# The Statistical Physics in Modeling Chromosome Movements During Nuclear Oscillation Stage of Fission Yeast



#### Wenwen Huang

Max Planck Institute for the Physics of Complex Systems
TU Dresden

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#### **Declaration**

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

Wenwen Huang November 2016

#### Acknowledgements

And I would like to acknowledge ...

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This is where you write your abstract ...

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