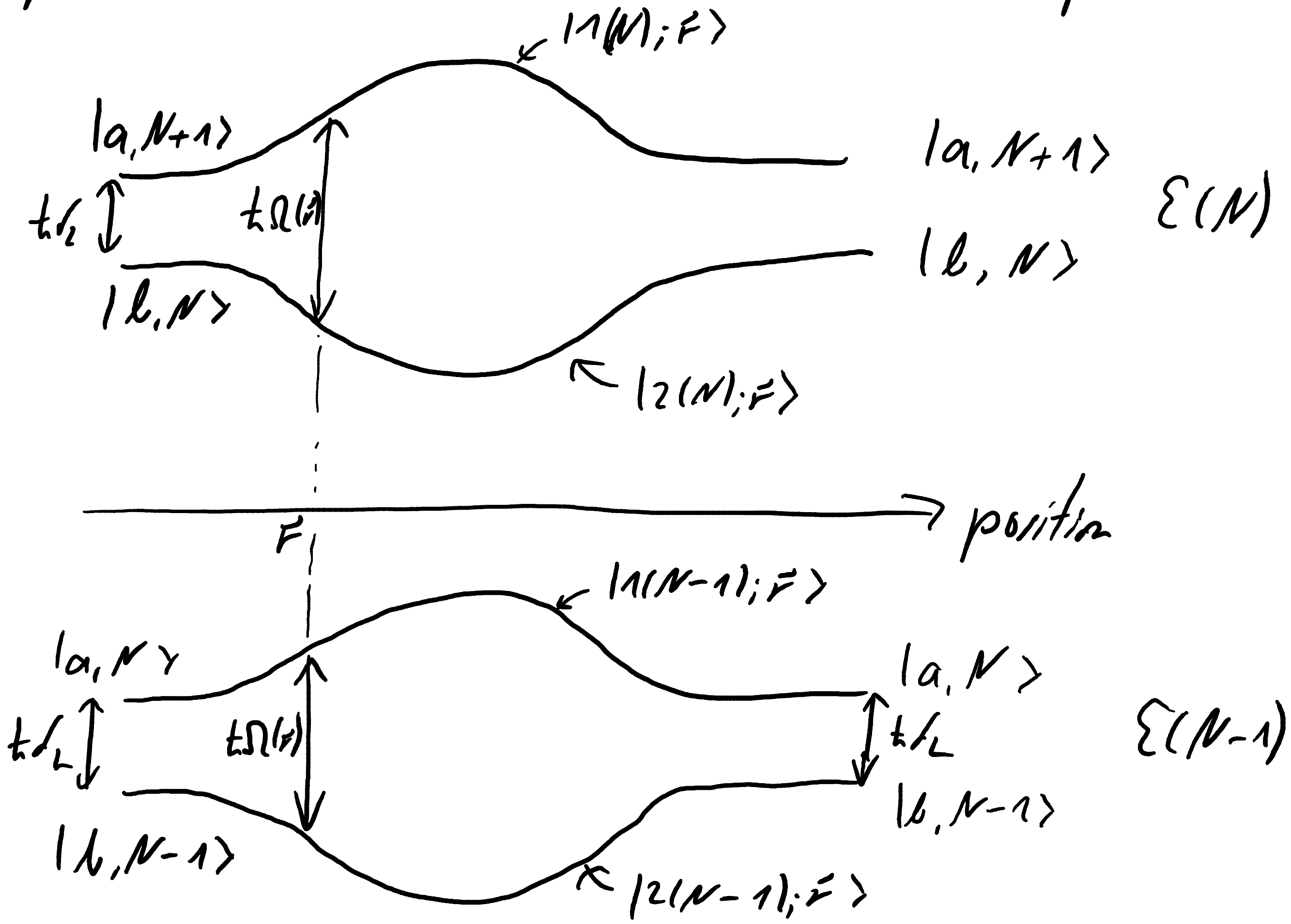
Dipole Forces within the Dressed atom picture



$$\hbar\Omega(\vec r) = \hbar\sqrt{\Omega_1^2(\vec r) + \delta_L^2}$$

Neglect spontaneous emission, atomic wavepacket is small, atom moves slowly $\rightarrow$ neglect non-adiabatic transition.

$\Rightarrow$ atom will stay in $|1(N)\rangle$ (or $|2(N)\rangle$)

$$\Rightarrow \vec F_1 = -\vec\nabla V_{1N}(\vec r) = -\frac{\hbar}{2}\vec\nabla\Omega(\vec r)$$

$$\vec F_2 = -\vec\nabla V_{2N}(\vec r) = +\frac{\hbar}{2}\vec\nabla\Omega(\vec r) = -\vec F_1$$

"optical Stern-Gerlach"

Spontaneous emission causes atom to change type of state (1 or 2)

$\rightarrow$ sign of the force changes abruptly.

