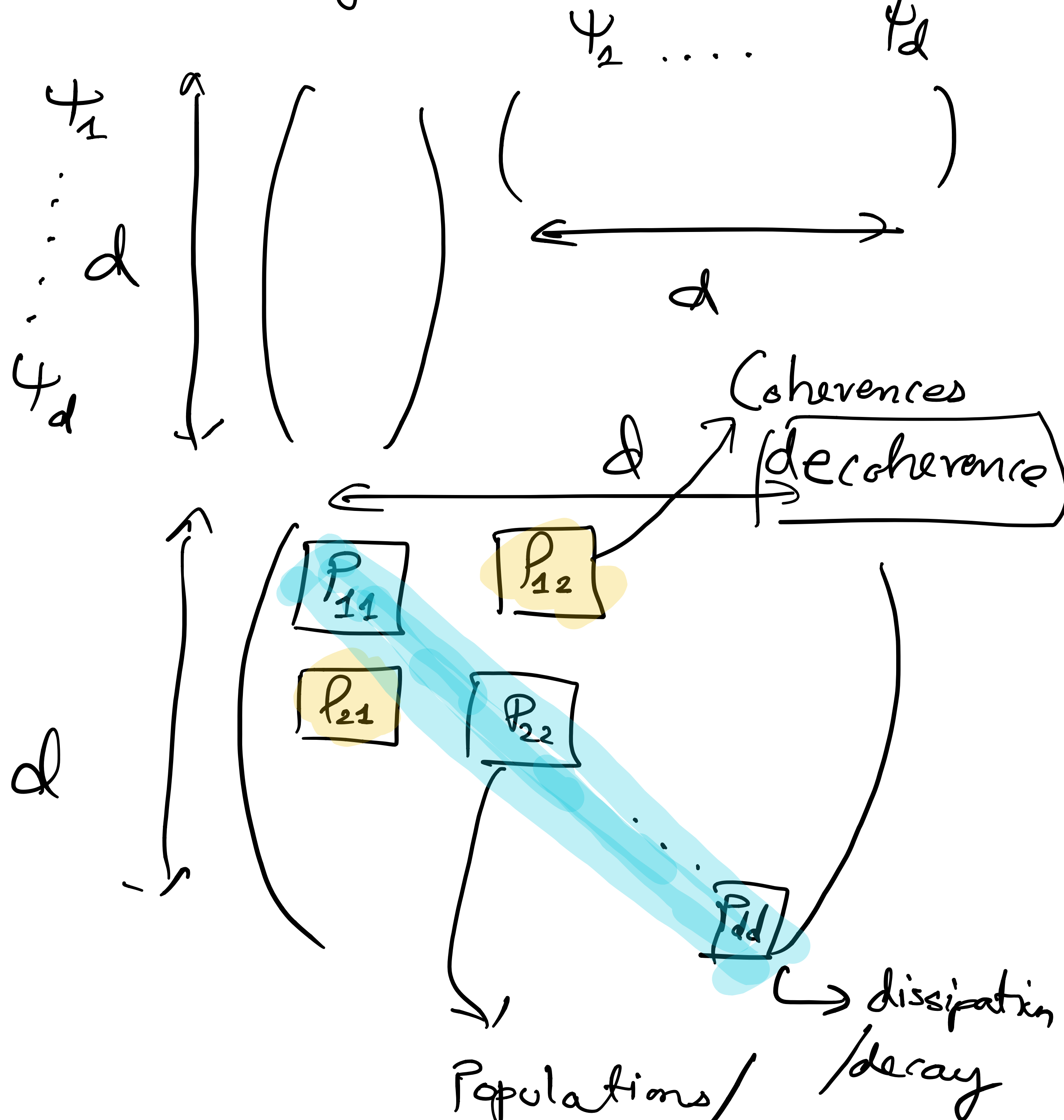
$$\hat{\rho}_{sys} = |\Psi\rangle\langle\Psi|$$



Probability of occupation of state $|\psi_2\rangle$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$\frac{1}{2}\,\underline{\underline{\left(|0\rangle + |1\rangle\right)}}\left(\langle 0| + \langle 1|\right)$$

$$\begin{pmatrix} 1/2 & \boxed{1/2} \\ 1/2 & 1/2 \end{pmatrix}$$

$$|0\rangle\langle 0| \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

no coherence!

