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Course: **8.421 - AMO I**
Problem set: **#9**
Due: Friday, April 15, 2022.

1. Transition Lifetimes and Blackbody Radiation.

(a) (i) The rate of *absorption* is given by the product of Einstein's B coefficient and the average number of photons per mode $\langle n\rangle_{\omega_0}$ where ω_0 is angular frequency associated with the (dominant) transition with $\lambda_0 = 590$ nm.

$$W = B\langle n\rangle = 1/60 \text{ s}^{-1}.$$

From lecture, we know that the Einstein's B coefficient can be written in terms of the (known) Einstein's $A = \Gamma_0 = 1/\tau$ coefficient:

$$B = \frac{\pi^2 c^3}{\hbar\omega_0^3} A = \frac{\pi^2 c^3}{\hbar\omega_0^3}\frac{1}{\tau}$$

With this, we can plug in the numbers to find

$$\langle n\rangle = \frac{W}{B} = \frac{W\hbar\omega_0^3\tau}{\pi^2 c^3} \approx 1.0324 \times 10^{-15}.$$

(ii) For blackbody radiation, we have

$$\langle n\rangle = \frac{1}{e^{\hbar\omega_0/k_B T} - 1} \implies T \approx 707.2 \text{ K}.$$

We see that in order for the absorption rate to reach 1 photon per minute, the blackbody temperature has to be $\sim$ 700 K, which is way above room temperature (of course the higher the temperature, the higher the absorption rate, and vice versa). As a result, there is no need to shield the vacuum system from room temperature radiation or col the vacuum system to cryogenic temperatures (as expected).

(b) Here we estimate the lifetime of hydrogen in the $F = 1$ hyperfine level of the $1S$ state.

(i) $F = 1 \to F = 0$ is a magnetic dipole transition.

(ii) To estimate the lifetime of the $F = 1$ state, we may assume that the (magnetic dipole) transition matrix element is μ_B. From here, we work entirely in the CGS unit system to find

$$\Gamma_0 = \frac{4\omega_0^3\mu_B^2}{3\hbar c^3} \approx 2.91 \times 10^{-15} \text{ s}^{-1}$$

where the numerical values for the fundamental constants can be found on Wikipedia. The lifetime is

$$\tau = \frac{1}{\Gamma_0} \approx 1.1 \times 10^7 \text{ years}.$$

(c) Now we look at a hydrogen BEC in the $F = 1$ state.

(i) To find the average number of photons per mode from blackbody radiation at the 21 cm line at 300 K and 4 K, we simply calculate

$$\langle n\rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

at the corresponding temperatures and angular frequency. The answers are

$$T = 300 \text{ K}, \qquad \langle n\rangle \approx 4375$$
$$T = 4 \text{ K}, \qquad \langle n\rangle \approx 57.84.$$

(ii) Similar to what we did before (but reversed), we can find the transition rates at $T = 300$ K and $T = 4$ K. We will also need the lifetime $\tau \approx 1.1 \times 10^7$ years for this calculation.

$$T = 300 \text{ K}, \qquad W = 1.484 \times 10^{11} \text{ s}^{-1}$$
$$T = 4 \text{ K}, \qquad W = 1.962 \times 10^{9} \text{ s}^{-1}.$$

(iii) It is clear that we should be concerned about blackbody radiation from the environment limiting our experiment with hydrogen in the $F = 1$ state if we need a trapping times on the order of seconds/minutes.

(d) The lifetime is much longer for the case of a magnetic dipole transition in hydrogen where there are more photons per mode mainly because the lifetime τ scales as $1/\omega_0^3$. The ratio between the sodium wavelength of 590 nm versus the 21 cm of hydrogen is $\sim 10^{-6}$. This gives a reduction factor of 10^{-18}. On top of this, there is also another factor of α^2 reduction when replacing the electric with magnetic dipole matrix element.

2. Saturation Intensity.

(a)

(b)

3. Saturation of Atomic Transitions.

(a)

(b)

(c)

(d)

(e)

(f)

(g)