PY 711 Fall 2010
Homework 4: Due Tuesday, September 21

1. In class we defined ψ_L as the "left-handed" Weyl spinor formed by the upper two components of the Dirac bispinor in the Weyl representation,

$$\psi(x) = \begin{bmatrix} \psi_L(x) \\ \psi_R(x) \end{bmatrix}. \tag{1}$$

Let ψ_L^* be the complex conjugate of ψ_L. The Majorana equation is given by

$$i\bar{\sigma} \cdot \partial\psi_L(x) - im\sigma^2\psi_L^*(x) = 0. \tag{2}$$

In our notation σ^2 is the second Pauli matrix,

$$\sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \tag{3}$$

$\bar{\sigma}^\mu = (1, -\vec{\sigma})$, and m is known as a Majorana mass.

   (a) (7 points) Starting from the transformation properties of ψ_L under rotations, show explicitly that the Majorana equation is invariant under any infinitesmal rotation.

   (b) (8 points) Starting from the transformation properties of ψ_L under Lorentz boosts, show explicitly that the Majorana equation is invariant under any infinitesmal boost.

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IN OUR NOTATION σ^2 IS THE SECOND PAULI MATRIX,

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$\bar{\sigma} = (1, -\vec{\sigma})$, AND m IS THE MAJORANA MASS.

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a. STARTING FROM THE TRANSFORMATION PROPERTIES OF ψ_L UNDER ROTATIONS, SHOW EXPLICITLY THAT THE MAJORANA EQUATION IS INVARIANT UNDER ANY INFINITESIMAL ROTATION.

$$\psi_L \to (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} - \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\psi_L$$

$$\sigma^2\psi_L^* \to (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} + \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\sigma^2\psi_L^*$$

From our discussion in class about Lorentz invariance of the Dirac equation, we know

$$\psi(x) \to \Lambda_{1/2}\,\psi(\Lambda_4^{-1}x)$$

$$\partial_\mu\psi(x) \to (\Lambda_4^{-1})^{\alpha}{}_{\mu}\,\partial_\alpha\psi(\Lambda_4^{-1}x)$$

For infinitesmial rotations,

$$\psi_L(x) \to (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2})\psi_L(\Lambda_4^{-1}x)$$

$$\sigma^2\psi_L^*(x) \to (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2})\sigma^2\psi_L(\Lambda_4^{-1}x)$$

a. CONTINUED

$$-im\sigma^2\psi_L^*(x) \to (1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(-im\sigma^2\psi_L^*(\Lambda_4^{-1}x))$$

Now, we need to show that $i\bar{\sigma}\cdot\partial\psi_L(x) \to (1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda_4^{-1}x))$

$$i\bar{\sigma}\cdot\partial\psi_L(x) \to i\bar{\sigma}^\mu(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda^{-1}{}_4)^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda_4^{-1}x)$$

$$= i\underbrace{(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(1+i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})}_{=1}\bar{\sigma}^\mu(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda^{-1}{}_4)^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda_4^{-1}x)$$

Look at

$$(1+i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})\bar{\sigma}^\mu(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2}) = \bar{\sigma}^\mu + \tfrac{i}{2}\vec{\theta}\cdot\vec{\sigma}\,\bar{\sigma}^\mu - \tfrac{i}{2}\vec{\theta}\cdot\bar{\sigma}^\mu\vec{\sigma}$$

$$= \bar{\sigma}^\mu - \tfrac{i}{2}\vec{\theta}\,[\bar{\sigma}^\mu, \vec{\sigma}]$$

To get a better idea what $[\bar{\sigma}^\mu, \vec{\sigma}]$ means, I'm going to look at $[\bar{\sigma}^\mu, \sigma^3]$.

$$[\bar{\sigma}^0, \sigma^3] = [1, \sigma^3] = 0$$

$$[\bar{\sigma}^1, \sigma^3] = [-\sigma^1, \sigma^3] = 2i\sigma^2 = -2i\bar{\sigma}^2$$

$$[\bar{\sigma}^2, \sigma^3] = [-\sigma^2, \sigma^3] = -2i\sigma^1 = 2i\bar{\sigma}^1$$

$$[\bar{\sigma}^3, \sigma^3] = [-\sigma^3, \sigma^3] = 0$$

$$(\bar{\sigma}^1)' = \bar{\sigma}^1 - \theta\bar{\sigma}^2$$

$$(\sigma^2)' = \bar{\sigma}^2 + \theta\bar{\sigma}^1$$

Similarly for σ^1 and σ^2.

