Notes on spontaneous emission:

- $P_b = e^{-\Gamma t}$

  (not like $1 - \Gamma T + \frac{\Gamma^2 t^2}{2} + \dots$, as perturbation theory)

- Q: What about shifts of the excited level b?

  A - There is a shift

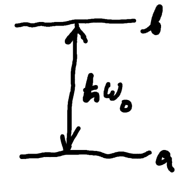
  B - There is no shift

  $$\sum_k \frac{|V_{ik}|^2}{E_i - E_k + i\eta} = \Delta - i\frac{\hbar\Gamma}{2}$$

- Γ can be modified! Dan Kleppner PRL 47, 233

  e.g. use cavity $\Rightarrow$ discretizes spectrum

  If one of the modes is resonant

  $|\varphi_i\rangle = |b, 0\rangle$; $|\varphi_f\rangle = |a, 1\rangle$

  $\Rightarrow$ Rabi oscillations

  If all modes are different: atom cannot emit!

- Consider absorption of a monochromatic wave

  $$|\varphi_i\rangle = |a; N\vec{k}_0 \vec{\varepsilon}_0\rangle \longleftrightarrow |\varphi_f\rangle = |b, (N-1)\vec{k}_0\vec{\varepsilon}_0\rangle \longleftrightarrow |\varphi_g\rangle = |a, (N-1)\vec{k}_0\vec{\varepsilon}_0, \vec{k}\vec{\varepsilon}\rangle$$

  $V_{if} = \frac{\hbar\Omega}{2}$ $\qquad$ V_{fg}

$V_{fg} = 0$: Rabi oscillations $\Omega \propto dE \propto d\sqrt{N}$

$V_{if} = 0$: $|\varphi_f\rangle$ decays spontaneously $e^{-\Gamma t}$

both on:

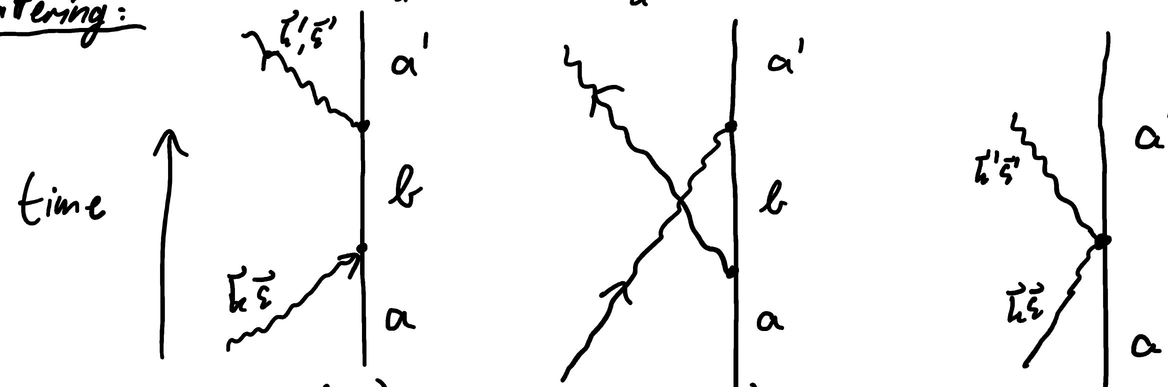
$\Omega \gg \Gamma$: atom oscillates between a and b
oscillations slowly damped with $\tau \sim \frac{1}{\Gamma}$

$\Omega \ll \Gamma$: $|\varphi_f\rangle$ "dissolved" in the continuum $|\varphi_g\rangle$
density of states of $|\varphi_f\rangle$ in $|\varphi_g\rangle$ is $\frac{1}{\hbar\Gamma}$

$|\varphi_i\rangle$ decays via $\Gamma_i = \frac{2\pi}{\hbar}|V_{if}|^2 \cdot \frac{1}{\hbar\Gamma} \approx \frac{\Omega^2}{\Gamma}$

Scattering:

$E_{a'} + \hbar\omega' = E_a + \hbar\omega$ $\qquad$ $E_a + \hbar\omega = E_{a'} + \hbar\omega'$



(α) $\qquad$ (β) $\qquad$ (γ)

$$\tau_{fi}^{\alpha} = \sum_b \lim_{\eta\to 0^+} \frac{\langle a', \vec{k}'\vec{\varepsilon}' | H_{I_1} | b, 0 \rangle\langle b, 0 | H_{I_1} | a, \vec{k}\vec{\varepsilon}\rangle}{E_a + \hbar\omega - E_b + i\eta}$$

$$\tau_{fi}^{\beta} = \sum_b \lim_{\eta\to 0^+} \frac{\langle a', \vec{k}'\vec{\varepsilon}' | H_{I_1} | b, \vec{k}\vec{\varepsilon}\, \vec{k}'\vec{\varepsilon}' \rangle\langle b, \vec{k}\vec{\varepsilon}\, \vec{k}'\vec{\varepsilon}' | H_{I_1} | a, \vec{k}\vec{\varepsilon}\rangle}{E_a - \hbar\omega' - E_b + i\eta}$$

$$\tau_{fi}^{\gamma} = \langle a', \vec{k}'\vec{\varepsilon}' | H_{I_2} | a; \vec{k}\vec{\varepsilon}\rangle$$

(using $-\vec{d}\cdot\vec{E}_\perp$ only α and β exists)

intermediate state in β is never discrete due to 2 photons

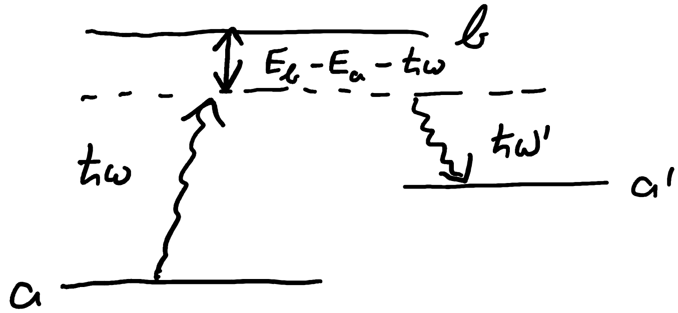
" " in α could be discrete if b is discrete.

