BCS order parameter, critical temperature and bandgap

Selfconsistency equation for Δ

$$b_k = \langle \hat{c}_{-k\downarrow} \hat{c}_{k\uparrow} \rangle = u_k v_k \langle 1 - \hat{a}^+_{k0} \hat{a}_{k0} - \hat{a}^+_{k1} \hat{a}_{k1} \rangle$$

Quasiparticles are fermions:

$$\langle \hat{a}^+_{ki} \hat{a}_{ki} \rangle = f(E_k)$$

Fermi-Dirac distribution

Order parameter

$$\Delta_k = \sum_{\vec{k}'} V_{k,k'} b_k$$

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Ground state satisfies:

$$a_{k,0} | g_s \rangle = 0, \quad a_{k,1} | g_s \rangle = 0$$

This gives:

$$| g_s \rangle = \prod_k (u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow}) | 0 \rangle$$

BCS critical temperature

$$\Delta_k = -\sum_{k'} V_{k,k'} \frac{\Delta_{k'}}{2E_{k'}} \tanh\left(\frac{E_{k'}}{2k_B T}\right)$$

Apprx: k-independent interaction
(s-wave scattering) $V_{k,k'} = -U \quad (-\omega_D < \epsilon_k, \epsilon_{k'} < \omega_D)$

Gap eqn
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$$\boxed{\frac{1}{U} = \sum_k \frac{\tanh\left(E_k / 2k_B T\right)}{2E_k}}$$

$$\sum_k \ldots = \int_{-\omega_D}^{\omega_D} \ldots \nu(\epsilon) d\epsilon$$

$$\approx \nu(\epsilon_F) \int_{-\omega_D}^{\omega_D} \ldots d\epsilon$$

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Critical temperature: set Δ=0

$$k_B T_c \ll \hbar\omega_D$$

$$\frac{1}{U\nu(\epsilon_F)} = \int_{-\omega_D}^{\omega_D} \frac{d\epsilon}{2\epsilon} \tanh\left(\frac{\epsilon}{2k_B T_c}\right) = \int_0^{\omega_D/2k_B T} \frac{\tanh x}{x} dx \approx \ln\left(\frac{2\gamma}{\pi} \frac{\omega_D}{2k_B T_c}\right)$$

$$\gamma = 1.78\ldots$$

Euler constant

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$$\boxed{k_B T_c = 1.13 \hbar \omega_D \exp\left(-\frac{1}{U\nu(\epsilon_F)}\right)}$$

Temperature dependence, exper tests

Determine the value Δ at T=0

$$\frac{1}{U\nu(\epsilon_F)}=\int_{-\omega_D}^{\omega_D}\frac{d\epsilon}{2\sqrt{\epsilon^2+\Delta^2}}\approx\ln\left(\frac{2\omega_D}{\Delta}\right)$$

gives

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A universal relation (testable):

$$\boxed{2\Delta_{T=0} \approx 3.52 k_B T_c}$$

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Temperature dependence $\Delta(T)$:

* Second-order phase transition, $\Delta(T)$ vanishes at T=Tc
* measured in microwave absorption experiments