

Measurement-assisted variational simulation of non-trivial quantum states

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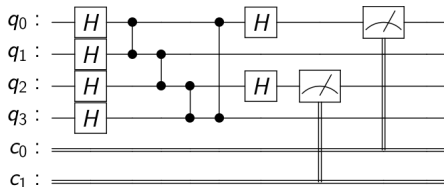


- Efficient variational simulation of nontrivial quantum states with QAOA [HH19]

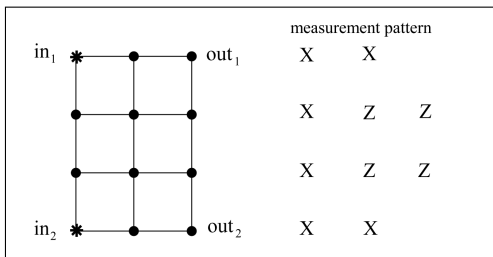
- Measurement-based quantum computing (MBQC)
- Variational simulation of non-trivial quantum state
- Research question: Measurement-assisted QAOA as an efficient/better simulation?

MBQC: One-way quantum computer [RB01]

Conventional quantum circuit models:



Cluster state: [Joz06]



MBQC: One-way quantum computer

Quantum teleportation = Entanglement + Measurement

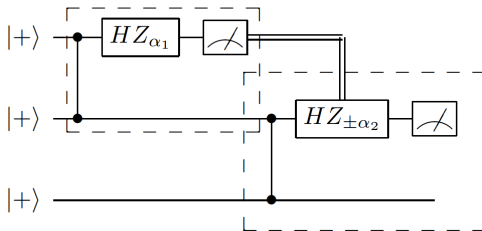
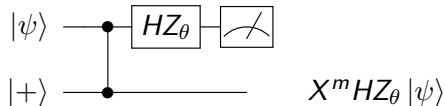


Figure: From [Nie06]

MBQC: One-way quantum computer

Universality: Quantum circuit model \equiv Cluster state formulation.



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- The CNOT gate can be implemented in a “T” configuration



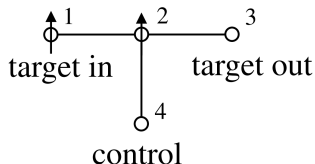
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qubit number	1	2	3	4	5
states	$ \psi\rangle$	$ +\rangle$	$ +\rangle$	$ +\rangle$	$ +\rangle$
entangle with CZ	*	•	•	•	•
measurements	X	$M(-\xi(-1)^{s_1})$	$M(-\eta(-1)^{s_2})$	$M(-\zeta(-1)^{s_3+s_4})$	
outcomes	s_1	s_2	s_3	s_4	

(a) From [Joz06]



(b) From [RB01]

Variational simulation of non-trivial quantum states

QAOA [FGG14]: Quantum approximate optimization algorithm

- Principle: Quantum adiabatic theorem on $H = H_2 + H_1$
- Variational ansatz (modified in (2))

$$|\psi(\gamma, \beta)\rangle = \underbrace{e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2}}_{p \text{ layers}} |\psi_1\rangle \quad (1)$$

- $(\gamma, \beta) = (\gamma_p, \dots, \gamma_1, \beta_p, \dots, \beta_1)$
- $|\psi_1\rangle = \text{ground state of } H_1 \text{ (easy to prepare)}$
- Cost function:

Overlap: $|\langle\psi_0|\psi(\gamma, \beta)\rangle|^2$, or Energy: $\langle\psi(\gamma, \beta)| H |\psi(\gamma, \beta)\rangle$.

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Variational simulation of the GHZ state

Example: GHZ state $\sim |0\rangle^{\otimes L} + |1\rangle^{\otimes L}$

$$H_{GHZ} = - \sum_{i=1}^L Z_i Z_{i+1} = - \underbrace{\sum_{i=1}^L Z_i Z_{i+1}}_{H_2} - 0 \underbrace{\sum_{i=1}^L X_i}_{H_1}, \quad |GS_{H_1}\rangle = \bigotimes_{i=1}^L |+\rangle$$

\Rightarrow Perfect fidelity, $p \sim L/2$.

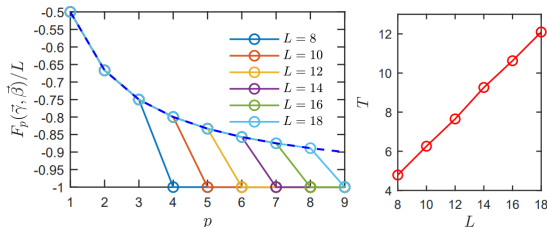


Figure: GHZ state simulation. Fidelity & p vs. L , [HH19]

Variational simulation of TFIM ground state

Example: Transverse field Ising model

$$H := H_2 + H_1 = - \sum_{i=1}^L Z_i Z_{i+1} - g \sum_{i=1}^L X_i$$

\Rightarrow Perfect fidelity, $p \sim L/2$

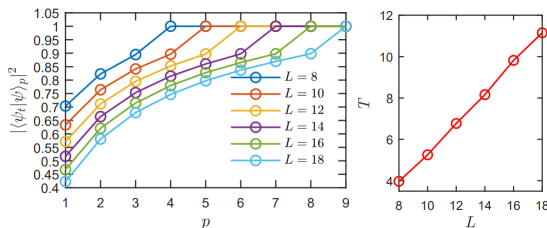


Figure: TFIM state simulation. Fidelity & p vs. L , [HH19]

Limitations of protocol in [HH19]:

- $p \sim L$.
- MERA construction [Vid08]: $p \sim \log(L)$,
but non-local unitaries required.

