清華大學
Tsinghua University

Interdisciplinary Investigation

"Complex (soft) hydrodynamics"

From single paradigms to X-paradigm

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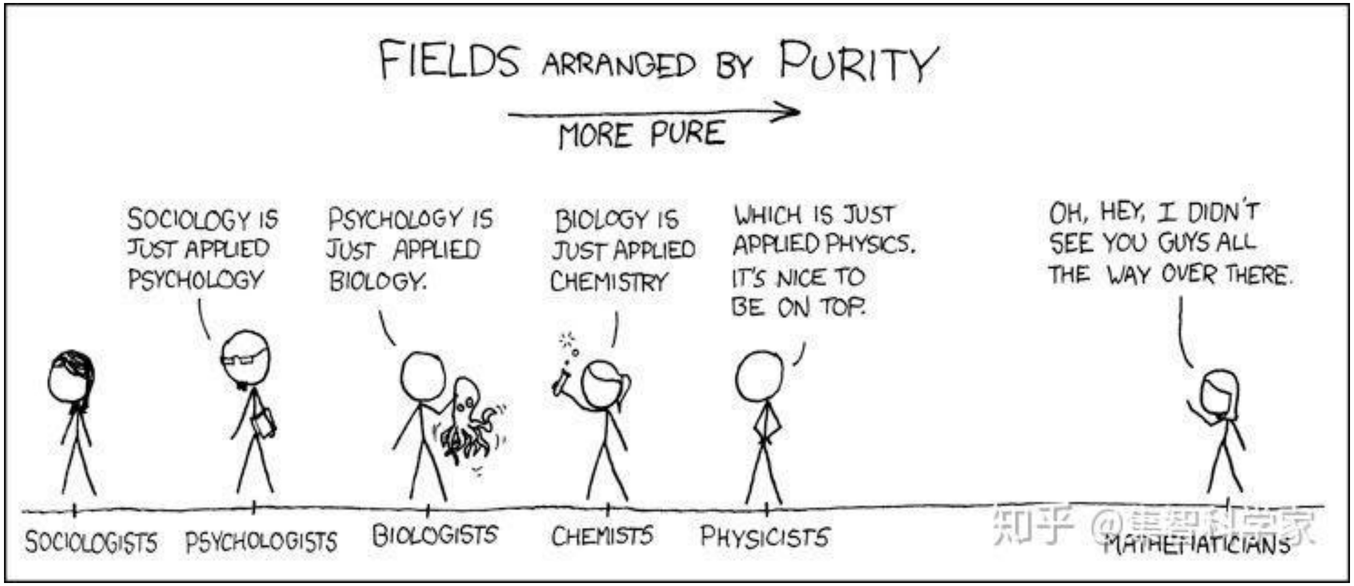
2024.11.13

“MORE IS DIFFERENT”

- P.W. Anderson (1923-2020, USA; 1977 Nobel-Phys)

  “for their fundamental theoretical investigations of the electronic structure of magnetic and disordered systems” (shared with Mott, van Vleck)



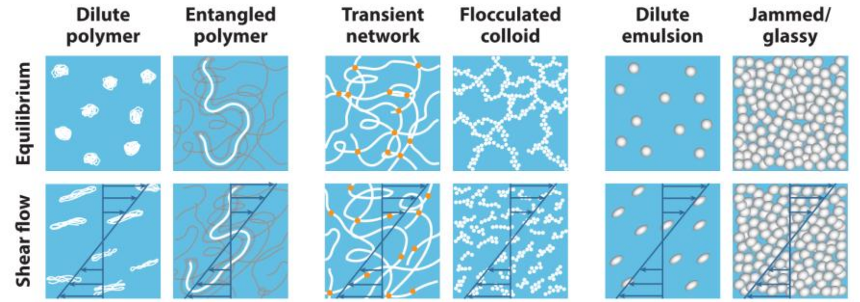
Emergence (涌现论/演生论) v.s. Reductionism (还原论)

1. P.W. Anderson. More is different: broken symmetry and the nature of the hierarchical structure of science, *Science*, 1972
2. 段远源，专业基础课《高等工程热力学》，文献翻译作业

Emergence … of what? What can be actually <u>observed and measured</u>?

找到长时间可观测的、可复现的规律[宏观守恒量，模式/斑图]

When hydrodynamics meets "complex"

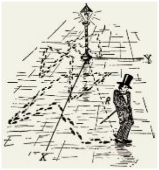


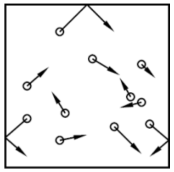
- What is ***Hydrodynamics***? Why we need it?
  - ▫ **quasi-particles as collective excitations**
  - ▫ **long-wavelength (small wavevector) limit**
  - ▫ **observables describing long-distance prop**

- One way to develop a **macroscopic** theory with densities of physical quantities and the corresponding currents, is to combine
  - **continuity equations** (manifesting conservation laws), with
  - **thermodynamic arguments** ("constitutive relations"), between the macroscopic currents and the external bias to close the equations, to identify
    - how the entropy of the system responds to <u>local density fluctuations of the conserved quantities</u>
    - which requires the total entropy production rate to be non-negative
- They are **phenomenological** since they provide no means of calculating the coefficients in the constitutive relations, which is justified **at distances that are much larger than any "microscopic" scattering length scales**, the condition that is very often satisfied in experiments.

1. Narozhny, Hydrodynamics approach to 2D electron systems, 2022
2. 朗道，场论、流体动力学、物理动理学；谢多夫，连续介质力学
3. Ewoldt, Designing complex fluids, *Annal Rev Fluid Mech*, 2022

random





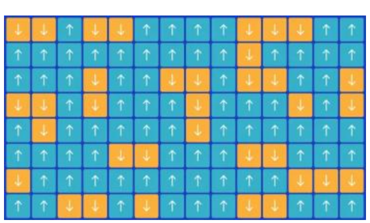
short-range interaction

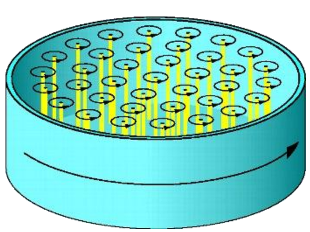
For "<u>strongly-correlated</u>" systems, does the hydrodynamic description still hold?

粒子之间存在长程关联[电磁作用、物质波相干性、交联/位阻作用…]

Contents

- **“Strongly-correlated” systems: Figures, Problems and Stories**
  - ***“Where to start … with something new”***
  - **Multiphase interface of immiscible electrolyte solutions**
  - **Transport properties of correlated quantum systems**
- Complex hydrodynamics as X-paradigm: What, Why, How, and Which?
  - Example: Emergence of quantum hydrodynamics
  - New physics originated from “interface” of perspective
  - Hallmark: Merging kinetic behaviors into hydrodynamics





Li [2022-23]

“Where to start … with something new”

- P.W. Anderson (1923-2020, USA; 1977 Nobel-Phys)
  - I think **<u>most people entering research find that by far the most difficult question is where to start, especially when confronted with something that is actually new</u>**. This, … , is the kind of question a book like this should be designed to answer.
  - Many books are simply compendia of methods that have already been used or of techniques for calculating a little better something that is already understood. I am writing this immediately after the experience of having been confronted by the new phases of He; **<u>faced with such a genuinely novel problem it is far more important to have some idea of what the relevant questions are than it is to do any one calculation with great accuracy or rigor</u>**.
  - This is one of the reasons why I suggest that the two most important principles of condensed matter physics for our purposes are, first, **broken symmetry**, which tells us that what the **<u>order parameter</u>** is and what **<u>symmetry it breaks</u>** are the most vital questions; and, second, the **continuity principle**, which tells us to search for the **<u>right simple problem</u>** when **<u>confronted with a complicated one</u>**. To my way of thinking, detailed perturbation methods, and even Green's function and fluctuation-dissipation ideas, are somewhat less important, because they emphasize computation rather than understanding.

1. Anderson. *Basic notions of condensed matter physics*, 1984.
2. 朗道，统计物理学 (I, II)、物理动理学.

field	convection? interface?
dynamics	instability? pattern?
particle	viscosity? vortex?

How to choose the <u>order parameter</u> and <u>adiabatic approximation</u>?

