Bose-Fermi Pair Correlations in Attractively Interacting Bose-Fermi Atomic Mixtures

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We study static properties of attractively interacting Bose-Fermi mixtures of uniform atomic gases at zero temperature. Using Green's function formalism we calculate boson-fermion scattering amplitude and fermion self-energy in the medium to lowest order of the hole line expansion. We study ground state energy and pressure as functions of the scattering length for a few values of the boson-fermion mass ratio m_b/m_f and the number ratio N_b/N_f. We find that the attractive contribution to energy is greatly enhanced for small values of the mass ratio. We study the role of the Bose-Fermi pair correlations in the mixture by calculating the pole of the boson-fermion scattering amplitude in the medium. The pole shows a standard quasiparticle dispersion for a Bose-Fermi pair, for $m_b/m_f \geq 1$. For small values of the mass ratio, on the other hand, a Bose-Fermi pair with a finite center-of-mass momentum experiences a strong attraction, implying large medium effects. In addition, we also study the fermion dispersion relation. We find two dispersion branches with the possibility of the avoided crossings. This strongly depends on the number rario N_b/N_f.

I. INTRODUCTION

Recent developments in the field of cold atomic gases have proven that this system provides an ideal laboratory for the studies of quantum many-body systems[1]. This is due to the experimental facilities which allow to control various parameters characterizing the many-body system, e.g., external potentials including optical lattices, choice of atoms obeying Bose or Fermi statistics and their mixtures, variable particle densities, etc. The use of Feshbach resonances, in particular, makes the atomic gases an extremely flexible system as it provides a means to control atomic interactions[2, 3]. One thus was, for instance, able to study the BEC-BCS crossover process in the two-component Fermi system, which has been under intense investigation for decades[4]. By changing the resonance energies through the external magnetic field, one can in principle change the magnitude and the sign of the scattering length of the interacting particles, keeping track all the way from a resonating fermion pair to a bound composite particle, a bosonic molecule.

The aim of the present paper is the study of pair correlations in a different system, a Bose-Fermi(BF) mixture. Degenerate mixtures of bosons and fermions have been created since several years, and studies of static and dynamic properties have been performed[5]. Among those are the studies of attractively interacting BF systems, where one finds a sudden loss of fermions as the BF attractive interaction is effectively increased[6]. Detailed studies of the dynamics of this system are still missing, however. Recently the finding of Feshbach resonances and formation of the boson-fermion molecules have been reported[7, 8]. It is thus expected that by controling the BF interaction one may realize an analog of the process found in two-component Fermi systems. What should be expected if one replaced fermion pairs in the BEC-BCS crossover process by BF pairs? Such studies have indeed been performed theoretically[9] (see also [10].) By adopting a Cooper type two-particle problem on top of the boson-fermion degenerate system, it was shown that a stable correlated BF pair is created even before the threshold for the BF bound state. In contrast to the BCS case, however, the system allows only one correlated BF pair with a given center-of-mass (CM) momentum because of the fermionic nature of the composite particle. It is then suggested that by increasing the BF attractive interaction, one may create BF pairs with different CM momentum stepwise, until finally a new Fermi sea of the BF pairs is completed.

In Ref.[9] a separable BF interaction has been adopted to elucidate the mechanism of the creation of BF pairs. In the present paper we adopt a standard pseudopotential for the interaction, and calculate energy and pressure of the system for various values of input parameters. We use Green's function formalism for this system and calculate perturbatively relevant diagrams to lowest order of the hole-line expansion. Such formalism has been developed in [11] together with the calculation of the energies including Bose-Bose (BB) interaction. Our formulation is similar to [11], but we use the renormalization procedure of [12] in relating the pseudopotential strength to the S-wave scattering length. This allows us to formally take the limit $|a| \to \infty$, the unitarity limit [13], which is necessary when one considers a (nearly) bound state of a pair of atoms. We then calculate the poles of the BF pair scattering amplitude in the BF medium, which may be compared with the results of [9]. Studies of the behavior of the poles as a function of input parameters give us suggestions on the role of the BF pair correlations in the static properties of the system.

The content of the paper is as follows: In the next section we present our model based on the Hamiltonian without Bose-Bose interaction. We calculate the BF scattering amplitude in the BF mixture in ladder approximation, and give formulas for physical quantities in terms of the amplitude. In section 3 we show numerical results for the ground state energy and pressure for various choices of the boson/fermion masses and the values of the Bose-Fermi interaction. We then study Bose-Fermi pair correlation in Section 4 by focusing on the pole structure of the boson-fermion scattering amplitude in the mixture. We also calculate the pole of the single fermion Green's function and study the role of the Bose-Fermi pair and its dispersion. We summarize our results in section 5 together with a comment on the effects of the Bose-Bose interaction. Detailed expressions for the scattering amplitude are given in the appendix.

II. FORMULATION

We consider a uniform system of a polarized Bose-Fermi mixture of atomic gases with attractive boson-fermion interaction. The model Hamiltonian of the system is given by

$$\begin{aligned} H &= T_b + T_f + H_{bf}, \\ T_b &= \int d^3\mathbf{x}\phi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_b} - \mu_b\right)\phi(\mathbf{x}), \\ T_f &= \int d^3\mathbf{x}\psi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_f}\right)\psi(\mathbf{x}), \\ H_{bf} &= g_{bf}\int d^3\mathbf{x}\phi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\phi(\mathbf{x}), \end{aligned} \quad (1)$$

where ψ and ϕ are the boson and fermion field operators, respectively, T_b and T_f denote bosonic and fermionic kinetic energies, while H_{bf} denotes boson-fermion interaction with strength g_{bf} (< 0) of the boson-fermion pseudopotential. Effects of the boson-boson interaction will be mentioned later, while the fermion-fermion interaction is omitted throughout as we consider one-component (polarized) fermions. We will adopt the Bogoliubov approximation in treating the Bose-Einstein condensate (BEC), and therefore include in T_b the bosonic chemical potential μ_b.

A. Green's function formalism in the Bose-Fermi mixture

To treat condensed bosons, we adopt the conventional Bogoliubov method by separating the zero momentum mode from the remainder :

$$\phi(\mathbf{x}) = \sqrt{n_0} + \varphi(\mathbf{x}) \quad (2)$$

together with its conjugate. $n_0 = N_0/V$ is the number density of bosons with momentum $\mathbf{k} = 0$. As usual we omit the fluctuation of the boson number in the condensate. The boson number operator $\hat{N}_b$ writes

$$\hat{N}_b = N_0 + \int d^3\mathbf{x}\varphi^\dagger(\mathbf{x})\varphi(\mathbf{x}). \quad (3)$$

and the Hamiltonian takes the form

$$H = H_0 + H_{bf}, \quad (4)$$

where

$$\begin{aligned} H_0 = &\int d^3\mathbf{x}\varphi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_b} - \mu_b\right)\varphi(\mathbf{x}) \\ &+ \int d^3\mathbf{x}\psi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_f}\right)\psi(\mathbf{x}) \\ &- \mu_b N_0 \end{aligned} \quad (5)$$

and

$$\begin{aligned} H_{bf} = &\, n_0 g_{bf}\int d^3\mathbf{x}\psi^\dagger(\mathbf{x})\psi(\mathbf{x}) \\ &+ \sqrt{n_0}g_{bf}\int d^3\mathbf{x}\psi^\dagger(\mathbf{x})\psi(\mathbf{x})(\varphi^\dagger(\mathbf{x}) + \varphi(\mathbf{x})) \\ &+ g_{bf}\int d^3\mathbf{x}\psi^\dagger(\mathbf{x})\varphi^\dagger(\mathbf{x})\varphi(\mathbf{x})\psi(\mathbf{x}). \end{aligned} \quad (6)$$

Physical quantities can be expressed in terms of Green's functions. We define the boson and fermion Green's functions by

$$iG^f(x-y) = \frac{\langle\Psi_0|T\left[\psi_H(x)\psi_H^\dagger(y)\right]|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}, \quad (7)$$

$$iG^b(x-y) = \frac{\langle\Psi_0|T\left[\varphi_H(x)\varphi_H^\dagger(y)\right]|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}, \quad (8)$$

where $\psi_H(x), \varphi_H(x)$ are the field operators in the Heisenberg picture, and $|\Psi_0\rangle$ represents the interacting ground state.