Mean dipole Force:

$$\langle \vec{F}_{dip} \rangle = \vec{F}_1 \pi_1^{st} + \vec{F}_2 \pi_2^{st}$$
$$= -\frac{\hbar}{2} \vec{\nabla} \Omega(\vec{r}) \left(\pi_1^{st} - \pi_2^{st} \right)$$

$$\pi_1^{st} = \frac{\Gamma_{2\to 1}}{\Gamma_{1\to 2} + \Gamma_{2\to 1}} = \frac{\sin^4\vartheta}{\sin^4\vartheta + \cos^4\vartheta}$$

$$\pi_2^{st} = \frac{\Gamma_{1\to 2}}{\Gamma_{1\to 2} + \Gamma_{2\to 1}} = \frac{\cos^4\vartheta}{\sin^4\vartheta + \cos^4\vartheta}$$

$$\tan 2\vartheta = -\frac{\Omega_1}{\delta_L}$$

$$\cos^2 = 1 - \sin^2 = 1 - \tan^2 \cos^2 = \frac{1}{1+\tan^2} \; ; \; \sin^2 = \frac{\tan^2}{1+\tan^2}$$

$$\cos^4\vartheta - \sin^4\vartheta = \left(\cos^2\vartheta - \sin^2\vartheta\right)\left(\cos^2\vartheta + \sin^2\vartheta\right)$$
$$= \cos(2\vartheta) = \frac{1}{(1+\tan^2(2\vartheta))^{1/2}}$$

$$\cos^4\vartheta + \sin^4\vartheta = \left(\cos^2\vartheta + \sin^2\vartheta\right)^2 - 2\sin^2\vartheta\cos^2\vartheta$$
$$= 1 - \frac{1}{2}\sin^2(2\vartheta)$$
$$= 1 - \frac{1}{2}\frac{\tan^2(2\vartheta)}{1+\tan^2(2\vartheta)} = \frac{1+\frac{1}{2}\tan^2(2\vartheta)}{1+\tan^2(2\vartheta)}$$

$$\frac{\cos^4\vartheta - \sin^4\vartheta}{\cos^4\vartheta + \sin^4\vartheta} = \frac{\sqrt{1+\frac{\Omega_1^2}{\delta_L^2}}}{1+\frac{1}{2}\frac{\Omega_1^2}{\delta_L^2}} = \frac{\delta_L\sqrt{\Omega_1^2+\delta_L^2}}{\delta_L^2+\frac{1}{2}\Omega_1^2}$$

$$\nabla\Omega(\vec{r}) = \frac{\nabla\Omega_1^2}{2\sqrt{\Omega_1^2+\delta_L^2}}$$

Same as OBE-result!

$\nabla\Omega^2$ ... $\Omega \gg \Gamma$

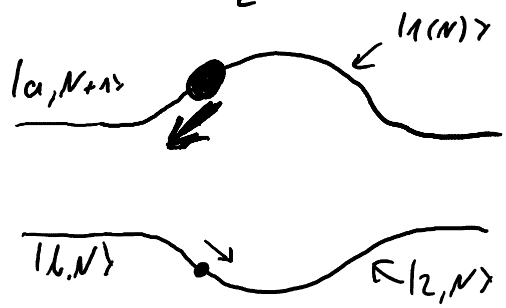
$$\nabla\Omega(\vec{r}) = \frac{\nabla\Omega_1}{2\sqrt{\Omega_1^2+\delta_L^2}}$$

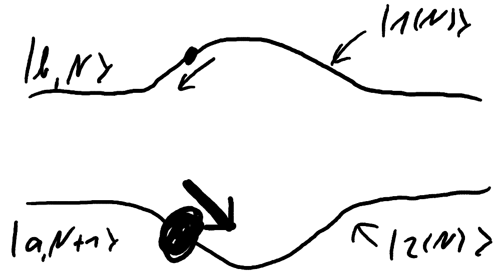
Same as OBE-result!

$$\Rightarrow -\frac{\hbar}{2}\vec{\nabla}\Omega(\vec{r})\left(\pi_1^{st}-\pi_2^{st}\right) = \boxed{-\frac{\hbar}{4}\delta_L\frac{\nabla\Omega_1^2}{\delta_L^2+\frac{\Omega_1^2}{2}+\frac{\Gamma^2}{4}}}$$

neglect $\Omega_1 \gg \Gamma$

$\delta_L > 0$ $\qquad\qquad$ $\delta_L < 0$



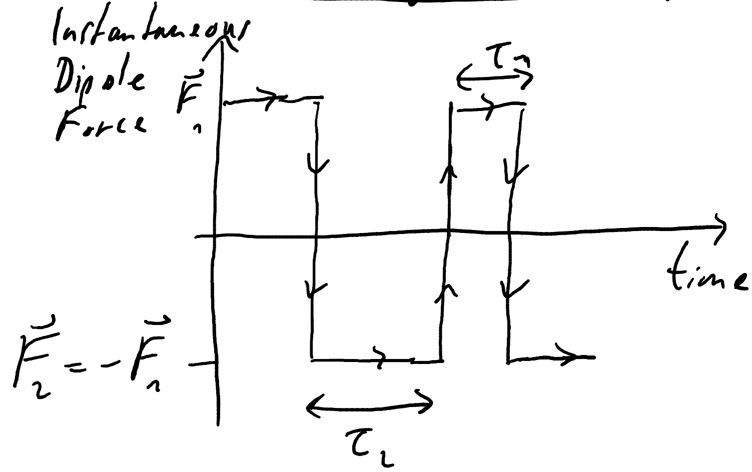


$\Pi_1^{st} > \Pi_2^{st}$ $\qquad\qquad$ $\Pi_1^{st} < \Pi_2^{st}$

→ repulsive force $\qquad\qquad$ → attractive force.

