

Physics 8.321, Fall 2020

Midterm

You have **75 minutes** to complete the exam and a grace period of an additional 15 minutes to get your solutions uploaded to canvas. You may use your books and notes including the notes on canvas from the course, and you may use symbolic manipulation tools like mathematica and matlab, but you may not consult other online resources, and you may not communicate with other people in any way while doing the midterm. You also may not communicate any information about the exam to anyone after you have completed it until the exam period is over at the end of the day on 10/23/20.

Note: you are expected to upload your completed exam to the canvas website immediately after completing the exam and within 75 minutes or less of downloading it. You have a few extra minutes in case of technical complications. The system will log your download and upload times. If for some reason you have difficulty uploading your exam after completion, please email it immediately to one of the course staff.

1. (30 points)

A *spin-1* particle is associated (in a specific choice of basis) with angular momentum operators

$$S_x = \hbar/\sqrt{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \hbar/\sqrt{2} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

A particle is in a state that can be described in this basis as $|\psi\rangle = (1/2, \sqrt{2}/2, 1/2)^T$ (the T just denotes the transpose to make this a column vector, not a row vector).

- (a) What are the expectation values $\langle O \rangle = \langle \psi | O | \psi \rangle$ in this state of the three spin operators S_x, S_y, S_z ?
- (b) What are the expectation values $\langle S_x^2 \rangle$ and $\langle S_z^2 \rangle$?
- (c) Compute both sides of the uncertainty relation

$$\langle \Delta S_x^2 \rangle \langle \Delta S_z^2 \rangle \geq \frac{1}{4} |\langle [S_x, S_z] \rangle|^2$$

and confirm that the inequality holds.

2. (30 points)

Consider a combined system of one spin-1/2 particle as described in class and one spin-1 particle as described above.

- (a) What are the possible values in the combined system of the observable associated with the operator

$$S_z = S_z^{(1/2)} + S_z^{(1)}?$$

Identify the states associated with each eigenvalue in a basis of your choice.

- (b) A particle is in the state $|\psi\rangle = |+, 1\rangle$, where $|\pm, s^{(1)}\rangle$ denotes the state with eigenvalue of $S_z^{(1/2)}$ given by $\pm\hbar/2$ and eigenvalue of $S_z^{(1)}$ given by $s^{(1)}\hbar$. The spin of the spin-1/2 particle along the x axis is measured. What are the possible values of this spin and what is the probability of each outcome?
- (c) After performing the measurement in the previous part, the value of $S_x^{(1)}$ is measured. What are the possible outcomes and probabilities?
- (d) How does your answer for (c) differ from what you would get if the initial state was again the state $|\psi\rangle = |+, 1\rangle$, i.e. the measurement from part (b) had not been performed first?

3. (40 points)

In this problem we choose units where $\hbar = m = 1$. Consider a particle in a 2D potential

$$V(x, y) = \begin{cases} \frac{1}{2}x^2, & \text{when } |y| < 1 \\ \infty, & \text{when } |y| > 1. \end{cases}$$

Describe the eigenvalue spectrum of the Hamiltonian and the associated eigenstates. What are the energy eigenvalues for the lowest-lying 5 states of the system?