.6065	0.5038			
	0.002		0.9568		0.9825	0.9788	0.7519	0.4976	0.8121	0.5018	0.6477	0.5034	0.1144	0.5118			
d = 1	0.001		0.9346		0.9623	0.9651	0.0445	0.4850	0.8614	0.4987	0.1676	0.4897	0.2653	0.4917			
	0.0005		<b>0.9380</b> 0.9693		0.9446	0.9422 0.9655	0.3095	0.4924	0.5943	0.5040	0.7309	0.4974	0.2594	0.4916			
	0.0002 0.0001		0.9693		<b>0.9282</b> 0.9610	0.9633 0.9530	0.9067 0.8204	0.4991 0.5017	0.4938 0.9610	0.5051 0.4996	0.1021 0.9196	0.4878 0.5008	0.9205 0.6786	0.5007 0.4969			
	0.01	0.9979	0.9988	0.9918	0.9894	1.0207	0.2747	0.5081	0.2338	0.5089	0.8688	0.5012	0.3148	0.5075			
	0.005		0.9762		0.9825	0.9917	0.8852	0.5011	0.1460	0.5108	0.5882	0.5040	0.2836	0.5080			
	0.002		0.9675		0.9557	0.9877	0.7730	0.4979	0.4953	0.4949	0.1776	0.4900	0.6302	0.5036			
d=2	0.001		0.9995		0.9672	0.9817	0.0655	0.5137	0.7319	0.4974	0.5542	0.4956	0.9392	0.4994			
	0.0005		0.9663		0.9806	0.9777	0.0864	0.5128	0.1931	0.5097	0.0339	0.5158	0.0245	0.5168			
	0.0002		1.0034		0.9392	0.9428	0.1068	0.5120	0.7450	0.4976	0.1525	0.4893	0.2421	0.4913			
	0.0001		0.9792		0.9673	0.9556	0.9347	0.5006	0.4807	0.4947	0.8478	0.5014	0.7290	0.4974			
	0.01		1.1231		1.1084	1.0795	0.0005	0.5260	0.0149	0.5182	0.0038	0.5216	0.1723	0.5102			
	0.005		1.0744		1.0665	1.1051	0.5259	0.5047	0.0861	0.5128	0.9106	0.4992	0.0198	0.5174			
1 _ 2	0.002 0.001		1.0235 1.0343		1.0746 1.0548	1.0499 1.0551	0.7553 0.6355	0.5023 0.5035	0.5115 0.6821	0.5049 0.5031	0.0372	0.5155 0.5104	0.1239	0.5115 0.5099			
d=3	0.001		1.0043		1.0348	1.0077	0.8080	0.5033	0.8859	0.5081	0.1631 0.3116	0.5104	0.1841 0.8003	0.5019			
	0.0003		1.0017		1.0113	1.0077	0.8180	0.3018	0.2839	0.3080	0.7378	0.3075	0.7097	0.3019			
	0.0002		1.0073		0.9905	0.9824	0.9596	0.4996	0.2742	0.4918	0.9740	0.5002	0.5906	0.4960			
	0.01		1.3211		1.3035	1.3037	0.0023	0.5227	0.1548	0.5106	0.0181	0.5176	0.0014	0.5238			
	0.005		1.2614		1.2633	1.2192	0.0025	0.5165	0.1348	0.5094	0.0131	0.5170	0.2278	0.5090			
	0.002		1.1809		1.1524	1.1212	0.0205	0.5103	0.2072	0.5157	0.3352	0.5072	0.8672	0.4988			
d = 4	0.001		1.1137		1.1401	1.1370	0.0935	0.5125	0.1586	0.5105	0.0229	0.5170	0.0257	0.5166			
	0.0005		1.1117		1.0980	1.1065	0.1022	0.5122	0.3322	0.5072	0.2515	0.5085	0.0720	0.5134			
