(a)

$$\begin{cases} -kx = m\ddot{x} \\ -kz - mg = m\ddot{z} \end{cases} \qquad \boxed{\vec{F} = m\vec{a}}$$

(b)

$$\begin{cases} x = r\sin\theta \\ z = -r\cos\theta \end{cases}$$

$$\Rightarrow \begin{cases} -kr\sin\theta = m\left[(2\dot{r}\dot{\theta} + r\ddot{\theta})\cos\theta + (\ddot{r} - r\dot{\theta}^2)\sin\theta\right] \\ kr\cos\theta - mg = m\left[(-\ddot{r} + r\dot{\theta}^2)\cos\theta + (2\dot{r}\dot{\theta} + r\ddot{\theta})\sin\theta\right] \end{cases}$$

(c)

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{z}^2) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$$

$$V = \frac{1}{2}k(x^2 + z^2) + mgz = \frac{1}{2}kr^2 - mgr\cos z$$

$$L = T - V$$

$$\begin{cases} m\ddot{r} = -kr + mg\cos\theta + mr\dot{\theta}^2 \\ mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} = -mgr\sin\theta \end{cases}$$

$$\Rightarrow \begin{cases} m\ddot{r} = mg\cos\theta - kr + mr\dot{\theta}^2 \\ r\ddot{\theta} = -g\sin\theta - 2\dot{r}\dot{\theta} \end{cases} \qquad \boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0}$$

(d)

$$\boxed{\begin{array}{l} p_i = \dfrac{\partial L}{\partial \dot{q}_i}, \quad H(\{q_i, p_i\}) = \sum_i p_i\dot{q}_i - L \\ \text{EOMs are} \quad \dot{p}_i = -\dfrac{\partial H}{\partial q_i}, \quad \dot{q}_i = \dfrac{\partial H}{\partial p_i} \end{array}}$$

$$p_r = m\dot{r}, \quad p_\theta = mr^2\dot{\theta}, \quad H = m(\dot{r}^2 + r^2\dot{\theta}^2) - L$$

$$= \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{1}{2}kr^2 - mgr\cos\theta$$

$$\begin{cases} \dot{p}_r = -kr + mg\cos\theta + \dfrac{p_\theta^2}{mr^3}, \quad \dot{r} = \dfrac{p_r}{m} \\ \dot{p}_\theta = -mg\sin\theta, \quad \dot{\theta} = \dfrac{p_\theta}{mr^2} \end{cases}$$

(e)

$$V = \frac{1}{2}k(r-b)^2 - mgr\cos\theta$$

$$\begin{cases} m\ddot{r} = mg\cos\theta - k(r-b) + mr\dot{\theta}^2 \\ r\ddot{\theta} = -g\sin\theta - 2\dot{r}\dot{\theta} \end{cases}$$

$$\begin{cases} \dot{p}_r = -k(r-b) + mg\cos\theta, & \dot{r} = \frac{p_r}{m} \\ \dot{p}_\theta = -mg\sin\theta, \quad \dot{\theta} = \frac{p_\theta}{mr^2} \end{cases}$$

(f) $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2}m\left[\dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2)\right]$

$V = \frac{1}{2}k(x^2 + z^2) + mgz = \frac{1}{2}kr^2 - mgr\cos z$

$L = T - V$

$$\boxed{\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = -r\cos\theta \end{cases}}$$

EOMs:

$$\begin{cases} m\ddot{r} - mg\cos\theta + kr - mr(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2) = 0 \\ mr\ddot{\theta} + 2m r\dot{r}\dot{\theta} + mgr\sin\theta - mr^2\sin\theta\cos\theta\,\dot{\varphi}^2 = 0 \\ mr^2\sin^2\theta\,\ddot{\varphi} + 2mr\dot{r}\sin^2\theta\,\dot{\varphi} + 2mr^2\sin\theta\cos\theta\,\dot{\theta}\dot{\varphi} = 0 \end{cases}$$

$p_r = m\dot{r}, \quad p_\theta = mr^2\dot{\theta}, \quad p_\varphi = m\sin^2\theta\,\dot{\varphi}$

$$\mathcal{H} = p_r\dot{r} + p_\theta\dot{\theta} + p_\varphi\dot{\varphi} - L$$
$$= \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_\varphi^2}{2mr^2\sin^2\theta} + \frac{1}{2}kr^2 - mgr\cos\theta$$

EOMs:
$$\begin{cases} \dot{p}_r = -kr + mg\cos\theta \\ \dot{p}_\theta = -mgr\sin\theta + \frac{p_\varphi^2\cos\varphi}{mr^2\sin^3\varphi} \\ \dot{p}_\varphi = 0 \end{cases}$$