$$\hat{\rho}_{sys} = |\Psi\rangle\langle\Psi|$$

$$\begin{array}{cc} & \begin{array}{ccc}\psi_1 & \cdots & \psi_d\end{array} \\ \begin{array}{c}\psi_1 \\ \vdots \\ \psi_d\end{array} & \begin{pmatrix} \\ \\ \\ \end{pmatrix} \begin{pmatrix} & \quad & \end{pmatrix} \end{array}$$

$d \qquad d$

$$\underbrace{\begin{pmatrix} \boxed{\rho_{11}} & \boxed{\rho_{12}} & & \\ \boxed{\rho_{21}} & \boxed{\rho_{22}} & & \\ & & \ddots & \\ & & & \boxed{\rho_{dd}} \end{pmatrix}}_{d \times d}$$

Coherences / decoherence

dissipation / decay

Populations / Probability of occupation of state $|\psi_i\rangle$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

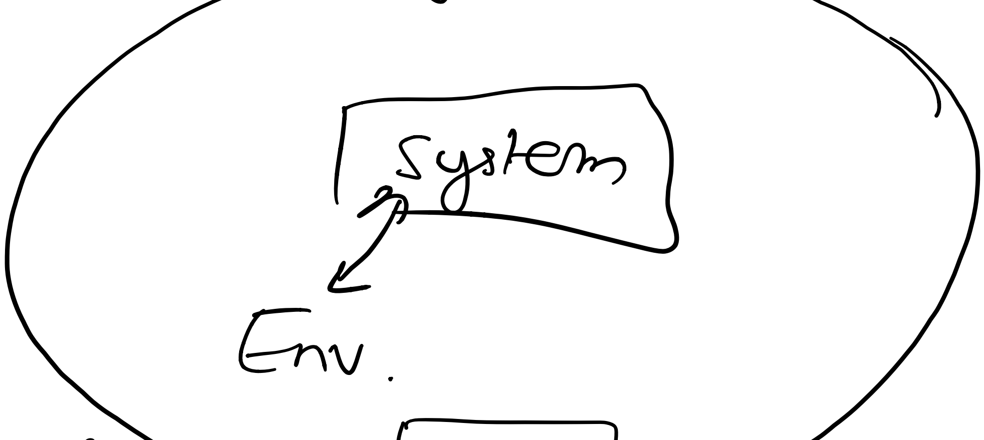
$$\frac{1}{2}\underline{\underline{\left(|0\rangle + |1\rangle\right)}}\left(\langle 0| + \langle 1|\right)$$

$$\begin{pmatrix} 1/2 & \boxed{1/2} \\ 1/2 & 1/2 \end{pmatrix}$$

$$|0\rangle\langle 0| \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

no coherence!

Open Quantum Systems



$$\hat{\mathcal{H}} = \mathcal{H}_S + \mathcal{H}_E + \boxed{\mathcal{H}_{int}}$$

$$\hat{\rho}_{tot}(0) = \hat{\rho}_{sys}(0) \otimes \hat{\rho}_{Env}(0)$$

$$\begin{array}{c|cccc} & |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \hline \langle 00| & & \times & & \times \\ \langle 01| & \times & & & \\ \langle 10| & & \times & & \times \\ \langle 11| & \times & & \times & \end{array}$$