$i,j = 1,2,3$ for rotations

So, $\bar{\sigma}^\mu \to \bar{\sigma}^\mu - i\theta[J^{ij}_{(4)}]^\mu{}_\nu\bar{\sigma}^\nu$ $\quad [J^{ij}_4]^\mu{}_\nu = i(g^{i\mu}\delta^j{}_\nu - g^{j\mu}\delta^i{}_\nu)$

or, $(1+i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})\bar{\sigma}^\mu(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2}) = (\Lambda_4)^\mu{}_\nu\bar{\sigma}^\nu$

good

$\bar{\sigma}^\mu$ transforms like a 4-vector a CONTINUED

Now,

$$i\bar{\sigma}\cdot\partial\psi_L(x) \to i(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(1+i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})\,\bar{\sigma}^\mu(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda^{-1}_4)^\alpha{}_\mu\,\partial_\alpha\psi_L(\Lambda^{-1}_4 x)$$

$$= i(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda_4)^\mu{}_\beta\,\bar{\sigma}^\beta(\Lambda^{-1}_4)^\alpha{}_\mu\,\partial_\alpha\psi_L(\Lambda^{-1}_4 x)$$

$$= (1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})\,i\,\delta^\alpha{}_\beta\,\bar{\sigma}^\beta\,\partial_\alpha\psi_L(\Lambda^{-1}_4 x)$$

$$= (1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda^{-1}_4 x))$$

Finally, we find that the Majorana equation

$$i\bar{\sigma}\cdot\partial\psi_L(x) - im\sigma^2\psi_L^*(x) = 0$$

becomes in the new frame

$$(1-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda^{-1}_4 x) - im\sigma^2\psi_L^*(\Lambda^{-1}_4 x)) = 0.$$

So, the Majorana equation is invariant under infinitesimal rotation.

b. STARTING FROM THE TRANSFORMATION PROPERTIES OF ψ_L UNDER LORENTZ BOOSTS, SHOW EXPLICITLY THAT THE MAJORANA EQUATION IS INVARIANT UNDER ANY INFINITESIMAL BOOST.

$$-im\sigma^2\psi_L^*(x) \to (1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(-im\sigma^2\psi_L^*(\Lambda_4^{-1}x))$$

Now, we need to show $i\bar{\sigma}\cdot\partial\psi_L(x) \to (1+\vec{\beta}\cdot\frac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda_4^{-1}x))$

$$i\bar{\sigma}\cdot\partial\psi_L(x) \to i\sigma^\mu(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda_4^{-1})^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda_4^{-1}x)$$

$$= i(1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})\bar{\sigma}^\mu(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda_4^{-1})^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda_4^{-1}x)$$ ✓

Look at

$$(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})\bar{\sigma}^\mu(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2}) = \bar{\sigma}^\mu - \tfrac{1}{2}\vec{\beta}\cdot\vec{\sigma}\,\bar{\sigma}^\mu - \tfrac{1}{2}\vec{\beta}\cdot\bar{\sigma}^\mu\vec{\sigma}$$

$$= \bar{\sigma}^\mu - \tfrac{1}{2}\vec{\beta}\{\bar{\sigma}^\mu, \vec{\sigma}\}$$

✓

Again, look at $\{\bar{\sigma}^\mu, \sigma^3\}$.

$$\{\bar{\sigma}^0, \sigma^3\} = 2\sigma^3 = -2\bar{\sigma}^3$$

$$\{\bar{\sigma}^1, \sigma^3\} = 0$$

$$\{\bar{\sigma}^2, \sigma^3\} = 0$$

$$\{\bar{\sigma}^3, \sigma^3\} = -2\bar{\sigma}^0$$

$$(\bar{\sigma}^0)' = \bar{\sigma}^0 + \beta\bar{\sigma}^3$$

$$(\bar{\sigma}^3)' = \bar{\sigma}^3 + \beta\bar{\sigma}^0$$

So, $\bar{\sigma}^\mu \to \bar{\sigma}^\mu - i\vec{\beta}[J_4^{0i}]^\mu{}_\nu\bar{\sigma}^\nu$

Check: $\bar{\sigma}^0 \to \bar{\sigma}^0 - i\beta(i(g^{00}\delta^3{}_\nu - g^{30}\delta^0{}_\nu))\bar{\sigma}^\nu$

$$= \bar{\sigma}^0 + i\beta\bar{\sigma}^3$$

✓

So $(1-\vec{\beta}\cdot\frac{\vec{\sigma}}{2})\bar{\sigma}^\mu(1-\vec{\beta}\cdot\frac{\vec{\sigma}}{2}) = (\Lambda_4)^\mu{}_\nu\bar{\sigma}^\nu$

b CONTINUED

So now

$$i\bar{\sigma}\cdot\partial\psi_L(x) \to i(1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})\bar{\sigma}^\mu(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda^{-1}_4)^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda^{-1}_4 x)$$

$$= (1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i(\Lambda_4)^\mu{}_\nu\,\bar{\sigma}^\nu(\Lambda^{-1}_4)^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda^{-1}_4 x))$$

$$= (1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\delta^\alpha{}_\nu\,\bar{\sigma}^\nu\partial_\alpha\psi_L(\Lambda^{-1}_4 x))$$

$$= (1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda^{-1}_4 x))$$

✓

We find now that the Majorana equation

$$i\bar{\sigma}\cdot\partial\psi_L(x) - im\sigma^2\psi_L^*(x) = 0$$

in the boosted frame is

$$(1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda^{-1}_4 x) - im\sigma^2\psi_L^*(\Lambda^{-1}_4 x)) = 0.$$

