AMO Physics 1, MIT OCW

(PH 8.421 – Wolfgang Ketterle)

Oct 31, 2021

Ilnan O. Rui

AMO II : 8.421

Prof. Wolfgang Ketterle

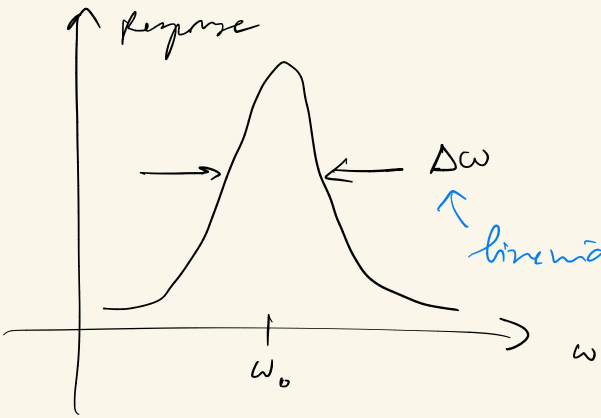
Source : YouTube/MIT OCW

2021 - 2022

Oct 31, 2021

Video 1

RESONANCE 1



Quality factor

$$\boxed{Q = \frac{\omega_0}{\Delta\omega}}$$

Shape : Lorentzian = img part of

$$\frac{1}{\omega - \omega_0 \pm i\Gamma/2}$$

$\Gamma \rightsquigarrow$ decay rate

$e^{-i\omega_0 t - \Gamma t}$

$\tau = 1/\Gamma \rightsquigarrow$ linewidth

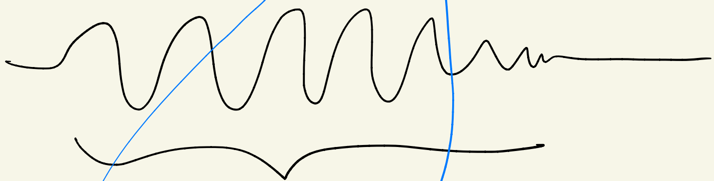
$$\boxed{\Gamma = \Delta\omega = \text{FWHM}}$$

Video 2

RESONANCE 2

How precisely can you measure freq?

Fourier Thm $\boxed{\Delta\omega\, \Delta t \geq 1/2}$ (Heisenberg Uncertainty Relation)



Δt

Variance in distribution of frequency components

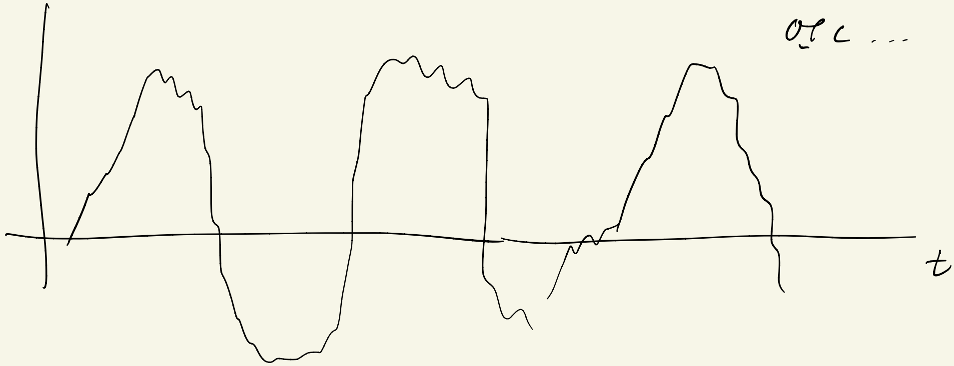
Equality realized for Gaussian wave packets

