

GENERAL RELATIVITY & COSMOLOGY

A Quick Guide

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1 Overview and Review

What is general relativity? It's a theory of gravity.

Replaces Newton's law of gravity, for heavy masses and high precision.

Keep in mind, GR is not compatible with Quantum Mechanics.

Question in Physics: how to reconcile GR and QM?

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2 Review of Multivariable and Vector Calculus

3 Flat 3-dimensional space

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9.1 Large-scale geometry of the universe

9.1.1 Cosmological principle

9.1.2 Robertson-Walker (flat, open, closed) geometries

9.1.3 Expansion of the universe

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9.2.4 A matter-dominated universe ($\Lambda = 0$) [Friedmann models]

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