$\omega_L = \omega_0 \quad (\delta_L = 0) \qquad \Pi_1^{st} = \Pi_2^{st}, \quad \langle \vec{F}_{dip} \rangle = 0$

Fluctuations of the dipole force:



$\int_0^\infty d\tau\, F(t-\tau) \quad F(t)$

$$D_{dip} = \left\langle \frac{d}{dt}\left(\frac{\Delta p^2}{2}\right)\right\rangle = \frac{1}{2}\frac{d}{dt}\left(\langle p^2\rangle - \langle p\rangle^2\right) = \langle p\dot{p}\rangle - \langle p\rangle\langle\dot{p}\rangle$$

$$= \int_0^\infty d\tau \left[\langle F_{dip}(t)\, F_{dip}(t+\tau)\rangle - \langle F_{dip}\rangle^2\right]$$

Order of magnitude on resonance: $D_{dip} \sim \frac{F_1^2}{\Gamma}$

$$F_{dip} \sim -\hbar \vec{\nabla}\Omega_1$$

$$D_{dip} \sim \frac{\hbar^2 (\vec{\nabla}\Omega_1)^2}{\Gamma} \simeq \underbrace{\hbar^2 k^2 \Gamma}_{D_{spont}} \frac{\Omega_1^2}{\Gamma^2} \xrightarrow[\frac{\Omega_1}{\Gamma}\to\infty]{} \infty$$

Far away: atom remains in g and $D \to 0$.

Atomic motion in an intense standing wave

Redistribution processes become predominant in comparison with fluorescence cycles.

Velocity dependent force which has sign change when $I_L \nearrow$.

$$F(v) = -\alpha v \quad \text{for } v \text{ small}$$

For $\delta_L < 0$ $(\omega_L < \omega_0)$

$\alpha > 0$ at low I_L (Doppler cooling)
$\alpha < 0$ at high I_L

For $\delta_L > 0$ $(\omega_L > \omega_0)$

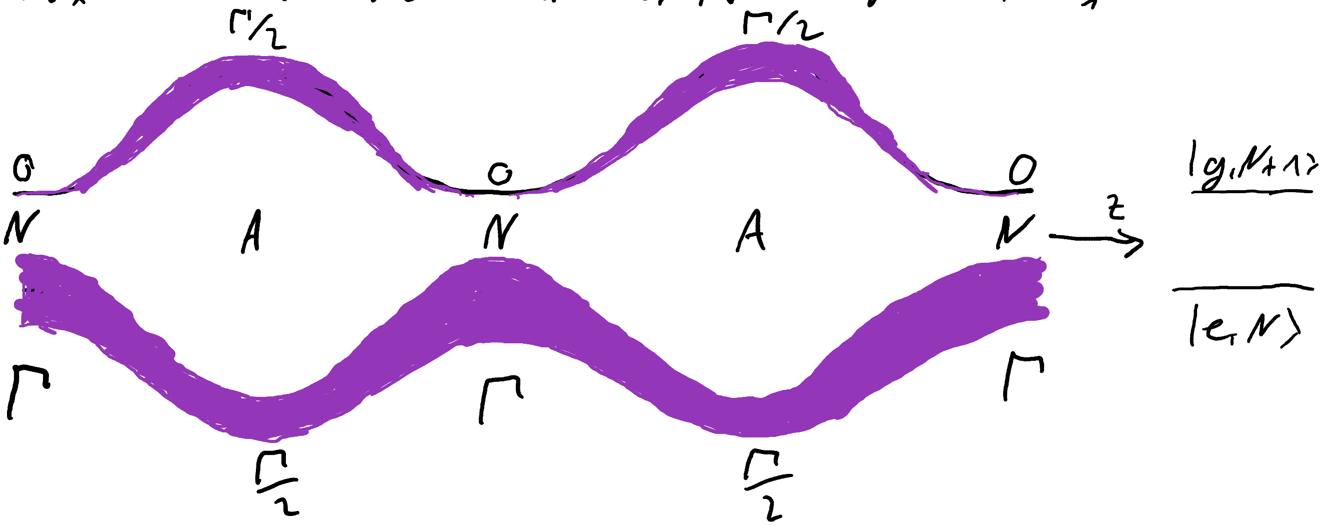
$\alpha < 0$ at low I_L (Doppler "anti-cooling")
$\alpha > 0$ at high I_L "Stimulated blue molasses"

Dressed atom interpretation:
High-intensity Sysiphus effect.

2-level atom in a blue-detuned intense standing wave ($\delta > 0$)

Splitting $\hbar\Omega(z) = \hbar\sqrt{\delta^2 + \Omega_1^2 \sin^2(kz)}$

- Minimum at the nodes N: $\hbar\delta$
- Maximum at the antinodes A: $\hbar\sqrt{\delta^2+\Omega_1^2}$



