8.321 Recitation 5-6

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1 Eigenvalue Solving Exercise

1.1 Classical Systems

- CO_2 molecule: $m_1 - m_2 - m_1$
- Eigenvalues: $\omega^2 = 0, \frac{k}{m_1}, \frac{k}{m_1} + \frac{2k}{m_2}$. Eigenvectors: $(A_1, A_2, A_3) = (1,1,1), (1,0,-1), \left(1, -2\frac{m_1}{m_2}, 1\right)$
- Ionic crystal: $... - m - M - m - M - ...$
- Eigenvalues: $\omega^2(q) = \frac{k}{Mm}\left(M + m \pm \sqrt{(M+m)^2 - 4Mm\sin^2 qa}\right)$. Eigenvectors: $(A_m, A_M) = \left(1, -\frac{m}{M}\right), (1,1)$ when $q \to 0$
- Acoustic and optical bands

1.2 Quantum Systems

- Landau levels
- Monolayer graphene: $\hat{H} = v_F\left(\hat{p}_x\sigma_x + (\hat{p}_y + eB\hat{x})\sigma_y\right)$
- Eigenvalues: $E = \pm v_F\sqrt{2\hbar eBn}$. Eigenvectors: $|\psi_n\rangle = e^{ip_y y/\hbar}\begin{pmatrix} |n-1\rangle \\ |n\rangle \end{pmatrix}$
- Bilayer graphene: $\hat{H} = \frac{1}{2m}\begin{pmatrix} 0 & (\hat{p}_x - i(\hat{p}_y + eB\hat{x}))^2 \\ (\hat{p}_x + i(\hat{p}_y + eB\hat{x}))^2 & 0 \end{pmatrix}$
- Eigenvalues: $E = \pm\frac{\hbar eB}{m}\sqrt{n(n-1)}$. Eigenvectors: $|\psi_n\rangle = e^{ip_y y/\hbar}\begin{pmatrix} |n-2\rangle \\ |n\rangle \end{pmatrix}$