Open Quantum Systems

System

Env.

$$\hat{\mathcal{H}} = \mathcal{H}_S + \cancel{\mathcal{H}_E} + \boxed{\mathcal{H}_{int}}$$

$$\hat{\rho}_{tot}(0) = \hat{\rho}_{sys}(0) \otimes \hat{\rho}_{Env}(0)$$

$$\begin{array}{c|cccc} & |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \hline \langle 00| & & \times & & \times \\ \langle 01| & \times & & & \\ \langle 10| & & \times & & \times \\ \langle 11| & \times & & \times & \end{array}$$

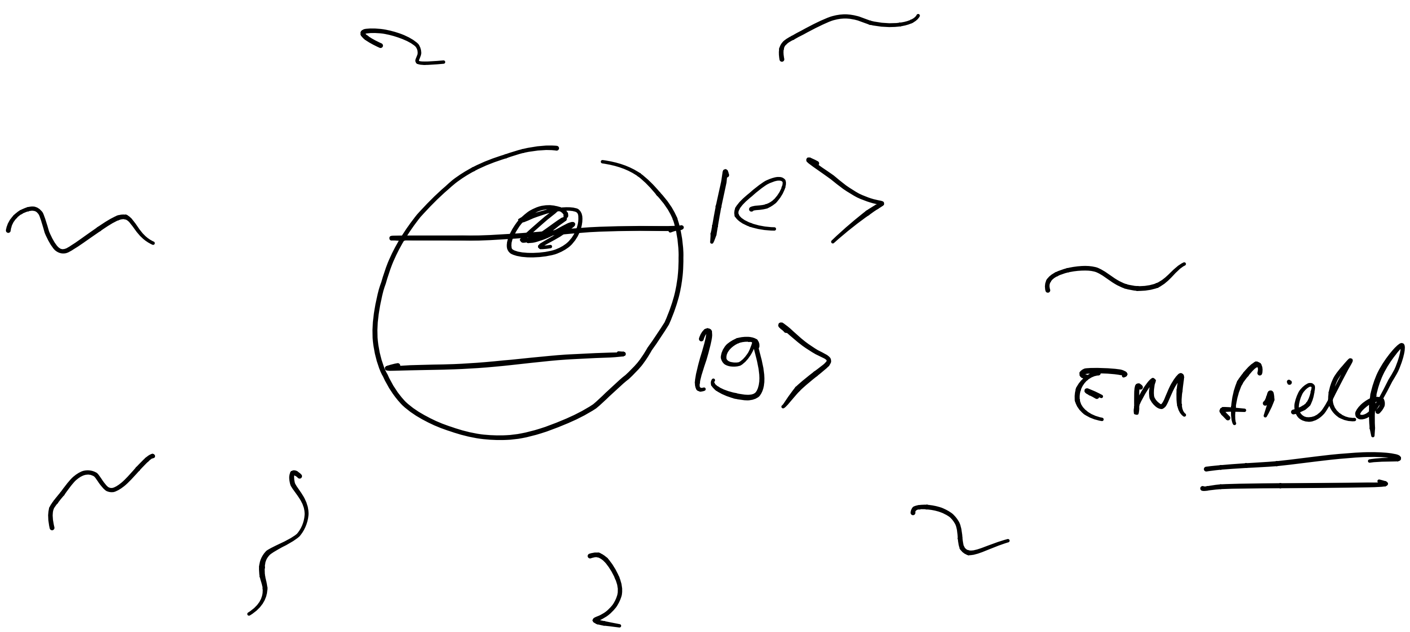
$$\mathrm{Tr}_{Env.}\left[\hat{\rho}_{tot}(t)\right]$$

$$= \sum_{n \in \mathcal{H}_{env.}} \langle n| \hat{\rho}_{tot.}(t) |n\rangle$$

Averaging over the state of env.

