Financial Econometrics

Final Project

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An overview of Vanguard Funds*

1. `VBISX` – Short-Term Bond Index Fund Investor Shares

Vanguard Short-Term Bond Index Fund employs a "passive management" investment approach designed to track the performance of the Bloomberg Barclays Capital U.S. 1-5 year government/cred-it float adjusted index by investing in government, corporate and international dollar denominated bonds that have maturities between 1 and 5 years.

2. `VBLTX` – Long-Term Bond Index Fund

Vanguard Long-Term Bond Index Fund aims to track the performance of the Bloomberg Barclays Capital U.S. Long Government/Credit Float Adjusted Index, a market weighted bond in-dex that includes investment-grade bonds with a dollar-weighted average maturity of 15 to 30 years.

3. `VFINX` – 500 Index Fund Investor Shares

Vanguard 500 Index Fund seeks track the performance of the Standard & Poor's 500 Index, which is dominated by the stocks of large U.S. companies by investing substantially all of its assets in the stocks that make up the Index.

4. `VIVAX` – Value Index Fund Investor Shares

Vanguard Value Index Fund is an open-end fund incorporated in the USA. The Fund aims to track the investment performance of the CRSP U.S. Large Cap Value Index, a broadly diversified index of value stocks of predominantly large U.S. companies. The Fund invests all or substantially all of its assets in the stocks that make up the Index.

5. `VIGRX` – Growth Index Fund Investor Shares

Vanguard Growth Index Fund is an open-end fund incorporated in the USA. The Fund aims to track the performance of the CRSP US Large Cap Growth Index, a broadly diversified index of growth stocks of predominantly large U.S. companies. The Fund invests all or substantially all of its assets in the stocks that make up the index.

6. `VEXMX` – Extended Market Index Fund Investor Shares

Vanguard Extended Market Index Fund is an open-end fund incorporated in the USA. The Fund's objective is to track the performance of the Wilshire 4500 Completion Index, a broadly diversified index of stocks of small and medium-size U.S. companies. The Fund invests all or substantially all of its assets in the 1,200 largest capitalization stocks in its target index.

7. `NAESX` – Small-Cap Index Fund Investor Shares

Vanguard Small-Cap Index Fund seeks the performance of a benchmark index that measures the investment return of small-capitalization stocks by employing an indexing investment approach designed to track the performance of the CRSP US Small Cap Index, a broadly diversified index of stocks of small U.S. companies. [FIGI BBG000BCBC01]

8. `VEIEX` – Emerging Markets Stock Index Fund

Vanguard Emerging Markets Stock Index Fund seeks the performance of the Select Emerging Markets Index by investing in the common stocks included in the Select Emerging Markets Index, which includes approximately 595 common stocks of companies located throughout the world.

9. `VEURX` – European Stock Index Fund Investor Shares

Vanguard European Stock Index Fund is an open-end fund incorporated in the USA. The Fund's objective is to match the performance of the FTSE Developed Europe Index. The Fund invests in more than 500 stocks across the European region, which makes up roughly half of the non-U.S. equity marketplace.

10. `VPACX` – Pacific Stock Index Fund Investor Shares

Vanguard Pacific Stock Index Fund is an open-end fund incorporated in the USA. The Fund aims to match the performance of the FTSE Developed Asia Pacific Index. The Fund invests in about 840 stocks throughout the Asia Pacific region, which makes up roughly a quarter of the non-U.S. equity marketplace.

*overview extracted from Bloomberg terminal

Section One: Return Calculations and Sample Statistics

Descriptive Statistics and Normality

	Mean	Variance	SD	Skewness (adjusted)	Kurtosis (adjusted)	JB-stat
NAESX	0.0100	0.0012	0.0348	-0.1653	2.8270	0.264606
VBISX	0.0007	**0.0000**	0.0037	0.1898	3.3024	0.436654
VBLTX	0.0034	**0.0005**	0.0232	-0.1673	3.5470	0.700405
VEIEX	0.0032	**0.0019**	**0.0433**	0.1549	3.0541	0.244489
VEURX	0.0051	0.0013	0.0365	0.0217	2.1607	-1.49805
VEXMX	0.0101	0.0012	0.0346	-0.2858	3.1136	0.834162
VFINX	**0.0104**	0.0008	0.0282	-0.0317	3.1035	0.013059
VIGRX	**0.0118**	0.0010	0.0313	0.0498	2.7594	-0.01022
VIVAX	0.0092	0.0008	0.0279	-0.0967	2.9717	0.095094
VPACX	0.0062	0.0010	0.0321	0.0768	3.3169	0.140871
Risk Free Asset	0.0003	0.0000	0.0004	1.3382	3.6111	18.78603

Table 1 Simple Monthly Returns Statistics

	Mean	Variance	SD	Skewness (adjusted)	Kurtosis (adjusted)	JB-stat
NAESX	0.0093	0.0012	0.0346	-0.2562	2.8678	0.661441
VBISX	0.0007	**0.0000**	0.0037	0.1775	3.2972	0.387026
VBLTX	0.0032	**0.0005**	0.0232	-0.2520	3.5649	1.104038
VEIEX	0.0023	**0.0019**	**0.0431**	0.0312	2.9240	0.008806
VEURX	0.0044	0.0013	0.0363	-0.0404	2.1697	-1.43808
VEXMX	0.0095	0.0012	0.0345	-0.3894	3.2379	1.575536
VFINX	**0.0100**	0.0008	0.0280	-0.1173	3.1017	0.142549
VIGRX	**0.0113**	0.0010	0.0309	-0.0301	2.7775	-0.0188
VIVAX	0.0088	0.0008	0.0277	-0.1760	2.9858	0.314818
VPACX	0.0056	0.0010	0.0320	-0.0311	3.3203	0.093372
Risk Free Asset	0.0003	0.0000	0.0004	1.3378	3.6096	18.76985

Table 2 Continuously Compounded Monthly Returns

1.1 Notable Returns

Among the risky assets, the Growth Index Fund Investor Shares (VIGRX) yielded the highest expected return and relatively low variance and standard deviation. We can attribute the high and relatively stable returns to the component assets tracked by the VIGRIX, top holdings including tech heavyweights FAANGs – Facebook, Apple, Amazon, Netflix and Google, and other notable holdings being large cap US stocks Boeing and Philip Morris – all of which have historically generated huge returns and demonstrated impressive growth. It therefore makes sense that the VIGRIX generated the highest returns. Its relative stability could be explained by the fact that these stocks are large cap stocks traded in the US market, where price action is less volatile.

Tightly following is the returns of the Vanguard 500 Index Fund (VFINX), which tracks the S&P 500 Index in the US, with top holdings similar to those in VIGRIX: the FAANGs tech heavyweights, key financials such as JPMorgan Chase, Bank of America, as well as strong consumer and healthcare names like Pfizer and AT&T. Like VIGRIX, the large cap and strong growth of the top holdings explains the high returns. Indeed, the SPX had a generally stable upward trend from 2013 to 2017, except for dips in 2H15 and 1Q16. However, we acknowledge the relative volatility, and attribute that to dips from late 2015 to early 2016, which was when China was actively trying to deleverage in the midst of stock market turbulence, which could have affected the SPX heavyweights, especially the tech giants.

The Long Term Bond Index Fund (VBLTX) had the second lowest variance and reasonable returns. This makes sense, as the fund tracks the Bloomberg Barclays US Long Government/Credit Float Adjusted Index, which is a market weighted bond index that includes investment grade bonds with a dollar-weighted maturity of 15-30 years. The investment grade bonds explains the size of returns, which are not exceptionally high but also not exceptionally low. The low variance, or in other words, stability of its performance could be attributed to the fact that these bonds have long maturity and should therefore be quite stable.

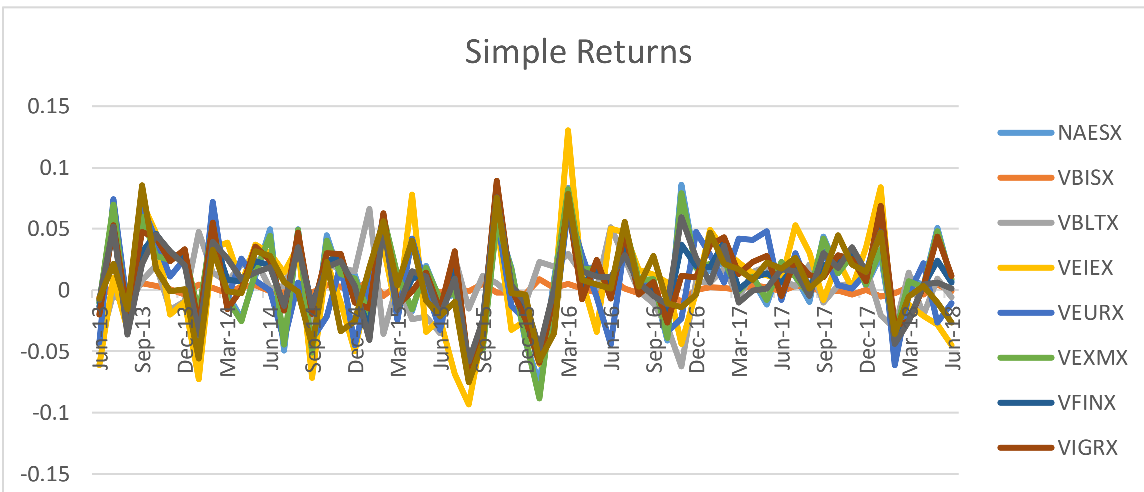


Table 3 Simple Monthly Returns

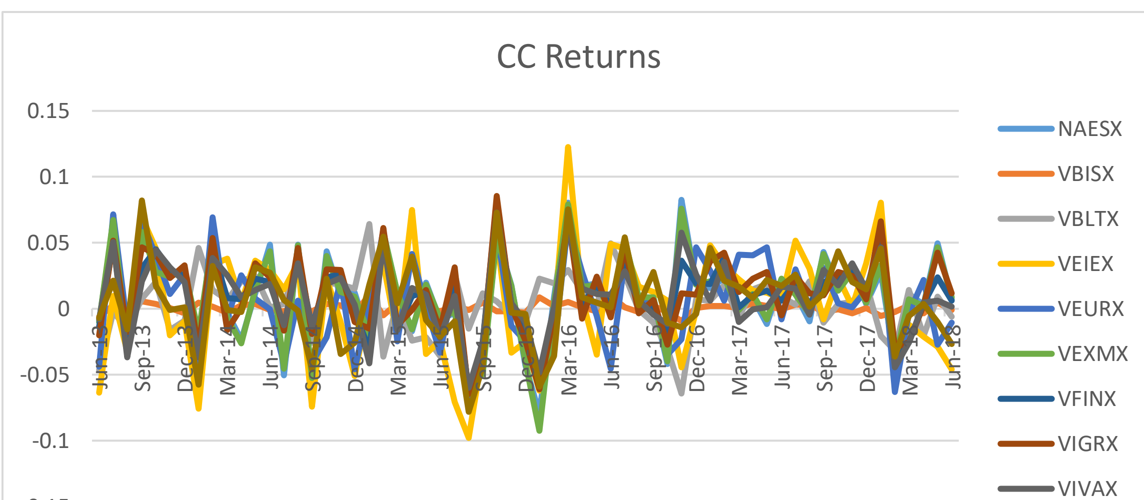


Table 4 CC Monthly Returns

1.2 Irregularities and News Events: The 2015 Chinese Stock Market Crash

Table 5 Chinese stock indices SHCOMP and CHINEXYT

Referencing Tables 3 and 4, we observe substantial fluctuations in returns from the 3rd quarter of 2015 to the 1st quarter of 2016. Recall that Chinese state officials actively tried to deleverage with Yuan devaluations, but efforts were futile as the popping of the stock market bubble on 12 June 2015 caused stock market turbulence until the Spring of 2016. As we observe in Figure 3, both the Shanghai Composite Index (commonly acknowledged to be backed by "National Team") and Shenzhen Chinext (indicator of real market sentiment due to huge retail base) ramped up to historic highs and plunged incredibly in the later months of 2015. The huge selloff had a massive impact on global markets, from developed US markets to more volatile emerging markets.

1.3 Normality and the Constant Expected Return Model

	Simple Statistics			CC Statistics		
	Skewness	**Kurtosis**	**JB-stat**	**Skewness**	**Kurtosis**	**JB-stat**
NAESX	-0.1653	2.8270	0.264606	-0.2562	2.8678	0.661441
VBISX	0.1898	3.3024	0.436654	0.1775	3.2972	0.387026
VBLTX	-0.1673	3.5470	0.700405	-0.2520	3.5649	1.104038
VEIEX	0.1549	3.0541	0.244489	0.0312	2.9240	0.008806
VEURX	0.0217	2.1607	-1.49805	-0.0404	2.1697	-1.43808
VEXMX	-0.2858	3.1136	0.834162	-0.3894	3.2379	1.575536
VFINX	-0.0317	3.1035	0.013059	-0.1173	3.1017	0.142549
VIGRX	0.0498	2.7594	-0.01022	-0.0301	2.7775	-0.0188
VIVAX	-0.0967	2.9717	0.095094	-0.1760	2.9858	0.314818
VPACX	0.0768	3.3169	0.140871	-0.0311	3.3203	0.093372

Table 6 Normality of Returns

We observe that the distributions of the returns of the various Vanguard funds are normal. Normality is defined by whether skewness is close to 0, and whether kurtosis is close to 3. Having observed the data, we see that the assets' skewness is more or less close to 0, while their kurtosis are reasonably close to 3.

A more precise estimation would be garnered as we put the assets to the Jarque-Bera test.

The Jarque-Bera test is a goodness-of-fit test of whether the sample data have the skewness and kurtosis matching a normal distribution. The null hypothesis is a joint hypothesis of the skewness being zero and the kurtosis of 3. Any deviation from this increases the JB statistic. The JB statistic is defined as follows:

$$JB = \frac{n-k+1}{6}(S^2 + \frac{1}{4}(C-3)^2)$$

Where n is the number of observations, S is the sample skewness. C is the sample kurtosis, and k is the number of regressors.

Following the computation of adjusted skewness, kurtosis, and JB statistics, we compare the JB statistics to its critical value of 5.99. As we observe from we cannot reject the null hypothesis that the asset return distributions are normal. We can therefore infer that asset returns are normal.

1.4 Comparison with the Constant Expected Return Model

The Constant Expected Return Model (CERM) assumes that an asset's return over time is normally distributed with a constant mean and variance. It also assumes that the correlations between asset returns are constant over time. As we have shown above, an interpretation of the sample statistics and the JB test would suggest some commonalities with the CERM. However, we are yet to test whether the asset returns are correlated and constant over time.

1.5 Estimates and standard errors

	Estimates			Estimated SE Values		
	Mean	**Variance**	**SD**	**Mean hat**	**Variance hat**	**SD hat**
NAESX	0.0093	0.0012	0.0346	0.0044	0.0002	0.0031
VBISX	0.0007	0.0000	0.0037	0.0005	0.0000	0.0003
VBLTX	0.0032	0.0005	0.0232	0.0030	0.0001	0.0021
VEIEX	0.0023	0.0019	0.0431	0.0055	0.0003	**0.0039**
VEURX	0.0044	0.0013	0.0363	0.0046	0.0002	0.0033
VEXMX	0.0095	0.0012	0.0345	0.0044	0.0002	0.0031
VFINX	0.0100	0.0008	0.0280	0.0036	0.0001	0.0025
VIGRX	0.0113	0.0010	0.0309	0.0040	0.0002	0.0028
VIVAX	0.0088	0.0008	0.0277	0.0035	0.0001	0.0025
VPACX	0.0056	0.0010	0.0320	0.0041	0.0002	0.0029
Market	**1.0731**	**8.4720**	**2.9107**	**0.0001**	**0.0000**	**0.0000**

Figure 1 Estimates and Standard Errors (CC Returns)

	Approximate 95% Confidence Intervals					
	Mean		**Variance**		**Standard Deviation**	
	lower	**upper**	**lower**	**upper**	**lower**	**upper**
NAESX	0.0005	0.0182	0.0008	0.0016	0.0283	0.0409
VBISX	-0.0003	0.0016	0.0000	0.0000	0.0031	0.0044
VBLTX	-0.0028	0.0091	0.0003	0.0007	0.0190	0.0274
VEIEX	-0.0088	0.0133	0.0012	0.0025	0.0353	0.0509
VEURX	-0.0049	0.0137	0.0008	0.0018	0.0297	0.0429
VEXMX	0.0006	0.0183	0.0008	0.0016	0.0283	0.0408
VFINX	0.0028	0.0172	0.0005	0.0011	0.0229	0.0331
VIGRX	0.0034	0.0192	0.0006	0.0013	0.0253	0.0365
VIVAX	0.0017	0.0159	0.0005	0.0010	0.0227	0.0327
VPACX	-0.0026	0.0138	0.0007	0.0014	0.0262	0.0377
Market	**0.0002**	**0.0004**	**0.0000**	**0.0000**	**0.0003**	**0.0005**

Figure 2 Approximation with 95% Confidence Intervals (CC Returns)

With reference to Figures 4 and 5, the Variance and Standard Deviations of the Vanguard funds are all below those of market returns. This makes sense, as the Vanguard fund component assets are handpicked to maximize returns and minimize risks, while the market component assets include all traded assets within the market – including those that are very risky.