找到模式（及转换）的核心控制量［物理量（无量纲数）］ 从熟悉的系统过渡［极限情况做起］

Two figures: Onsager & Landau

- L. Onsager (1903-1976, USA; 1968 Nobel-Chem)

◆ ES: electrolyte solution	◆ **IR: irreversible process**	◆ IS: 2D Ising model
◆ ES_1: liquid crystal theory	◆ MF: Mathew function	◆ IS_1: electron theory in metal
◆ ES_2: dielectric theory	◆ IR_1: fluctuation theory	◆ IS_2: vortex in superfluid He II
◆ ES_3: reaction-rate theory	◆ **IR_2: turbulence statistics**	◆ IS_3: superconducting in He II

Extra topics: ice's electrical theory, radiochemistry, **molecular biology (life in the early days)**

- L.D. Landau (1908-1968, Russia; 1962 Nobel-Phys)

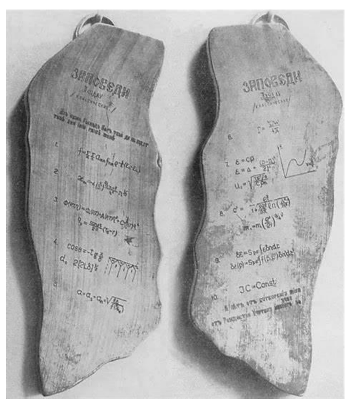
(QM) 量子力学的密度矩阵和统计物理学, 1927

(Mag) 自由电子抗磁性的理论, 1930

(Mag) 二级相变的研究, 1936-1937

(Mag) 铁磁性磁畴理论和反铁磁性解释, 1935

(CM) 超导体的混合态理论, 1934



1937, 原子核的几率理论 (PP)

1940-1941, 氦II超流性量子理论 (CM)

1954, 基本粒子的电荷约束理论 (PP)

1956, 费米液体的量子理论 (CM)

1957, 弱相互作用的 CP 不变性 (PP)

The "Ten Commandments" (朗道十诫)

理论物理学教程 (朗道十卷)

Contributions: Symmetry **(ES-IR)** & its breaking **(IS; Mag-PP)**, correlation function **(ES)** & quasi-particle **(CM-QM)**

【找到模式（及转换）的核心控制量［物理量（无量纲数）］】 【从熟悉的系统过渡［极限情况做起］】

Two typical correlated systems: Challenges

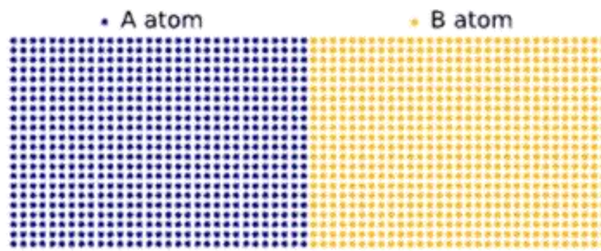
Electrolytes in solutions

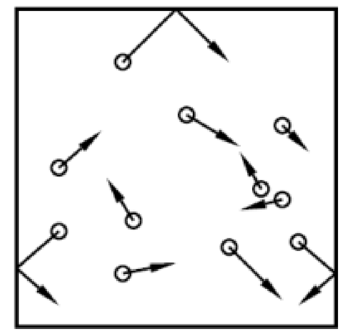
- Nature
  - ✓ dissolved
    - hydrodynamic
    - chemical
- Correlation
  - ✓ steric
  - ✓ **electrostatic**
  - ✓ flow-induced

Quantums in solids

- Nature
  - ✓ statistics
    - boson/fermion
  - ✓ wave nature
- Correlation
  - ✓ coherent
  - ✓ **electro-** $(e,l)^2$
  - ✓ magneto- (↑,↓)

Familiar ones: **gas molecule transport by Prof. (99'-04')]**





WHY

decomposition of

ion transport still stands?

- ◻ **Advection,**
- ◻ **Diffusion,**
- ◻ **Electro-migration**

WHY

particle description of

electron transport still stands?

- ◻ **Excitation,**
- ◻ **Propagation,**
- ◻ **Scattering**

Two typical correlated systems: Typical scale

Electrolytes in solutions

- **Mean distance (background)**
  - $n_\infty d_M^3 \sim 1$
  - concentration
- **Radius of ion ("coherence")**
- **Debye length (screening)**
  - $\varepsilon\,(\delta\varphi/\lambda_D)^2 \sim (\delta n_\infty)\,k_B T$
  - concentration
  - solvent permittivity
- **Bjerrum length (correlated)**
  - $k_B T \sim e^2/\varepsilon d_B$
  - solvent permittivity
- **Relaxation time ("scattering")**
  - $\tau \sim \lambda_D^2 / (k_B T/\mu_m)$
  - frictional force [μ, E, …]
  - viscosity effect (gas/self?)
  - dissociation effect (weak?)

Quantums in solids

- **Lattice periodicity (background)**
  - interaction energy
  - steric effect (entropy)
- **Coherent length (coherence)**
  - wavepacket width **[volume]**
  - wavepacket energy **[life]**
- **Debye length (screening)**
  - $\varepsilon\,(\delta\varphi/\lambda_D)^2 \sim (\delta n_\infty)\,k_B T$
  - concentration
  - lattice permittivity
- **Bjerrum length (correlated)**
  - $k_B T \sim e^2/\varepsilon d_B$
  - lattice permittivity
- **Mean free path (scattering)**
  - X-X (normal)
  - **X-X (umklapp), X-Y**

Onsager: The motion of ions in salt solutions

- **Electrolyte**: dissolved (hydrodynamics, chemical), [electrostatic, steric] correlation

WHY

decomposition of ion transport stands?

- **Advection,**
- **Diffusion,**
- **Electro-migration**

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Zur Theorie der Elektrolyte. I.

Von Lars Onsager.

der Elektrolyte. I. Physik.Zeitschr.XXVII,1926.

$$\Lambda = \Lambda_0(1-\alpha\sqrt{c})$$

start from P. Debye's work

Total force balance, including electromigration & Stokes (viscous)

Viscous forces on the ion with superposition principle: apparent migration & electrophoretic

12 December 1969, Volume 166, Number 3911 **SCIENCE**

The Motion of Ions: Principles and Concepts

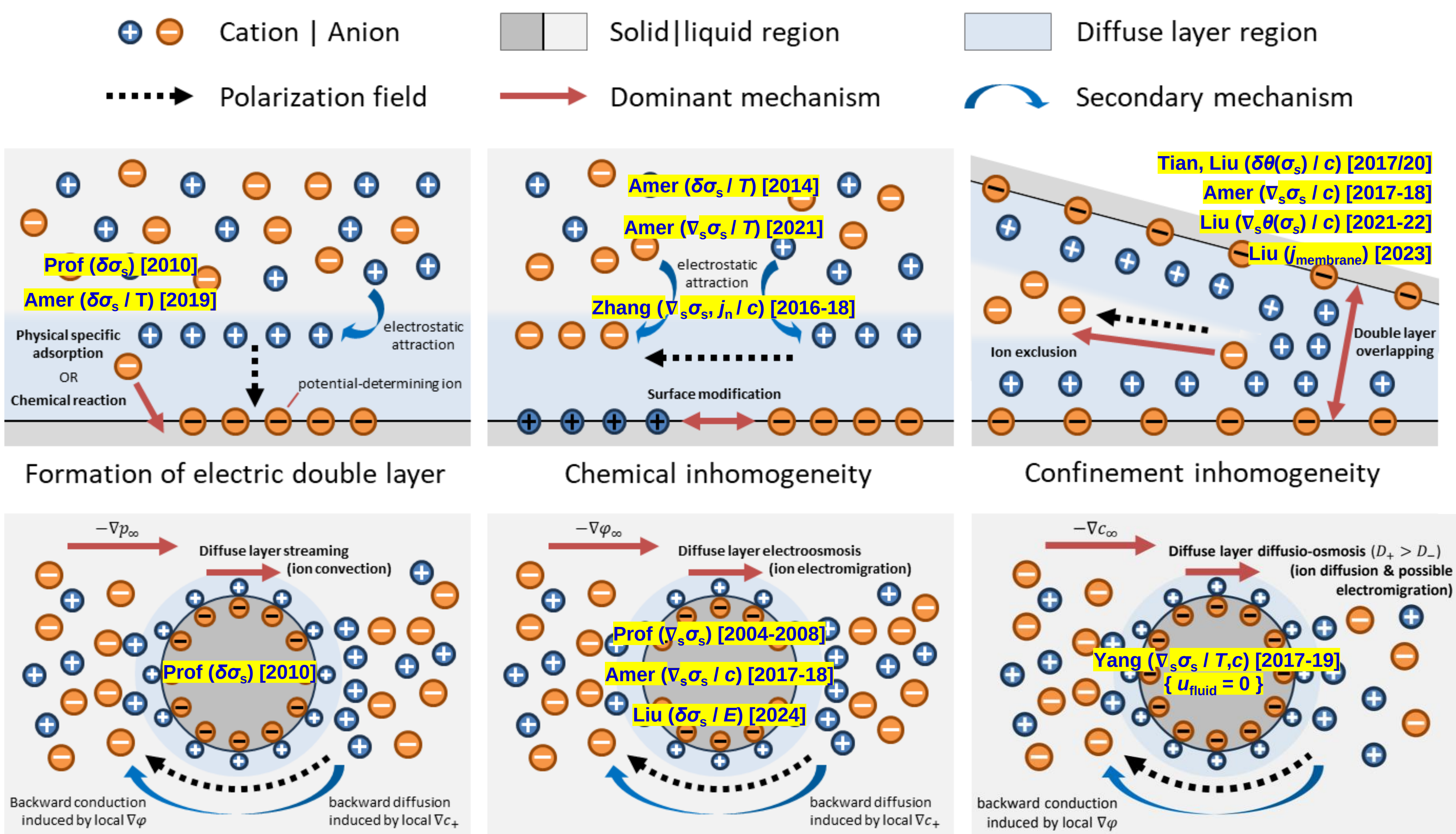
"Anomalies" of strong electrolytes from the ideal additive behavior: Electrical conductivity, Freezing point depression, Electromotive force

This article is the lecture he delivered in Sweden, when he received the Novel Prize in chemistry.