$\longrightarrow$ resonant scattering $\rightarrow$ later

$a' = a$: $E_a = E_{a'}$; $\omega' = \omega \Rightarrow$ elastic scattering

$a' \neq a$: inelastic ; $\hbar\omega' - \hbar\omega = E_a - E_{a'}$

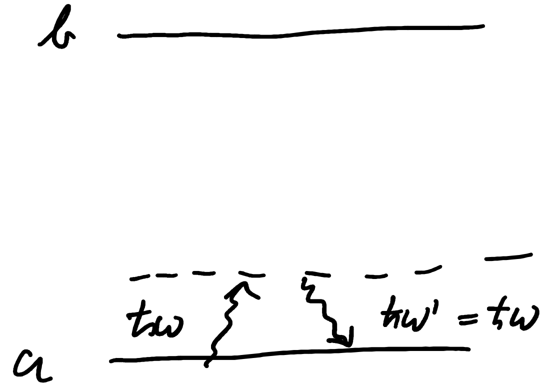
Other diagrammatic rep:



Rayleigh scattering

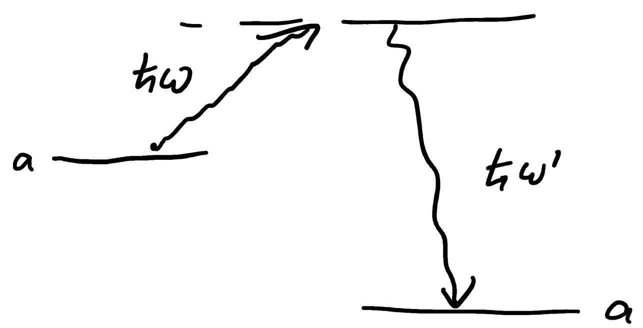
$\hbar\omega \ll |E_b - E_a|$

$\sigma \propto \omega^4$



Raman scattering

Stokes

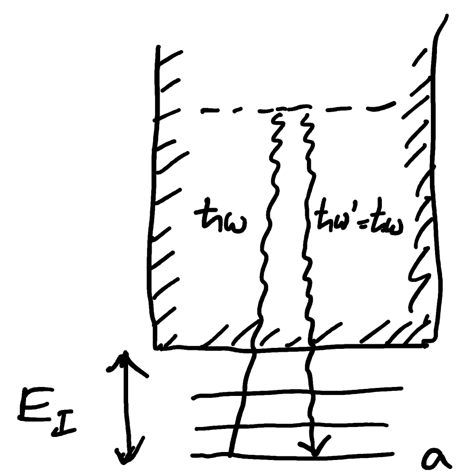


Anti - Stokes

Thompson scattering

$\hbar\omega \gg E_I$ (ionization energy)

process γ dominates
(no $\frac{1}{\hbar\omega}$ denominator)



Van-der-Waals and Casimir forces

$e^- - e^- \qquad \frac{e^2}{r}$

$e^- -$ atom

$\vec{E}$ induces dipole moment $\vec{d} = \alpha \vec{E}$ in atom

Interaction: $\quad -\vec{d}\cdot\vec{E} = -\alpha |\vec{E}|^2$

$$|\vec{E}| = \frac{e}{r^2}$$

$$\Rightarrow \quad V_{a-e^-} = -\alpha \frac{e^2}{r^4}$$

atom - atom ; mean value $\langle \vec{d} \rangle = 0$

In second order we find

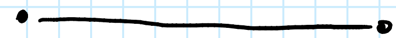
$$V_{aa} = \frac{\left(\frac{\vec{d}\cdot\vec{d}'}{r^3}\right)^2}{\Delta E}$$

$$\Delta E \sim \frac{e^2}{a_0} \qquad |\vec{d}| \sim e a_0$$

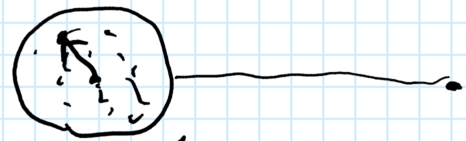
$$V_{aa} \sim \frac{\left(\frac{(e a_0)^2}{r^3}\right)}{\frac{e^2}{a_0}} \sim -\frac{e^2}{a_0} \cdot \frac{a_0^{\,6}}{r^6}$$

Van-der-Waals and Casimir forces

$e^- - e^-$ $\qquad \dfrac{e^2}{r}$



e^- - atom



$\vec{E}$ induces dipole moment $\vec{d} = \alpha \vec{E}$ in atom.