→ Spontaneous emission

Gerry & Knight - Introductory Q Optics



Cohen-Tannoudji

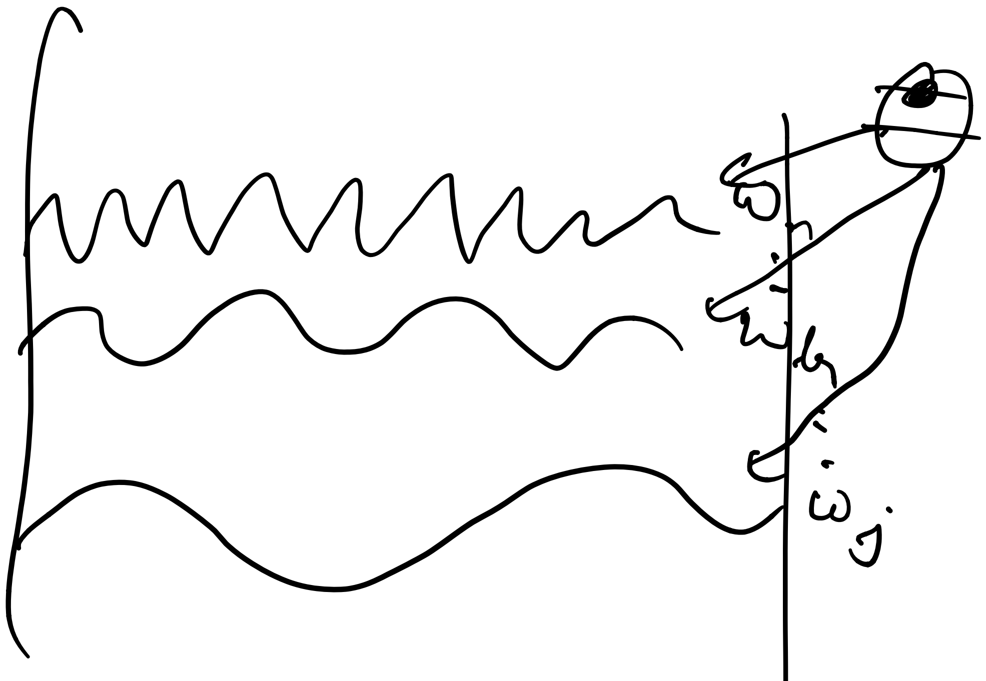
$$\hat{\rho}_{at} \quad \begin{array}{c|cc} & |e\rangle & |g\rangle \\ \hline \langle e| & \rho_{ee} & \rho_{eg} \\ \langle g| & \rho_{ge} & \rho_{gg} \end{array}$$

$$\hat{\rho}_{at} = \rho_{ee}\,|e\rangle\langle e| + \rho_{eg}\,|e\rangle\langle g| + \rho_{ge}\,|g\rangle\langle e| + \rho_{gg}\,|g\rangle\langle g|$$

$$\mathcal{H}_A = \hbar\omega_0\, \hat{\sigma}^+ \hat{\sigma}^- = \hbar\omega_0 |e\rangle\langle e|$$

$$\mathcal{H}_F = \sum_k \hbar\omega_k\, \underbrace{\hat{a}^\dagger(\omega_k)}_{\text{creation operator}}\, \underbrace{\hat{a}(\omega_k)}_{\text{annihilation operator}}$$

"ω" → harmonic oscillator



Quantum Harmonic Oscillator

$$\hat{a}^\dagger(\omega_k)\, |0\rangle$$

→ vacuum EMF

$$\downarrow$$

$$|\{1\}_{\omega_k}\rangle$$

$$\mathcal{H}_{int} = -\underline{\hat{d}} \cdot \underline{\hat{E}}$$

$$= g\left[\hat{\sigma}_+ \hat{a}(\omega) + \hat{\sigma}_- \hat{a}^\dagger(\omega)\right]$$

raising op. $\boxed{\hat{\sigma}^{+}} = |e\rangle\langle g|$

$\hat{\Pi}_e = |e\rangle\langle e|$ $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

lowering op. $\hat{\sigma}^{-} = |g\rangle\langle e|$ $\rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\left(|e\rangle\langle g|\right)\left(|g\rangle\langle e|\right)$$