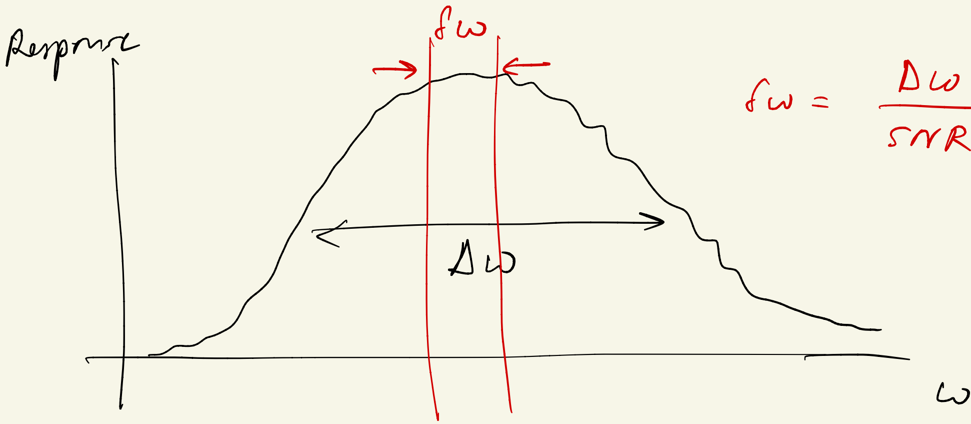
$\boxed{Q1}$ Does $\Delta\omega \Delta t \geq \frac{1}{2}$ hold in classical system?

YES!

(if you have good SNR ratio)

etc ...





$$\delta\omega = \frac{\Delta\omega}{SNR}$$

"splitting the line"

Factor of 100 $\longrightarrow$ straightforward

1000 $\longrightarrow$ challenge ...

$\boxed{Q2}$ Can you measure the freq of a QM harmonic osc in a time Δt to better than $\Delta\omega = 1/2\Delta t$?

YES (will explain later ...)

$\boxed{Q3}$ Same, but observe a laser pulse ?

Is laser classical or quantum oscillator ?

↳ quantum phenomenon, but in classical limit ....

Beat note with another stable laser ...

on PD → get high SNR by using strong local oscillator ...

↳ current out of PD is classical ...

→ So what is the answer ?

(Q3) What about Heisenberg uncertainty relation?

↳ well ... this is about a single measurement.

So... single photon then $\Delta\omega \Delta t \geq 1/2$

But can get higher accuracy by

either
- repeated measurements
- OR multiple photons

Rule of thumb SNR is $\sqrt{n}$ ⟶ $\boxed{n \text{ uncorr. photons}}$

$$\Rightarrow \boxed{\delta\omega = \frac{1}{2\Delta t} \frac{1}{\sqrt{n}}}$$

$\boxed{\text{Correlated photons ... } \quad \delta\omega = \frac{1}{2\Delta t} \frac{1}{n}}$

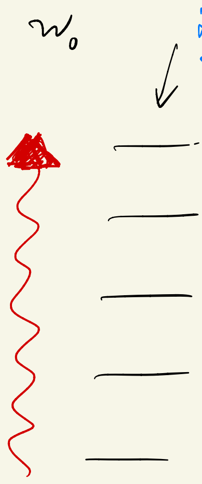
Heisenberg limit.

$\boxed{\text{Q2}}$ quantum HO

→ YES, if have high photon at frequency ω_0 which interact with QHO @ ω_0

NO, if

⇒ we bring in non linear process $\omega = n\omega_0$



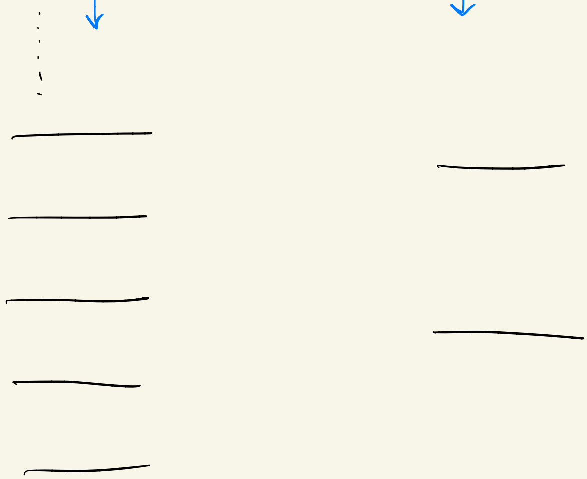
$$\Delta E = \hbar/2\Delta t$$

$$\boxed{\Delta\omega_0 = \Delta E/n = \frac{\hbar}{2\Delta t \cdot n}}$$

non-linear process

Basically go to n^{th} level, measure Energy (E) there with some uncertainty, but then divide E by n to find energy difference between two levels ...