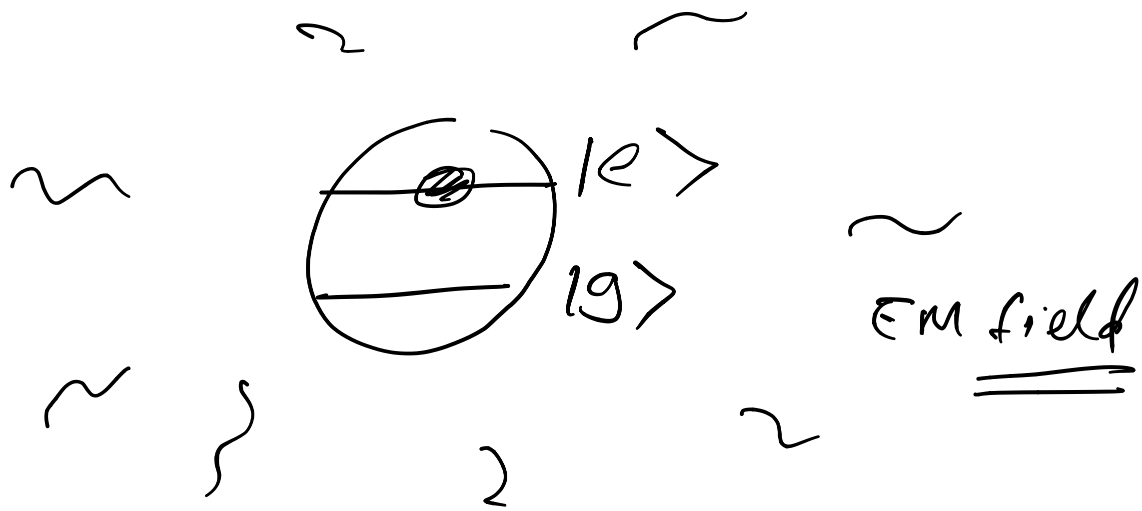
$$\mathrm{Tr}_{Env.}\left[\hat{\rho}_{tot}(t)\right]$$

$$= \sum_{n \in \mathcal{H}_{Env.}} \langle n| \hat{\rho}_{tot.}(t) |n\rangle$$

Averaging over the state of env.

→ Spontaneous emission

Gerry & Knight – Introductory Q Optics



Cohen-Tannoudji

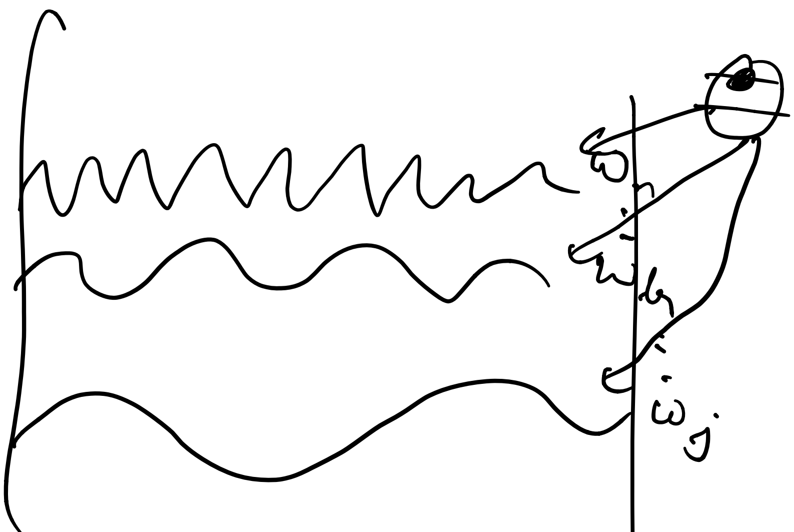
$$\hat{\rho}_{at} \quad \begin{array}{c|cc} & |e\rangle & |g\rangle \\ \hline \langle e| & \rho_{ee} & \rho_{eg} \\ \langle g| & \rho_{ge} & \rho_{gg} \end{array}$$

$$\hat{\rho}_{at} = \rho_{ee}\,|e\rangle\langle e| + \rho_{eg}\,|e\rangle\langle g| + \rho_{ge}\,|g\rangle\langle e| + \rho_{gg}\,|g\rangle\langle g|$$

$$\mathcal{H}_A = \hbar\omega_0 \, \hat{\sigma}^+ \hat{\sigma}^- = \hbar\omega_0 |e\rangle\langle e|$$

$$\mathcal{H}_F = \sum_k \hbar\omega_k \, \underbrace{\hat{a}^\dagger(\omega_k)}_{\text{creation operator}} \, \underbrace{\hat{a}(\omega_k)}_{\text{annihilation operator}}$$