In terms of precision, VEIEX is perhaps the least precise estimate, as it has the highest standard error of standard deviation, the measure of risk. On the other hand, VBISX, with the lowest standard error, could be said to be the most precise.

1.6 Covariance and Correlation of Vanguard Funds

Covariance of two assets measures the direction of linear dependence between the two variables, while the correlation of two assets measures both the direction and strength of the linear relationship between the two assets. Correlation is a scaled version of covariance.

	NAESX	*VBISX*	*VBLTX*	*VEIEX*	*VEURX*	*VEXMX*	*VFINX*	*VIGRX*	*VIVAX*	*VPACX*
NAESX	0.0012									
VBISX	0.0000	0.0000								
VBLTX	0.0000	0.0001	0.0005							
VEIEX	0.0007	0.0000	0.0003	0.0018						
VEURX	0.0008	0.0000	0.0001	0.0010	0.0013					
VEXMX	0.0012	0.0000	0.0000	0.0007	0.0008	0.0012				
VFINX	0.0008	0.0000	0.0000	0.0008	0.0008	0.0008	0.0008			
VIGRX	0.0009	0.0000	0.0001	0.0008	0.0009	0.0009	0.0008	0.0009		
VIVAX	0.0008	0.0000	0.0000	0.0007	0.0007	0.0008	0.0007	0.0007	0.0008	
VPACX	0.0007	0.0000	0.0001	**0.0011**	0.0009	0.0007	0.0006	0.0007	0.0006	0.0010

Figure 3 Covariance of Vanguard Assets

	NAESX	*VBISX*	*VBLTX*	*VEIEX*	*VEURX*	*VEXMX*	*VFINX*	*VIGRX*	*VIVAX*	*VPACX*
NAESX	1									
VBISX	-0.0881	1.0000								
VBLTX	0.0194	0.7900	1.0000							
VEIEX	0.4883	0.2078	0.2632	1.0000						
VEURX	0.6093	0.0616	0.1158	0.6713	1.0000					
VEXMX	0.9951	-0.0998	0.0126	0.5094	0.6418	1.0000				
VFINX	0.8454	-0.1091	0.0359	0.6426	0.7618	0.8540	1.0000			
VIGRX	0.8099	0.0042	0.1156	0.6374	0.7844	0.8332	**0.9546**	1.0000		
VIVAX	0.8362	-0.2067	-0.0368	0.6053	0.6905	0.8336	**0.9646**	0.8446	1.0000	
VPACX	0.6336	0.0320	0.0934	0.8431	0.7609	0.6617	0.7329	0.7599	0.6647	1.0000

Figure 4 Correlation of Vanguard Assets

With reference to Figures 6 and 7, the Emerging Markets Stock Index Fund (VEIEX) and Pacific Stock Index Fund Investor Shares (VPACX) have the highest covariance. The 500 Index Fund Investor Shares (VFINX), Growth Index Fund Investor Shares (VIGRX) and Value Index Fund Investor Shares (VIVAX) have the highest positive correlation. The Short-term Bond Index Fund Investor Shares (VBISX) and Value Index Fund Investor Shares (VIVAX) have the highest negative correlation.

The **Emerging Markets Stock Index Fund (VEIEX) and Pacific Stock Index Fund Investor Shares (VPACX) have the highest covariance**. As we observe the top holdings in both funds, we notice certain commonalities. First, they have nearly the same asset allocation structure of equity, cash, and government. Second, the top holdings are regional tech giants and financial heavyweights. For VEIEX, they are Chinese tech giants Tencent, Alibaba, TSMC, and Baidu. For VPACX, they are Samsung Electronics and Sony Corp. We notice that the above names have very similar risk exposures, in that their performance are very similarly affected by news on semiconductor/chips supply, global regulations on software safety, and global demand for tech components and gadgets. Financial heavyweights in VEIEX are ICBC and Ping An, while VPACX include AIA, MUFG and SoftBank. These are commonly known as large cap blue chips within the region that help hold up the regional indices.

VEIEX US Equity | Report | Page 3/4 Security Description

1) Profile | 2) Performance | 3) Holdings | 4) Organizational

Holdings As Of 10/31/2018 Portfolio Stats As Of 10/31/2018 Alloc As Of 10/31/2018

6) Top Holdings | MHD »

Name	Position	% Net	Value
10) Tencent Holdings Ltd	23.75M	4.017%	813.69M
11) Alibaba Group Holding Ltd	4.39M	3.085%	624.80M
12) Taiwan Semiconductor Manufactu	61.85M	2.293%	464.33M
13) Naspers Ltd	1.78M	1.537%	311.39M
14) Taiwan Semiconductor Manufactu	7.83M	1.473%	298.37M
15) China Construction Bank Corp	372.85M	1.461%	295.87M
16) Industrial & Commercial Bank o	327.48M	1.097%	222.19M
17) Baidu Inc	1.15M	1.081%	219.05M
18) China Mobile Ltd	21.94M	1.015%	205.57M
19) Ping An Insurance Group Co of	21.63M	1.010%	204.54M

Top Asset Allocation	
Equity	97.50%
Cash and Other	2.42%
Government	.08%

Top Ind. Group Allocation	
Banks	15.29%
Internet	11.22%
Oil&Gas	7.52%
Semiconductors	4.91%
Diversified Fin...	4.86%
Telecommunic...	4.70%

7) Holdings Analysis | PORT »

Top 10 Hldings % Port	18.07	Average P/C	8.19
Median Mkt Cap	17.88B	Average P/S	1.08
Avg Wtd Mkt Cap	65.03B	Average P/E	12.37
Avg Div Yield	3.15	Average P/B	1.52

Top Geo. Allocation	
China	29.09%
Taiwan	14.05%
India	10.78%
Brazil	8.75%

Table 7 VEIEX Holdings

VPACX US Equity | Report | Page 3/4 Security Description

1) Profile | 2) Performance | 3) Holdings | 4) Organizational

Holdings As Of 10/31/2018 Portfolio Stats As Of 10/31/2018 Alloc As Of 10/31/2018

6) Top Holdings | MHD »

Name	Position	% Net	Value
10) Samsung Electronics Co Ltd	2.04M	2.619%	76.42M
11) Toyota Motor Corp	1.09M	2.197%	64.12M
12) AIA Group Ltd	5.24M	1.366%	39.85M
13) Commonwealth Bank of Australia	765.03k	1.289%	37.62M
14) Mitsubishi UFJ Financial Group	5.47M	1.134%	33.10M
15) BHP Billiton Ltd	1.40M	1.104%	32.21M
16) Sony Corp	548.01k	1.016%	29.66M
17) SoftBank Group Corp	368.99k	1.001%	29.20M
18) Westpac Banking Corp	1.49M	.968%	28.26M
19) CSL Ltd	196.25k	.898%	26.20M

Top Asset Allocation	
Equity	97.55%
Cash and Other	2.41%
Government	.04%

Top Ind. Group Allocation	
Banks	10.20%
Auto Manufactu...	4.40%
Semiconductors	4.36%
Insurance	4.18%
Pharmaceutica...	4.04%
Food	3.96%

7) Holdings Analysis | PORT »

Top 10 Hldings % Port	13.59	Average P/C	8.01
Median Mkt Cap	18.72B	Average P/S	.86
Avg Wtd Mkt Cap	34.62B	Average P/E	12.05
Avg Div Yield	3.09	Average P/B	1.21

Top Geo. Allocation	
Japan	58.93%
Australia	15.36%
South Korea	11.13%
Hong Kong	7.11%

Table 8 VPACX Holdings

The **500 Index Fund Investor Shares (VFINX), Growth Index Fund Investor Shares (VIGRX) and Value Index Fund Investor Shares (VIVAX) have the highest positive correlation**. Again, the asset allocations are very similar in equity, cash, and government. As we juxtapose the three funds, we also see a lot of overlaps in top holding assets. For VFINX and VIGRX, common top holdings are US based FAANG names – Apple, Amazon and Alphabet. For VFINX and VIVAX, top overlaps are large caps Microsoft, key industrials Exxon Mobil, as well as financials JPM and Berkshire. These component stocks are all based in the US and therefore exposed to very similar market risks.

VFINX US Equity | Report | Page 3/4 Security Description

1) Profile | 2) Performance | 3) Holdings | 4) Organizational

Holdings As Of 10/31/2018 Portfolio Stats As Of 10/31/2018 Alloc As Of 10/31/2018

6) Top Holdings | MHD »

Name	Position	% Net	Value
10) Apple Inc	66.18M	4.361%	14.48B
11) Microsoft Corp	110.59M	3.557%	11.81B
12) Amazon.com Inc	5.91M	2.843%	9.44B
13) Berkshire Hathaway Inc	27.09M	1.675%	5.56B
14) Johnson & Johnson	38.69M	1.631%	5.42B
15) JPMorgan Chase & Co	48.47M	1.591%	5.28B
16) Facebook Inc	34.78M	1.590%	5.28B
17) Exxon Mobil Corp	61.06M	1.465%	4.87B
18) Alphabet Inc	4.44M	1.439%	4.78B
19) Alphabet Inc	4.31M	1.416%	4.70B

Top Asset Allocation	
Equity	99.57%
Cash and Other	.42%
Government	.02%

Top Ind. Group Allocation	
Internet	8.72%
Banks	7.15%
Software	6.95%
Pharmaceutica..	6.24%
Computers	6.02%
Retail	5.47%

7) Holdings Analysis | PORT »

Top 10 Hldings % Port	21.57	Average P/C	12.21
Median Mkt Cap	103.85B	Average P/S	2.03
Avg Wtd Mkt Cap	228.17B	Average P/E	20.86
Avg Div Yield	1.95	Average P/B	3.29

Top Geo. Allocation	
U.S.	97.18%
Ireland	1.13%
U.K.	.84%
Switzerland	.40%

Table 9 VFINX Holdings

VIGRX US Equity | Report | Page 3/4 Security Description

1) Profile | 2) Performance | 3) Holdings | 4) Organizational

Holdings As Of 10/31/2018 Portfolio Stats As Of 10/31/2018 Alloc As Of 10/31/2018

6) Top Holdings | MHD »

Name	Position	% Net	Value
10) Apple Inc	17.28M	8.455%	3.78B
11) Amazon.com Inc	1.65M	5.888%	2.63B
12) Facebook Inc	9.59M	3.253%	1.46B
13) Alphabet Inc	1.19M	2.897%	1.30B
14) Alphabet Inc	1.18M	2.846%	1.27B
15) Visa Inc	7.06M	2.177%	973.82M
16) Home Depot Inc/The	4.55M	1.789%	800.12M
17) Boeing Co/The	2.17M	1.721%	770.06M
18) Mastercard Inc	3.67M	1.621%	725.12M
19) Comcast Corp	18.18M	1.550%	693.40M

Top Asset Allocation	
Equity	99.50%
Cash and Other	.48%
Government	.02%

Top Ind. Group Allocation	
Internet	17.89%
Computers	9.76%
Retail	7.97%
Software	6.37%
Diversified Fin..	5.80%
Healthcare-Pro..	5.56%

7) Holdings Analysis | PORT »

Top 10 Hldings % Port	32.20	Average P/C	16.33
Median Mkt Cap	92.40B	Average P/S	3.39
Avg Wtd Mkt Cap	254.05B	Average P/E	23.72
Avg Div Yield	1.22	Average P/B	6.01

Top Geo. Allocation	
U.S.	95.99%
Ireland	2.11%
U.K.	1.28%
Canada	.13%

Table 10 VIGRX Holdings

VIVAX US Equity | Report | Page 3/4 Security Description

1) Profile | 2) Performance | 3) Holdings | 4) Organizational

Holdings As Of 10/31/2018 Portfolio Stats As Of 10/31/2018 Alloc As Of 10/31/2018

6) Top Holdings | MHD »

Name	Position	% Net	Value
10) Microsoft Corp	17.89M	6.540%	1.91B
11) Berkshire Hathaway Inc	4.56M	3.206%	936.54M
12) Johnson & Johnson	6.26M	2.999%	876.14M
13) JPMorgan Chase & Co	7.84M	2.926%	854.80M
14) Exxon Mobil Corp	9.88M	2.694%	786.95M
15) Bank of America Corp	22.14M	2.084%	608.88M
16) Pfizer Inc	13.68M	2.016%	588.91M
17) UnitedHealth Group Inc	2.25M	2.009%	586.91M
18) Verizon Communications Inc	9.64M	1.884%	550.32M
19) Wells Fargo & Co	10.11M	1.843%	538.30M

Top Asset Allocation	
Equity	99.78%
Cash and Other	.20%
Government	.02%

Top Ind. Group Allocation	
Banks	12.98%
Pharmaceutica..	9.51%
Software	7.92%
Insurance	7.02%
Oil&Gas	6.59%
Telecommunic..	5.81%

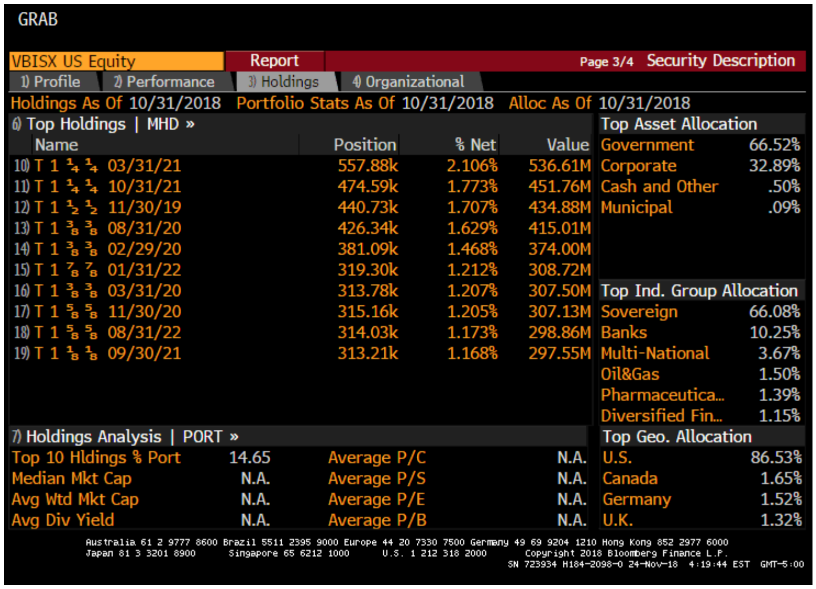
7) Holdings Analysis | PORT »

Top 10 Hldings % Port	28.20	Average P/C	10.03
Median Mkt Cap	107.21B	Average P/S	1.48
Avg Wtd Mkt Cap	187.62B	Average P/E	18.92
Avg Div Yield	2.74	Average P/B	2.33

Top Geo. Allocation	
U.S.	98.44%
Switzerland	.71%
U.K.	.26%
Ireland	.23%

Table 11 VIVAX Holdings

The **Short-term Bond Index Fund Investor Shares (VBISX) and Value Index Fund Investor Shares (VIVAX) have the highest negative correlation**. The reason for this is simple: VBISX underlying assets are investment grade bonds, while VIVAX are large cap blue chips in the US market.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VBISX US Equity | Report | Page 3/4 Security Description

1) Profile | 2) Performance | 3) Holdings | 4) Organizational

Holdings As Of 10/31/2018 Portfolio Stats As Of 10/31/2018 Alloc As Of 10/31/2018

6) Top Holdings | MHD »

	Name	Position	% Net	Value
10)	T 1 ¼ ¼ 03/31/21	557.88k	2.106%	536.61M
11)	T 1 ¼ ¼ 10/31/21	474.59k	1.773%	451.76M
12)	T 1 ½ ½ 11/30/19	440.73k	1.707%	434.88M
13)	T 1 ⅜ ⅜ 08/31/20	426.34k	1.629%	415.01M
14)	T 1 ⅜ ⅜ 02/29/20	381.09k	1.468%	374.00M
15)	T 1 ⅞ ⅞ 01/31/22	319.30k	1.212%	308.72M
16)	T 1 ⅜ ⅜ 03/31/20	313.78k	1.207%	307.50M
17)	T 1 ⅝ ⅝ 11/30/20	315.16k	1.205%	307.13M
18)	T 1 ⅝ ⅝ 08/31/22	314.03k	1.173%	298.86M
19)	T 1 ⅛ ⅛ 09/30/21	313.21k	1.168%	297.55M

Top Asset Allocation	
Government	66.52%
Corporate	32.89%
Cash and Other	.50%
Municipal	.09%

Top Ind. Group Allocation	
Sovereign	66.08%
Banks	10.25%
Multi-National	3.67%
Oil&Gas	1.50%
Pharmaceutica...	1.39%
Diversified Fin...	1.15%

7) Holdings Analysis | PORT »

Top 10 Hldings % Port	14.65	Average P/C	N.A.
Median Mkt Cap	N.A.	Average P/S	N.A.
Avg Wtd Mkt Cap	N.A.	Average P/E	N.A.
Avg Div Yield	N.A.	Average P/B	N.A.

Top Geo. Allocation	
U.S.	86.53%
Canada	1.65%
Germany	1.52%
U.K.	1.32%

Table 12 VBISX Holdings

As we interpret the above observations, we believe diversification can reduce risks. Portfolio risks can be broken down into two parts: market "systematic" risks, such as global inflation rate and changes in GDP. Systematic risks cannot be diversified away, as they affect all assets within a market. What leaves us is the firm specific risks, which can indeed by diversified away. These can be industry specific or even regional specific factors, such as global demand for semiconductor and chips, the production of iPhones, regional politics, and changes in firm management. As we add in more assets, the firm-specific factors can be cancelled out, thereby reducing risks.

