

8.321 Recitations 19-20

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1 Imaginary Time Path Integral

- $\tau = it$, $\langle x_f | e^{-H\tau} | x_i \rangle = \int Dx e^{-S_E/\hbar}$
- Ground state energy and wavefunction: $\langle x_f | e^{-H\tau} | x_i \rangle \xrightarrow{\tau \rightarrow \infty} e^{-E_0\tau/\hbar} \psi_0(x_f) \psi_0^*(x_i)$
- Euclidean action: $S_E = \int d\tau (\frac{1}{2}m\dot{x}^2 + V(x))$, potential is flipped and tunneling becomes rolling
- Instanton: tunneling/rolling classical solution localized in τ , reproduce WKB
- Path integral as sum of dilute instanton gas
- Tunneling contribution to energy, lifting degeneracy

2 Berry Phase

- Geometric phase: $\gamma = i \int_C \langle \psi(\mathbf{R}) | \nabla_{\mathbf{R}} \psi(\mathbf{R}) \rangle \cdot d\mathbf{R}$
- Example: spin-1/2 particle with rotating magnetic field. $\gamma = -\frac{1}{2}\Omega$ sourced by Berry monopole with charge 1
- Reproduce Aharonov-Bohm effect
- Integer quantum Hall effect: $\sigma_{xy} = \frac{e^2}{h}\nu$ where ν is number of Berry monopole