The energy of the system can be expressed in terms of the Green's functions. The fermion and boson kinetic energies are calculated according to the standard procedure [14] as:

$$\langle T_f\rangle = \left\langle\frac{-\nabla^2}{2m_f}\right\rangle = -iV\int\frac{d^4q}{(2\pi)^4}\epsilon^f_{\mathbf{q}}G^f(q)e^{iq_0\eta}, \quad (9)$$

$$\langle T_b\rangle = \left\langle\frac{-\nabla^2}{2m_b}\right\rangle = iV\int\frac{d^4q}{(2\pi)^4}\epsilon^b_{\mathbf{q}}G^b(q)e^{iq_0\eta}, \quad (10)$$

where $G(q)$'s are the Fourier transform of the Green's functions, η is a positive infinitesimal, and we set $\epsilon^{b,f}_{\mathbf{q}} = \mathbf{q}^2/2m_{b,f}$. The different signs in the two expressions come from the ordering of the field operators.

To calculate the interaction energy, we first consider the Heisenberg equation of motion for the fermion field:

$$
\begin{aligned}
i\frac{\partial}{\partial t}\psi_H(x) =&[\psi_H(x), H] \\
=&(-\frac{\nabla^2}{2m_f})\psi_H(x) + n_0 g_{bf}\psi_H(x) \\
&+ \sqrt{n_0}g_{bf}\psi_H(x)(\varphi_H^\dagger(x)+\varphi_H(x)) \\
&+ g_{bf}\psi_H(x)\varphi_H^\dagger(x)\varphi_H(x). \qquad (11)
\end{aligned}
$$

Multiplying by $\psi_H^\dagger(x')$ and integrating over $\mathbf{x}$, we obtain

$$
\begin{aligned}
\langle H_{bf}\rangle &= -i\int d^3\mathbf{x} \lim_{\substack{\mathbf{x}'\to\mathbf{x}\\ t'\to t}} \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m_f}\right) G^f(t\mathbf{x}, t'\mathbf{x}') \\
&= -iV\int \frac{d^4q}{(2\pi)^4}(q_0 - \epsilon_{\mathbf{q}}^f)G^f(q)e^{iq_0\eta}. \qquad (12)
\end{aligned}
$$

We now introduce fermion and boson self-energies $\Sigma^f(q)$ and $\Sigma^b(q)$ through

$$
G^f(q) = \frac{1}{q_0 - \epsilon_{\mathbf{q}}^f - \Sigma^f(q)}, \qquad (13)
$$

and

$$
G^b(q) = \frac{1}{q_0 - \epsilon_{\mathbf{q}}^b + \mu_b - \Sigma^b(q)}. \qquad (14)
$$

In the integrand of Eq.(12) one may use the relation from Eq.(13)

$$
\left(q_0 - \epsilon_{\mathbf{q}}^f\right) G^f(q) = 1 + \Sigma^f(q)G^f(q), \qquad (15)
$$

and finds

$$
\langle H_{bf}\rangle = -iV\int \frac{d^4q}{(2\pi)^4}\Sigma^f(q)G^f(q)e^{iq_0\eta}, \qquad (16)
$$

the first term in the r.h.s. in Eq.(15) giving a null contribution to the integral. The total energy E of the system is finally obtained as

$$
\begin{aligned}
E =&\langle T_f\rangle + \langle T_b\rangle + \langle H_{bf}\rangle \\
=& -iV\int \frac{d^4q}{(2\pi)^4}\left(\epsilon_{\mathbf{q}}^f + \Sigma^f(q)\right) G^f(q)e^{iq_0\eta} \\
&+ iV\int \frac{d^4q}{(2\pi)^4}\epsilon_{\mathbf{q}}^b G^b(q)e^{iq_0\eta}. \qquad (17)
\end{aligned}
$$

The thermodynamic potential at zero temperature is given by

$$
\Omega(N_f, N_0, \mu_b) = \langle H\rangle = E - \mu_b\langle \hat{N}_b\rangle, \qquad (18)
$$

where

$$
\langle \hat{N}_b\rangle = N_0 + iV\int \frac{d^4q}{(2\pi)^4}G^b(q)e^{iq_0\eta}. \qquad (19)
$$

The system is characterized by the boson and fermion particle numbers, N_b and N_f. The number of bosons satisfies the thermodynamic relation

$$
\frac{\partial\Omega}{\partial\mu_b} = -N_b. \qquad (20)
$$

The parameter N_0 should be chosen to minimize the thermodynamic potential

$$
\frac{\partial\Omega}{\partial N_0} = 0 \qquad (21)
$$

which leads to an explicit expression for μ_b as shown below.

We also will calculate the pressure to discuss the stability of the system: As usual, it is obtained from the thermodynamic relation

$$
P = \frac{\partial E}{\partial V}. \qquad (22)
$$

B. Self-energy in the ladder approximation

To obtain the total energy of the system we calculate the fermion self-energy Σ^F in ladder approximation. Here the self-energy is expressed in terms of the two-particle scattering amplitude, $\Gamma(\mathbf{q},\mathbf{q}',P)$, in the medium of the Bose-Fermi mixture as shown in fig.1. The interaction energy is accordingly calculated up to the lowest two-particle correlation diagram, fig.2, in the spirit of the hole-line expansion [14].

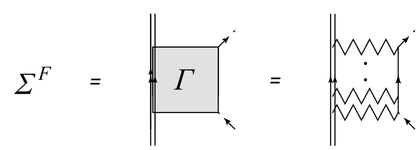

FIG. 1: Fermion self-energy in ladder approximation. The double solid lines represent fermion propagation, while single solid line represents noncondensed free boson propagation. The arrows denote condensed bosons and are associated with the factor $\sqrt{n_0}$. The zigzag lines represent the boson-fermion interaction g_{bf}.

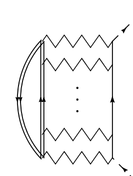

FIG. 2: Ladder diagram contribution to the interaction energy. The downward double solid line indicates a hole propagation. Otherwise as in fig.1

The scattering amplitude Γ in the present model obeys the integral equation [9, 11, 14, 15],

$$\Gamma(\mathbf{q},\mathbf{q}',P)=g_{bf}+ig_{bf}\int\frac{d^4k}{(2\pi)^4}G_0^f\left(\frac{m_f}{m_f+m_b}P+k\right)G_0^b\left(\frac{m_b}{m_f+m_b}P-k\right)\Gamma(\mathbf{k},\mathbf{q}',P), \tag{23}$$

where P denotes a four-momentum of the center-of-mass motion of the interacting particles, while $\mathbf{q}$ and $\mathbf{q}'$ are the relative three-momentum in the final and intial states. The boson and fermion free Green's functions in medium are given by

$$G_0^f(p)=\frac{\theta\left(|\mathbf{p}|-k_F\right)}{p_0-\epsilon_{\mathbf{p}}^f+i\eta}+\frac{\theta\left(k_F-|\mathbf{p}|\right)}{p_0-\epsilon_{\mathbf{p}}^f-i\eta}, \tag{24}$$

$$G_0^b(p)=\frac{1}{p_0-\epsilon_{\mathbf{p}}^b+\mu_b+i\eta}, \tag{25}$$

where the Fermi momentum k_F is fixed by the fermion density N_f/V. After the integration over k_0, Eq.(23) becomes

$$\Gamma(\mathbf{q},\mathbf{q}',P)=g_{bf}+g_{bf}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\theta\left(\left|\tilde{\mathbf{P}}_f+\mathbf{k}\right|-k_F\right)}{P_0-\epsilon_{\tilde{\mathbf{P}}_f+\mathbf{k}}^f-\epsilon_{\tilde{\mathbf{P}}_b-\mathbf{k}}^b+\mu_b+i\eta}\Gamma(\mathbf{k},\mathbf{q}',P) \tag{26}$$

