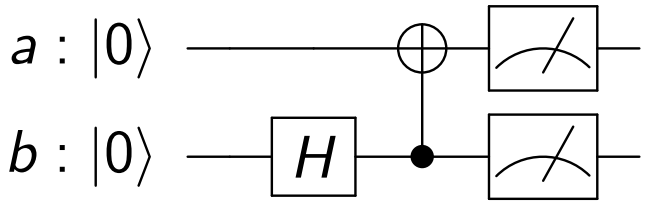
Matrices in Quantum Computing

Huan Q. Bui

Matrix Analysis

Professor Leo Livshits

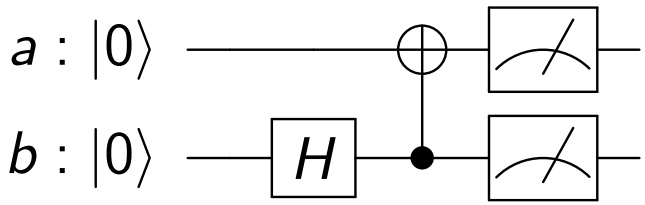
CLAS, May 2, 2019


$a : |0\rangle$
$b : |0\rangle$
H

Presentation layout

1. Background
2. Matrices in an entanglement circuit
3. Recap

Background



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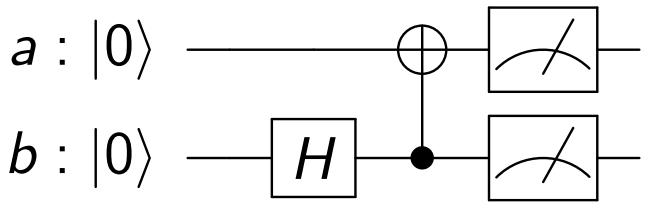
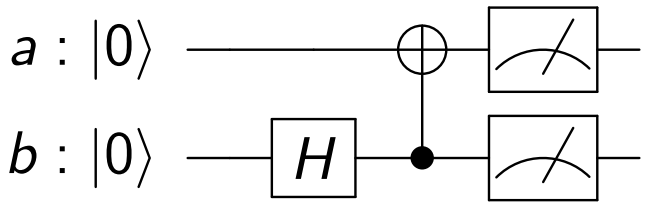


Components:

Background



Components:

1. Quantum bits - Qubits

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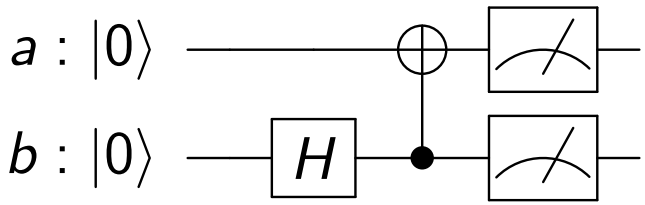

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2. Quantum gates: single and multiple-qubit gates

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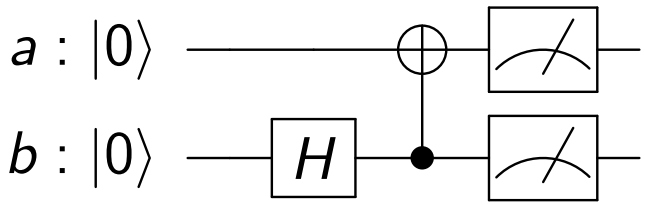