1. L. Onsager, *Physikalische Zeitschrift*, 1926.
2. L. Onsager, *J Chem Phys*, 1931; *Science*, 1969.

"for the discovery of the reciprocal relations bearing his name, which are fundamental for the thermodynamics of irreversible processes"

Electrolyte transport at multiphase interface (S)



Pressure gradient upon an enclosure | Electric field upon an enclosure | Concentration gradient upon an enclosure

序参数组成

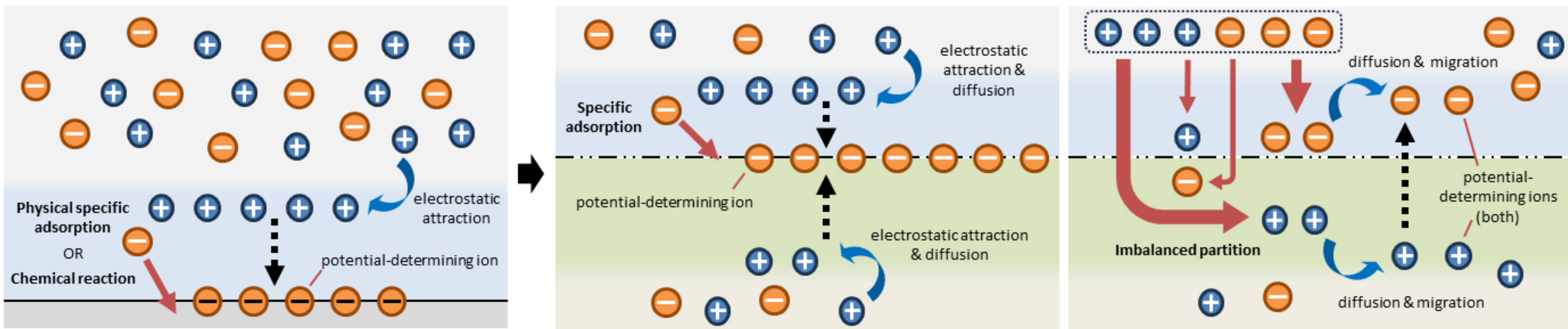
λ_D, ζ_s/σ_s, α_h
[β_E, $T^e{}_s$, Pe]
[j_s, j_n, $\nabla_s\sigma_s$]

电动(单相)流体力学：固液界面－双电层形成－额外对称性破缺－电&浓度极化

Electrolyte transport at multiphase interface (L)

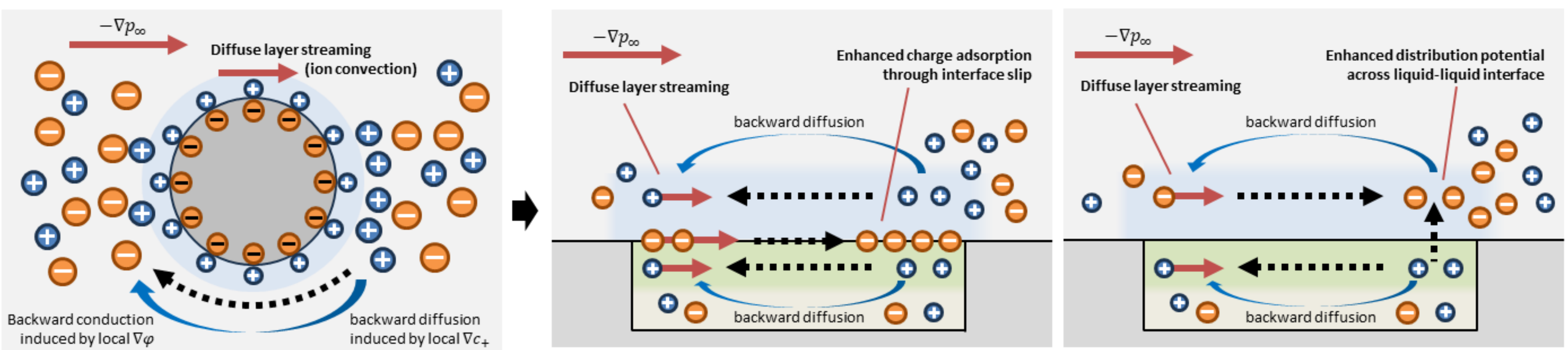
Cation | Anion — Solid|liquid|liquid regions — Diffuse layer regions

Polarization field — Dominant mechanism — Secondary mechanism



Formation of electric double layer

Feature 1: Complex charging mechanisms **Huang ($\delta\sigma_s$) [2023-24]**



Pressure gradient upon an enclosure

Feature 2: Slip-induced inhomogeneous charging **Huang (Ongoing)**

序参数组成

$\lambda_{D,i}$, σ_s, Φ_∞, $\alpha_{h,i}$

[β_E, $T^e_{s,i}$, Pe_i], Ca_θ 电动多相流体力学：液液界面－双侧双电层－额外对称性破缺－双侧极化耦合

[$j_{s,i}$, j_n, $\nabla_s\sigma_s$, $\nabla_s\Phi_\infty$]

Landau: The motion of electrons in solids

- **Quantums**: statistics & wave nature, [electro $(e,l)^2$ | magneto (↑,↓) | coherent] correlation

WHY

particle

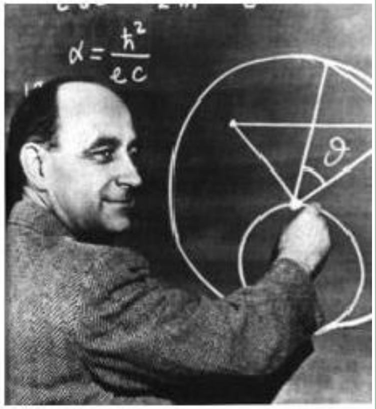
description of

electron transport

still stands?

- ☐ **Excitation,**
- ☐ **Propagation,**
- ☐ **Scattering**

Landau Fermi-liquid

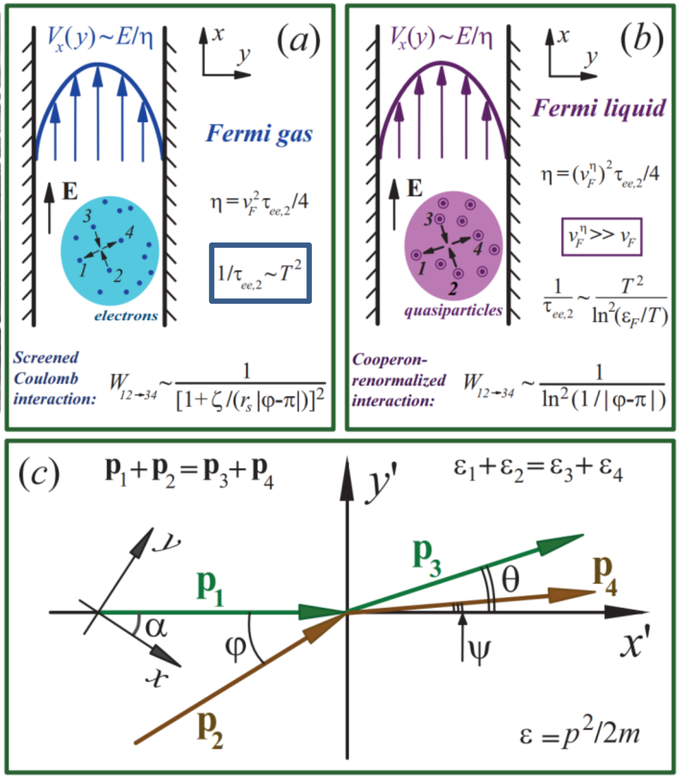


In 1956, Landau developed a theory of interacting spin-1/2 fermions[1]

The Landau-Fermi liquid theory successfully describes metals, nuclear matter, $\boxed{\text{liquid He-3}}$...

At low temperatures, the average excitation energy is ~ $k_B T$

From Fermi gas to Fermi liquid

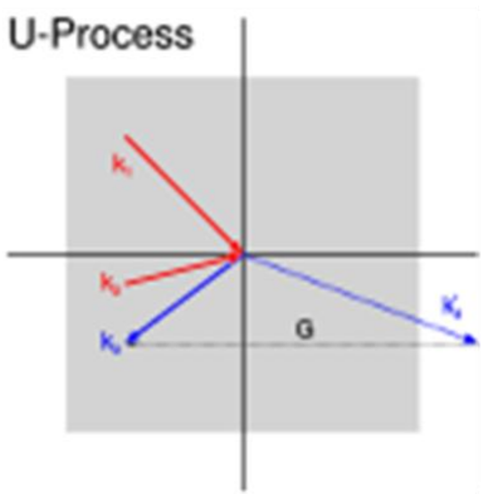


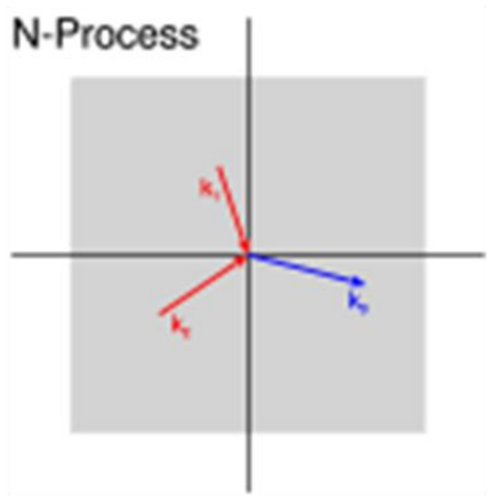
1. L. D. Landau, The theory of Fermi liquids, *Zh Eskp Teor Fiz*, 1956
2. P.S. Alekseev & A.P. Dmitriev, Viscosity of two-dimensional electrons, *Phys Rev B*, 2020