Interaction $\quad -\vec{d}\cdot\vec{E} = -\alpha |\vec{E}|^2$

$$|\vec{E}| = \frac{e}{r^2}$$

$$\Rightarrow \quad V_{a-e^-} = -\alpha \frac{e^2}{r^4}$$

atom-atom $\quad$ mean value $\langle \vec{d} \rangle = 0$

However, in second order we find

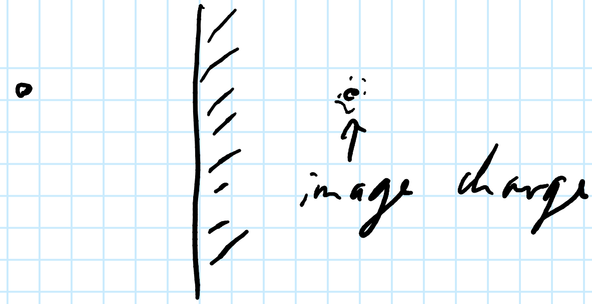
$$V_{aa} = \frac{\left(\vec{d}\cdot\vec{d}'/r^3\right)^2}{\Delta E}$$

$\uparrow$ characteristic exc. energy

$$\Delta E \sim \frac{e^2}{a_0} \qquad \vec{d} \sim e a_0$$

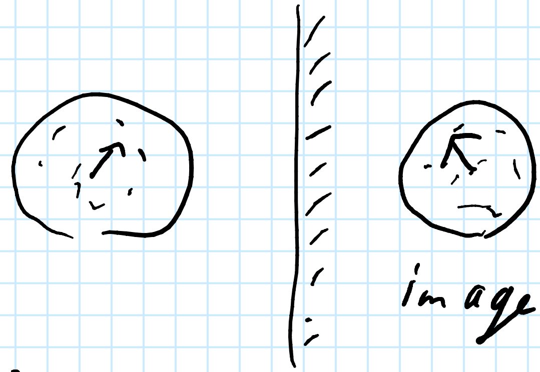
$$V_{aa} - \frac{\left((e a_0)^2/r^3\right)^2}{e^2/a_0} \sim \frac{e^2}{a_0}\,\frac{a_0^6}{r^6}$$

electron - wall:



$$V_{e\,wall} = (-)\,\frac{e^2}{z}$$

atom - wall



$$V = \frac{|d|^2}{z^3}$$

$$\sim \frac{(e a_0)^2}{z^3}$$

No van-der-Waals in classical theory!

ground-state of atom = "e^- bound by spring" is $\vec{d} = 0$. No fluctuations.

$\Rightarrow$ Need "zero-point" fluctuations of QM to get correlated $\vec{d}$'s: $\langle d_a d_b \rangle \neq 0$

General argument at large distances

Two polarizable systems

external field E_0 induces $\vec d_1, \ \vec d_2 = \alpha \vec E_0$.

Interaction

$$V(\vec x_1, \vec x_2, t) = \int_0^\infty d\omega \, N(\omega) \; \vec d_1 \cdot \vec E_{2\to 1}(\omega, t)$$

$$N(\omega)\, d\omega = 2 V d^3k = 2\omega^2 d\omega \, \sin\vartheta \, d\vartheta \, d\varphi \, \frac{V}{c^3}$$

$$\vec E_{2\to 1} \propto \underbrace{\frac{\vec d_2 - 3 d_{2z}\hat z}{r^3} + \frac{\dot{\vec d}_2 - 3\dot d_{2z}\hat z}{c r^2}}_{} + \frac{\ddot{\vec d}_2 - \ddot d_{2z}\hat z}{c^2 r}$$

$\longrightarrow 1 - 3\cos^2\vartheta \quad \longrightarrow$ zero | radiation term!

radiation term $\propto \alpha_2 \vec E_0 \dfrac{\omega^2}{c^2 r}$

$$V(\vec x_1, \vec x_2) = \frac{V}{c^5 r} \int_0^{c/r} \alpha_1(\omega)\, \alpha_2(\omega)\, \omega^2 \, |E_0(\omega, \vec x_1)|^2 \omega^2 d\omega$$

$$|E|^2 \sim u(\omega) \sim \frac{\hbar\omega}{V}$$

So $$V(\vec x_1, \vec x_2) = \frac{\hbar}{c^5 r} \int_0^{c/r} \alpha_1(\omega)\, \alpha_2(\omega)\, \omega^5 \, d\omega$$

atom = e^- on a spring

$$\alpha(\omega) = \frac{e^2/m}{\omega_0^2 - \omega^2} \longrightarrow \frac{e^2}{m\omega_0^2}\frac{a_0^3}{a_0^3} = a_0^3$$

if $r > \frac{c}{\omega_0}$, relevant $\omega = ck = \frac{c}{r}$ is $\ll \omega_0$.

e^- has no ω_0^2

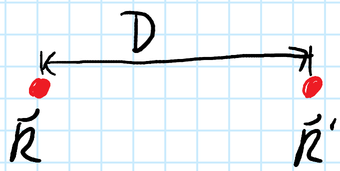
$$V_{ee} \sim \frac{\hbar e^4}{c^3 m^2 r^3}$$

$$V_{ae} \sim \frac{\hbar e^2 \alpha}{m c r^5}$$

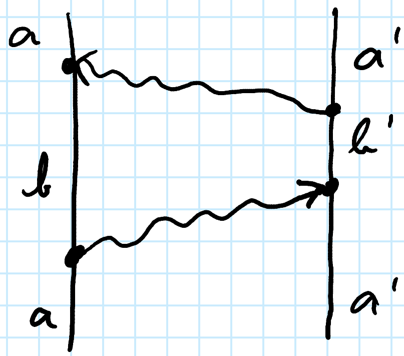
$$V_{aa} \sim \frac{\hbar c \alpha_1 \alpha_2}{r^7}$$

Interaction by photon exchange

— Van-der-Waals interaction



$$H_I' = -\vec{d}\cdot\vec{E}_\perp(\vec{R}) - \vec{d}'\cdot\vec{E}_\perp(\vec{R}')$$



Conservation of energy $\Rightarrow$ first process coupling the two atoms is a fourth order process

$\Rightarrow$ exchange of pairs of photons between the two atoms.