z-dependence of the radiative widths

- At a node N, $|1(N)\rangle = |g, N+1\rangle$; $|2(N)\rangle = |e,N\rangle$

  $\hookrightarrow \Gamma_1 = 0 \qquad \Gamma_2 = \Gamma$

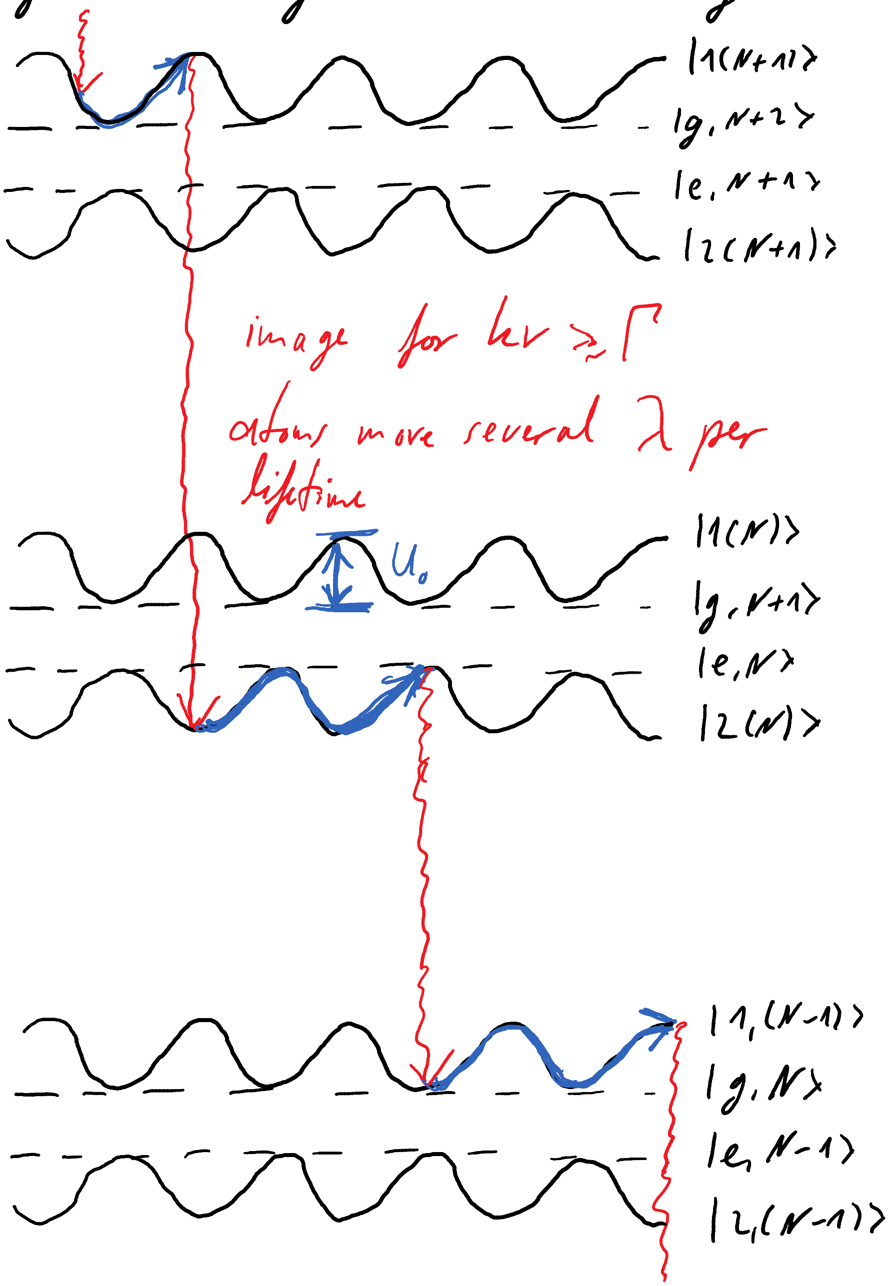
- At an antinode, if $\Omega_1 \gg \delta$ and Γ
  $|1(N)\rangle$ and $|2(N)\rangle$ are equally contaminated by $|e,N\rangle$

  $\Rightarrow \Gamma_1 \approx \Gamma_2 \simeq \frac{\Gamma}{2}$.

An atom moving along the z-axis and remaining in a given dressed state "sees" a series of hills & valleys. The departure rate is maximum at the top, regardless of the dressed state.

High intensity Sysiphus Cooling

Atom moving along the standing wave.
The radiative cascade no longer occurs for a fixed value of z.



For $\delta_L = \omega_L - \omega_0 > 0$

Transitions $|1(N+1)\rangle \rightarrow |2(N)\rangle$ occur preferentially at the antinodes

Transitions $|2(N)\rangle \longrightarrow |1(N-1)\rangle$ occur preferentially at the nodes

Cooling Rate:

One hill-valley per $\frac{1}{\Gamma_{2\to 1}}$: $F \cdot v \approx U_0 \, \Gamma_{2\to 1}$

Like Sysiphus, the atom is running uphill more frequently than downhill

J. Dalibard, C. Cohen-Tannoudji, JOSA B2, 1707 (1985)

Stimulated Molasses

- Between 2 spontaneous emission processes, the total atomic energy (kinetic + potential) is conserved.

  When the atom climbs up a hill, its kinetic energy is transformed into potential energy by stimulated emission processes which redistribute photons between the 2 counterpropagating waves.

  Atomic momentum is transferred to laser photons.

