

Measurement-based Quantum Computing & Efficient variational simulation of non-trivial quantum states

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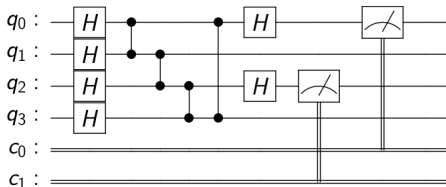


- Measurement-based quantum computing (MBQC)
- Variational simulation of non-trivial quantum state
- Research question: MBQC as an efficient/better simulation?



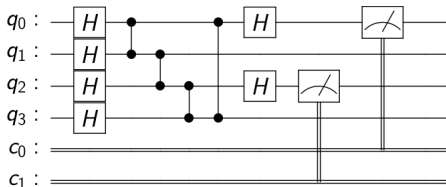
MBQC: One-way quantum computer [RB01]

Conventional quantum circuit models:

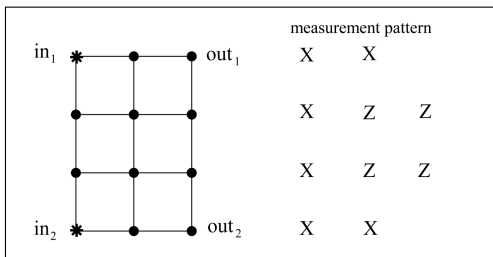


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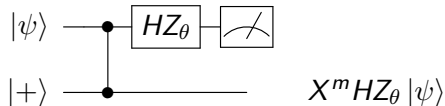


Cluster state: [Joz06]



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Quantum teleportation = Entanglement + Measurement



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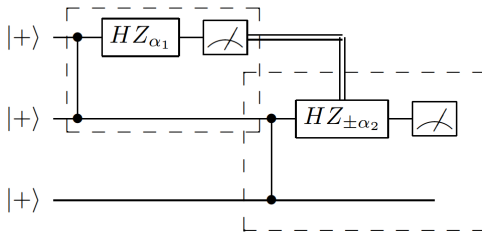
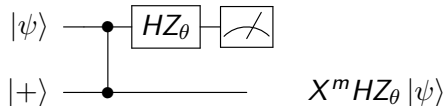


Figure: From [Nie06]

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Universality: Quantum circuit model \equiv Cluster state formulation.



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- The CNOT gate can be implemented in a “T” configuration



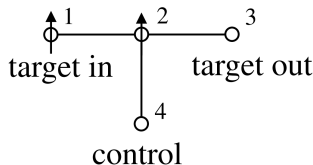
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qubit number	1	2	3	4	5
states	$ \psi\rangle$	$ +\rangle$	$ +\rangle$	$ +\rangle$	$ +\rangle$
entangle with CZ	*	•	•	•	•
measurements	X	$M(-\xi(-1)^{s_1})$	$M(-\eta(-1)^{s_2})$	$M(-\zeta(-1)^{s_3+s_4})$	
outcomes	s_1	s_2	s_3	s_4	

(a) From [Joz06]



(b) From [RB01]

Variational simulation of non-trivial quantum states

QAOA [FGG14]: Quantum approximate optimization algorithm



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- Principle: Quantum adiabatic theorem on $H = H_2 + H_1$



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- Variational ansatz (modified in (2))

$$|\psi(\gamma, \beta)\rangle = \underbrace{e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2}}_{p \text{ layers}} |\psi_1\rangle \quad (1)$$

- $(\gamma, \beta) = (\gamma_p, \dots, \gamma_1, \beta_p, \dots, \beta_1)$
- $|\psi_1\rangle = \text{ground state of } H_1 \text{ (easy to prepare)}$



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- $|\psi_1\rangle = \text{ground state of } H_1 \text{ (easy to prepare)}$
- Cost function:

Overlap: $|\langle\psi_0|\psi(\gamma, \beta)\rangle|^2$, or Energy: $\langle\psi(\gamma, \beta)| H |\psi(\gamma, \beta)\rangle$.

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Variational simulation of non-trivial quantum states

Example: GHZ state $\sim |0\rangle^{\otimes L} + |1\rangle^{\otimes L}$

$$H_{GHZ} = -\sum_{i=1}^L Z_i Z_{i+1} = -\underbrace{\sum_{i=1}^L Z_i Z_{i+1}}_{H_2} - 0 \underbrace{\sum_{i=1}^L X_i}_{H_1}, \quad |GS_{H_1}\rangle = \bigotimes_{i=1}^L |+\rangle$$



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\Rightarrow Perfect fidelity, $p \sim L/2$.

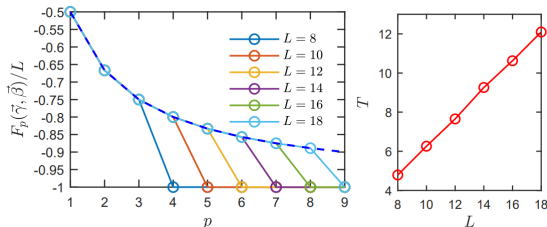


Figure: GHZ state simulation. Fidelity & p vs. L , [HH19]



Variational simulation of TFIM ground state

Example: Transverse field Ising model

$$H := H_2 + H_1 = - \sum_{i=1}^L Z_i Z_{i+1} - g \sum_{i=1}^L X_i$$



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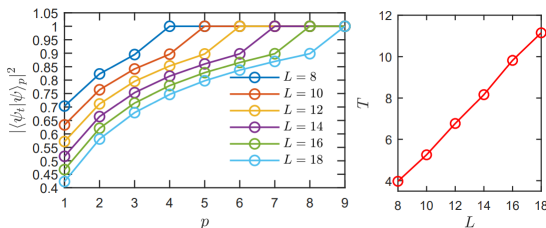


Figure: TFIM state simulation. Fidelity & p vs. L , [HH19]



Variational simulation of TFIM ground state

Limitations of protocol in [HH19]:

- Requires non-local unitaries.
- $p \sim L$.
- MERA construction [Vid08]: $p \sim \log(L)$,
but non-local unitaries required.