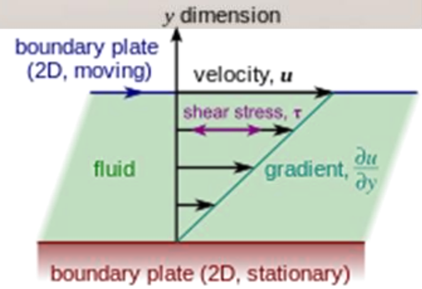
"for his pioneering theories for condensed matter, especially liquid helium"

Transport in correlated quantum systems (ph)

Room temperature ⟶ Low temperature







Classical model	Hydrodynamic model	Viscous fluid model
Phonon ⟷ Phonon (U)	Phonon ⟷ Phonon (N) ⟷	Molecular⟷Molecular

Umklapp-induced resistance
(Fourier law)

Boundary-induced resistance
(Hydrodynamic flow)

Guo (Meso & Macro) [2013-18]；Miao, Ran (Meso) [2016-22]

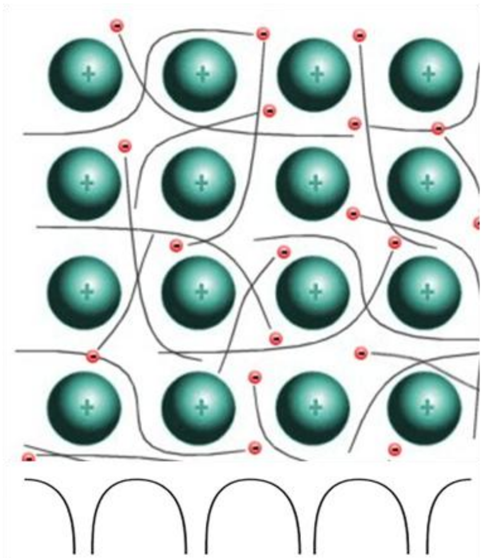
Phonon hydrodynamics!

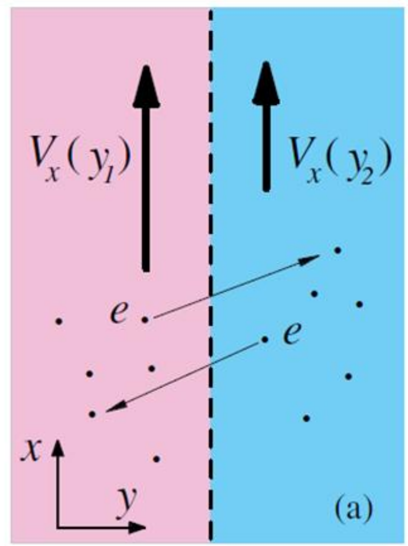
序参数组成 [l_{tri}], l_N, l_R, W, [l_{dis}] 声子水动力学：散射截面 – 守恒散射 – 声子集体运动 – 声子粘性

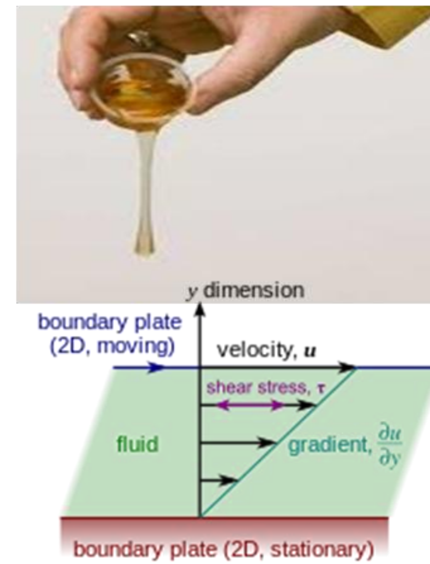
Transport in correlated quantum systems (e)

Room/ultralow temperature ⟶ Low temperature & low dimension & ultrapure

- Metal, Si|Ga[Al]As, Doped-S|BLG, Pt|$PdCoO_2$
- SLG, semi-metal (MoP, WP_2), topo-insulator







Nearly Free electron model
Electron⟷ Lattice*

Viscous electron model
Electron⟷Electron** ⟷

Viscous fluid model
Molecular⟷Molecular

Lattice-induced resistance
(Ohmic flow)

Boundary-induced resistance
(Hydrodynamic flow)

Miao (Meso) [2017-22]

Huang (Meso & Macro) [2017-21]；Meng (Macro) [2019]

Electron hydrodynamics!

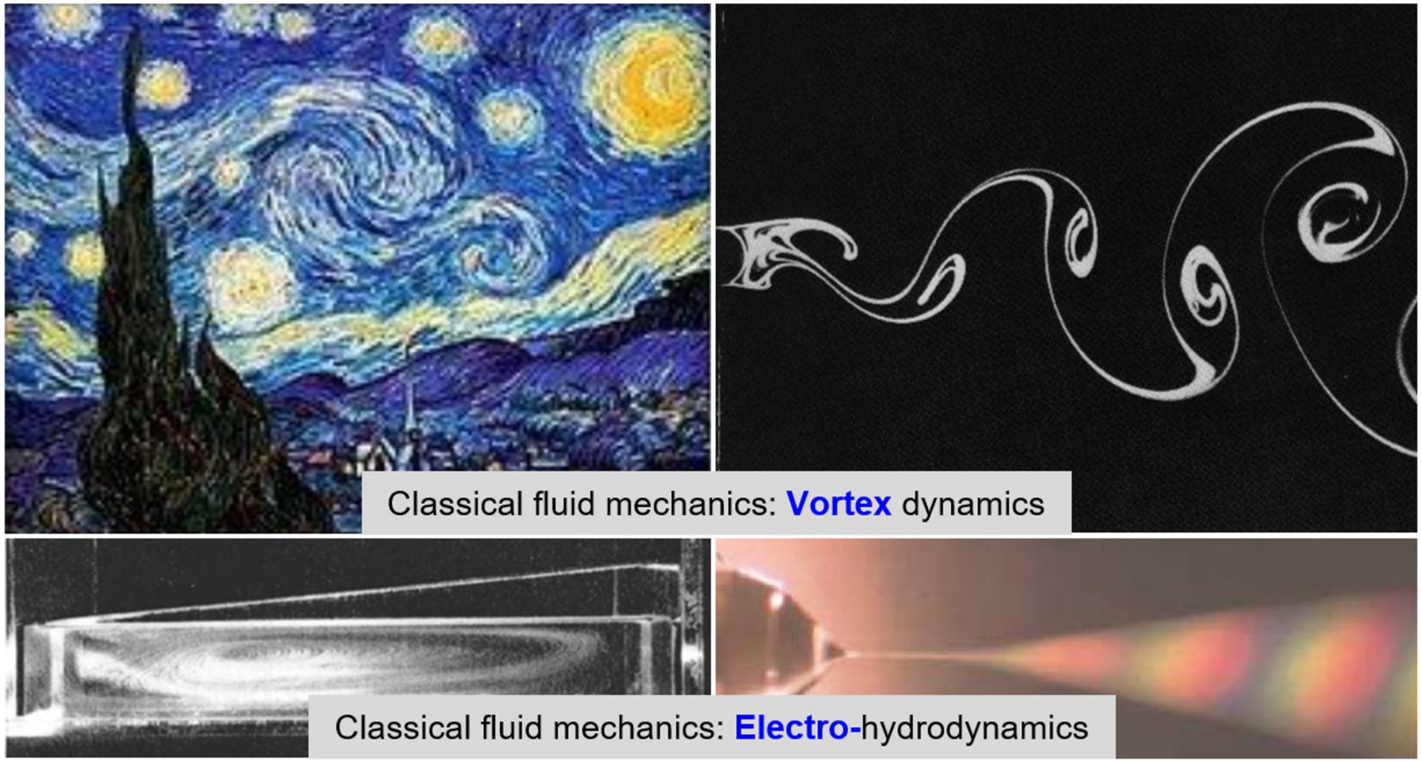
序参数组成 $[\lambda_D]$, l_N, l_R, W, $[l_{dis}]$, E_F, U_{corr}, $\beta_{E,B}$

电子水动力学：散射截面－守恒散射－电子集体运动－电子粘性

Contents

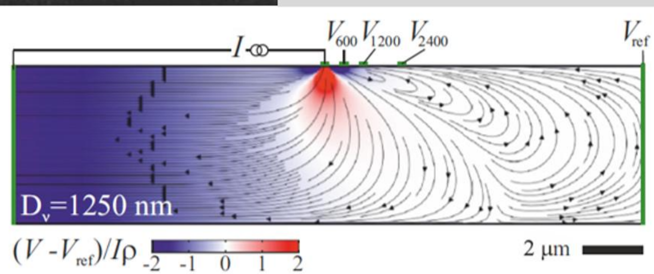
- “Strongly-correlated” systems: Figures, Problems and Stories
  - *“Where to start … with something new”*
  - Multiphase interface of immiscible electrolyte solutions
  - Transport properties of correlated quantum systems
- **Complex hydrodynamics as X-paradigm: What, Why, How, and Which?**
  - **Example: Emergence of quantum hydrodynamics**
  - **New physics originated from “interface” of perspective**
  - **Hallmark: Merging kinetic behaviors into hydrodynamics**

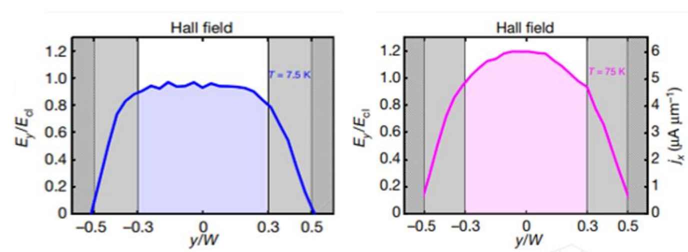
From one paradigm (electro-hydrodynamics) …



Classical fluid mechanics: **Vortex** dynamics

Classical fluid mechanics: **Electro-**hydrodynamics





Novel phenomena in 2D materials: **Electron backflow** | From **Ballistic** to **Viscous** regime

Note for top 2 figures. Van Gogh's *The Starry Night*. Taylor pump and Taylor cone.

