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MA355 : Combinatorics

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# HW #1 : Feb 19, 2021

**P5 p.3**

Drink : 5 choices
Sandwich : 15 choices (problem?)
Fruit : 4 choices

$\Rightarrow$ Total of $5 \cdot 15 \cdot 4 = 300$ choices of the 3 items.

**P13 p.7**

(a) For each input there are 12 possible outputs. Since there are 3 inputs, and the function is generic, there are

$$12 \cdot 12 \cdot 12 = 12^3 \text{ such functions.}$$

(b) Conjecture: The number of functions is $\boxed{n^m}$

Proof: By phrasing the product principle in terms of a sequence of decisions, we have that for each one of the $m$ inputs, there are $n$ choices.

And so the number of possible function has to be $\underbrace{n \cdot n \ldots n}_{m \text{ times}} = n^m$. $\square$

(c) $N^M$ is good notation because it behaves the same way as exponential notation. If the domain is a disjoint union of 2 sets, say $M = M_1 \sqcup M_2$ (where $M_i$ denotes sets) then ~~the answer~~ $|M| = |M_1| + |M_2|$ and, we have

$N^{|M|} = N^{|M_1|+|M_2|} = N^{|M_1|} N^{|M_2|}$, where the $2^{nd}$ equality follows from the fact that choosing $f : M \to N$ is the same as choosing $f_1 : M_1 \to N$ and $f_2 : M_2 \to N$ (thus, we multiply $N^{M_1}$ & $N^{M_2}$).

P 15 p.8 . $n$ rows, each seat has capacity = 2.
$\Rightarrow$ $2n$ total seats.

. If $n$ men & $n$ women sit in seat, there is a man & a woman in each row, then how many ways?.

$\rightarrow$ WLOG, let men board first, then there are $2^n n!$ ways.
~~For each seat, there are then~~

Sequentially, there are $n-i$ possible choices of the women to sit in the seat $i$. ~~Each way, there are 2 ways to sit~~

$\rightarrow$ Total $= \underline{2^n n!\, n!} = \boxed{(n!)^2 2^n}$

P17 p8 $\{k \text{ pieces of fruit}\} \longrightarrow \{n \text{ children}\}$

This the same as counting the # of functions from a $k$-element set to an $n$-element set. For each piece of fruit, there are $n$ choices...
$\Rightarrow$ Total is $\underbrace{n \cdot n \cdot n \cdot \dots n}_{k \text{ times}} = \boxed{n^k}$

P19, p8 $\underline{k \le n}$. Further restriction: Each child gets at most one.

$\rightarrow$ the total is equal to the # of ways to choose $k$ children out of $n$ to have fruit, which is $\boxed{\dfrac{n!}{k!(n-k)!}}$

$\underline{k > n}$ The total is $\boxed{0}$, because there exists at least one child with more than 1 piece of fruit.

26 p.11 (a) One-to-one from finite $X$ to finite $Y$:
no two arrows from $X$ go to the same vertex in $Y$.

(b) Onto: each vertex in $Y$ has some arriving arrow from $X$.

(c) One-to-one & Onto: each arrow from $X$ goes to exactly one vertex in $Y$.

27 p11 # of permutations an $n$-element set has is $\boxed{n!}$

28 p11 $m = a_1 2^{k-1} + a_2 2^{k-2} + \dots + a_k 2^0$

Bijection between the binary representations of integers between $0$ & $2^n - 1$ & the subsets of an $n$-element set $F$

Any number $c \in \mathbb{Z}$ between $0$ & $2^n-1$ can be written as

$$c = a_1 2^{n-1} + a_2 2^{k-2} + \dots + a_n 2^0$$

where $a_i \in \{0, 1\}$. A bijection can be defined as follows

$f(c) = S \in F$ ~~such that~~ where the elements of $S$ are the ith elements of $F$ for which $a_i = 0$.

gotten from the binary representation of $c$.

~~Supp Ex 1, p. 29~~ Can't read...

~~• $n = \underbrace{1 + 1 + \dots + 1}_{n \text{ times}}$ ⇒ use $\boxed{n-1 \text{ "+" signs}}$~~

~~• How many ways can we write $n$ as a sum of a list of $k$ positive numbers?~~

~~↳ if we treat each "+" sign as distinct based on their location, then we see that the total is just the # of subsets of such $(n-1)$ plus signs~~

~~⇒ there are $\boxed{2^{n-1} \text{ ways}}$~~

~~• If we don't include $n = n$, then there are $\boxed{2^{n-1}-1}$ ways~~

$\boxed{\text{Supp Ex 1, p. 29}}$ Write $n$ as a sum of ones ⇒ use $\boxed{n-1}$ plus signs.

To write $n$ as sum of $k$ positive numbers, ~~there~~ are $k-1$ plus signs and so there are $\boxed{\binom{n-1}{k-1}}$ ways to do this.

$\boxed{\text{Supp Ex 2, p. 30}}$ → note that this is <u>not</u> the same as "partitions".

Total number of <u>compositions</u> of $n$?

$$\# = \sum_{k=1}^{n} \binom{n-1}{k-1} = \sum_{k'=0}^{n-1} \binom{n-1}{k'} = \boxed{2^{n-1}}$$

If we don't accept writing $n$ as $n$, then it's <u>$2^{n-1}-1$</u>.