	0.0002		1.0521		1.0487	1.0510	0.5517	0.5044	0.0073	0.5200	0.4581	0.5055	0.5910	0.5040			
	0.0001		1.0837		1.0589	1.0509	0.2403	0.5088	0.9512	0.5005	0.6854	0.5030	0.6532	0.5033			
	0.01		1.5695		1.5603	1.5243	0.0003	0.5273	0.0012	0.5241	0.0007	0.5253	0.0206	0.5173			
	0.005		1.4785		1.4385	1.4456	0.0002	0.5278	0.0032	0.5219	0.0154	0.5181	0.0030	0.5222			
, -	0.002		1.3549		1.3691	1.3510	0.0000	0.5332	0.0001	0.5288	0.0000	0.5324	0.0000	0.5365			
d = 5	0.001		1.2964		1.3005	1.2110	0.0002	0.5282	0.0015	0.5237	0.0000	0.5312	0.0569	0.5142			
	0.0005 0.0002		1.2559 1.1746		1.2361	1.1938 1.1708	0.0006	0.5256	<b>0.0107</b> 0.3808	0.5190	0.0092	0.5194 0.4992	0.1488	0.5108 0.5054			
	0.0002		1.1740		<b>1.1562</b> 1.1553	1.1708 1.1074	0.4006 0.6017	0.5063 0.5039	0.2233	0.5065 0.5091	0.9149 0.1501	0.4992	0.4725 0.5762	0.3034			
	0.01	2.0945	2.4049	2.3487	2.4215	2.3374	0.0000	0.5377	0.0000	0.5366	0.0000	0.5400	0.0000	0.5339			
	0.005		2.3711		2.3313	2.2386	0.0000	0.5443	0.0000	0.5396	0.0000	0.5387	0.0000	0.5333			
	0.002		2.1827		2.1710	2.0722	0.0000	0.5515	0.0000	0.5491	0.0000	0.5432	0.0000	0.5380			
d = 8	0.001	1.7151	2.0761	2.1198	2.0804	1.9098	0.0000	0.5534	0.0000	0.5613	0.0000	0.5519	0.0000	0.5372			
	0.0005		1.9976		1.9881	1.8038	0.0000	0.5626	0.0000	0.5643	0.0000	0.5633	0.0000	0.5368			
	0.0002		1.7979		1.8695	1.6424	0.0000	0.5528	0.0000	0.5546	0.0000	0.5619	0.0000	0.5303			
	0.0001	1.4719	1.7575	1.7223	1.7567	1.5938	0.0000	0.5539	0.0000	0.5450	0.0000	0.5512	0.0037	0.5216			
	0.01		2.5080		2.5738	2.5994	0.0271	0.5165	0.0069	0.5201	0.0002	0.5277	0.0000	0.5320			
	0.005		2.8216		2.7286	2.7150	0.0000	0.5437	0.0000	0.5313	0.0000	0.5428	0.0000	0.5379			
1 10	0.002		2.8479		2.8899	2.6569	0.0000	0.5561	0.0000	0.5735	0.0000	0.5577	0.0000	0.5387			
d = 10	0.001		2.8835		2.8322	2.6249	0.0000	0.5657	0.0000	0.5743	0.0000	0.5672	0.0000	0.5439			
	0.0005		2.7033		2.6135	2.4393	0.0000	0.5710	0.0000	0.5743	0.0000	0.5602	0.0000	0.5408			
	0.0002		2.4606		2.5065	2.2301	0.0000	0.5711	0.0000	0.5723	0.0000	0.5738	0.0000	0.5446			
	0.0001	1.7778	2.0912	2.3241	2.3366	2.1341	0.0000	0.5445	0.0000	0.5708	0.0000	0.5744	0.0000	0.5553			

