Measurement-assisted variational simulation of non-trivial quantum states

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Layout

- Motivation
- Measurement-based quantum computing (MBQC)
- Variational simulation of non-trivial quantum state
- Research question: Measurement-assisted QAOA as an efficient/better simulation?

ΡΙ

Motivation

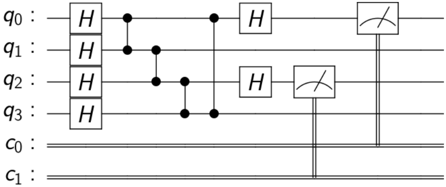
- Efficient variational simulation of nontrivial quantum states with QAOA [HH19] requires $\mathcal{O}(L)$ circuit depth

  $\boxed{\text{Why?}} \implies$ local unitaries spread correlations slowly, making nontrivial states expensive to prepare

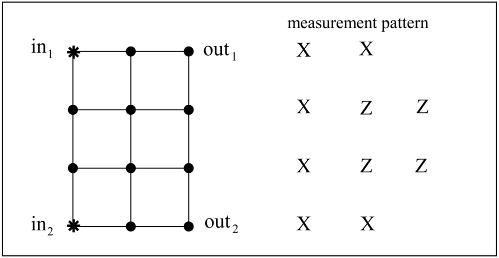
- Entanglement + measurements can rapidly spread correlations (e.g. simulated the GHZ state with $\mathcal{O}(1)$ layer of measurements)

$\implies$ Entanglement + Measurements + Local unitaries = Speedup?

Conventional quantum circuit models:

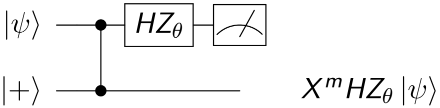


Cluster state: [Joz06]



MBQC: One-way quantum computer

Quantum teleportation = Entanglement + Measurement



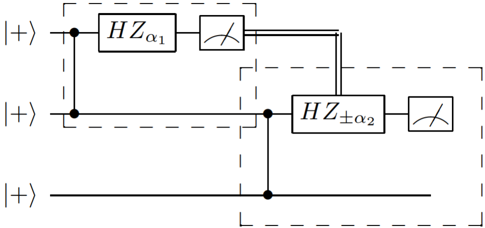
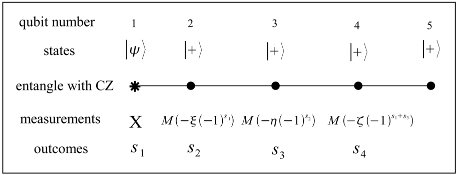


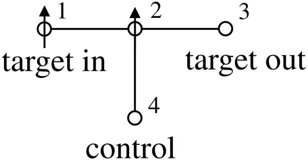
Figure: From [Nie06]

Universality: Quantum circuit model $\equiv$ Cluster state formulation. How?

- Transfer of information by teleportation
- Any qubit rotation can be done on a chain of qubits
- The CNOT gate can be implemented in a "T" configuration



(a) From [Joz06]



(b) From [RB01]

Variational simulation of non-trivial quantum states

QAOA [FGG14]: Quantum approximate optimization algorithm

- Principle: Quantum adiabatic theorem on $H = H_2 + H_1$
- Variational ansatz (modified in (2))

$$\boxed{|\psi(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle = \underbrace{e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2}}_{p \text{ layers}} |\psi_1\rangle} \tag{1}$$

  - $(\boldsymbol{\gamma},\boldsymbol{\beta}) = (\gamma_p, \dots, \gamma_1, \beta_p, \dots, \beta_1)$
  - $|\psi_1\rangle$ = ground state of H_1 (easy to prepare)
- Cost function:

$$\text{Overlap: } |\langle\psi_0|\psi(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle|^2, \quad \text{or} \quad \text{Energy: } \langle\psi(\boldsymbol{\gamma},\boldsymbol{\beta})| H |\psi(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle .$$

Variational simulation of the GHZ state

Example: GHZ state $\sim |0\rangle^{\otimes L} + |1\rangle^{\otimes L}$

$$H_{GHZ} = -\sum_{i=1}^{L} Z_i Z_{i+1} = \underbrace{-\sum_{i=1}^{L} Z_i Z_{i+1}}_{H_2} \underbrace{-0\sum_{i=1}^{L} X_i}_{H_1}, \qquad |GS_{H_1}\rangle = \bigotimes_{i=1}^{L} |+\rangle$$

$\implies$ Perfect fidelity, $p \sim L/2$.

