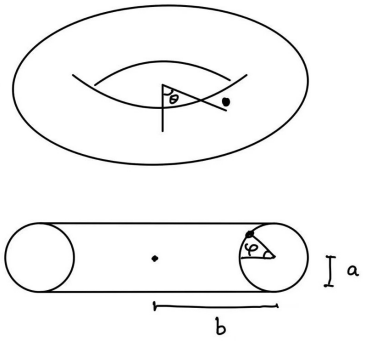
8.(3)09 Section 3

September 24, 2021

1 Particle on a torus

Consider a particle constrained to move on a torus with big radius b and small radius a. Let's consider motion under no external potential. We need two coordinates to specify the position: θ the angle around the big radius, and φ the angle around the small radius.



(a)

Write down the Lagrangian for the system.

(b)

Calculate p_θ and p_φ. Obtain the Hamiltonian.

(c)

Find all the conserved quantities in the system. You can then write p_φ^2 in terms of coordinates (θ and/or φ) and the conserved quantities.

(d)

Describe what the path of the particle may look like.... (In order to do this, we can first examine $\frac{\partial p_\varphi}{\partial \varphi}$ around $\varphi = \pi$, which should look like a restoring force. So we can get path that oscillate around $\varphi = \pi$. However, if p_φ is big enough, the particle will circle the torus. What separates these two cases?)