GENERAL RELATIVITY & COSMOLOGY

A Quick Guide

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1 Overview and Review

What is general relativity? It's a theory of gravity.

Replaces Newton's law of gravity, for heavy masses and high precision.

Keep in mind, GR is not compatible with Quantum Mechanics.

Question in Physics: how to reconcile GR and QM?

1.1 Review of Special Relativity

1.2 The Equivalence Principle

1.3 Versions of the Equivalence Principle

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2 Review of Multivariable and Vector Calculus

3 Flat 3-dimensional space

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3.2 Contravariant and covariant vectors

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4 Flat spacetime

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7 Einstein's field equations

7.1 The stress-energy tensor $T^{\mu\nu}$

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8 Predictions and tests of general relativity

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9.1.1 Cosmological principle

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9.2 Dynamical evolution of the universe

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9.2.2 The cosmological constant Λ

9.2.3 Equations of state

9.2.4 A matter-dominated universe ($\Lambda = 0$) [Friedmann models]

9.2.5 A flat, metter-dominated universe ($\Lambda = 0$) [old favorite model]

9.3 Observational cosmology

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9.3.3 Matter densities and dark matter

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9.4.3 Concordance model [new favorite model]

9.4.4 Open questions