You may find the following information helpful:

Physical Constants

Electron mass	$m_e \approx 9.1 \times 10^{-31} kg$	Proton mass	$m_p \approx 1.7 \times 10^{-27} kg$
Electron Charge	$e \approx 1.6 \times 10^{-19} C$	Planck's const./2π	$\hbar \approx 1.1 \times 10^{-34} Js^{-1}$
Speed of light	$c \approx 3.0 \times 10^{8} ms^{-1}$	Stefan's const.	$\sigma \approx 5.7 \times 10^{-8} Wm^{-2}K^{-4}$
Boltzmann's const.	$k_B \approx 1.4 \times 10^{-23} JK^{-1}$	Avogadro's number	$N_0 \approx 6.0 \times 10^{23} mol^{-1}$

Conversion Factors

$1atm \equiv 1.0 \times 10^5 Nm^{-2}$ $\quad$ $1\mathring{A} \equiv 10^{-10} m$ $\quad$ $1eV \equiv 1.1 \times 10^4 K$

Thermodynamics

$dE = TdS + đW$ $\quad$ For a gas: $đW = -PdV$ $\quad$ For a wire: $đW = Jdx$

Mathematical Formulas

$\int_0^\infty dx\ x^n\ e^{-\alpha x} = \frac{n!}{\alpha^{n+1}}$ $\quad$ $\left(\frac{1}{2}\right)! = \frac{\sqrt{\pi}}{2}$

$\int_{-\infty}^{\infty} dx \exp\left[-ikx - \frac{x^2}{2\sigma^2}\right] = \sqrt{2\pi\sigma^2} \exp\left[-\frac{\sigma^2 k^2}{2}\right]$ $\quad$ $\lim_{N\to\infty} \ln N! = N \ln N - N$

$\left\langle e^{-ikx} \right\rangle = \sum_{n=0}^{\infty} \frac{(-ik)^n}{n!} \langle x^n \rangle$ $\quad$ $\ln \left\langle e^{-ikx} \right\rangle = \sum_{n=1}^{\infty} \frac{(-ik)^n}{n!} \langle x^n \rangle_c$

$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$ $\quad$ $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$

Surface area of a unit sphere in d dimensions $\quad$ $S_d = \frac{2\pi^{d/2}}{(d/2-1)!}$

1. *Poisson brackets:* Consider the integral over a multidimensional phase space $\Gamma \equiv [\mathbf{p}, \mathbf{q}]$:

$$I = \int d\Gamma A\{B, C\}\,,$$

where $A(\mathbf{p}, \mathbf{q})$, $B(\mathbf{p}, \mathbf{q})$, and $C(\mathbf{p}, \mathbf{q})$ are functions over phase space, and

$$\{B, C\} \equiv \left(\frac{\partial B}{\partial \mathbf{q}} \cdot \frac{\partial C}{\partial \mathbf{p}} - \frac{\partial B}{\partial \mathbf{p}} \cdot \frac{\partial C}{\partial \mathbf{q}}\right)\,,$$

denotes the Poisson bracket of B and C.

(a) Prove the following identity (which you can use in subsequent parts of this problem)

$$I = \int d\Gamma A\{B, C\} = \int d\Gamma B\{C, A\}\,.$$

(b) Show that when $C(\mathbf{p}, \mathbf{q}) = F(A(\mathbf{p}, \mathbf{q}))$, where $F(x)$ denotes any function of x,

$$\int d\Gamma A\{B, C\} = 0\,.$$

(c) The phase space density $\rho(\Gamma, t)$ satisfies the equation $\partial_t \rho = \{H, \rho\}$, and an associated entropy is given by $S(t) = -\int d\Gamma \rho(\Gamma, t) \ln \rho(\Gamma, t)$. Prove that $dS/dt = 0$.

(d) The average of function $A(\mathbf{p}, \mathbf{q})$ is given by $\langle A \rangle(t) = \int d\Gamma \rho(\Gamma, t) A(\mathbf{p}, \mathbf{q})$. Prove that

$$\frac{d\langle A \rangle}{dt} = \langle \{A, \mathcal{H}\} \rangle\ .$$

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2. *Three gas mixture:* Consider a mixture of three gases (a), (b) and (c), in a box.

(a) Write down the Boltzmann equations for the one particle densities f_a, f_b and, f_c, in terms of the Liouville operators $\mathcal{L}_\alpha \equiv [\partial_t + (\vec{p}_\alpha/m_\alpha) \cdot \nabla]$, and appropriate collision operators

$$C_{\alpha,\beta} = -\int d^3\vec{p}_2 d^2\vec{b}_{\alpha\beta}\ |\vec{v}_1 - \vec{v}_2|\left[f_\alpha(\vec{p}_1, \vec{q}_1) f_\beta(\vec{p}_2, \vec{q}_1) - f_\alpha(\vec{p}_1{}', \vec{q}_1) f_\beta(\vec{p}_2{}', \vec{q}_1)\right],$$

for $\alpha, \beta = a, b, c$.

(b) If there are no interactions between particles of different species, i.e. $C_{\alpha,\beta} = 0$ for $\alpha \neq \beta$, write down the most general zeroth order solution for the densities f_a, f_b and, f_c.

(c) How does including interactions between (a) and (b) particles, but no interactions between the (a) and (c) or (b) and (c) particles modify the form of f_a, f_b and, f_c?

(d) What is the corresponding form of f_a, f_b and, f_c upon including interactions between (a) and (b) particles, (c) and (b) particles, but no interactions between the (a) and (c) particles?

(e) Including interactions among all particles, i.e. with all $C_{\alpha,\beta} \neq 0$, what are the slow (hydrodynamic) modes of this gas mixture?

(f) Starting with a configuration of N_a, N_b, and N_c particles in a box of volume V, what are the final (equilibrium) forms of f_a, f_b and, f_c?

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