All intermediate states off-resonant.

few low-frequency modes ($k^2 dk$ small, $\langle\cdot\rangle \propto \sqrt{\omega}$)

large wave-vectors interfere $e^{i\vec{k}\cdot\vec{R}}$, $e^{i\vec{k}\cdot\vec{R}'}$

$\Rightarrow$ dominant $k \sim \frac{1}{D}$.

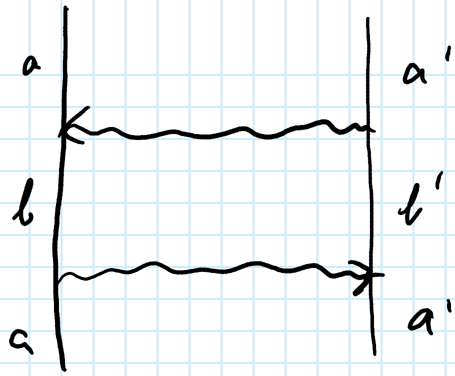
Two limits:

$D \ll \lambda_{ab} = \frac{\hbar c}{|E_a - E_b|}$: $\hbar\omega \gg \hbar\omega_0$.

photons exist for very short time.

$\Rightarrow$ horizontal photon lines

$D \ll \lambda_{ab}$



defect $\approx \hbar\omega$

It's like both atoms excited simultaneously.

effective Hamiltonian δV with

$\langle b, b' | \delta V | a, a' \rangle =$

$$= -\sum_{k\vec{\varepsilon}} \frac{1}{\hbar\omega} \langle b, b', 0 | H_I | b, a'; k\vec{\varepsilon} \rangle \langle b, a'; k\vec{\varepsilon} | H_I | a, a', 0 \rangle$$

$$= -\sum_{k\vec{\varepsilon}} \frac{1}{2\varepsilon_0 L^3} (\vec{d} \cdot \vec{\varepsilon})(\vec{d}' \cdot \vec{\varepsilon}) e^{i\vec{k}(\vec{R}-\vec{R}')} + h.c.$$

$$= -\frac{1}{\varepsilon_0} \sum_{ij} d_i d_j \, \delta_{ij}^{\perp}(\vec{R}-\vec{R}')$$

$$\delta_{ij}(\vec{R}-\vec{R}') = \frac{-\delta_{ij} + 3n_i n_j}{4\pi D^3} \qquad \sum_{ij} \varepsilon_i \varepsilon_j = \delta_{ij} - \frac{k_i k_j}{k^2}$$

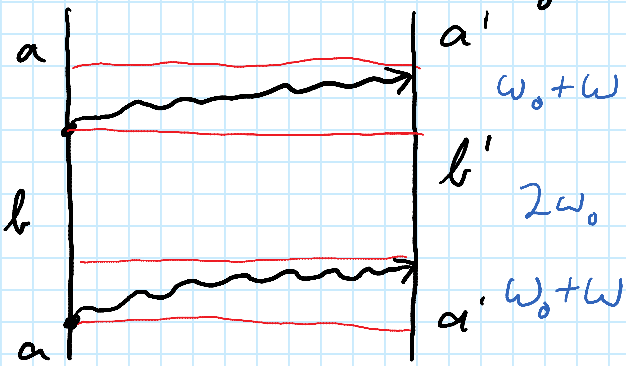
$\Rightarrow$ dipole-dipole interaction! $\quad |a, a'\rangle \to |b, b'\rangle$

$$\Delta E = \sum_{bb'} \frac{\langle a a' | \delta V | b b' \rangle \langle b b' | \delta V | a a' \rangle}{E_a + E_{a'} - E_b - E_{b'}}$$

$$= \underline{\underline{-\frac{C_6}{D^6}}}$$

short distance $D \ll \lambda_{ab}$

$\omega \gg \omega_0$



energy defects

$$\frac{1}{\omega^2 \omega_0}$$

large distance $D \gg \lambda_{ab}$

$\omega \ll \omega_0$

energy defects

$$\frac{1}{\omega_0^2 \omega}$$

So for $D \gg \lambda_{ab}$ we get one more factor of ω

$\omega \propto \frac{1}{\lambda} \propto \frac{1}{D}$

$$\frac{1}{D^6} \xrightarrow[\text{long range}]{} \frac{1}{D^7}$$

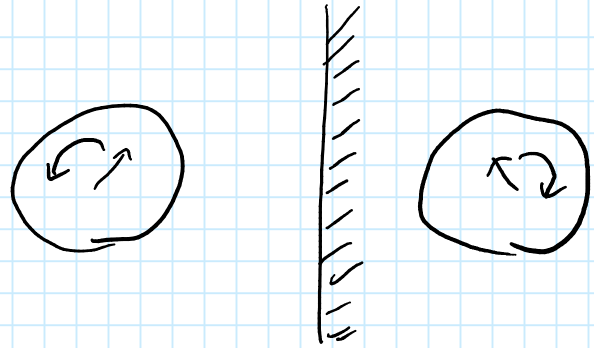
instantaneous $\longrightarrow$ retarded potentials

