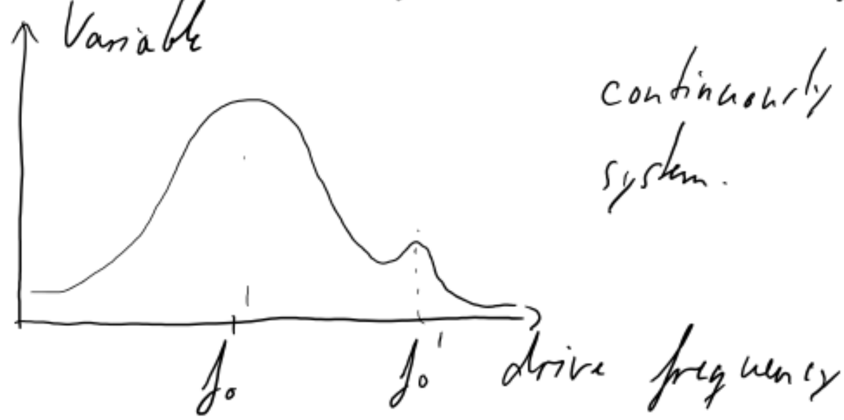
A resonance in a system shows itself if one or more variables of a system change periodically, even after external drives are switched off:

undriven system after some initial excitation

Equivalently, the response of the <u>driven</u> system as a function of drive frequency exhibits a peaked structure.

Maximum response gives resonance frequency



continuously driven system.

The simplest systems have a single resonance

frequency (i.e. harmonic motion).
Damping results in a finite time $\Delta t < \infty$ for the undriven system, and a finite width $\Delta f > 0$ of the resonance curve of the driven system. Δf and Δt are related by a Fourier transform:
Any oscillation with time-varying amplitude must be made of a superposition of different frequency components, giving the resonance curve a finite width in frequency space.

Low dissipation $\Rightarrow$ long Δt
$\Rightarrow$ small Δf

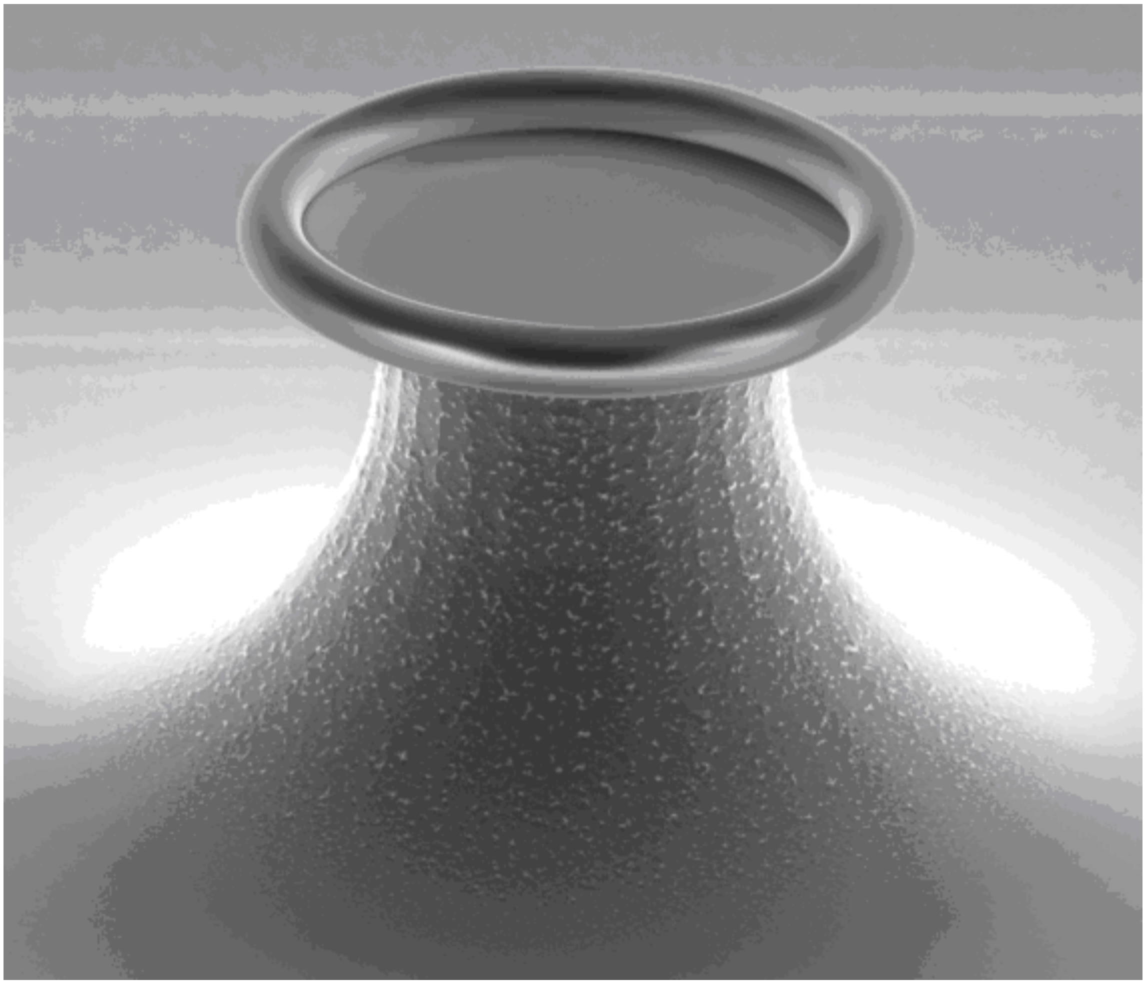
Quality factor:

$$Q = \frac{f_0}{\Delta f}$$

Atomic Physics deals with isolated atomic systems in vacuum $\Rightarrow$ high quality factors!
e.g. Doppler-broadened optical transition in room-temperature gas:

$$Q = \frac{10^{15}\,\mathrm{Hz}}{10^{9}\,\mathrm{Hz}} = 10^6$$

Typically much worse in solids,
mechanical or electrical Qs of 10^3 at
room temperature, $\sim 10^6$ at ~ 1 Kelvin.
Exception: Optomechanical resonators
(whispering gallery modes in spheres
or cylinders of high-purity glass)
$Q \sim 10^9$ (Vahalla group)



Earth rotation
$$Q \sim 10^7$$

Pulsar $Q \sim 10^{10}$

AMO: "Useful resonances"

- Reproducible
- Connected by theory to fundamental constants or other parameters of interest.

Atoms are identical

- change of fundamental constants $< 10^{-15}$ per year (age of the universe ~ 14 billion years)
- Surprises: Anomalous Zeeman effect → Spin (Uhlenbeck & Goudsmit 1925)
  Lamb Shift → QED.
  (~ 1 GHz between $^2S_{1/2}$ and $^2P_{1/2}$)
- Resonances are a tool for control

Classical harmonic Oscillator:

$$\ddot{q} + \gamma \dot{q} + \omega_0^2 q = 0$$

(mass on a spring, charge, voltage or current in RLC circuit...)

Weak damping ($\gamma^2 < 4\omega_0^2$)

$$q \sim e^{-\frac{\gamma}{2}t} e^{\pm i\omega' t}$$

$$\omega' = \omega_0 \sqrt{1 - \frac{\gamma^2}{4\omega_0^2}}$$

$$\approx \omega_0$$

Decay rate constant for <u>amplitude</u> is $\gamma/2$

for <u>stored energy</u> is γ

$\rightarrow$ stored energy decays in $\tau \sim \frac{1}{\gamma}$.

If HO is driven at a frequency ω close to resonance, the energy absorbed $\propto |q_0|^2$ is Lorentzian: Try $q = q_0 e^{i\omega t}$

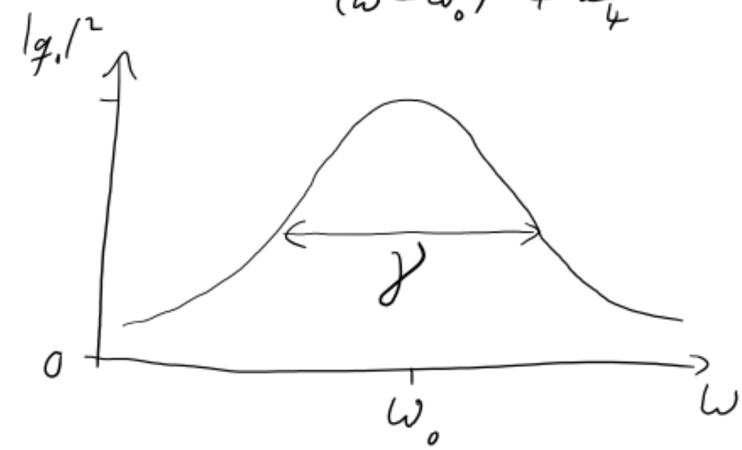
$$-\omega^2 q_0 + i\omega\gamma q_0 + \omega_0^2 q_0 = F_0/m$$

