Matrix exponential

- $\exp : M_n(\mathbb{C}) \to GL(n,\mathbb{C})$
  $X \mapsto \sum_{k=0}^{\infty} \frac{X^k}{k!}$ ← Converges (absolutely) for all X

- exp is surjective

- $e^{AXA^{-1}} = A e^X A^{-1} \quad (A \in GL(n,\mathbb{C}))$

- $\det(e^X) = e^{\operatorname{tr} X}$

- $e^0 = \mathbb{1}$ ← zero matrix
- $(e^X)^* = e^{X^*}$
- $(e^X)^T = e^{X^T}$
- $(e^X)^{-1} = e^{-X}$
- $e^{(\alpha+\beta)X} = e^{\alpha X} e^{\beta X} \quad (\alpha, \beta \in \mathbb{C})$
- if $XY = YX$ then $e^{X+Y} = e^X e^Y = e^Y e^X$
- in general $e^{X+Y} = \lim_{k\to\infty} \left(e^{X/k} e^{Y/k}\right)^k$ (Lie product formula)

- $\frac{d}{d\varepsilon} e^{\varepsilon X} = X e^{\varepsilon X} = e^{\varepsilon X} X \quad (\varepsilon \in \mathbb{R})$
- $\left.\frac{d}{d\varepsilon}\right|_{\varepsilon=0} e^{\varepsilon X} = X$
- $e^{\varepsilon X} = e^{\varepsilon Y} \;\; \forall \varepsilon \in \mathbb{R} \iff X = Y$