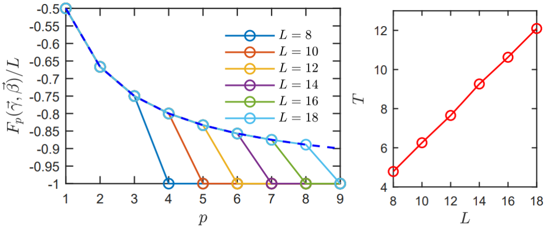


Figure: GHZ state simulation. Fidelity & p vs. L, [HH19]

Variational simulation of TFIM ground state

Example: Transverse field Ising model

$$H := H_2 + H_1 = -\sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=i}^{L} X_i$$

$\implies$ Perfect fidelity, $p \sim L/2$

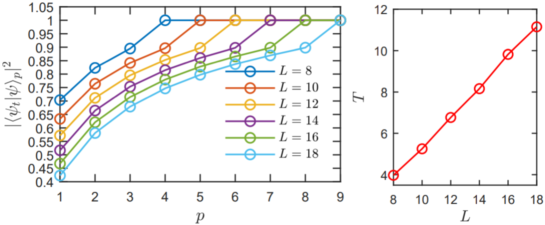


Figure: TFIM state simulation. Fidelity & p vs. L, [HH19]

Variational simulation...

Limitations of protocol in [HH19]:

- $p \sim L$.
- MERA construction [Vid08]: $p \sim \log(L)$,

  but non-local unitaries required.

$\Longrightarrow$ Is a measurement-assisted QAOA scheme a solution?

Measurement-based simulation of the GHZ state

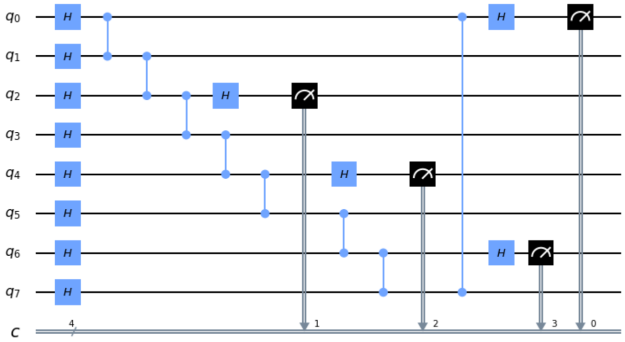


Figure: Preparing a 4-qubit GHZ state with a 8-qubit cluster state

Resulting state: $\sim |0\rangle^{\otimes 4} + |1\rangle^{\otimes 4}$ (up to one layer of Pauli corrections.)

Measurement-based QAOA for TFIM

Hamiltonian

$$H := H_2 + H_1 = -\sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=i}^{L} X_i$$

QAOA ansatz:

$$|\psi(\boldsymbol{\gamma}, \boldsymbol{\beta})\rangle = e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2} |\psi_1\rangle$$

MBQC is universal $\implies$ Measurement-based QAOA ansatz is possible.

Ingredients: Z, X-rotations, & $CNOT$.

♣ Scheme can be simplified by changing measurement pattern.

But...

Limitations:

- $p \sim L$, where p is the number of layers of measurements.
- Non-local unitaries required

$\downarrow\downarrow\downarrow$

Two possibilities:

- QAOA is insufficient; need a completely new algorithm.
- QAOA is sufficient, but need better MBQC implementation. $\Longleftarrow$

2nd possibility: How robust is QAOA?

