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Exam Midterm #1

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①

| p. 1

$$(a) \quad dE = TdS - PdV$$

$$\Rightarrow dS = \frac{dE + PdV}{T} = \frac{1}{T} \{ dE + PdV \}$$

$$\text{Now, } dE = d(DpV) = D[Vdp + PdV]$$

$$\underline{\text{So}}, \quad dS = \frac{1}{T} \{ D[Vdp + PdV] + PdV \}$$

$$= \frac{1}{T} \{ DVdp + P(D+1)dV \}$$

$$\underline{\text{So}} \quad \boxed{dS(p, V) = \frac{1}{c p^n} \{ DVdp + P(D+1)dV \}}$$

② • Since S is a function of state, we have

$$\frac{\partial^2 S}{\partial p \partial V} = \frac{\partial^2 S}{\partial V \partial p}$$

$$\bullet \text{ Now, } dS(p, V) = \left. \frac{\partial S}{\partial p} \right|_V dp + \left. \frac{\partial S}{\partial V} \right|_p dV$$

$$\underline{\text{So}} \quad \begin{cases} \frac{\partial S}{\partial p} = \frac{DV}{T} = \frac{DV}{c p^n} = \frac{DV}{c} p^{-n} \Rightarrow \frac{\partial^2 S}{\partial p \partial V} = \frac{D}{c} p^{-n} \\ \frac{\partial S}{\partial V} = \frac{P(D+1)}{T} = \frac{P(D+1)}{c p^n} = \left(\frac{D+1}{c} \right) p^{-n+1} \Rightarrow \frac{\partial^2 S}{\partial V \partial p} = \frac{D+1}{c} (1-n) p^{-n} \end{cases}$$

$$\Rightarrow D = (D+1)(1-n) \Rightarrow \boxed{D = \frac{1-n}{n}}$$

(c) Form of adiabatic curve is $p_s(V)$.

p_s

$$\text{Adiabatic} \Rightarrow \delta Q = 0 \Rightarrow dE = -P dV$$

$$\Rightarrow D \{ P dV + V dP \} = -P dV$$

$$\Rightarrow P V dP = -(D+1) P dV$$

$$\Rightarrow \int \frac{dP}{P} = \int \underbrace{\left(-\frac{D+1}{D} \right)}_D \frac{dV}{V}$$

$$\Rightarrow \ln \frac{P_f}{P_i} = -D \ln \frac{V_f}{V_i} = \cancel{\ln \frac{V_i}{V_f}}$$

$$\Rightarrow \frac{P_f}{P_i} = \left(\frac{V_f}{V_i} \right)^{-D} \Rightarrow P_f V_f^{+D} = \text{constant} = G$$

\Rightarrow Form of adiabatic ~~curve~~ curve is ...

$$P = G V^{-D} \text{ where } D = \frac{D+1}{D} = \frac{1}{1-n}$$

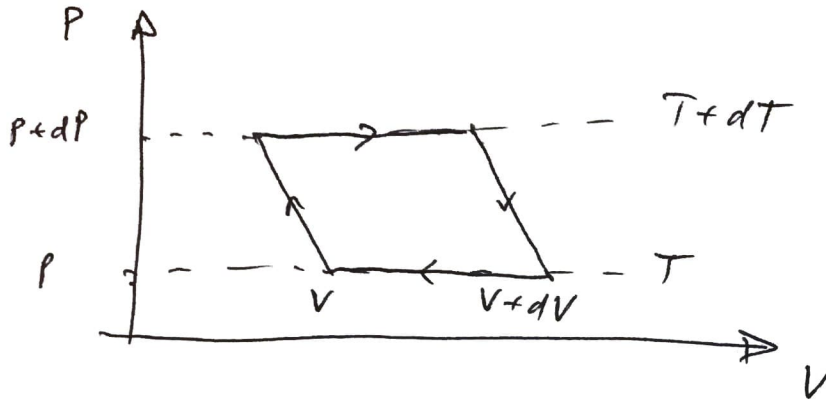
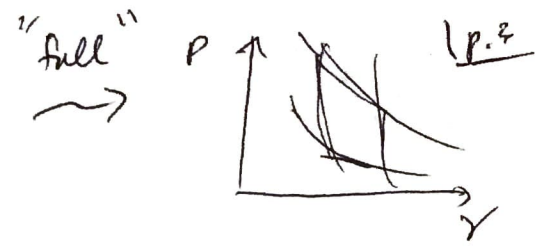
$$\boxed{P_s(V) = G V^{\rightarrow 1/1-n}}$$

\uparrow
constant.