And another paradigm (physical kinetics) …

- **Boltzmann transport equation (BTE) – semi-classical description**

collective moving velocity

$$\boxed{\frac{\mathrm{d}f}{\mathrm{d}t}} = \frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{r}} + \frac{\boldsymbol{F}}{m}\cdot\frac{\partial f}{\partial \boldsymbol{v}} = \boxed{C(f)} \begin{cases} \text{Resistive scattering} & f_0(\boldsymbol{r}, \varepsilon_{\boldsymbol{k}}) \\ \text{Conservative scattering} & f_0(\boldsymbol{r}, \varepsilon_{\boldsymbol{k}} - \boldsymbol{u}\cdot\boldsymbol{p}) \end{cases}$$

Change of particle state ⟸ **Particle scattering** **Different mechanisms** ⟹ **Different (quasi-) equilibrium states**

f: the non-equilibrium distribution function of the particle cluster around $(\boldsymbol{r}, \boldsymbol{v})$

Solving BTE: Deterministic | Stochastic (particle nature)

Upscaling BTE: Hydrodynamic description (macroscopic)

Beyond BTE: *wave nature, strong correlation, scattering rate*

<u>Coherence</u>: Quantum transport in low-D system

<u>Localization</u>: Strong disordered/correlated system

<u>Super/Magneto/Topo</u>: Strong (spin/Coulomb) correlated system

1. Rammer. *Quantum transport theory*. Perseus, 1998.
2. Datta. *Quantum transport – Atom to transistor*. Cambridge University Press, 2005
3. Nazarov. *Quantum transport – Introduction to nanoscience*. Cambridge University Press. 2009.
4. G. Chen. *Nanoscale energy transport and conversion*, Tsinghua University Press, 2014

Into X-paradigm (quantum hydrodynamics) …

- **Boltzmann transport equation (BTE) – Chapman-Enskog expansion**

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{r}} + \frac{\boldsymbol{F}}{m}\cdot\frac{\partial f}{\partial \boldsymbol{v}} = C(f) \qquad f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \ldots$$

- **Classical N-S equation**

$$\frac{\mathrm{D}\boldsymbol{P}}{\mathrm{D}t} + \nabla\cdot\boldsymbol{\sigma} = \rho\boldsymbol{g} \qquad \boldsymbol{\sigma} = p\boldsymbol{I} - \mu\left(\frac{2}{3}(\nabla\cdot\boldsymbol{u})\boldsymbol{I} + \nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right)$$

- **Phonon N-S equation**

$$\boxed{\frac{\partial \boldsymbol{q}}{\partial t}} + \nabla\cdot\boldsymbol{Q} = -\frac{\boldsymbol{q}}{\tau_{\mathrm{R}}} \qquad \boldsymbol{Q} = \frac{1}{3}v_{\mathrm{g}}^2 e\boldsymbol{I} - \frac{1}{5}v_{\mathrm{g}}^2\tau_{\mathrm{N}}\left((\nabla\cdot\boldsymbol{q})\boldsymbol{I} + \nabla\boldsymbol{q} + (\nabla\boldsymbol{q})^T\right)$$

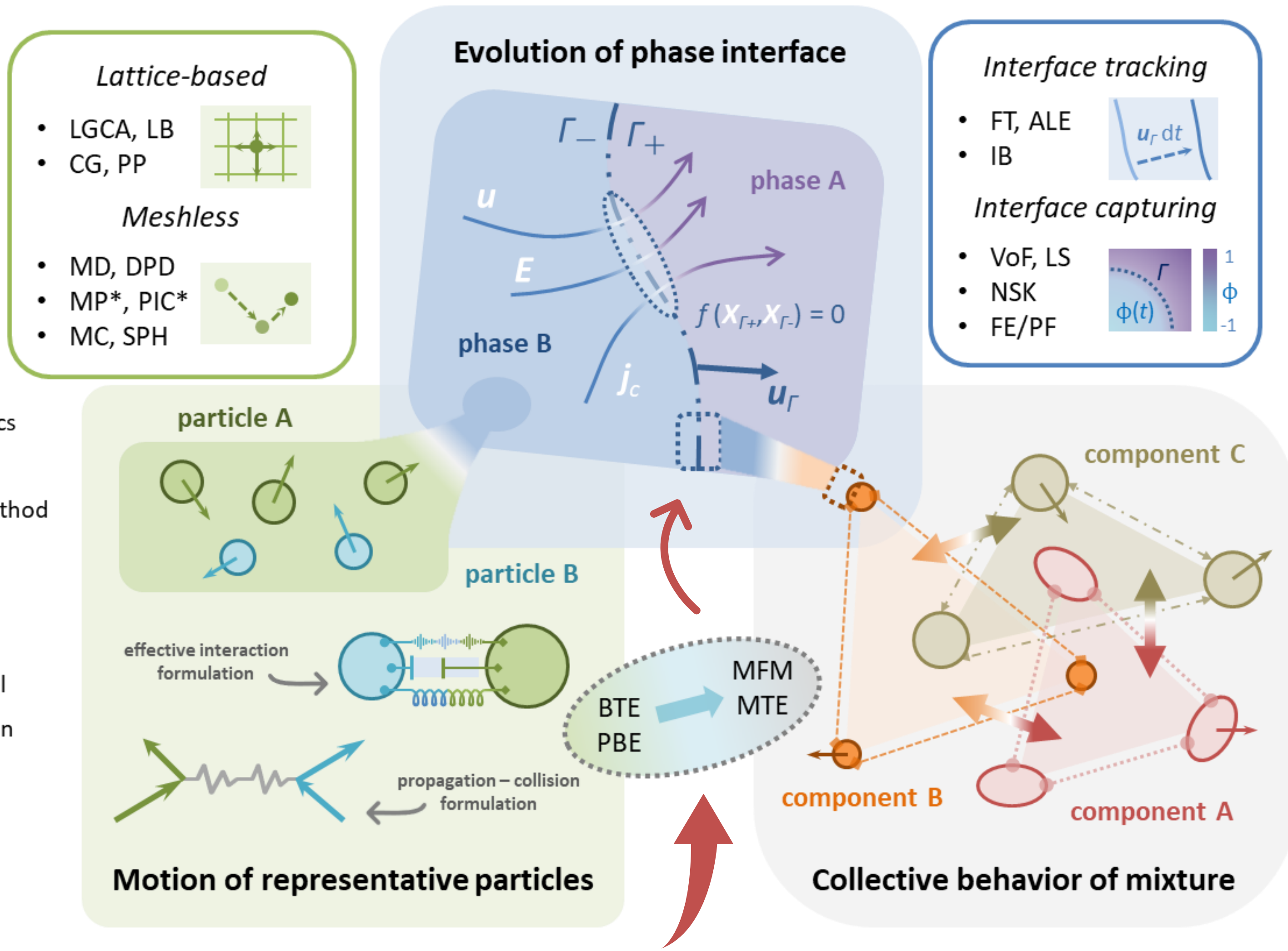
- **Electron N-S equation**

$$\frac{\partial \boldsymbol{P}}{\partial t} + \boxed{\nabla\cdot\boldsymbol{T}} = -\frac{\boldsymbol{P}}{\tau_{\mathrm{MR}}} + m^*\boldsymbol{F}_{\mathrm{macro}} \qquad \boldsymbol{T} = \mathcal{P}\boldsymbol{I} - \mu_{\mathrm{e}}\left((\nabla\cdot\boldsymbol{u})\boldsymbol{I} + \nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right)$$

1. Y.Y. Guo. 非傅里叶导热的宏观声子输运模型及非平衡热力学. 博士学位论文, 2018.
2. Y.F. Huang. 微纳尺度低维电子输运的水动力学研究. 本科毕业论文, 2019.

“Interface” from the perspective of multiscale …

LGCA: lattice gas cellular automaton
LB: lattice Boltzmann model
CG: color gradient model
PP: pseudo-potential model
MD: molecular dynamics
DPD: dissipative particle dynamics
MP: material point method
PIC: particle-in-cell method
MC: Monte-Carlo method
SPH: smoothed particle hydrodynamics
FT: front-tracking method
ALE: arbitrary Lagrangian-Eulerian method
IB: immersed boundary method
VoF: volume-of-fluid model
LS: level-set model
FE/PF: free energy / phase field model
NSK: Navier-Stokes-Korteweg equation
BTE: Boltzmann transport equation
PBE: population balance equation
MFM: multi-fluid model
MTE: momentum transport equation



新物理源自新研究对象(视角)：新界面 → 新尺度 → 新机理 → 新描述 → 新方法

X-paradigm: What, Why, How, and Which ?