Section Two Basic Risk-Return and Value-at-Risk (VaR) Calculations

Value-at-Risk is a measure used to estimate how the value of an asset or a portfolio of assets could decrease over a certain time period under usual conditions. In practice, it is used for measuring the market risk or volatility risk of institutions' asset portfolios. It is narrowly defined as market risk, a risk of loss from the change in the value of a tradeable asset.

2.1 Standard Deviation as a measure of risk

Monthly ER and SD estimates into annual estimates

	Monthly Estimates		Annual Estimates	
	Mean	**SD**	**Mean**	**SD**
NAESX	0.0100	0.0348	0.1194	0.4175
VBISX	0.0007	0.0037	0.0080	0.0449
VBLTX	0.0034	0.0232	0.0410	0.2786
VEIEX	0.0032	**0.0433**	0.0384	**0.5195**
VEURX	0.0051	**0.0365**	0.0611	**0.4374**
VEXMX	0.0101	**0.0346**	0.1213	**0.4157**
VFINX	0.0104	0.0282	0.1251	0.3390
VIGRX	**0.0118**	0.0313	**0.1416**	0.3750
VIVAX	0.0092	0.0279	0.1104	0.3349
VPACX	0.0062	0.0321	0.0738	0.3857

Table 13 Monthly and Annual Estimates

The annual estimates of expected return are found by multiplying the monthly estimated expected return by 12, while the annual estimates of standard deviation are found by multiplying the monthly estimates of standard deviation by 12^1/2. The annual mean returns are highest for VIGRX and tightly followed by VFINX and VEXMX. The annual standard deviations are highest for VIVAX and VFINX, and lowest for VBISX and VBLTX.

Table 13 suggests that VIGRX would garner the highest rates of returns, according to both the monthly and annual estimates, with a reasonable level of risk, as depicted by the standard deviations.

Notably, VEIEX generates limited returns as compared with the highest risks, as it has the highest standard deviation among all assets. That is to say, VEIEX is the riskiest investment among the Vanguard funds.

Other notably risky investments are VEURX and VEXMX, with the latter having higher rates of return.

The table below illustrates the value of a $1 investment in the Vanguard funds in 5 years:

	Annual Mean	Present ($)	In 5 years ($)
NAESX	0.1194	1.00	1.76
VBISX	0.0080	1.00	1.04
VBLTX	0.0410	1.00	1.22
VEIEX	0.0384	1.00	1.21
VEURX	0.0611	1.00	1.34
VEXMX	0.1213	1.00	1.77
VFINX	0.1251	1.00	**1.80**
VIGRX	0.1416	1.00	**1.94**
VIVAX	0.1104	1.00	1.69
VPACX	0.0738	1.00	1.43

Table 14 Investments in 5 Years

Table 14 shows us that VIGRX seems to garner the highest returns, followed by VFINX.

2.2 Sharpe ratio as a measure of risk and return

The Sharpe ratio is a measure of risk premium on the asset per unit of risk as measured by the standard deviation. It is the slope of the combination line between a risky asset and a risk-free asset such as T-bills. With the annualized estimate on monthly risk-free rate, the Sharpe ratio for each asset is as follows:

	Mean	SD	Sharpe Ratio	Risk Free Rate
NAESX	0.1194	0.4175	0.2777	0.0034
VBISX	0.0080	0.0449	0.1017	
VBLTX	0.0410	0.2786	0.1348	
VEIEX	0.0384	0.5195	0.0672	
VEURX	0.0611	0.4374	0.1316	
VEXMX	0.1213	0.4157	0.2834	
VFINX	0.1251	0.3390	**0.3587**	
VIGRX	0.1416	0.3750	**0.3683**	
VIVAX	0.1104	0.3349	0.3192	
VPACX	0.0738	0.3857	0.1823	

Table 15 Sharpe Ratio

As we compute the Sharpe Ratio for the Vanguard Funds, VIGRX has the highest Sharpe Ratio, tightly followed by VFINX. If we were to interpret the Sharpe Ratio as in indicator of returns as well as the reward to volatility, we can say that the VIGRX and VFINX have the highest performance for investors to their respective risk tolerance levels. That is to say, VFINX and VIGRIX would be the “fairest” assets of choice for investors.

2.3 Historic Simulation Approach to VaR

The Value-at-Risk measure is calculated as the difference between the (fund) portfolio's expected value and the lower bound of a standard confidence interval with a certain probability of the lower tail. Assuming we have an initial investment of $100,000, the 5% VaR for the Vanguard funds are calculated as follows:

	Initial Wealth	**Mean**	**SD**	**Quantiles**		**Value-at-Risk (absolute)**		**Value-at-Risk (relative)**	
				q(0.01)	**q(0.05)**	**1% VaR**	**5% VaR**	**1% VaR**	**5% VaR**
NAESX	100000	0.1118	0.4152	-0.8540	-0.5711	57430.1839	43509.3722	79321.7064	65400.8948
VBISX	100000	0.0080	0.0449	-0.0964	-0.0658	9189.1309	**6370.2072**	10089.9015	**7270.9778**
VBLTX	100000	0.0378	0.2785	-0.6100	-0.4202	45665.2622	34310.6725	53623.3612	42268.7715
VEIEX	100000	0.0274	0.5173	-1.1760	-0.8235	69150.1053	**56111.0944**	86635.7625	73596.7516
VEURX	100000	0.0531	0.4355	-0.9601	-0.6633	61713.9435	48483.2876	77664.1656	64433.5096
VEXMX	100000	0.1137	0.4143	-0.8501	-0.5678	57262.2988	43320.3715	79341.5885	**65399.6612**
VFINX	100000	0.1198	0.3360	-0.6618	-0.4328	48408.0790	35133.5093	67679.7109	54405.1412
VIGRX	100000	0.1352	0.3708	-0.7274	-0.4747	51684.0957	37794.4031	74300.6424	60410.9498
VIVAX	100000	0.1054	0.3326	-0.6685	-0.4418	48751.1782	35711.0707	66183.6778	53143.5703
VPACX	100000	0.0676	0.3834	-0.8244	-0.5631	56152.3083	43058.2102	66183.6778	53143.5703

Table 16 VaR, CC Returns

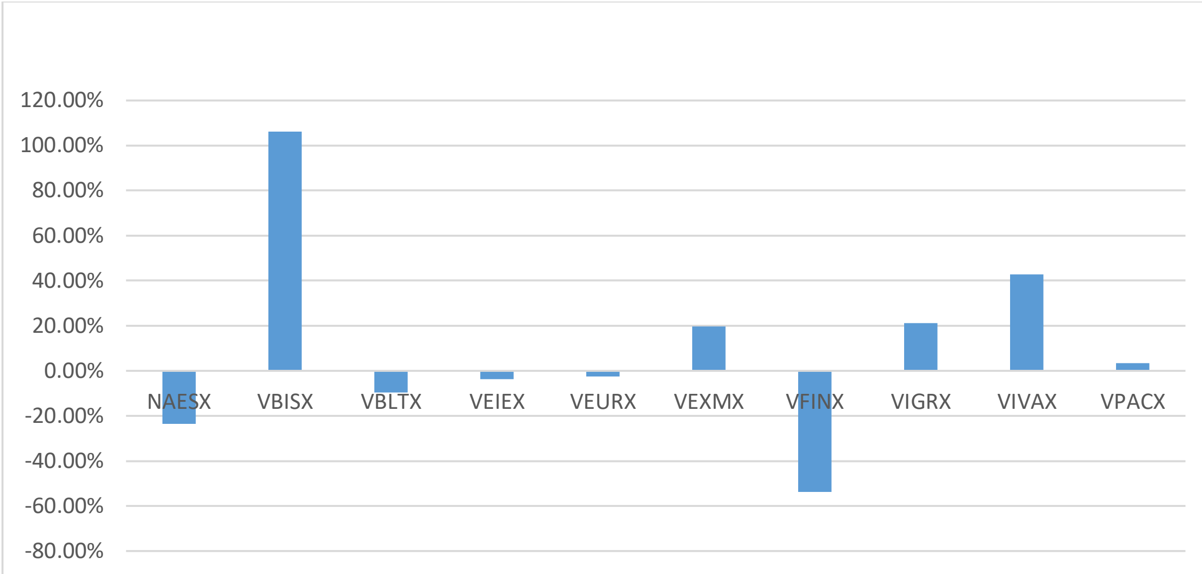
VEIEX has the highest 5% absolute VaR, while VEXMX has the highest 5% relative VaR. This means that with the same initial wealth, investing in these two funds would have your investment with the highest amount at risk. That is also to say that these two funds are riskiest among the others. We attribute this to the relatively high standard deviations of returns, which we believe has to do with the underlying assets of these funds being riskier, as most of them are traded in the emerging market. Their exposure to risks is relatively higher. These funds would be attractive investments for aggressive investors with very low risk aversion.

VBISX has the lowest 5% absolute and relative VaR. This means investing in this fund would be the "safest", that the value at risk relative to the initial investment amount is the smallest. This makes sense, as the underlying assets in this fund are investment grade T-bills. These are comparable to risk-free assets, and would be attractive for very risk-averse investors.

Section 3 Mean Variance Portfolio Theory and Asset Allocation

The table and graph below show the result of global minimum portfolio. As we can see, to achieve the minimum variance portfolio, we want to short sell some assets that have the highest risk and fluctuation. To see how closely the short selling is related to return's standard deviation, we sorted the standard deviation from the largest to the smallest and found out that the first three assets with the highest SD are exactly the short selling ones. For VBLTX, the fund ranking as the second last in terms of standard deviation, it has relatively low standard deviation and a low expected return as well. To make a trade off between risk and return, we can introduce the concept of sharp ratio, which is the forth column. We found that the sharp ratio of VBLTX is very low. The ratios are low for the first two assets as well.

global minimum weight		Standard Deviation	Expected return	return/standard deviation
VEIEX	-3.79%	4.2939%	0.3199%	6.77%
VEURX	-2.48%	3.6153%	0.5089%	13.27%
NAESX	-23.46%	3.4503%	0.9951%	28.00%
VEXMX	19.72%	3.4358%	1.0108%	28.58%
VPACX	3.31%	3.1880%	0.6150%	18.38%
VIGRX	21.12%	3.0996%	1.1801%	37.14%
VFINX	-53.73%	2.8016%	1.0422%	36.16%
VIVAX	42.91%	2.7680%	0.9200%	32.19%
VBLTX	-9.65%	2.3028%	0.3420%	13.59%
VBISX	106.04%	0.3711%	0.0671%	10.25%

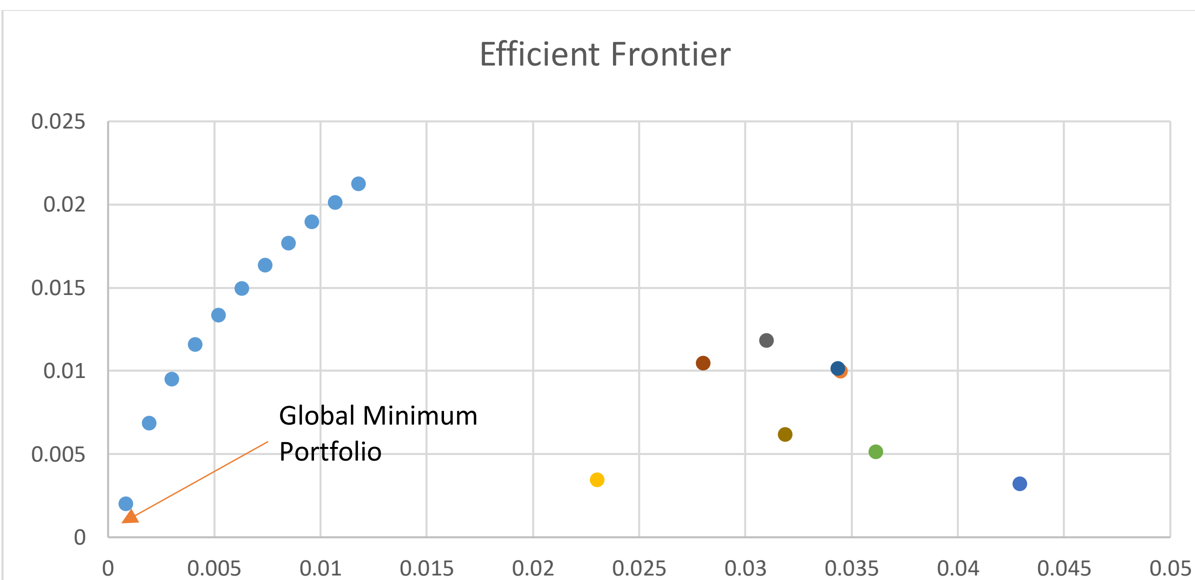


From the result above, we worked out the expected return and standard deviation. The SD is lower than any individual SD.

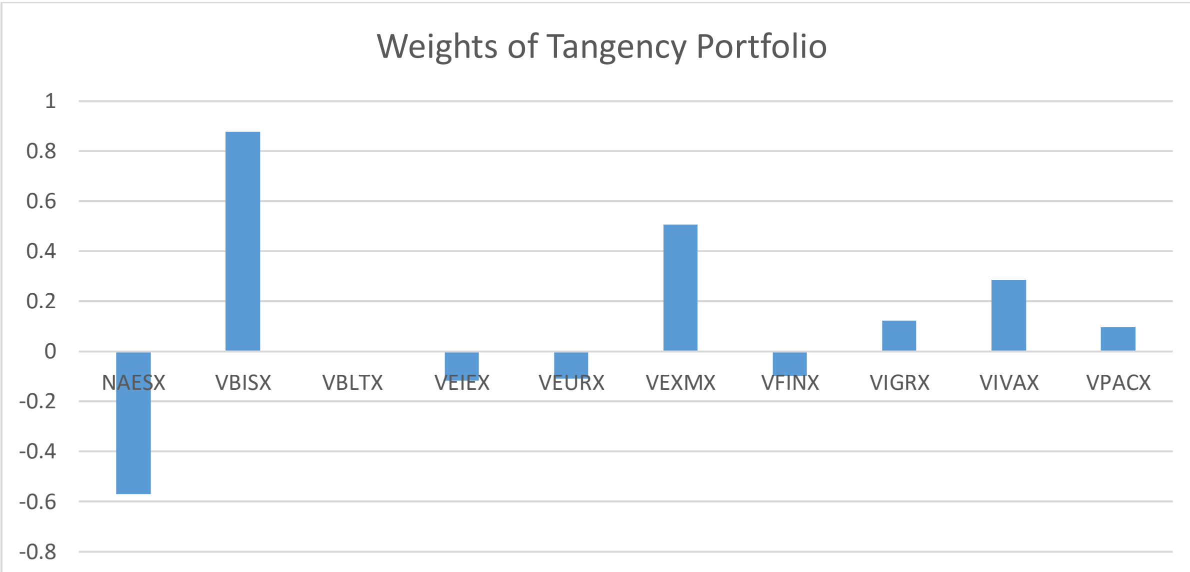
E(Rp) = 0.0837%

SD(Rp) = 0.20%

The efficient frontier is shown below, with the different color dots indicating the locations of risky assets



If we use monthly risk-free rate equal to the average monthly yield on the risk-free asset, we can determine the weights of tangency portfolio and the corresponding expected return and standard deviation.



Given the weights like this, the expected return and standard deviation are 0.28% and the standard deviation is 0.430%. To achieve the maximum sharp ratio, the portfolio should invest most on the second asset, VBISX. The weigh bar chart here is very similar to the chart of global minimum weight, which means it also considers short selling the asset that has the relatively high variance and low expected return.

Assuming the return follows the identical and independent normal distribution, the annual return and SD would be:

	Global Minimum	Tangency	Rf
r	0.0837%	0.2810%	0.00029
SD	0.20%	0.43%	0
annual r	1.00%	3.37%	0.35%
annual SD	0.69%	1.49%	0
sharp ratio	0.95	2.03	
Initial wealth	100000		
VaR	135.74		

The 5% VaR of the $100,000 investment over a one-month investment horizon is 135.74, which is relatively low.

If we want to achieve a target expected return of 1% per month using a combination of the risk-free and risky assets, we will borrow at the interest rate of 0.003% per month and invest 385.26% of our wealth on the tangency portfolio. The weights of each individual asset are shown as followed:

	Tangency	**Rf**
r	0.2810%	0.03%
required return	1%	
SD	2%	
weight	3.85	-2.85
VaR	1722.13	
weights of individual asset		
NAESX	-219.03%	
VBISX	337.79%	
VBLTX	-0.65%	
VEIEX	-44.40%	
VEURX	-41.82%	
VEXMX	195.48%	
VFINX	-37.72%	
VIGRX	47.84%	
VIVAX	110.44%	
VPACX	37.32%	

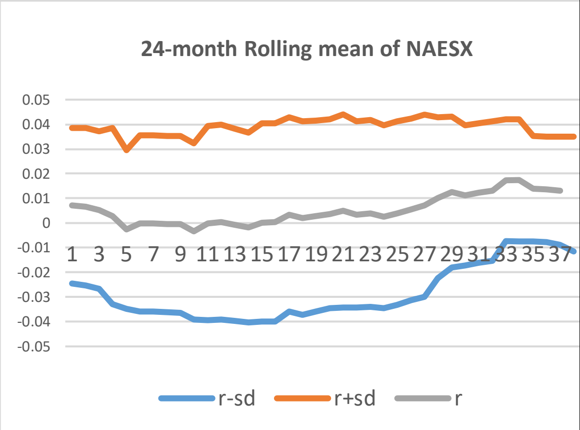
From the table we can see, the VaR is so much higher than the VaR we estimate before. This is sensible because now we are requiring a relatively high return, which comes along with the higher risk.

