Spring, 2021

Physics 312: Physics of Fluids
Assignment #2

Background Reading

Friday, Feb. 19: Tritton 5.1 - 5.3, 5.5, 6.1, 6.2,
Kundu & Cohen 3.1 - 3.5

Monday, Feb. 22: Kundu & Cohen 2.1 - 2.4, 3.6, 3.7

Wednesday, Feb. 24: Kundu & Cohen 2.5, 2.7 - 2.9

Informal Written Reflection

Due: Thursday, February 25 (8 am)

Same overall approach, format, and goals as before!

Formal Written Assignment

Due: Friday, February 26 (in class)

1. Explain why particle paths, streamlines, and streaklines are identical for *steady flow*. Do this in three steps...

   (a) First, using the definition of steady flow, explain why fluid particles always move along streamlines.

   (b) If a particle is moving along one streamline, can it ever move onto a different streamline? Why or why not?

   (c) Next, using a similar argument, explain why the fluid particles that make up a streaklines are all following the same path.

2. Both of your textbooks note that a flow that is steady in one reference frame may not be steady in another. Read through Kundu and Cohen sections 3.4 and 3.5 and Tritton sections 6.1 and 6.2. Then give a rough interpretation of the flow fields plotted in Kundu and Cohen's figure 3.8. Why, for example, does the velocity point upstream in one frame and downstream in another?

3. Developing a sense of comfort with index notation is an important part of your basic training in fluid dynamics. Indeed, without this notation, we would have had a much harder time deriving and understanding the fluid dynamical expressions of fundamental conservation laws. In this problem, you will get some more practice working with indices...

   Consider each of the following tensor expressions. Which are allowed under the standard rules of index notation? If any of these expressions are not allowed, explain what the problem is.

   (a) $a = b_i c_{ij} d_j$

   (b) $a = b_i c_i + d_j$

   (c) $a_i = \delta_{ij} b_i + c_i$

   (d) $a_k = b_k c + d_i e_{ik}$

   (e) $a_i = b_i + c_{ij} d_{ji} e_i$

4. If A is a second-order tensor, show that the quantity A_{ii} is invariant under rotation of the coordinate axes. This quantity has a special name in linear algebra... Do you recognize it?

   (Hint: Use the second-order transformation rule,

$$A'_{mn} = R_{mi} R_{nj} A_{ij},$$

   and the fact that R is an orthogonal matrix: the inverse of R is equal to its transpose. Show that $A'_{ii} = A_{ii}$.)

(assignment continued on next page...)

5. Show that δ_{ij} is an isotropic tensor. That is, show that rotation of the coordinate axes does not change any of the components of δ_{ij}.

   (Hint: See the hint for previous problem. . .)