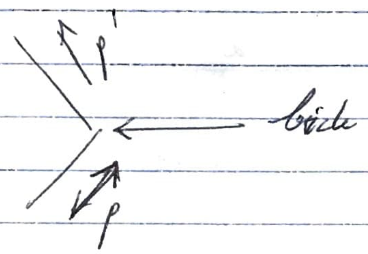
Radiative Corrections

Soft Brehmsstrahlung → low freq radiation when e^- undergoes sudden acceleration

Classical picture

@ $t=0$, $x=0$, e^- is given a momentum kick.



→ look at radiation of Maxwell's eqn... once we know the $j^\mu(x,t)$...

Recall for particle @ rest..

$$j^\mu = e\cdot(\text{particle density}, \vec{0}) = (1,0,0,0)\cdot e\cdot\delta^3(\vec{x})$$

$$\Rightarrow j^\mu(x) = \int dt'\,(1,0,0,0)^\mu\, e\,\delta^{(4)}(x-y(t'))$$

where $y(t') = (t',0,0,0)$.

world line of particle.

In general... $y^\mu(\tau)$

$$j^\mu(x) = e\int d\tau\,\frac{dy^\mu}{d\tau}\,\delta^{(4)}(x-y(\tau))$$

picks τ such that $y^0(\tau) = \tau$

At τ we have $\delta^{(3)}(\vec{x}-\vec{y}(\tau))$ and 4-velocity $\dfrac{dy^\mu(\tau)}{d\tau}$

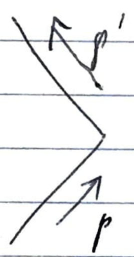
Check that j^μ is conserved...

Let $f(x)$ be fn such that $f(x) \to 0$ as $x \to \infty$

then $\int d^4x\, f(x)\, \partial_\mu j^\mu(x) = \int d^4x\, f(x)\, e \cdot \int d\tau \frac{dy^\mu}{d\tau} \partial_\mu \delta^{(4)}_x(x - y(\tau))$

$$= -e \int d\tau \frac{dy^\mu}{d\tau}(\tau) \frac{\partial f(x)}{\partial x}\Big|_{x = y(\tau)}$$

$$= -e\, f(y^\mu(\tau))\Big|_{\tau=-\infty}^{\tau=\infty} = 0 \quad \checkmark$$



$\Rightarrow$ world line looks like...

$$y^\mu(\tau) = \begin{cases} \frac{p^\mu}{m}\tau & \tau < 0 \\ \frac{p'^\mu}{m} & \tau > 0 \end{cases}$$

$$\rightarrow \quad j^\mu(x) = e\int_0^\infty d\tau\, \frac{p'^\mu}{m} \delta^{(4)}(x - p'/m\, \tau) + e\int_{-\infty}^0 d\tau\, \frac{p^\mu}{m} \delta^{(4)}(x - p/m\, \tau)$$

FT...

$$\tilde{j}^\mu(k) = ie \left\{ \frac{p'^\mu}{k\cdot p' + i\varepsilon} - \frac{p^\mu}{k\cdot p - i\varepsilon} \right\}$$

Maxwell...

$$\partial_\mu \partial^\mu A^\nu = j^\nu \Rightarrow -k^2 \tilde{A}^\mu(k) = \tilde{j}^\mu(k)$$

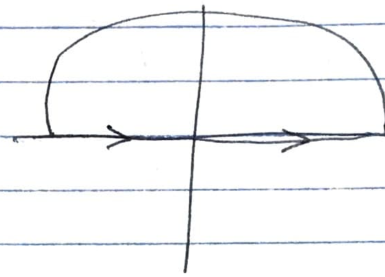
$$\Rightarrow \tilde{A}^\mu(k) = \frac{-ie}{k^2}\left(p'^\mu/_{k\cdot p' + i\varepsilon} - p^\mu/_{k\cdot p - i\varepsilon} \right)$$

So

$$A^\mu(x) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot x}\, \frac{(-ie)}{k^2}\left(\frac{p'^\mu}{k\cdot p'+i\epsilon} - \frac{p^\mu}{k\cdot p-i\epsilon}\right)$$

When $x^0<0$, momentum $= p^\mu$ ⇒ the term p'^μ cannot contribute.

