8.(3)09 Section 1

September 10, 2021

1 Pendulum with a spring

Consider a pendulum consisting of a massless spring with zero relaxed length with spring constant k and a point mass m at the end moving in two dimensions. Let the length of the spring be r and the angle from vertical be θ. Gravity acts with acceleration g in the $-\hat{z}$ direction.

(a) Newton's second law in Cartesian coordinates

Using Newton's second law, write down the equations of motion for this system using Cartesian coordinates x and z, where $\hat{z}$ is vertically upward in the figure and $\hat{x}$ is to the right in the figure.

(b) Newton's second law in polar coordinates

Using Newton's second law, write down the equations of motion for the system in the polar coordinates r and θ.

(c) Lagrangian in polar coordinates

Write the Lagrangian for the system in terms of the coordinates r and θ and given constants k, m, and g. Use the Euler-Lagrange equations to obtain the equations of motion. Verify that they match those obtained in (b).

(d) Hamiltonian in polar coordinates

Legendre transform the Lagrangian to obtain the Hamiltonian. Use Hamilton's equations to obtain the equations of motion.

(e) Non-zero relaxed spring length

Repeat (a)-(d) assuming that the spring has a relaxed length b.

(f) Three-dimensions

Repeat (a)-(e) allowing the system to move in three dimensions. Promote polar coordinates to spherical coordinates by adding the azimuthal angle φ around the $\hat{z}$ axis (measured, not that it matters, from the $\hat{x}$ axis).