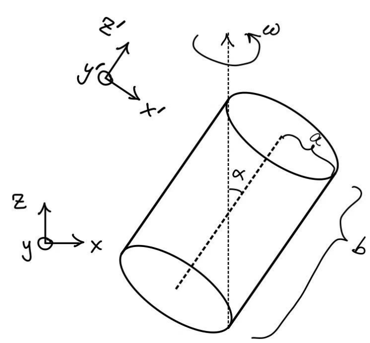
8.(3)09 Section 4

October 1, 2021

1 Moment of Inertia of an Off-Balance, Non-Uniform Cylinder

Consider a cylinder spinning:



Take the cylinder to have non-uniform density given by the form $\rho(r) = \rho_0(r/a)^2$, where r is the perpendicular distance from the z' axis.

(a)

Find the moment of inertial of the cylinder around the center of mass in the primed axes.

(b)

Perform a rotation to find the moment of inertia of the cylinder in the unprimed axes.

(c)

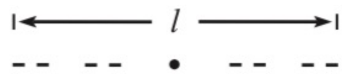
Find the cylinder's kinetic energy.

2 Moment of Inertia of Fractals (Morin Problem 8.8)

Find the moment of inertia of each of the following objects.

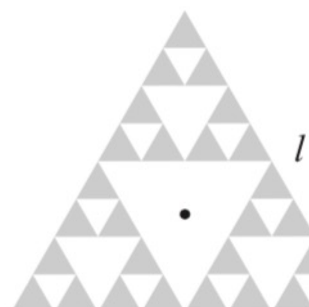
(a)

Consider a stick of length ℓ and remove the middle third. Then remove the middle third from each of the remaining two pieces. Continue removing the middle third from each remaining piece. Repeat infinitely many times (you may recognize this as analogous to the Cantor set). Let the final object have mass m and let the axis be through the center, perpendicular to the stick.



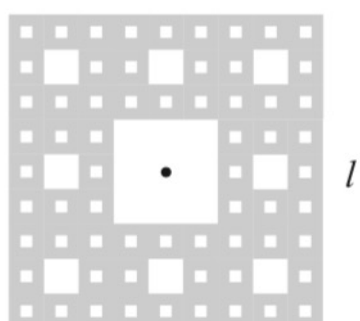
(b)

Take an equilateral triangle of side ℓ and remove the "middle" triangle (1/4 of the area). Then remove the "middle" triangle from each of the remaining three triangles, and so on, forever. Let the final object have mass m and the axis be through the center, perpendicular to the plane.



(c)

Take a square of side ℓ and move the "middle" square (1/9) of the area. Then remove the "middle" square from each of the remaining eight squares, and so on, forever. Let the final object have mass m and let the axis be through the center, perpendicular to the plane.



(d)

Define the fractal dimension of an object to be the number d for which r^d is the increase in "volume" when the dimensions are increased by a factor r. Find the fractal dimensions of the objects described in (a), (b), and (c).