- 何为**力学 (工科基础)** 相关的交叉学科？
  - 《科学革命的范式》范式及其**转换**
  - 《论技术科学》例外**来自/创造**工程需求
  - 交叉特征：界面接触 + 内核融合 + 进化独立
  - 现实投影：工业需求 + 大牛烙饼 + 众人推广
  - 典型案例：软物质物理，计算神经科学
- 原学科范式 – 力学 / 物理 / 化学
  - 经典[电]流体力学：[electro-]hydrodynamics
  - 凝聚态物理(粒子散射)：physical kinetics
  - 电化学/胶体科学(离子吸附)：chemical kinetics

		电动多相流体力学	量子水动力学
a)	出现新“界面”：	液液界面	散射截面
b)	涌现新“尺度”：	双侧双电层	守恒散射强度
c)	新“研究对象”：	电动双侧耦合	量子集体运动
d)	引入新“机理”：	界面极化	量子粘性
e)	要求新“描述”：	有效边界	水动力学
f)	亟需新“方法”：	摄动展开	升尺度展开

- 新学科范式 – 或仅作为名词归纳
  - 复杂流体力学：complex (soft) hydrodynamics
  - 研究对象：含内部额外(强关联)自由度的流体体系
  - 复杂机理举例：奇界面，多物理，子结构 “序参数”
    - 子分支 1：multiphase electrokinetic hydrodynamics
    - 子分支 2：quantum hydrodynamics
    - 子分支 3：soft matter physics (of its transport)
- 关键图像 – merge kinetics into hydrodynamics
  - 传统手段：物理实验 → 理论模型 → 数值模拟(实验)
  - 新兴手段：理论建构 与 数据处理 的结合

多尺度系统的对策与挑战

奇界面：摄动理论-时空分区，参数要求高、难延拓

子结构：粗粒理论-代表单元，大尺度模型、难整合

多物理：有效理论-机理提取，时空强关联、难解耦

反问题：低维理论-去除冗余，优化成本高、难规划

涌现/演生 (emergence)

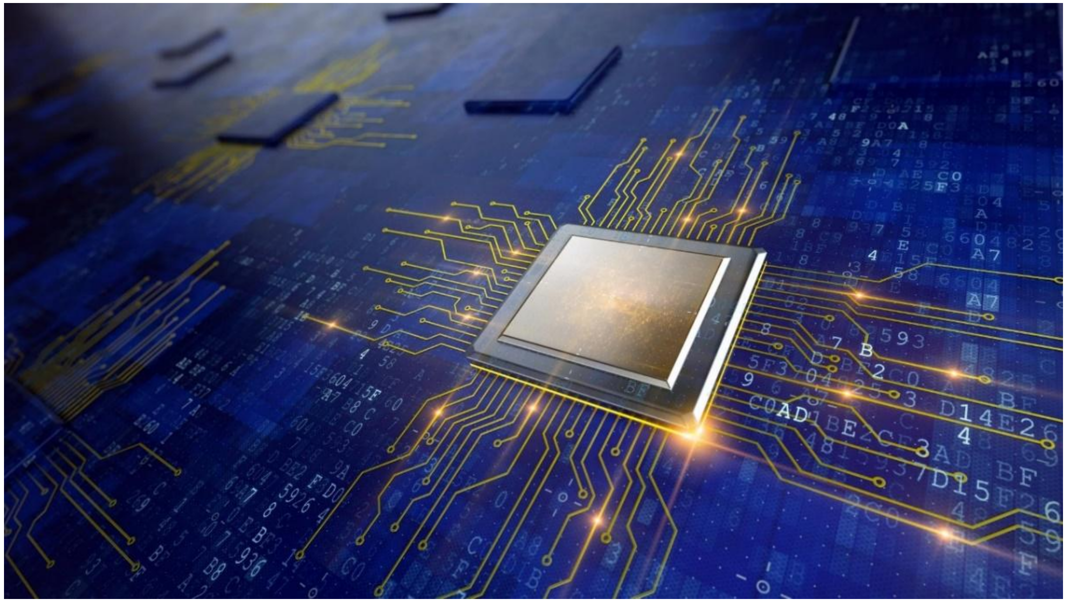
系统不同尺度的模型可能完全不同
并且互相几乎“不可通约”
多尺度模拟通常仍只是“黑箱”！

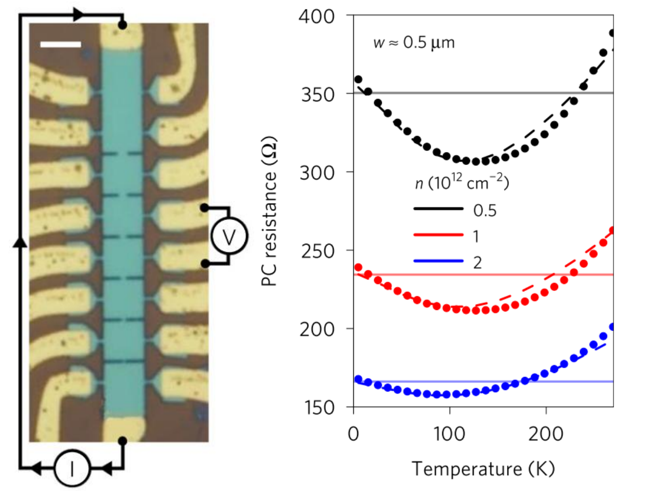
Last but not at least …

- "Strongly-correlated" systems: Figures, Problems and Stories
  - ***"Where to start … with something new"***
  - Multiphase interface of immiscible electrolyte solutions
  - Transport properties of correlated quantum systems
- **Complex hydrodynamics as X-paradigm**: What, Why, How, and Which?
  - **Example:** Birth of Huang's Undergrad/PhD proposals
  - **New physics originated from:** jump out of **local consensus**
  - **Hallmark:** The first paper (易上手 – 避免强耦合，抓本质 – 敢于做假设)
  - **Positive feedback requiring:** 良师益友，敢想敢干，博观约取，厚积薄发

Thank you

Electron hydrodynamics: transit – σ_e





Super-ballistic flow in confined graphene

High electron conductivity

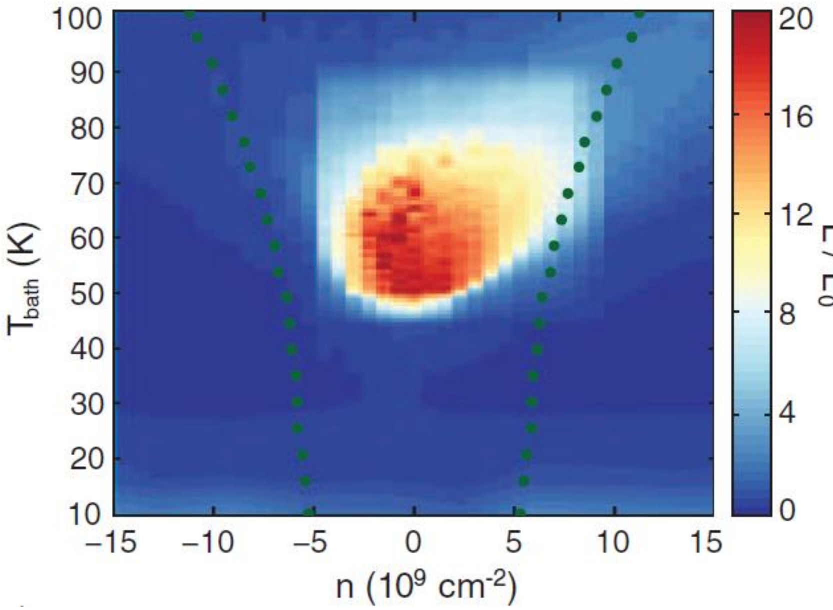
Low heat generation → **High** current transport capability

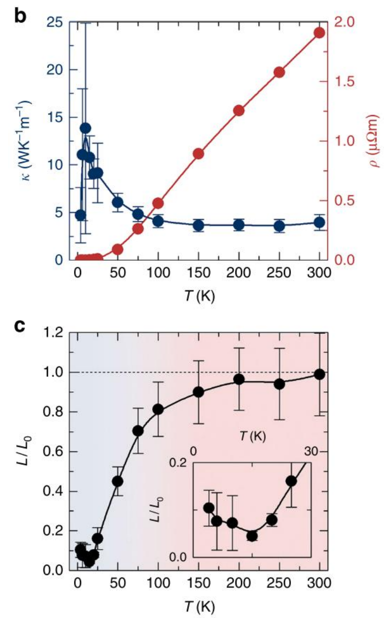
High cutoff frequency → **Low** switching time of the transistor

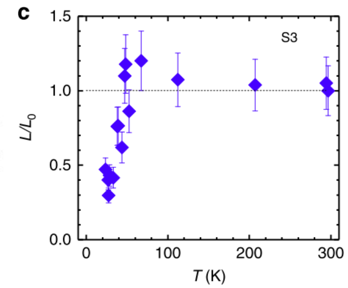
- ✓ System: 2D materials, ultra-pure, microscale and low-T
- ✓ Phenomena: high mobility, may improve performance of semiconductors

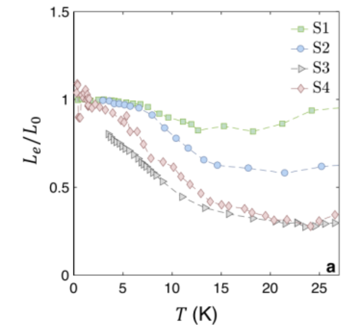
Kumar K.R. et al. *Nat Phys*, **13**(12): 1182-5, 2017.