with $\tilde{\mathbf{P}}_f=m_f/(m_f+m_b)\mathbf{P}$ and $\tilde{\mathbf{P}}_b=m_b/(m_f+m_b)\mathbf{P}$. We dropped the hole propagation part in accordance with the present approximation. With respect to the T-matrix equation in [9], we notice that there the phase space factor in (26) is replaced by $\theta\left(\left|\tilde{\mathbf{P}}_f+\mathbf{k}\right|-k_F\right)\to\theta\left(\left|\tilde{\mathbf{P}}_f+\mathbf{k}\right|-k_F\right)+N_0$. This is natural, because in [9] the shift operation (2) for the bosons has not been performed and therefore the free boson occupancy N_0 appears additionally. The two formulations are, however, essentially equivalent. From the structure of Eq.(26), one easily finds that Γ depends only on the variable P, and we hereafter write simply $\Gamma(P)$. One also finds that the integral in Eq.(26) requires a momentum cutoff, which originates from the use of the zero-range interaction. We can remedy this shortcoming by employing the observable S-wave scattering length a, instead of the pseudopotential coupling constant g_{bf}. We perform this renormalization following the procedure adopted in [12] (see also, [16]), slightly different from the one in [11]. The S-wave scattering length is related to the two-particle scattering amplitude Γ_0 in vacuum by the relation

$$\Gamma_0(\mathbf{q}=\mathbf{q}'=P=0)=\frac{2\pi a}{\nu}, \tag{27}$$

where ν is the reduced mass, and Γ_0 obeys the equation similar to Eq.(23) with G_0 replaced with the free Green's function in vacuum. By solving the equation for Γ_0 one obtains

$$\frac{2\pi a}{\nu}=\frac{g_{bf}}{1+g_{bf}\displaystyle\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\epsilon_{\mathbf{k}}^f+\epsilon_{\mathbf{k}}^b}}, \tag{28}$$

where the integral in the denominator involves again the implicit momentum cutoff. Now one may combine the above expression with Eq.(26), and eliminate g_{bf} in favor of the scattering length a, and finally obtains

$$\Gamma(P)=\frac{2\pi a}{\nu}\left[1-\frac{2\pi a}{\nu}I\left(P_0,|\mathbf{P}|\right)\right]^{-1} \tag{29}$$

with

$$I\left(P_0,|\mathbf{P}|\right)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left\{\frac{\theta\left(\left|\tilde{\mathbf{P}}_f+\mathbf{k}\right|-k_F\right)}{P_0-\epsilon_{\tilde{\mathbf{P}}_f+\mathbf{k}}^f-\epsilon_{\tilde{\mathbf{P}}_b-\mathbf{k}}^b+\mu_b+i\eta}+\frac{1}{\epsilon_{\mathbf{k}}^f+\epsilon_{\mathbf{k}}^b}\right\}. \tag{30}$$

Since the integral in the denominator is convergent at large $|\mathbf{k}|$, we can let the momentum cutoff go to infinity.

The expression (29) involves all orders in the scattering length and allows us to formally take the unitarity limit $|a| \to \infty$ in the following section. This limit has been studied for two-component Fermi systems in relation with the BEC to BCS crossover phenomenon. If a similar phenomenon is expected or not for Bose-Fermi pairs will be studied in the next section.

Using above vertex function, we can calculate the proper self-energies for the fermion and the boson as

$$\Sigma^f(p) = n_0\Gamma(p) \tag{31}$$

$$\Sigma^b(p) = -i\int \frac{d^4p'}{(2\pi)^4} G_0^f(p')\Gamma(p+p'). \tag{32}$$

Expression (26) implies that $\Gamma(p+p')$ is analytic in the upper half p_0' plane, and Eq.(32) reduces to

$$\Sigma^b(p) = \int \frac{d^3\mathbf{p}'}{(2\pi)^3}\theta(k_F - |\mathbf{p}'|)\Gamma(p+p'). \tag{33}$$

with $p_0' = \epsilon^f_{\mathbf{p}'}$. This shows that $\Sigma^b(p)$, and hence, also $G^b(p)$ is analytic in the upper half p_0 plane. One then finds from Eq.(20) that

$$N_b = N_0. \tag{34}$$

III. RESULTS FOR TOTAL ENERGY AND PRESSURE

We calculate the energy of the system in the leading order of the hole-line expansion, that is we replace Green's functions in Eq.(17) with the free one $G_0^{f,b}$ in Eq.(25), and obtain

$$\begin{aligned} E &\sim -iV\int \frac{d^4q}{(2\pi)^4}\left(\epsilon^f_{\mathbf{q}} + \Sigma^f(q)\right) G_0^f(q) e^{iq_0\eta} \\ &= E_0 N_f + N_0\int \frac{d^3\mathbf{p}}{(2\pi)^3}\theta(k_F - |\mathbf{p}|)\Gamma(\epsilon^f_{\mathbf{p}}, \mathbf{p}), \end{aligned} \tag{35}$$

where, $E_0 = 3/5E_F$ and E_F is the Fermi energy. Within the same approximations, the thermodynamic potential at zero temperature is given by

$$\Omega = E_0N_f + N_0\int \frac{d^3\mathbf{p}}{(2\pi)^3}\theta(k_F - |\mathbf{p}|)\Gamma(\epsilon^f_{\mathbf{p}}, \mathbf{p}) - \mu_b N_0. \tag{36}$$

Thus, the equilibrium condition (21) for Ω leads to the integral equation for μ_b,

$$\mu_b = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\theta(k_F - |\mathbf{p}|)\Gamma(\epsilon^f_{\mathbf{p}}, \mathbf{p}), \tag{37}$$

where Γ depends also on μ_b. The total energy of the system is then finally given by

$$E = E_0N_f + \mu_b N_b. \tag{38}$$

Details of the calculation and the analytic expression for Γ are given in the appendix.

We may rewrite Eq.(37) in a scaled form as

$$\tilde{\mu}_b = 2\left(1 + \frac{1}{\zeta}\right)\int_0^1 d\tilde{p}\,\tilde{p}^2\tilde{\Gamma}(\tilde{p}, \tilde{\mu}_b, \zeta), \tag{39}$$

where we introduced tilde (dimensionless) quantities through $\Gamma(p, \mu_b) = 2\pi^2/\nu k_F \tilde{\Gamma}(\tilde{p}, \tilde{\mu}_b)$, $\tilde{a} = k_F a$, $\tilde{p} = |\mathbf{p}|/k_F$, $\tilde{\mu}_b = \mu_b/E_F$. The expression shows that the scaled chemical potential $\tilde{\mu}_b$ depends only on the mass ratio $\zeta = m_b/m_f$ and the dimensionless scattering length $\tilde{a} = k_F a$. We solved Eq.(39) for $\tilde{\mu}_b$ numerically as a function of the boson-fermion mass ratio ζ for different values of the interaction strength represented by $\tilde{a}$. In terms of the scaled quantities, the ground state energy per particle is expressed from Eq.(38) as

$$\frac{E}{N_f} = \frac{3}{5}E_F(1+\beta), \tag{40}$$

where the dimensionless parameter β is given by

$$\beta = \frac{5}{3}\tilde{\mu}_b\frac{N_b}{N_f}. \tag{41}$$

We first show the results for energy and pressure in the unitarity limit, $|a| \to \infty$. In this limit, assuming S-wave scattering and neglecting effective range, we are left with only one length scale, k_F^{-1}, or $n_f^{-1/3}$ in terms of the density $n_f = N_f/V$ [13, 17] for a given mass ratio ζ. Note that the chemical potential $\tilde{\mu}_b$ has no k_F dependence in the unitarity limit, and the parameter β depends only on the number ratio N_b/N_f. Thus the ground state energy per particle, Eq.(40), is proportional to E_F, and the dependence on the parameters are all absorbed in a simple multiplicative factor $(1+\beta)$.

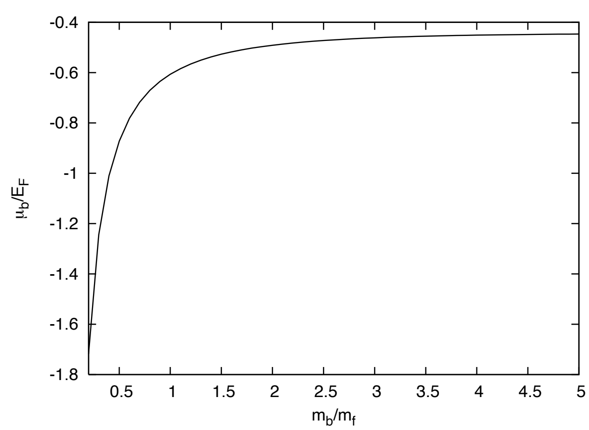

FIG. 3: Scaled chemical potential $\tilde{\mu}_b = \mu_b/E_F$ as a function of the boson-fermion mass ratio $\zeta = m_b/m_f$ in the unitarity limit.

