- Counterintuitive conceptual framework
- Nondeterministic (w/ measurements)
- (Apparently) irreversible dynamics

Both classical + quantum conceptual frameworks
useful in certain regimes

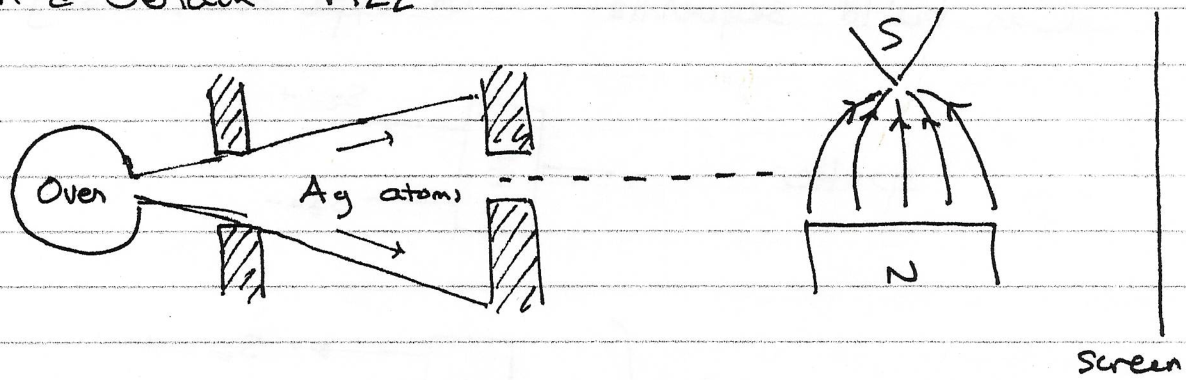
Classical picture not fundamental — replaced by QM

Is QM fundamental?

Perhaps not, but describes all physics relevant to current society & technology
[emergent QM, emergent geometry of space-time?]

Simple example of QM: 2-state system (spin ½ particle)

Stern & Gerlach 1922



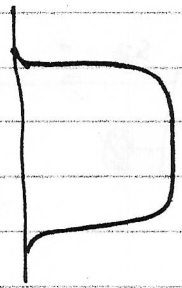
Silver: 47 electrons, angular momentum $\Rightarrow \mu$ (mag. dipole moment)
from spin of 47^{th} electron.

Energy $U = -\mu \cdot B$

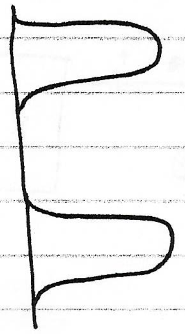
$$F_z = -\frac{\partial U}{\partial z} = \mu_z \frac{\partial B_z}{\partial z}$$

Gives force along $\hat{z}$ depending on μ_z.

Classically, expect



Actually see



$$\mu_z \approx \pm \frac{e\hbar}{2mc} \approx \frac{e}{mc} S_z$$

$$S_z = \pm \hbar/2 \quad \left[\hbar = 1.0546 \times 10^{-2} \text{ erg}\cdot\text{s}\right.$$

So measuring $S_z \Rightarrow$ discrete values (2 states)

PI

QM: $\mathcal{H}$ = 2D cpx vector space

$$= \left\{ \begin{pmatrix} C_+ \\ C_- \end{pmatrix}, \ C_\pm \in \mathbb{C} \right\} \underset{\text{phys notation}}{\Rightarrow} \left\{ |\alpha\rangle = C_+|+\rangle + C_-|-\rangle \right\}$$

(Dirac) "ket" notation

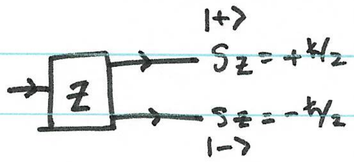
Unit norm condition: $\langle\alpha|\alpha\rangle = |C_+|^2 + |C_-|^2 = 1$

$\Big(\underset{\text{"bra"}}{\langle\alpha|} = (C_+^* \ \ C_-^*)$ [dual vector] $\Big)$.

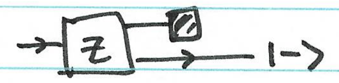
$e^{i\theta}|\alpha\rangle \sim |\alpha\rangle$ physically equivalent

Sequential S-G experiments, w/ components

splitter



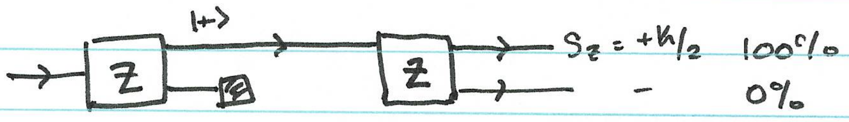
filters



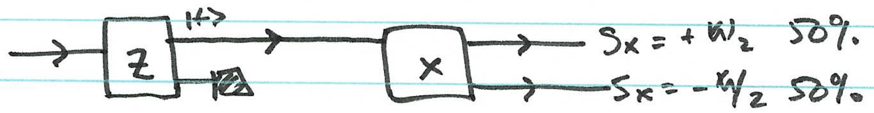
Similar on eg. x-axis

Single-particle experiments

i)



ii)



Spin - 1/2 particle observables

Operators :

(P2) $S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \equiv \frac{\hbar}{2}\sigma_z$ measures spin along z-axis

(P4) eg. $S_z|+\rangle = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}1\\0\end{pmatrix} = \hbar/2\,|+\rangle$

and $S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \equiv \frac{\hbar}{2}\sigma_x$

$S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \equiv \frac{\hbar}{2}\sigma_y$

More generally, for axis $\hat{n}$ $S_n = \underline{S}\cdot\underline{\hat{n}}$ (HW)

For each S_n, $\exists$ 2D basis of eigenstates

$$S_n|S_n;\pm\rangle = \pm\hbar/2\,|S_n;\pm\rangle$$

eig. $|S_x;\pm\rangle = \frac{1}{\sqrt{2}}|+\rangle \pm \frac{1}{\sqrt{2}}|-\rangle$ in S_z basis.

