Problem Set 7

For this problem set you have more time:
Due: Friday 11:59pm, April 7th via Canvas upload
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Office hour: TBA

1 Long-range interaction between an excited atom and a ground-state atom

(this is the carry-over from last pset)
Consider the case where one atom is excited and the other atom is in its ground state. For simplicity model each atom as a two level system with one ground state and one excited state.

a) Assume you have two atoms a and b with almost (but not quite) degenerate ground $\leftrightarrow$ excited state transition energies $(E_i^{(a)} - E_g^{(a)}) \approx (E_i^{(b)} - E_g^{(b)})$. How does the energy of the state $|i_a g_b\rangle$ change as a function of the separation R for large distances? What about state $|g_a i_b\rangle$? For what separation does perturbation theory become invalid?

b) Now assume you have two identical (i.e. same transition energy) atoms. Calculate the long-range interaction potential curves for the case of one excited atom and one ground state atom.

c) For case (b) what is the relation between the spontaneous decay rate of the atom and its long-range interaction coefficient?

2 Casimir model of the electron

Model the electron as two parallel plates of area a^2, separated by distance a and carrying charge $q = \frac{e}{2}$. Balance the Casimir and electrostatic forces and from this determine a value for the fine-structure constant $\alpha \equiv \frac{e^2}{\hbar c}$ (cgs units).
Note:Casimir's original idea to obtain α by balancing electrostatic and Casimir forces led to tour de force calculations e.g. for spherical shells by the likes of Schwinger et al., whose outcome was Casimir repulsion, not attraction (see article by Spruch on Canvas). More recently, it appears this might be an artefact of assumptions and regularization, see e.g. Graham, Quandt, Weigel, "Attractive electromagnetic Casimir stress on a spherical dielectric shell", Phys. Lett. B 846 (2013), and Leonhardt, Simpson, "Exact solution for the Casimir stress in a spherically symmetric medium", PRD 84, 081701(R) (2011).