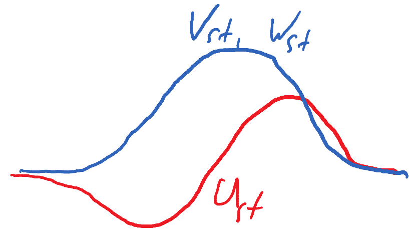
Steady-state regime:

$$u_{st} = \frac{\Omega_1}{2} \frac{\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4} + \frac{\Omega_1^2}{2}}$$



$$v_{st} = \frac{\Omega_1}{2} \frac{\Gamma/2}{\delta_L^2 + \frac{\Gamma^2}{4} + \frac{\Omega_1^2}{2}}$$

$$w_{st} + \frac{1}{2} = \sigma_{bb}^{st} = \frac{\Omega_1^2}{4} \frac{1}{\delta_L^2 + \frac{\Gamma^2}{4} + \frac{\Omega_1^2}{2}}$$

v_{st} (component in quadrature to drive) and population in $|b\rangle$ vary as Lorentzian with FWHM $\sqrt{\Gamma + 2\Omega_1^2}$ – power broadening.

u_{st} and v_{st} $\propto \Omega_1$ at low intensity

$\propto \frac{1}{\Omega_1}$ at high intensity

$\rightarrow$ mean dipole $\rightarrow 0$ at large intensity.

population $\sigma_{bb}^{st} \propto \Omega_1^2$ at low intensity

$\rightarrow \frac{1}{2}$ at high intensity

$\rightarrow$ high intensity equalizes populations of two levels

$\Rightarrow$ transition is "saturated"

Saturation parameter $s = \dfrac{\frac{\Omega_1^2}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$.

$$u_{st} = \frac{\delta_L}{\Omega_1} \frac{s}{1+s} \; ; \; v_{st} = \frac{\Gamma}{2\Omega_1} \frac{s}{1+s} \; ; \; \sigma_{bb}^{st} = \frac{1}{2} \frac{s}{1+s}$$

Note: These are not perturbative in Ω_1! Ω_1 is in denominator

Work done by driving field as atom moves:

$$dW = q\,\mathcal{E}_0 \cos(\omega_L t)\cdot d\vec{r}$$

$$\Rightarrow \frac{dW}{dt} = \mathcal{E}_0 \cos(\omega_L t)\cdot \langle \dot{\vec{d}} \rangle \qquad \vec{d} = q\vec{r}$$

$$\overline{\langle \frac{dW}{dt} \rangle} = -2 d_{ab}\,\mathcal{E}_0\,\omega_L \left[\overline{\cos^2(\omega_L t)}\, v + \overline{\sin\omega_L t\cdot\cos(\omega_L t)}\, u\right]$$

$$= \hbar\,\Omega_1\,\omega_L\, v$$

mean absorbed power only related to quadrature component v of the mean atomic dipole

mean number of photons absorbed per unit time:

$$\langle \frac{dN}{dt} \rangle = \Omega_1 v$$

$\Rightarrow$ simple interpretation of 3^{rd} Bloch equation:

$$w = \frac{1}{2}(\sigma_{bb} - \sigma_{aa}) = \sigma_{bb} - \frac{1}{2}$$

$$\dot{\sigma}_{bb} = \langle \frac{dN}{dt} \rangle - \Gamma\sigma_{bb}$$

Any disappearance of an incident photon leads to a transition of the atom from $a \rightarrow b$.

Second term describes spontaneous emission from b.

Steady state: $\dot{\sigma}_{bb} = 0$, and $\boxed{\langle \frac{dN}{dt} \rangle_{st} = \Gamma\sigma_{bb}^{st}}$

External Degrees of Freedom. Mean Radiative Forces

Include translation of atom:

$$H = \frac{\vec{P}^2}{2M} + H_A + H_R - \vec{d}\cdot\left(\vec{E}_\ell(\vec{R},t) + \vec{E}_\perp(\vec{R})\right)$$

Heisenberg equations for $\vec{R}$ and $\vec{P}$:

$$\dot{\vec{R}} = \frac{\partial H}{\partial \vec{P}} = \frac{\vec{P}}{M}$$

$$\dot{\vec{P}} = M\ddot{\vec{R}} = -\frac{\partial H}{\partial \vec{R}} = \sum_{j=x,y,z} d_j \nabla_{\vec{R}}\left[E_{\ell_j}(\vec{R},t) + E_{\perp_j}(\vec{R})\right]$$

$$\rightarrow M\langle\ddot{\vec{R}}\rangle = \sum_j \langle d_j \nabla_{\vec{R}}\left(E_{\ell_j}(\vec{R},t) + E_{\perp_j}(\vec{R})\right)\rangle$$

let $\vec{r}_G \equiv \langle\vec{R}\rangle$ center of atomic wavepacket.