$$= \left(|e\rangle\langle e|\right) = \underline{\underline{\hat{\Pi}_e}}$$

$$\mathcal{H} = \mathcal{H}_A + \mathcal{H}_F + \mathcal{H}_{int}$$

$$|\Psi(t)\rangle_{AF} = e^{-i\mathcal{H}t}\,|e\rangle \otimes |\{0\}\rangle$$

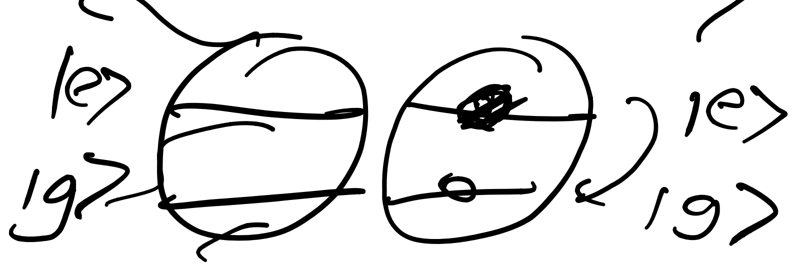
$$\rho_{AF}(t) = |\Psi(t)\rangle_{AF}\ {}_{AF}\langle\Psi(t)|$$

$$\mathrm{Tr}_F\ \hat{\rho}_{AF}(t) = \hat{\rho}_A(t)$$

$$e^{-\Gamma_0 t}$$

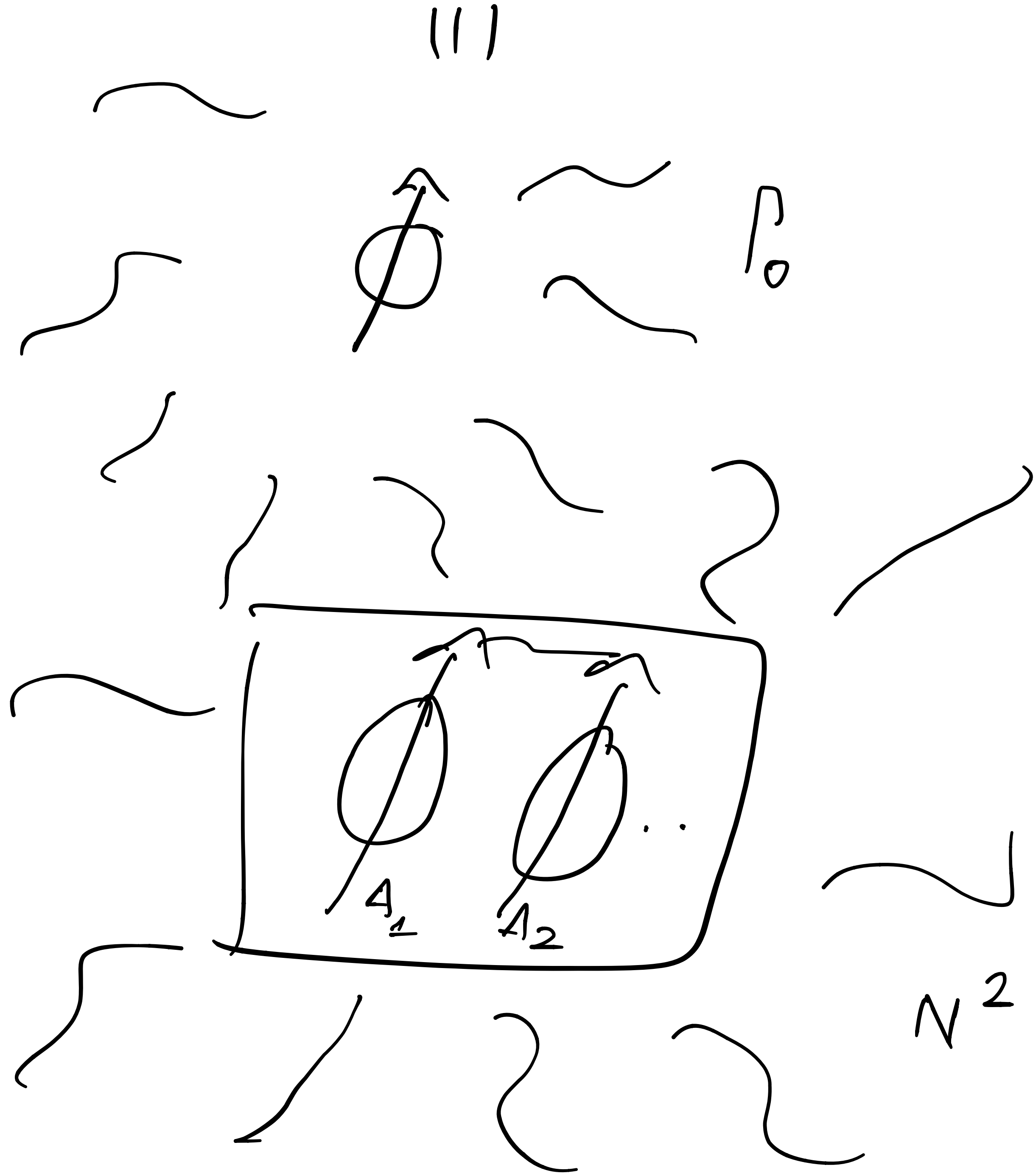
spontaneous emission rate

$$\Gamma_0 \sim |\vec{d}|^2$$



$$\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)$$

$$\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$$



$$I = |A_1 \pm A_2|^2 \qquad A_1 = A_2 = A$$

$$= 4|A|^2 \, , \, \boxed{0}$$

radiated intensity by classical dipoles ~ radiated intensity by atomic dipoles

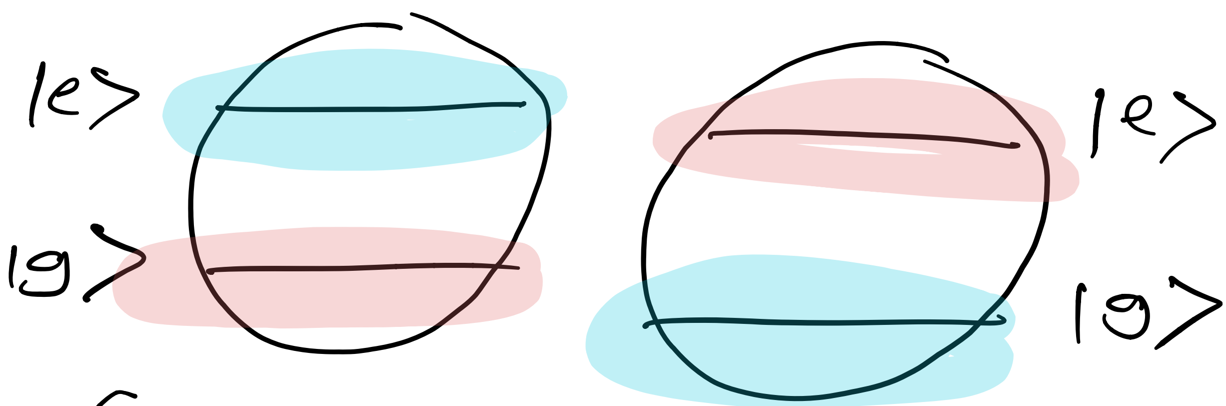
$$I = |A_1|^2 + |A_2|^2 + \ldots |A_N|^2$$

$$= N\,|A|^2$$