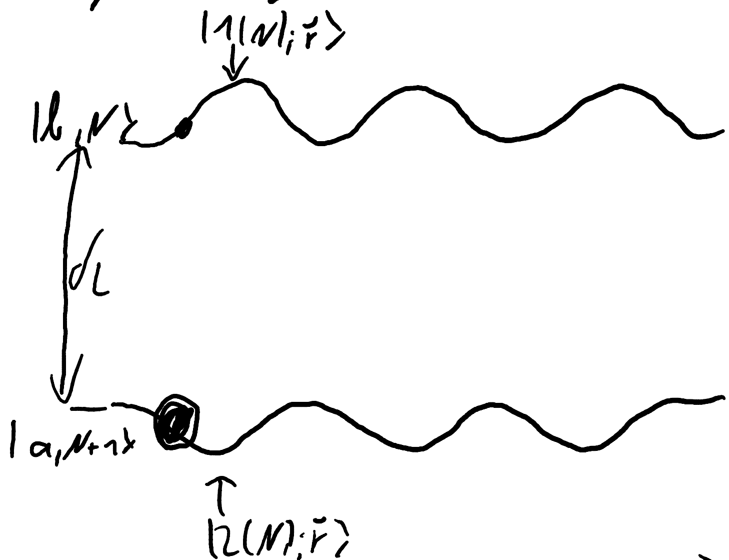
- The total atomic energy is then dissipated by spontaneous emission photons which carry away the gain of atomic potential energy.

  No saturation when $\Omega_1 \nearrow$

Observed: Aspect, Dalibard, Heidmann, Salomon, Cohen-Tannoudji
PRL 57, 1688 (1986)

Fluctuations of dipole force for large detuning:

Say $\omega_L < 0$:



$$|1(N);\vec r\rangle = |b,N;\vec r\rangle + \frac{\Omega_1(\vec r)}{\delta_L}|a,N+1\rangle$$

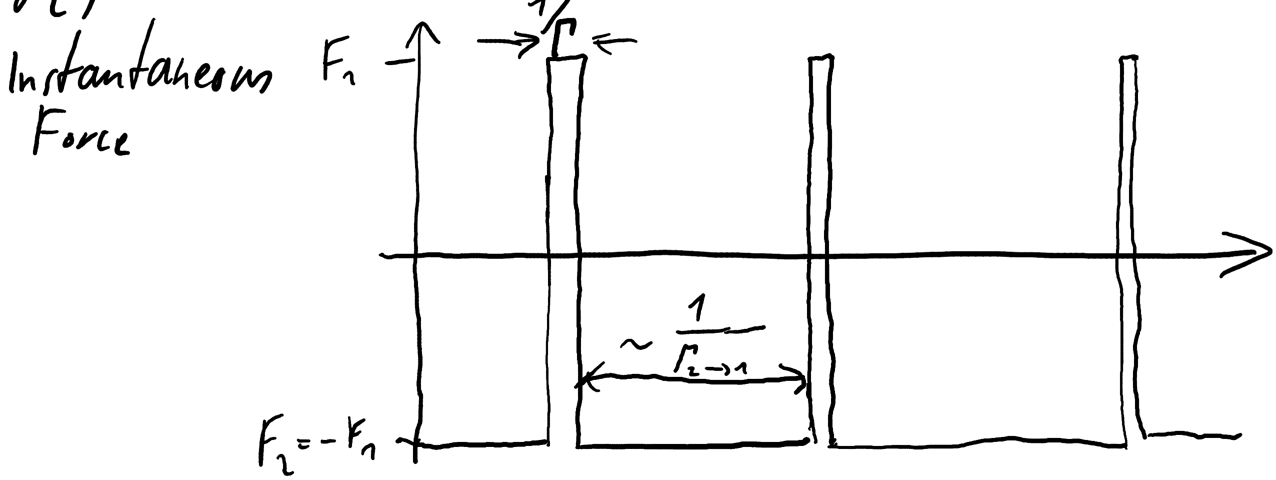
$$|2(N);\vec r\rangle \approx |a,N+1;\vec r\rangle - \frac{\Omega_1(\vec r)}{\delta_L}|b,N\rangle$$

$$\Rightarrow \Gamma_{1\to 2} \simeq \Gamma$$

$$\Gamma_{2\to 1} \approx \Gamma \cdot \left(\frac{\Omega_1}{\delta_L}\right)^4$$

$\frac{\Gamma_{2\to 1}}{\Gamma} = \left(\frac{\Omega_1}{\delta}\right)^4$: probability to find state $|2(N)\rangle$ in the unstable state $|b,N\rangle$ times probability to find state $|1(N-1)\rangle$ in stable state $|a,N\rangle$ into which $|b,N\rangle$ can decay.

$\left(\frac{\Omega_1}{\delta_L}\right)^4 \ll 1$ for large detuning.



Momentum diffusion coefficient:

$$D_{dip} = \int_0^\infty d\tau \left(\langle F(t) F(t+\tau) \rangle - \langle F(t) \rangle^2 \right)$$

$$= \frac{F_1^2}{\Gamma} \cdot \text{probability that atom is in state } |1(N)\rangle$$

$$= \frac{F_1^2}{\Gamma} \cdot \frac{\frac{1}{\Gamma}}{\frac{1}{\Gamma_{2\to1}}}$$

$$= \frac{(\hbar \nabla \Omega)^2}{\Gamma^2} \Gamma_{2\to1} \qquad \nabla\Omega \sim \frac{\nabla \Omega_1^2}{\delta_L} \text{ large } \delta_L$$

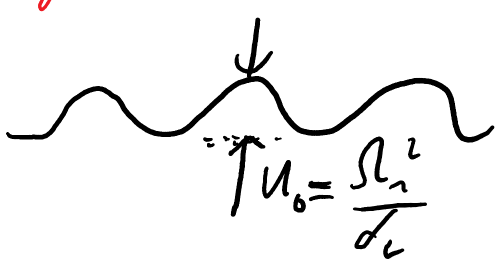
$$\sim \hbar^2 k^2 \Gamma \frac{\Omega_1^4}{\Gamma^2 \delta^2} \cdot \frac{\Omega_1^4}{\delta^4}$$

$$= D_{spont} \cdot \frac{\Omega_1^2}{\Gamma^2} \left(\frac{\Omega_1}{\delta_L} \right)^6 \xrightarrow[\delta_L \gg \Omega_1]{} 0$$

