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MA355 : HW9

Supp #1 p74

(a) $\binom{n+k-1}{k}$

(b) n^k

(c) $\binom{n}{k}$

(d) $n^{\underline{k}}$

(e) none of the above

(f) $\binom{k-1}{n-1}$

Supp #2 p74

(a) $s^{\underline{r}}$

(b) s^r

(c) $\binom{s}{r}$

(d) $r^{\underline{s}}$

(e) $\binom{s+r-1}{s}$

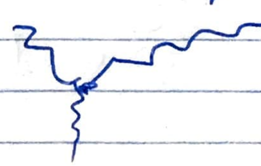
(f) $(r+s-1)^{\underline{r}}$

(g) $(r)^{\underline{s}}(r-1)^{\underline{r-s}}$

(h) $\binom{r-1}{s-1}$

144

need tree of exactly 3 vertices of deg = 1

→ tree looks like  → 1 vertex of deg 3

→ need to break $n-1$ into 3 parts ⇒ there are $\binom{n-2}{2}$ of these (by star-and-bar argument).

$n!$ ways to label, but need to divide by $3!$ because the orientation of the branches doesn't matter

→ $\boxed{\frac{1}{6}n!\binom{n-2}{2}}$

146 9 distinct sandwiches
3 distinct bags
each bag gets at least one:

$\hookrightarrow$ $\boxed{S(9,3)\cdot 3!}$

147 9 distinct sandwiches
3 distinct bags
each gets exactly 3:

$\boxed{\binom{9}{3}\binom{6}{3}}$

149 $f : K \to N$

K ↓ k elements, N ↓ n elements.

$\Rightarrow$ f defines a multiset from N

$\to$ to count # functions $f : K \to N$, we have to add up all the ways to label the elements of a k-element set with n distinct labels so that label i is used j_i times..

$$\to \# = \sum_{\sum j_i = n} \binom{k}{j_1, j_2, j_3, \ldots, j_n}$$

$$= \sum_{\sum j_i = n} \binom{k}{j_1, j_2 \ldots, j_n} x_1^{j_1} x_2^{j_2} x_3^{j_3} \ldots x_n^{j_n} \Big|_{x_i = 1\ \forall i}$$

$$= (x_1 + \ldots + x_n)^k \Big|_{x_i = 1\ \forall i}$$

$$= n^k.$$

150 onto $\Rightarrow$ The total is $n!\, S(k,n)$ (problem 143)

$$n!\, S(k,n) = \sum_{\substack{\sum j_i = n \\ j_i \geq 1 \,\forall i}} \binom{k}{j_1, j_2, \ldots, j_n}$$

158 $P(6,3) = 3$

$6 = 4+1+1$
$= 3+2+1$
$= 2+2+2$ $\rightarrow$ $\boxed{3}$ ways to put 6 identical apples into three ~~parts~~ id. bags so that each bag has at least 1.

161 partition of k obviously sums to k

Vectors ~~set.~~ $(x_1, x_2, \ldots, x_n)$ s.t. $\sum i x_i = k$ ~~are exactly partitions of k.~~ describes the partitions of k

i.e. they represent the partitions of k. Each x_i is the multiplicity of i in the partition.

$(2,1) \equiv (2,1,0,0)$

$x_1 = 2$
$x_2 = 1$ $\rightarrow$ $\boxed{\sum i x_i = 1 \cdot 2 + 2 \cdot 1 = 4}$.

Decreasing list representation for 4 in this case is

$\boxed{4 = 2+1+1}$

162 # partitions of k+1 where smallest part is one is equal to the # of partitions of k.

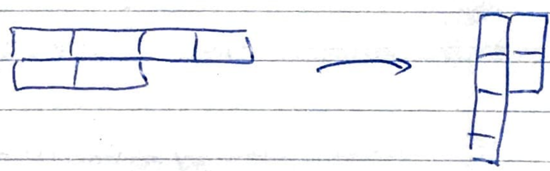
↳ I think this is fairly obvious, so I won't explain much further (there's really nothing to explain).

166 # of partitions of k into even parts is equal to the # of partitions of k into parts of even multiplicity

This is easy to see via conjugation.

Since conjugation is bijective, each partition into even parts is transformed into a partition w/ even multiplicity.

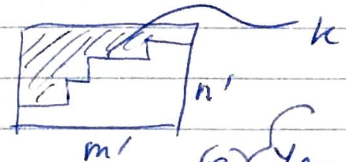
Ex



each row appears an even # times b/c the differences between parts ~~in the~~ in the original partition are even.

168 (a) Because it is a decreasing list ~~representation~~ → it represents a partition of some number.

(b) $\boxed{m'n' - k}$



(c) $\boxed{\begin{matrix} m' > m \\ n' = n \end{matrix}}$

(f) Yes.

(e) Yes, $n' = n$, $m' - n$ = smallest prt

(d) $m' - m$ = ~~smallest~~ smallest part, $n' = n$ $\boxed{\text{OR}}$ $m' = m$, $n' > n$