Long-range potentials

	$r < \lambda_{ab} = 137\, a_0$	$r > \lambda_{ab} = 137\, a_0$
atom-atom	$\frac{e^2}{a_0} \frac{a_0^6}{r^6}$	$\frac{\hbar c \; a_0^6}{r^7}$
atom-wall	$\frac{(e a_0)^2}{z^3}$	$\frac{\hbar c}{z^4} a_0^3$
wall-wall	$\frac{\hbar c}{z^4}$	

Atom - Wall

$z \ll \lambda_{ab} \approx \frac{a_0}{\alpha} \approx 137\, a_0$:



$\Rightarrow$ correlated dipoles

$$\Rightarrow \quad V_{a-w} = \frac{(e a_0)^2}{z^3}$$

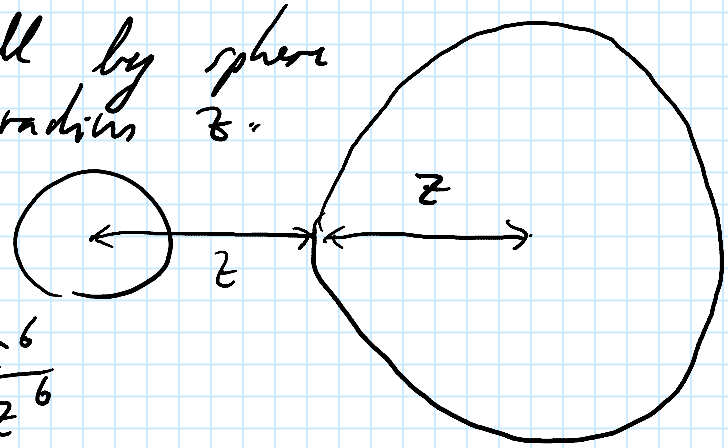
$z \gg \lambda_{ab} = 137\, a_0$

Use Spruch's formula

$$V(r) = \frac{\hbar}{c^5 r} \int_0^{c/r} d\omega \; \alpha_1(\omega)\, \alpha_2(\omega)\, \omega^5$$

but trick: replace wall by sphere of radius z.

$\alpha_{sphere} \propto z^3$



$$\Rightarrow V_{a-w} = \frac{\hbar}{c^5 z}\, a_0^3 z^3 \, \frac{c^6}{z^6}$$

$$= \boxed{\frac{\hbar c}{z^4}\, a_0^3}$$

Other way:

Polarizable system in presence of background field:

energy $\alpha(\omega) |E_0|^2(\omega, \vec{x}) = \alpha(\omega)\, u(\omega, \vec{x})$

$u(\omega, \vec{x})$ = energy density $= \frac{\hbar\omega}{V}$ for vacuum.

$$\Rightarrow \quad \mathcal{E} = \int_0^\infty d\omega\, N(\omega)\, u(\omega, \vec{x})\, \alpha(\omega)$$

$$= \frac{\hbar}{c^3} \int_0^\infty d\omega\, \alpha(\omega)\, \omega^3 \quad = \text{ infinite}$$

Now: atom at distance z from an ideal wall

$\Rightarrow$ fluctuation with $\omega \gg \frac{c}{z}$ not affected by presence of the wall.

fluctuation with $\omega \ll \frac{c}{z}$ greatly affected.

$\Rightarrow$ While self-energy $\mathcal{E}(z)$ is also infinite

$\mathcal{E} - \mathcal{E}(z)$ is finite

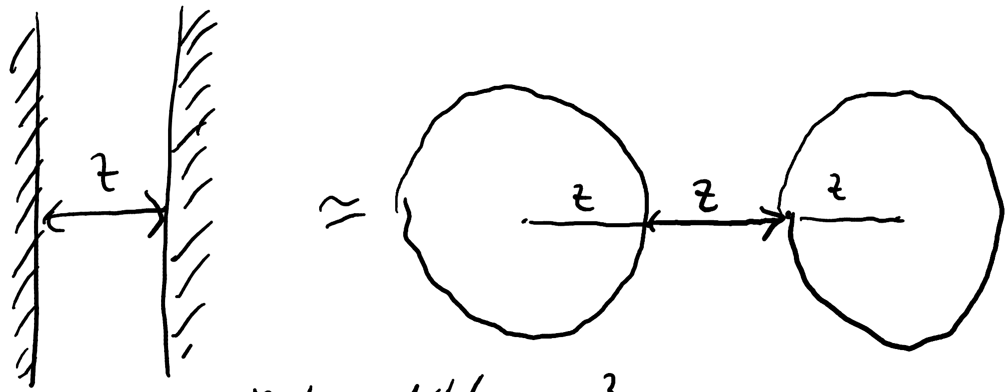
contributions $\omega \gg \frac{c}{z}$ cancel