We show in fig.3 the chemical potential μ_b and in fig.4 the beta parameter as functions of the mass ratio

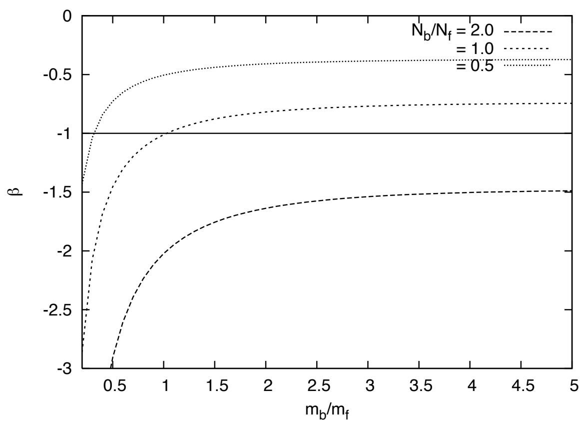

FIG. 4: β parameter as a function of the boson-fermion mass ratio in the unitarity limit for $N_b/N_f = 2.0, 1.0, 0.5$ from top to bottom.

$\zeta = m_b/m_f$, both in the unitarity limit. Note that the results are independent of the magnitude of the individual mass parameters as we show dimensionless quantities scaled with E_F. Figure 3 shows that the boson chemical potential is always negative. This fact reflects the attractive boson-fermion interaction in the unitarity limit, in accordance with Eq.(28) which implies negative g_{bf}. The behavior of β in fig.4 simply follows the one of the chemical potential. The results suggest that the attractive interaction becomes more effective for small values of the mass ratio ζ, and the effect is greatly enhanced as the particle number ratio N_b/N_f becomes larger, that is as the number of bosons increases with respect to the fermions. The dependence on ζ may partly be understood by noting that the relative phase space available for the intermediate states in the two-body scattering in the mixture will be larger for small ζ, i.e., for a relatively larger m_f, because of the lower Fermi energy and higher level density.

We next consider the pressure to study the stability of the system. Since the total energy takes a universal form and the β parameter has no volume dependence in the unitarity limit, the pressure is simply given by

$$P = \frac{\partial E}{\partial V} = \frac{2}{5}\frac{N_f}{V}E_F(1+\beta) \qquad \text{(unitarity limit)}. \qquad (42)$$

For $\beta < -1$ the pressure becomes negative, and the system collapses. This happens especially for larger values of N_b/N_f, where the pressure becomes always negative irrespective of the mass ratio ζ. In actual experiments, e.g., for the ^{40}K-^{87}Rb mixture which has $\zeta \sim 2.3$, the number ratio is typically $\mathcal{O}(1) \sim \mathcal{O}(10^3)$ and the system would collapse in the unitarity limit. This is not in contradiction to recent experimental results [6].

Another feature in the unitarity limit seen from fig.3 and fig.4 is that the boson chemical potential and the ground state energy saturate when the mass ratio becomes large. It is natural that the bosonic degree of freedom gets frozen and a universal fermionic description appears in this case, since the fermionic effects on the bosons would become negligible, and the bosons would act as a static external field for fermions. This behavior at large ζ also has been discussed in [11].

We now study the case with an arbitrary value of the scattering length. First, we show the chemical potential μ_b as a function of $(k_F a)^{-1}$ in fig.5, where the mass ratio ζ and N_b/N_f are set to 1. Then, we show the energy and pressure as a function of $(k_F a)^{-1}$ in figs.6, 7 and 8, for the mass ratio $\zeta = 0.8, 1.0, 1.2$ with different values of $N_b/N_f = 0.5, 1.0, 2.0$. For $a \neq 0$, the pressure is given by

$$P = \frac{2}{5}\frac{N_f}{V}E_F\left[1+\frac{5}{3}\left\{2\left(1+\frac{1}{\zeta}\right)\frac{N_b}{N_f}\int_0^1 d\tilde{p}\,\tilde{p}^2\left(\tilde{\Gamma}(\tilde{p},\tilde{\mu}_b,\zeta)-\frac{\pi}{\tilde{a}}\tilde{\Gamma}^2(\tilde{p},\tilde{\mu}_b,\zeta)\right)\right\}\right], \qquad (43)$$

where the term dependent on Γ^2 reflects that the chemical potential, and hence the β parameter, depends on V. From the results on energies we see that the system becomes more attractive at smaller values of the mass ratio ζ as in the unitarity limit, although the effect is not large in this parameter range. We find a strong increase of the attraction as the parameter $(k_F a)^{-1}$ passes through zero, the unitarity limit, from negative to positive. This is in accord with a naive picture where the positive values of the scattering length imply a newly formed bound state. One should however note that even in the present approximation the effects of the medium modify the two-body scattering amplitude, and a simple picture of independent bound pairs does not hold in general. Turning now to the pressure, a comparison of Eq.(43) with the corresponding expression (42) in the unitarity limit shows that the strong attraction for $a > 0$ comes from the large negative values of Γ as well as from the coherence of the two terms in the integrand.

IV. BOSE-FERMI PAIR CORRELATION

Results of the previous section indicate that the strong attraction in the mixture will show up especially for positive a, which may eventually lead to a collapse of the

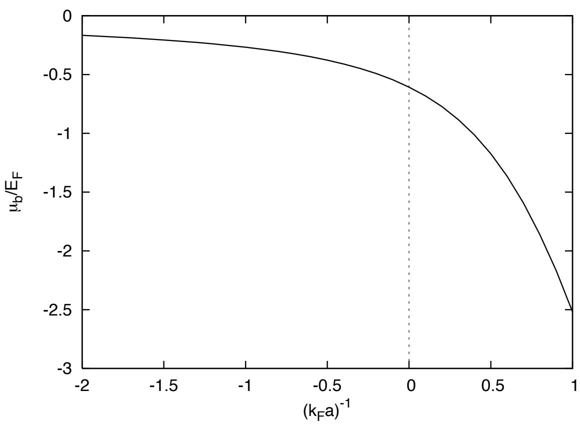

FIG. 5: Scaled chemical potential $\tilde{\mu}_b = \mu_b/E_F$ as a function of $(k_F a)^{-1}$ for $m_b/m_f = N_b/N_f = 1$.

system. We now consider another scenario for the attractively interacting mixture, the possibility of a Bose-Fermi pair formation[9, 10]. For this purpose we study in this section the behavior of the pole of the Bose-Fermi scattering amplitude $\Gamma(P)$ in the mixture.

In unpolarized (or two-component) Fermi systems, an infinitesimal attraction around the Fermi surface leads to formation of Cooper pairs with a center-of-mass (CM) momentum $\mathbf{P} = \mathbf{0}$, causing a transition to the BCS state. In a Bose-Fermi mixture, on the other hand, the difference in the momentum distribution of the two particles and the fermionic character of the Bose-Fermi pair, in particular, predict quite a different scenario for the formation of the pairs in the mixture. It requires consideration of the balance of the kinetic energies of different kinds of particles and the magnitude of the attractive interaction.

A. Preliminary considerations

Let us give a brief picture on the formation of Bose-Fermi pairs in the mixture. Following the idea of [9], we may take a Bose-Fermi mixture with $N_b = N_f$, focusing only one pair of a boson and a fermion with CM momentum $\mathbf{P}$, putting other particles as a free background. The Hamiltonian in this system may be written as

$$H = \frac{\mathbf{p}_b^2}{2m_b} + \frac{\mathbf{p}_f^2}{2m_f} + V_{bf} + H_{\rm bg}, \quad (\mathbf{p}_b + \mathbf{p}_f = \mathbf{P}) \quad (44)$$

where we explicitly write the kinetic energy and the interaction for the two particles, while the background Hamiltonian $H_{\rm bg}$ acts only to impose a Pauli-principle constraint. We neglect here the effect of the boson chemical potential for simplicity. If the effect of V_{bf} on the pair were negligible, the energy of the Bose-Fermi pair with CM momentum $\mathbf{P}$ would be simply

$$\epsilon_{\rm free}(|\mathbf{P}|) = \frac{\mathbf{P}^2}{2m_f}, \quad (45)$$

which has been called *free branch* in [9], since the boson will remain at $\mathbf{p}_b = 0$ in the condensate. When the effect of the interaction becomes important, one may rewrite Eq.(44) as

$$H = \frac{\mathbf{P^2}}{2(m_b + m_f)} + H_{\rm rel} + H_{\rm bg}, \quad (46)$$

where the interaction V_{bf} is contained in the Hamiltonian $H_{\rm rel}$ of the relative motion. By replacing the latter with its eigenvalue $E_{\rm rel}$ (the effect of the medium may be included here), one obtains a different dispersion curve for the Bose-Fermi pair, the *collective branch*, which is given by

$$\epsilon_{\rm coll}(|\mathbf{P}|) = \frac{\mathbf{P^2}}{2(m_b + m_f)} + E_{\rm rel}. \quad (47)$$

