8.311 Electromagnetic Theory

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Pset 5, due April 6, 2022

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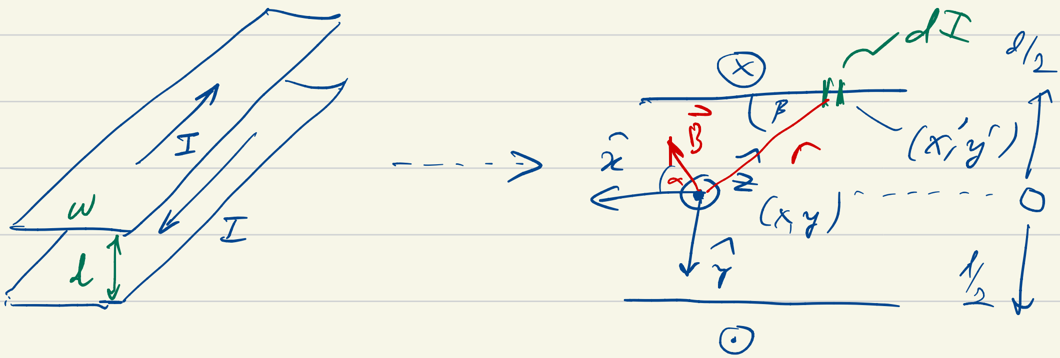


Have $dI = \frac{I}{w} dw$

dB (due to dI) $= \frac{\mu_0 \, dI}{2\pi r}$

$$= \frac{\mu_0 \, dI}{2\pi}\left[\frac{1}{(x-x')^2 + (y-y')^2}\right]^{1/2}$$

So $\quad dB_x = dB \cos\alpha = dB \sin\beta$

$$= \frac{\mu_0 dI}{2\pi} \frac{y-y'}{(x-x')^2+(y-y')^2}$$

$$dB_y = - dB \sin\alpha$$

$$= -\frac{\mu_0 dI}{2\pi} \frac{x-x'}{(x-x')^2+(y-y')^2}$$

2 plates

$$\Rightarrow \quad B_x\Big|_{y=0} = 2\int dB_x = 2\int \frac{\mu_0}{2\pi} \frac{y-y'}{(x-x')^2+(y-y')^2} \frac{I}{w} \, d\overset{dx'}{w}$$

$$= 2\int_{x'=\frac{w}{2}}^{x'=\frac{-w}{2}} \frac{\mu_0}{2\pi} \frac{-d/2}{(x-x') + (d/2)^2} \frac{I}{w} \, dx'$$

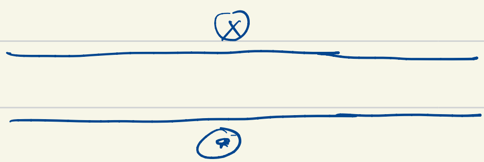
$$= \frac{\mu_0 I}{\pi w} \left\{ \tan^{-1} \frac{x-w/2}{d/2} - \tan^{-1} \frac{x+w/2}{d/2} \right\}$$

$B_y\big|_{y=0} = 0$ due to symmetry.

So

$$\boxed{\vec{B}(x,y,0) = \frac{\mu_0 I}{\pi w}\left\{\tan^{-1}\frac{x-w/2}{d/2} - \tan^{-1}\frac{x+w/2}{d/2}\right\}}$$

(2) $d \ll w$ ... then



(i) $B_x \longrightarrow \frac{\mu_0 I}{\pi w}\left\{+\frac{\pi}{2} + \frac{\pi}{2}\right\}$ (?)

I'm getting confused about the signs here, but the answer is that

$$\boxed{B_x \to \frac{\mu_0 I}{w}}$$ in $\longleftarrow$ direction

(ii) Vector potential ?

Have $\vec{B} = +\frac{\mu_0 I}{w} \hat{x}$ uniform...

So can take $\vec{A} = B y \hat{z}$

Check $\vec{\nabla} \times \vec{A} = \partial_y A_z \hat{z} = B \hat{z}$ ✓

So take $\boxed{\vec{A} = +\frac{\mu_0 I}{w} y \hat{z} + \text{constant}}$

(iii) magnetic force per unit length on each strip...

