Problem Set 6

Due: Friday 5pm, Mar 18, via Canvas upload or in envelope outside 26-255
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1 Symmetries and Permanent Dipole Moments

The molecule HCl is reported to have a permanent electric dipole moment (EDM) of 0.41 ea_o. There is more than one way to answer each of the three parts.

a) Considering electromagnetic interactions only, the Hamiltonian of HCl is rotationally invariant in the lab frame. Explain the connection between this and HCl's observed electric dipole moment. (Hint: This question can be answered by thinking about both parity and rotational invariance.)

b) The fact that a "permanent" EDM value is quoted for HCl indicates that there is an experimental regime where the observed dipole moment is independent of the electric field. This can be observed as a linear Stark effect. Given the parity considerations from part (a), where does this measured linear Stark effect come from?

c) Justify why atoms can have permanent magnetic dipole moments, but not EDM's. A clearly explained diagram will suffice.

2 The Stark Effect in Hydrogen

Episode 1 - The Stark splitting of the $2S$ and $2P$ levels

(i) Find the energies and eigenfunctions of the $n = 2$ states of hydrogen in an applied electric field $\mathbf{E} = E\hat{\mathbf{z}}$. You should simplify the problem to a two level system by considering only the essential physics of the two closest interacting states. In order to do this you must justify what assumptions you have made and why they are reasonable.

(ii) How large (in V/cm) must the electric field be for the Stark shift to be linear? Calculate the linear shift in the lab units of MHz/(V/cm).

Suggestions:

- The $n = 2$ level of hydrogen has a total of 8 states when both electron charge and spin are included in the Hamiltonian.

- The fine structure splitting raises the four $2P_{3/2}$ states about 10 GHz above the two $2P_{1/2}$ and two $2S_{1/2}$ states.
- The Lamb shift raises $2S_{1/2}$ above $2P_{1/2}$ by 1.06 GHz.
- Both $2S_{1/2}$ and $2P_{1/2}$ have total angular momentum J = 1/2. Your answer to 1c will be useful in arguing that the two substates for each level remain degenerate and can be treated as one state for this problem.
- The electric dipole moment between the $2S$ and $2P$ states is $3ea_o$. Use this to construct your interaction hamiltonian.
- You may find it helpful to express the eigenvectors in terms of an angle θ defined by $\tan\theta = 2V/\omega_0$, where V is the off-diagonal matrix element in the interaction Hamiltonian. (Do you recognize the geometric interpretation of θ?)

Stay tuned for episode 2 - Stark Quenching!