"ω" → harmonic oscillator



Quantum Harmonic Oscillator

$\hat{a}^\dagger(\omega_k) \, |0\rangle$ → vacuum EMF

$$\mathcal{H}_{int} = -\hat{\underline{d}} \cdot \hat{\underline{E}}$$

$$= g\left[\hat{\sigma}_+ \hat{a}(\omega) + \hat{\sigma}_- \hat{a}^\dagger(\omega)\right]$$

$$|\{1\}_{\omega_k}\rangle$$

raising op. $\boxed{\hat{\sigma}^{+}} = |e\rangle\langle g|$

$\hat{\Pi}_e = |e\rangle\langle e| \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

lowering op. $\hat{\sigma}^{-} = |g\rangle\langle e| \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$(|e\rangle\langle g|)\,(|g\rangle\langle e|)$$

$$= (|e\rangle\langle e|) = \underline{\underline{\hat{\Pi}_e}}$$

$$\mathcal{H} = \mathcal{H}_A + \mathcal{H}_F + \mathcal{H}_{int}$$

$$|\Psi(t)\rangle_{AF} = e^{-i\mathcal{H}t}\,|e\rangle \otimes |\{0\}\rangle$$

$\downarrow$

$$\rho_{AF}(t) = |\underline{\Psi}(t)\rangle_{AF}\ {}_{AF}\langle\underline{\Psi}(t)|$$

$$\mathrm{Tr}_F\ \hat{\rho}_{AF}(t) = \underline{\underline{\hat{\rho}_A(t)}}$$

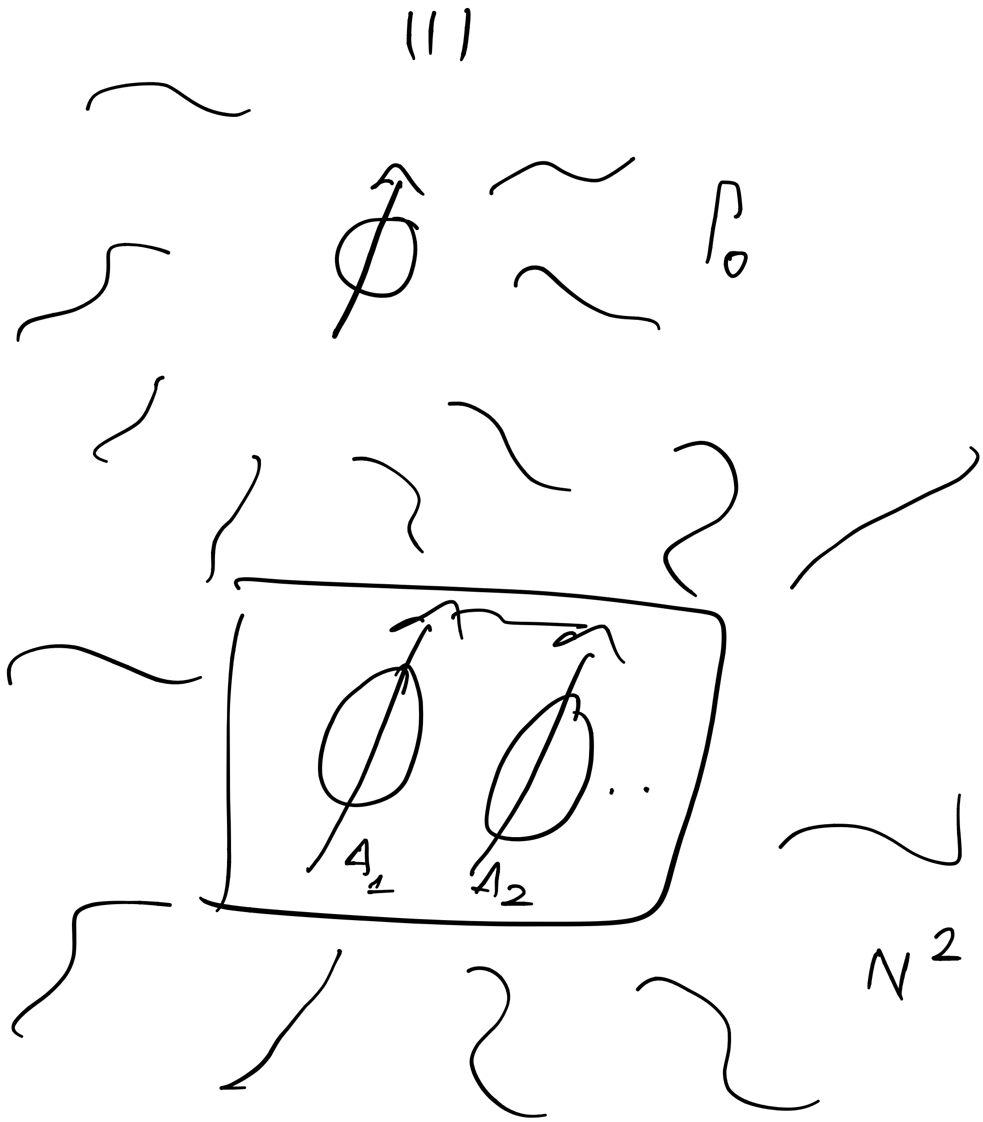
spontaneous emission rate

$$e^{-\Gamma_0 t}$$

$$\Gamma_0 \sim |\vec{d}|^2$$

$$\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)$$

$$\underline{\underline{\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)}}$$