The calculation for the model with a separable interaction in [9] shows that the two dispersion curves, Eqs.(45) and (47), coexist except for the region of CM momentum $\mathbf{P}_c$ given by a solution of the equation

$$\epsilon_{\rm free}(|\mathbf{P}_c|) = \epsilon_{\rm coll}(|\mathbf{P}_c|). \quad (48)$$

There is a mixture of the two branches around $|\mathbf{P}| = P_c$, and the dispersion curve of the pair deviates from $\epsilon_{\rm free}$ and $\epsilon_{\rm coll}$. For a sufficiently attractive interaction, the solution of Eq.(48) satisfies the condition $|\mathbf{P}_c| \le k_F$, which occurs for

$$E_{\rm rel} \le \frac{m_b}{m_b + m_f} E_F. \quad (49)$$

One may thus expect that the system will lower the energy by converting the free Bose-Fermi particles in the range $|\mathbf{P}_c| \le |\mathbf{P}| \le k_F$ into *collective* pairs. (Note that we neglect the interaction of the pairs in this simple argument.) If V_{bf} is so strong as to allow for a bound state of the pair, i.e., $E_{\rm rel} < 0$, all the free bosons and fermions would be replaced with the bound B-F pairs, and the new Fermi sea of the pairs will be formed.

This picture may be modified even in this simple model, however, for a system with many Bose-Fermi pairs. As the pairs can occupy low-momentum states having lower energies without changing the total momentum of the system, the formation of the collective pairs may start if the condition

$$E_{\rm rel} = \epsilon_{\rm coll}(|\mathbf{P}| = 0) \le \epsilon_{\rm free}(k_F) = E_F \quad (50)$$

is satisfied, even before the condition (49). Similarly, the formation of the Fermi sea of the pair will be completed only when

$$\epsilon_{\rm coll}(k_F) \le \epsilon_{\rm free}(|\mathbf{P}| = 0) = 0 \quad (51)$$

is satisfied.

We note that the collective branch (47) may suffer a Laudau type damping for sufficiently large $E_{\rm rel}$. This is

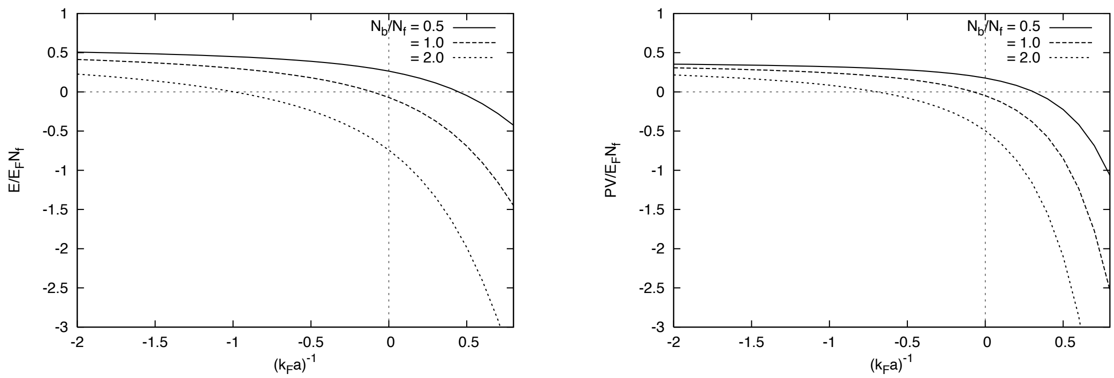

FIG. 6: Scaled energy $E/E_F N_f$ (left panel) and pressure $PV/E_F N_f$ (right panel) as a function of $(k_F a)^{-1}$ for $m_b/m_f = 0.8$. Three curves corresponds to $N_b/N_f = 0.5, 1.0, 2.0$ from top to bottom. The vertical line indicates the unitarity limit $a = \infty$.

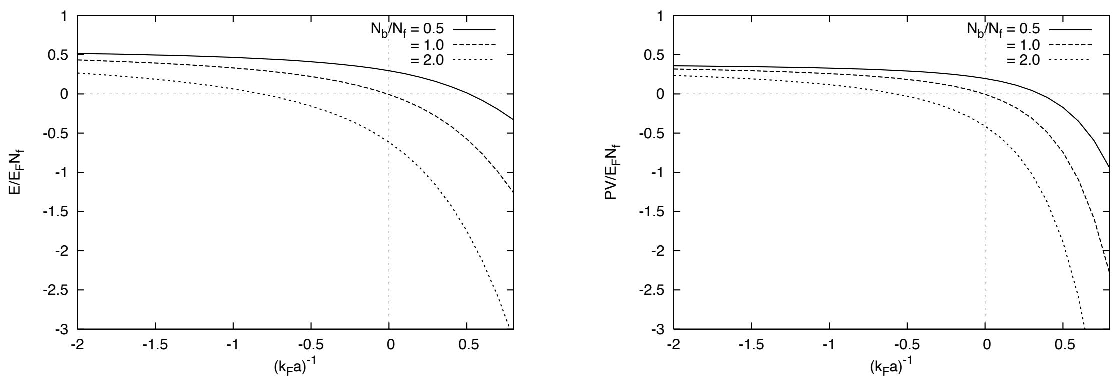

FIG. 7: Same as fig.6, but for $m_b/m_f = 1.0$.

because the free Bose-Fermi mixture with Fermi momentum k_F has a continuum of the boson-fermion excitation with momentum $\mathbf{P} = \mathbf{p}_b + \mathbf{p}_f$ which starts at the energy [9]

$$\epsilon_{\rm th}(|\mathbf{P}|) = \frac{(|\mathbf{P}| - k_F)^2}{2m_b} + E_F. \tag{52}$$

This implies that the collective branch remains undamped only when

$$\epsilon_{\rm coll}(|\mathbf{P}|) \leq \epsilon_{\rm th}(|\mathbf{P}|), \tag{53}$$

a condition which is approximately realized for the pole of the boson-fermion scattering amplitude as shown below.

B. Pole behavior of the two-particle scattering amplitude

Now we study the behavior of the pole of $\Gamma(P)$. A first study concerns the pole condition. That is

$$\frac{\nu}{2\pi a} = I\left(P_0, |\mathbf{P}|\right). \tag{54}$$

For $|\mathbf{P}| = 0$, we show in fig.9 the right hand side of (54) as a function of P_0 for $m_b/m_f = 1$. We see the development of a logarithmic divergency as P_0 aproaches $k_F^2/2\nu - \mu_B$. This stems from the fact that above dispersion integral has exactly the same structure as the one encountered in the problem of Cooper for a fermion pair in a Fermi-sea. We want, however, to point out that the pole corresponds to a composite fermion what has important consequences for the physics. Nevertheless the fact is there that a stable collective B-F pair developes for any infinitesimal attraction, i.e, even in the limit $a \to -0$ quite in analogy to the original Cooper pole [14].

Let us now discuss the collective pole contained in $\Gamma(P)$. Be P_0^c the pole of $\Gamma(P)$ with CM momentum $\mathbf{P}$. $P_0^c(|\mathbf{P}|)$ represents the total energy of a boson-fermion pair corresponding to the *collective branch* aside from the chemical potential. One may then define a $|\mathbf{P}|$-dependent binding energy (including total kinetic energy) measured from the last filled free Bose-Fermi pair by

$$\Delta_{pair}(|\mathbf{P}|) = \epsilon_{\rm free}(k_F) - \mu_b - P_0^c(|\mathbf{P}|), \tag{55}$$

see fig.10. Positive value of $\Delta_{pair}(|\mathbf{P}|)$ would signal a formation of the Bose-Fermi pair.