$\boxed{\text{HO vs 2-level system}}$

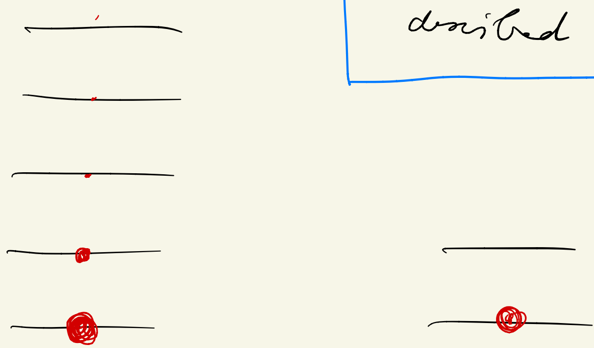


2-level system

$\boxed{\text{weak excitation then atom can be described by HO}}$

(classical)

Difference ...



Put something in the excited state

$\Rightarrow$ 2 level system $\Rightarrow$ to 1st excited state

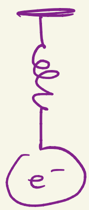
HO $\rightarrow$ same to 2nd state ....

because energy spacings are the same.

more differences ...

- 2 level system → has saturation
- HO → no saturation, large amplitudes

$\boxed{\text{Lorentz oscillator model}}$

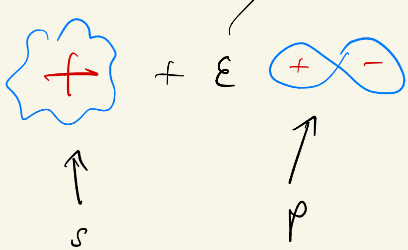


→ get **exact** descriptions like

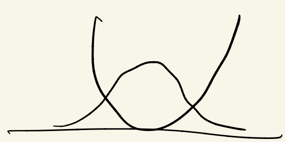
- polarizability
- index of refraction for gas ...

More differences ...

"small" excitation



→ oscillating dipole



$\boxed{\text{Small ??}}$

↳ population of 2^{nd} excited state is negligible

$p_1/p_0 \ll 1 \Rightarrow p_2$ can be neglected

[Ex] Cavity QED

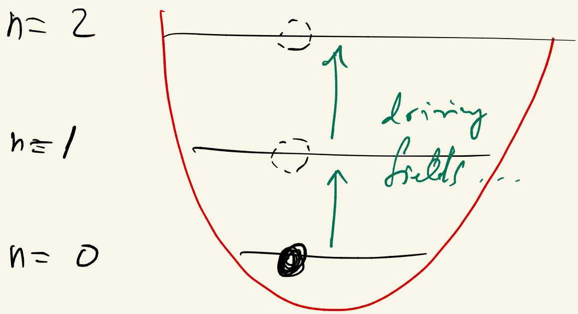
Suppose want 100% population in $n=1$

$\hookrightarrow$ Fock state (cannot be gotten from HO because ... needs to be driven

$\downarrow$

But driven means final state is in superposition ...

$\rightarrow$ can't get 100% in $n=1$)

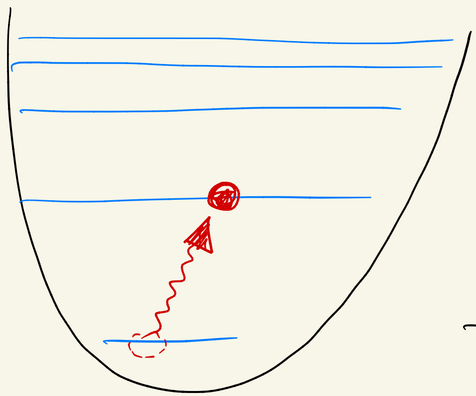


$n=1$ cannot be created @ 100% because of coherent state = superposition of many states ...

But possible with 2 lvl system ... $\rightarrow$ $\boxed{\pi \text{ pulse}}$

• Possible in cavity QED due to anharmonicity

or non linearity



Energy spacings are not constant ...

→ "2-level system"

⇒ By adding an atom to cavity, get

Rabi frequency $\propto \sqrt{n} \to$ # of photon

↓

level splitting etc → get anharmonicity

etc ...

Quantumness of 2 level system comes from

saturation. HO → you can drive

a hard as you can

and response will

still be linear ...

ROTATING SYSTEMS

Are they more classical or $\boxed{\text{quantum}}$?