$\omega \ll \frac{c}{z}$ roughly comparable

$$\Rightarrow \quad V_{a\text{-wall}}(z) = \frac{\hbar}{c^3} \int_0^{c/z} d\omega\, \alpha(\omega)\, \omega^3$$

$$= \frac{\hbar c}{z^4}\, a_0^3$$

Wall-wall: Trick: replace walls by spheres of radius z



Polarizability z^3

Need $\frac{\text{Force}}{\text{unit area}} = \frac{1}{z^2}\frac{\partial V}{\partial z} \simeq \frac{V}{z^3}$

$$V = \frac{\hbar}{c^5 z} z^6 \int_0^{c/z} d\omega\, \omega^5$$

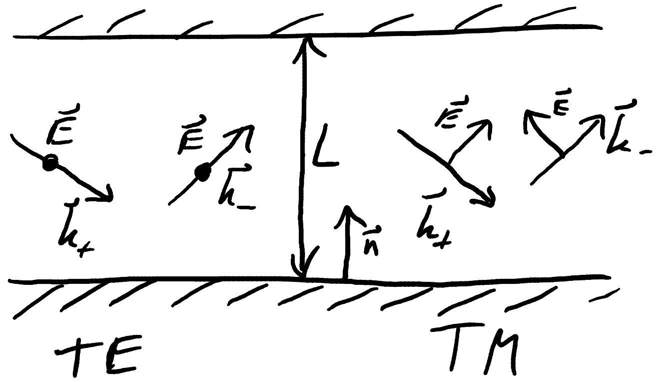
$$= \frac{\hbar c}{z}$$

So $\frac{F}{A} = \frac{\hbar c}{z^4}$ Casimir 1948

Casimir Effect – Full calculation

see Serge Haroche, Les Houches Summer School Lecture 1990

Cavity



TE and TM modes exist in cavity.

superpositions of plane waves with

$$\vec{k}_{\pm} = \mp l \vec{n} + k \vec{\varphi}$$

$\vec{n}$ – normal
$\vec{\varphi}$ – parallel to mirror

$$\frac{\omega^2}{c^2} = l^2 + k^2$$

Total electric field along mirrors and magnetic field normal to surfaces must vanish at $z=0$ and $z=L$.

$\Rightarrow$ l is quantized

$$l = \frac{m\pi}{L}.$$

$$\Rightarrow \quad \omega^2 = m^2 \omega_0^2 + k^2 c^2$$

with $\omega_0 = \frac{c\pi}{L}$

Field distributions

$$\vec{\alpha}^{E}_{m,k,p}(z,\vec{\rho}) = \sqrt{\frac{2}{V}}\,\sin\left(\frac{m\pi z}{L}\right) e^{ik\vec{\rho}\cdot\vec{\rho}}\ \vec{\rho}\times\vec{n}$$

$$\vec{\alpha}^{M}_{m,k,p}(z,\rho) = \sqrt{\frac{\beta_m}{V}}\left[\frac{ck}{\omega}\cos\left(\frac{m\pi z}{L}\right)\vec{n} - i\frac{m\omega_0}{\omega}\sin\left(\frac{m\pi z}{L}\right)\vec{\rho}\right] e^{ik\vec{\rho}\cdot\vec{\rho}}$$

$$\beta_m = 1 \text{ if } m=0$$
$$\quad = 2 \text{ if } m>0$$

$V = L\,a^2$, a arbitrary side length.

$m=0$ modes have no spatial variation along z.
$\Rightarrow$ since transverse electric field must vanish at $z=0$ and $z=L$, it must vanish everywhere

$\Rightarrow$ No TE mode with $m=0$.

TM with $m=0$ has $\sqrt{\frac{1}{V}}$ normalization

all others (for which $\langle \sin^2\rangle = \langle\cos^2\rangle = \frac{1}{2}$) have $\sqrt{\frac{2}{V}}$.

Vector potential $\vec{A}(z,\vec{\rho}) = \vec{A}^E(z,\vec{\rho}) + \vec{A}^M(z,\vec{\rho})$

$$\vec{A}^E(z,\vec{\rho}) = \sum_{mkp}\left\{\sqrt{\frac{\hbar}{2\varepsilon_0\omega}}\,\vec{\alpha}^E_{mkp}(z,\vec{\rho})\,a^E_{mkp} + h.c.\right\}$$

$$\vec{A}^M(z,\vec{\rho}) = \sum_{mkp}\left\{\sqrt{\frac{\hbar}{2\varepsilon_0\omega}}\,\vec{\alpha}^M_{mkp}(z,\vec{\rho})\,a^M_{mkp} + h.c.\right\}$$

Counting modes:

cyclic boundary conditions $\Rightarrow$ k_x, k_y quantized in units of $\frac{2\pi}{a}$.

# of modes for given m between k and $k+dk$ is

$$d^2k \cdot \frac{a^2}{(2\pi)^2} = \frac{a^2}{2\pi} k\,dk = \frac{a^2}{2\pi c^2}\,\omega\, d\omega$$

$$\uparrow \; \frac{\omega^2}{c^2} = l^2 + k^2$$

A given ω can be obtained for $m = 0$ to $\mathrm{Int}\left(\frac{\omega}{\omega_0}\right)$

