SYNCHRONIZATION IN LARGE-SCALE COMPLEX NETWORKS: EXPLOTING THE DYNAMICAL ASPECTS

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Synchronization of large-scale complex networks is the functional basis of some real systems, such as brains, and also usually the aim of some engineering designs.

To achieve this goal, the prevailing paradigm is to modify the network structure, such as to break the hub node into a group of subnodes, to make networks weighted or directed, or to compensate with negative links etc. However, the structure of networks in real-world cases are well developed not allowing major structural modifications.

Here we attack this problem in another direction, i.e., instead of structure manipulations, by considering some factors in dynamical aspects the stable synchronization of large networks can also be facilitated considerably.

Consider a network of N oscillator with arbitrary complex network with

$$\dot{x}_i = F(x_i) + \varepsilon(t)\sum_{j=1}^{N} G_{ij} H(x_j(\tau)),$$

where $G = (G_{ij})$ describes the topology of the complex networks, and ε and τ are coupling strength and time delay, respectively (in the most studies, ε is time constanst and no time delay). By linearization of the equations, the synchronous solution of the coupled systems is stable if the following relationship is satisfied

$$R \equiv \frac{\lambda_N}{\lambda_2} < \frac{\alpha_2}{\alpha_1} \equiv S.$$

Here, R is the eigenratio characterizing the synchronizability of complex network, and S is the threshold of master stability function (MSF) ratio, only determined by dynamical aspects, including the oscillator dynamics, coupling scheme, time delay etc. Within this framework, most of previous studies try to reduce the value of R by modifying networks. Here, instead, we try to increase S, while R is kept unchange, to make the above relationship being better satisfied.

The coupled chaotic Rössler oscillators

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FIG. 1: Master stability function as a function of coupling strength α and on-off ratio θ with on-off period T=2s, 3s, 6s, 9s for (a)-(d) (Left). The corresponding threshold ratio S, time step=0.01 (Right). Note that in the limit of time step$\rightarrow 0$, S goes to infinity.

An example within foodweb networks

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FIG. 2: Time evolution of synchronization error δ and time series (inset). Parameter $T = 1$, $\theta = 0.1$, $\varepsilon = 100$. The results means that intensive interaction among commnunities in animal's active season for just few months can synchroniz e very large spatial zone. Notice that, for traditional constant coupling, there is a size instability for this coupled systems, while on-off coupling (accounted for seasonality) can avoid this.

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Dynamic coupling strength

$$\varepsilon_{ij} = \varepsilon_0 S(\frac{\Delta w_{ij}}{\Delta w'_{ij}} - l),$$

where Δw_{ij} is the difference of the arbitrary weight coupling between nodes i and j, and $\Delta w'_{ij}$ corresponding to the rest, l is the threshold that control coupling switching on/off. $S(x) = 1/(1+e(-\alpha x))$ is a sigmoid function.

This dynamic coupling can be considered generalized on-off coupling (the sigmoid function becomes Heaviside function as α approaches positive infinity), and can apply to a more variety of chaotic oscillators.

Notice that, this form is now beyond the MSF framework because each coupling strength is determined by the two node state, thus not uniform.

An example is given as follows:

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FIG. 3: (a) The transversal Lyapunov exponent as function of ε_0 for $z-$coupled Lorenz oscillators.(b) Time series for $\varepsilon_0 = 10$ in noisy backgroud ($\sim 10^{-5}$). upper panel: constant coupling; lower pannel: dynamic coupling leading to the symmetry broken for Lorenz equation $(-x, -y, z) \rightarrow (x, y, z)$ gets stable synchronization. The threshold $l = 2$.

Self-organized functional networks in synchronization process with this dynamic coupling. Physically, the network is globally connected, and the functional connectivity between to two nodes exist if $\varepsilon_{ij} > \varepsilon_c$ (e.g., $0.01\varepsilon_0$), vice versa.

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FIG. 4: (a) Degree distribution for the self-organized functional networks, with a tail $p(k) \sim k^{-3.1}$. Data from networks of size $N = 1000$ over 100 realizations. (b) An examplified network with $N = 100$.

In this case, there is a uniform time delay in the the transmission of signal, as well as in their own signal.The resulting synchronous manifold remains the same as for the isolated individual.

By including intermediate amount of time-delay, the stable region emerges. This comes from the phase structure of coupled dynamics.

Interestingly, Fig. 5(b) shows that for this time-delayed network stable synchronization can survive in "all negative links".

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FIG. 5: Master stability function as function of coupling strength α and uniform time delay τ for the nondiagnal coupling $x \rightarrow y$ (a) and $y \rightarrow x$ (b) respectively.

By including some simple dynamical factors, such as dynamic coupling and time delay, the networked system allows better synchronization performance. Thus, the study opens a new view to consider the problem of synchronization in networked system.

Meanwhile, we have to admit that the uniform on-off and uniform time delay adopted here may not be so realistic in real world. A more suitable treatment should be under the framework of adaptive networks.

The research emphasizes the role of exploiting the dynamical structure of coupled units, which could actually be as much important as the role of structure part played in network synchronization.