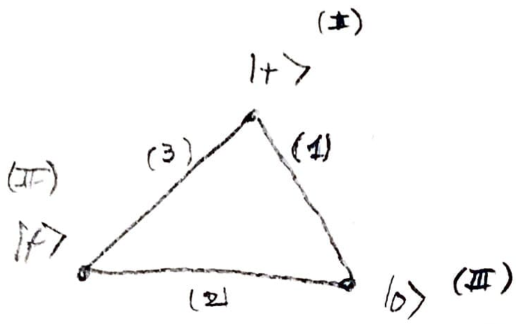

(1): controlled-not-phase

(2): controlled-phase on 1

(3): controlled-phase on 0

$\boxed{(1)}$: $(C_{x,\beta})\ \frac{1}{2}\left(|0\rangle+|1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + e^{i\beta}|11\rangle\right)$

$\Rightarrow$ total state is $\frac{1}{\sqrt{2}}\left(|0+0\rangle + e^{i\beta}|1+1\rangle\right)$

$\boxed{(2)}$ controlled-phase (on 1):

$$(C_{2\beta,1})\left(|0\rangle\underbrace{(|0\rangle+|1\rangle)}|0\rangle + e^{i\beta}|1\rangle\underbrace{(|0\rangle+|1\rangle)}|1\rangle\right)$$

$$= |0\rangle(|0\rangle+|1\rangle)|0\rangle + e^{i\beta}|1\rangle\left(|0\rangle+e^{2i\beta}|1\rangle\right)|1\rangle$$

$\boxed{(3)}$ controlled-phase (on 0):

$$(C_{2\beta,0})\left(|0\rangle\underbrace{(|0\rangle+|1\rangle)}|0\rangle + e^{i\beta}|1\rangle\underbrace{\left(|0\rangle+e^{2i\beta}|1\rangle\right)}|1\rangle\right)$$

$$= \boxed{|0\rangle\left(e^{2i\beta}|0\rangle+|1\rangle\right)|0\rangle + e^{i\beta}|1\rangle\left(|0\rangle+e^{2i\beta}|1\rangle\right)|1\rangle}$$

$=$

call this $|\psi\rangle$

now, write $|\psi\rangle$ as

$$|\psi\rangle = |0\rangle\left(e^{2i\beta}|0\rangle + |1\rangle\right)|0\rangle + e^{i\beta}|1\rangle\left(|0\rangle + e^{2i\beta}|1\rangle\right)|1\rangle$$

$$= \cancel{e^{2i\beta}}\left\{ |0\rangle\left(|0\rangle + e^{-2i\beta}|1\rangle\right)|0\rangle + e^{i\beta}|1\rangle\left(e^{-2i\beta}|0\rangle + |1\rangle\right)|1\rangle \right\}$$

$$= \left[|0\rangle\left(|0\rangle + e^{-2i\beta}|1\rangle\right) + |1\rangle\left(|1\rangle + e^{-2i\beta}|0\rangle\right)\right] \otimes \left(|0\rangle + e^{i\beta}|1\rangle\right)$$

$$+ \left[|0\rangle\left(|0\rangle + e^{-2i\beta}|1\rangle\right) - |1\rangle\left(|1\rangle + e^{-2i\beta}|0\rangle\right)\right] \otimes \left(|0\rangle - e^{i\beta}|1\rangle\right)$$

make measurement on 3^{rd} qubit in $M(\beta) = \{|0\rangle \pm e^{i\beta}|1\rangle\}$ basis

we find

if get $|0\rangle + e^{i\beta}|1\rangle$ then the other 2 qubits acquire state

- $|0\rangle\left(|0\rangle + e^{-2i\beta}|1\rangle\right) + |1\rangle\left(|1\rangle + e^{-2i\beta}|0\rangle\right)$

$$\propto e^{i\beta\,\sigma_z \otimes \sigma_z}|{+}{+}\rangle$$

if get $|0\rangle - e^{i\beta}|1\rangle$ then the other 2 qubits acquire state

- $|0\rangle\left(|0\rangle + e^{-2i\beta}|1\rangle\right) - |1\rangle\left(|1\rangle + e^{-2i\beta}|0\rangle\right)$

$$\propto (Z \otimes I)\, e^{i\beta\,\sigma_z \otimes \sigma_z}|{+}{+}\rangle$$