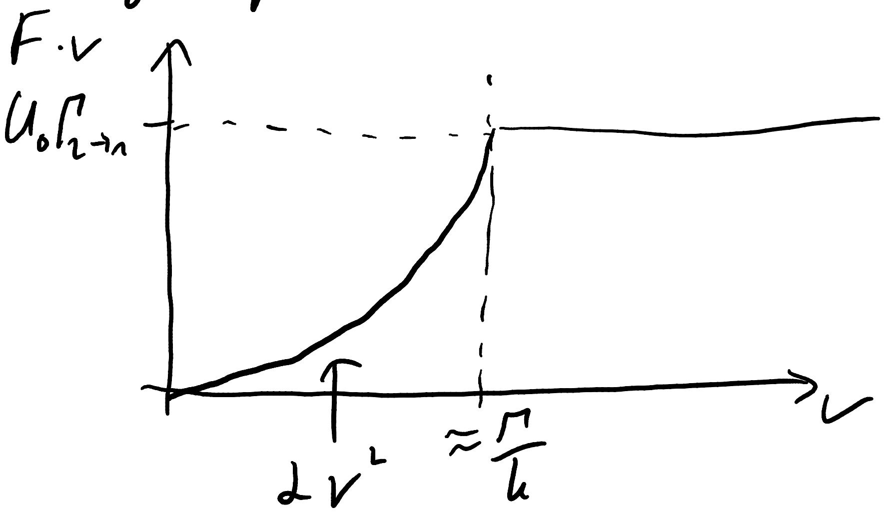
Cooling power: $F \cdot v \simeq U_0 \Gamma_{2\to1} = \hbar \frac{\Omega_1^2}{\delta_L} \cdot \Gamma \left(\frac{\Omega_1}{\delta_L} \right)^4$

(if $kv \gtrsim \Gamma$)

$$= \left(\frac{\Omega_1}{\delta_L} \right)^6 \cdot \hbar \Gamma \cdot \delta_L$$



Cooling power vs v:



Estimate:

$$\alpha = \lim_{v \to 0} \frac{F(v) \cdot v}{v^2} = \frac{U_0 \Gamma_{2\to 1}}{(\Gamma/k)^2}$$

$$\approx \hbar k^2 \left(\frac{\Omega_1}{\delta_L}\right)^6 \frac{\delta_L}{\Gamma}$$

Heating $\left\langle \frac{\Delta \dot{p}^2}{2m} \right\rangle = F \cdot v = \alpha v^2 =$ Cooling

$$\frac{D}{m} = \frac{\alpha}{m} k_B T$$

$$\Rightarrow k_B T = \frac{D}{\alpha} \approx \hbar \frac{\Omega_1^2 \Gamma}{\Gamma \delta} = \hbar \frac{\Omega_1^2}{\delta}$$

$$= U_0$$

Note: For $U_0 < \Gamma$ we have to include heating by atomic recoil.

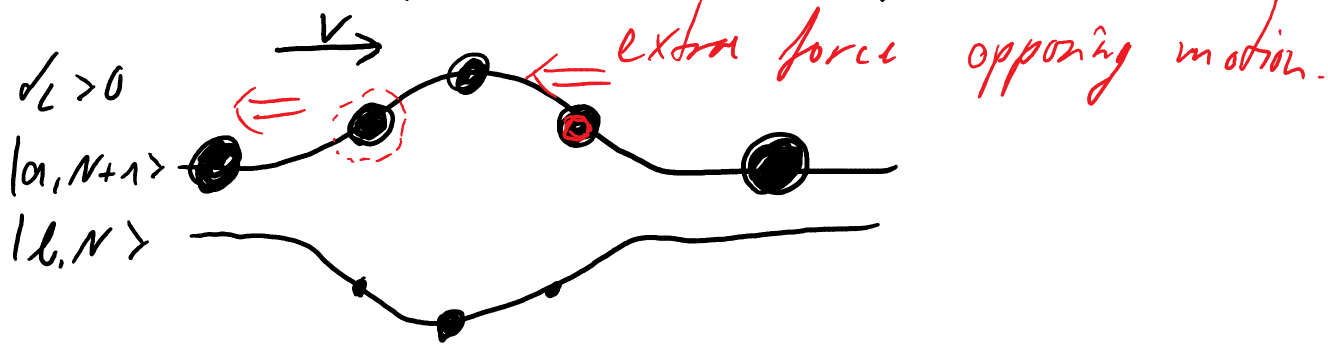
Blue molasses cannot cool below the Doppler limit: $\frac{\Gamma}{2}$

Another way to get cooling force for $\delta_L > 0$

Let atom move with velocity v

There is a time lag of $\tau_{pop} = \Gamma_{pop}^{-1}$ to establish equilibrium.

$$\Rightarrow \quad \Pi_i = \Pi_i^{st}(\vec{r} - \vec{v}\,\tau_{pop})$$



as atom moves from a region where $\Omega_1 = 0$ to the peak, the population has not quite enough time to adjust. The force is thus more opposing than would be the case in equilibrium. Same is true on the downhill leg, the force is less advancing than in equilibrium.