We solved $P_0^c(|\mathbf{P}|)$ numerically. Here we include chemical potential μ_b, and show $\Delta_{pair}(|\mathbf{P}|)$ in units of E_F for $\zeta = 0.8, 1.0, 1.2$ in figs.11, 12 and 13. The number ratio is fixed at $N_b/N_f = 1$. Left panel of each figure

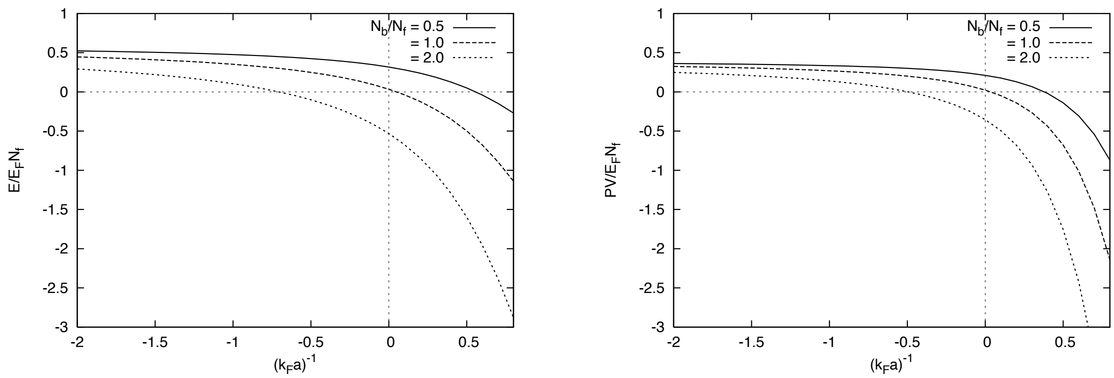

FIG. 8: Same as fig.6, but for $m_b/m_f = 1.2$.

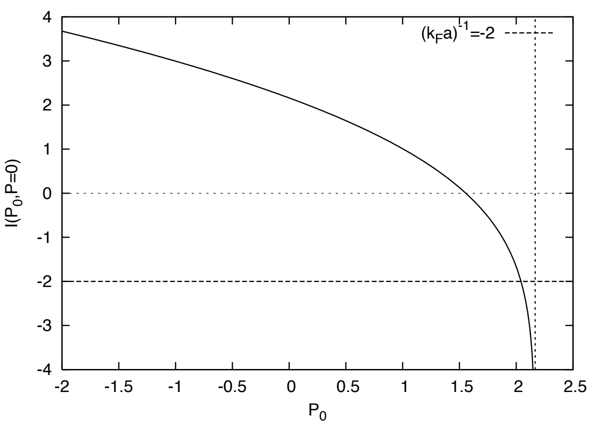

FIG. 9: The behavior of the right hand side of eq.(54). The horizontal line is the left hand side. Here, $m_b/m_f = 1$. The vertical line shows the the position of the logarithmic divergence.

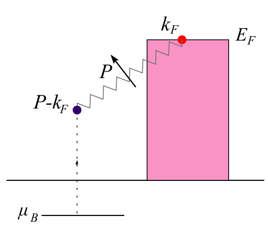

FIG. 10: Rough sketch of the formation of the Bose-Fermi pair with CM momentum $\mathbf{P}$. The dots represent a boson (left) and a fermion(right). The shaded box indicates a fermi sphere.

shows results for several values of $1/k_F a$. One finds that each line extends up to a maximum value of $|\mathbf{P}|$, the energy of which corresponds to $\epsilon_{\text{th}}(|\mathbf{P}|)$ in Eq.(52), i.e., the point where the pole hits the continuum and obtains a finite lifetime due to the non-vanishing imaginary part. In the right panels of these figures, we show the dispersion curves at the threshold values of $1/k_F a$, where $\Delta_{pair}(|\mathbf{P}|)$ turns from negative to positive for the first time.

We see from these figures that the effective binding increases as the interaction becomes more attractive, and eventually leads to a formation of the Bose-Fermi pair. The threshold value of $1/k_F a$ is lower for a smaller value of $\zeta = m_b/m_f$ in agreement with the result for the total energy. We find also that Δ_{pair} is always negative in the unitarity limit, suggesting that the Bose-Fermi pair may not be formed in this limit.

A peculiar feature seen from the figures is the non-monotonic dependence of Δ_{pair} against CM momentum $|\mathbf{P}|$, which is in contrast to the P^2 dependence of Eq.(47) in the simple picture. This is particularly apparent for small values of $\zeta = m_b/m_f$. Missing in the simple model is the P-dependence of the pair binding energy E_{rel} which should be present due, e.g., to the phase space available for the two-body scattering in medium. The medium effect becomes stronger for larger (positive) values of $(k_F a)^{-1}$, especially for small ζ as seen from the figure. It may be mentioned in addition, that the results in the next subsection suggest a strong interplay of the collective and free branches for the Fermion dispersion relation.

C. Fermion dispersion and level crossing

Let us now investigate the pole structure of the single particle Green's function G^f of (13). The pole condition reads,

$$E_{\mathbf{p}} = \epsilon^f_{\mathbf{p}} + \Sigma^f(E_{\mathbf{p}}, \mathbf{p}). \tag{56}$$

In order to be consistent with our publication in [9], we here neglect the chemical potential μ_b in $\Gamma(P)$ of (29). This means that we replace in $\Gamma(P)$ the Boson and Fermion propagators by totally free ones with the kinetic energies of boson and fermion and $\mu_b = 0$ which is the uncorrelated value.

With the definition of Σ^f in (13) with $\mu_b = 0$ and the expression Γ of (29), we easily can investigate the poles of G^f for various system parameters. We first consider cases, like in [9], where $N_b \ll N_f$.

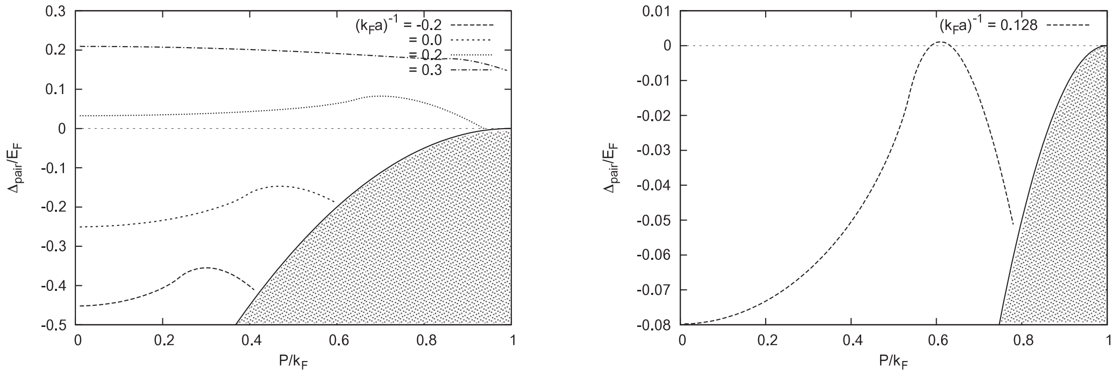

FIG. 11: $\Delta_{pair}(P)$ as a function of center of mass momentum $|\mathbf{P}|$ for various $(k_F a)^{-1}$ with $m_b/m_f = 0.8$ and $N_b/N_f = 1$. The shaded area shows the contiuum of the excitation which starts at $\epsilon_{\rm th}$, Eq.(52). The right panel shows the enlarged figure at the threshold value of $(k_F a){-}1$ (see text). Note the scale difference in the vertical axis between left and right panels.

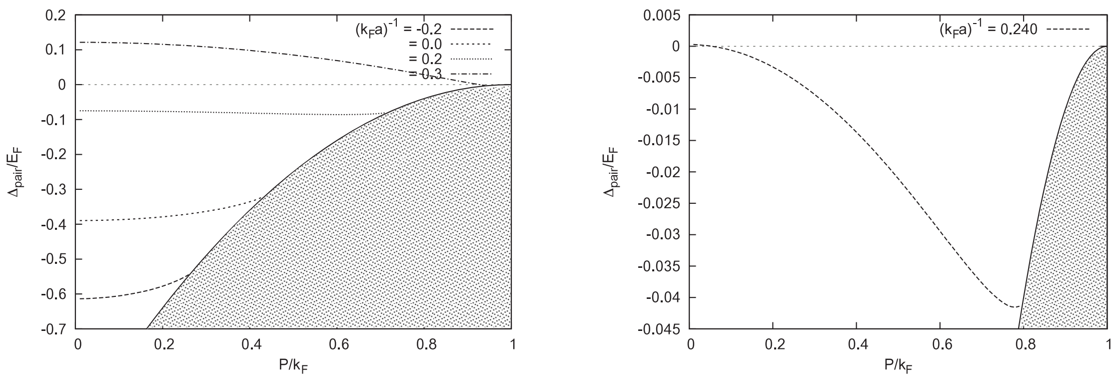

FIG. 12: Same as Fig.11 with $m_b/m_f = 1.0$.