Electron hydrodynamics: energy – ZT_e









Breakdown of W-F law in graphene (left) & WP_2 (mid) & MoP (r-top) & Sb (r-bot)

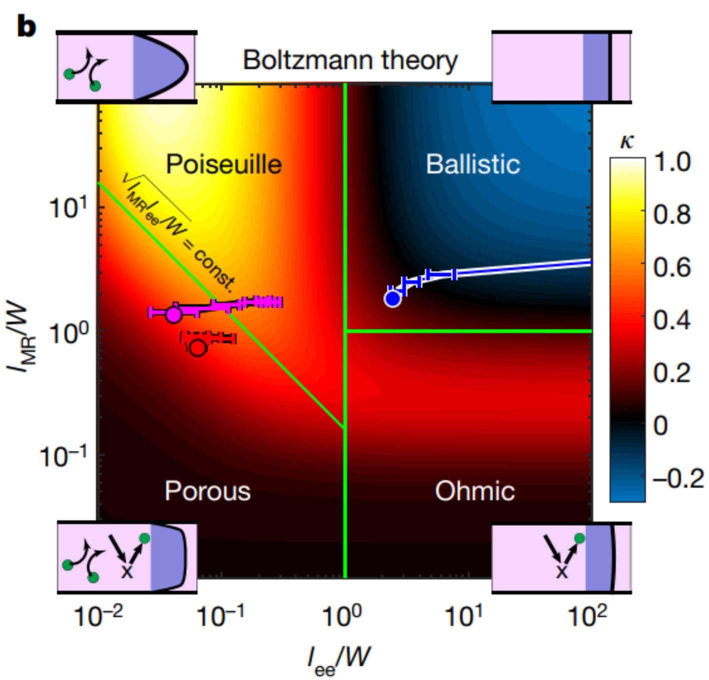
$$\mathrm{ZT} = S^2/\mathcal{L} \quad \mathcal{L} \equiv \frac{\kappa_e}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 \equiv \mathcal{L}_0$$

higher/lower κ_e than W-F law

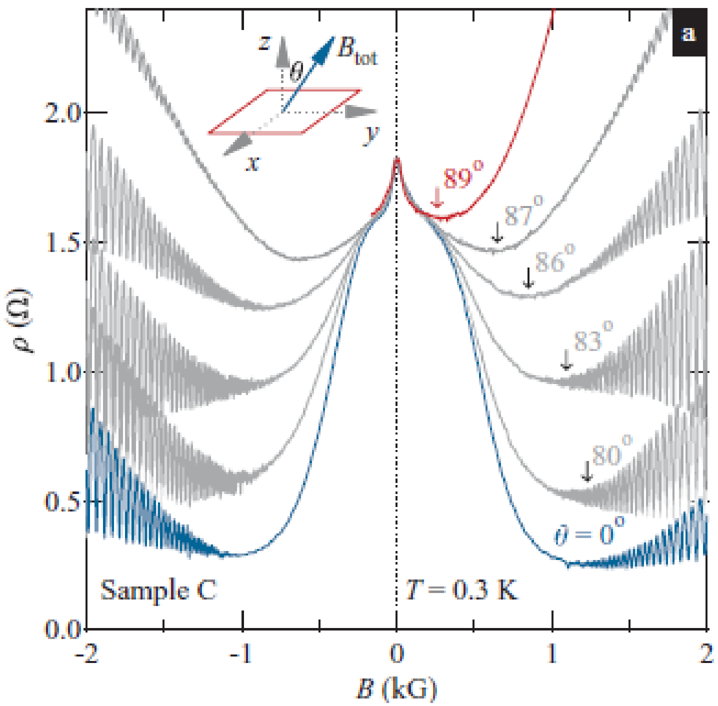
- ✓ System: 2D material, ultrapure, low T
- ✓ Phenomena: Anomalous transport of 2D electrons
- ✓ Impact: low heat loss in thermoelectric materials

1. Crossno J. *et al*. *Science*, **351**: 1058-1061, 2016.
2. Gooth, J., *et al*., *Nat Comm*. **9**(1): 4093, 2018.
3. Jaoui, A., *et al*., *npj Quantum Materials*, **3**(1): 64, 2018.
4. Kumar, N., *et al*., *Nat Comm*, **10**(1): 2475, 2019.
5. Jaoui, A., B. Fauqué, and K. Behnia, *Nat Comm*, **12**(1): 195, 2021.

Electron hydrodynamics: info – $\sigma_e(B,T)$



Conduction regime with different mechanisms
(confinement)

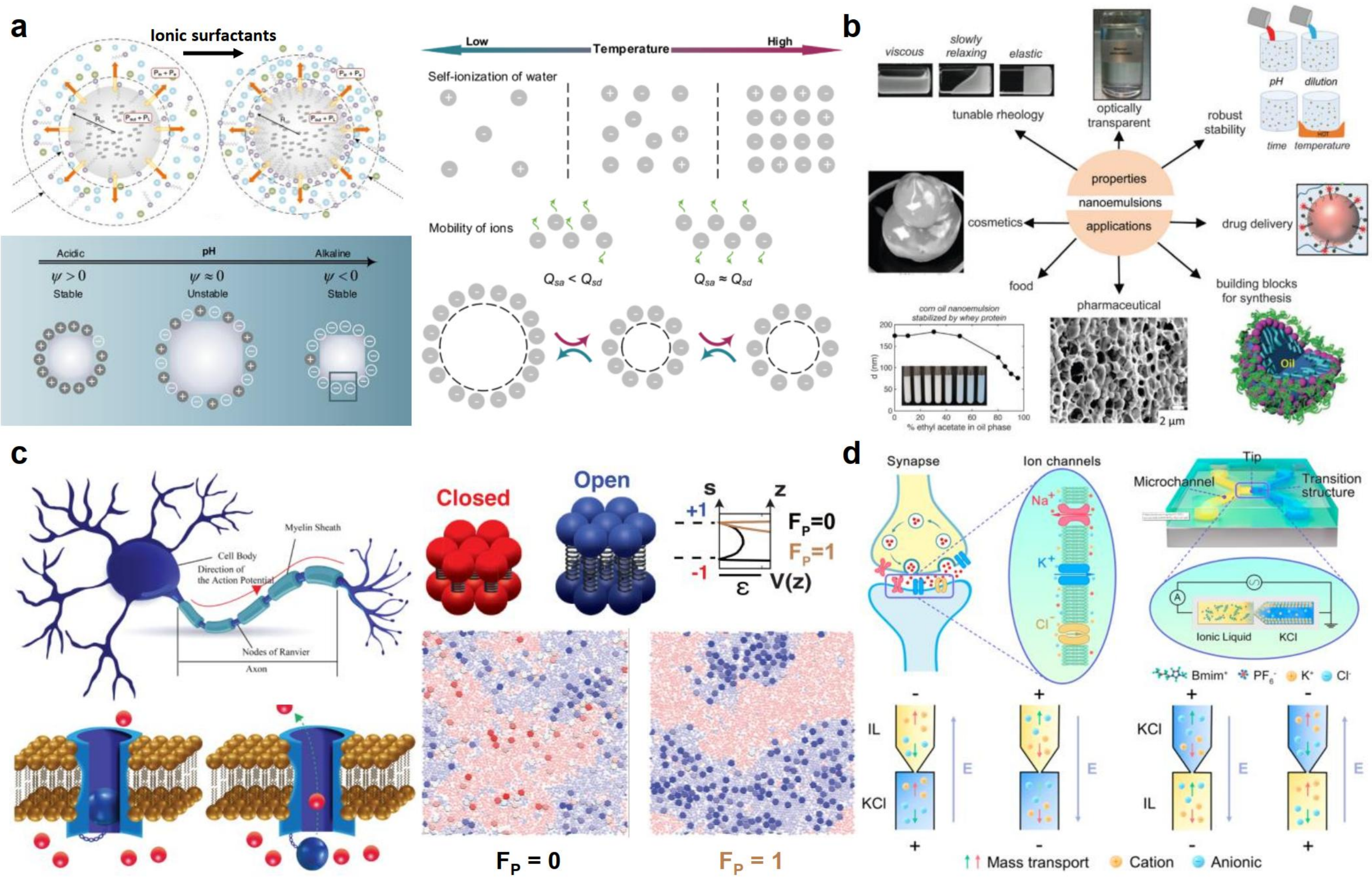


Giant negative magneto-resistance in GaAs/AlGaAs
(magnetic field)

Magnetic field will dramatically impact the electric resistance in confined materials.

