Polymer Maps to Particle Diffusion and Back in 1D

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I. INTRODUCTION

Many biological processes can be modelled by idealized physical concepts and qualitatively studied through the law of physics and methods of mathematics. A good example is the movement of DNA. Polymer models are often utilized to describe DNA[], characterized the long chain property of its chemical structure. In our study of chromosome alignment in meiotic fission yeast, a freely jointed bead rod ring model is adopted[]. Chromosome movements during the stage of horsetail oscillation of nucleus are translated to pinned polymer loop in an external field[]. To understand to statistics of distance between loci, we first formulate the problem in 1D, i.e. rods can only pointing to right or left. Amazingly, we found this simple model can maps to a 1D particle diffusion problem, well known as single file diffusion (SFD)[]. On the other hand, SFD is a paradigmatic model in non-equilibrium statistics with many applications[].

In this paper, we will show how to map from a pinned polymer model describing chromosomes to a single file particle diffusion. The equivalence between polymer and particle means that we solve one case in one picture automatically solves the correspondence in the other. We thus show a example that the pinned polymer loop in constant external field can be solved analytically in 1D. We demonstrate that the famous Fermi-Dirac statistics serves as an asymptotic approximation of statistics of rod orientation. Exact solution is also feasible by solving the fermion number partition problem. Thus the particle picture correspondence, which is SFD in external force field with reflecting boundary condition, is also solved and to our knowledge is solved for the first time. Results are verified by numerical simulations. To further demonstrate the power of this mapping method, we also discuss the of some other applications. In particular, the dynamics of polymer is discussed by mapping back from SFD dynamics.

The next section we will describe how to build the mapping from polymer and particle. In section III, some of the applications are discussed. Finally, we give our conclusion remarks and outlook in section IV.

II. STATISTICS OF ONE-DIMENSIONAL PINNED POLYMER LOOP

A. Brownian bridge

B. Fermi-Dirac statistics of rod orientations

C. Fermion integer number partition theory

D. From rods to a polymer, beauty of Gaussian statistics

III. ASYMMETRIC EXCLUSION PROCESS

IV. TOWARDS UNDERSTANDING DYNAMICS

V. CONCLUSIONS

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