Calculation of the cooling (for $\delta_L > 0$) force:

$$\Pi_i(\vec r) \simeq \Pi_i^{st}(\vec r - \vec v\, \tau_{pop})$$

$$\vec F_{dip} \approx \vec F_{dip}^{\,st} - \left(-\frac{\hbar \vec\nabla \Omega}{2}\right)\left(\vec\nabla\left(\Pi_1^{st} - \Pi_2^{st}\right)\cdot \vec v\right)\tau_{pop}.$$

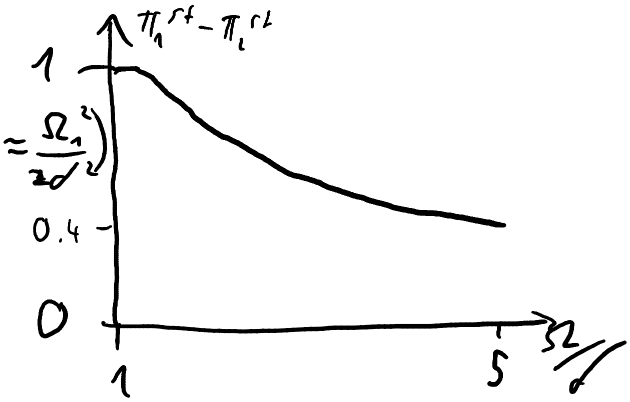
$$\Pi_1^{st} - \Pi_2^{st} = \frac{\cos^4\vartheta - \sin^4\vartheta}{\cos^4\vartheta + \sin^4\vartheta} = \frac{\cos(2\vartheta)}{1 - \frac{1}{2}\sin^2(2\vartheta)} = \frac{\frac{1}{\cos(2\vartheta)}}{\frac{1}{\cos^2(2\vartheta)} - \frac{1}{2}\tan^2(2\vartheta)}$$

$$= \frac{\sqrt{1+\tan^2(2\vartheta)}}{1 + \frac{1}{2}\tan^2(2\vartheta)} = \frac{\delta\sqrt{\delta^2 + \Omega_1^2}}{\delta^2 + \frac{1}{2}\Omega_1^2}$$

$$= 2\frac{\delta\Omega}{\delta^2 + \Omega^2} = 2\frac{1}{\frac{\delta}{\Omega} + \frac{\Omega}{\delta}}$$

$$= 2\frac{1}{x + \frac{1}{x}} \quad \text{with} \quad x = \frac{\Omega}{\delta} > 1$$

Note: Taylor expansion near $x = 1$: set $x = 1 + \varepsilon$ $\left(\varepsilon \simeq \frac{\Omega_1^2}{2\delta^2}\right)$



$$\Pi_1^{st} - \Pi_2^{st} = 2\frac{1}{1+\varepsilon + \frac{1}{1+\varepsilon}}$$

$$= 2\frac{1}{1+\varepsilon + 1 - \varepsilon + \varepsilon^2}$$

$$= \frac{1}{1 + \frac{\varepsilon^2}{2}} \approx 1 - \frac{\varepsilon^2}{2} \quad \text{starts quadratically in } \varepsilon$$

$$= 1 - \frac{\Omega_1^4}{8\delta^4} \quad \text{for } \Omega_1 \ll \delta_L$$

Then $\boxed{\vec\nabla\left(\Pi_1^{st} - \Pi_2^{st}\right) \approx -\frac{\Omega_1^4}{2\delta^4}\vec\alpha}$ with $\vec\alpha = \frac{\vec\nabla\Omega_1}{\Omega_1}$

$$\vec\nabla\Omega \approx \frac{\Omega_1}{\delta}\nabla\Omega_1 = \frac{\Omega_1^2}{\delta}\vec\alpha \;;\quad \tau_{pop} \simeq \frac{1}{\Gamma}\;;$$

$$\Rightarrow \vec{F}_{dip} \approx \vec{F}_{dip}^{\,rL} - \frac{1}{4}\,\frac{\hbar\,\delta}{\Gamma}\left(\frac{\Omega_1}{\delta}\right)^6 (\vec{\alpha}\cdot\vec{v})\,\vec{\alpha}$$

More exactly (arbitrary Ω_1/δ):

$$\Pi_1^{st} - \Pi_2^{st} = 2\,\frac{1}{x + \frac{1}{x}} \qquad x = \frac{\Omega}{\delta} > 1$$

$$\vec{\nabla}\left(\Pi_1^{st} - \Pi_2^{st}\right) = -2\,\frac{1}{\left(x + \frac{1}{x}\right)^2}\left(\vec{\nabla}x - \frac{\vec{\nabla}x}{x^2}\right)$$

$$= -2\,\frac{x^2 - 1}{(x^2+1)^2}\,\vec{\nabla}x$$

$$= -2\,\frac{\Omega_1^2\,\delta}{(\Omega_1^2 + 2\delta^2)^2}\,\vec{\nabla}\Omega$$

$$= -2\,\frac{\Omega_1^2\,\delta}{(\Omega_1^2 + 2\delta^2)^2}\,\frac{\Omega_1^2}{\Omega}\,\vec{\alpha}$$

$$= -2\,\frac{\Omega_1^4}{(\Omega_1^2 + 2\delta^2)^2}\,\frac{\delta}{\Omega}\,\vec{\alpha}$$