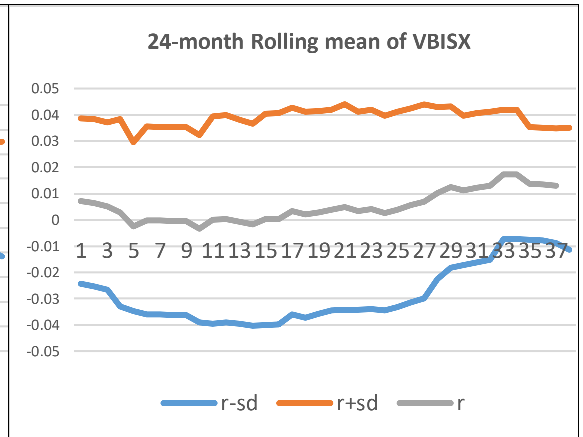
Section 4 Constant Expected Return Model

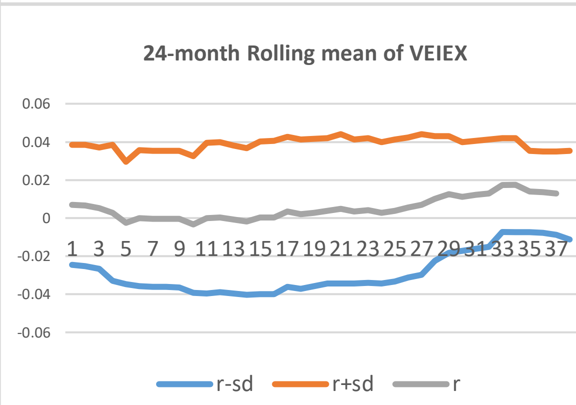
The Constant Expected Return Model (CERM) is a simple model that aims to capture an asset's return. It assumes that an asset's return over time is normally distributed with a constant (time invariant) mean and variance. The model also assumes that the correlations between asset returns are constant over time. The simplicity of the model lays the ground for more sophisticated analysis and estimates.

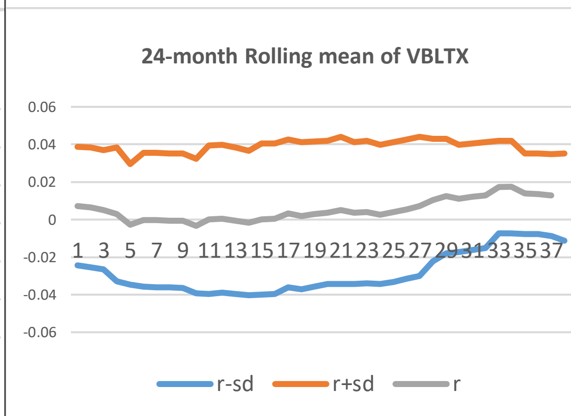
To see how appropriate is for constant expected return model to be used in our analysis, we computed 24-month rolling estimates of the mean and standard deviation of the continuously compounded returns and graphed these rolling estimates together with corresponding returns.

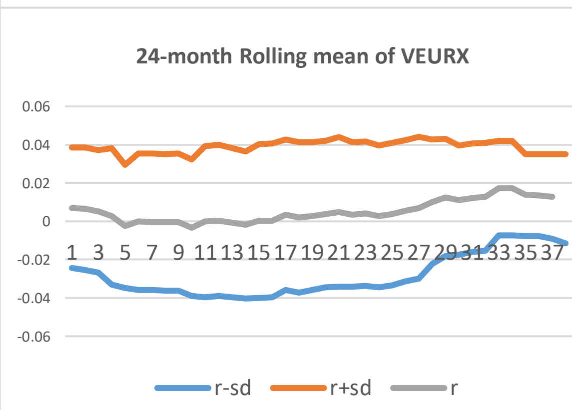
Consider the tables below. From the rolling mean and rolling standard deviation graphs, we acknowledge that the continuously compounding return is quite stable over time and the standard deviation is more like a constant. Hence we are comfortable with the analysis we have done before and the CERM is suitable here for these ten assets.

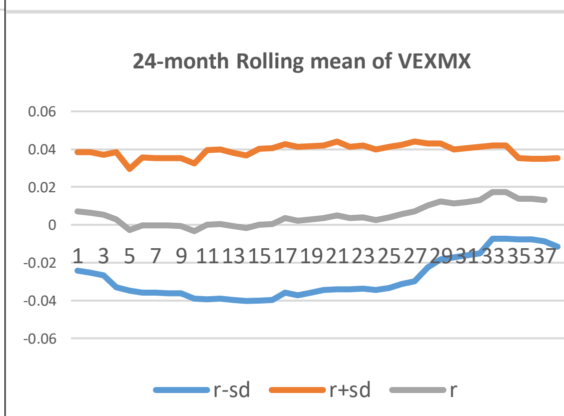

24-month Rolling mean of NAESX
r-sd
r+sd
r

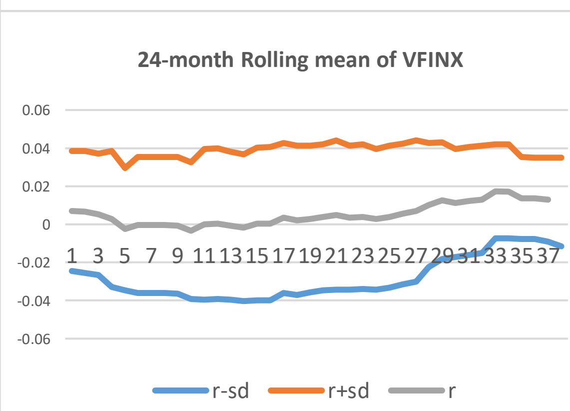

24-month Rolling mean of VBISX
r-sd
r+sd
r

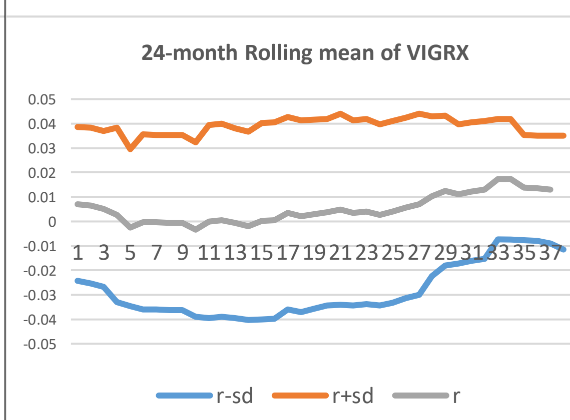

24-month Rolling mean of VEIEX
r-sd
r+sd
r

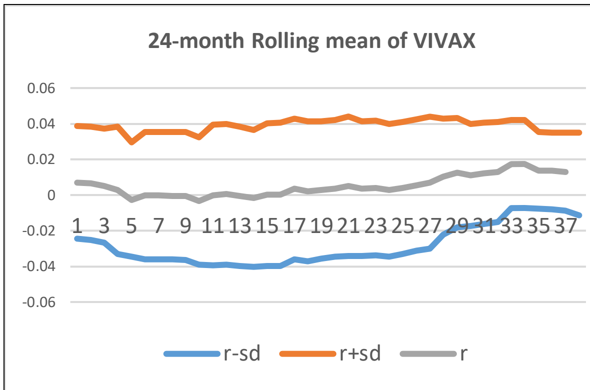

24-month Rolling mean of VBLTX
r-sd
r+sd
r

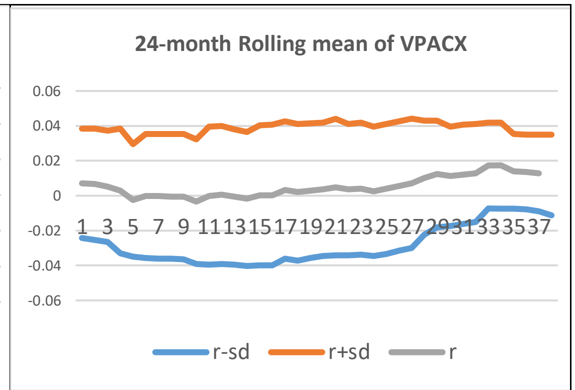

24-month Rolling mean of VEURX
r-sd
r+sd
r


24-month Rolling mean of VEXMX
r-sd
r+sd
r

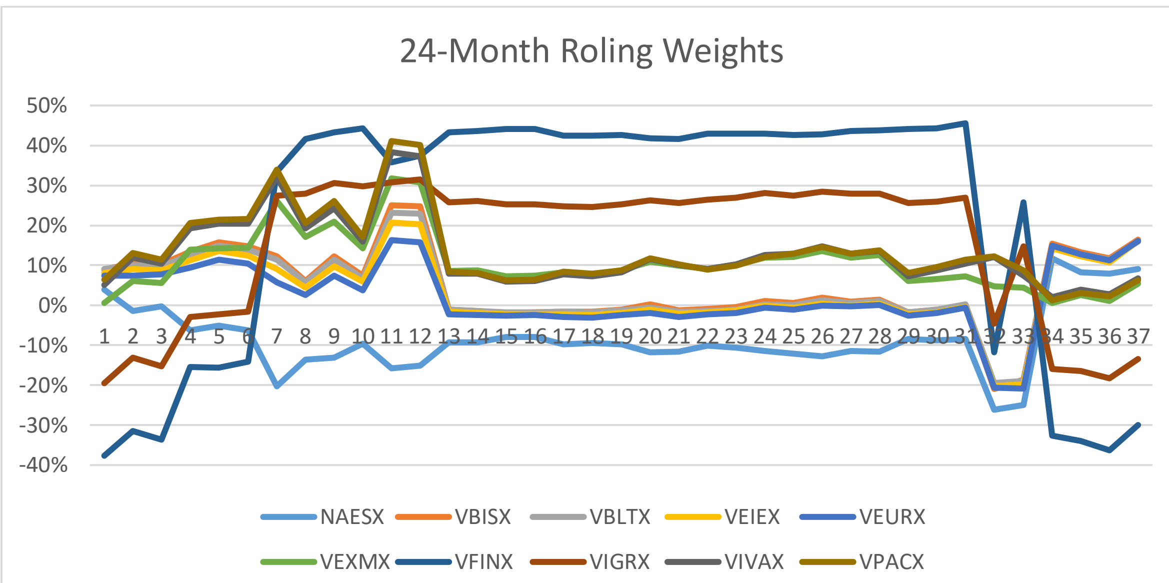

24-month Rolling mean of VFINX
r-sd
r+sd
r


24-month Rolling mean of VIGRX
r-sd
r+sd
r





After examining the stability of the return and standard deviation, we want to check whether the tangency weights are relatively stable over the time. Particularly, we estimated 37 times the 24-month rolling tangency weights and graphed the result as follows:



This graph shows that the portfolio weights are very unstable. Specifically, the fluctuations tend to move together at the same time, for most of the weights remaining stable in the middle. NAESX is relatively stable when comparing with other assets. The weights to invest in VIGRX and VFINX change the most, which have negative weights initially and highest positive weights when rolling forward, and finally fall back to below-zero territory.

The result illustrates that the portfolio weight in each asset is not actually constant over time, which is sensible because portfolio manager needs to reallocate the fund and capital according to assets' past performance and future predictions on the risk free rate and other economic factors.

Section 5 Single Index Model

The Single Index Model, as proposed by William Sharpe (1963), is also known as the market model or single factor model. It is a purely statistical model to explain the behavior of asset returns. It is a generalization of the constant expected return model (CERM) to account for systematic factors that may affect an asset's return. However, we note that despite close relations, the Single Index Model is different from the Capital Asset Pricing Model (CAPM), which will be discussed in Section 6.

5.1 Estimated equations

NAESX	$\hat{r} = -0.001579022 + 1.074465881rm$ $(0.002099226)\ \ (0.068162724)$
VBISX	$\hat{r} = 0.000862798 + -0.017915798rm$ $(0.000510414)\ \ (0.016573363)$
VBLTX	$\hat{r} = 0.003480426 + -0.005606416rm$ $(0.003198)\ \ (0.1038541)$
VEIEX	$\hat{r} = -0.006738307 + 0.926009rm$ $(0.004667)\ \ (0.1515467)$
VEURX	$\hat{r} = -0.00503 + 0.943395rm$ $(0.0033026)\ \ (0.1072379)$
VEXMX	$\hat{r} = -0.00149989 + 1.08169rm$ $(0.00199)\ \ (0.064639791)$
VFINX	$\hat{r} = 0.000098977 + 0.961948rm$ $(0.000515)\ \ (0.0167375)$
VIGRX	$\hat{r} = 0.000872 + 1.018420888rm$ $(0.001364)\ \ (0.04429857)$
VIVAX	$\hat{r} = -0.000679282 + 0.920614378rm$ $(0.001075)\ \ (0.0349179)$
VPACX	$\hat{r} = -0.002502706 + 0.806318498rm$ $(0.0030256)\ \ (0.098243703)$
Tangent Portfolio (SS, without SS constraint)	$\hat{r} = 0.001871827 + 0.08747007rm$ $(0.00048267)\ \ (0.01567)$
Tangent Portfolio (No SS)	$\hat{r} = 0.000791428 + 0.290212695rm$ $(0.00054359)\ \ (0.01765)$

Table 17 Estimated Equations

Interval	NAESX	VBISX	VBLTX	VEIEX	VEURX	VEXMX	VFINX	VIGRX	VIVAX	VPACX	SS	No SS
Beta	1.07	-0.02	-0.01	0.93	0.94	1.08	0.96	1.02	0.92	0.81	0.09	0.29
Lower 95%	0.94	-0.05	-0.21	0.62	0.73	0.95	0.93	0.93	0.85	0.70	0.06	0.25
Upper 95%	1.21	0.02	0.20	1.23	1.16	1.21	1.00	1.11	0.99	1.00	0.12	0.33

Table 18 Beta and estimates

The estimated slope betas below represent the sensitivities of different funds to the systematic factors, referring to the market return in this single index model. Excepting that the return of VBISX and VBLTX are negatively correlated with the market return since they're bond index funds, other returns are positively related to market changes. The return of NAESX and VIGRX shows the strong positive relations to the market returns. However, tangency portfolio has much less market sensitivity compared to other funds.

Estimate	NAESX	VBISX	VBLTX	VEIEX	VEURX	VEXMX	VFINX	VIGRX	VIVAX	VPACX	SS	No SS
R^2	0.8081	0.0194	0.0001	0.3876	0.5674	0.8260	0.9825	0.8996	0.9218	0.5331	0.3455	0.8209
SE/SD(e)	0.0154	0.0037	0.0234	0.0342	0.0242	0.0146	0.0038	0.0100	0.0078	0.0222	0.0035	0.0040

Table 19 R-square and estimates

The R square, on one hand, shows the goodness of the fit. On the other hand, it depicts the degree of variation of these returns could be explained mainly by the market return's variation. It can be seen from the graph above that large parts of the volatility of NAESX, VEXMX, VFINX, VIGRX, VIVAX and Tangency portfolio (No SS) are caused by the systematic risks. Also, standard deviations of the residuals vary. VEIEX, VELTX and VEURX show the higher standard errors of the regression compared to others.

1. Hypothesis Test (95% confidence intervals):

$$H_0: \beta = 0$$

$$H_1: \beta \neq 0$$

	NAESX	VBISX	VBLTX	VEIEX	VEURX	VEXMX	VFINX	VIGRX	VIVAX	VPACX	SS	No SS
t Stat	15.76	-1.08	-0.05	6.11	8.80	16.73	57.47	22.99	26.37	8.21	5.58	16.44
P-value	0.00	0.28	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Reject	Yes	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 20 Hypothesis test, null: Beta=0

$$H_0: \beta = 1$$

$$H_1: \beta \neq 1$$

	NAESX	VBISX	VBLTX	VEIEX	VEURX	VEXMX	VFINX	VIGRX	VIVAX	VPACX	SS	No SS
t Stat	1.09	-61.42	-9.68	-0.49	-0.53	1.26	-2.27	0.42	-2.27	-1.97	-58.22	-40.21
P-value	0.28	0.00	0.00	0.63	0.60	0.21	0.03	0.68	0.03	0.05	0.00	0.00
Reject	No	Yes	Yes	No	No	No	Yes	No	Yes	No	Yes	Yes

Table 21 Hypothesis Test, null: Beta = 1

The hypotheses test results above have showed the statistical relationship between the return of various funds and the market portfolio, and determine what outcomes would lead to the rejection of $\beta = 0$ or $\beta = 1$ for a pre-specified 95% level of confidence. If the β equals to zero, it means the changes in fund's return are not related to market's and the single index model become the constant return model. If β are 1, it means for every 1% percent change in market return, the funds would change for 1%.