Assume $\frac{h}{Mv} = \lambda_{dB} \ll \lambda$. Last term $=0$.

$$M\ddot{\vec{r}}_G = \underbrace{\sum_{j=x,y,z} \langle d_j\rangle \vec{\nabla} E_{\ell_j}(\vec{r}_G,t)}_{\text{force governing } \vec{r}_G}$$

Two timescales of evolution: $T_{int} = \frac{1}{\Gamma}$ $\left(\text{or } \frac{1}{\Omega_1}\right)$

slow atoms: $v\,T_{int} = \frac{v}{\Gamma} \ll \lambda$ $\qquad v \ll 50\,\frac{m}{s}$

external timescale: velocity evolves over timescales

$$T_{ext} = \frac{\hbar}{E_{rec}}, \quad E_{rec} = \frac{\hbar^2 k^2}{2M} \quad \text{recoil energy}$$

Typically $\hbar\Gamma \gg E_{rec} \Rightarrow T_{int} \ll T_{ext}$

Sodium: $\hbar\Gamma = 400\,E_{rec}$.

$T_{ext} \gg T_{int} \Rightarrow \langle \vec{d} \rangle$ has time to reach steady-state before atom moves appreciably under the influence of the radiative force.

<u>Forces for $\vec{v} = \vec{0}$ at $\vec{r} = \vec{0}$:</u>

$$\vec{E}_\ell(\vec{r},t) = \vec{e}\ \mathcal{E}_0(\vec{r}) \cos(\omega_L t + \phi(\vec{r}))$$

$\uparrow$ polarization independent on $\vec{r}$

Choose time origin s.t. $\phi(\vec{0}) = 0$

$$\vec{\nabla} E_{\ell_j} = e_j \left[\cos(\omega_L t) \vec{\nabla} \mathcal{E}_0 - \sin(\omega_L t)\, \mathcal{E}_0 \vec{\nabla}\phi \right]$$

$$\langle d_j \rangle = 2(\vec{d}_{ab})_j \left[u_{st} \cos(\omega_L t) - v_{st} \sin(\omega_L t) \right]$$

$$\vec{F} = \sum_j \overline{\langle d_j \rangle \vec{\nabla} E_{\ell_j}} = (\vec{e} \cdot \vec{d}_{ab}) \left[u_{st} \vec{\nabla} \mathcal{E}_0 + v_{st}\, \mathcal{E}_0 \vec{\nabla}\phi \right]$$

$$\vec{F}_{reactive} = (\vec{e} \cdot \vec{d}_{ab}) u_{st} \vec{\nabla} \mathcal{E}_0 \quad \propto \text{ in-phase comp. of atomic dipole}$$

$$\vec{F}_{dissipative} = (\vec{e} \cdot \vec{d}_{ab}) v_{st}\, \mathcal{E}_0 \vec{\nabla}\phi \quad \propto \text{ quadrature comp. of atomic dipole}$$

$$\Omega_1 = -\vec{d}_{ab} \cdot \vec{\varepsilon}\ \mathcal{E}_0 / \hbar$$

$$\vec{F}_{react} = -\hbar \Omega_1 u_{st} \vec{\alpha} \qquad \text{with } \vec{\alpha} = \frac{\vec{\nabla}\Omega_1}{\Omega_1}$$

$$\vec{F}_{dissip} = -\hbar \Omega_1 v_{st} \vec{\beta} \qquad \text{with } \vec{\beta} = \vec{\nabla}\phi$$

Dissipative Force. Radiation Pressure

plane wave, wave vector $\vec{k}_L$

$$E_L(\vec{r},t) = \vec{e}\,\mathcal{E}_0 \cos(\omega t - \vec{k}_L \cdot \vec{r})$$

$$\phi(\vec{r}) = -\vec{k}_L \cdot \vec{r}$$

$$\vec{\beta} = \vec{\nabla}\phi = -\vec{k}_L \qquad \vec{\nabla}\mathcal{E}_0 = \vec{0}$$

$$F_{dissip} = \Omega_1 v_{st}\, \hbar k_L$$

$$= \left\langle \frac{dN}{dt} \right\rangle_{st} \hbar \vec{k}_L$$

Interpretation: atom absorbs momentum $\hbar k_L$ from laser. If emission is stimulated, it loses this momentum again to the laser field. If emission is spontaneous, loss of momentum is zero on average, as opposite emission directions are equally probable

Also: $$F_{dissip} = \Gamma \sigma_{bb}^{st}\, \hbar k_L$$

$= \hbar k_L \times$ number of photons spontaneously emitted per unit time.

$$F_{dissip} = \hbar k_L \frac{\Gamma}{2} \frac{\Omega_1^2/2}{(\omega_L-\omega_0)^2 + \frac{\Gamma^2}{4} + \frac{\Omega_1^2}{2}}$$

- Lorentzian centered on $\omega_L = \omega_0$, FWHM $\sqrt{\Gamma^2 + 2\Omega_1^2}$
- low intensity, $F_{dissip} \propto$ Intensity
- high intensity, $F_{sat} = \hbar k_L \frac{\Gamma}{2}$ limit indep. on intensity.