Test QAOA with TFIM without translation invariance:

$$\mathcal{H} = \sum_j J_j Z_j Z_{j+1} + \sum_j g_j X_j \tag{2}$$

Modified QAOA ansatz (reference (1))

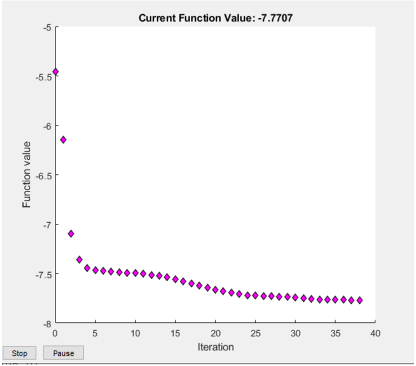
- p layers
- Each layer is parameterized by $(\boldsymbol{\gamma}, \boldsymbol{\beta})_k = (\gamma_1, \ldots, \gamma_L, \beta_1, \ldots, \beta_L)_k$.

Conjecture

This modified QAOA can target any point in the phase diagram with perfect fidelity for at most $p = L/2$. In which case, the total number of parameters is L^2.

2nd possibility: How robust is QAOA?

Conjecture holds for $L = 2, 4, \dots, 14$, even at lower p:



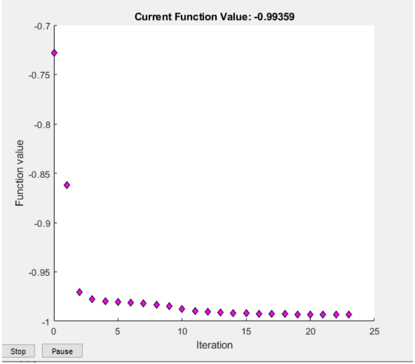
(a) $N = 6,\ p = 3$

```
Ground state energy
  -7.7800e+00

Energy minimum
  -7.7707e+00

Optimal angles
    7.7305e-02    5.1819e-01
    6.7580e-01    9.8622e-01
    3.7412e-01    5.4148e-01
    5.8479e-01    4.3484e-01
    3.6534e-01    4.1250e-01
    2.3284e-01    8.5791e-02

Fidelity by Energy: 99.8804%
Time taken : 00:00:18
Time in sec: 17.7538
```

(b) 99.9% fidelity at low iteration count.

2nd possibility: How robust is QAOA?

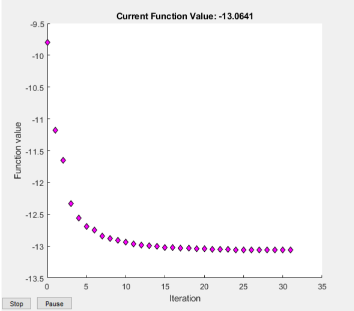
(Using the overlap as the cost function in this case)



(a) $N = 8,\ p = 4$

```
Ground state energy
  -1.1520e+01

Optimal angles
     2.1895e-01    2.2117e-01
     5.6762e-01    5.4324e-01
     3.0717e-01    3.4677e-01
     4.2687e-01    4.5373e-01
     3.5788e-01    2.8814e-01
     4.5521e-01    4.6618e-01
     3.9238e-01    3.7951e-01
     2.6419e-01    2.9289e-01

Fidelity by Overlap: 99.359%
Time taken : 00:04:20
Time in sec: 260.2146
```

(b) 99.4% fidelity at low iteration count.



(a) $N = 10$, $p = 2$ only

```
Ground state energy
  -1.3535e+01

Energy minimum
  -1.3064e+01

Optimal angles
    5.0595e-01   9.0266e-02
    5.9734e-01   3.9420e-01
    5.5699e-01   1.8317e-01
    3.5894e-01   4.5162e-01

Fidelity by Energy: 96.5225%
Time taken : 00:37:22
Time in sec: 2241.7264
```

(b) 96% fidelity, not bad for $p = 2$.

$\implies$ Can't get to higher L's due to large parameter space ($\sim L^2$) and limitations in computing power and algorithm efficiency.

Summary & Questions

- MBQC
- Variational non-trivial state simulation & QAOA
- Measurement-based QAOA. It is a good idea?
- Robustness of QAOA, tested on TFIM with non-constant g_i.
- Is it possible, in principle, to get speedup with MBQC + QAOA?
- In particular, can we target the critical ground state ($g \equiv 1$) with sublinear circuit depth?

References I

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- Robert Raussendorf and Hans J. Briegel, *A one-way quantum computer*, Phys. Rev. Lett. **86** (2001), 5188–5191.

References II

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