$$\text{“}I_{sup}\text{''} = N^2\,|A|^2 \rightarrow$$ superradiant collection of dipoles

$$= |A_1 + A_2 \ldots A_N|^2$$

$$A_1 = A_2 = \ldots A_N = A$$



$$\frac{1}{\sqrt{2}}\left(|eg\rangle \pm |ge\rangle\right)$$

R.H. Dicke 1954

Coherence in sp. emission processes

Mandel and Wolf Quantum Optics textbook Chapter 16

$$\mathcal{H} = -\underline{\hat{\vec{d}}} \cdot \hat{\vec{E}}$$

N atoms

$$\sqrt{N}|\vec{d}|^2 = -\sum_{n=1}^{N} \hat{\vec{d}}_n \cdot \hat{\vec{E}}(x_0)$$

$$= -\left[\sum_{n=1}^{N} \hat{\vec{d}}_n\right] \cdot \hat{\vec{E}}(x_0)$$

$$\Gamma_N \sim \left| \left\langle \sum_{n=1}^{N} \hat{\vec{d}}_n \right\rangle_{|eg\rangle \pm |ge\rangle} \right|^2$$

$$\hat{\vec{d}}_n = \vec{d}\left(\hat{\sigma}_n^+ + \hat{\sigma}_n^-\right)$$

$$\hat{\vec{d}}_1 + \hat{\vec{d}}_2 = \vec{d}\left(\hat{\sigma}_1^+ + \hat{\sigma}_1^- + \hat{\sigma}_2^+ + \hat{\sigma}_2^-\right)$$