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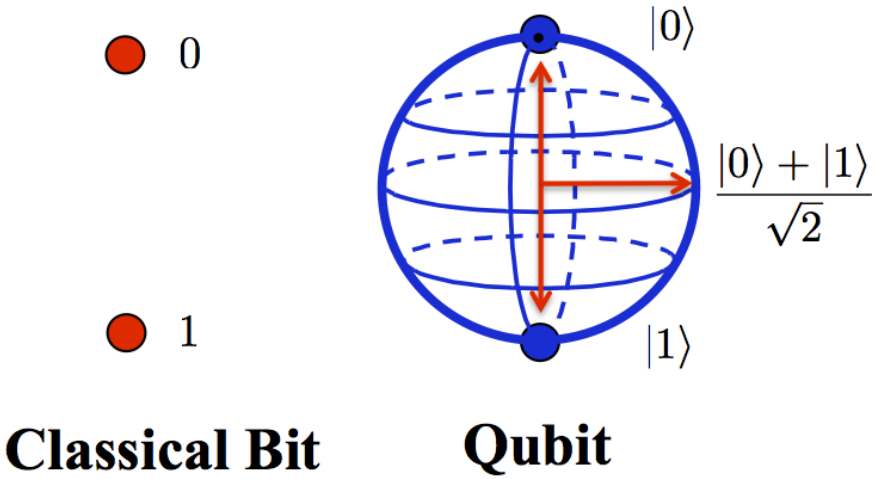
2. Quantum gates: single and multiple-qubit gates
3. Measurement

Quantum Bits - Qubits



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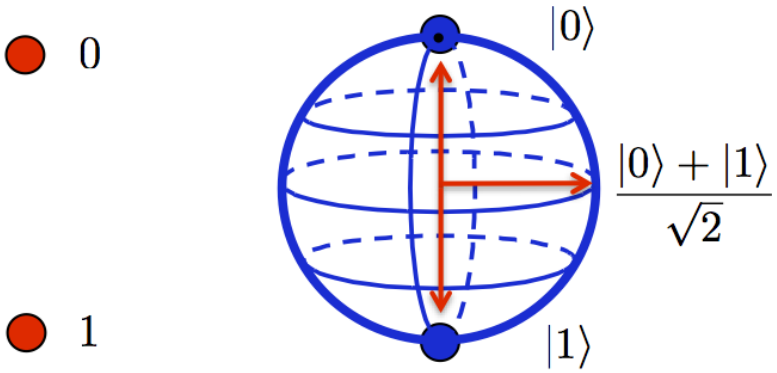
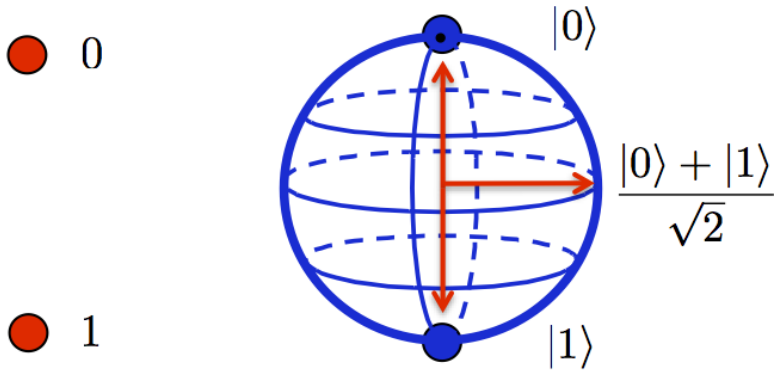


Classical Bit **Qubit**

$$a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Quantum Bits - Qubits



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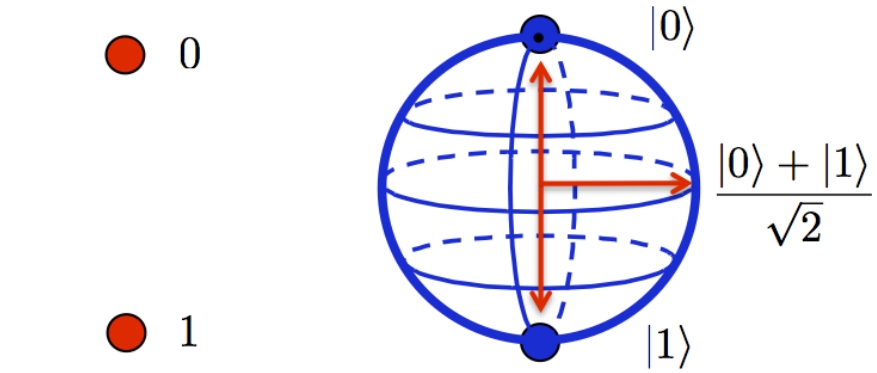
$$a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}0\\1\end{bmatrix} \qquad |a|^2 + |b|^2 = 1$$

Quantum Bits - Qubits



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$$\boxed{a\,|0\rangle + b\,|1\rangle}$$

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$\rightarrow$ linear transformations on one or many qubits.

