

1 Supplement Materials

1.1 Simulation

To control the number of free parameters, we choose the ideal bead-rod polymer model to describe the dynamics of chromosomes in nucleus, without considering the exclusive volume effect. As we will see later that the system can be reduced to a one free parameter model, i.e. T_{eff} .

The simulation of model bead connected with rigid rod utilize the technique of Brownian Dynamics[1]. The dynamical equation of beads representing chromosome loci is

$$\dot{\mathbf{r}}_i = \frac{1}{\xi}(\mathbf{F}_i^b + \mathbf{F}_i^c + \mathbf{F}_i^e + \mathbf{F}_i^{pseudo}) \quad (1)$$

where \mathbf{r}_i is the position vector of the i th bead, ξ is friction coefficient, \mathbf{F}_i^b is random force, \mathbf{F}_i^c is constraint force caused by rigid rod constraints, \mathbf{F}_i^e is external force and \mathbf{F}_i^{pseudo} is pseudo force added to mimic the statistics of bead-spring. Notice that the statistics of bead-rod is not exactly same as bead-spring. Subtle differences are caused by the intrinsic "rigid" of rods[2, 1].

The random force, which characterize the fluctuation origin of beads dynamics, is a typical Brownian force satisfies the following conditions

$$\langle \mathbf{F}_i^b \rangle = \mathbf{0}; \langle \mathbf{F}_i^b(t) \mathbf{F}_j^b(t') \rangle = 2k_B T_c \xi \delta_{ij} \delta(t - t') \quad (2)$$

k_B is Boltzmann constant and T_c is *characterizing temperature* characterize the level of randomness arise from the thermal motion of solvent molecules and some sorts of interactions between chromosome and proteins.

The constraint force for a specific bead in a bead-rod ring writes

$$\mathbf{F}_i^c = \lambda_i \mathbf{u}_i - \lambda_{i-1} \mathbf{u}_{i-1} \quad (3)$$

where λ_i is strength of tension on the rod between i th and $(i + 1)$ th bead, \mathbf{u}_i is the unit vector along this rod and $i = N$ ends to $i = 0$.

In case of constant force field, the external force is constant $\mathbf{F}_i^e = -\xi \mathbf{v}$ acting on every bead except the pinned one representing SPB.

The pseudo force is calculated using

$$\mathbf{F}_i^{pseudo} = -\frac{\partial U_{met}}{\partial \mathbf{r}_i}; U_{met} = \frac{1}{2} k_B T_c \ln(\det G) \quad (4)$$

where G is the metric matrix of the bead-rod system[4].

Since the rods present in our model are rigid rods, additional constrained equations are needed to keep the rod length unchangeable

$$(\mathbf{r}_{i+1} - \mathbf{r}_i)^2 - a^2 = 0 \quad (5)$$

where a is rod length.

Parameters above can be eliminated and dimensionless term of the dynamical equations can be obtained by the scaling $\mathbf{r}' \rightarrow \mathbf{r}/a$; $t' \rightarrow t/(\xi a^2/k_B T_c)$;

$\mathbf{F}' \rightarrow \mathbf{F}/(k_B T_c/a)$. Our only free parameter *effective temperature* which is also dimensionless is defined as

$$T_{eff} = \frac{Fa}{k_B T_c} = \frac{\xi v a}{k_B T_c} \quad (6)$$

Numerical scheme employed to solved the set of constrained differential equations (1) and (5) is predictor-corrector algorithm used widely in bead-rod simulation[1, 5, 3]. Basic steps include calculating a prediction of $\mathbf{r}_i(t + \delta t)$ without considering the constraint force followed by a correction step, i.e. solving the algebra constraint equations to get constraint forces and re-plugin to the original equations for the corrected $\mathbf{r}_i(t + \delta t)$. The differential equations are solved using Euler iterative method with a time step $dt = 10^{-4}$. Statistical results are all obtained based on the ensemble of 10^{10} steps after equilibrium.

References

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