(1) $\frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right)$ sup.

$$\frac{1}{\sqrt{2}}\left(\langle eg| \boxed{+} \langle ge|\right)\left(\underline{\hat{\vec{d}}_1} + \underline{\hat{\vec{d}}_2}\right)$$

$$\frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right)$$

$$= 2\vec{d}$$

(2) $\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$ subradiant

$$= \frac{1}{\sqrt{2}}(\langle eg| - \langle ge|)(\vec{\hat{d}}_1 + \vec{\hat{d}}_2)\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$$

$$= 0$$

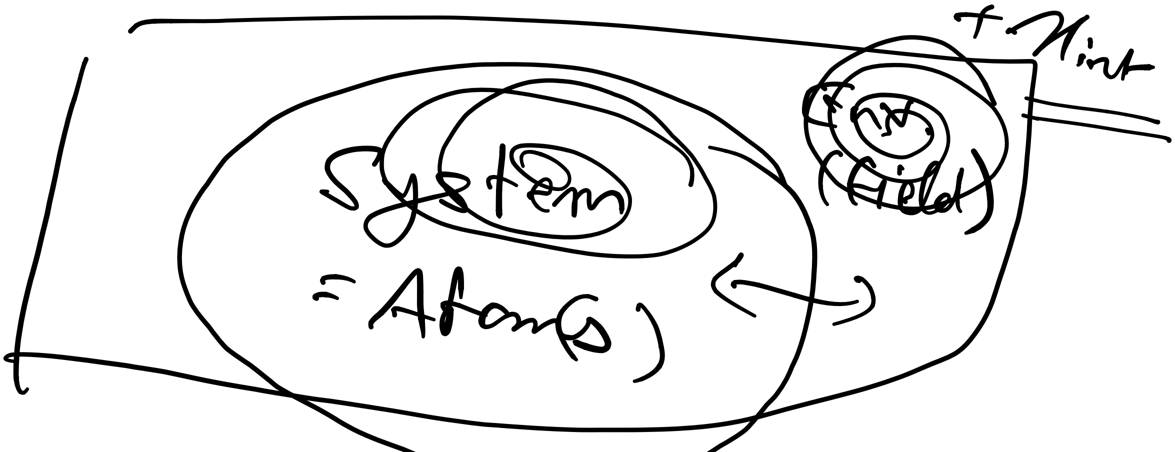
Schrodinger eq^n

$$i\hbar \frac{d|\psi\rangle}{dt} = \mathcal{H}|\psi\rangle$$

$\hookrightarrow$ total Hamiltonian

$$\hat{\rho}_{AF} = |\psi\rangle_{AF}\langle\psi|$$

$\mathcal{H} = \mathcal{H}_A + \mathcal{H}_F + \mathcal{H}_{int}$



$$\frac{d\hat{\rho}_{AF}}{dt} = \frac{d(|\psi\rangle)}{dt}\langle\psi| + |\psi\rangle\frac{d(\langle\psi|)}{dt}$$

$$= \left[\frac{-i}{\hbar}\mathcal{H}|\psi\rangle\right]\langle\psi| + |\psi\rangle\left(\frac{+i}{\hbar}\langle\psi|\mathcal{H}\right)$$

$$\frac{d\hat{\rho}_{AF}}{dt} = \frac{-i}{\hbar}\Big[\mathcal{H}|\psi\rangle\langle\psi| - |\psi\rangle\langle\psi|\mathcal{H}\Big]$$

$$\Rightarrow \frac{d\hat{\rho}_{AF}}{dt} = \frac{-i}{\hbar}\left[\hat{\mathcal{H}}\hat{\rho}_{AF} - \hat{\rho}_{AF}\hat{\mathcal{H}}\right]$$

$$\underline{\langle\psi(t)|}\,\hat{O}\,|\psi(t)\rangle \longleftrightarrow \langle\psi|\underbrace{\mathcal{U}^\dagger\hat{O}\,\mathcal{U}}|\psi\rangle$$

$$\longleftrightarrow \langle\psi|\,\underline{\hat{O}(t)}\,|\psi\rangle$$

$$\Rightarrow \boxed{\frac{d\hat{\rho}_{AF}}{dt} = \frac{-i}{\hbar}\left[\mathcal{H}, \hat{\rho}_{AF}\right]}$$

Master eqn.

