8.422 AMO II - Slides on Photon detection and correlation

February 27, 2023

Detecting photons

Electric Field operator

$$\begin{aligned}\vec{E}(\vec{r},t) &= \sum_j \mathcal{E}_j \vec{\epsilon}_j \left(\hat{a}_j e^{-i\vec{k}_j\cdot\vec{r}-i\omega_j t} + \hat{a}_j^\dagger e^{i\vec{k}_j\cdot\vec{r}+i\omega_j t} \right) \\ &= \vec{E}^{(+)}(\vec{r},t) + \vec{E}^{(-)}(\vec{r},t)\end{aligned}$$

$\vec{E}^{(+)}$ - positive frequency part $\Rightarrow$ photon annihilation
$\vec{E}^{(-)}$ - negative frequency part $\Rightarrow$ photon creation
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$$\boxed{P = \langle i|\vec{E}^{(-)}(\vec{r},t)\vec{E}^{(+)}(\vec{r},t)|i\rangle}$$

Detecting photons

A more general measurement records photon absorption at two (or more) different places and/or times (delayed coincidences).
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Probability:

$$\begin{aligned} P &= \sum_f |\langle f|\vec{E}^{(+)}(\vec{r}', t')\vec{E}^{(+)}(\vec{r}, t)|i\rangle|^2 \\ &= \langle i|\vec{E}^{(-)}(\vec{r}, t)\vec{E}^{(-)}(\vec{r}', t')\vec{E}^{(+)}(\vec{r}', t')\vec{E}^{(+)}(\vec{r}, t)|i\rangle \end{aligned}$$

Detecting photons

Generalizing to a density matrix ρ describing our knowledge of the initial state of the system, e.g. $\rho = \sum_i p_i |i\rangle\langle i|$ for a mixed state, we need

$$P \;=\; \sum_i p_i \langle i|\vec{E}^{(-)}(\vec{r},t)\vec{E}^{(+)}(\vec{r},t)|i\rangle$$

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$$\begin{aligned} P &= \sum_i p_i \langle i|\vec{E}^{(-)}(\vec{r},t)\vec{E}^{(+)}(\vec{r},t)|i\rangle \\ &= \mathrm{Tr}\left(\rho \vec{E}^{(-)}(\vec{r},t)\vec{E}^{(+)}(\vec{r},t)\right) \end{aligned}$$

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\end{aligned}
$$

or for coincidences

$$
P = \mathrm{Tr}\left(\rho \vec{E}^{(-)}(\vec{r},t)\vec{E}^{(-)}(\vec{r}',t')\vec{E}^{(+)}(\vec{r}',t')\vec{E}^{(+)}(\vec{r},t)\right)
$$

Correlation functions

A more general function, relevant to describe non-ideal photon detectors, but also conceptually to understand coherence, is the first-order correlation function

$$G^{(1)}(\vec{r}t, \vec{r}'t') = \mathrm{Tr}\left(\rho \vec{E}^{(-)}(\vec{r}, t)\vec{E}^{(+)}(\vec{r}', t')\right)$$

and the second-order correlation function

$$G^{(2)}(\vec{r}_1 t_1 \vec{r}_2 t_2, \vec{r}_3 t_3 \vec{r}_4 t_4) =$$

$$\mathrm{Tr}\left(\rho \vec{E}^{(-)}(\vec{r}_1, t_1)\vec{E}^{(-)}(\vec{r}_2, t_2)\vec{E}^{(+)}(\vec{r}_3, t_3)\vec{E}^{(+)}(\vec{r}_4, t_4)\right)$$

Generalization to general n-photon coherence functions is straightforward.

Correlation functions

More convenient for quantitative analysis are the normalized versions

$$g^{(1)}(\vec{r}t, \vec{r}'t') = \frac{G^{(1)}(\vec{r}t, \vec{r}'t')}{\left(G^{(1)}(\vec{r}t, \vec{r}t)G^{(1)}(\vec{r}'t', \vec{r}'t')\right)^{1/2}}$$

and

$$g^{(2)}(\vec{r}_1 t_1 \vec{r}_2 t_2, \vec{r}_3 t_3 \vec{r}_4 t_4) = \frac{G^{(2)}(\vec{r}_1 t_1 \vec{r}_2 t_2, \vec{r}_3 t_3 \vec{r}_4 t_4)}{\prod_{j=1}^{4} \left(G^{(1)}(\vec{r}_j t_j, \vec{r}_j t_j)\right)^{1/2}}$$

Application: Two-photon correlation measurements, e.g. Hanbury Brown-Twiss experiment. Tells us about coherent vs chaotic vs non-classical light.