For $m > 0$ we have TE and TM mode

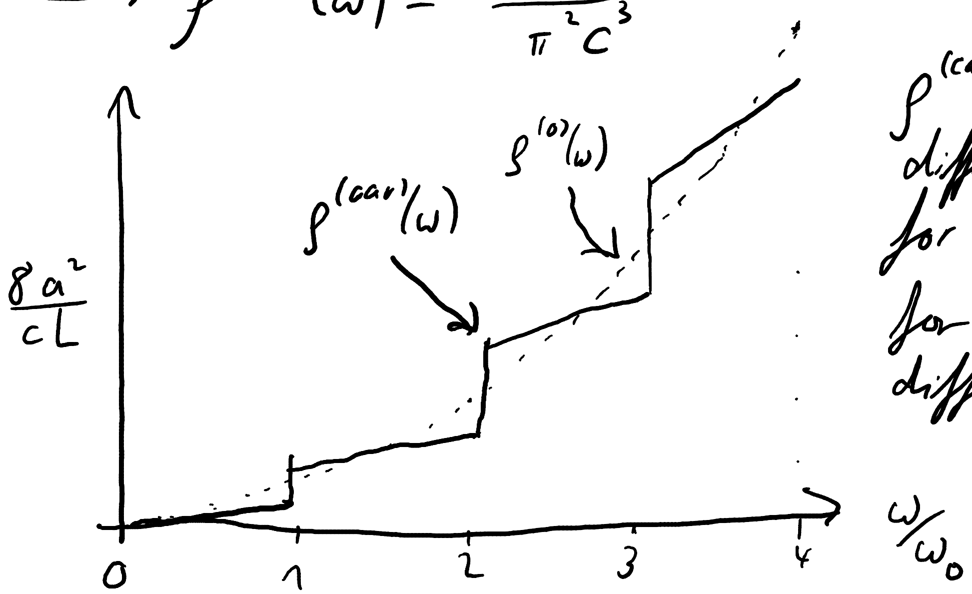
$m = 0$ one TM mode.

$$\Rightarrow \rho^{(car)}(\omega) = \frac{a^2\omega}{2\pi c^2}\left[1 + 2\,\mathrm{Int}\left(\frac{\omega}{\omega_0}\right)\right]$$

$$= \frac{V\omega\,\omega_0}{2\pi^2 c^3}\left[1 + 2\sum_{m=1}^{\infty}\Theta\left(\frac{\omega}{\omega_0} - m\right)\right]$$

In free space $L \to \infty$, $\omega_0 \to 0$

$$\longrightarrow \rho^{(0)}(\omega) = \frac{V\omega^2}{\pi^2 c^3}$$



$\rho^{(car)}$ and $\rho^{(0)}$ differ significantly for $\omega \lesssim$ few ω_0.

for $\omega \gg \omega_0$, difference is negligible

(constructed by adding mode by mode)

Casimir effect: The variation of $\rho^{(cav)}(\omega)$ with L leads to a variation of the total field vacuum energy with L

$\Rightarrow$ force that pulls mirrors together

$$W(L) = \sum_{\text{modes}} \frac{\hbar\omega}{2} = \int_0^\infty d\omega \, \frac{\hbar\omega}{2} \rho^{(cav)}(\omega)$$

$$= \frac{a^2\hbar}{4\pi c^2} \left[I_0 + 2 \sum_{m=1}^\infty I_m \right]$$

with $I_m = \int_{m\omega_0}^\infty d\omega \, \omega^2$

$W(L)$ diverges. I_0 divergence doesn't depend on L.

for I_m $(m \neq 0)$ introduce converging term $e^{-\lambda\omega/c}$.

$$I_m = \int_{m\pi c/L}^\infty \omega^2 e^{-\lambda\omega/c} d\omega$$

$$= c^2 \frac{\partial^2}{\partial\lambda^2} \int_{m\pi c/L}^\infty e^{-\lambda\omega/c} d\omega$$

$$= c^3 \frac{\partial^2}{\partial\lambda^2} \left[- \frac{e^{-m\pi\lambda/L}}{\lambda} \right]$$

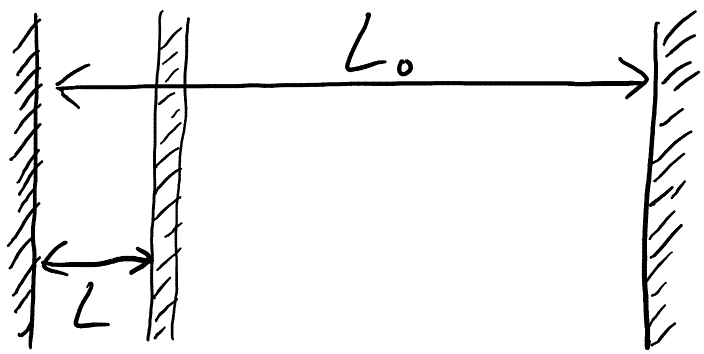
$$\sum_{m=1}^\infty I_m = - c^3 \frac{\partial^2}{\partial\lambda^2} \left\{ \sum_{m=1}^\infty \frac{e^{-m\pi\lambda/L}}{\lambda} \right\}$$

$$= \frac{c^3\pi}{L} \frac{\partial^2}{\partial\lambda^2} \left\{ \frac{L}{\pi\lambda} \frac{1}{e^{\frac{\pi\lambda}{L}} - 1} \right\}$$

$$\frac{L}{\pi\lambda}\frac{1}{e^{\pi\lambda/L}-1} = \frac{L^2}{(\pi\lambda)^2} - \frac{L}{2\pi\lambda} + \frac{1}{12} - \frac{1}{720}\left(\frac{\pi\lambda}{L}\right)^2 + \dots$$

$$W(L) = \frac{a^2\hbar}{4\pi c^2} I_0 + \frac{a^2\hbar c}{2}\left[\frac{6L}{\pi^2\lambda^4} - \frac{1}{\pi\lambda^3} - \frac{2\pi^2}{720L^3} + \dots\right]$$

Trick: Embed mirror in between larger gap:



Total energy

$$W_T(L) = \frac{a^2\hbar}{2\pi c^2} I_0 + \frac{a^2\hbar c}{2}\left[\frac{6L_0}{\pi^2\lambda^4} - \frac{2}{\pi\lambda^3} - \frac{2\pi^2}{720L^3} + \dots\right]$$

