1. Recall that to do teleportation, as described in class, Alice and Bob start with the state $\frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$, and Alice measured the bit she wished to teleport and her half of the EPR pair in the Bell basis. She then tells the measurement result to Bob, and he applies a Pauli matrix according to the following table:

Alice's measurement result	Bob's unitary transform
$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$	id
$\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$	σ_x
$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$	σ_z
$\frac{1}{\sqrt{2}}(\|01\rangle - \|10\rangle)$	σ_y

Suppose Alice and Bob start with the Bell state $\frac{1}{\sqrt{2}}(|01\rangle_{AB} - |10\rangle_{AB})$. What table should they use?

2. The Deutsch-Jozsa algorithm distinguishes between functions which are balanced and functions which are constant. Suppose it is handed a function which is neither balanced nor constant; that is, suppose the function has r values for which $f(x) = 0$ and s values for which $f(x) = 1$, where $r + s = 2^n$. What is the probability that the Deutsch-Jozsa algorithm will output that the function is constant?

3. (20 points)

Consider the following four states in the two-qubit system where one qubit is held by Alice and the other by Bob:

$$|0\rangle_A \otimes |0\rangle_B\,,\quad |0\rangle_A \otimes |1\rangle_B\,,\quad |1\rangle_A \otimes |+\rangle_B\,,\quad |1\rangle_A \otimes |-\rangle_B\,,$$

(a) Explain why these four states form an orthonormal basis.

(b) Suppose Alice and Bob are in different laboratories connected only by a telephone line. Show that they can identify these four states unambiguously.

(c) Now, suppose Alice and Bob are forced to make simultaneous measurements. That is, they agree beforehand on some measurement strategy. They go to their labs and make measurements (during this time they cannot communicate with each other). They then pick up the telephone and try to decide which of the four original states they had.

They would like to find a way to identify these four states unambiguously under this restriction. Unfortunately, it is the case that they cannot. Give as good an explanation as you can for why they cannot.

(d) Now, suppose Alice and Bob share an EPR pair in the state $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$. They again are required to measure their states simultaneously (possibly involving the EPR pair in their measurements) and then communicate afterwards to determine their states. Can they identify their four states unambiguously now?

Hint: it might help for Alice or Bob to apply CNOT gates to their qubits.

4. Suppose Alice, Bob, and Charlie hold the GHZ state

$$\frac{1}{\sqrt{2}}\big(\,|000\rangle_{ABC} + |111\rangle_{ABC}\,\big).$$

   (a) Show that if they are not allowed to communicate, there is no way for them to arrange for Alice and Charlie to end up sharing the EPR state

$$\frac{1}{\sqrt{2}}\big(\,|00\rangle_{AC} + |11\rangle_{AC}\,\big).$$

   (b) Show if they are allowed to communicate classical information, but not quantum states, Alice and Charlie can end up with the EPR state

$$\frac{1}{\sqrt{2}}\big(\,|00\rangle_{AC} + |11\rangle_{AC}\,\big).$$