2. Normality test:

$$H_0: e_i \sim i.i.d.N(\mu, \sigma^2)$$

$$H_1: e_i \sim not\ (i.i.d.)normal$$

	NAESX	VBISX	VBLTX	VEIEX	VEURX	VEXMX	VFINX	VIGRX	VIVAX	VPACX	SS	No SS
JB Stat	0.28	0.06	0.07	0.34	0.33	0.27	0.32	0.30	0.35	0.49	0.27	7.99
Reject	No	No	No	No	No	No	No	Yes	Yes	No	No	Yes

Table 22 Normality test

The normality test results above (all residual distribution histogram attached in the appendix) describe the residual of the Tangency portfolio with short-sell constraint do not comply to the normal distributions. This result could be observed more intuitively by the residual histogram below:

Residual Histogram of the Tangency Portfolio (No SS)

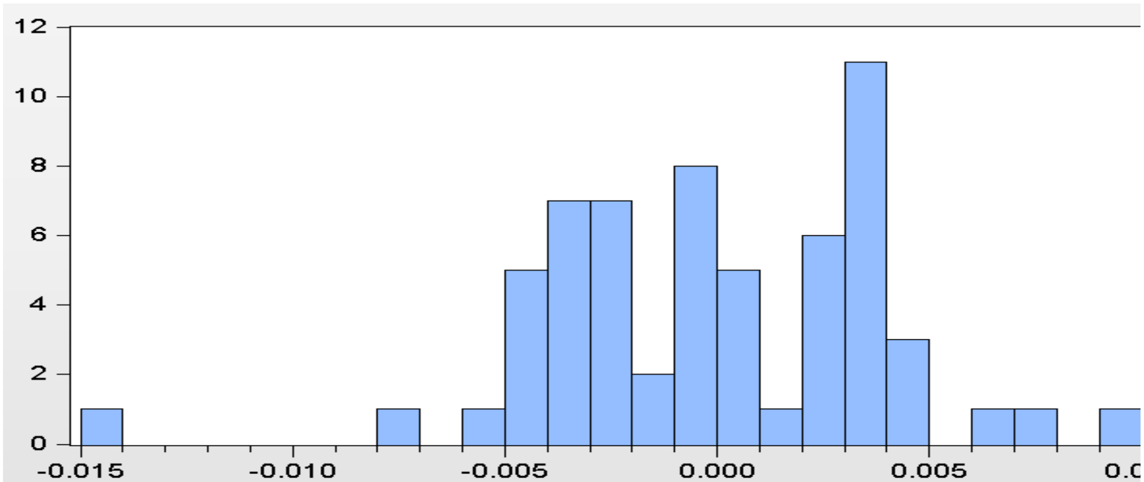


Table 23 Residual Histogram

3. Autocorrelation Test (LM Test):

$$H_0: no\ autocorrelation\ in\ ei\ \ up\ to\ p\ lag$$

$$H_1: autocorrelation\ in\ ei\ exists$$

There are many ways to test the serial correlation. LM test is more suitable in this case compared to DW test because there's higher orders-the 6/12-month lag. After conducting LM tests (all test results are attached in the appendix) of funds and the tangency portfolios with 1/6/12-month lag, autocorrelations do exist in two samples. Among these, their null hypotheses should be rejected given 95% confidence level by looking at the p-value of nR^2, which means they have serial correlation problems. These two test results are below:

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	5.320676	Prob. F(1,58)	0.0247
Obs*R-squared	5.125675	Prob. Chi-Square(1)	0.0236

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 00:40
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MKT	0.007320	0.065873	0.111125	0.9119
C	-8.88E-05	0.002027	-0.043829	0.9652
RESID(-1)	-0.290261	0.125836	-2.306659	0.0247

R-squared	0.084027	Mean dependent var	-1.24E-18
Adjusted R-squared	0.052442	S.D. dependent var	0.015239
S.E. of regression	0.014834	Akaike info criterion	-5.535808
Sum squared resid	0.012763	Schwarz criterion	-5.431994
Log likelihood	171.8421	Hannan-Quinn criter.	-5.495122
F-statistic	2.660338	Durbin-Watson stat	1.989191
Prob(F-statistic)	0.078450		

Table 24 NAESX 1-month lag

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.489607	Prob. F(6,53)	0.0339
Obs*R-squared	13.41224	Prob. Chi-Square(6)	0.0369

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 01:43
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000729	0.002837	0.257024	0.7982
MKT	-0.019707	0.094642	-0.208227	0.8358
RESID(-1)	0.124164	0.142718	0.870001	0.3882
RESID(-2)	-0.302912	0.141049	-2.147570	0.0363
RESID(-3)	0.159450	0.135371	1.177877	0.2441
RESID(-4)	-0.400564	0.135638	-2.953186	0.0047
RESID(-5)	-0.114942	0.141698	-0.811177	0.4209
RESID(-6)	-0.107147	0.142446	-0.752198	0.4553

R-squared	0.219873	Mean dependent var	3.19E-18
Adjusted R-squared	0.116837	S.D. dependent var	0.021965
S.E. of regression	0.020642	Akaike info criterion	-4.801296
Sum squared resid	0.022582	Schwarz criterion	-4.524460
Log likelihood	154.4395	Hannan-Quinn criter.	-4.692801
F-statistic	2.133949	Durbin-Watson stat	1.994677
Prob(F-statistic)	0.055631		

Table 25 VPACX 6-month lag

Section 6 Sharpe-Lintner-Mossin Capital Asset Pricing Model

Part1

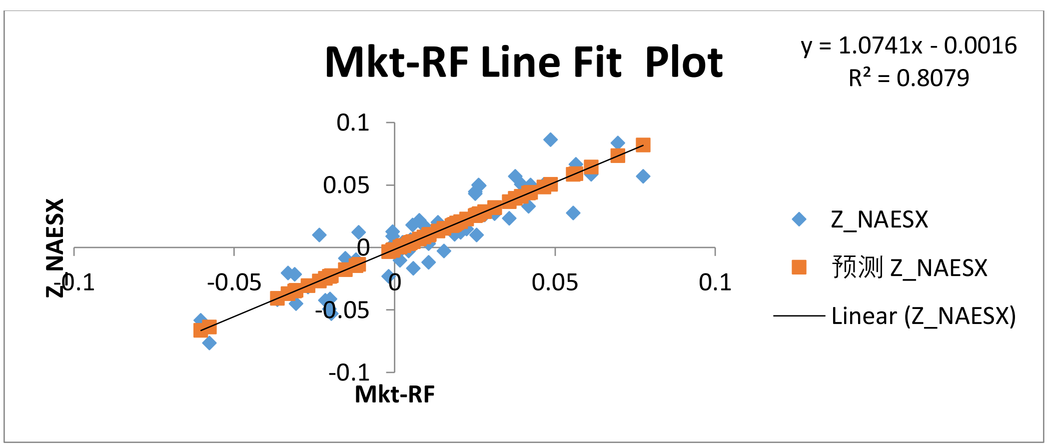
(1) See the Excel file "CAPM", sheet "Regression Data", and the weights for these two tangency portfolios are listed below:

Weights	E (r)	SS	No-SS
w-NAESX	0.00995123	-0.568586772	0
w-VBISX	0.000670541	0.876788344	0.66110125
w-VBLTX	0.003420262	-0.001676242	0.030990023
w-VEIEX	0.003198836	-0.115238526	0
w-VEURX	0.005088869	-0.108543055	0
w-VEXMX	0.010107902	0.507472415	0
w-VFINX	0.010421787	-0.097580599	0.200984123
w-VIGRX	0.011801213	0.124026245	0.106924604
w-VIVAX	0.009199967	0.28647523	0
w-VPACX	0.006150016	0.09686296	0
Sum	-	1	1

(2) Each CAPM regression is constructed in the corresponding sheet (from "Z_NAESX" to "Z_No SS"), the estimated equations, corresponding standard errors and R-Squares are listed as follows:

(i) Z_NAESX

$$y = 1.0741x - 0.0016$$
$$(0.0682) \quad (0.0021)$$
$$R^2 = 0.8079$$



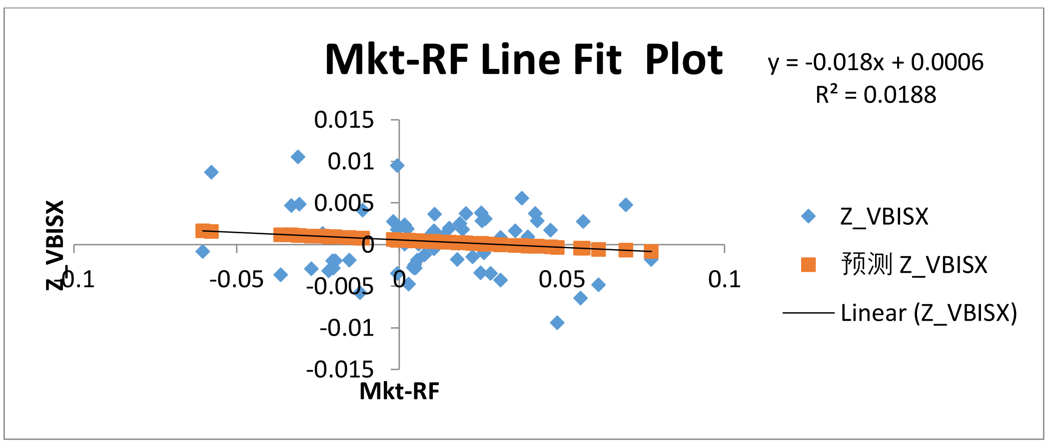
Interpretation of Beta: the slope of regression is the risky asset's beta, which measures the responsiveness of a stock's price to changes in the overall stock market. Z_NAESX has beta of 1.0741 which is slightly bigger than 1, indicating that the security's price is theoretically more volatile than the market.

Precision of the estimates: The standard error of beta is only 0.6% of the parameter, which is quite small, implying a precise estimation; while the SE magnitude of alpha is bigger than the parameter, indicating a rough estimation.

Estimated SE for the regression is 0.0154, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 80.79 % of variability of Z_NAESX is explained by the variability in market premium. $1 - R^2 = 0.1921$ so that 19.21% of Z_NAESX's return is not explained by the market.

(ii) Z_VBISX

$$y = -0.018x + 0.0006$$
$$(0.0169) \quad (0.0005)$$
$$R^2 = 0.0188$$



Interpretation of Beta: Z_VBISX has a beta of -0.018 which is close to 0, since a zero-beta portfolio is constructed to have no systematic risk, so its performance is not correlated to swings in the broader market

Precision of the estimates: The magnitudes of SE(beta) and SE(alpha) are nearly the same as the corresponding parameters, both indicating a rough estimation.

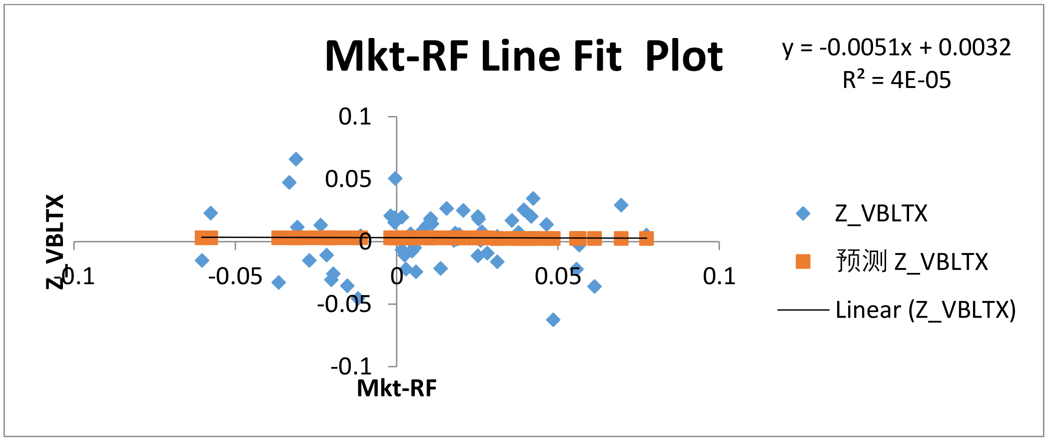
Estimated SE for the regression is 0.0038, which is quite small, implying the short distance of a dot from the regression line. However, from R^2 we know that only 1.88 % of variability of Z_VBISX is explained by the variability in market premium, so that 98.12 % of its return is not explained by the market, rather due to non-systematic risk, when compared with the first regression we can conclude that beta is estimated much more precisely for a diversified portfolio.

(iii) Z_VBLTX

$y = -0.0051x + 0.0032$

(0.1040) (0.0032)

R^2 = 4E-05



Z_VBLTX has a beta of -0.051 which is close to 0, so its performance is not correlated to swings in the broader market.

Precision of the estimates: The magnitude of SE(beta) is twice that of the coefficient and the magnitude of SE(alpha) is nearly the same as the corresponding parameter, both indicating a rough estimation.

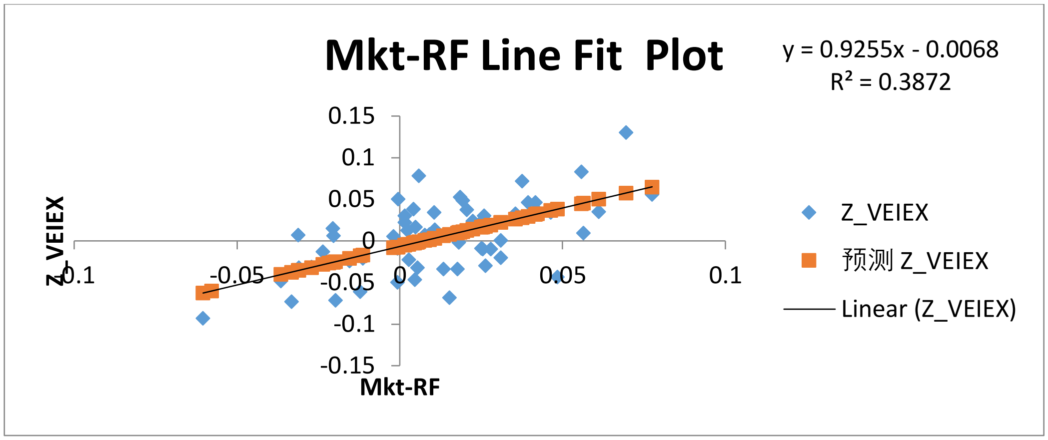
Estimated SE for the regression is 0.0234, which is quite small, implying the short distance of a dot from the regression line. However, from R^2 we know that 4E-03 % of variability of Z_VBLTX is explained by the variability in market premium, so that nearly 100.00% of its return is not explained by the market, almost all due to non-systematic risk.

(iv) Z_VEIEX

$y = 0.9255x - 0.0068$

(0.1516) (0.0047)

$R^2 = 0.3872$



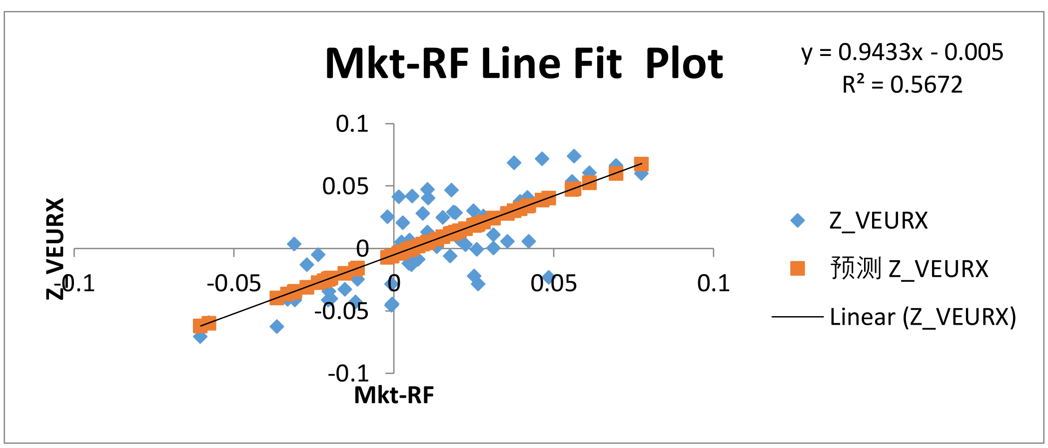
Z_VEIEX has beta of 0.9255 which is close to 1, indicating that the security's price is theoretically at the similar volatile level as the market.

The standard error of beta is 16% of the parameter, and SE(alpha) is over 2/3 of its parameter, both are not small, implying a relatively rough estimation.

Estimated SE for the regression is 0.0342, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 38.72 % of variability of Z_VEIEX is explained by the variability in market premium, which is typical for stocks, and $1 - R^2$ = 61.28 % of its return is not explained by the market.

(v) Z_VEURX

$$y = 0.9433x - 0.005$$
$$(0.1073) \quad (0.0033)$$
$$R^2 = 0.5672$$



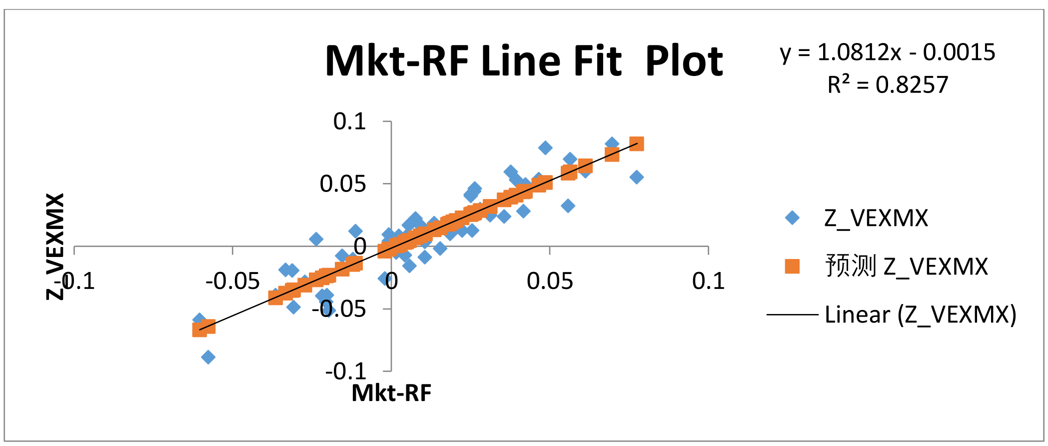
Z_VEURX has beta of 0.9433 which is close to 1, indicating that the security's price is theoretically at the similar volatile level as the market.