## 3.1.2 Answer to RQ1 - Part 2: Results of Experiments

Table 5 provides the detailed experimental data for the comparisons among VPP-ART and other different algorithms with the 22 subject programs.

#### 3.2 Comparisons of Efficiency

## 3.2.1 Answer to RQ2 - Part 1: Results of Simulations

Figure 1 provides the test case generation times of VPP-ART, FSCS-ART, and the three KDFC-ART algorithms, for various

test suite sizes, in different dimensional input domains (d = 1, 2, 3, 4, 5, 8, 10). In the figures, the x-axis shows the size of the test suite (N), and the y-axis shows the time taken to generate the N test cases.

## 3.2.2 Answer to RQ2 - Part 2: Results of Experiments

Table 6 reports the average time taken to detect the first failure (*F-time*) in each program, by each algorithm. In addition, this table also provides the pairwise statistical analysis of VPP-ART against other ART algorithms.

		1	F-measu	re			Statistical Analysis												
Program	А								vs. RT	710 1	SCS-ART				Bal-KDEC	vs. LimBal-KDi			
Tiogram	d	VPP- ART	RT	FSCS- ART	Naive- KDFC	SemiBal- KDFC		v-value			effect size								
airy	1	707 20	1448.70	816 N2	806.91	803.29		0.0000	0.6248	0.1793	0.5100	0.3791	0.5066	0.4055	0.5062	0.3163	0.5075		
bessi0	1		758.91		443.44	440.31	449.13	0.0000	0.5246	0.1793	0.5100	0.9175	0.5008	0.4033	0.3062	0.5561	0.5044		
erfcc	1			1054.65					0.6067	0.0301	0.5162	0.3323	0.5072	0.1319	0.5112	0.2828	0.5080		
probks	1			1469.21			1475.86		0.6212	0.4970	0.5051	0.9523	0.5004	0.9427	0.5005	0.4633	0.5055		
tanh	1	312.42		319.82	306.91	309.85	309.36	0.0000	0.6133	0.3084	0.5076	0.3982	0.4937	0.7942	0.4981	0.4720	0.4946		
bessj	2	462.96	784.93	452.49	457.52	457.60	461.69	0.0000	0.6152	0.4585	0.4945	0.9216	0.5007	0.6456	0.4966	0.5489	0.4955		
gammq	2					1100.74			0.5269	0.2777	0.5081	0.8889	0.5010	0.0757	0.5132	0.0011	0.5243		
sncndn	2		629.03		629.63	649.75	655.74		0.4997	0.5157	0.5048	0.9029	0.4991	0.3750	0.5066	0.1915	0.5097		
golden	3					1816.82 1618.40			0.5142 0.6003	0.4552 0.0076	0.5056 0.4801	0.9594 0.1738	0.4996 0.4899	0.5315 0.0951	0.5047 0.4876	<b>0.0038</b> 0.6289	0.5216 0.4964		
plgndr cel						1593.71			0.6478	0.5594	0.4956	0.1736	0.4899	0.7584	0.4977	0.6289	0.4964		
e12	4				710.58	714.01		0.0000	0.6381	0.0003	0.4730	0.0009	0.4753	0.0001	0.4702	0.0429	0.4849		
calDay	5	1101.30	1394.67	1312.37	1295.35	1262.40	1226.75	0.0000	0.5556	0.0000	0.5380	0.0000	0.5380	0.0000	0.5354	0.0006	0.5256		
complex						1150.68			0.4934	0.0002	0.5278	0.9074	0.5009	0.8793	0.5011	0.7912	0.5020		
pntLinePos						1490.97			0.4812	0.0000	0.5431	0.5160	0.5048	0.0299	0.5162	0.0980	0.5123		
triangle line						1389.04 3269.93			0.5014 0.4917	0.4483 0.1796	0.4943 0.5100	0.7890 0.7428	0.4980 0.5024	0.1772 0.3340	0.4899 0.4928	<b>0.0175</b> 0.2840	0.4823 0.5080		
pntTrianglePos						4955.54			0.4917	0.1798	0.5100	0.7428	0.5024	0.3528	0.5069	0.3202	0.4926		
TwoLinesPos						8430.90			0.4540	0.0000	0.5922	0.0000	0.5304	0.0000	0.5337	0.0000	0.5522		
nearestDistance									0.6518	0.0000	0.5489	0.0176	0.5177	0.4080	0.5062	0.1908	0.5098		
calGCD						1017.74			0.5180	0.5202	0.5048	0.6552	0.5033	0.3601	0.5068	0.0242	0.5168		
select	11	5490.70	4759.86	5599.86	5907.50	5634.03	5808.85	0.0000	0.4596	0.6502	0.5034	0.0368	0.5156	0.5461	0.5045	0.0885	0.5127		
-  → FSCS-ART → VPP-ART → Naive-KDFC → SemBal-KDFC → LimBal-KDFC  → FSCS-ART → VPP-ART → VPP-AR									50 45 40 0 0 3 35 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	DFC	00 6500 800 The nu	3.0 2.5 2.0 1.5 1.0 0.5 0.000 200	CSART VPP.	(c) d:	20000		000 20000		
	15										10 2000 4000 6000 8000 10000 12000 16000 16000 18000 200  The number of test cases (N)  → FSCS ART → VPF-ART → Naive KDPC → Smilled KDPC → LiniBal KDPC  (g) d = 10								

Fig. 1. Generation times for various test suite sizes in different dimensions.

