Physics 8.321, Fall 2020
Homework #6

Due **Friday, November 12** by 8:00 PM.

1. [Sakurai and Napolitano Problem 16, Chapter 2 (page 151)]

Consider a function, known as the **correlation function**, defined by

$$C(t) = \langle x(t)x(0)\rangle ,$$

where $x(t)$ is the position operator in the Heisenberg picture. Evaluate the correlation function explicitly for the ground state of a one-dimensional simple harmonic oscillator.

2. [Modified from Sakurai and Napolitano Problem 17, Chapter 2 (page 152)]

Consider again a one-dimensional simple harmonic oscillator. Do the following algebraically – that is, without using wavefunctions.

(a) Construct a (normalized) linear combination of $|0\rangle$ and $|1\rangle$ such that $\langle x\rangle$ is as large as possible.

(b) Suppose the oscillator is in the state constructed in (a) at $t = 0$. What is the state vector for $t > 0$ in the Schrödinger picture? Evaluate the expectation value $\langle x\rangle$ as a function of time for $t > 0$, using (i) the Schrödinger picture and (ii) the Heisenberg picture. Evaluate $\langle p\rangle$ as a function of time as well and confirm Ehrenfest's theorem giving the classical equations of motion.

(c) Evaluate $\langle(\Delta x)^2\rangle$ as a function of time using either picture.

3. Consider a simple harmonic oscillator of frequency ω which begins in the state

$$|\psi(0)\rangle = c_0 e^{\phi_0 a^\dagger} |0\rangle$$

where $\phi_0 = \alpha + i\beta$ is an arbitrary complex number and $c_0 = \exp\left(-|\phi_0|^2/2\right)$.

(a) Solve the equation of motion for $|\psi(t)\rangle$.

(b) Evaluate $\langle x\rangle$, $\langle p\rangle$ as functions of time.

(c) Describe the wavefunction associated with $|\psi(t)\rangle$ in terms of modulus $\rho(x)$ and phase $S(x)$. Give the physical interpretation of the modulus and phase. Describe qualitatively what happens to the wavefunction over time. Compare with the time-development of a free particle given an initial Gaussian state.