Answer:

$$\boxed{P_s(V) = G V^{+1/n-1}}$$

(d) Infinitesimal
Carnot cycle in (p, V) coords



(e) The work done is $\delta W = dP dV$

~~$\delta Q = dE + P dV = T dS$ (expanding along constant pressure)~~

~~$\Rightarrow DV dp + P(D+1) dV$~~

0

~~$P(D+1) dV$~~

Carnot Efficiency: $\eta = \frac{\delta W}{\delta Q} = \frac{dT}{T} = \frac{dP dV}{\delta Q}$

$\Rightarrow \frac{1}{T} d(cp^n) = \frac{dP dV}{\delta Q}$

$\rightarrow \frac{1}{T^n} \cdot dp^n \cdot n dp = \frac{dP dV}{Q} \Rightarrow \boxed{\delta Q = (dV) \left(\frac{P}{n} \right)}$

$\left(\frac{n}{P} \right) = \frac{dV}{Q}$

(2)

p.4

$$(a) \quad C_1 (T_1 - T_f) = C_2 (T_f - T_2)$$

$$C_1 T_1 - C_1 T_f = C_2 T_f - C_2 T_2$$

$$C_1 T_1 + C_2 T_2 = (C_1 + C_2) T_f$$

$$\Rightarrow \boxed{T_f = \frac{C_1 T_1 + C_2 T_2}{C_1 + C_2}}$$

$$\Delta S = S_f - S_i$$

$$= (S_{1,f} + S_{2,f}) - (S_{1,i} + S_{2,i})$$

$$\text{where } T_f = \frac{C_1 T_1 + C_2 T_2}{C_1 + C_2}$$

$$= \Delta S_1 + \Delta S_2 = \boxed{C_1 \ln \frac{T_f}{T_1} + C_2 \ln \frac{T_f}{T_2}}$$

Note can
simplify, but
expression
still clunky

(b) If Carnot engine is used then $\Delta S = 0$

$$\Rightarrow C_1 \ln \frac{T_f}{T_1} = -C_2 \ln \frac{T_f}{T_2}$$

$$\frac{T_f^{C_1}}{T_1^{C_1}} = \frac{T_2^{C_2}}{T_f^{C_2}} \Rightarrow T_f^{C_1 + C_2} = T_1^{C_1} T_2^{C_2}$$

$$\Rightarrow \boxed{T_f = \sqrt[C_1 + C_2]{T_1^{C_1} T_2^{C_2}}}$$

Work done is given by conservation of energy:

$$W = (C_1 T_1 + C_2 T_2) - (C_1 + C_2) T_f$$

$$\boxed{W = (C_1 T_1 + C_2 T_2) - (C_1 + C_2) \sqrt[C_1 + C_2]{T_1^{C_1} T_2^{C_2}}}$$

3 (a) $p(x) = \frac{1}{2b} \exp\left(-\frac{|x-a|}{b}\right) \quad x \in \mathbb{R}.$

$$\langle e^{-ikx} \rangle = \int_{-\infty}^{\infty} e^{-ikx} p(x) dx$$

$$= \int_{-\infty}^{\infty} e^{-ikx} \frac{1}{2b} \exp\left(-\frac{|x-a|}{b}\right) dx$$

$$= \int_{-\infty}^a e^{-ikx} \frac{1}{2b} \exp\left(\frac{+(x-a)}{b}\right) dx$$

$$+ \int_a^{\infty} \frac{1}{2b} \exp\left(-\frac{(x-a)}{b}\right) dx$$

$$= \frac{e^{-iak}}{2(1+ibk)} + \frac{ie^{-iak}}{2(i+bk)} = \boxed{\frac{e^{-iak}}{1+b^2k^2}}$$

$$\approx 1 - iak + \left(\frac{-a^2}{2} - b^2\right)k^2 + \dots$$

So $\boxed{\langle x \rangle = a}$

$$\langle x^2 \rangle = a^2 + 2b^2 \Rightarrow \boxed{\sigma^2} = \langle x^2 \rangle - \langle x \rangle^2$$

$$= a^2 + 2b^2 - a^2$$

$$= \boxed{2b^2}$$

← Variance

$$(1) \quad p(x) = \frac{|x|}{2a^2} \exp\left(-\frac{|x|}{a}\right) \quad -\infty < x < \infty$$

$$\langle e^{-ikx} \rangle = \int_{-\infty}^{\infty} e^{-ikx} p(x) dx$$

$$= \int_{-\infty}^0 \frac{-x}{2a^2} \exp\left(\frac{x}{a}\right) e^{-ikx} dx + \int_0^{\infty} \frac{x}{2a^2} \exp\left(-\frac{x}{a}\right) e^{-ikx} dx$$

$$= \frac{-1}{2(i+ak)^2} + \frac{-1}{2(-i+ak)^2}$$

$$= \boxed{\frac{1-a^2k^2}{(1+a^2k^2)^2}} \approx 1 - 3a^2k^2 + 5a^4k^4 + \dots$$

$$\boxed{\langle x \rangle = 0}$$

$$\langle x^2 \rangle = 6a^2 \Rightarrow$$

$$\boxed{\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = 6a^2}$$

grade: 15/15 nice!