PH312: Physics of Fluids (Prof. McCoy) – Reflection
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Reflection on the readings:

1. Kundu & Cohen 4.1 - 4.3; Tritton 5.4,

2. Kundu & Cohen 4.7

3. Kundu & Cohen 4.10, 4.11; Tritton 5.6, 5.7

I think we've finally gotten to the "meat" of fluid dynamics this week. Starting with conservation of mass, we then talked about conservation of momentum, and finally deriving the Navier-Stokes equation.

Even though I am familiar with a lot of these ideas (conservation laws, Newton's second law, and vector calculus), I find that putting them together to describe the same laws to describe fluid flow is actually very complicated and very subtle. One reason, I think, is that we have to work with fields in fluid dynamics (rather than point-particles), and so the calculus is much more involved. Another reason is interpreting the math. As we have seen, the calculus used to describe fluids is almost identical to that used in electromagnetism (things like conservation laws, Laplace/Poisson equations, stress tensor, etc. appear in both contexts), yet in fluid dynamics, we also talk about decomposing tensors into various parts responsible for different "actions" on our fluid element. Moreover, we have things like pressure, viscosity, and the Newtonian-ness of fluids, and so on. It's possible that (classical) electromagnetism is somewhat simpler than fluid dynamics because both electric and magnetic fields propagate at the speed of light, whereas the fluid flow speed can vary as a function of many factors.

I found this nice comparison between hydrodynamic and electromagnetic variables from arXiv:1202.4611:

TABLE 1. Comparison between hydrodynamic and electromagnetic variables.

Hydrodynamics	Electromagnetism
Hydrodynamic fluid velocity, $\mathbf{v}_f$	Magnetic vector potential, $\mathbf{A}$
Hydrodynamic fluid acceleration ($\mathbf{a}_f$), $d\mathbf{v}_f/dt$	Induced electric field ($\mathbf{E}_i$), $d\mathbf{A}/dt$
(Induced) Mass of the fluid, m_f	Electric charge, q
(Induced) Fluid density, ρ_f	Charge density, ρ_q
Virtual mass, $m_{virtual}$	Relativistic mass, $m_{relativist}$
Acoustic gamma factor, γ_a	Lorentz Gamma factor, γ
Acoustic impedance, Z_a	Electromagnetic impedance, Z_w
Compressibility of the fluid, β	Permittivity of the vacuum, ε_0
(Surrounding) Fluid density, ρ_f	Permeability of the vacuum, μ_0
Acceleration Inertia Force, $-\rho_f(\partial\mathbf{v}_f/\partial t)$	Acceleration Inertia Force, $-\rho_q(\partial\mathbf{A}/\partial t)$
Virtual mass increase, $-\rho_f(\mathbf{v}_f \cdot \nabla)\mathbf{v}_f$	Relativistic mass increase, $-\rho_q(\mathbf{v}\cdot\nabla)\mathbf{A}$
Hydrodynamic Inertia Force, $-\rho_f(d\mathbf{v}_f/dt)$	Electromagnetic Inertia Force, $-\rho_q(d\mathbf{A}/dt)$