You may find the following information helpful:

Physical Constants

Electron mass	$m_e \approx 9.1 \times 10^{-31} kg$	Proton mass	$m_p \approx 1.7 \times 10^{-27} kg$
Electron Charge	$e \approx 1.6 \times 10^{-19} C$	Planck's const./2π	$\hbar \approx 1.1 \times 10^{-34} Js^{-1}$
Speed of light	$c \approx 3.0 \times 10^{8} ms^{-1}$	Stefan's const.	$\sigma \approx 5.7 \times 10^{-8} Wm^{-2}K^{-4}$
Boltzmann's const.	$k_B \approx 1.4 \times 10^{-23} JK^{-1}$	Avogadro's number	$N_0 \approx 6.0 \times 10^{23} mol^{-1}$

Conversion Factors

$1atm \equiv 1.0 \times 10^5 Nm^{-2}$ $\quad$ $1\mathring{A} \equiv 10^{-10} m$ $\quad$ $1eV \equiv 1.1 \times 10^4 K$

Thermodynamics

$dE = TdS + \text{đ}W$ $\quad$ For a gas: $\text{đ}W = -PdV$ $\quad$ For a wire: $\text{đ}W = Jdx$

Mathematical Formulas

$\int_0^\infty dx\ x^n\ e^{-\alpha x} = \frac{n!}{\alpha^{n+1}}$ $\qquad$ $\left(\frac{1}{2}\right)! = \frac{\sqrt{\pi}}{2}$

$\int_{-\infty}^{\infty} dx \exp\left[-ikx - \frac{x^2}{2\sigma^2}\right] = \sqrt{2\pi\sigma^2} \exp\left[-\frac{\sigma^2 k^2}{2}\right]$ $\qquad$ $\lim_{N\to\infty} \ln N! = N \ln N - N$

$\left\langle e^{-ikx} \right\rangle = \sum_{n=0}^{\infty} \frac{(-ik)^n}{n!} \langle x^n \rangle$ $\qquad$ $\ln \left\langle e^{-ikx} \right\rangle = \sum_{n=1}^{\infty} \frac{(-ik)^n}{n!} \langle x^n \rangle_c$

$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$ $\qquad$ $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$

Surface area of a unit sphere in d dimensions $\qquad$ $S_d = \frac{2\pi^{d/2}}{(d/2-1)!}$

1. *Gas:* The temperature of a gas is found do depend only on its pressure as $T(p, V) = c\, p^n$, while its internal energy is given by $E(p, V) = D\, pV$, where c, D and n are constants.

(a) Give the expression for the differential changes in entropy as $dS(p, V)$.

(b) Noting that entropy is a function of state, find the relation between n and D.

(c) Find the form of adiabatic curves as $p_S(V)$.

(d) Draw an infinitesimal Carnot cycle in the (p, V) coordinates.

(e) How much heat is extracted in the above Carnot cycle at the temperature $T(p)$?

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2. *Constant heat capacities:* Consider two bodies with temperature independent heat capacities C_1 and C_2, and initial temperatures $T_1 > T_2$.

(a) If the two bodies are brought into contact such that the only heat exchange is between them, what is the final temperature T_F, and what is the change in entropy.

(b) What is the final temperature if a Carnot engine is used to transfer heat between the two bodies? What is the amount of work done by the engine in this case?

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3. *Probability:* Calculate the characteristic function, the mean, and the variance of the following probability density functions:

(a)

$$p(x) = \frac{1}{2b} \exp\left(-\frac{|x-a|}{b}\right) \quad \text{for} \quad -\infty < x < \infty.$$

(b)

$$p(x) = \frac{|x|}{2a^2} \exp\left(-\frac{|x|}{a}\right) \quad \text{for} \quad -\infty < x < \infty.$$

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