$\Rightarrow$ explains 1-part expts.

eg. if in state $C_+|+\rangle + C_-|-\rangle$, prob $S_z = \pm\hbar/2$ is $|C_\pm|^2$.

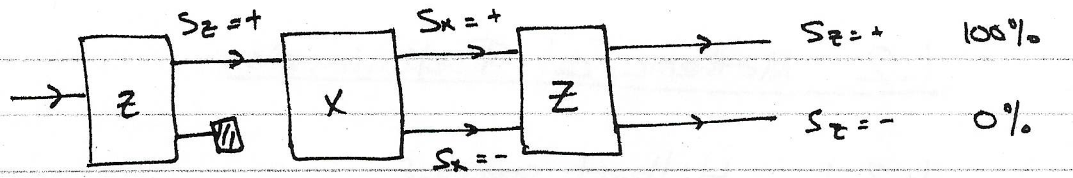
~~eg.~~ if in state $|+\rangle$, prob $S_x = +\hbar/2$ given by

$$|\langle S_x;+|+\rangle|^2 = 1/2 = 50\%,$$

since $\langle S_x;\pm|\alpha\rangle = C_\pm^{(x)}$ just as $\langle\pm|\alpha\rangle = C_\pm$

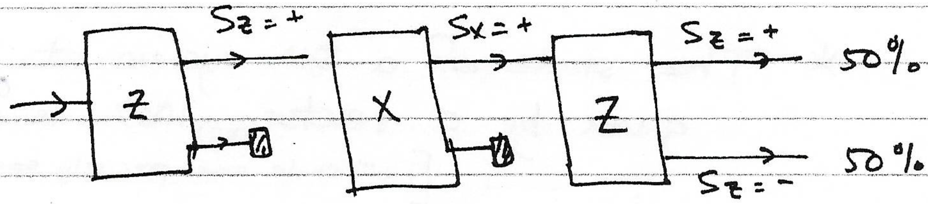
More generally, if system in state $|i\rangle$ (initial state, normalized), and $|f\rangle$ is normalized (non-degenerate) eigenstate of $\mathcal{O}$ with $\mathcal{O}|f\rangle = \lambda|f\rangle$ then prob $(\mathcal{O}=\lambda)$ is $|\langle f|i\rangle|^2$.

iii)

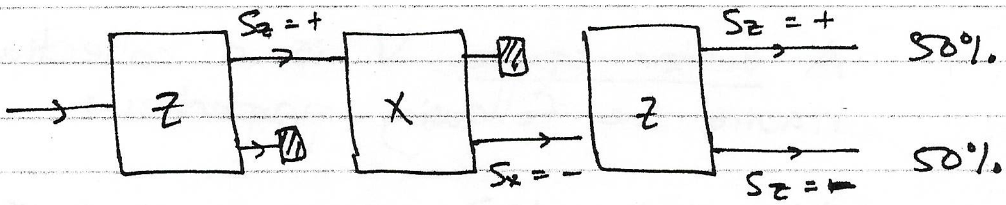


BUT

iv)



v)



Cannot simultaneously measure S_z, S_x

"incompatible observables" $S_z S_x \neq S_x S_z$

Analogous to 2-slit experiment for photons.

In iii), not measuring S_x.
in iv), v) measuring S_x.

iii) needs "interference" of probability wave
— no classical interpretation.

iii), iv) ⇒ Irreversible, nondeterministic dynamics (assuming locality)

1.2 Mathematical Preliminaries

1.2.1 Hilbert spaces

First postulate of QM:

* The state of a QM system at time t is given by a vector (ray) $|\alpha\rangle$ in a complex Hilbert space $\mathcal{H}$.
[will state more precisely soon.]

Vector spaces

A vector space V is a collection of objects ("vectors") $|\alpha\rangle$ having the following properties:

A1: $|\alpha\rangle + |\beta\rangle$ gives a unique vector $|\gamma\rangle$ in V.

A2: (commutativity) $|\alpha\rangle + |\beta\rangle = |\beta\rangle + |\alpha\rangle$

A3: (associativity) $(|\alpha\rangle + |\beta\rangle) + |\gamma\rangle = |\alpha\rangle + (|\beta\rangle + |\gamma\rangle)$

A4: $\exists$ vector $|\phi\rangle$ such that $|\phi\rangle + |\alpha\rangle = |\alpha\rangle \; \forall |\alpha\rangle$

A5: For all $|\alpha\rangle$ in V, $-|\alpha\rangle$ is also in V so that $|\alpha\rangle + (-|\alpha\rangle) = |\phi\rangle$.

[(A1-A5): V is a commutative group under $+$]

For some field F (i.e., $\mathbb{R}, \mathbb{C}$, with $+, *$ defined) scalar multiplication of any $c \in F$ with any $|\alpha\rangle \in V$ gives a vector $c|\alpha\rangle \in V$. Scalar multiplication has the following properties

M1: $c(d|\alpha\rangle) = (cd)|\alpha\rangle$

M2: $1|\alpha\rangle = |\alpha\rangle$