Quiz 1

Thursday, January 27, 2022 8:42 AM

All following problems are concerned with cylinders of radius R and infinitely long along z direction.

1. Calculate the electric field $\boldsymbol{E}(\boldsymbol{r})$ and electrostatic potential $\phi(\boldsymbol{r})$ of a uniformly charged solid cylinder with 3D charge density ρ, for $\boldsymbol{r}$ over all space. (10 pts)

2. Calculate the electric field $\boldsymbol{E}(\boldsymbol{r})$ and electrostatic potential $\phi(\boldsymbol{r})$ of a cylindrical conductor with charge per unit length λ along z direction, for $\boldsymbol{r}$ over all space. (10 pts)

3. Calculate the mutual capacitance per unit length C between two parallel cylindrical conductors separated at a distance $d \gg R$. (20 pts)

Solution

(1) Gauss law

$$2\pi r\, E(r) = \int_0^r \rho\; 2\pi r' dr' / \varepsilon_0$$

$$= \begin{cases} \pi r^2 \rho/\varepsilon_0 & \text{if } r \le R \\ \pi R^2 \rho/\varepsilon_0 & \text{if } r > R \end{cases}$$

$$\Rightarrow \vec{E}(\vec{r}) = \begin{cases} \dfrac{r\rho}{2\varepsilon_0}\hat{r} & \text{if } r \le R \\ \dfrac{R^2\rho\,\hat{r}}{2\varepsilon_0 r} & \text{if } r > R \end{cases}$$

$$\vec{E}(\vec{r}) = -\nabla\phi(r)$$

$$\Rightarrow \phi(r) = -\frac{R^2}{2\varepsilon_0}\rho \log r + \text{const} \qquad \text{for } r > R$$

$$= \frac{R^2}{2\varepsilon_0}\rho \log\left(\frac{R}{r}\right) + \underline{\underline{\text{const}}}$$

$$\phi(r) = \left(\frac{R^2 - r^2}{2\rho\varepsilon_0}\right) + \underline{\underline{\text{const}}} \qquad \text{for } r < R$$

the same const ensures $\phi(r)$ is continuous at $r = R$

(2) $\vec{E}(\vec{r}) = 0$ for $r < R$

for $r > R$: Gauss law

$$2\pi r \cdot E(r) = \sigma/\varepsilon_0$$

$$E(r) = \frac{\sigma}{2\pi r \varepsilon_0}$$

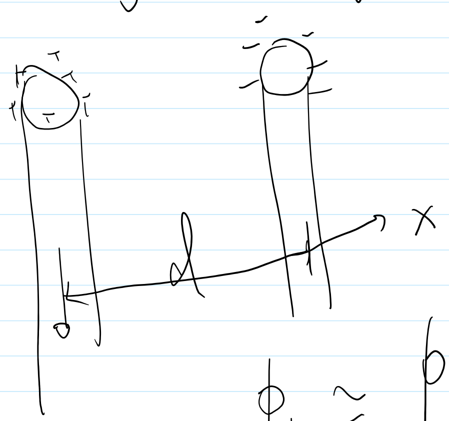
$$\ldots (R) + \underline{\underline{\text{const}}} \qquad \text{for } r > R$$

$$\phi(r) = \frac{\sigma}{2\pi\epsilon_0}\log\left(\frac{R}{r}\right) + \text{const} \quad \text{for } r>R$$

$$\phi(r) = \text{const} \quad \text{for } r<R$$

Same const ensures $\phi(r)$ is continuous at R

13, When two cylinderal conductors are far away charge is nearly uniform on the surface of each



$$\phi(x) \approx \phi_1(x) + \phi_2(x)$$

$$= \frac{\sigma}{2\pi\epsilon_0}\log\left(\frac{R}{x}\right) - \frac{\sigma}{2\pi\epsilon_0}\log\left(\frac{R}{d-x}\right)$$

$$\phi_1 \simeq \phi(x=R) = +\frac{\sigma}{2\pi\epsilon_0}\log\left(\frac{d-R}{R}\right) \simeq \frac{\sigma}{2\pi\epsilon_0}\log\left(\frac{d}{R}\right)$$

$$\phi_2 \simeq \phi(x=d-R) = -\frac{\sigma}{2\pi\epsilon_0}\log\left(\frac{d-R}{R}\right) \simeq -\frac{\sigma}{2\pi\epsilon_0}\log\left(\frac{d}{R}\right)$$

$$V = \phi_1 - \phi_2 = \frac{\sigma}{\pi\epsilon_0}\log\left(\frac{d}{R}\right)$$

$$C = \frac{\sigma}{V} = \frac{\pi\epsilon_0}{\log\left(\frac{d}{R}\right)}$$