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Example: Hadamard gate.

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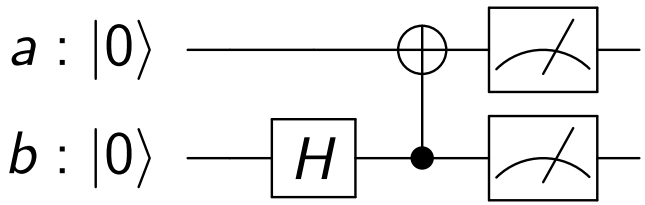

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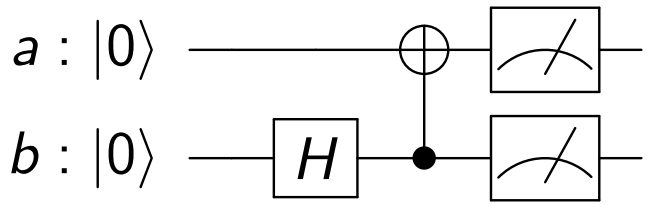

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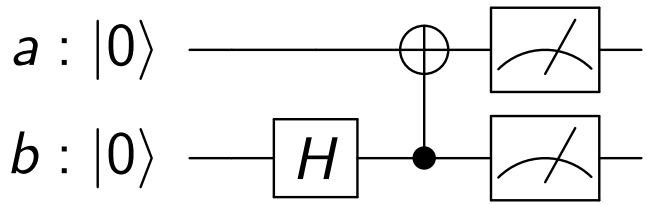

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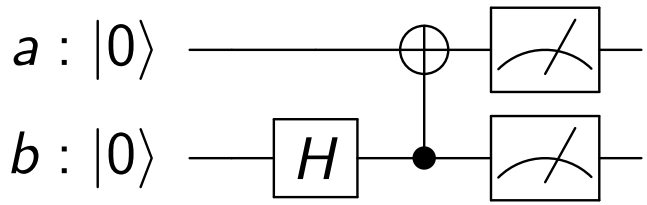

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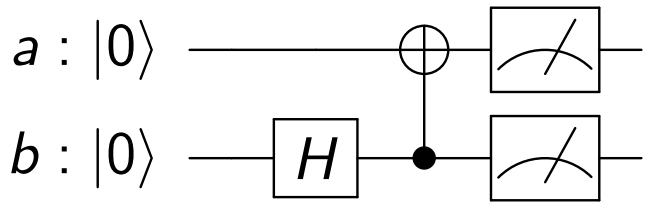

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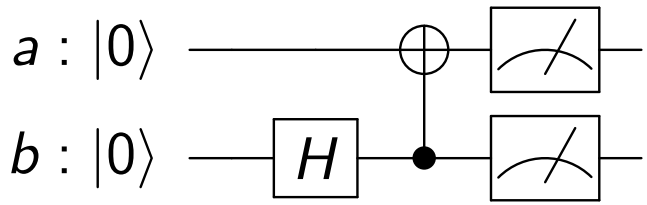

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$$H|0\rangle = H \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Multiple Qubits

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$$\text{Qubit 1: } a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix} \qquad \text{Qubit 2: } c|0\rangle + d|1\rangle = \begin{bmatrix} c \\ d \end{bmatrix}$$

