Electromagnetic Theory

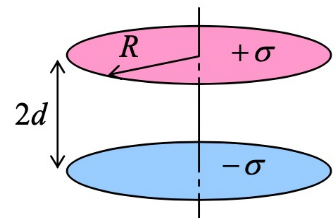
Problem Set #1 (8.311)

[Due Wed. Feb 9, 2022]

Problem 1.1. Calculate the electric field created by a thin, long, straight filament, electrically charged with a constant linear density $\lambda$, using two approaches:

(i) directly from the Coulomb law, and
(ii) using the Gauss law.

Problem 1.11. Two similar thin, circular, coaxial disks of radius $R$, separated by distance $2d$, are uniformly charged with equal and opposite areal densities $\pm\sigma$ – see the figure on the right. Calculate and sketch the distribution of the electrostatic potential and the electric field of the disks along their common axis.




Problem 1.17. Prove the following *reciprocity theorem of electrostatics*:[14] if two spatially-confined charge distributions $\rho_1(\mathbf{r})$ and $\rho_2(\mathbf{r})$ induce, respectively, distributions $\phi_1(\mathbf{r})$ and $\phi_2(\mathbf{r})$ of the electrostatic potential, then

$$\int \rho_1(\mathbf{r})\phi_2(\mathbf{r})d^3r = \int \rho_2(\mathbf{r})\phi_1(\mathbf{r})d^3r .$$

[14] *Hint*: Consider integral $\int \mathbf{E}_1 \cdot \mathbf{E}_2 d^3r$.

Problem 1.19. Calculate the electrostatic energy $U$ of a (generally, thick) spherical shell, with charge $Q$ uniformly distributed through its volume – see the figure on the right. Analyze and interpret the dependence of $U$ on the inner cavity's radius $R_1$, at fixed $Q$ and $R_2$.