8.421 AMO I LECTURE NOTES

Preface

The original incarnation of these notes was developed to accompany the lectures in the MIT graduate courses in atomic physics. AMO I was created in the late 1960s as a one-term introductory course to prepare graduate students for research in atomic physics in the Physics department. Over the years Dan Kleppner and David Pritchard changed the contents of the course to reflect new directions of research, though the basic concepts remained as a constant thread. With the growth of interest in atom cooling and quantum gases, a second one-term course, AMO II, was designed by Wolfgang Ketterle in the late 1990s and presented with AMO I in alternating years. We still teach AMO I in the traditional way. These lecture notes combine the (g)olden notes of Dan and Dave. As part of the Joint Harvard/MIT Center for Ultracold Atoms summer school in Atomic Physics in 2002, John Doyle got involved and improved the notes. They have been circulated since and at some point were put into the form of an AMO Wiki. At this moment in time, I'd like to resurrect them in their traditional paper form, and only carefully add topics as I see fit.

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1. CHAPTER 1
THE TWO-STATE SYSTEM: RESONANCE

1.1. **Introduction.** The cornerstone of major areas of contemporary Atomic, Molecular and Optical Physics (AMO Physics) is the study of atomic and molecular systems through their resonant interaction with applied oscillating electromagnetic fields. The thrust of these studies has evolved continuously since Rabi performed the first resonance experiments in 1938. In the decade following World War II the edifice of quantum electrodynamics was constructed largely in response to resonance measurements of unprecedented accuracy on the properties of the electron and the fine and hyperfine structure of simple atoms. At the same time, nuclear magnetic resonance and electron paramagnetic resonance were developed and quickly became essential research tools for chemists and solid state physicists. Molecular beam magnetic and electric resonance studies yielded a wealth of information on the properties of nuclei and molecules, and provided invaluable data for the nuclear physicist and physical chemist. With the advent of lasers and laser spectroscopy these studies evolved into the creation of new species, such as Rydberg atoms, to studies of matter in ultra intense fields, to fundamental studies in the symmetries of physics, to new types of metrology. With the advent of laser cooling and trapping, these techniques led to the creation of novel atomic quantum fluids, from Bose-Einstein condensates to strongly interacting Fermi gases.

Resonance techniques may be used not only to learn about the structure of a system, but also to prepare it in a precisely controlled way. Because of these two facets, resonance studies have led physicists through a fundamental change in attitude - from the passive study of atoms to the active control of their internal quantum state and their interactions with the radiation field. The chief technical legacy of the early work on resonance spectroscopy is the family of lasers which have sprung up like the brooms of the sorcerer's apprentice. The scientific applications of these devices have been prodigious. They caused the resurrection of physical optics - what we now call quantum optics - and turned it into one of the liveliest fields in physics. They have had a similar impact on atomic and molecular spectroscopy. In addition, lasers have led to new families of physical studies such as single particle spectroscopy, multiphoton excitation, cavity quantum electrodynamics, and laser cooling and trapping. This chapter is about the interactions of a two-state system with a sinusoidally oscillating field whose frequency is close to the natural resonance frequency of the system. The term "two-level" system is sometimes used, but this is less accurate than the term two-state, because the levels could be degenerate, comprising several states. However, its misusage is so widespread that we adopt it anyway. The oscillating field will be treated classically, and the linewidth of both states will be taken as zero until near the end of

the chapter where relaxation will be treated phenomenologically. The organization of the material is historical because this happens to be also a logical order of presentation. The classical driven oscillator is discussed first, then the magnetic resonance of a classical spin, and then a quantized spin. The density matrix is introduced last and used to treat systems with damping - this is a useful prelude to the application of resonance ideas at optical frequencies and to the many real systems which have damping.
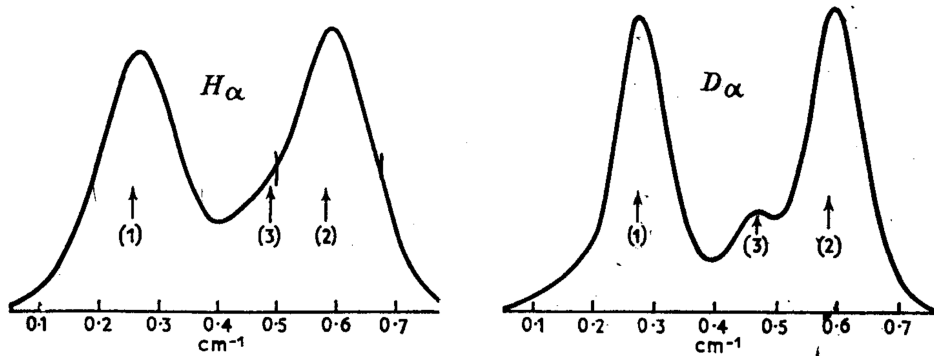

FIGURE 1. Spectral profile of the H_α line of atomic hydrogen by conventional absorption spectroscopy. Components 1) and 2) arise from the fine structure splitting. The possibility that a third line lies at position 3) was suggested to indicate that the Dirac theory might need to be revised. (From "The Spectrum of Atomic Hydrogen"- Advances. G.W. Series ed., World Scientific, 1988).