Assume:

$$\boxed{Tr_F}\left[\hat{\rho}_{AF}\right] \equiv \hat{\rho}_A$$

$$|\psi\rangle_{AF} = |\psi\rangle_A \otimes |\psi\rangle_F$$

$$\hat{\rho}_{AF} = |\psi\rangle_A |\psi\rangle_F \otimes \langle\psi|_A \otimes \langle\psi|_F$$

$$Tr_F[\circ] \longrightarrow \sum_{n \in H_F} \langle n|[\circ]|n\rangle$$

orthonormal basis in the Hilbert space of field

$$\frac{d}{dt}\, Tr_F\left[\hat{\rho}_{AF}\right] = \frac{-i}{\hbar}\, Tr_F\left[\hat{\mathcal{H}}, \hat{\rho}_{AF}\right]$$

linear operator

$$\Rightarrow \frac{d\hat{\rho}_A}{dt} = \frac{-i}{\hbar}\, Tr_F\left\{\left[\hat{\mathcal{H}}, \hat{\rho}_{AF}\right]\right\}$$

dynamics of the atomic system

$$|\psi\rangle = C_e|e\rangle + C_g|g\rangle$$

$$\hat{\rho}_A = \begin{array}{c|cc} & |e\rangle & |g\rangle \\ \hline \langle e| & \boxed{\rho_{ee}(t)} & \rho_{eg}(t) \\ \langle g| & \rho_{ge}(t) & \rho_{gg}(t) \end{array}$$

$$\rho_{eg}(t) \Rightarrow C_e^* C_g$$

$$\mathcal{H}_A = \hbar\omega_0\, |e\rangle\langle e|$$

$$= \underline{\rho_{ee}}\, |e\rangle\langle e| + \rho_{eg}\, |e\rangle\langle g| + \rho_{ge}\, |g\rangle\langle e| + \rho_{gg}\, |g\rangle\langle g|$$

$$E = Tr_A \left[\hat{\rho}_A \, \hat{\mathcal{H}}_A \right] \quad \begin{pmatrix} \hbar\omega_0 & 0 \\ 0 & 0 \end{pmatrix}$$

energy

Hamiltonian

$$= \rho_{ee} \, \hbar\omega_0 \rightarrow \text{dissipation}$$

Expectation value of an observable $\hat{O}$ $= Tr_S \left[\hat{\rho}_S \, \hat{O} \right]$

$$= \sum_n \langle n | \left[|\psi\rangle_S \langle\psi|_S \hat{O} \right] | n \rangle$$

$$\hat{O} = \sum O_n |n\rangle\langle n|$$

$$= \langle \psi |_S \, \hat{O} \, | \psi_S \rangle$$

(completeness property the basis $|n\rangle$)

$$|\psi\rangle_S = \sum_{n_1} \psi_{n_1} | n_1 \rangle$$

$$= \sum_n \langle n | \left(\sum_{n_1} \psi_{n_1} | n_1 \rangle \right) \left(\sum_{n_2} \psi^*_{n_2} \langle n_2 | \right)$$

$$\left(\sum_m \mathcal{O}_m | m \rangle \langle m | \right) | n \rangle$$

$$= \sum_{n, n_1, n_2, m} \mathcal{O}_m \psi_{n_1} \boxed{\delta_{n, n_1}} \psi^*_{n_2} \boxed{\delta_{n_2, m}} \boxed{\delta_{mn}}$$

$n = n_1$ $\qquad m = n_2 \qquad m = n$

$$\Rightarrow n = m = n_2 = n_1$$

$$= \sum_n \mathcal{O}_n \psi_n \psi_n^*$$

$$= \mathrm{Tr}_S \left[\hat{\rho}_S \hat{\mathcal{O}}_S \right]$$

Dissipation : average energy of the system is lost to the environ.

as a function of time.

Decoherence: loss of coherence of

$$\underline{\rho_{ee,ee}} \ , \quad \underline{\rho_{ee,gg}} \ , \quad \underline{\rho_{gg,ee}}$$

$$E \equiv Tr_A \left[\rho_{AB} \otimes \mathcal{H}_A \right]$$

$$e^{-\gamma t} \ , \quad e^{-2\gamma t} \ , \quad 1$$

normal super sub