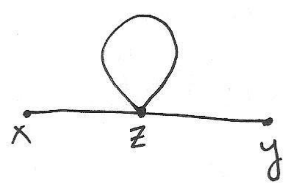
From last lecture:

One additional comment about symmetry factors: If there is an internal vertex with a line coming out <u>and</u> going into the vertex, then there is a symmetry factor of 2.

For example:

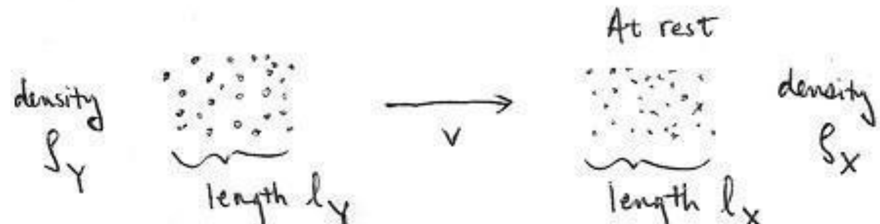


$S = 2$

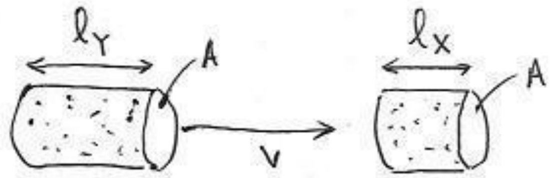
Cross Sections + S-matrix

We consider the collision of two beams of particles with relatively well-defined momenta.

Consider a target of particles of type X at rest and incoming particles of type Y moving with speed v towards the target.



Let ρ_Y and ρ_X be the particle densities as observed from rest. Let l_Y and l_X be the length of the the particle bunches as observed from rest. Let A be the cross-sectional area of overlap.



The total numbers of particles are

$$N_X = \rho_X \ell_X A \quad \text{and} \quad N_Y = \rho_Y \ell_Y A$$

So then the total number of scatterings is proportional to $N_X \cdot N_Y$.

Let the total number of scatterings be

$$N_X \cdot N_Y \cdot \frac{\sigma}{A}$$

where $\frac{\sigma}{A}$ is the probability one particular X particle and one particular Y particle collide.

We call σ the effective area or cross section of the scattering process.

When $N_X = 1$ (one target particle)

$$\text{the total number of scatterings} = N_Y \frac{\sigma}{A} = \rho_Y \cdot \ell_Y \cdot \sigma$$

So $\sigma = \dfrac{\text{total \# scatterings}}{\rho_Y \cdot l_Y}$

Suppose we measure for a small time interval Δt.

Then

$$\sigma = \frac{\left(\frac{\Delta \text{ \# scatterings}}{\Delta t}\right)}{\rho_Y \underbrace{\left(\frac{\Delta l_Y}{\Delta t}\right)}_{= v}} = \frac{\text{scatterings per unit time}}{\underbrace{\rho_Y \cdot v}_{\text{particle flux}}}$$

The differential cross section is the portion of σ in which the final particle momenta lie inside some window of momenta. We can write this as

$$\frac{d\sigma}{d^3\vec{p}_1 \cdots d^3\vec{p}_n}$$

final particle momenta

If there are only two final particles then there are only two free parameters. Why?

two spatial momenta $\sim$ 6 parameters

four-momentum conservation $\sim$ 4 constraints
(energy + spatial momenta)

We can take these two parameters to be orientation angles θ and ϕ.

So then we can measure $\frac{d\sigma}{d\Omega}(\theta,\phi)$,

where $d\Omega$ is the solid angle differential

$$d\Omega = d\cos\theta \, d\phi .$$

In most cases the "differential cross-section" name refers to $\frac{d\sigma}{d\Omega}$

Peskin + Schroeder use wave packets to produce normalizable states. This is a little complicated. We consider instead a periodic box with length L on all sides.

The spatial momentum modes are now discrete

$$\vec{k} = \frac{2\pi}{L} \cdot (\underbrace{n_x, n_y, n_z}_{\text{integers}})$$

The a's + $a^\dagger$'s satisfy

$$[a_{\vec{k}}, a^\dagger_{\vec{k}'}] = \delta_{\vec{k},\vec{k}'} \cdot V$$

where $V = L^3$ (volume)

The connection with the infinite volume case is...

as $V \to \infty$:

$$\delta_{\vec{k},\vec{k}'} \cdot V = \int_0^L\!\!\int_0^L\!\!\int_0^L dx_1 dx_2 dx_3 \, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}$$

$$\to \iiint d^3\vec{x} \, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} = (2\pi)^3 \delta^{(3)}(\vec{k}-\vec{k}')$$

In our periodic box

$$\phi(x) = \sum_{\vec{k}} \frac{\left(\frac{2\pi}{L}\right)^3}{(2\pi)^3\sqrt{2E_{\vec{k}}}} \left(a_{\vec{k}} e^{-ik\cdot x} + a^\dagger_{\vec{k}} e^{+ik\cdot x}\right)$$

$$= \sum_{\vec{k}} \frac{1}{V\sqrt{2E_{\vec{k}}}} \left(a_{\vec{k}} e^{-ik\cdot x} + a^\dagger_{\vec{k}} e^{ik\cdot x}\right)$$

Now imagine starting with free field theory