$$I = |A_1 \pm A_2|^2 \qquad A_1 = A_2 = A$$

$$= 4|A|^2, \ \boxed{0}$$

radiated intensity

by classical dipoles ~ radiated intensity by atomic dipoles

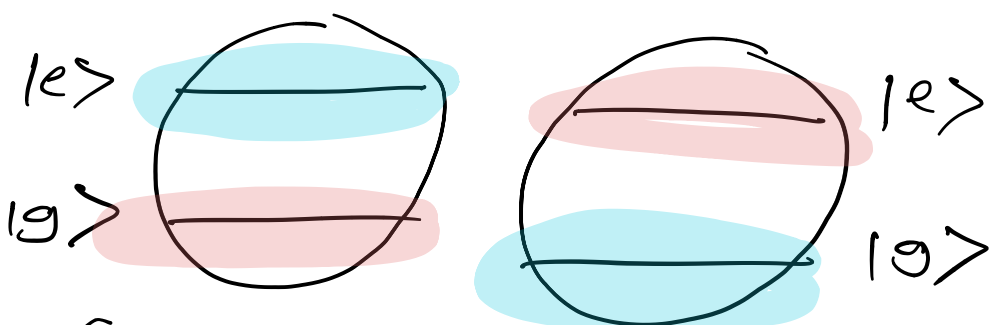
$$I = |A_1|^2 + |A_2|^2 + \ldots |A_N|^2$$

$$= N |A|^2$$

$$\text{“}I_{sup}\text{”} = N^2 |A|^2$$ → superradiant collection of dipoles

$$= |A_1 + A_2 \ldots\ldots A_N|^2$$

$$A_1 = A_2 = \ldots A_N = A$$



$$\frac{1}{\sqrt{2}}\left(|eg\rangle \pm |ge\rangle\right)$$

R.H. Dicke 1954

Coherence in sp. emission processes

Mandel and Wolf Quantum Optics textbook Chapter 16

$$\mathcal{H} = -\hat{\vec{d}}\cdot\hat{\vec{E}}$$

N atoms

$$\Gamma \sim |\vec{d}|^2$$

$$= -\sum_{n=1}^{N} \hat{\vec{d}}_n \cdot \hat{\vec{E}}(x_0)$$

$$= -\left[\sum_{n=1}^{N} \hat{\vec{d}}_n\right] \cdot \hat{\vec{E}}(x_0)$$

$$\Gamma_N \sim \left| \left\langle \sum_{n=1}^{N} \hat{\vec{d}}_n \right\rangle_{|eg\rangle \pm |ge\rangle} \right|^2$$

$$\hat{\vec{d}}_n = \vec{d}\left(\hat{\sigma}_n^+ + \hat{\sigma}_n^-\right)$$

$$\hat{\vec{d}}_1 + \hat{\vec{d}}_2 = \vec{d}\left(\hat{\sigma}_1^+ + \hat{\sigma}_1^- + \hat{\sigma}_2^+ + \hat{\sigma}_2^-\right)$$

(1) $\frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right)$ sup.

$$\frac{1}{\sqrt{2}}\left(\langle eg| \boxed{+} \langle ge|\right)\left(\underline{\hat{\vec{d}}_1} + \underline{\hat{\vec{d}}_2}\right)$$

$$\frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right)$$

$$= 2\vec{d}$$

(2) $\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$ subradiant

$$= \frac{1}{\sqrt{2}}\left(\langle eg| - \langle ge|\right)\left(\hat{\vec{d}}_1 + \hat{\vec{d}}_2\right)\frac{1}{\sqrt{2}}\left(|eg\rangle - |ge\rangle\right)$$

$$= 0$$