Field on one strip due to the other is

$$B_F = \frac{1}{2} B = \frac{\mu_0 I}{2w}$$

Force per unit length...

$\sim l \cdot A$

$$\frac{F}{l} = \frac{1}{l} \int_{strip} j \, B_F \, d^3r$$

$$= \frac{1}{l} B_F \int j \, d^3r = I B_F = \boxed{\frac{\mu_0 I^2}{2w}}$$

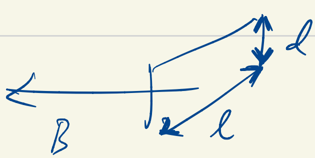
(iv) magnetic energy per unit length...

$$\frac{U}{l} = \frac{1}{l} \int_{gap} \frac{B^2}{2\mu_0} d^3r = \frac{B^2}{2\mu_0} \frac{1}{l} \cdot l d w = \boxed{\frac{\mu_0 d I^2}{2w}}$$

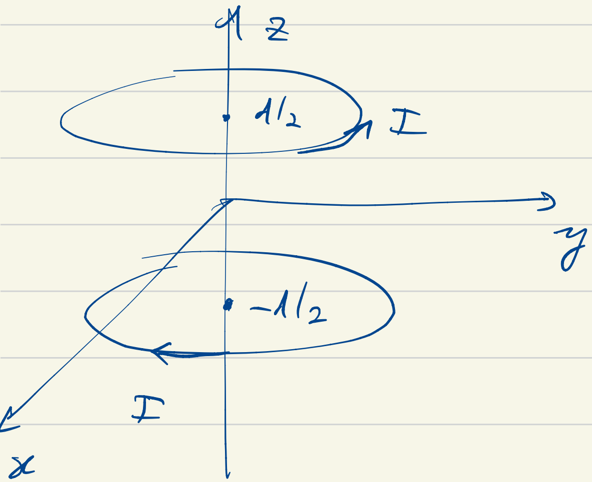
$\frac{\mu_0 I}{w}$

Self inductance...

$$\frac{L}{l} = \frac{\Phi}{I} \frac{1}{l} = \frac{1}{Il} \int_{gap} B_n \, d^2\sigma = \frac{\mu_0 I l d}{I l w} = \boxed{\frac{\mu_0 d}{w}}$$



③ Calculate $\tilde{B}$ near centre of anti-Helmholtz config



We know (from notes) that

$$B(0,0,z) = \frac{\mu_0 I}{2}\left\{\frac{-R^2}{\left(R^2+\left(d/2+z\right)^2\right)^{3/2}} + \frac{R^2}{\left(R^2+\left(d/2-z\right)^2\right)^{3/2}}\right\}$$

Taylor expand this near $z = 0$ we get

$$\vec{B}(0,0,z) \simeq \frac{48\, dR^2 \mu_0 I\, z}{(d^2+4R^2)^{5/2}}\, \hat{z}$$

To find B_x, B_y we use rotational symmetry + the fact that $\vec{\nabla}\cdot\vec{B} = 0$

$$\Rightarrow \quad \partial_x B_x + \partial_y B_y = -\partial_z B_z$$

$$\Rightarrow \quad \partial_x B_x = \partial_y B_y = -\frac{1}{2}\partial_z B_z$$

$$= -24 \frac{d R^2 \mu_0 I}{(d^2+4R^2)^{5/2}}$$

<u>S_1</u>

$$\boxed{\vec{B}\Big|_{\vec{r}\sim 0} = \frac{48 d R^2 \mu_0 I}{(d^2+4R^2)^{5/2}} \left(-\frac{x}{2}\hat{x} - \frac{y}{2}\hat{y} + z\hat{z} \right)}$$