→ $x^0<0$



Can have poles at $-|\vec{k}| \pm i\delta$ & $|\vec{k}| \pm i\delta$, but if pole at $\mp|\vec{k}| + i\delta$ → contribution from p'^μ

⇒ both of these must be in the lower half plane

→ $x^0<0$ ⇒ residue @ $k\cdot p = i\epsilon$.

→ $$A^\mu(x) = \int \frac{d^3k}{(2\pi)^4}\, e^{+i\vec{k}\cdot\vec{x}}\, e^{-i\vec{k}\cdot\vec{p}/p^0\, t}\, (2\pi i)\, \frac{ie}{k^2}\, \frac{p^\mu}{p^0}$$

In rest frame ... $p^0 = m$, $\vec{p}=0$

⇒ $$A^\mu(x) = e\int \frac{d^3k}{(2\pi)^4}\, e^{i\vec{k}\cdot\vec{x}}\, \frac{(1,0,0,0)}{|\vec{k}|^2}$$

↑

Coulomb potential ~~for~~ in the $\mu=0$ component & zero for other components.

→ Same thing with $x^0>0$ ($k\cdot p' = -i\epsilon$ ← residue)

~~Ithe~~ the interesting Bremsstrahlung radiation comes from the other 2 poles ... at $k^0 = |\vec{k}| - i\delta$
$k^0 = -|\vec{k}| - i\delta$

Residues give...

$$A^\mu_{rad}(x) = \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{-e}{2|\vec{k}|} e^{+i\vec{k}\cdot\vec{x}} \left(\frac{p'^\mu}{k\cdot p'} - \frac{p^\mu}{k\cdot p} \right) \bigg|_{k^0 = |\vec{k}|} e^{-i|\vec{k}|\cdot t} \right.$$

$$\left. + \frac{e}{2|\vec{k}|} e^{+i\vec{k}\cdot\vec{x}} \left(\frac{p'^\mu}{k\cdot p'} - \frac{p^\mu}{k\cdot p} \right) \bigg|_{k^0 = -|\vec{k}|} \cdot e^{+i|\vec{k}|\cdot t} \right\}$$

$$2^{nd} \text{ term} = \int \frac{d^3k}{(2\pi)^3} \frac{-e}{2|\vec{k}|} e^{-i\vec{k}\cdot\vec{x}} \left(\frac{p'^\mu}{k\cdot p'} - \frac{p^\mu}{k\cdot p} \right) \bigg|_{k^0 = |\vec{k}|} e^{+i|\vec{k}|\cdot t}$$

$\parallel$

complex conjugate of 1^{st} term...

$$\Rightarrow A^\mu_{rad}(x) = Re\left\{ \int \frac{d^3k}{(2\pi)^3} \underbrace{\mathcal{Q}^\mu(\vec{k})}_{\frac{-e}{|\vec{k}|} \left(\frac{p'^\mu}{k\cdot p'} - \frac{p^\mu}{k\cdot p} \right) \big|_{k^0 = |\vec{k}|}} e^{+i\vec{k}\cdot\vec{x}} e^{-i|\vec{k}|\cdot t} \right\}$$

Now, recall that

$$E^i(x) = -F^{0i} = -\partial_0 A^i - \partial_i A^0 = -\partial_0 \vec{A} - \vec{\nabla} A^0$$

$$B^i(x) = \vec{\nabla} \times \vec{A}$$

Choose frame s.t. $p^0 = p'^0 = E$.

Let $k^\mu = (|\vec{k}|, \vec{k})$

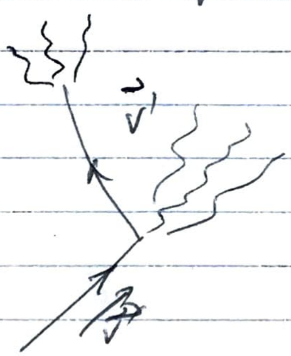
$p^\mu = (E, E\vec{v})$ $\quad p'^\mu = (E, E\vec{v}')$

then

$$\frac{1}{k\cdot p'} = \frac{1}{E|\vec{k}|(1 - \hat{k}\cdot\vec{v}')}$$

$$\frac{1}{k\cdot p} = \frac{1}{E|\vec{k}|(1 - \hat{k}\cdot\vec{v})}$$

→ Radiation peaked when $\vec{k}$ points in the same direction as $\vec{v}$ or $\vec{v}'$.



also the note that $k_\mu a^\mu = 0$.