Summary

Note adaptive measurement required

Rotation in X

	$\|\psi\rangle$	$\|+\rangle$	$\|+\rangle$	
measure	X	$\dashrightarrow M((-1)^{s_1} 2\theta)$		acquires
outcome	s_1	s_2		$X^{s_2} Z^{s_1} e^{i\theta\sigma_x}\|\psi\rangle$

■ Let $|\psi\rangle = a|0\rangle + b|1\rangle$ then

- $$e^{i\theta\sigma_x}|\psi\rangle = (a\cos\theta + bi\sin\theta)|0\rangle + (ai\sin\theta + b\cos\theta)|1\rangle$$
$$\cong \left[(a+b) + e^{-2i\theta}(a-b)\right]|0\rangle + \left[(a+b) - e^{-2i\theta}(a-b)\right]|1\rangle$$

■ Cluster state produces

s_1	s_2	out	Correction
$0 \Rightarrow M(2\theta)$	0	$\left[(a+b)+e^{-2i\theta}(a-b)\right]\|0\rangle + \left[(a+b)-e^{-2i\theta}(a-b)\right]\|1\rangle$	$Z^0 X^0$
$0 \Rightarrow M(2\theta)$	1	$\left[(a+b)-e^{-2i\theta}(a-b)\right]\|0\rangle + \left[(a+b)+e^{-2i\theta}(a-b)\right]\|1\rangle$	$Z^0 X^1$
$1 \Rightarrow M(-2\theta)$	0	$\left[(a-b)+e^{2i\theta}(a+b)\right]\|0\rangle + \left[(a-b)-e^{2i\theta}(a+b)\right]\|1\rangle$	$Z^1 X^0$
$1 \Rightarrow M(-2\theta)$	1	$\left[(a-b)-e^{2i\theta}(a+b)\right]\|0\rangle + \left[(a-b)+e^{2i\theta}(a+b)\right]\|1\rangle$	$Z^1 X^1$

$\square$

Rotation in Z

$|\psi\rangle$ —— $|+\rangle$ —— $|+\rangle$

measure $\quad M(2\theta) \dashrightarrow X$ $\quad\rightarrow$ acquires state

outcome $\quad s_1 \qquad s_2$ $\qquad X^{s_2} Z^{s_1} e^{i\theta\sigma_z}|\psi\rangle$

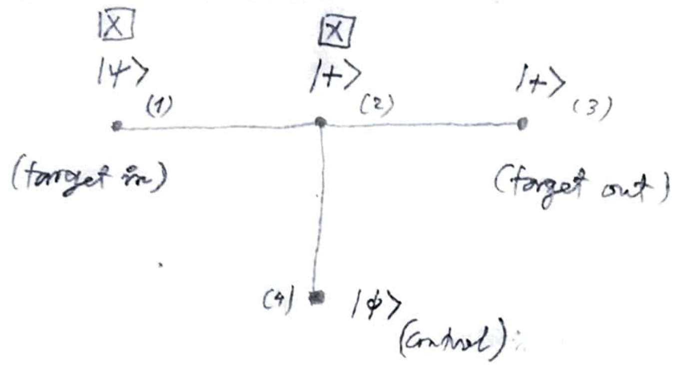
- Let $|\psi\rangle = a|0\rangle + b|1\rangle$ then
  - $e^{i\theta\sigma_z}|\psi\rangle = a e^{+i\theta}|0\rangle + b e^{-i\theta}|1\rangle$

- Cluster state produces

s_1	s_2	out	Correction
0	0	$a\|0\rangle + e^{-2i\theta} b\|1\rangle \cong a e^{i\theta}\|0\rangle + b e^{-i\theta}\|1\rangle$	$Z^0 X^0$
0	1	$e^{-2i\theta} b\|0\rangle + a\|1\rangle \cong b e^{-i\theta}\|0\rangle + a e^{i\theta}\|1\rangle$	$Z^0 X^1$
1	0	$a\|0\rangle - e^{-i\theta} b\|1\rangle \cong a e^{i\theta}\|0\rangle - b e^{-i\theta}\|1\rangle$	$Z^1 X^0$
1	1	$-e^{-2i\theta} b\|0\rangle + a\|1\rangle \cong -e^{-i\theta} b\|0\rangle + a e^{i\theta}\|1\rangle$	$Z^1 X^1$

$\square$

Controlled-NOT



Note → measurements can be simultaneous

measure (1) & (2) in $\boxed{X}$ → outcomes s_1, s_2

so that (3),(4) acquires state $\boxed{Z_4^{s_1} X_3^{s_1} Z_3^{s_2} \, \mathrm{CNOT} |\phi\rangle |\psi\rangle}$ (4) (3)

- Let $|\psi\rangle = a|0\rangle + b|1\rangle$; $|\phi\rangle = c|0\rangle + d|1\rangle$

  then $\mathrm{CNOT}|\phi\rangle|\psi\rangle = ac|00\rangle + bc|01\rangle + ad|11\rangle + bd|10\rangle$

- What cluster state produces

s_1	s_2	out	correction
0	0	$ac|00\rangle + ad|11\rangle + bc|01\rangle + bd|10\rangle$	$Z_4^0 Z_3^0 X_3^0$
0	1	$ac|01\rangle + ad|10\rangle + bc|00\rangle + bd|11\rangle$	$Z_4^0 Z_3^0 X_3^1$
1	0	$ac|00\rangle + ad|11\rangle - bc|01\rangle - bd|10\rangle$	$Z_4^1 Z_3^1 X_3^0$
1	1	$ac|01\rangle + ad|10\rangle - bc|00\rangle - bd|11\rangle$	$Z_4^1 Z_3^1 X_3^1$