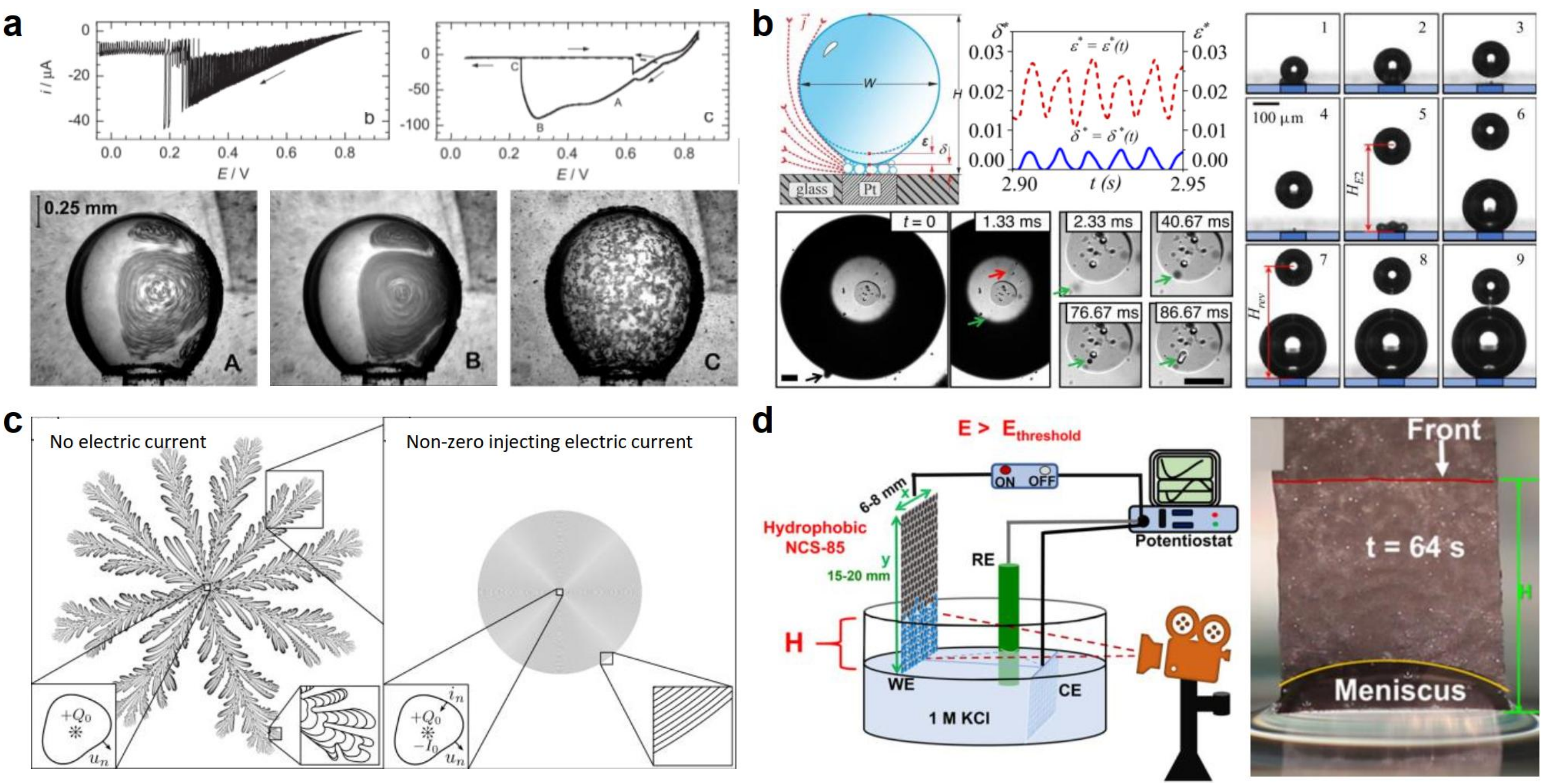
1. Hatke A.T. *et al*. *Phys Rev B*, **85**: 081304, 2012.
2. Alekseev P. S. and Semina M. A. *Phys Rev Lett*, **98**: 165412, 2018.
3. Chandra M, Kataria G, Sahdev D, *et al*. *Phys Rev B*, **99**: 165409, 2019.
4. Mandal I, Lucas A. *Phys Rev B*, **101**(4): 045122, 2020.

Electrokinetic multiphase flow: interface charging



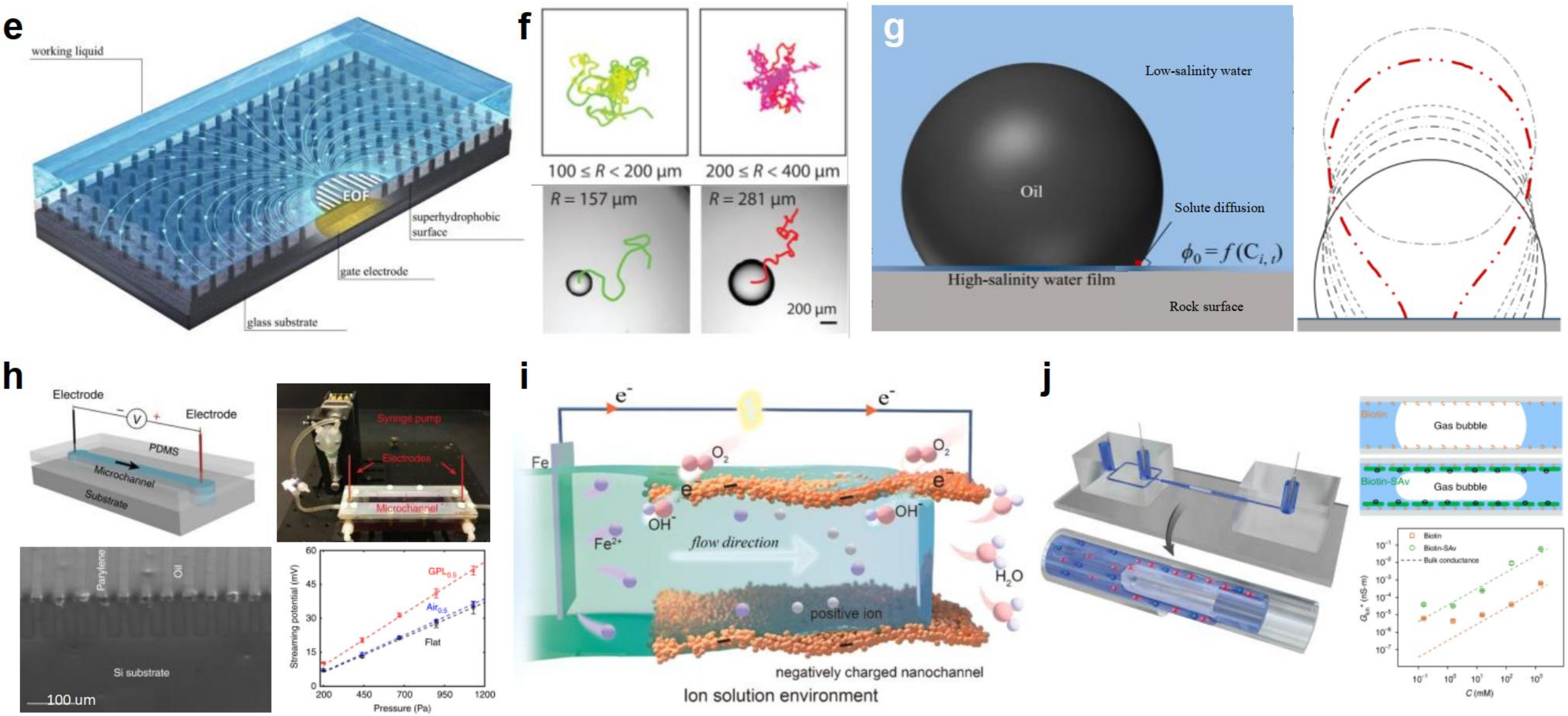
a) Li *et al.*, *JCIS*, 2021; Ma *et al.*, *JCIS*, 2022 / 2024.
b) Gupta *et al.*, *Soft Matter*, 2016.
c) Torbati *et al.*, *Rev Mod Phys*, 2022; Suma *et al.*, *PRX Life*, 2024.
d) Li *et al.*, *Nano Lett*, 2024.

Electrokinetic multiphase flow: field-driven flow



a) Trojanek *et al.*, *Electrochem Comm*, 2017.
b) Bashkatov *et al.*, *Phys Rev Lett* / *Phys Chem Chem Phys* / *Journal of Fluid Mech*, 2019/2022/2023/2024.
c) Mirzadeh *et al.*, *Phys Rev Lett*, 2017.
d) Pan *et al.*, 2023.

Electrokinetic multiphase flow: ion-driven transport



e) Dehe *et al*, *Phys Rev Fluids*, 2020.
f) Suda *et al*, *Phys Rev Lett*,2021; Michelin, *Annal Rev Fluid Mech*, 2023.
g) An *et al*, *Fuel*, 2022.
h) Fan *et al*, *Nat Comm*, 2018.
i) Li *et al*, *Nano Energy*, 2021.
j) Ma *et al*, *Nat Comm*, 2020.