You may find the following information helpful:

Physical Constants

Electron mass	$m_e \approx 9.1 \times 10^{-31} kg$	Proton mass	$m_p \approx 1.7 \times 10^{-27} kg$
Electron Charge	$e \approx 1.6 \times 10^{-19} C$	Planck's const./2π	$\hbar \approx 1.1 \times 10^{-34} Js^{-1}$
Speed of light	$c \approx 3.0 \times 10^{8} ms^{-1}$	Stefan's const.	$\sigma \approx 5.7 \times 10^{-8} Wm^{-2}K^{-4}$
Boltzmann's const.	$k_B \approx 1.4 \times 10^{-23} JK^{-1}$	Avogadro's number	$N_0 \approx 6.0 \times 10^{23} mol^{-1}$

Conversion Factors

$1atm \equiv 1.0 \times 10^5 Nm^{-2}$ $\quad$ $1\mathring{A} \equiv 10^{-10} m$ $\quad$ $1eV \equiv 1.1 \times 10^4 K$

Thermodynamics

$dE = TdS + đW$ $\quad$ For a gas: $đW = -PdV$ $\quad$ For a wire: $đW = Jdx$

Mathematical Formulas

$\int_0^\infty dx\ x^n\ e^{-\alpha x} = \frac{n!}{\alpha^{n+1}}$ $\quad$ $\left(\frac{1}{2}\right)! = \frac{\sqrt{\pi}}{2}$

$\int_{-\infty}^{\infty} dx \exp\left[-ikx - \frac{x^2}{2\sigma^2}\right] = \sqrt{2\pi\sigma^2} \exp\left[-\frac{\sigma^2 k^2}{2}\right]$ $\quad$ $\lim_{N\to\infty} \ln N! = N \ln N - N$

$\left\langle e^{-ikx} \right\rangle = \sum_{n=0}^\infty \frac{(-ik)^n}{n!} \langle x^n \rangle$ $\quad$ $\ln \left\langle e^{-ikx} \right\rangle = \sum_{n=1}^\infty \frac{(-ik)^n}{n!} \langle x^n \rangle_c$

$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$ $\quad$ $\ln(1-x) = -\sum_{n=1}^\infty \frac{x^n}{n}$

Surface area of a unit sphere in d dimensions $\quad$ $S_d = \frac{2\pi^{d/2}}{(d/2-1)!}$

1. *Attractive shell potential:* Consider a gas of particles in three dimensions interacting through a pair-wise central potential, $\mathcal{V}(r)$, where

$$\mathcal{V}(r) = \begin{cases} +\infty & \text{for } 0 < r < a, \\ -\varepsilon & \text{for } a < r < b, \\ 0 & \text{for } b < r < \infty. \end{cases}$$

(a) Calculate the second virial coefficient $B_2(T)$.

(b) Find the limiting behavior of $B_2(T)$ at high temperature (including the first correction to order of β), and comment on the low temperature behavior of $B_2(T)$.

(c) In the high temperature limit, reorganize the equation of state into the van der Waals form $(P + an^2)(V - Nb) = Nk_BT$, and identify the van der Waals parameters a and b.

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2. *Interacting point particles:* Consider a system of N classical point particles at temperature T, in a volume V. Unspecified interactions between the particles modify the energy of any configuration by $-NU(V/N)$, where $U(v)$ if some function of the inverse density $v = V/N$. The partition function is thus given by

$$Z(T, N, V) = Z_{\text{ideal gas}}(T, N, V) \times \exp\left[\beta N U(v)\right] ,$$

where $Z_{\text{ideal gas}}(T, N, V)$ is the partition function of a classical gas, and $\beta = (k_BT)^{-1}$,

(a) The ideal gas partition function depends on volume V and temperature T as $Z_{\text{ideal gas}}(T, N, V) \propto V^x T^y$. What are the values of x and y?

(b) Using the partition function, or otherwise, compute the energy $E = \langle \mathcal{H} \rangle$.

(c) Find the heat capacity C_V at constant volume.

(d) Using the partition function, or otherwise, compute the pressure $P(n, T)$, as a function of the density $n = N/V$.

(e) Compute the isothermal compressibility $\kappa_T(n) = -\frac{1}{V} \left.\frac{\partial V}{\partial P}\right|_T$.

(f) What is the necessary condition for $U(v)$ for stability of the system of particles.

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