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but non-local unitaries required.

\Rightarrow Is a measurement-based QAOA scheme a solution?



Measurement-based QAOA for TFIM

Hamiltonian

$$H := H_2 + H_1 = - \sum_{i=1}^L Z_i Z_{i+1} - g \sum_{i=1}^L X_i$$

QAOA ansatz:

$$|\psi(\gamma, \beta)\rangle = e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2} |\psi_1\rangle$$

MBQC is universal \implies Measurement-based QAOA ansatz is possible.

Ingredients: Z , X -rotations, & $CNOT$.

♣ Scheme can be simplified by changing measurement pattern.



But...

Limitations:

- $p \sim L$, where p is the number of layers of measurements.
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Two possibilities:

- QAOA is insufficient; need a completely new algorithm.
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- QAOA is sufficient, but need better MBQC implementation. \Leftarrow



2nd possibility: How robust is QAOA?

Test QAOA with TFIM without translation invariance:

$$\mathcal{H} = \sum_j Z_j Z_{j+1} + \sum_j g_j X_j \quad (2)$$



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Modified QAOA ansatz (reference (1))

- p layers
- Each layer is parameterized by $(\gamma, \beta)_k = (\gamma_1, \dots, \gamma_L, \beta_1, \dots, \beta_L)_k$.



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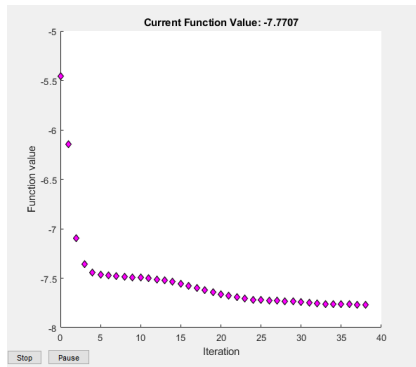
Conjecture

This modified QAOA can target the critical ground state with perfect fidelity for $p = L/2$. In which case, the total number of parameters is L^2 .



2nd possibility: How robust is QAOA?

Conjecture holds for small $N = 4, 6, 8, 10$, even at lower p :



(a) $N = 6$, $p = 3$

Ground state energy

-7.7800e+00

Energy minimum

-7.7707e+00

Optimal angles

7.7305e-02 5.1819e-01

6.7580e-01 9.8622e-01

3.7412e-01 5.4148e-01

5.8479e-01 4.3484e-01

3.6534e-01 4.1250e-01

2.3284e-01 8.5791e-02

Fidelity by Energy: 99.8804%

Time taken : 00:00:18

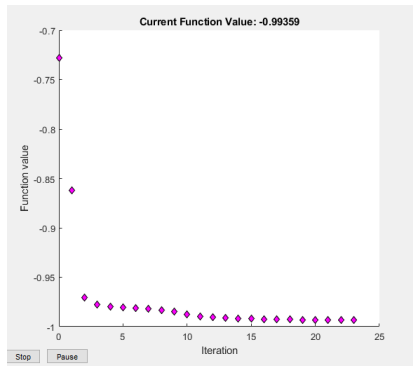
Time in sec: 17.7538

(b) 99.9% fidelity at low iteration count.



2nd possibility: How robust is QAOA?

(Using the overlap as the cost function in this case)



(a) $N = 8, p = 4$

Ground state energy

-1.1520e+01

Optimal angles

2.1895e-01 2.2117e-01

5.6762e-01 5.4324e-01

3.0717e-01 3.4677e-01

4.2687e-01 4.5373e-01

3.5788e-01 2.8814e-01

4.5521e-01 4.6618e-01

3.9238e-01 3.7951e-01

2.6419e-01 2.9289e-01

Fidelity by Overlap: 99.359%

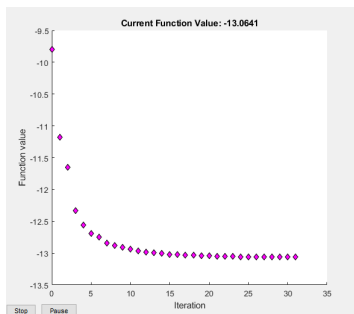
Time taken : 00:04:20

Time in sec: 260.2146

(b) 99.4% fidelity at low iteration count.



2nd possibility: How robust is QAOA?



(a) $N = 10$, $p = 2$ only

Ground state energy
-1.3535e+01

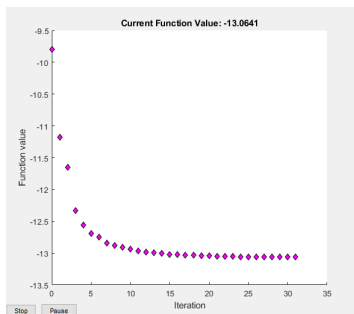
Energy minimum
-1.3064e+01

Optimal angles
5.0595e-01 9.0266e-02
5.9734e-01 3.9420e-01
5.5699e-01 1.8317e-01
3.5894e-01 4.5162e-01

Fidelity by Energy: 96.5225%
Time taken : 00:37:22
Time in sec: 2241.7264

(b) 96% fidelity, not bad for $p = 2$.

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\Rightarrow Small N due to large parameter space ($\sim L^2$) and limitations in computing power and algorithm efficiency.








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- **Next?** Jordan-Wigner transform \implies Reduce problem size

References I

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