Name: ______________________________

Statistics 482 Spring 2020
Final

15 May 2020

- This is an open-book, open-note, closed-internet exam. I have also provided you with properties of common distributions.
- Do not use Wolfram Alpha to obtain integrals. Enough work must be shown for me to tell that the answer was not just copied.
- All work must be your own. You may not give or receive any kind of aid, either verbally, visually, or otherwise, during this exam. No other sources may be consulted, except as specified above.
- **The exam has 100 possible points. There are 6 questions and 11 pages, including this cover page. You have 3 hours to complete the exam so plan your time accordingly. I have included the possible points next to each problem.**
- Some questions are more difficult than others, and the questions may not be in order of difficulty. Don't spend too much time on any one question; if you get stuck, go on and try another part.
- Whenever possible, show your work and explain your reasoning. In case you make a mistake, I can more easily give you partial credit if you explain your steps.
- Some parts of a question may require the answer to an earlier part of the question. If you can't solve the earlier part, you can still receive partial credit for the latter parts: make up a reasonable answer for the earlier part and use that in solving for the latter parts.
- **Please upload your answers to Moodle in a single document with filename LASTNAME_482_Final.pdf.**

Question 1 (18 points total)

Suppose that X_1, X_2,…,X_n are independent and identically distributed from a distribution with density function,

$$f(x\,|\,\theta) = \begin{cases} \left(\frac{1}{\theta}\right) e^{-x/\theta}; & x > 0 \\ 0; & \text{elsewhere} \end{cases}$$

where θ is the unknown parameter.

A. (4 points) Find the maximum likelihood estimator of the parameter, θ.

B. (8 points) Is the maximum likelihood estimator in (A) efficient? Show or explain.

Question 1 continued…

C. (6 points) Now find the MVUE for θ.

Question 2 (10 points total)
Consider a random sample, X_1, X_2,…, X_n, from a Rayleigh distribution with density function,

$$f(x) = \begin{cases} \left(\dfrac{2x}{\theta}\right) e^{-x^2/\theta}; & x > 0 \\ 0; & \text{elsewhere} \end{cases}$$

A. (3 points) Find a sufficient statistic for θ.

B. (5 points) Using the sufficient statistic in (A), find the MVUE for θ. Hint: Start by taking the expectation of the sufficient statistic.

C. (2 points) What would you need to show in order to prove that the MVUE you found in (B) is unique?

Question 3 (26 points total)
Suppose *Y* is a random sample of size 1 (a single random observation) from a population with distribution,

$$f(y|\theta) = \begin{cases} \theta y^{\theta-1}; & 0 \le y \le 1 \\ 0; & elsewhere \end{cases}$$

A. **(8 points)** Find a most powerful test of $H_0: \theta = 1 \; vs. H_A: \theta = 2.$ Find the form of the rejection region, we well as making sure to show your work. Also find the specific values of your test statistic for which the null hypothesis would be rejected if $\alpha = 0.05$.

B. **(2 points)** Is the test in (A) uniformly most powerful for alternatives of the form, $\theta > 1$? Show or explain.

Question 3 continued…

C. (6 points) Find the power for the test in (A).

D. (10 points) Find the likelihood ratio test for $H_0: \theta = 1 \; vs. H_A: \theta > 1$. Make sure to find the values of the test statistic for which you would reject if $\alpha = 0.05$.

Question 4 (12 points total)

Consider a random variable, X, with a Poisson(λ) distribution with probability function,

$p(x|\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$; x = 0, 1, 2,…

A. (4 points) Show that the random variable *X* belongs to the exponential class of distributions. Specify the functions: $p(\theta), k(x), H(x), \text{and } q(\theta)$.

B. (8 points) Now show that the complete sufficient statistic for this distribution also belongs to the exponential family.

Question 5 (20 points total)

Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables from a Uniform(0, θ) distribution.

A. (5 points) Find the maximum likelihood estimator for both $E[X_1]$ and $Var[X_1]$.

B. (3 points) Find a minimal sufficient statistic, *Y*, for θ.

C. (5 points) Show that, if $\theta > 0$, then the sufficient statistic you found in (B) is complete. Note that the density function for the max is given by,

$$\boldsymbol{g_{(n)}(y) = \left(\frac{n}{\theta^n}\right) y^{n-1}; \quad 0 \le y \le \theta.}$$

Question 5 continued…

D. **(5 points)** Show that if $\theta > 1$, then the sufficient statistic you found in (B) is *not* complete.

Question 6 (14 points total)
For each of the following statements, say whether the statement is true or false.

A. A minimax loss function helps you to find the decision that provides you with the best “worst-case” scenario.

True **False**

B. The maximum likelihood estimator will have the smallest variance of all estimators if the sample size is large.

True **False**

C. A minimal statistic is always unique.

True **False**

D. An ancillary statistic, by itself, provides us with no information about the unknown parameter.

True **False**

E. A Wald test evaluates the distance from the null value, θ_0, to the value of the estimate of θ that maximizes the likelihood function.

True **False**

F. Maximum likelihood estimation provides you with a valid estimate only if regularity conditions are satisfied.

True **False**

G. Likelihood, score, and Wald tests are asymptotically equivalent.

True **False**