- Order of magnitude of acceleration for sodium:

$$a = \frac{F_{sat}}{M} = 10^6 \frac{m}{s^2} = 10^5 g$$

- Find T_{ext}:

$$a T_{ext} = \frac{\hbar k_L}{M} \Gamma T_{ext} = v_m$$

v_m is velocity when atom gets out of resonance due to the Doppler effect:

$$k_L v_m \simeq \Gamma$$

$$\Rightarrow \quad T_{ext} \sim \frac{M}{\hbar k_L^2} \sim \frac{\hbar}{E_{rec}}$$

Reactive Force. Dipole Force

Simplest example: Standing wave

$$\vec{E}_L(\vec{r},t) = \vec{e}_x \, \mathcal{E}_0 \cos(k_L z)\cos(\omega t)$$

phase is constant. Amplitude changes in space. (with period $\lambda = \frac{2\pi}{k_L}$)

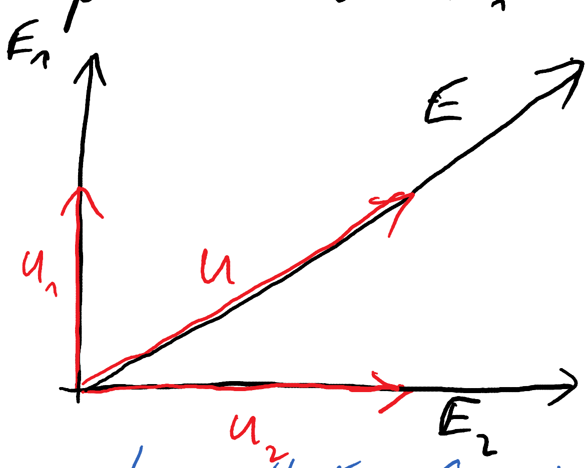
$F_{react} \propto u_{st}$ does not involve any exchange of energy between atom and light field.

However, there can be energy redistribution among the light waves making up standing wave.

$$\xrightarrow{\hbar k_L} \quad \bullet \quad \xleftarrow{-\hbar k_L}$$

atom can absorb from $+\hbar k_L$ and be stimulated to emit into $-\hbar k_L$. Total momentum of field has changed by $-2\hbar k_L$, atom has gained $+2\hbar k_L$.

pick a point where E_1 and E_2 are in quadrature



u shown in phase with E: $\delta_L < 0$!

u_1 does not absorb from E_1
u_2 " " " E_2
but u_1 absorbs from E_2
u_2 delivers to E_1
$|E_1||u_2| = |E_2||u_1|$
energy lost by E_1 = energy gained by E_2.

Coherent nature of redistribution: Only depends on relative phases of the two waves.

If u 180° out of phase with E ($\delta_L > 0$), sign of reactive force is opposite to the case u in phase.

$$F_{react} = -\frac{\hbar(\omega_L - \omega)}{4} \frac{\nabla(\Omega_1^2)}{(\omega_L - \omega_0)^2 + \frac{\Gamma^2}{4} + \frac{\Omega_1^2}{2}}.$$

changes sign with detuning $\omega_L - \omega_0$ as a dispersion curve.

$\omega_L < \omega_0$ (red detuning) attractive

$\omega_L > \omega_0$ (blue detuning) repulsive

max force at δ_L that depends on Ω_1^2.

characteristic value $|\delta_L| \approx |\Omega_1|$

$$|F_{react}| \sim \frac{\hbar|\nabla\Omega_1^2|}{\Omega_1} \sim \hbar|\nabla\Omega_1|$$

reactive force increases without bound with intensity!

$\nabla\Omega_1$ is at most $k_L\Omega_1$, so $|F_{react}| < \hbar k_L \Omega_1$.

photon exchanges occur at best at a rate Ω_1, as it should be for absorption - stimulated emission cycles.

Note $\vec{F}_{react} = -\vec{\nabla} U$

$$U = \frac{\hbar(\omega_L - \omega_0)}{2} \ln\left[1 + \frac{\Omega_1^2/2}{(\omega_L - \omega_0)^2 + \frac{\Gamma^2}{4}}\right]$$

for red detuning: can trap atoms in intensity maxima.
U - dipole potential