1.2. **Resonance Studies and Q.E.D..** One characteristic of atomic resonance is that the results, if you can obtain them at all, are generally of very high accuracy, so high that the information is qualitatively different from other types. The hydrogen fine structure is a good example.

In the late 1930s there was extensive investigation of the Balmer series of hydrogen, ($n > 2 \rightarrow n = 2$). The Dirac Theory was thought to be in good shape, but some doubts were arising. Careful study of the Balmer-alpha line ($n = 3 \rightarrow n = 2$) showed that the line shape might not be given accurately by the Dirac Theory.

Pasternack, in 1939, suggested that the $2\,^2S_{1/2}$ and $2\,^2P_{1/2}$ states were not degenerate, but that the energy of the $2\,^2S_{1/2}$ state was greater than the Dirac value by $\sim .04\,\mathrm{cm}^{-1}$ (or, in frequency, $\sim 1200\,\mathrm{MHz}$). However, there was no rush to throw out the Dirac theory on such flimsy evidence.

In 1947, Lamb found a splitting between the $^2S_{1/2}$ and $^2P_{1/2}$ levels using a resonance method. The experiment is one of the classics of physics. Although his

very first observa- tion was relatively crude, it was nevertheless accurate to one percent. He found

$$S_H = \frac{1}{h} \left[E(^2S_{1/2}) - E(^2P_{1/2}) \right] = 1050(10)\,\text{MHz} \tag{1.1}$$

The inadequacy of the Dirac theory was inescapably demonstrated.

The magnetic moment of the electron offers another example. In 1925, Uhlenbeck and Goudsmit suggested that the electron has intrinsic spin angular momentum $s = 1/2$ (in units of $\hbar$) and magnetic moment

$$\mu_e = \frac{e\hbar}{2m} = \mu_B \tag{1.2}$$

where μ_B is the Bohr magneton. The evidence was based on studies of the multiplicity of atomic lines (in particular, the Zeeman structure). The proposal was revolutionary, but the accuracy of the prediction that $\mu_e = \mu_B$ was poor, essentially one significant figure. According to the Dirac theory, $\mu_e = \mu_B$, exactly. However, our present understanding is

$$\frac{\mu_e}{\mu_B} - 1 = 1.1596521884(43) \times 10^{-3} \qquad \text{(experiment, U. of Washington)} \tag{1.3}$$

This result is in good agreement with theory, the limiting factor in the comparison being possible doubts about the value of the fine structure constant.

The Lamb shift and the departure of μ_e from μ_B resulted in the award of the 1955 Nobel prize to Lamb and Kusch, and provided the experimental basis for the theory of quantum electrodynamics for which Feynman, Schwinger and Tomonaga received the Nobel Prize in 1965.

1.2.1. *The language of resonance: a classical damped system.* Because the terminology of classical resonance, as well as many of its features, are carried over into quantum mechanics, we start by reviewing an elementary resonant system. Consider a harmonic oscillator composed of a series RLC circuit. The charge obeys

$$\ddot{q} + \gamma \dot{q} + \omega_0^2 q = 0 \tag{1.4}$$

where $\gamma = R/L$, $\omega_0^2 = 1/LC$. Assuming that the system is underdamped (i.e. $\gamma^2 < 4\omega_0^2$), the solution for q is a linear combination of

$$\exp\left(-\frac{\gamma}{2}\right) \exp\left(\pm i\omega' t\right) \tag{1.5}$$

where $\omega' = \omega_0 \sqrt{1 - \gamma^2/4\omega_0^2}$. If $\omega_0 \gg \gamma$, which is often the case, we have $\omega' \equiv \omega_0$. The energy in the circuit is

$$W = \frac{1}{2C} q^2 + \frac{1}{2} L \dot{q}^2 = W_0 e^{-\gamma t} \tag{1.6}$$

where $W_0 = W(t = 0)$. The decay time of the stored energy is $\tau = \frac{1}{\gamma}$. If the circuit is driven by a voltage $E_0 e^{i\omega t}$, the steady state solution is $q_0 e^{i\omega t}$ where

$$q_0 = \frac{E_0}{2\omega_0 L} \frac{1}{(\omega_0 - \omega + i\gamma/2)}. \tag{1.7}$$

(We have made the usual resonance approximation: $\omega_0^2 - \omega^2 \approx 2\omega_0(\omega_0 - \omega)$.) The average power delivered to the circuit is

$$P = \frac{1}{2}\frac{E_0^2}{R}\frac{1}{1 + \left(\frac{\omega - \omega_0}{\gamma/2}\right)^2} \tag{1.8}$$

