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MA355 #8

(125) Same as the # of ways to distribute k identical books on the shelves of a bookcase with n shelves

By the "stars and bars" argument, we have $(k+n-1)$ spaces and $(n-1)$ dividers,

so $\boxed{\text{Ans} = \binom{k+n-1}{n-1}}$

(126) Bijection is between the multisets of size k of a set of size n <u>and</u> the subsets of size $(n-1)$ of a set of size $(n+k-1)$.

↳ But this is just what the "stars and bars" argument is about. $(n+k-1)$ = # of slots, and we choose $(n-1)$ slots to put the "bars".

(127) m = # shelves
r = # books
x_m = # books on shelf m

$$\to \boxed{\text{Ans} = \binom{m+r-1}{m-1} = \binom{m+r-1}{r}}$$

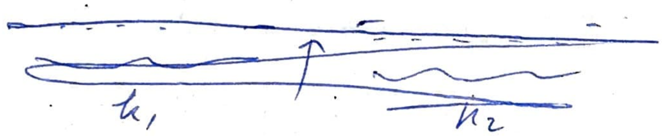
(131)

130 given $\binom{k-1}{n-1}$

The bijection is between compositions of k into n parts and subsets of size $(n-1)$ of a set of size $(k-1)$.

Given a composition, $k_1 + k_2 + \dots + k_n = k$

then the subset of size $(n-1)$ consists of the k elements ~~of the form \{ in the locations $k_i + 1$ for $i = 1, \dots, n-1$~~ whose location is specified by the k_i's.



Ex: $1 + 1 + 3 = 5$ – | – | – – –

→ the subset is the first 2 spaces of the $(k-1)$ spaces.

Given a subset, we can also get a composition.

Again, this is "stars" and "bars"

Ex   – – – – | – – – | – – ⟶ $4 + 3 + 2 = 9 = (k)$

$n = 3$

(132) Put k identical books onto n shelves, each with at least one book ...

$\Rightarrow \binom{k-1}{n-1}$

Composition of k into n parts

They're the same picture ... The shelves are the dividers ... → we're decomposing k books into the k shelves ...

(134)

$k \backslash n$	0	1	2	3	4	5
0	1	0				
1	0	1				
2	0	3	1			
3	0	7	6	1		
4	0	15	25	10	1	
5	0					
6						
7						
8						
9						

crap... missing a column

~~k = 9~~

~~n = 3~~

$S(k,n) = S(k-1,n-1) + n\,S(k-1,n)$

(136)

$k \backslash n$	0	1	2	3
0	1	0	0	0
1	0	1	0	0
2	0	1	1	0
3	0	1	3	1
4	0	1	7	6
5	0	1	15	25
6	0	1	31	90
7	0	1	63	301
8	0	1	127	966
9	0	1	255	$\boxed{3025}$

k = 9

n = 3

$S(k,n) = S(k-1,n-1) + n\,S(k-1,n)$

$\boxed{S(9,3) = 3025}$

(1.37) 9 different sandwiches into 3 identical bags...
s.t each has exactly 3 ...

$$\frac{1}{3!}\binom{9}{3}\binom{6}{3} = \frac{84 \cdot 20}{6} = \boxed{280}$$

(138) $S(k, k-1) = ?$

↳ distribute k things to $k-1$ identical recipients
each with at least one thing

→ one recipient has 2.

→ need to pick 2 out of k things & done.

$$\boxed{S(k, k-1) = \binom{k}{2}}$$