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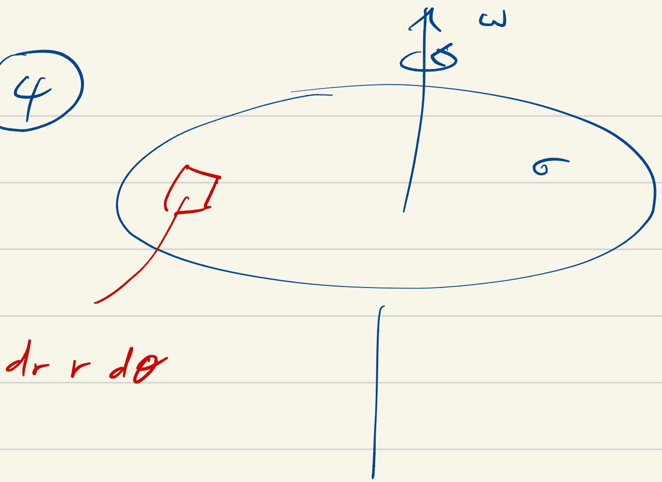
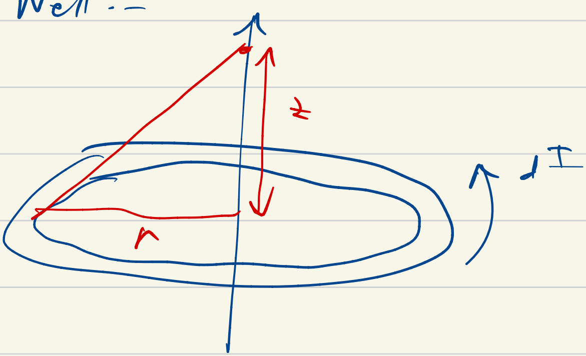


$dI = \sigma r \omega \, dr$

(i) induced $\vec{B}$ field on axis ...

Well ..



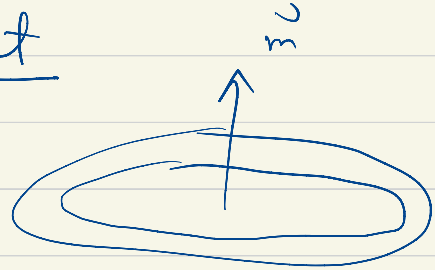
$$dB = \frac{\mu_0 \, dI}{2} \frac{r^2}{(r^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 \sigma \omega}{2} \frac{r^3}{(r^2+z^2)^{3/2}} \, dr$$

$$\rightarrow \quad B = \int_{r=0}^{r=R} \frac{\mu_0 \sigma \omega}{2} \frac{r^3 \, dr}{(r^2+z^2)^{3/2}} = (\ldots \text{mathematica})$$

$$\boxed{B = \mu_0 \sigma \omega \left\{ -|z| + \frac{R^2 + 2z^2}{2(R^2+z^2)^{1/2}} \right\}}$$

(ii) <u>magnetic moment</u>



$$m = \int dm$$

$$dm = A\,dI = \pi r^2 (\sigma r \omega)\,dr$$

$$= \pi \sigma \omega r^3\,dr$$

So $\boxed{m} = \int_{r=0}^{r=R} \pi \sigma \omega r^3\,dr = \boxed{\frac{\pi \sigma \omega R^4}{4}}$

<u>Relate m to B?</u>

Compare B @ ∞ to $\vec{B}$ due to m ...

B @ ∞ ... $\longrightarrow$ $B = \mu_0 \sigma \omega |z| \left\{ -1 + \dfrac{1 + R^2/2z^2}{\sqrt{1 + R^2/z^2}} \right\}$

expand $R/z \ll 1$ ... to get

$$\boxed{B = \sigma \mu_0 \omega |z| \frac{1}{8} \frac{R^4}{|z|^4} = \frac{\mu_0 \sigma \omega R^4}{8|z|^3}}$$

Now field due to magnetic dipole moment...

$$\boxed{\vec{B}} = \left(\frac{3\vec{r}\,(\vec{m}\cdot\vec{r}) - m r^2}{r^5} \right) \frac{\mu_0}{4\pi}$$

and $\quad \vec{r} = |z|\, \vec{m}/m$

$$\sim \frac{3m|z|^2 - m|z|^2}{|z|^5} \frac{\mu_0}{4\pi}$$

$$= \frac{\mu_0}{4\pi} \frac{2m}{|z|^3} = \frac{\mu_0}{4\pi} \cdot \frac{2}{|z|^3} \cdot \frac{\pi \sigma \omega R^4}{4} = \boxed{\frac{\mu_0 \sigma \omega R^4}{8|z|^3}}$$