\implies Is a measurement-assisted QAOA scheme a solution?

Measurement-based simulation of the GHZ state



Measurement-based QAOA for TFIM

Hamiltonian

$$H := H_2 + H_1 = - \sum_{i=1}^L Z_i Z_{i+1} - g \sum_{i=1}^L X_i$$

QAOA ansatz:

$$|\psi(\gamma, \beta)\rangle = e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2} |\psi_1\rangle$$

MBQC is universal \implies Measurement-based QAOA ansatz is possible.

Ingredients: Z , X -rotations, & $CNOT$.

♣ Scheme can be simplified by changing measurement pattern.



But...

Limitations:

- $p \sim L$, where p is the number of layers of measurements.
- Non-local unitaries required



Two possibilities:

- QAOA is insufficient; need a completely new algorithm.
- QAOA is sufficient, but need better MBQC implementation.

2nd possibility: How robust is QAOA?

Test QAOA with TFIM without translation invariance:

$$\mathcal{H} = \sum_j J_j Z_j Z_{j+1} + \sum_j g_j X_j \quad (2)$$

Modified QAOA ansatz (reference (1))

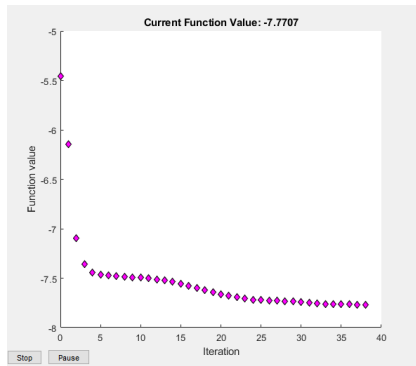
- p layers
- Each layer is parameterized by $(\gamma, \beta)_k = (\gamma_1, \dots, \gamma_L, \beta_1, \dots, \beta_L)_k$.

Conjecture

This modified QAOA can target any point in the phase diagram with perfect fidelity for at most $p = L/2$. In which case, the total number of parameters is L^2 .

2nd possibility: How robust is QAOA?

Conjecture holds for $L = 2, 4, \dots, 14$, even at lower p :



(a) $N = 6, p = 3$

Ground state energy

-7.7800e+00

Energy minimum

-7.7707e+00

Optimal angles

7.7305e-02 5.1819e-01

6.7580e-01 9.8622e-01

3.7412e-01 5.4148e-01

5.8479e-01 4.3484e-01

3.6534e-01 4.1250e-01

2.3284e-01 8.5791e-02

Fidelity by Energy: 99.8804%

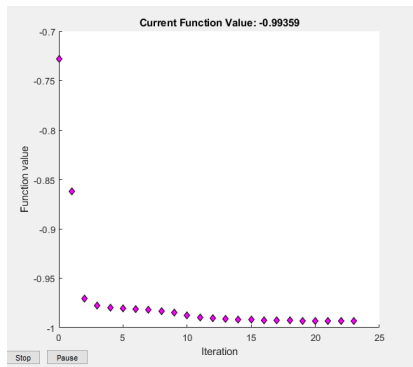
Time taken : 00:00:18

Time in sec: 17.7538

(b) 99.9% fidelity at low iteration count.

2nd possibility: How robust is QAOA?

(Using the overlap as the cost function in this case)



(a) $N = 8, p = 4$

Ground state energy

-1.1520e+01

Optimal angles

2.1895e-01 2.2117e-01

5.6762e-01 5.4324e-01

3.0717e-01 3.4677e-01

4.2687e-01 4.5373e-01

3.5788e-01 2.8814e-01

4.5521e-01 4.6618e-01

3.9238e-01 3.7951e-01

2.6419e-01 2.9289e-01

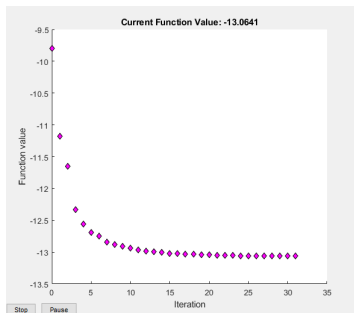
Fidelity by Overlap: 99.359%

Time taken : 00:04:20

Time in sec: 260.2146

(b) 99.4% fidelity at low iteration count.

2nd possibility: How robust is QAOA?



(a) $N = 10$, $p = 2$ only

Ground state energy
-1.3535e+01

Energy minimum
-1.3064e+01

Optimal angles
5.0595e-01 9.0266e-02
5.9734e-01 3.9420e-01
5.5699e-01 1.8317e-01
3.5894e-01 4.5162e-01

Fidelity by Energy: 96.5225%
Time taken : 00:37:22
Time in sec: 2241.7264






(b) 96% fidelity, not bad for $p = 2$.

⇒ Can't get to higher L 's due to large parameter space ($\sim L^2$) and limitations in computing power and algorithm efficiency.

Summary & Questions

- MBQC
- Variational non-trivial state simulation & QAOA
- Measurement-based QAOA. It is a good idea?
- Robustness of QAOA, tested on TFIM with non-constant g_i .
- Is it possible, in principle, to get speedup with MBQC + QAOA?
- In particular, can we target the critical ground state ($g \equiv 1$) with sublinear circuit depth?

References I

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