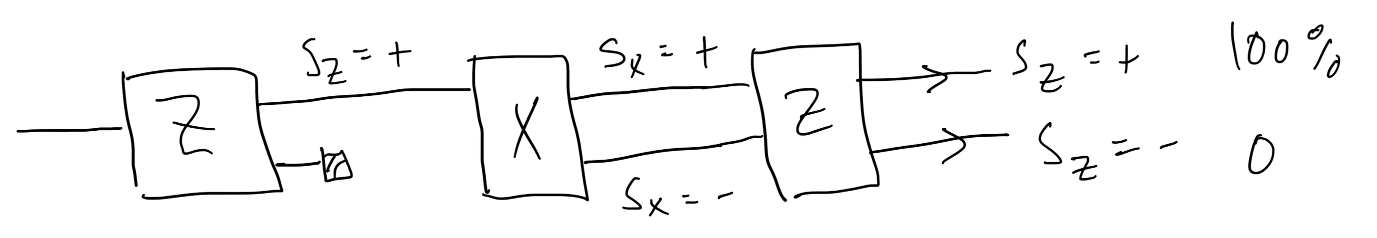
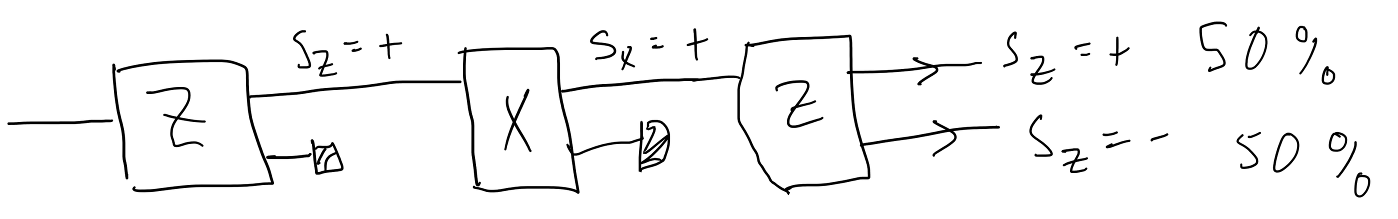
Friday September 17

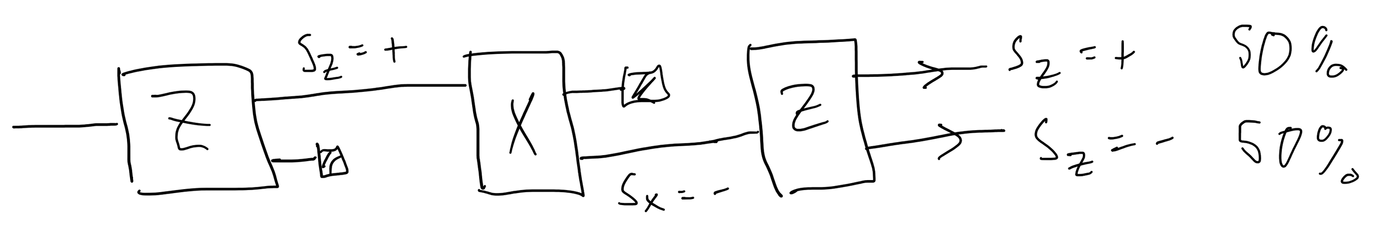
Thursday, September 16, 2021 6:07 PM

QM is strange!

Recall from class:







- We see that measurement of S_x changes the final result of expt.
- To get exactly $S_z=+$ or $S_z=-$ we need to keep both $S_x=+$ and $S_x=-$ coming out ~~of~~ the filter.
- The states $|S_z=\pm\rangle$ are "coherent superpositions" of $|S_x=+\rangle$ and $|S_x=-\rangle$

What is $|S_x=+\rangle$?

Recall $|S_x=+\rangle$ is the state that has definite value of $\hat{S}_x$. Thus, $|S_x=+\rangle$ must be an <u>eigenvector</u> of $\hat{S}_x$.

$\hookrightarrow \hat{S}_x|S_x=+\rangle = (\#)\cdot|S_x=+\rangle$

How do we find eigenvectors?

Recall that in the standard basis $|S_z=+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|S_z=-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ we have

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

Any state in this system can be written as $|\psi\rangle = \psi_+|S_z=+\rangle + \psi_-|S_z=-\rangle = \begin{pmatrix}\psi_+\\\psi_-\end{pmatrix}$

$$\Rightarrow |S_x=+\rangle = \begin{pmatrix}x_+\\x_-\end{pmatrix} \text{ for some } x_\pm \in \mathbb{C}$$

Eigenvalues of $\hat{S}_x$ (λ_i)

λ_i s.t. $\hat{S}_x|\lambda_i\rangle = \lambda_i|\lambda_i\rangle$

$$\Rightarrow (\hat{S}_x - \lambda_i\cdot\mathbb{1})\underbrace{|\lambda_i\rangle}_{\neq 0} = 0$$

Can only get 0 from nonzero vector if $\boxed{\det(\hat{S}_x-\lambda_i\mathbb{1})=0}$

Quick Group Exercise

- Compute $\det(\hat{S}_x - \lambda_i\cdot\mathbb{1})$
- Solve for eigenvalues

A: $|\hat{S}_x - \lambda_i\mathbb{1}| = \begin{vmatrix}-\lambda_i & \frac{\hbar}{2}\\ \frac{\hbar}{2} & -\lambda_i\end{vmatrix} = \lambda_i^2 - \left(\frac{\hbar}{2}\right)^2$

$\Rightarrow \lambda_i^2 - \left(\frac{\hbar}{2}\right)^2 = 0 \Rightarrow \lambda_i = \pm\frac{\hbar}{2}$

- Now Describe $|S_x=+\rangle$ in z basis
  - $\rightarrow$ recall $|S_x=+\rangle = \begin{pmatrix}x_+\\x_-\end{pmatrix}$

$$\hat{S}_x|S_x=+\rangle = \frac{\hbar}{2}|S_x=+\rangle$$

A: $\frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}x_+\\x_-\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}x_+\\x_-\end{pmatrix}$

$\Rightarrow \begin{pmatrix}x_-\\x_+\end{pmatrix} = \begin{pmatrix}x_+\\x_-\end{pmatrix}$ or $x_+ = x_-$

Normalization: $|x_+|^2 + |x_-|^2 = 2|x_+|^2 = 1$

$\Rightarrow x_+ = \frac{1}{\sqrt{2}}e^{i\phi_+} = x_-$

$\Rightarrow |S_x=+\rangle = \frac{1}{\sqrt{2}}e^{i\phi_+}\begin{pmatrix}1\\1\end{pmatrix}$

Overall phase not fixed. Recall "states" are really <u>rays</u> in Hilbert space.

Recall S_x and S_z are "incompatible observables"

$\rightarrow$ i.e. there is no state with definite S_x and S_z

If this was true then we would have $|\lambda_x, \lambda_z\rangle$ s.t.

$\hat{S}_x|\lambda_x,\lambda_z\rangle = \lambda_x|\lambda_x,\lambda_z\rangle$

$\hat{S}_z|\lambda_x,\lambda_z\rangle = \lambda_z|\lambda_x,\lambda_z\rangle$

$\therefore \hat{S}_x\hat{S}_z|\lambda_x,\lambda_z\rangle = \hat{S}_z\hat{S}_x|\lambda_x,\lambda_z\rangle$

<u>Quick Q</u>: what are both equal to?

It turns out that compatible observables need to have $\hat{A}\hat{B}|\psi\rangle = \hat{B}\hat{A}|\psi\rangle$ for all $|\psi\rangle$