Ex Gyroscope ... rotating + precessing ...

Rotating system, under excitation, has a maximum amplitude → more like a two level system.

- Precessing gyroscope has a bound on amplitude it can be excited ...
- Indeed, the motion of classical magnetic moment ...

  Excitation spectrum has limit ...

  unlike harmonic oscillator ...

  ↳ actually captures, provides EXACT models essentially all features of QM 2 level system.

$\boxed{?}$ What aspect of 2-level system will we not find? when does this analogy break down?
(Even at low degs?)

Superpositions ... Probabilistic ... Projective measurements ... things like these ...

So, good only upto expectation values ...

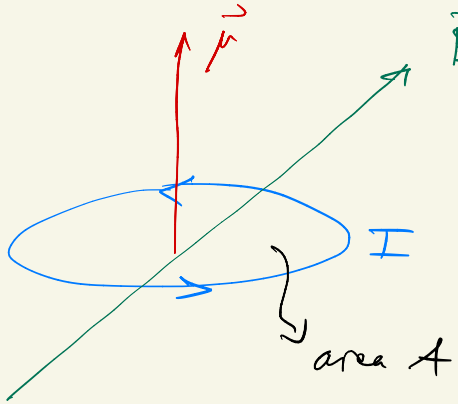
MAGNETIC RESONANCE

$\boxed{?}$ Classical magnetic moment in static / time varying fields (which we assume are spatially <u>uniform</u>)

<u>Interaction energy</u> ...

- $\mathcal{U} = -\vec{\mu} \cdot \vec{B}$
- $\vec{F} = -\vec{\nabla} \mathcal{U} = 0$ for uniform fields
- $\vec{\tau} = \vec{\mu} \times \vec{B}$

Magnetic moment ... Think about



$$\vec{\mu} = A \cdot I\, \hat{e}_z$$

Torque → angular momentum $\vec{L}$

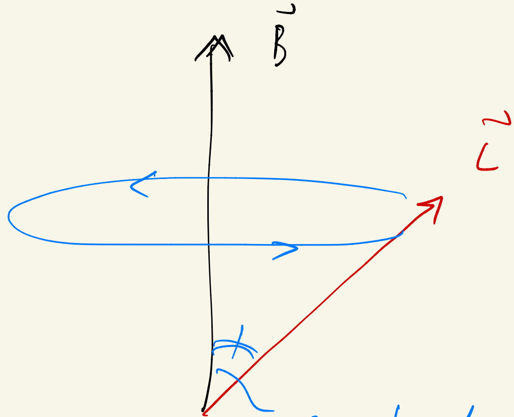
$$\frac{d}{dt}\vec{L} = \vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\boxed{\vec{\mu} = \gamma \vec{L}}$$ ⟶ assume this

gyromagnetic ratio

Then $$\boxed{\dot{\vec{L}} = \gamma \vec{L} \times \vec{B}}$$

↳ pure precession around axis of $\vec{B}$.



precession at LARMOR freq

$$\boxed{\vec{\Omega}_L = -\gamma \vec{B}}$$

angular freq.

constant tipping angle

Ex/ • e^- $\boxed{\gamma_e = 2\pi \cdot 2.8 \text{ MHz}/G}$

• classical charge distribution ...

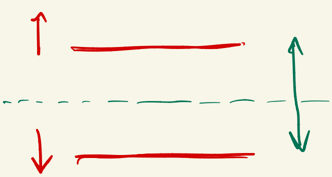
$$\boxed{\Omega_c = \frac{e}{2m} B}$$

$$\boxed{\gamma = 2\pi \cdot \underbrace{1.4 \text{ MHz}}_{\mu_B \text{ (Bohr magneton)}}/G}$$

• proton

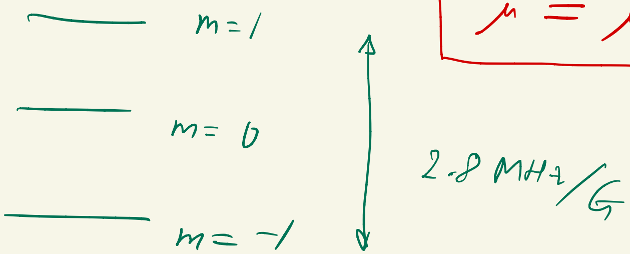
$$\boxed{\gamma_p = 2\pi \cdot 4.2 \text{ kHz}/G}$$

Note: e^-, $s = 1/2$, $\boxed{\mu_e = 1\mu_B}$ → magnetic moment of e^-



$2.8 \text{ MHz}/G = 1.4 - (-1.4)$

e^- vs classical current distribution



$\boxed{\mu = \mu_B}$ as well

- <u>What is precession freq?</u>

2.8 MHz/G
$\boxed{1.4 \text{ MHz/G}}$
0.7 MHz/G

why 1.4 MHz/G ?

