| Step | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
|---|---|
| 1a | $\{C = \widehat{C}\}$ |
| 4 | $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$, $A \rightarrow \left(\begin{array}{c\|c} A_L & A_R \end{array}\right)$ where $C_{TL}$ is $0 \times 0$, $A_L$ has 0 columns |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_0 & a_1 & A_2 \end{array}\right)$ where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 column |
| 6 | $\left\{ \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|cc} A_0^T A_0 + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \hline \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$ |
| 8 | $c_{10}^T := a_1^T A_0 + c_{10}^T$ <br> $\gamma_{11} := a_1^T a_1 + \gamma_{11}$ |
| 7 | $\left\{ \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{cc\|c} A_0^T A_0 + \widehat{C}_{00} & A_0^T a_1 + \widehat{c}_{10}^T & \widehat{C}_{02} \\ a_1^T A_0 + \widehat{c}_{10}^T & a_1^T a_1 + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \hline \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_0 & a_1 & A_2 \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{C = A^T A + \widehat{C}\}$ |

| Step | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
|---|---|
| 1a | $\{ \qquad \}$ |
| 4 | where |
| 2 | $\left\{ \qquad \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \qquad \wedge \qquad \right\}$ |
| 5a | where |
| 6 | $\left\{ \qquad \right\}$ |
| 8 | |
| 7 | $\left\{ \qquad \right\}$ |
| 5b | |
| 2 | $\left\{ \qquad \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \qquad \wedge \neg( \qquad ) \qquad \right\}$ |
| 1b | $\{ \qquad \}$ |

| Step | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
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| 8 | |
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| 5b | |
| 2 | $\left\{ \right\}$ |
| | endwhile |
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| Step | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
|---|---|
| 1a | $\{C = \widehat{C}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while do |
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| 5a | where |
| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg( \quad ) \right\}$ |
| 1b | $\{C = A^T A + \widehat{C}$ $\}$ |

| Step | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
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| 4 | where |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$ |
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| 8 | |
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| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{C = A^T A + \widehat{C} \}$ |

| Step | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
|---|---|
| 1a | $\{C = \widehat{C}$ $\}$ |
| 4 | $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ , $A \rightarrow \left(\begin{array}{c\|c} A_L & A_R \end{array}\right)$ <br> where $C_{TL}$ is $0 \times 0$, $A_L$ has 0 columns |
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| 3 | while $m(C_{TL}) < m(C)$ do |
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| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
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| 1b | $\{C = A^T A + \widehat{C}$ $\}$ |

| Step | Algorithm: $[C] := \textsc{syrk\_lt\_unb\_var1}(A, C)$ |
|---|---|
| 1a | $\{C = \widehat{C}\}$ |
| 4 | $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ , $A \rightarrow \left(\begin{array}{c\|c} A_L & A_R \end{array}\right)$ <br> where $C_{TL}$ is $0 \times 0$, $A_L$ has 0 columns |
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| 5a | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_0 & a_1 & A_2 \end{array}\right)$ <br> where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 column |
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| 8 | |
| 7 | $\{ \}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_0 & a_1 & A_2 \end{array}\right)$ |
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| 1b | $\{C = A^T A + \widehat{C}\}$ |

| Step | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
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| 8 | |
| 7 | $\{ \}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_0 & a_1 & A_2 \end{array}\right)$ |
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| 8 | |
| 7 | $\left\{ \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{cc\|c} A_0^T A_0 + \widehat{C}_{00} & A_0^T a_1 + \widehat{c}_{10}^T & \widehat{C}_{02} \\ a_1^T A_0 + \widehat{c}_{10}^T & a_1^T a_1 + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \hline \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_0 & a_1 & A_2 \end{array}\right)$ |
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| | endwhile |
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<table>
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<tr><td>3</td><td>while $m(C_{TL}) < m(C)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$</td></tr>
<tr><td>5a</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c|c} A_L & A_R \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_0 & a_1 & A_2 \end{array}\right)$<br>where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 column</td></tr>
<tr><td>6</td><td>$\left\{ \left(\begin{array}{c|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c|cc} A_0^T A_0 + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \hline \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$</td></tr>
<tr><td>8</td><td>$c_{10}^T := a_1^T A_0 + c_{10}^T$<br>$\gamma_{11} := a_1^T a_1 + \gamma_{11}$</td></tr>
<tr><td>7</td><td>$\left\{ \left(\begin{array}{cc|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{cc|c} A_0^T A_0 + \widehat{C}_{00} & A_0^T a_1 + \widehat{c}_{10}^T & \widehat{C}_{02} \\ a_1^T A_0 + \widehat{c}_{10}^T & a_1^T a_1 + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \hline \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_0 & a_1 & A_2 \end{array}\right)$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{ C = A^T A + \widehat{C} \}$</td></tr>
</table>

| | |
|---|---|
| | Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$ |
| | |
| | $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$, $A \rightarrow \left(\begin{array}{c\|c} A_L & A_R \end{array}\right)$ where $C_{TL}$ is $0 \times 0$, $A_L$ has 0 columns |
| | |
| | while $m(C_{TL}) < m(C)$ do |
| | |
| | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_0 & a_1 & A_2 \end{array}\right)$ where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 column |
| | |
| | $c_{10}^T := a_1^T A_0 + c_{10}^T$<br>$\gamma_{11} := a_1^T a_1 + \gamma_{11}$ |
| | |
| | $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_0 & a_1 & A_2 \end{array}\right)$ |
| | |
| | endwhile |
| | |
| | |

Algorithm: $[C] := \text{SYRK\_LT\_UNB\_VAR1}(A, C)$

$$C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right), A \rightarrow \left(\begin{array}{c|c} A_L & A_R \end{array}\right)$$

where $C_{TL}$ is $0 \times 0$, $A_L$ has 0 columns

**while** $m(C_{TL}) < m(C)$ **do**

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c|c} A_L & A_R \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_0 & a_1 & A_2 \end{array}\right)$$

where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 column

$$c_{10}^T := a_1^T A_0 + c_{10}^T$$

$$\gamma_{11} := a_1^T a_1 + \gamma_{11}$$

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_0 & a_1 & A_2 \end{array}\right)$$

**endwhile**