So, the Majorana equation is invariant under infinitesimal boosts.

b CONTINUED

So now

$$i\bar{\sigma}\cdot\partial\psi_L(x) \to i(1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})\bar{\sigma}^\mu(1-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(\Lambda_4^{-1})^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda_4^{-1}x)$$

$$= (1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i(\Lambda_4)^\mu{}_\nu\,\bar{\sigma}^\nu(\Lambda_4^{-1})^\alpha{}_\mu\partial_\alpha\psi_L(\Lambda_4^{-1}x))$$

$$= (1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\,\delta^\alpha{}_\nu\,\bar{\sigma}^\nu\partial_\alpha\psi_L(\Lambda_4^{-1}x))$$

$$= (1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda_4^{-1}x))$$

✓

We find now that the Majorana equation

$$i\bar{\sigma}\cdot\partial\psi_L(x) - im\sigma^2\psi_L^*(x) = 0$$

in the boosted frame is

$$(1+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\psi_L(\Lambda_4^{-1}x) - im\sigma^2\psi_L^*(\Lambda_4^{-1}x)) = 0.$$

So, the Majorana equation is invariant under infinitesimal boosts.

PY711 Solutions #4

1. (a) Under an infinitesmal rotation $\Lambda(\vec{\theta})$,

$$\Lambda : \vec{x} \to \vec{x} + \vec{\theta} \times \vec{x}$$

For a scalar function $f(t, \vec{x})$

$$\Lambda : f\Big|_{t,\vec{x}} \to f\Big|_{t,\vec{x}-\vec{\theta}\times\vec{x}}$$

For the gradient of a function there is an extra term coming from the rotation of gradient components

$$\Lambda : \vec{\nabla} f\Big|_{t,\vec{x}} \to \vec{\nabla} f\Big|_{t,\vec{x}-\vec{\theta}\times\vec{x}} + \vec{\theta}\times\vec{\nabla} f\Big|_{t,\vec{x}-\vec{\theta}\times\vec{x}}$$

For this extra piece we use the notation $\Delta(\vec{\nabla}) = \vec{\theta}\times\vec{\nabla}$.

For a two-component spinor function $\chi(t,\vec{x})$ there is an extra term from the rotation of spinor components

$$\Lambda : \chi\Big|_{t,\vec{x}} \to \chi\Big|_{t,\vec{x}-\vec{\theta}\times\vec{x}} + (-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})\chi\Big|_{t,\vec{x}-\vec{\theta}\times\vec{x}}$$

For this extra piece we write $\Delta(\chi) = (-i\vec{\theta}\cdot\frac{\vec{\sigma}}{2})\chi$.

From here on we omit writing $\Big|_{t,\vec{x}-\vec{\theta}\times\vec{x}}$.

We now consider the transformation of

$$i\bar{\sigma}\cdot\partial\chi = i\partial_0\chi - i\vec{\sigma}\cdot\vec{\nabla}\chi$$

For $i\partial_0\chi$ we find
$$\Delta(i\partial_0\chi) = i\partial_0(-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})\chi = (-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(i\partial_0\chi)$$

For $-i\vec{\sigma}\cdot\vec{\nabla}\chi$ we have

$$\begin{aligned}\Delta(-i\vec{\sigma}\cdot\vec{\nabla}\chi) &= -i\vec{\sigma}\cdot\Delta(\vec{\nabla})\chi - i\vec{\sigma}\cdot\vec{\nabla}(\Delta(\chi)) \\ &= -i\vec{\sigma}\cdot(\vec{\theta}\times\vec{\nabla})\chi - i\vec{\sigma}\cdot\vec{\nabla}(-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})\chi\end{aligned}$$

Using the identity $\sigma^k\sigma^l = \sigma^l\sigma^k + [\sigma^k,\sigma^l] = \sigma^l\sigma^k + 2i\varepsilon^{klm}\sigma^m$, sum over m

we get

$$\begin{aligned}\Delta(-i\vec{\sigma}\cdot\vec{\nabla}\chi) &= \cancel{-i\vec{\sigma}\cdot(\vec{\theta}\times\vec{\nabla})\chi} + (-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(-i\vec{\sigma}\cdot\vec{\nabla})\chi \\ &\quad + \cancel{2i(\vec{\theta}\times\vec{\nabla})\cdot\tfrac{\vec{\sigma}}{2}\chi} \\ &= (-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(-i\vec{\sigma}\cdot\vec{\nabla})\chi\end{aligned}$$