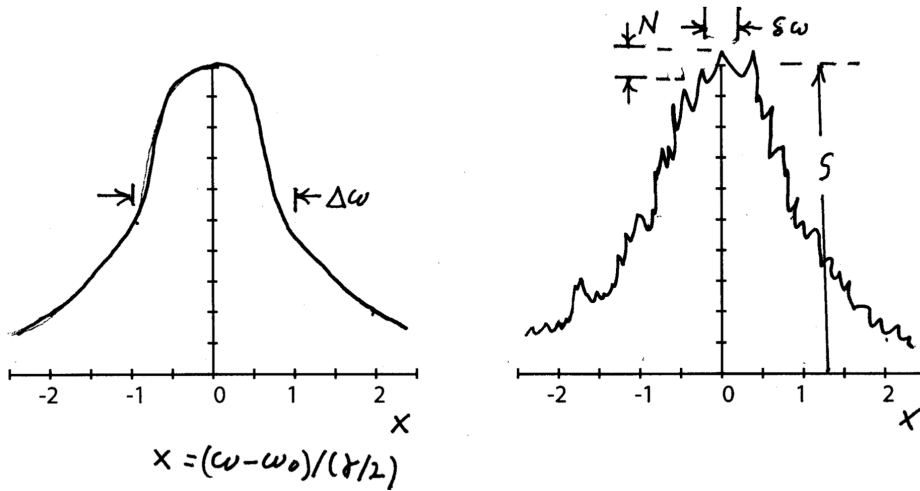

$$x = (\omega - \omega_0)/(\gamma/2)$$

FIGURE 2. Sketch of a Lorentzian curve, the universal response curve for damped oscillators and for many atomic systems. The width of the curve (full width at half maximum) is $\Delta\omega = \gamma$, where γ is the decay constant. The time constant for decay is $\tau = \gamma$. In the presence of noise (right), the frequency precision with which the center can be located, $\delta\omega$, depends on the signal-to-noise ratio, S/N: $\delta\omega = \Delta\omega/(S/N)$.

The plot of P vs ω (Fig. 2) is universal resonance curve often called a "Lorentzian curve". The full width at half maximum ("FWHM") is $\Delta\omega = \gamma$. The quality factor of the oscillator is

$$Q = \frac{\omega_0}{\Delta\omega} \tag{1.9}$$

Note that the decay time of the free oscillator and the linewidth of the driven oscillator obey

$$\tau \Delta\omega = 1 \tag{1.10}$$

This can be regarded as an uncertainty relation. Assuming that energy and frequency are related by $E = \hbar\omega$ then the uncertainty in the energy is $\Delta E = \hbar\Delta\omega$ and

$$\tau\Delta E = \hbar \tag{1.11}$$

It is important to realize that the Uncertainty Principle merely characterizes the spread of individual measurements. Ultimate precision depends on the experimenter's skill: the Uncertainty Principle essentially sets the scale of difficulty for his or her efforts.

The precision of a resonance measurement is determined by how well one can "split" the resonance line. This depends on the signal to noise ratio (S/N) (see Fig. 2). As a rule of thumb, the uncertainty $\delta\omega$ in the location of the center of the line is

$$\delta\omega = \frac{\Delta\omega}{S/N} \tag{1.12}$$

In principle, one can make $\delta\omega$ arbitrarily small by acquiring enough data to achieve the required statistical accuracy. In practice, systematic errors eventually limit the precision. Splitting a line by a factor of 10^4 is a formidable task which has only been achieved a few times, most notably in the measurement of the Lamb shift. A factor of 10^3, however, is not uncommon, and 10^2 is child's play.

1.3. Magnetic Resonance: Classical Spin in Time-Varying B-Field.

1.3.1. *The classical motion of spins in a static magnetic field.* Note: angular momentum will always be expressed in a form such as $\hbar\boldsymbol{J}$, where the vector $\boldsymbol{J}$ is dimensionless. The interaction energy and equation of motion of a classical spin in a static magnetic field are given by

$$W = -\boldsymbol{\mu} \cdot \boldsymbol{B}, \tag{1.13}$$

$$\boldsymbol{F} = -\boldsymbol{\nabla}W = \boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \boldsymbol{B}), \tag{1.14}$$

$$\text{torque} = \boldsymbol{\mu} \times \boldsymbol{B} \tag{1.15}$$

In a uniform field, $\boldsymbol{F} = 0$. The torque equation $(\mathrm{d}\hbar\boldsymbol{J}/\mathrm{d}t = \text{torque})$ gives

$$\frac{\mathrm{d}\hbar\boldsymbol{J}}{\mathrm{d}t} = \boldsymbol{\mu} \times \boldsymbol{B}. \tag{1.16}$$

Since $\boldsymbol{\mu} = \gamma\hbar\boldsymbol{J}$ (where γ is called the gyromagnetic ratio - not to be confused with the different meaning of γ in the previous section), we have

$$\frac{\mathrm{d}\boldsymbol{J}}{\mathrm{d}t} = \gamma\boldsymbol{J} \times \boldsymbol{B} = -\gamma\boldsymbol{B} \times \boldsymbol{J}. \tag{1.17}$$

To see that the motion of $\boldsymbol{J}$ is a pure precession about $\boldsymbol{B}$, imagine that $\boldsymbol{B}$ is along $\hat{\boldsymbol{z}}$ and that the spin, $\boldsymbol{J}$, is tipped at an angle θ from this axis, and then rotated at an angle $\phi(t)$ from the x-axis (i.e., θ and ϕ are the conventionally chosen angles in