$$\tau_{pop} = \Gamma^{-1}\left(\cos^4\vartheta + \sin^4\vartheta\right)^{-1} = \Gamma^{-1}\left(1 - \tfrac{1}{2}\sin^2(2\vartheta)\right)^{-1}$$

$$= \Gamma^{-1}\,\frac{\frac{1}{\cos^2(2\vartheta)}}{\frac{1}{\cos^2(2\vartheta)} - \frac{1}{2}\tan^2(2\vartheta)} = \Gamma^{-1}\,\frac{1 + \tan^2(2\vartheta)}{1 + \frac{1}{2}\tan^2(2\vartheta)}$$

$$= \Gamma^{-1}\,2\,\frac{\Omega_1^2 + \delta^2}{\Omega_1^2 + 2\delta^2} = \frac{2\,\Omega^2}{\Gamma\,(\Omega_1^2 + 2\delta^2)}$$

$$\vec{\nabla}\Omega = \frac{\Omega_1^2}{\Omega}\,\vec{\alpha}$$

$$\Rightarrow \boxed{\vec{F}_{dip}(\vec{r},\vec{v}) \approx \vec{F}_{dip}^{st} - \frac{2\hbar\delta}{\Gamma}\left(\frac{\Omega_1^2}{\Omega_1^2 + 2\delta^2}\right)^3 (\vec{\alpha}\cdot\vec{v})\,\vec{\alpha}}$$

See Eq. 3.31 in Dalibard, Cohen-Tannoudji, JOSA B 2, 1707 (1985)

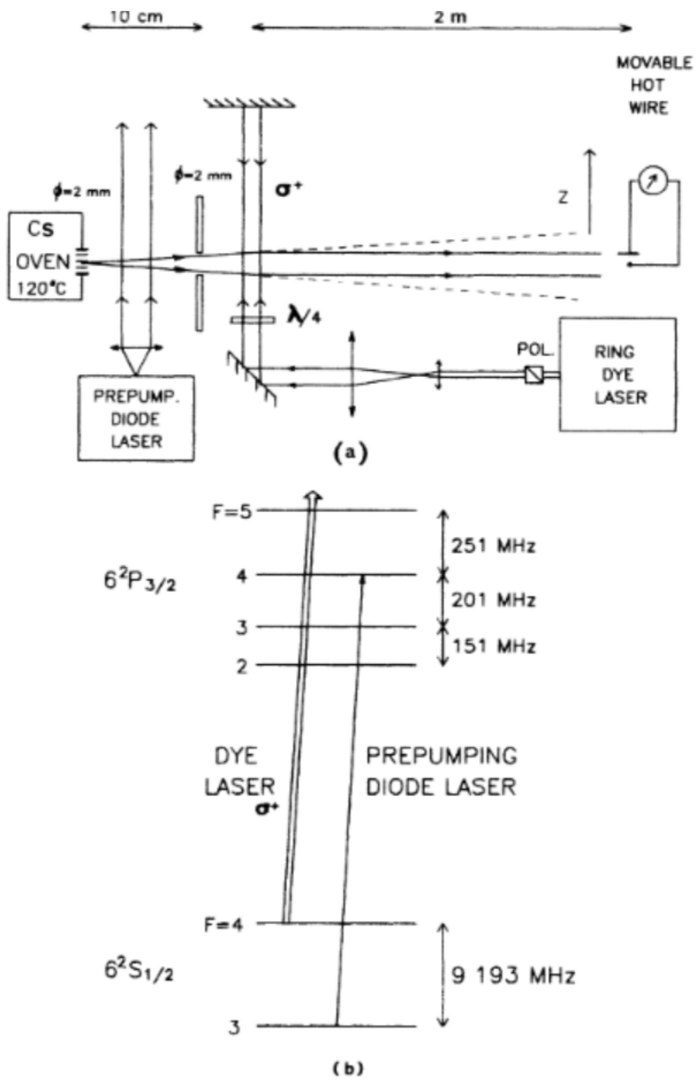


FIG. 2. (a) Experimental setup, (b) relevant cesium energy levels.

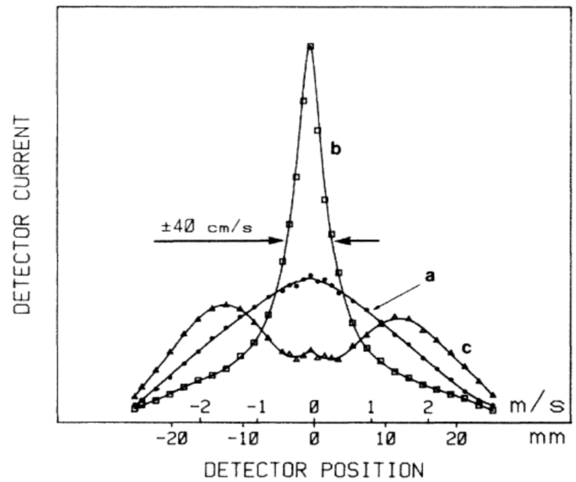


FIG. 3. Detector current vs position of the hot-wire detector. The corresponding transverse atomic velocities are given in m/s. Peak current is 2.2×10^9 atoms/s. The full lines are intended merely as visual aids. Curve *a*, laser beam off (HWHM 2 m/s); curve *b*, laser beam on with a positive detuning ($\delta/2\pi = +30$ MHz); curve *c*, laser beam on with a negative detuning ($\delta/2\pi = -30$ MHz).