$\Longrightarrow$ Same μ, but precession freq is <u>beat</u> <u>note</u>, so if we want to drive ... have to drive at 1.4 MHz/G.

ROTATING COORDINATE TRANSFORMATION

- $\vec{A}$ rotating at angular freq $\vec{\Omega}$ then

$$\boxed{\dot{\vec{A}} = \vec{\Omega} \times \vec{A}}$$

- $$\boxed{\dot{\vec{A}}_{in} = \dot{\vec{A}}_{rot} + \vec{\Omega} \times \vec{A}}$$

2 limiting cases:
$$\begin{cases} \dot{\vec{A}}_{rot} = 0 \Rightarrow \dot{\vec{A}} = \vec{\Omega} \times \vec{A} \\ \vec{\Omega} = 0 \Rightarrow \dot{\vec{A}}_{in} = \dot{\vec{A}}_{rot} \end{cases}$$

operator eqn ...

$$\boxed{\left.\frac{d}{dt}\right|_{rot} = \left.\frac{d}{dt}\right|_{in} - \vec{\Omega} \times}$$

<u>apply this to $\vec{L}$ ...</u>

$$\dot{\vec{L}}_{rot} = \dot{\vec{L}}_{in} - \vec{\Omega} \times \vec{L} = \vec{L} \times (\mu \vec{B} + \vec{\Omega})$$

$$= \gamma \vec{L} \times \left(\vec{B} + \vec{\Omega}/\gamma\right)$$

So ... going to rotating frame, $\vec{B}$ gets replaced by effective $\vec{B}$ field ...

fictitious

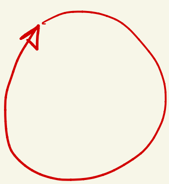
$$\boxed{\vec{B} \longrightarrow \vec{B} + \vec{\Omega}/\gamma \equiv \underbrace{\vec{B} + \vec{B}_r}_{\vec{B}_{eff}}}$$

If we choose $\vec{\Omega} = -\gamma \vec{B}$ then $\vec{B}_{eff} = 0$

$\Rightarrow$ $\vec{L}$ is constant in rotating frame

$\Rightarrow$ To know $\vec{L}$ in lab frame, just transform rotation back ...

Q e^- in $\vec{B}$ $\longrightarrow$ get cyclotron motion



$$\boxed{\Omega_L = \frac{e}{2m} B}$$ Larmor freq.

Cyclotron motion freq ...

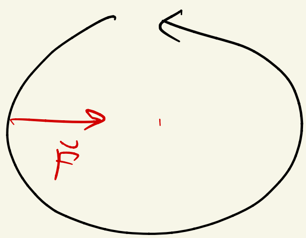
$$\boxed{\Omega_c = 2\Omega_L = \frac{eB}{m}}$$

Larmor's Thm (apply to isolated charges)

↳ can transform away effect of $\vec{B}$ by going to Larmor's frequency...

(Factor of 2 difference between cyclotron motion freq & Larmor freq is due to the difference between free charge vs moments...)

Recall cyclotron motion...



$$\vec{F} = q\vec{v}\times\vec{B} = m\vec{a} = mv^2/r$$

$$\Rightarrow \quad qvBr = mv^2$$

$$qBr = mv$$

$$\boxed{r_{cyc} = mv/qB}$$

$$T = \frac{2\pi r_{cyc}}{v} = \frac{2\pi \cdot m\cancel{v}}{\cancel{v}\, qB}$$

$$\Rightarrow \quad f_{cyc} = qB/2\pi m$$

$$\Rightarrow \quad \boxed{\Omega_{cyc} = qB/m}$$

Video 3

$\boxed{\text{RESONANCE 3}}$

Nov , 2021