(neglect $\frac{1}{(L_0-L)^3} \ll \frac{1}{L^3}$)

For a different configuration L' we get $W_T(L')$.

$$W_T(L') - W_T(L) = -\frac{a^2\pi^2\hbar c}{720}\left(\frac{1}{L'^3} - \frac{1}{L^3}\right)$$

$$\Rightarrow U(L) = -\frac{\pi^2\hbar c}{720}\frac{a^2}{L^3}$$

Pressure $P_{vac} = \frac{1}{a^2}\frac{\partial U}{\partial L} = \frac{\pi^2\hbar c}{240}\frac{1}{L^4}$

$P_{vac} = 10^{-3}$ Pa for $L = 1$ μm. $\hat{=}$ one electron per L^2

$= 10^{-5}$ mbar

Physical justification for λ:
Plasma frequency!
Above ω_{pl} mirrors are transparent.

Two ways to calculate Casimir effect:

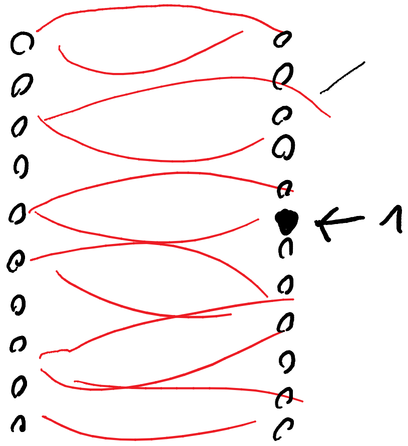
$$\Delta E = \sum \frac{1}{2}\hbar\omega - \sum \frac{1}{2}\hbar\omega_0$$

or S-matrix approach

$$\langle 0 | U(t) | 0 \rangle \sim |\langle \psi | 0 \rangle|^2 e^{-i\Delta E t/\hbar}$$

$\rightarrow$ no reference to vacuum energy needed.
See Jaffe.

Other derivation:



$\sum V_{\text{atom-atom}}$ or $\sum F_{\text{atom-atom}}$

Force on atom 1 from all atoms of other wall:

$$\int \frac{d^2x}{a_0^2} \frac{\hbar c\, a_0^6}{(z^2+x^2+y^2)^4} \longrightarrow \frac{\hbar c\, a_0^4}{z^6}$$

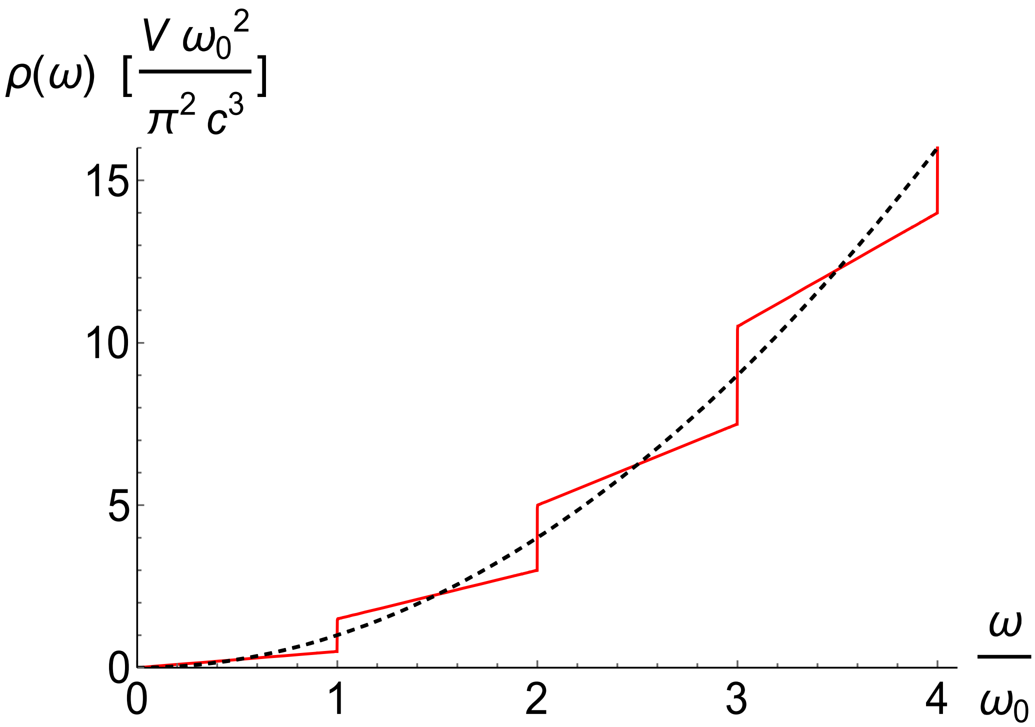
Add all forces on all atoms in one wall:

$$\int \frac{d^2x}{a_0^2} \frac{\hbar c\, a_0^4}{z^6} \frac{z}{\sqrt{z^2+x^2+y^2}} \longrightarrow \frac{\hbar c\, a_0^2}{z^4}$$

$\uparrow$ only component normal to wall

$$\Rightarrow \quad \frac{\text{Force}}{\text{unit area}} = \frac{\hbar c}{z^4}.$$

Plot of the cavity density of states (red) and the free space density of states



Zoom in onto the region from $\omega = 0\ to\ \omega = \omega_0$:
It is nice to see how here the cavity density of states is clearly falling behind the one of free space: