Classical Mechanics III (8.09 & 8.309) Fall 2021
Assignment 8

Massachusetts Institute of Technology
Physics Department
Mon. November 15, 2021

Due Mon. November 22, 2021
6:00pm

Announcements

This week we will continue our study of fluids, now including viscosity.

- On this problem set, **8.09 students** should do six problems 1,3,4,5,6,7 and **8.309 students** should do all 7 problems. Problem 1 is carried out slightly differently for the two groups, as described below. Note that the last two problems are short but important examples of the application of dimensional analysis.

Reading Assignment

- For ideal fluids and sound waves, read sections 8.6–8.10, 8.13, and 8.14 from the Mechanics book by **Symon** (scanned and available on the website).
- For fluids with viscosity the readings are from Landau and Lifshitz, Fluid Mechanics, Chapter II, sections 15-17 and 19-20 (which is available on the website).
- Further reading on fluids, including lots of worked examples and plenty of problems, can be found in the book "A physical introduction to Fluid Mechanics" by Alexander Smits.

Problem Set 8

In these problems we explore both ideal and viscous fluids. The last two problems use dimensional analysis.

1. **Flow Geometries** [everyone: 12 points for 8.09, 6 points for 8.309]

   8.09 students do (a),(b),(c) for 4 points each. 8.309 students should PICK ANY TWO of the three parts to do, for 3 points each.

   The velocity potential for certain potential flows of an ideal fluid is given in cylindrical coordinates (r, θ, z) by a function $\phi = \phi(r, \theta)$. Find the geometry of a fixed surface that is consistent with each of these potential flows:

   (a) $\phi = Cr^2 \cos(2\theta)$

   (b) $\phi = Cr^4 \cos(4\theta)$

   (c) $\phi = Cr^{2/3} \cos\left(\dfrac{2\theta}{3}\right)$

   Identify any stagnation points at finite r, and roughly sketch a streamline near the surface for each case.

2. **The Stream Function** [8.309 ONLY, 6 points]

   Consider an ideal incompressible fluid in two-dimensions (the x-y plane). The stream-function $\psi(x, y)$ is defined so that $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$, which guarantees that $\nabla \cdot \vec{v} = 0$.

   (a) [2 points] Show that lines of constant ψ are streamlines (hence ψ's name).

   (b) [4 points] Show that if the flow is also irrotational that ψ satisfies Laplace's equation. For $\psi(r, \theta) = \frac{2}{3} r^{3/2} \sin(\frac{3}{2}\theta)$ with polar coordinates (r, θ), what is the corresponding velocity potential $\phi(r, \theta)$?

3. **Ideal Fluid Flow around a Cylinder** [everyone, 16 points]

   Consider an infinitely long solid cylinder of radius R with its axis along $\hat{z}$, which is immersed in an incompressible irrotational ideal fluid of density ρ. At $x = -\infty$ the fluid flows with a constant velocity $\vec{u} = u\hat{x}$.

   (a) [6 points] Using Laplace's equation find the steady potential flow around the cylinder. Solve for the velocity field $\vec{v}$. Be sure that you satisfy the boundary conditions.

   (b) [4 points] Derive a result for the pressure on the cylinder.

   (c) [6 points] Derive a differential equation for the streamlines for this flow. Show that $x^2 + y^2 = R^2/(1 - K/y)$ is a solution of your equation, where K is an arbitrary constant.

4. **A Spherical Sound Wave** [10 points, everyone]

   A sound wave is generated in air by a point source.

   (a) [3 points] Show by direct computation that the spherical pressure wave

   $$p = \frac{1}{r}\, f(r - c_s t)$$

   satisfies the sound wave equation for any differentiable function f.

   (b) [4 points] Find the corresponding velocity field $\vec{v}$. To facilitate grouping the time dependence of your solution into the combination $r - c_s t$, let $g(x) = \int^x dx' f(x')$.

   (c) [3 points] Cross check your answer in (b) by verifying that your $\vec{v}$ also satisfies the wave equation. Note that in spherical coordinates the Laplacian on a vector field $\nabla^2(\phi(r)\hat{r}) = [\nabla^2\phi(r)]\hat{r} - \frac{2}{r^2}\phi(r)\hat{r}$.

5. **Viscous Fluid Velocity between Coaxial Cylinders** [10 points, everyone]

   Two very long cylinders are coaxial with $\hat{z}$. The inner cylinder is held at rest and has a radius R_1. The outer cylinder has radius R_2 and moves parallel to its axis with a constant velocity $u\hat{z}$. A viscous fluid between the two cylinders exhibits a steady flow, and there is no pressure gradient in the z direction. What is the velocity profile of the fluid? What is the friction force per unit length for each cylinder?

6. **Vortex Shedding with Dimensional Analysis** [6 points, everyone]

   Wind blows over a cylindrical chimney and vortices are shed into the wake. The frequency of vortex shedding f depends on the chimney diameter D, its length L, the wind velocity V, and the kinematic viscosity of air ν.

   (a) [3 points] Use dimensional analysis to write down a general functional relation for f in terms of the other parameters.

   (b) [1 point] If on another day we observe a second chimney whose diameter and length are $1/2$ those of the original one, then what must the velocity of the wind be for there to be a chance to use a similarity analysis for the two chimneys?

   (c) [2 points] Taking the velocity of wind for the second chimney as in (b), what is the frequency of vortex shedding for the second chimney relative to the first?

7. **Golf Ball Drag with Dimensional Analysis** [6 points, everyone]

   The drag force F_D on a golf ball depends on its velocity V, its diameter D, its spin rate ω measured in radians/sec, the air density ρ and air viscosity η, and the speed of sound c_s.

   (a) [3 points] Use dimensional analysis to find a general functional relation for F_D in terms of the other parameters.

(b) [3 points] Suppose that the speed of sound is not important. How does your result in (a) change? With this assumption, if an experiment is carried out with air in a wind tunnel at velocity $2V$, what diameter and spin rate are needed for the results to be dynamically similar to those for the velocity V?