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$$\text{Qubit 1: } a\,|0\rangle + b\,|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix} \qquad \text{Qubit 2: } c\,|0\rangle + d\,|1\rangle = \begin{bmatrix} c \\ d \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} \boxtimes \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a \begin{bmatrix} c \\ d \end{bmatrix} \\ b \begin{bmatrix} c \\ d \end{bmatrix} \end{bmatrix} = \begin{bmatrix} ac \\ ad \\ bc \\ bd \end{bmatrix}.$$

Multiple Qubits

Do this for the basis states

$$|0\rangle \boxtimes |0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad |0\rangle \boxtimes |1\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad |1\rangle \boxtimes |0\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad |1\rangle \boxtimes |1\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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Notation:

$$|00\rangle = |0\rangle \boxtimes |0\rangle \qquad |01\rangle = |0\rangle \boxtimes |1\rangle$$
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Can see that we have a basis for describing the combined state.

$$\begin{bmatrix} a \\ b \end{bmatrix} \boxtimes \begin{bmatrix} c \\ d \end{bmatrix} = ac\,|00\rangle + ad\,|01\rangle + bc\,|10\rangle + bd\,|11\rangle\,.$$

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Ex: $p(x) \cdot q(y)$ is a "combined state." But there are NO $p(x), q(y)$ s.t.

$$p(x) \cdot q(y) = xy + 1,$$

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Kronecker Product

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If

$$\mathcal{A} = \begin{bmatrix} m & n \\ o & p \end{bmatrix} \quad \text{and } \mathcal{B} = \begin{bmatrix} q & r & s \\ t & u & v \\ w & x & y \end{bmatrix}$$

Kronecker Product

then

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$$\mathcal{A} \otimes \mathcal{B} = \begin{bmatrix} m \begin{bmatrix} q & r & s \\ t & u & v \\ w & x & y \end{bmatrix} & n \begin{bmatrix} q & r & s \\ t & u & v \\ w & x & y \end{bmatrix} \\ o & p \end{bmatrix}$$

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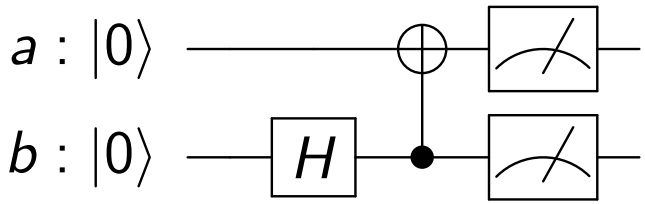
$$= \begin{bmatrix} mq & mr & ms & nq & nr & ns \\ mt & mu & mv & nt & nu & nv \\ mw & mx & ms & nw & nx & ny \\ oq & or & os & pq & pr & ps \\ ot & ou & ov & pt & pu & pv \\ ow & ox & oy & pw & px & py \end{bmatrix}$$

Kronecker Product

Check that $I\left|0\right\rangle \boxtimes H\left|0\right\rangle = (I \otimes H)\left|00\right\rangle$:



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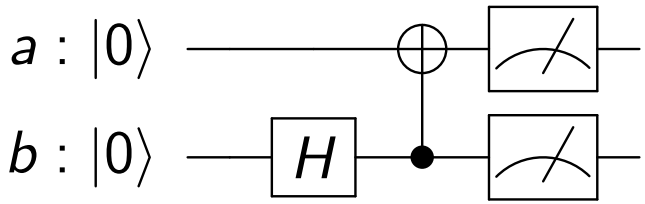

LHS:

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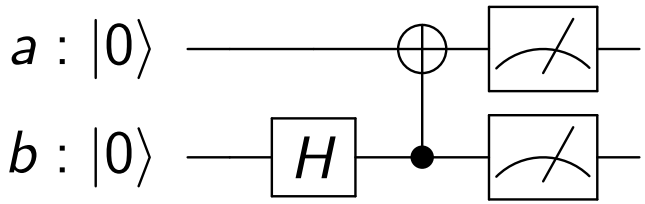