We expect two branches: one which corresponds in week coupling to $E_{\mathbf{p}} \sim \epsilon^f_{\mathbf{p}}$ and one which corresponds to the collective branch, i.e. the pole contained in $\Gamma(p)$.

In fig.14, we show on the left panel the case of $N_b/N_f = 1/1000$ and on the right panel the case of $N_b/N_f = 1/100$. From top to botom, we have $(k_F a)^{-1} = -0.2, 0.2, 0.5$. Here mass ratio $m_b/m_f = 1$. The dotted area is the region where the imaginary part of Σ^f is different from zero. As in our ealier work [9], we see an avoided crossing of the two branches at some finite value of p/k_F. We also see that the interaction between the two branches becomes stronger as N_b/N_f increases.

In fig.15, we show the case $(k_f a)^{-1} = 0.5$ and $N_b/N_f = 1/10$ for left panel and $N_b/N_f = 1$ for right panel. We see that no crossing feature is visible any longer and the two branches probably become completely hybridised. In a future publication we intend to investigate in a systematic way the nature of the two branches as a function of the system parameters.

V. SUMMARY, DISCUSSION AND CONCLUSION

We studied in this paper the static properties of the Bose-Fermi mixture in the lowest order of the hole-line expansion, i.e. in the T-matrix approximation. The interaction parameter is expressed in terms of the scattering length up to infinite order using the renormalization procedure of ref.[12], so as to allow for the calculation around the unitarity limit. The T-matrix approach is a common approximation often used in the past in Fermi and Bose systems [14]. It has, however, not been investigated very much in Bose-Fermi mixtures and, therefore, one has not much experience about its quality in that situation. Recently appeared, however, a study for a one dimensional system [18], from where exact Quantum Monte Carlo results are available for comparison. In ref [19] it is shown that the T-matrix approach yields quite reasonable results also in the Bose-Fermi case, even in a one dimensional case which is probably the worst situation possible.

With this background in mind, we first studied energy and pressure as functions of the inverse scattering length for several choices of the mass ratio $\zeta = m_b/m_f$ and the number ratio N_b/N_f. The energy of the system be-

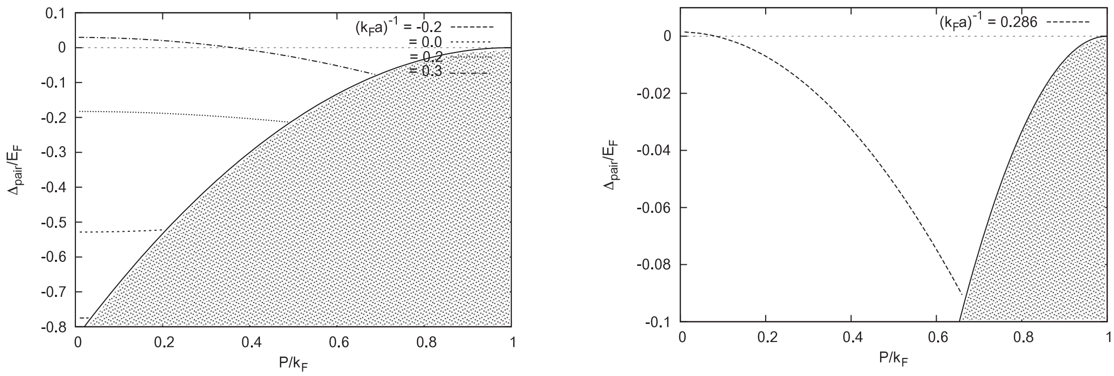

FIG. 13: Same as Fig.11 with $m_b/m_f = 1.2$.

comes strongly attractive as the inverse scattering length changes sign from negative to positive, i.e. around the unitary limit. As one increases the number of bosons with respect to the fermions, arrives a point where the pressure becomes negative, i.e. the system becomes unstable (collapse). The effect is stronger for small values of m_b/m_f. This is not in contradiction with experiments [6].

Next we studied the possibility of stable BF-pairs, as in [9]. We, indeed, also found in the present model that even for infinitesimal BF-attraction a stable BF-mode appears, reminiscent of the Cooper pole in a two component Fermi gas, since in both cases its origin stems from the presence of a sharp Fermi surface. However, in the BF case the BF-pair is a composite fermion, whereas in the original Cooper problem, one has a composite fermion pair, i.e. a boson-like cluster. In the latter case, many pairs can well be treated by the usual BCS formalism. On the other hand , the case of many BF-pairs still has to be worked out. This shall be done in future work. But we here studied the BF pair formation by observing its binding energy measured from the last filled free BF pair. For some finite values of the attractive interaction, there occurs the formation of stable BF pairs. For $\zeta \geq 1$ the pair shows a standard dispersion of a quasiparticle, while for $\zeta = 0.8$ the pair with finite center-of-mass momenta feel stronger attraction. This effect is not clearly seen in the energy or pressure, where the singular effect may have been averaged out.

Up to this point we assumed an ideal case where the Bose-Fermi interaction is dominant, while the Bose-Bose interaction was neglected. To compare with a real atomic gas system, the Bose-Bose interaction cannot be discarded even when the Bose-Fermi interaction is enhanced, e.g., through Feshbach resonances. We checked the effect of the Bose-Bose interaction on the energy of the system up to the first order in $N_b^{1/3} a_{bb}$. We took the BoseBose scattering length a_{bb} in the range $-0.3 \leq k_F a_{bb} \leq 0.3$, and the number ratio N_b/N_f =2.0, 1.0 and 0.5, and repeated the calculation of the β parameter and the pressure [20]. The calculation shows that the effect of the Bose-Bose interaction is small within the adopted parameter values: The β value at the unitarity limit, for instance, deviates less than 10% for $m_b = m_f$ and $N_b = N_f$, and does not change conclusions obtained at $a_{bb} = 0$.

In summary, we suggest that the energy gain in the Bose-Fermi mixture at positive values of a_{bf} is related to the formation of the resonant Bose-Fermi pairs, and that the center-of-mass momenta of the pairs are dependent on the ratio m_b/m_f due to the statistics of the two kinds of the particles.

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APPENDIX: EXPRESSION FOR THE SCATTERING AMPLITUDE

Let us calculate the $I(P_0, |\mathbf{P}|)$ (29) of the scattering amplitude $\Gamma(P)$

$$I(P_0, |\mathbf{P}|)) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left\{ \frac{\theta\left(\left|\tilde{\mathbf{P}}_f + \mathbf{k}\right| - k_F\right)}{P_0 - \epsilon^f_{\tilde{\mathbf{P}}_f+\mathbf{k}} - \epsilon^b_{\tilde{\mathbf{P}}_b-\mathbf{k}} + \mu_b + i\eta} + \frac{1}{\epsilon^f_{\mathbf{k}} + \epsilon^b_{\mathbf{k}}} \right\}. \quad (A.1)$$

