Spring, 2021

Physics 312: Physics of Fluids
Assignment #4 (Solutions)

Background Reading

Friday, Mar. 5: Kundu & Cohen 1.7

Wednesday, Mar. 10: Kundu & Cohen 1.6

Informal Written Reflection

Due: Thursday, March 11 (8 am)

Same overall approach, format, and goals as before!

Formal Written Assignment

Due: Friday, March 12 (in class)

1. In class, we mentioned several historically famous observations that can be understood as simple consequences of the properties of fluids at rest. In this problem, you'll confirm this understanding...

   (a) In his groundbreaking book, *Two New Sciences*, Galileo mentioned a curious observation that suction pumps seem to be unable to raise water to heights more than roughly 10 m. Derive this number using our analysis of pressure variation in a liquid.

   (b) Next, using the fact that mercury is much denser than water, derive a typical height of the mercury column in Torricelli's barometer. This is, again, a consequence of our analysis of pressure variation in a liquid.

(c) Finally, using the well-known result that pressure drops exponentially with height in a gas, show that the drop in atmospheric pressure at the top of 50 m building would indeed be measurable using Torricelli's barometer.

(Hint: To solve these problems, you'll only need to know standard atmospheric pressure, the density of water or mercury, and the relevant formula for pressure variation in a liquid or gas…)

Solution:

(a) The largest possible height is achieved by using atmospheric pressure to support all of the weight of a column of water, i.e., by setting $\rho g h$ equal to p_0, where h is the height of the column:

$$h = \frac{p_0}{\rho g} = \frac{(101.3 \times 10^3 \text{ N/m}^2)}{(10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 10 \text{ m.}$$

(b) This is essentially the same calculation, except the much greater density of mercury produces a much smaller value of h:

$$h = \frac{p_0}{\rho g} = \frac{(101.3 \times 10^3 \text{ N/m}^2)}{(13.534 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 0.763 \text{ m.}$$

(c) Combining the exponential result discussed in class with the calculation using in the previous parts of this problem, we can confirm that the mercury level drops a few cm when brought to the top of a 50-meter building. This is a measurable difference!

$$p = p_0 \exp(-0.050/7.3) = 100.608 \times 10^3 \text{ N/m}^2$$

$$h = \frac{p}{\rho g} = \frac{(100.608 \times 10^3 \text{ N/m}^2)}{(13.534 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 0.758 \text{ m.}$$

2. What fraction of the volume of an iceberg (density 917 $\mathrm{kg/m^3}$) floating in the ocean (salt water, density 1024 $\mathrm{kg/m^3}$) is submerged?

   (Hint: Use Archimedes' principle! The weight of the fluid displaced by the submerged ice must equal the total weight of the iceberg...)

   Solution:

   Let V_i be the volume of the iceberg and $V_s < V_i$ be the volume submerged. Then Archimedes' principle tells us that just under 90% of the iceberg rests below the surface:

$$\begin{aligned}\text{weight of iceberg} &= (917\ \mathrm{kg/m^2}) \times V_i\, g \\ &= (1024\ \mathrm{kg/m^2}) \times V_s\, g = \text{weight of fluid displaced}\end{aligned}$$

$$\Rightarrow\ \text{fraction submerged} \ = \frac{V_s}{V_i} = \frac{917}{1024} = 0.896$$

3. Many fishes have an internal, gas-filled organ that is closely-related, in evolutionary terms, to our lungs. This organ, called the swim bladder, illustrates many of the principles we've been studying...

   (a) Buoyancy control is thought to be a major function of the swim bladder: by adding gas to its bladder, a fish may adjust its density. If the density of a zebrafish with its swim bladder collapsed is 1.05 $\mathrm{g/cm^3}$, calculate how much volume the inflated bladder occupies when the fish is neutrally buoyant? Give your answer as a percentage of the fish's total volume.

   (b) The swim bladder has flexible walls that can expand or contract. Thus, to maintain neutral buoyancy at different depths, a fish may need to add or remove gas from its bladder. This is a simple consequence of the properties of a fluid at rest. Explain!

(c) Evolution has equipped fishes with a remarkable set of strategies for controlling gas content. Read about swim bladders online and, in a few sentences, summarize what you learned. Note that deeper-dwelling fishes with closed bladders cannot rise to the surface too quickly. Use your knowledge of fluids to explain why not.