The standard error of beta is 1/9 of the parameter, and SE(alpha) is over 2/5 of its parameter, both are not small, implying a relatively rough estimation.

Estimated SE for the regression is 0.0242, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 56.72 % of variability of Z_VEURX is explained by the variability in market premium, and $1-R^2$ = 43.28 % of its return is not explained by the market.

(vi) Z_VEXMX

$$y = 1.0812x - 0.0015$$
$$(0.0647) \quad (0.0020)$$
$$R^2 = 0.8257$$



Z_VEXMX has beta of 1.0812 which is slightly bigger than 1, indicating that the security's price is theoretically more volatile than the market.

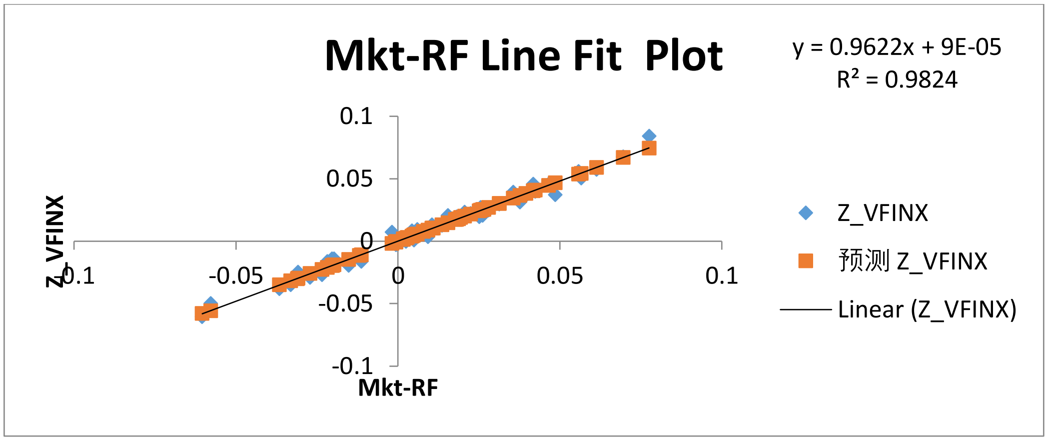
The standard error of beta is only 6% of the parameter, but SE(alpha) is larger than its parameter, implying a relatively rough estimation.

Estimated SE for the regression is 0.0146, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 82.57 % of variability of Z_VEXMX is explained by the variability in market premium, and $1-R^2$ = 17.43 % of its return is not explained by the market, implying that it's a well-diversified portfolio,

(vii) Z_VFINX

$$y = 0.9622x + 9E\text{-}05$$

(0.0168) (0.0005)
$R^2 = 0.9824$



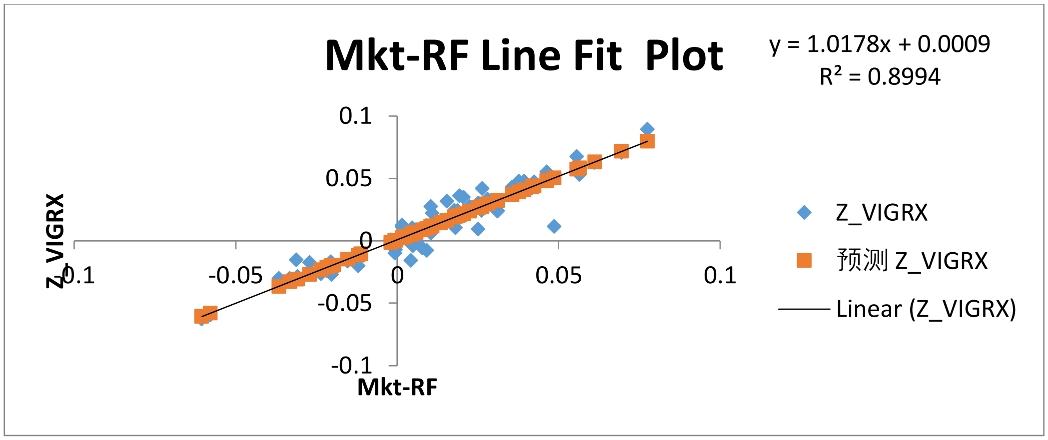
Z_VEURX has beta of 0.9622 which is close to 1, indicating that the security's price is theoretically at the similar volatile level as the market.

The standard error of beta is nearly 1/10 of the parameter, but SE(alpha) is quite larger than its parameter due to the almost-zero intercept, implying a relatively good estimation.

Estimated SE for the regression is 0.0038, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 98.24 % of variability of Z_VEURX is explained by the variability in market premium, indicating that it's a good tracking asset towards market.

(viii) Z_VIGRX

$y = 1.0178x + 0.0009$
(0.0443) (0.0014)
$R^2 = 0.8994$



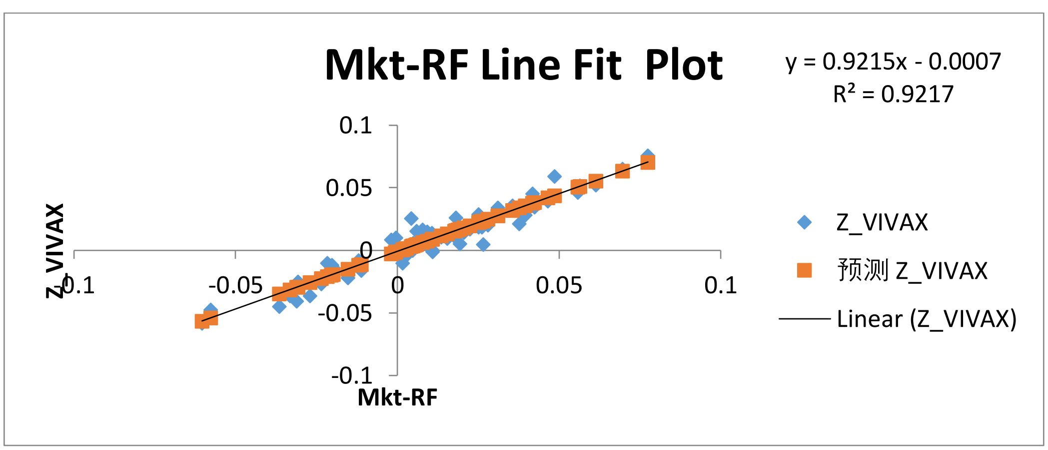
Z_VIGRX has beta of 1.0178 which is close to 1, indicating that the security's price is theoretically at the similar volatile level as the market.

The standard error of beta is nearly 4% of the parameter, but SE(alpha) is nearly twice its parameter due to the almost-zero intercept, implying a relatively good estimation.

Estimated SE for the regression is 0.0100, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 89.94 % of variability of Z_VIGRX is explained by the variability in market premium, indicating that it's a good tracking asset towards market.

(ix) Z_VIVAX

$$y = 0.9215x - 0.0007$$
$$(0.0350) \quad (0.0011)$$
$$R^2 = 0.9217$$



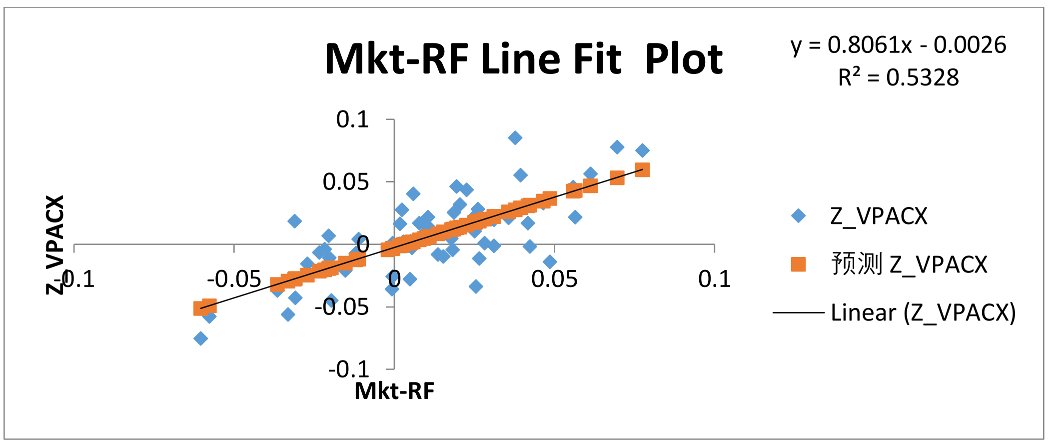
Z_VIVAX has beta of 0.9215 which is close to 1, indicating that the security's price is theoretically at the similar volatile level as the market.

The standard error of beta is below 4% of the parameter, but SE(alpha) is slightly larger than its parameter due to the almost-zero intercept, implying a relatively good estimation.

Estimated SE for the regression is 0.0079, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 92.17 % of variability of Z_VIGRX is explained by the variability in market premium, indicating that it's a good tracking asset towards market.

(x) Z_VPACX

$$y = 0.8061x - 0.0026$$
$$(0.0983) \quad (0.0030)$$
$$R^2 = 0.5328$$



Z_VPACX has beta of 0.8061 which is smaller than 1, indicating that the security's price is theoretically less volatile level as the market.

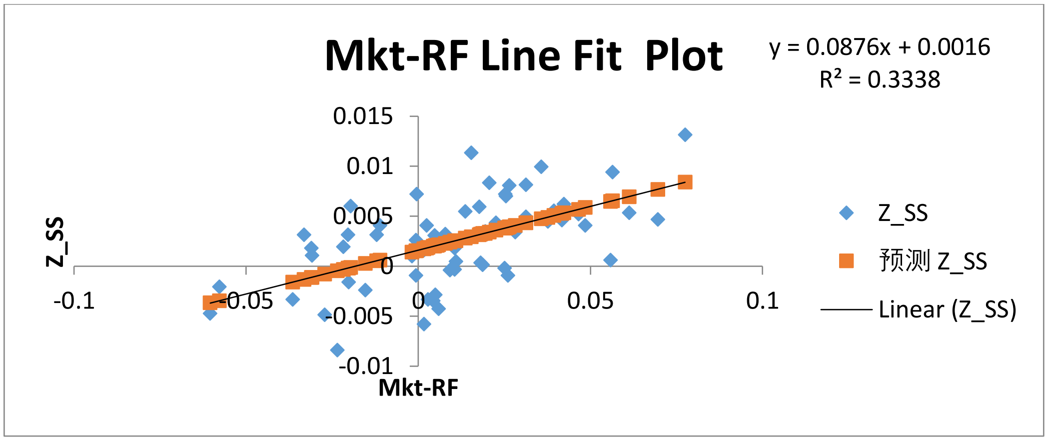
The standard error of beta is nearly 10% of the parameter, but SE(alpha) is slightly larger than its parameter due to the almost-zero intercept, implying a relatively good estimation.

Estimated SE for the regression is 0.0221, which is not so small, implying the relatively large distance of a dot from the regression line. What's more, from R^2 we know that 53.28 % of variability of Z_VPACX is explained by the variability in market premium, and $1 - R^2$ = 46.72 % of its return is not explained by the market.

(xi) Z_SS

$$y = 0.0876x + 0.0016$$

(0.0161) (0.0005)
$R^2 = 0.3338$



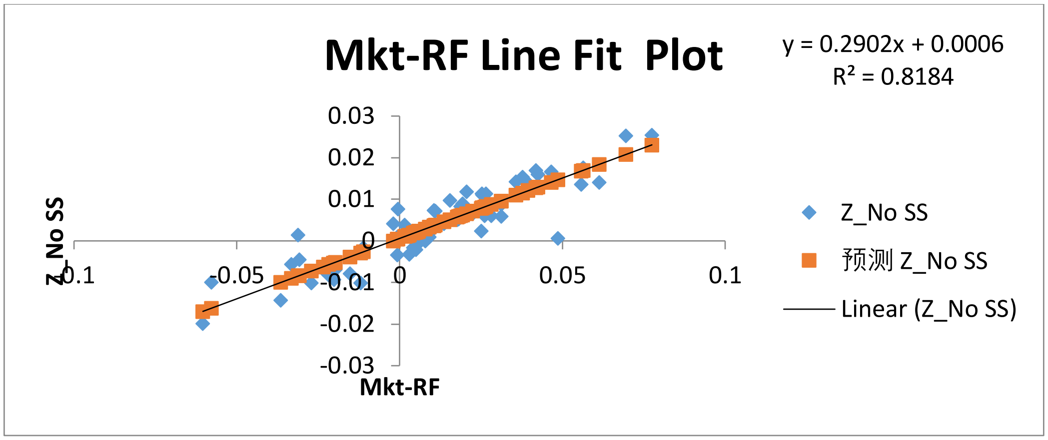
Z_SS has beta of 0.0876 which is close to 0, indicating that the security's price is theoretically not at the similar volatile level as the market.

The standard error of beta is nearly 1/8 of the parameter, and SE(alpha) is less than 1/3 of its parameter, both are not quite small, implying a relatively rough estimation.

Estimated SE for the regression is 0.0036, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 33.38 % of variability of Z_SS is explained by the variability in market premium, which is typical for stocks, and $1 - R^2$ = 66.62 % of its return is not explained by the market.

(xii) Z_No SS

$y = 0.2902x + 0.0006$
(0.0178) (0.0005)
$R^2 = 0.8184$



Z_No SS has beta of 0.2902, indicating that the security's price is theoretically not at the similar volatile level as the market.

The standard error of beta is nearly 6% of the parameter, and SE(alpha) is smaller than its almost-zero intercept, implying a relatively good estimation.

Estimated SE for the regression is 0.0040, which is quite small, implying the short distance of a dot from the regression line. What's more, from R^2 we know that 81.84 % of variability of Z_No SS is explained by the variability in market premium, indicating that it's a good tracking asset towards market.

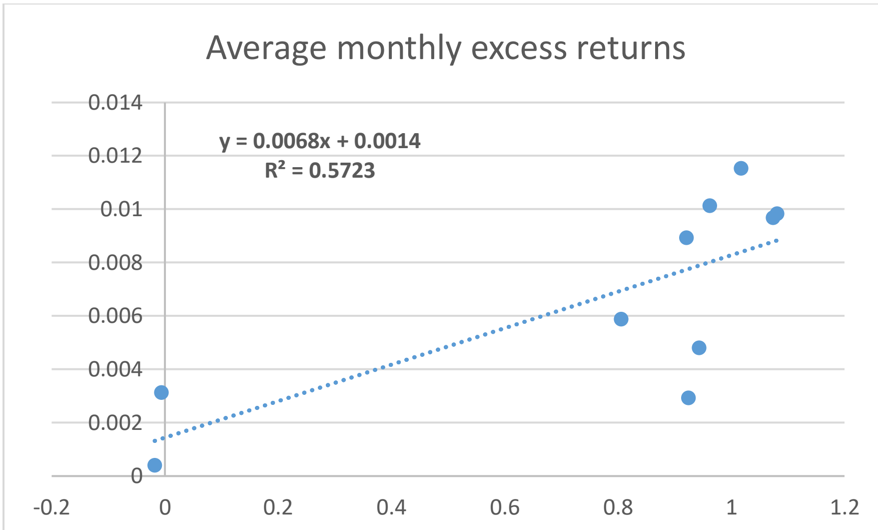
(3) After calculation, we can confirm that the beta of two tangency portfolios are the corresponding weighted average of the betas of the assets in the portfolio.

	Beta	Weight (SS)	Weight (No SS)
Z_NAESX	1.074061847	-0.568586772	0
Z_VBISX	-0.017959423	0.876788344	0.66110125
Z_VBLTX	-0.00505331	-0.001676242	0.030990023
Z_VEIEX	0.925526271	-0.115238526	0
Z_VEURX	0.943270021	-0.108543055	0
Z_VEXMX	1.081211721	0.507472415	0
Z_VFINX	0.96218329	-0.097580599	0.200984123
Z_VIGRX	1.017847787	0.124026245	0.106924604
Z_VIVAX	0.921501963	0.28647523	0
Z_VPACX	0.806058795	0.09686296	0
Z_SS	**0.087622074**	-	-
Z_No SS	**0.290186937**	-	-
Weighted Average		**0.087622074**	**0.290186937**

(4) According to the scatter plot of the average monthly excess returns for each asset against the beta estimates, we can estimate the cross-sectional linear equation and the its R-Square:

$$y = 0.0068x + 0.0014$$
$$R^2 = 0.5723$$

From the graph below, it appears to be a linear regression between average excess return and predicted beta.



