Path Integrals

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1 Anyons

(a) In two spatial dimensions we can also write the amplitude for two particles with initial relative position $\mathbf{q}$ to propagate to the relative position $\mathbf{q}'$ in a time T as

$$A(\mathbf{q}', T, \mathbf{q}, 0) = \sum_{\text{paths}} e^{iS[\mathbf{q}_1(t), \mathbf{q}_2(t)]} \tag{1}$$

As in lecture we will assume the particles cannot be at the same point due to some strong repulsive short range interaction. If the particles are identical, what is the configuration space? What is a natural covering space?

(b) As in three spatial dimensions the paths where the initial and final relative positions are the same $\mathbf{q} = \mathbf{q}'$ can be classified into topological classes. These paths can be characterized by the angular displacement $n\pi$ of the relative coordinate where n is an integer. Sketch some of these paths. Argue that two paths with angular displacements $n\pi$ and $m\pi$ are topologically distinct if $n \neq m$.

(c) Argue that in polar coordinates $\mathbf{q} = (q, \theta)$ the path integral in configuration space $A(\mathbf{q}, T, \mathbf{q}, 0) = A(q, \theta, T, q, \theta, 0)$ satisfies

$$A(q', \theta' + \pi, T, q, \theta, 0) = e^{i\phi} A(q', \theta', T, q, \theta, 0)\,. \tag{2}$$

(d) The path integral in configuration space $A(\mathbf{q}, T, \mathbf{q}, 0)$ can be written as a sum of path integrals in the covering space $\bar{A}_n(\mathbf{q}, T, \mathbf{q}, 0)$

$$A(\mathbf{q}, T, \mathbf{q}, 0) = \sum_{n=-\infty}^{\infty} \bar{A}_n(\mathbf{q}, T, \mathbf{q}, 0) \tag{3}$$

where only paths with angular displacement $n\pi$ contribute to the path integral $\bar{A}_n(\mathbf{q}, T, \mathbf{q}, 0)$. If we add phases C_n as in lecture, the amplitude in polar coordinates $\mathbf{q} = (q, \theta)$ becomes

$$A(q, \theta, T, q, \theta, 0) = \sum_{n=-\infty}^{\infty} C_n \bar{A}_n(q, \theta, T, q, \theta, 0)\,. \tag{4}$$

Use equation (2) to show that

$$C_n = e^{i\phi} C_{n-1}\,. \tag{5}$$

(e) Are there any constraints on the phase ϕ?