 ${\it TABLE~6} \\ {\it F-time~results~and~statistical~analysis~comparisons~among~VPP-ART~and~other~algorithms~with~the~$22~subject~programs $$$ 

				F-tim	ie			Statistical Analysis											
Program	d	VPP-	RT	FSCS-	Naive-	SemiBal-			vs. RT		SCS-ART				Bal-KDFC				
		ART		ART	KDFC	KDFC	KDFC	p-value	effect size	p-value	effect size	p-value	effect size	p-value	effect size	p-value	effect size		
airy	1	12.74	0.34	329.08	3.31	3.33	3.71	0.0000	0.0450	0.0000	0.8740	0.0000	0.2000	0.0000	0.2008	0.0000	0.2182		
bessj0	1	6.38	0.17	98.95	1.66	1.66	1.76	0.0000	0.0571	0.0000	0.8382	0.0000	0.2062	0.0000	0.2061	0.0000	0.2151		
erfcc	1	17.33	0.49	529.69	4.46	4.60	4.80	0.0000	0.0371	0.0000	0.8904	0.0000	0.1919	0.0000	0.1971	0.0000	0.2039		
probks	1	37.18	20.37	1081.17	18.11	17.95	18.57	0.0000	0.3139	0.0000	0.8845	0.0000	0.3000	0.0000	0.2973	0.0000	0.3057		
tanh	1	3.98	0.10	50.72	1.08	1.17	1.21	0.0000	0.0683	0.0000	0.8273	0.0000	0.2044	0.0000	0.2173	0.0000	0.2218		
bessj	2	7.55	0.70	132.59	8.43	3.08	3.97	0.0000	0.1331	0.0000	0.7889	0.0000	0.5326	0.0000	0.3214	0.0000	0.3647		
gammq	2	20.60	0.45	775.74	16.10	13.08	14.28	0.0000	0.0582	0.0000	0.8319	0.0000	0.4520	0.0000	0.4153	0.0000	0.4330		
sncndn	2	11.18	0.23	287.05	5.53	9.03	9.06	0.0000	0.0653	0.0000	0.7967	0.0000	0.3773	0.0000	0.4399	0.0000	0.4528		
golden	3	45.73	3.60	2417.30	45.16	72.10	66.76	0.0000	0.1211	0.0000	0.8518	0.1062	0.5120	0.0000	0.5623	0.0000	0.5760		
plgndr	3	42.52	7.79	1714.59	94.44	19.43	21.65	0.0000	0.2280	0.0000	0.8155	0.0000	0.6282	0.0000	0.3541	0.0000	0.3745		
cel	4	36.14	0.82	1858.20	155.37	23.46	28.03	0.0000	0.0459	0.0000	0.8601	0.0000	0.7358	0.0000	0.4126	0.0000	0.4457		
e12	4	15.42	0.46	379.12	25.32	29.34	28.23	0.0000	0.0703	0.0000	0.7910	0.0000	0.5933	0.0000	0.5869	0.0000	0.5968		
calDay	5	27.57	2.19	968.30	169.88	36.91	36.18	0.0000	0.1083	0.0000	0.8371	0.0000	0.7733	0.0004	0.5264	0.0088	0.5195		
complex	6	27.62	0.69	1317.92	136.16	136.54	113.95	0.0000	0.0525	0.0000	0.8536	0.0000	0.7358	0.0000	0.7446	0.0000	0.7431		
pntLinePos	6	34.46	0.52	1895.50	182.67	193.30	155.95	0.0000	0.0396	0.0000	0.8563	0.0000	0.7496	0.0000	0.7640	0.0000	0.7651		
triangle	6	35.23	0.61	1562.98	177.93	180.97	142.86	0.0000	0.0456	0.0000	0.8339	0.0000	0.7444	0.0000	0.7453	0.0000	0.7372		
line	8	102.64	1.57	10274.08	1542.17	1659.12	936.30	0.0000	0.0321	0.0000	0.8931	0.0000	0.8484	0.0000	0.8432	0.0000	0.8511		
pntTrianglePos	8	153.93	2.86	21846.93	2864.84	3077.41	1450.36	0.0000	0.0298	0.0000	0.8972	0.0000	0.8594	0.0000	0.8676	0.0000	0.8547		
TwoLinesPos	8	272.39	3.27	70499.32	6015.84	6919.79	3177.67	0.0000	0.0222	0.0000	0.9347	0.0000	0.8831	0.0000	0.8946	0.0000	0.8870		
nearestDistance	10	69.81	2.76	3493.54	3113.37	3467.71	1067.29	0.0000	0.0683	0.0000	0.8596	0.0000	0.8503	0.0000	0.8540	0.0000	0.8608		
calGCD	10	27.96	2.48	916.10	483.69	554.96	364.48	0.0000	0.1273	0.0000	0.8139	0.0000	0.8033	0.0000	0.8119	0.0000	0.8055		
select	11	216.93	5.93	26894.43	24325.93	23561.25	4338.21	0.0000	0.0446	0.0000	0.9035	0.0000	0.9088	0.0000	0.9086	0.0000	0.9039		