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Course: **8.421 - AMO I**
Problem set: **#8**
Due: Friday, April 8, 2022.

1. Optical Traps and Scattering. What are are the proper power and wavelength needed to trap an ultracold atomic gas? Consider an alkali atom with resonance frequency ω_0 on the principal $nS \to nP$ transition. A sample of atoms in the ground state nS are exposed to monochromatic radiation of intensity I and frequency $\omega_L < \omega_0$. Using the fact that essentially all of the oscillator strength out of the ground state comes from the $nS \to nP$ transition, we have

$$\alpha(\omega_L) \approx \frac{2e^2}{\hbar} |\langle nP| z |nS\rangle|^2 \frac{\omega_0}{\omega_0^2 - \omega_L^2} \implies |\langle nP| z |nS\rangle|^2 = \frac{\hbar\alpha(\omega_L)}{2e^2} \frac{\omega_0^2 - \omega_L^2}{\omega_0}.$$

(a) AC Stark shift:

(i) From lecture, the AC Stark shift U_i from time-dependent perturbation theory is given by

$$U_i = -\frac{1}{4}\alpha(\omega_L)\mathcal{E}^2 = -\frac{2I\alpha(\omega_L)}{4c\epsilon_0} = -\frac{I\alpha(\omega_L)}{2c\epsilon_0}.$$

(ii) Now, we want to use the rotating wave approximation to obtain the AC Stark shift. This can be done by first writing down the true (symmetrized) Hamiltonian:

$$\mathcal{H} = \frac{\hbar}{2}\begin{pmatrix} -\omega_0 & \omega_R e^{i\omega_L t} \\ \omega_R^* e^{-i\omega_L t} & \omega_0 \end{pmatrix}.$$

By going into the rotating frame, plus making the rotating wave approximation, we find that

$$\mathcal{H}_{\text{rot}}^{\text{RWA}} = \frac{\hbar}{2}\begin{pmatrix} -\delta & \omega_R \\ \omega_R & \delta \end{pmatrix}$$

where $\delta = \omega_0 - \omega_L$. The energy shifts can be obtained from the eigenvalues:

$$\Delta E = \pm\frac{\hbar}{2}\sqrt{\omega_R^2 + \delta^2} = \frac{\hbar}{2}\sqrt{\omega_R^2 + (\omega_0 - \omega_L)^2}.$$

In the limit where the Rabi frequency is much less than the detuning, we simply have that

$$|U_{ii}| = \frac{\hbar}{2}|\omega_0 - \omega_L|.$$

In particular, the energy of the lower state gets shifted down while the energy of the higher state gets shifted up (since we're red-detuning).

(iii) From the previous two parts, we find that

$$\frac{U_i}{U_{ii}} = \frac{I\alpha(\omega_L)}{2c\epsilon_0}\frac{2}{\hbar(\omega_0 - \omega_L)} = \frac{I}{c\epsilon_0}\frac{2e^2}{\hbar^2}|\langle nP| z |nS\rangle|^2 \frac{\omega_0}{(\omega_0 - \omega_L)^2(\omega_0 + \omega_L)}.$$

When $\omega_L \approx 0$, we have

$$\frac{U_i}{U_{ii}} \approx \frac{I}{c\epsilon_0}\frac{2e^2}{\hbar^2}|\langle nP| z |nS\rangle|^2 \frac{1}{\omega_0^2}\left(1 + \frac{\omega_L}{\omega_0} + \dots\right).$$

When $\omega_L \approx \omega_0$, we may write $\omega_L + \omega_0 = 2\omega_0$, so that

$$\frac{U_i}{U_{ii}} \approx \frac{I}{c\epsilon_0}\frac{2e^2}{\hbar^2}|\langle nP| z |nS\rangle|^2 \frac{1}{2(\omega_0 - \omega_L)^2}$$

We see that if the intensity has spatial structure, with the appropriate detuning, the AC Stark shift can have energy minima where the atoms can be trapped.

(b) Using time-dependent perturbation theory, we have that

$$blah$$

In the RWA picture, we know that

$$P_{e,ii}(t) = \frac{\omega_R^2}{\omega_R^2 + \delta^2} \sin^2\left(\frac{\sqrt{\omega_R^2 + (\omega_0 - \omega_L)^2}t}{2}\right).$$

Under time-averaging this is

$$P_{e,ii} = \frac{\omega_R^2}{2(\omega_R^2 + \delta^2)}$$

(c) (i)

(ii)

(iii)

(d) (i)

(ii)

2. Magic Wavelength Optical Trap.

(a) (i)

(ii)

(iii)

(b) (i)

(ii)

3. Species-Dependent and Spin-Dependent AC Stark shift

(a) (i)

(ii)

(b) (i)

(ii)