$$\Rightarrow \quad q_0 = \frac{F_0/m}{\omega_0^2 - \omega^2 + i\omega\gamma}$$

$$\underset{\omega \approx \omega_0}{\approx} \frac{\frac{\Gamma_0}{2m\omega_0}}{\omega_0 - \omega + i\frac{\gamma}{2}}$$

$$\propto \frac{\text{const.}}{-\delta + i\frac{\gamma}{2}} \qquad \delta = \omega - \omega_0$$

$$|q_0|^2 = \frac{\text{const.}}{(\omega-\omega_0)^2 + \frac{\gamma^2}{4}}$$



Full width half maximum (FWHM)

$$\Delta\omega = \gamma.$$

$\Rightarrow$ Quality factor $Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\gamma}$.

Note: ω, ω_0 is in $\frac{\text{rad}}{s}$, or short $\frac{1}{s}$.

not to be confused with

frequencies $f_0 = \frac{\omega_0}{2\pi}$

$$\Delta f = \frac{\Delta\omega}{2\pi} \quad \text{etc}$$

[illegible]

we often write explicitly:

$$\omega = 2\pi \cdot 1\ \text{MHz} \quad \text{rather than}$$

$$\omega = 6.28 \cdot 10^6\ \frac{1}{s}.$$

$\boxed{\text{Never}}$ write "$\omega = 6.28 \cdot 10^6\ \text{Hz}$"

For γ we'll write

$$\gamma = 10^4\ \frac{1}{s}, \quad \text{not } 2\pi \cdot 1.6\ \text{kHz}.$$

$$\tau = \frac{1}{\gamma} = 100\ \mu s.$$

Damping time and resonance linewidth obey

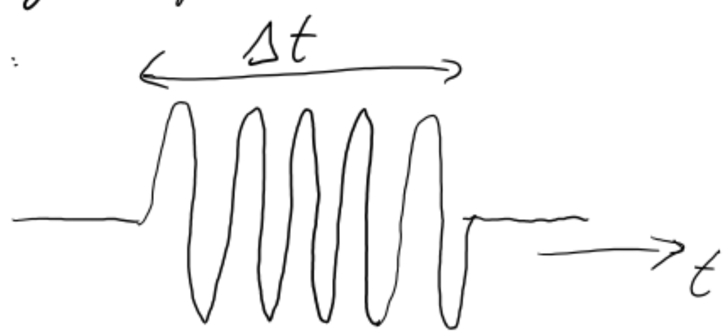
$$\Delta\omega \cdot \tau = 1.$$

or assuming $E = \hbar\omega$

$$\Delta E \cdot \tau = \hbar$$

Need finite range of frequencies to build up decaying pulse.

Similar to:



where according to Fourier, we need a finite spread of frequencies $\Delta\omega$ to describe this finite pulse:

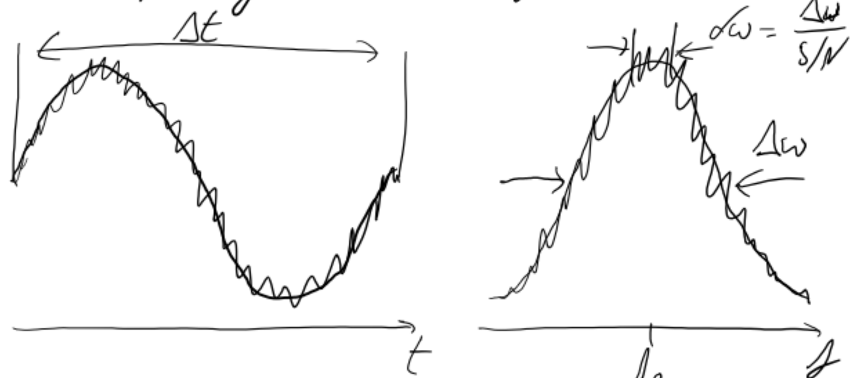
$$\Delta\omega\,\Delta t \geq \frac{1}{2}.$$

$$\Delta E\,\Delta t \geq \frac{\hbar}{2} \qquad \text{Heisenberg uncertainty}$$

$\boxed{\text{Q1:}}$ Can you measure the angular frequency of a classical HO in a time Δt better than

$$\Delta\omega = \frac{1}{2\Delta t}\ ?$$

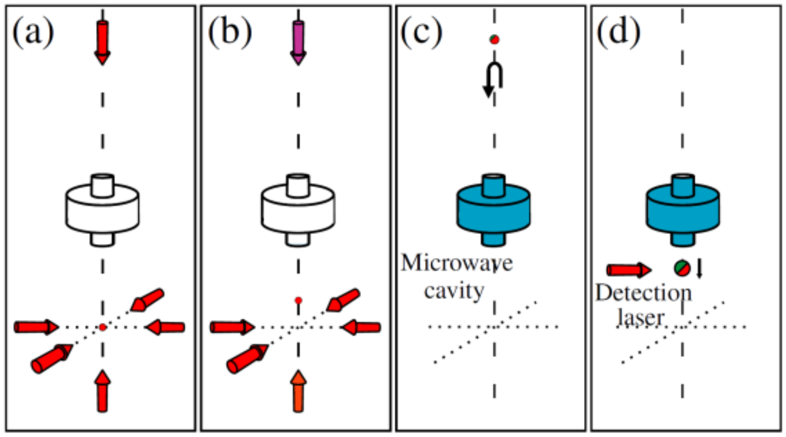
Yes, depending on the signal-to-noise ratio.



in time "splitting the line"

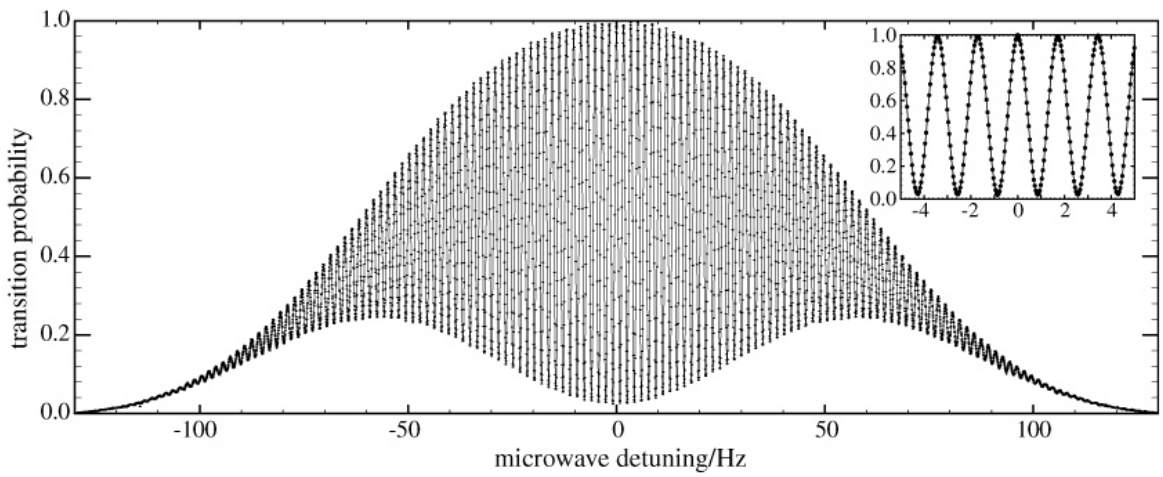
$\Longrightarrow$ extreme example of splitting the line:

Cs Fountain Clock



Principle of Fountain clocks (Wynands, Weyers, Metrologia **42** (2005))

Cs hyperfine transition frequency: **By definition:**
9,192,631,770 Hz



Measured Ramsey fringe pattern in the Physikalisch-Technische Bundesanstalt (PTB Braunschweig) CSF1 Cesium Fountain Clock

$\Delta t \simeq 1\,s$ (from time of flight) but $\frac{\delta f}{f_0} = 10^{-16}$ accuracy

(and not $\frac{1\,\text{Hz}}{10\,\text{GHz}} = 10^{-10}$)

Splitting line by 10^6!

Strontium Optical Atomic Clocks:

$$\frac{\delta f}{f_0} = 10^{-18}$$

$\Delta t \sim 100\,\text{ms}, \quad f_0 \sim 10^{15}\,\text{Hz}$

$$\frac{\Delta f}{f_0} = \frac{1}{f_0 \Delta t} \approx 10^{-14}$$

$\Rightarrow$ splitting the line by $> 10^3$

Q2: Can you measure the angular frequency of a quantum mechanical HO in a time Δt to better than $\Delta\omega = \frac{1}{2\Delta t}$?

Q3: Can you measure the angular frequency of a laser pulse lasting a time Δt to better than

$$\Delta\omega = \frac{1}{2\Delta t} \quad ?$$

Yes. E.g. beat laser with stable reference and record light on a photodiode. The stronger the pulse, the better the SNR.

How can we reconcile CM and QM?

Heisenberg makes a statement about the outcome of a single experiment on a single system.

$\Rightarrow$ Repeat! Or have many identical systems!

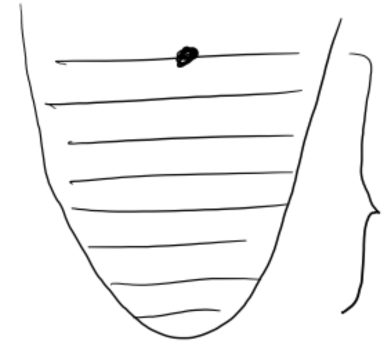
SNR for uncorrelated measurements $\propto \sqrt{N}$ N – photon number.

Correlated photons: SNR $\propto N$

$\Rightarrow$ Heisenberg limit $\Delta\omega = \dfrac{1}{2N\Delta t}$.

Single photon: $\Delta\omega\Delta t \geq \frac{1}{2}$ holds.

For Q2: Could prepare



$$\Delta E > \frac{\hbar}{2\Delta t}$$

but $\Delta\omega = \dfrac{\Delta E}{n\hbar} < \dfrac{1}{2\Delta t}$

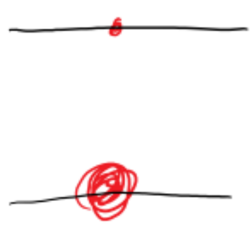
HO and two-level system

Lorentz model for atom

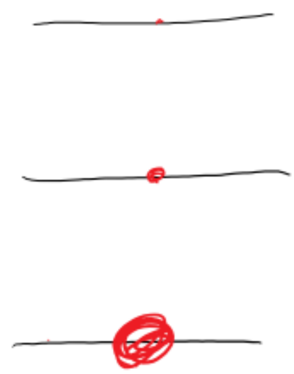
$\Rightarrow$ polarizability

$\Rightarrow$ index of refraction

Why does it work for 2-level systems?

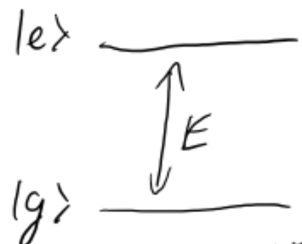


versus



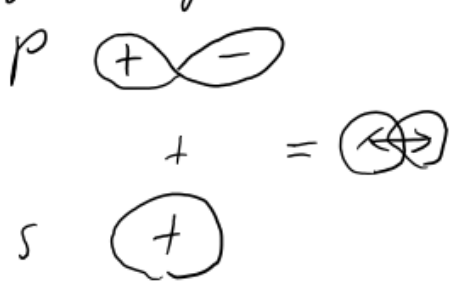
saturation

no saturation
large amplitude
$(q = A \cos(\omega t))$



$|g\rangle + e^{-iEt/\hbar}|e\rangle$

e.g. S and P states of hydrogen atom:



Coherent states
classical oscillation
poissonian distribution
the larger $\langle n \rangle$, the larger the amplitude.

When do classical HO models of two-level systems work?

systems work:

A. When saturation is negligible,

i.e. when population ratio $\frac{\rho_1}{\rho} \ll 1$

When you have saturation

$\Rightarrow$ Use classical periodic system that's naturally bounded: Rotation

Indeed, the motion of a classical magnetic moment captures almost all features of the two-level system (except projection onto one of two possible outcomes in a measurement.