Solution:

(a) This is similar to the iceberg question. With the bladder inflated, the average density of the fish drops from the given value to that of the fresh water it floats in...

$$\rho_0 = \frac{m}{V} = 1.05 \text{ g/cm}^3 \quad \text{and} \quad \rho_1 = \frac{m}{V + V_{bladder}} = 1.00 \text{ g/cm}^3$$

$$\Rightarrow\ 1.05\,V = (V + V_{bladder})\ \Rightarrow\ V_{bladder} = 0.050\,V$$

Thus, at neutral buoyancy, the percentage of the fish's total volume occupied by the inflated bladder is 4.8%:

$$\frac{V_{bladder}}{V + V_{bladder}} = \frac{0.050\,V}{1.05\,V} = 0.048$$

(b) As a fish dives deeper and deeper, the pressure exerted on the bladder by the surrounding fluid grows linearly. Thus, to maintain neutral buoyancy, this fish needs to increase the pressure in its bladder in proportion. That is, for the swim bladder to be useful at a variety of depths, a fish must have a mechanism for changing the density of gas in the bladder.

(c) Open bladder species can swallow and burp out gas as needed, but closed bladder species need internal gas production mechanisms. The general idea for the closed bladder, as I understand it, is similar to the molecular exchange that goes on in our lungs: gases diffuse into the swim bladder from the bloodstream and the capillary network is designed to allow large pressures to build up. Note that, as a consequence of this design, a fish ascending from the depths needs to allow time for accumulated gases to be absorbed back into the bloodstream. Lots more to say, of course!

4. How do trees move water from their roots to their leaves, which may be tens to hundreds of feet above the forest floor?!

   (a) Read through Kundu and Cohen's discussion of liquid rising in a narrow tube (pgs. 11 and 12). Then work through Kundu and Cohen, Chapter 1, Problem 1.

   (b) Tiny tubes in the xylem of trees transport water and nutrients large distances up the trunk. Calculate, using the same formulas used above, the height that water rising in a 25 μm radius tube of xylem achieves at static equilibrium

   (c) Is your answer in (b) larger or smaller than expected?... Do some reading about xylem online and, in a few sentences, summarize the actual mechanism used by trees to transport water up from the roots to their leaves.

   Solution:

   (a) Use Kundu and Cohen's formula for capillary rise:
   $$h = \frac{2 \times (0.073 \text{ N/m}) \times \sin(90^\circ)}{(10^3 \text{ kg/m}^3) \times (9.8 \text{ m/s}^2) \times (0.0015 \text{ m})} = 9.9 \text{ mm}$$

   (b) Lacking the correct values of σ and α for water on wood, we may as well use the values we used in (a) as estimates:
   $$h = \frac{2 \times (0.073 \text{ N/m}) \times \sin(90^\circ)}{(10^3 \text{ kg/m}^3) \times (9.8 \text{ m/s}^2) \times (2.5 \times 10^{-5} \text{ m})} = 60 \text{ cm}$$

   (c) The height of a typical tree is one to two orders of magnitude larger than our answer in (b). Capillary forces alone, it turns out, cannot pull water up to these great heights. The actual mechanism uses evaporation from the leaves and hydrogen bonding to create the suction required.

5. Two of the most familiar examples of surface tension phenomena are bubbles and droplets and, recently, the role they play in the spread of infectious disease has attracted considerable attention in the fluid dynamics community. Using our library's subscription to the Scopus database of peer-reviewed scientific publications, find and read a highly influential paper on bubbles or droplets and summarize something you learned. Note that Scopus will let you organize your search results by date, timed cited, and so on, which can be helpful when you're looking for recent papers, popular papers, etc. To keep things simple, limit your search to articles from the following ten physics-friendly sources:

   - Science
   - Nature
   - Nature Physics
   - Physical Review Letters
   - Physical Review E
   - Physical Review Fluids
   - Physics of Fluids
   - Journal of Fluid Mechanics
   - Annual Rev. of Fluid Mech.
   - Review of Modern Physics

   If possible, *be ready to discuss your efforts in our meeting on Thursday*! And, above all, *have fun* with your exploration!