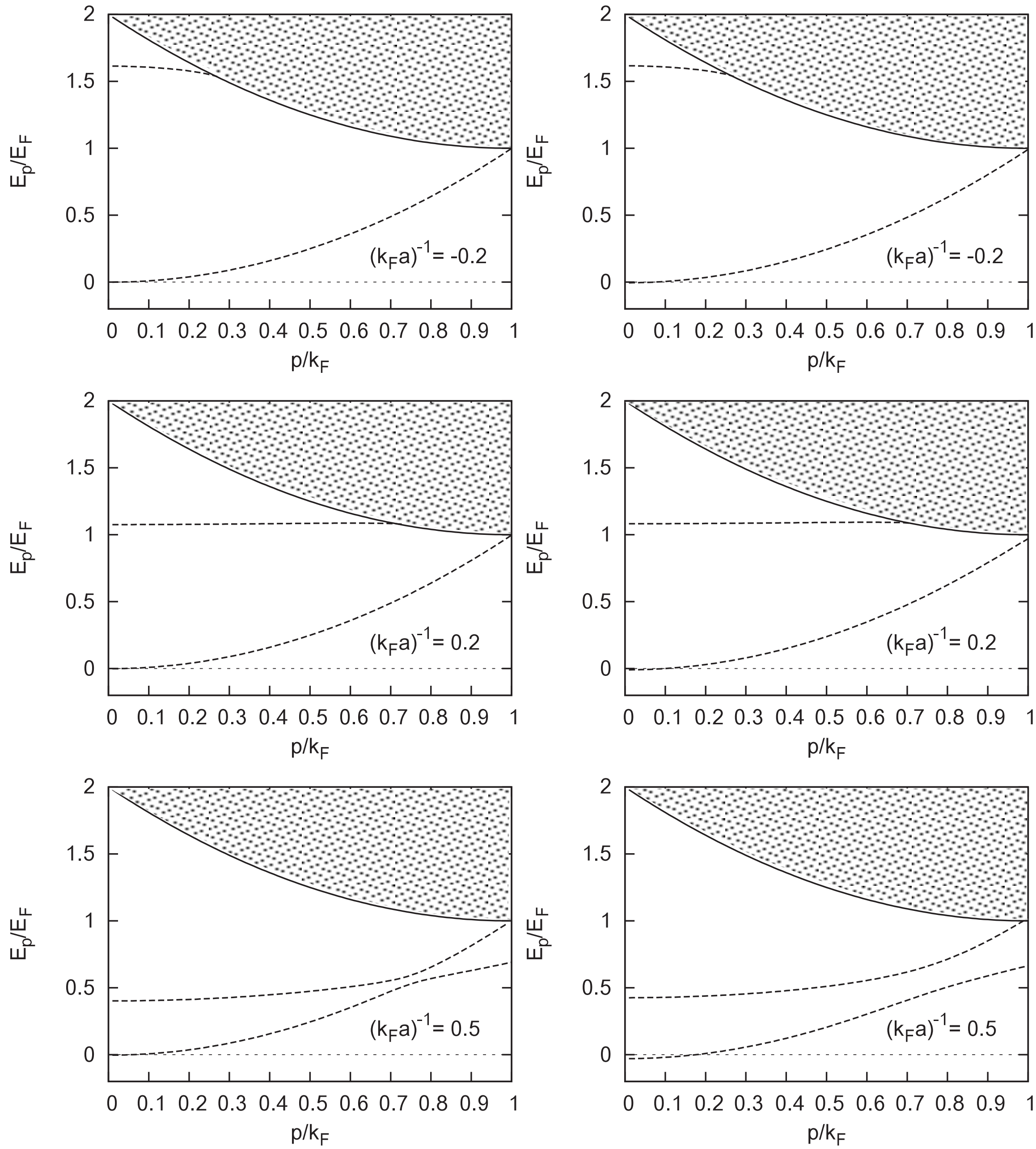

FIG. 14: Fermion dispersion curves. Left row corresponds $N_b/N_f = 0.001$ and right row corresponds $N_b/N_f = 0.01$. From top line to bottom,$(k_F a)^{-1}$ are -0.2,0.2 and 0.5. m_b/m_f is fixed to 1.0.

As we are interested in the real part of the pole of $\Gamma(P)$, we hereafter omit $i\eta$ in the denominator. Each term in the integrand shows an ultraviolet divergence, and we formally introduce a cutoff Λ which will be taken to infinity later.

$$I_1(P_0,|\mathbf{P}|) = \frac{1}{(2\pi)^2}\lim_{\Lambda\to\infty}\int_{k_F}^{\Lambda} dk\ \left[\left(-\frac{m_b}{|\mathbf{P}|}\right) k \ln\left|\frac{\frac{k^2}{2\nu}+\frac{|\mathbf{P}|k}{m_b}-P_0+\frac{\mathbf{P}^2}{2m_b}-\mu_b}{\frac{k^2}{2\nu}-\frac{|\mathbf{P}|k}{m_b}-P_0+\frac{\mathbf{P}^2}{2m_b}-\mu_b}\right| + 2k^2\frac{1}{\frac{k^2}{2\nu}}\right]. \tag{A.2}$$

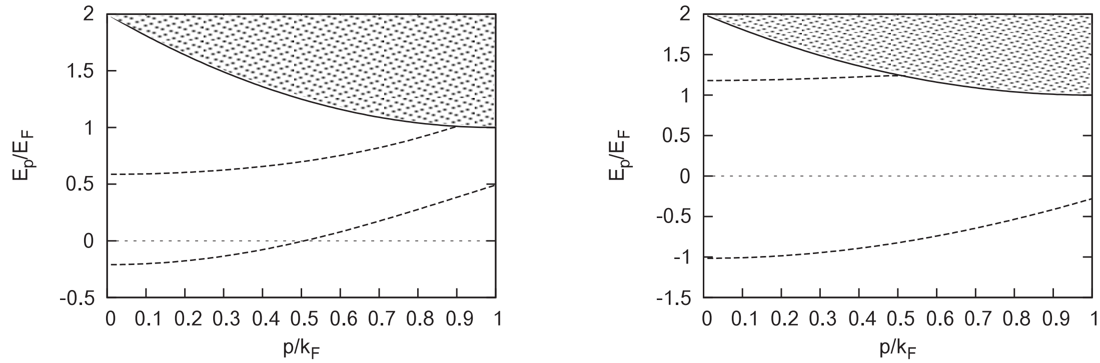

FIG. 15: Fermion dispersion curves. Left panel corresponds $N_b/N_f = 0.1$ and right one corresponds $N_b/N_f = 1.0$. $(k_F a)^{-1}$ are fixed to 0.5. And $m_b/m_f = 1.0$.

The divergent term at $\Lambda \to \infty$ coming from the first term in the integrand is cancelled out by the second term, and we obtain the finite quantit

$$I(P_0.|\mathbf{P}|) = \frac{1}{(2\pi)^2}\left(\frac{m_b}{2|\mathbf{P}|}\right)\left[\left\{k_F^2 - \left(\frac{\nu|\mathbf{P}|}{m_b}\right)^2 - A\right\}\ln\left|\frac{\left(k_F + \frac{\nu|\mathbf{P}|}{m_b}\right)^2 - A}{\left(k_F - \frac{\nu|\mathbf{P}|}{m_b}\right)^2 - A}\right| + \frac{4\nu|\mathbf{P}|}{m_b}k_F - \frac{4\nu|\mathbf{P}|}{m_b}AF(A)\right], \tag{A.3}$$

where

$$F(A) = \begin{cases} \frac{-1}{2\sqrt{A}}\ln\left|\frac{\left(k_F + \frac{\nu|\mathbf{P}|}{m_b} - \sqrt{A}\right)\left(k_F - \frac{\nu|\mathbf{P}|}{m_b} - \sqrt{A}\right)}{\left(k_F + \frac{\nu|\mathbf{P}|}{m_b} + \sqrt{A}\right)\left(k_F - \frac{\nu|\mathbf{P}|}{m_b} + \sqrt{A}\right)}\right| & (A > 0) \\ \frac{2k_F}{k_F^2 - \left(\frac{\nu|\mathbf{P}|}{m_b}\right)^2} & (A = 0) \\ \frac{\pi}{\sqrt{-A}} - \frac{1}{\sqrt{-A}}\arctan\left(\frac{k_F + \frac{\nu|\mathbf{P}|}{m_b}}{\sqrt{-A}}\right) - \frac{1}{\sqrt{-A}}\arctan\left(\frac{k_F - \frac{\nu|\mathbf{P}|}{m_b}}{\sqrt{-A}}\right) & (A < 0) \end{cases} \tag{A.4}$$

with

$$A = \left(\frac{\nu|\mathbf{P}|}{m_b}\right)^2 - 2\nu\left(-P_0 + \frac{\mathbf{P}^2}{2m_b} - \mu_b\right). \tag{A.5}$$

The final expression for $\Gamma(P)$ is given in terms of $I(P)$ by

$$\Gamma(P) = \frac{2\pi a}{\nu}\left[1 - \frac{2\pi a}{\nu}I(P_0, |\mathbf{P}|)\right]^{-1}. \tag{A.6}$$

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