So under rotations

$$\Delta(i\bar{\sigma}\cdot\partial\chi) = (-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\chi)$$

For $im\sigma^2\chi^*$ we have

$$\Delta(im\sigma^2\chi^*) = im\sigma^2(i\vec{\theta}\cdot\tfrac{\vec{\sigma}^*}{2})\chi^* = \underbrace{(-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})im\sigma^2\chi^*}_{\text{using identity } \sigma^2\vec{\sigma}^* = -\vec{\sigma}\sigma^2}$$

So then $i\bar{\sigma}\cdot\partial\chi - im\sigma^2\chi^*$ transforms homogeneously,

$$\Delta(i\bar{\sigma}\cdot\partial\chi - im\sigma^2\chi^*) = (-i\vec{\theta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\chi - im\sigma^2\chi^*),$$

and thus the Majorana equation is invariant under rotations.

(b) Under an infinitesmal boost $\Lambda(\vec{\beta})$,

$$\Lambda : t \to t + \vec{\beta}\cdot\vec{x}$$
$$\Lambda : \vec{x} \to \vec{x} + \vec{\beta}t$$

In this case we find (using same notation as before)

$$\Delta(\partial_0) = -\vec{\beta}\cdot\vec{\nabla}\,, \quad \Delta(\vec{\nabla}) = -\vec{\beta}\,\partial_0$$

The minus signs are due to the fact that $\vec{\nabla} = \frac{\partial}{\partial \vec{x}}$ is a lowered Lorentz index object.

For the left-handed spinor χ we found in lecture that

$$\Delta\chi = -\vec{\beta}\cdot\frac{\vec{\sigma}}{2}\chi$$

So we have

$$\begin{aligned}\Delta(i\partial_0\chi) &= i\Delta(\partial_0)\chi + i\partial_0\,\Delta(\chi)\\ &= i(-\vec{\beta}\cdot\vec{\nabla})\chi + i\partial_0(-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2}\chi)\\ &= (-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\partial_0\chi) - i(\vec{\beta}\cdot\vec{\nabla})\chi\end{aligned}$$

and

$$\begin{aligned}\Delta(-i\vec{\sigma}\cdot\vec{\nabla}\chi) &= -i\vec{\sigma}\cdot\Delta(\vec{\nabla})\chi + (-i\vec{\sigma}\cdot\vec{\nabla})\Delta(\chi)\\ &= -i\vec{\sigma}\cdot(-\vec{\beta}\partial_0)\chi + (-i\vec{\sigma}\cdot\vec{\nabla})(-\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})\chi\end{aligned}$$

Using the identity $\sigma^k\sigma^l = -\sigma^l\sigma^k + 2\delta^{kl}$,

$$\begin{aligned}\Delta(-i\vec{\sigma}\cdot\vec{\nabla}\chi) &= \vec{\beta}\cdot\vec{\sigma}\,(i\partial_0\chi) + (-i)(\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(\vec{\sigma}\cdot\vec{\nabla})\chi + 2i\,\vec{\beta}\cdot\vec{\nabla}\tfrac{\chi}{2}\\ &= (\vec{\beta}\cdot\vec{\sigma})(i\partial_0\chi) + (\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(-i\vec{\sigma}\cdot\vec{\nabla}\chi) + i\vec{\beta}\cdot\vec{\nabla}\chi\end{aligned}$$

Therefore
$$\begin{aligned}\Delta(i\bar{\sigma}\cdot\partial\chi) &= (\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\partial_0\chi) + (\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(-i\vec{\sigma}\cdot\vec{\nabla}\chi)\\ &= (\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\chi).\end{aligned}$$

For $im\sigma^2\chi^*$ we have

$$\Delta(im\sigma^2\chi^*) = im\sigma^2(-\vec{\beta}\cdot\tfrac{\vec{\sigma}^*}{2}\chi^*) = \underbrace{+(\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(im\sigma^2\chi^*)}_{\text{using identity } \sigma^2\vec{\sigma}^* = -\vec{\sigma}\sigma^2}.$$

So $i\bar{\sigma}\cdot\partial\chi - im\sigma^2\chi^*$ transforms homogeneously,

$$\Delta(i\bar{\sigma}\cdot\partial\chi - im\sigma^2\chi^*) = (+\vec{\beta}\cdot\tfrac{\vec{\sigma}}{2})(i\bar{\sigma}\cdot\partial\chi - im\sigma^2\chi^*),$$

and thus the Majorana equation is invariant under boosts.