(5) Testing on alpha

Upon 95% confidence level we can calculate each asset's confidence interval and test on the null hypothesis:

Note that: T-Critical= T.INV.2T(0.05,59)

	T-Stat	T-Critical	Reject or not?	Lower 95%	Upper 95%
Z_NAESX	-0.742183844	2.000995378	Not Reject	-0.005740756	0.00263436
Z_VBISX	1.096576926	2.000995378	Not Reject	-0.000468377	0.00160416
Z_VBLTX	0.996949853	2.000995378	Not Reject	-0.003205514	0.00957123
Z_VEIEX	-1.451960567	2.000995378	Not Reject	-0.016063655	0.00255419
Z_VEURX	-1.534002402	2.000995378	Not Reject	-0.011637273	0.00153735
Z_VEXMX	-0.741266674	2.000995378	Not Reject	-0.005442509	0.00250016
Z_VFINX	0.166274043	2.000995378	Not Reject	-0.000943247	0.00111421
Z_VIGRX	0.64978032	2.000995378	Not Reject	-0.001837686	0.00360512
Z_VIVAX	-0.663170394	2.000995378	Not Reject	-0.002858657	0.00143549
Z_VPACX	-0.847618746	2.000995378	Not Reject	-0.008590668	0.00347828
Z_SS	3.246591839	2.000995378	**Reject**	0.000615965	0.00259501
Z_No SS	1.07254894	2.000995378	Not Reject	-0.000507045	0.00167853

We can see that only the SS tangency portfolio's regression rejects the null hypothesis of "Alpha = 0", while the other 11 regressions cannot reject.

(6) Testing the normality of the residuals series
Note that: the denominator is adjusted to be n, and we are comparing with 5.99 = CHISQ.INV.RT(0.05,2))

	JB	JB-Critical	Reject or not?
Z_NAESX	0.308414877	5.99	Not Reject
Z_VBISX	0.368871923	5.99	Not Reject
Z_VBLTX	1.108379997	5.99	Not Reject
Z_VEIEX	0.170318197	5.99	Not Reject
Z_VEURX	0.275434495	5.99	Not Reject
Z_VEXMX	0.942709041	5.99	Not Reject
Z_VFINX	0.887521037	5.99	Not Reject
Z_VIGRX	25.02825729	5.99	**Reject**
Z_VIVAX	0.56390356	5.99	Not Reject
Z_VPACX	0.021680395	5.99	Not Reject
Z_SS	0.798732938	5.99	Not Reject
Z_No SS	6.354874424	5.99	**Reject**

We can see that only the residuals series of VIGRX and No-SS reject the normality test, while the other 10 series cannot reject.

Part 2

1. Autocorrelation Test (LM Test):

$$H_0: no\ autocorrelation\ in\ ei\ \ up\ to\ p\ lag$$

$$H_1: autocorrelation\ in\ ei\ exists$$

After conducting LM tests (all test results are attached in the appendix) of funds and the tangency portfolios with 1/6/12-month lag, autocorrelations do not exist in eight samples. Among these, their null hypotheses should not be rejected given 95% confidence level by looking at the p-value of nR^2. That is to say, other samples have serial correlation problems. These eight test results are below:

NAESX (6)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.231895	Prob. F(6,53)	0.3050
Obs*R-squared	7.465860	Prob. Chi-Square(6)	0.2799

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:24
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000504	0.002126	-0.236939	0.8136
ZMKT	0.026647	0.071598	0.372179	0.7112
RESID(-1)	-0.265999	0.138745	-1.917173	0.0606
RESID(-2)	0.005714	0.147612	0.038708	0.9693
RESID(-3)	-0.001543	0.146965	-0.010499	0.9917
RESID(-4)	-0.201135	0.161887	-1.242437	0.2195
RESID(-5)	0.031943	0.161323	0.198005	0.8438
RESID(-6)	-0.050893	0.159232	-0.319615	0.7505

R-squared	0.122391	Mean dependent var	3.13E-19
Adjusted R-squared	0.006481	S.D. dependent var	0.015271
S.E. of regression	0.015222	Akaike info criterion	-5.410482
Sum squared resid	0.012280	Schwarz criterion	-5.133646
Log likelihood	173.0197	Hannan-Quinn criter.	-5.301987
F-statistic	1.055910	Durbin-Watson stat	1.989781
Prob(F-statistic)	0.404548		

NAESX (12)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.138233	Prob. F(12,47)	0.3539
Obs*R-squared	13.73562	Prob. Chi-Square(12)	0.3179

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:25
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.001291	0.002181	-0.591820	0.5568
ZMKT	0.029600	0.074248	0.398668	0.6919
RESID(-1)	-0.323930	0.147143	-2.201460	0.0326
RESID(-2)	-0.045848	0.158035	-0.290116	0.7730
RESID(-3)	-0.066408	0.153307	-0.433170	0.6669
RESID(-4)	-0.312487	0.171798	-1.818924	0.0753
RESID(-5)	0.005267	0.170069	0.030969	0.9754
RESID(-6)	-0.098441	0.174751	-0.563322	0.5759
RESID(-7)	-0.180919	0.174989	-1.033887	0.3065
RESID(-8)	-0.152185	0.174563	-0.871805	0.3877
RESID(-9)	0.002145	0.178300	0.012029	0.9905
RESID(-10)	-0.298275	0.181042	-1.647547	0.1061
RESID(-11)	-0.280969	0.186627	-1.505513	0.1389
RESID(-12)	-0.141060	0.177012	-0.796893	0.4295

R-squared	0.225174	Mean dependent var	3.13E-19
Adjusted R-squared	0.010861	S.D. dependent var	0.015271
S.E. of regression	0.015188	Akaike info criterion	-5.338323
Sum squared resid	0.010842	Schwarz criterion	-4.853860
Log likelihood	176.8189	Hannan-Quinn criter.	-5.148457
F-statistic	1.050676	Durbin-Watson stat	2.000662
Prob(F-statistic)	0.422633		

VBISX (6)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.149713	Prob. F(6,53)	0.9883
Obs*R-squared	1.016639	Prob. Chi-Square(6)	0.9850

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:34
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000225	0.000753	0.298836	0.7662
ZMKT	0.311322	0.018041	17.25673	0.0000
RESID(-1)	-0.077620	0.053952	-1.438692	0.1561
RESID(-2)	0.014463	0.054969	0.263117	0.7935
RESID(-3)	0.074553	0.055496	1.343402	0.1849
RESID(-4)	-0.084458	0.056220	-1.502295	0.1390
RESID(-5)	-0.046087	0.058624	-0.786151	0.4353
RESID(-6)	-0.050873	0.057007	-0.892400	0.3762

R-squared	0.859453	Mean dependent var	0.002945
Adjusted R-squared	0.840891	S.D. dependent var	0.009803
S.E. of regression	0.003910	Akaike info criterion	-8.128714
Sum squared resid	0.000810	Schwarz criterion	-7.851878
Log likelihood	255.9258	Hannan-Quinn criter.	-8.020220
F-statistic	46.29984	Durbin-Watson stat	1.987669
Prob(F-statistic)	0.000000		

VBISX (12)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.514952	Prob. F(12,47)	0.8944
Obs*R-squared	7.088166	Prob. Chi-Square(12)	0.8517

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:34
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000377	0.000951	0.396829	0.6933
ZMKT	0.310696	0.018937	16.40665	0.0000
RESID(-1)	-0.069464	0.056414	-1.231330	0.2243
RESID(-2)	0.005455	0.057266	0.095255	0.9245
RESID(-3)	0.069348	0.057849	1.198776	0.2366
RESID(-4)	-0.065260	0.058514	-1.115289	0.2704
RESID(-5)	-0.053682	0.060706	-0.884299	0.3810
RESID(-6)	-0.084736	0.059866	-1.415432	0.1635
RESID(-7)	-0.038452	0.060344	-0.637210	0.5271
RESID(-8)	-0.042827	0.060383	-0.709261	0.4817
RESID(-9)	0.014633	0.060089	0.243524	0.8087
RESID(-10)	0.008408	0.060766	0.138367	0.8905
RESID(-11)	0.096252	0.060403	1.593488	0.1178
RESID(-12)	-0.057087	0.059883	-0.953311	0.3453

R-squared	0.873679	Mean dependent var	0.002945
Adjusted R-squared	0.838740	S.D. dependent var	0.009803
S.E. of regression	0.003937	Akaike info criterion	-8.038710
Sum squared resid	0.000728	Schwarz criterion	-7.554247
Log likelihood	259.1807	Hannan-Quinn criter.	-7.848845
F-statistic	25.00534	Durbin-Watson stat	1.939760
Prob(F-statistic)	0.000000		

VFINX (12)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.636386	Prob. F(12,47)	0.8004
Obs*R-squared	8.526052	Prob. Chi-Square(12)	0.7428

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:38
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000537	0.000894	0.600722	0.5509
ZMKT	-0.681688	0.019148	-35.60176	0.0000
RESID(-1)	0.024540	0.028314	0.866678	0.3905
RESID(-2)	-0.007969	0.028248	-0.282097	0.7791
RESID(-3)	-0.011241	0.028113	-0.399869	0.6911
RESID(-4)	0.021167	0.028721	0.736993	0.4648
RESID(-5)	-0.003103	0.029295	-0.105909	0.9161
RESID(-6)	0.051219	0.029680	1.725724	0.0910
RESID(-7)	0.008990	0.029880	0.300859	0.7648
RESID(-8)	-0.015339	0.029720	-0.516122	0.6082
RESID(-9)	-0.014837	0.030110	-0.492752	0.6245
RESID(-10)	-0.012355	0.030666	-0.402874	0.6889
RESID(-11)	-0.061527	0.030569	-2.012742	0.0499
RESID(-12)	0.047862	0.029861	1.602851	0.1157

R-squared	0.970262	Mean dependent var	-0.006806
Adjusted R-squared	0.962036	S.D. dependent var	0.019960
S.E. of regression	0.003889	Akaike info criterion	-8.063011
Sum squared resid	0.000711	Schwarz criterion	-7.578548
Log likelihood	259.9218	Hannan-Quinn criter.	-7.873146
F-statistic	117.9580	Durbin-Watson stat	1.967227
Prob(F-statistic)	0.000000		

TP SS (12)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.224920	Prob. F(12,47)	0.9962
Obs*R-squared	3.312771	Prob. Chi-Square(12)	0.9929

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:42
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000537	0.000741	-0.724532	0.4723
ZMKT	0.208156	0.018890	11.01944	0.0000
RESID(-1)	-0.109941	0.076892	-1.429818	0.1594
RESID(-2)	0.002572	0.078569	0.032737	0.9740
RESID(-3)	0.096561	0.079931	1.208050	0.2331
RESID(-4)	-0.099761	0.080661	-1.236784	0.2223
RESID(-5)	-0.103089	0.084796	-1.215732	0.2302
RESID(-6)	-0.103939	0.083043	-1.251628	0.2169
RESID(-7)	-0.058474	0.083795	-0.697822	0.4887
RESID(-8)	-0.078766	0.084521	-0.931908	0.3561
RESID(-9)	-0.009482	0.083679	-0.113318	0.9103
RESID(-10)	-0.007028	0.084539	-0.083128	0.9341
RESID(-11)	0.111531	0.084095	1.326245	0.1912
RESID(-12)	-0.061385	0.082932	-0.740190	0.4629

R-squared	0.757390	Mean dependent var	0.001095
Adjusted R-squared	0.690285	S.D. dependent var	0.007111
S.E. of regression	0.003957	Akaike info criterion	-8.028295
Sum squared resid	0.000736	Schwarz criterion	-7.543832
Log likelihood	258.8630	Hannan-Quinn criter.	-7.838429
F-statistic	11.28664	Durbin-Watson stat	1.914537
Prob(F-statistic)	0.000000		

Tan P No SS (6)

Tan P No SS (12)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.695702	Prob. F(6,53)	0.6541
Obs*R-squared	4.453527	Prob. Chi-Square(6)	0.6155

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:42
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.73E-05	0.000557	-0.031080	0.9753
ZMKT	0.003813	0.018874	0.202014	0.8407
RESID(-1)	-0.102837	0.140908	-0.729820	0.4687
RESID(-2)	0.032489	0.135922	0.239026	0.8120
RESID(-3)	0.131628	0.138354	0.951381	0.3457
RESID(-4)	-0.055927	0.137693	-0.406170	0.6863
RESID(-5)	-0.207311	0.139670	-1.484293	0.1437
RESID(-6)	0.030329	0.145182	0.208905	0.8353

R-squared	0.073009	Mean dependent var	-5.44E-19
Adjusted R-squared	-0.049424	S.D. dependent var	0.003977
S.E. of regression	0.004074	Akaike info criterion	-8.046457
Sum squared resid	0.000880	Schwarz criterion	-7.769621
Log likelihood	253.4169	Hannan-Quinn criter.	-7.937962
F-statistic	0.596316	Durbin-Watson stat	1.919509
Prob(F-statistic)	0.756033		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.605387	Prob. F(12,47)	0.8266
Obs*R-squared	8.166330	Prob. Chi-Square(12)	0.7720

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/24/18 Time: 12:43
Sample: 2013M06 2018M06
Included observations: 61
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.89E-05	0.000600	-0.048068	0.9619
ZMKT	0.009861	0.020227	0.487521	0.6282
RESID(-1)	-0.134676	0.148798	-0.905095	0.3700
RESID(-2)	0.033344	0.146787	0.227161	0.8213
RESID(-3)	0.129275	0.150856	0.856943	0.3958
RESID(-4)	-0.054277	0.147481	-0.368028	0.7145
RESID(-5)	-0.245529	0.151280	-1.623007	0.1113
RESID(-6)	0.008497	0.158197	0.053709	0.9574
RESID(-7)	-0.029842	0.157444	-0.189537	0.8505
RESID(-8)	-0.034397	0.153327	-0.224337	0.8235
RESID(-9)	-0.141310	0.153463	-0.920806	0.3619
RESID(-10)	-0.156542	0.159447	-0.981778	0.3312
RESID(-11)	0.072200	0.161237	0.447791	0.6564
RESID(-12)	0.162932	0.164066	0.993083	0.3258

R-squared	0.133874	Mean dependent var	-5.44E-19
Adjusted R-squared	-0.105692	S.D. dependent var	0.003977
S.E. of regression	0.004182	Akaike info criterion	-7.917650
Sum squared resid	0.000822	Schwarz criterion	-7.433187
Log likelihood	255.4883	Hannan-Quinn criter.	-7.727784
F-statistic	0.558818	Durbin-Watson stat	1.891533
Prob(F-statistic)	0.873570		

2. Box-Ljung Q-Stat

After computing all the Box-Ljung Q-statistics at up to 12 lags of 12 assets (ten funds and two tangency portfolios without and with short-sell constraint, observing the Q-Stats and check their corresponding P-Values, we found that the null hypothesis cannot be rejected given 95% confidence level and there're no serial correlations found in the samples, which might be a bit different from the LM test results. All Q-stats are shown below:

NAESX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	2.04	3.12	3.54	4.16	5.76	6.00	7.30	10.92	10.92	11.86	11.86	12.00

VBISX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	2.33	2.86	2.87	3.11	4.37	4.56	6.73	8.70	8.88	10.21	10.96	12.35

VBLTX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	2.32	2.82	2.83	3.10	4.29	4.47	6.65	8.53	8.72	10.05	10.85	12.27

VEIEX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	1.91	2.95	3.33	4.01	5.66	5.85	7.01	10.62	10.62	11.49	11.50	11.78

VEURX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Stat	1.92	2.96	3.34	4.10	5.68	5.87	7.05	10.67	10.67	11.54	11.55	11.81

VEXMX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	2.04	3.12	3.54	4.15	5.77	6.00	7.31	10.93	10.93	11.87	11.87	12.01

VFINX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	1.99	3.07	3.48	4.18	5.68	5.89	7.15	10.71	10.71	11.63	11.64	11.87

VIGRX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	2.02	3.10	3.51	4.17	5.73	5.95	7.23	10.82	10.83	11.75	11.60	11.94

VIVAX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	1.97	3.05	3.46	4.18	5.64	5.84	7.08	10.61	10.61	11.52	11.53	11.80

VPACX

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	1.82	2.82	3.19	4.09	4.47	5.61	6.69	10.18	10.19	11.02	11.05	11.53

SS

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	2.12	2.42	2.47	3.17	3.70	3.77	5.97	7.03	7.23	8.58	9.81	11.29

NO SS

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stat	0.65	0.65	1.90	2.17	4.52	5.09	5.89	6.03	6.13	6.90	7.12	8.73

3. Heteroskedasticity Test

White test can be used to examine the correlation between the explaining variable and the random error. This method could be executed by the following steps: First, regress the explained variable and the explaining variable using OLS so as to get the square of the random error. Second, construct the auxiliary regression model by putting in the square of the explaining variable and cross term (if any). Third, estimate and carry out the hypothesis testing. The third steps based on our samples are shown below:

$$Z_i = \beta_0 + \beta_{1i} Z_M + \mu_i$$

$$e_i^2 = \alpha_0 + \alpha_{1i} Z_M + \alpha_{2i} Z_M^2 + \varepsilon_i$$

Hypothesis test:

$$H_0: \alpha_1 = \alpha_2 = 0$$

$$H_1: in\ \alpha_1 and\ \alpha_2, at\ least\ one\ not\ equal\ to\ 0$$

After calculating all the statistics and corresponding p-values given 95% confidence level, we found that the null hypotheses are rejected in assets except the tangency portfolio with short-sell constraint. In other words, there is no Heteroskedasticity in tangency portfolio (No SS). All white test results are attached in the appendix file and the statistics are shown below:

White	NAESX	VBISX	VBLTX	VEIEX	VEURX	VEXMX	VFINX	VIGRX	VIVAX	VPACX	SS	No SS
nR^2	54.65	45.76	37.08	37.80	44.09	54.83	53.78	55.01	52.31	44.46	35.79	1.43
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23
Reject	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No

4. GRS Test

TS	NAESX	VBISX	VBLTX	VEIEX	VEURX	VEXMX	VFINX	VIGRX	VIVAX	VPACX	SS	No ss
beta	1.0741	-0.0180	-0.0051	0.9255	0.9433	1.0812	0.9622	1.0178	0.9215	0.8061	0.0876	0.2902
SE	0.0682	0.0169	0.1040	0.1516	0.1073	0.0647	0.0168	0.0443	0.0350	0.0983	0.0161	0.0178
alpha	-0.0016	0.0006	0.0032	-0.0068	-0.0050	-0.0015	0.0001	0.0009	-0.0007	-0.0026	0.0016	0.0006
SE	0.0021	0.0005	0.0032	0.0047	0.0033	0.0020	0.0005	0.0014	0.0011	0.0030	0.0005	0.0005
t (alpha)	-0.7422	1.0966	0.9969	-1.4520	-1.5340	-0.7413	0.1663	0.6498	-0.6632	-0.8476	3.2466	1.0725
R^2	0.8079	0.0188	0.0000	0.3872	0.5672	0.8257	0.9824	0.8994	0.9217	0.5328	0.3338	0.8184

Using estimates from the time series regression and constructing GRS test, the J-stat is 1.3117 with a 0.2432 p-value. The corresponding CHISQ-stat and p-value given 95% confidence level are 20.0034 and 0.0670 respectively, which shows that the CAPM cannot be rejected for these portfolios. Relevant data are shown below:

trans (f^bar)	Sigma_f^hat	J-stat numerator	J-stat denominator
0.0104	0.0008	0.3701	1.1288
J-statistic	**p-value (J-stat)**	**CHISQ-statistic**	**p-value (CHISQ-stat)**
1.3117	0.2432	20.0034	0.0670

Conducted the cross-sectional regression attaching results below:

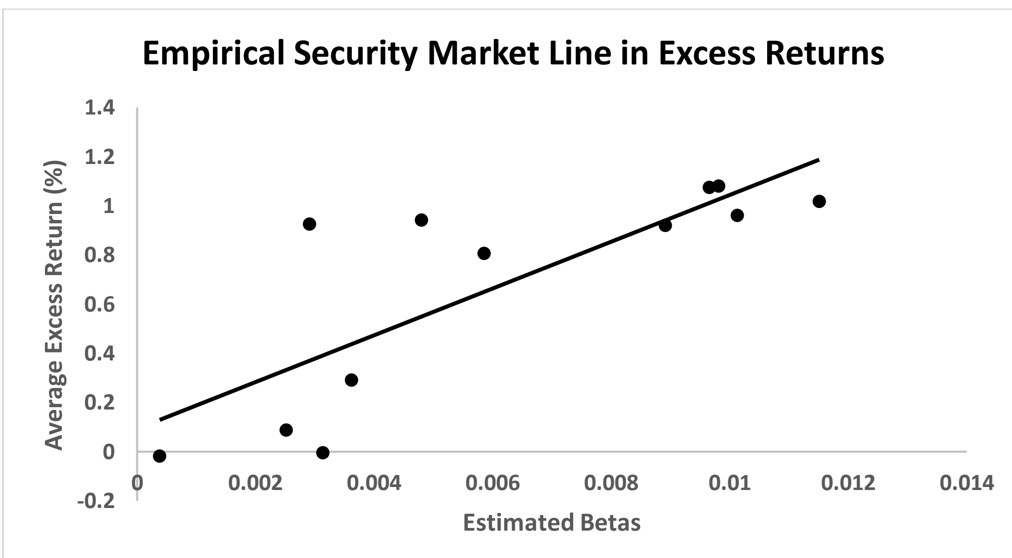
CS	Coefficients	SE	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.001606914	0.001275324	1.260004491	0.236277838	-0.001234685	0.004448514
Betas	0.006673124	0.001600217	4.170137307	0.001917547	0.003107619	0.01023863

$$\mathrm{E}\ (r_{i,t} - r_{f,t})\ = 0.001606914 + 0.006673124\beta_i + \alpha_i$$

(0.001275324) (0.001600217)

sq alpha	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
abs alpha	0.0016	0.0006	0.0032	0.0068	0.0050	0.0015	0.0001	0.0009	0.0007	0.0026	0.0016	0.0006

The estimated $\hat{\lambda}$ is 0.006673. The market risk premium E ($r_{m,t} - r_{f,t}$) is 0.0104. The estimated regression equation has a smaller $\hat{\lambda}$, which means the estimated one is less steep. The estimated intercept is 0.001607. Also, the time series alphas are calculated below the estimated equation. The root mean square alpha and the mean absolute alpha are 0.0028 and 0.0021 respectively. By comparing, we can find the estimated one is smaller than the time-series alphas. The SML line in excess returns is shown below:



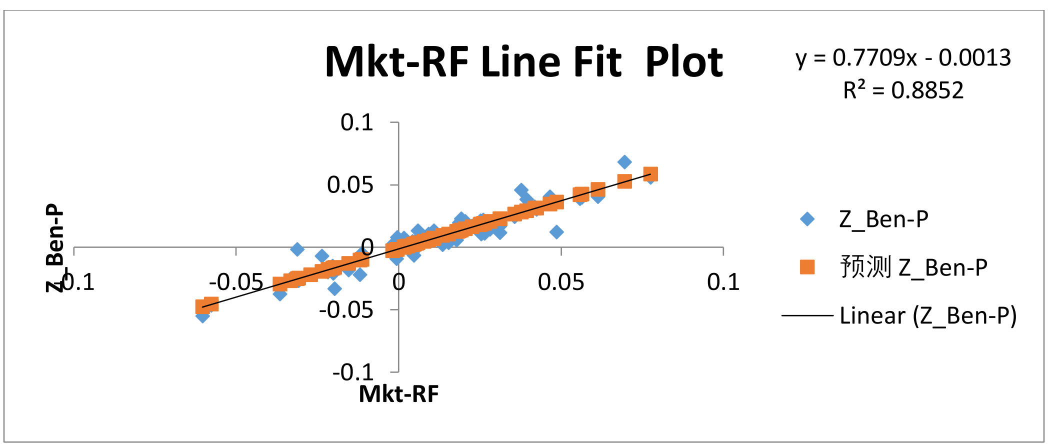
Section 7 Performance Evaluation

1. Assume short selling is allowed, and we assume the SD of risk-free asset is 0.

(i) **Benchmark portfolio**: Equally weighted

The ER, SD and other statistical data of the naively diversified portfolio are listed below:

Assets	E(r)	Weights	Beta		
NAESX	0.00995123	0.1	1.07406185	E(Ben-p)	0.00700106
VBISX	0.00067054	0.1	-0.0179594	Var(Ben-p)	0.00055947
VBLTX	0.00342026	0.1	-0.0050533	SD(Ben-p)	**0.023653**
VEIEX	0.00319884	0.1	0.92552627	Sharpe Ratio	0.28372069
VEURX	0.00508887	0.1	0.94327002	Beta(Ben-p)	**0.770865**
VEXMX	0.0101079	0.1	1.08121172	Regression Beta	**0.770865**
VFINX	0.01042179	0.1	0.96218329		
VIGRX	0.01180121	0.1	1.01784779		
VIVAX	0.00919997	0.1	0.92150196		
VPACX	0.00615002	0.1	0.80605879		
Sum	-	1	-		



We can see that the estimated beta equals the beta constructed from the "CAPM regression", which is reasonable since $\beta_p = \sum_{i=1}^{n} w_i \beta_i$.

(ii) **First efficient portfolio**: SS tangency portfolio & Risk-free asset (the same SD as benchmark)

The ER, SD, corresponding weights and other statistical data of the first efficient portfolio are listed below:

Assets	E(r)	Weights	Beta		
NAESX	0.00995123	-3.1308163	1.07406185	E(1st Eff-p)	0.01416799
VBISX	0.00067054	4.82787036	-0.0179594	Var(1st Eff-p)	0.00055947
VBLTX	0.00342026	-0.0092299	-0.0050533	SD(1st Eff-p)	**0.0236532**
VEIEX	0.00319884	-0.6345393	0.92552627	Sharpe Ratio	0.5867212
VEURX	0.00508887	-0.5976719	0.94327002	Beta(1st Eff-p)	**0.482474**
VEXMX	0.0101079	2.79430155	1.08121172		
VFINX	0.01042179	-0.5373092	0.96218329		
VIGRX	0.01180121	0.68292723	1.01784779		
VIVAX	0.00919997	1.57742205	0.92150196		
VPACX	0.00615002	0.5333577	0.80605879		
Risk-free	0.00029016	-4.5063122	0		
Sum	-	1	-		

We can notice that this efficient portfolio yields a higher (0.0142 > 0.0070) expected return than the benchmark (on average) and have the same SD (0.0237), further confirming the efficacy of the mean-variance portfolio.

(iii) **Second efficient portfolio**: Market portfolio & Risk-free asset (the same Beta as benchmark)

The ER, SD, corresponding weights and other statistical data of the second efficient portfolio are listed below:

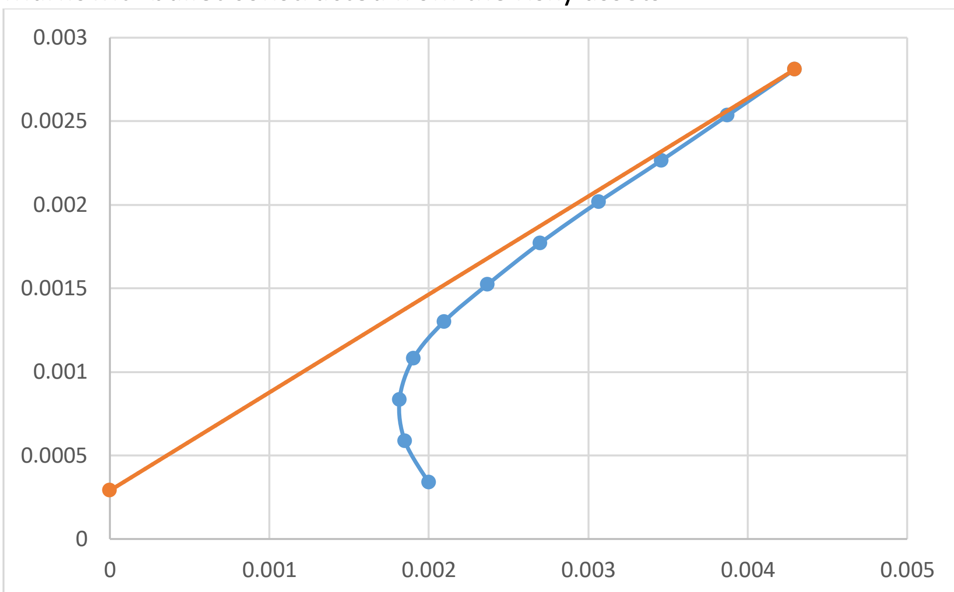
Assets	E(r)	SD	Weights	Beta		
Mkt Portfolio	0.01073115	0.02910676	0.7708649	1	E(2nd Eff-p)	0.00833875
Risk-free	0.00029016	0	0.2291351	0	Var(2nd Eff-p)	0.00050344
Sum	-	-	1	-	SD(2nd Eff-p)	0.02243738
					Sharpe Ratio	0.358713
					Beta(2nd Eff-p)	**0.770865**

When compared to the values for the benchmark, we can see that under the same beta of 0.7709, their ER and SD are quite close; when we put the three portfolios together, we can notice that and first efficient portfolio holds a relatively low SD of 0.0237, while computing the highest expected return of 0.0142, which is more than twice that of the benchmark, and nearly 1.7 times of the second effective portfolio.

Obviously in this case our tangency portfolio dose not equal to the market portfolio, however, we cannot simply conclude that CAPM is wrong, since our tangency portfolio is not computed from all assets in the market, and we have several low-correlated risky assets in it. Therefore, more adjustments should be applied to test CAPM.

2. A summary

(i) Markowitz bullet constructed from the risky assets



(ii) ER and SD

Portfolios	E(r)	SD
Mkt-p	0.01073115	0.02837586
SS Tan-p	0.00281051	0.00429565
Ben-p	0.00700106	0.0236532
1st Eff-p	0.01416799	0.0236532
2nd Eff-p	0.00833875	0.02243738

3. Analysis for the risky assets and the three portfolios constructed

(i) Jensen's Alpha & Test on "Jensen's Alpha = 0"

Assets	Jensen's Alpha	SE(Alpha)	T-Stat	T-critical	Reject or not?
NAESX	-0.0015532	0.00209274	-0.7421838	2.00099538	Cannot Reject
VBISX	0.0005679	0.00051788	1.09657693	2.00099538	Cannot Reject
VBLTX	0.00318286	0.0031926	0.99694985	2.00099538	Cannot Reject

VEIEX	-0.0067547	0.00465215	-1.4519606	2.00099538	Cannot Reject
VEURX	-0.00505	0.00329202	-1.5340024	2.00099538	Cannot Reject
VEXMX	-0.0014712	0.00198468	-0.7412667	2.00099538	Cannot Reject
VFINX	8.55E-05	0.00051411	0.16627404	2.00099538	Cannot Reject
VIGRX	0.00088372	0.00136002	0.64978032	2.00099538	Cannot Reject
VIVAX	-0.0007116	0.001073	-0.6631704	2.00099538	Cannot Reject
VPACX	-0.0025562	0.00301574	-0.8476187	2.00099538	Cannot Reject
Ben-p	-0.0013377	0.00110935	-1.2058345	2.00099538	Cannot Reject
1st Eff-p	0.00884031	0.00272295	3.24659184	2.00099538	**Reject**
2nd Eff-p	0	5.98E-19	0	2.00099538	Cannot Reject

From the above table we can see that among the 10 risky assets and 3 portfolios, all cannot reject the null hypothesis of "Jensen's Alpha = 0" except the first efficient portfolio, implying its nature of actively-managed portfolio and the ability for sustainable abnormal return.

(ii) Measurements: Jensen's Alpha & Treynor Index & Sharpe Ratio

Assets	Jensen's Alpha	Treynor Ratio	Sharpe Ratio
NAESX	-0.001553193	0.00899489	0.28000467
VBISX	0.000567895	-0.02118	0.1024988
VBLTX	0.003182864	-0.6194162	0.13592313
VEIEX	-0.006754729	0.00314273	0.06774043
VEURX	-0.005049958	0.00508731	0.13273289
VEXMX	-0.001471172	0.00908031	0.28575049
VFINX	8.54869E-05	0.01052983	0.36164161
VIGRX	0.000883721	0.01130921	0.37136695
VIVAX	-0.00071158	0.00966879	0.32188118
VPACX	-0.00255619	0.00726976	0.18381138
Ben-p	-0.001337685	0.00870568	0.28372069
1st Eff-p	0.008840308	0.02876383	0.58672045
2nd Eff-p	0	0.01044099	0.35871353

(iii) Ranking of assets and portfolios

Assets	By Jensen's Alpha	Assets	By Treynor Ratio	Assets	By Sharpe Ratio
1st Eff-p	0.008840308	**1st Eff-p**	0.02876383	**1st Eff-p**	0.586720452
VBLTX	0.003182864	VIGRX	0.01130921	VIGRX	0.371366952

VIGRX	0.000883721	VFINX	0.01052983	VFINX	0.36164161
VBISX	0.000567895	2nd Eff-p	0.01044099	2nd Eff-p	0.358713525
VFINX	8.54869E-05	VIVAX	0.00966879	VIVAX	0.32188118
2nd Eff-p	0	VEXMX	0.00908031	VEXMX	0.285750495
VIVAX	-0.00071158	NAESX	0.00899489	Ben-p	0.283720685
Ben-p	-0.001337685	Ben-p	0.00870568	NAESX	0.280004669
VEXMX	-0.001471172	VPACX	0.00726976	VPACX	0.183811382
NAESX	-0.001553193	VEURX	0.00508731	VBLTX	0.135923129
VPACX	-0.00255619	VEIEX	0.00314273	VEURX	0.132732886
VEURX	-0.005049958	VBISX	-0.02118	VBISX	0.102498799
VEIEX	-0.006754729	VBLTX	-0.6194162	VEIEX	0.067740433

According to the 3 ranking criteria, the first efficient portfolio has the highest Jensen's alpha of 0.0088, the highest Treynor ratio of 0.0288, and the highest Sharpe ratio of 0.5867, therefore, it is the best portfolio according to the three measure.

(iv) Fama-French 3-factor model & Test on "Alpha = 0"

Since we only have one significant non-zero alpha for the first efficient portfolio, now we re-run the regression for it using the Fama-French 3-factor model:

	Coefficients	SE	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.00806006	0.00257474	**3.13043381**	0.00275216	0.00290423	0.01321589
Mkt-RF	0.50309534	0.08474202	5.93678707	1.8252E-07	0.33340237	0.67278831
SMB	-0.1120208	0.09988879	-1.1214555	0.26679638	-0.3120447	0.08800302
HML	-0.2923109	0.10577125	-2.7636137	0.0076848	-0.5041142	-0.0805076

We can observe that using a 5% significance level, the intercept still significantly rejects the null hypothesis of "Alpha = 0" (3.1304 > 2), implying the inefficiency of the market.