LHS:

$$I|0\rangle \boxtimes H|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \boxtimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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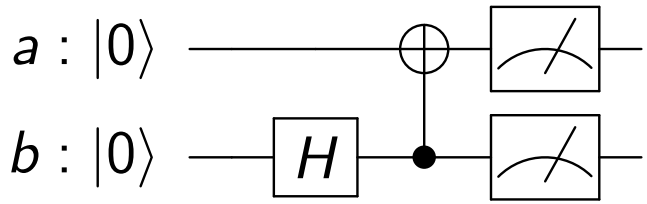

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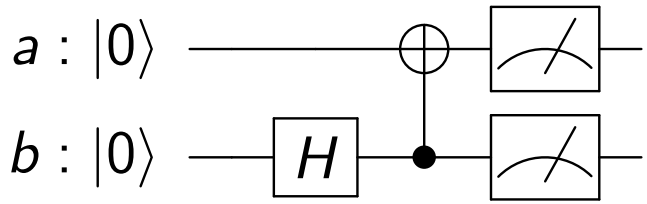

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$$
\begin{aligned}
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&= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \boxtimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
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$$

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$$(I \otimes H)(|00\rangle) = \begin{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & \mathcal{O} \\ \mathcal{O} & \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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Some properties & Elementariness revisited

$\otimes$ and $\boxtimes$ are very much alike.

1. Bilinear
2. Distributive.

$$\begin{bmatrix} a \\ b \end{bmatrix} \boxtimes \begin{bmatrix} c \\ d \end{bmatrix} = (a\,|0\rangle + b\,|1\rangle) \boxtimes (c\,|0\rangle + d\,|1\rangle)$$

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Some properties and Elementariness revisited

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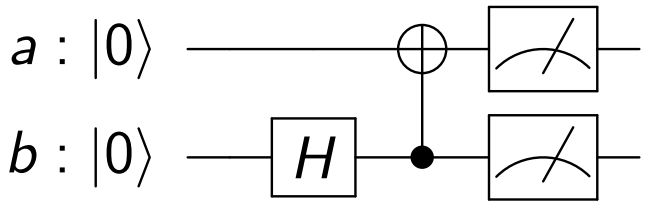
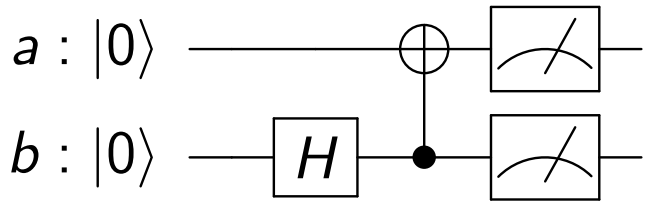
Ex:



Some properties and Elementariness revisited

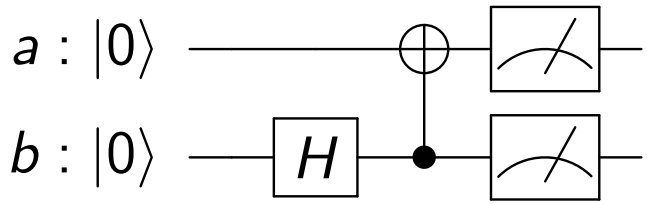
Ex:



The Control-NOT gate:

Some properties and Elementariness revisited

Ex:

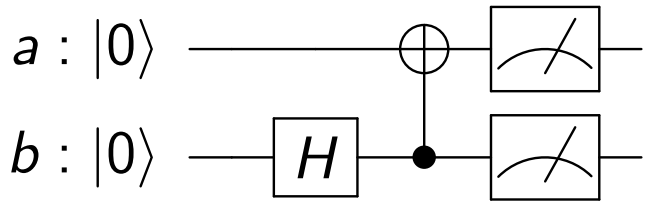


The Control-NOT gate:

$$CNOT_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Some properties and Elementariness revisited

Ex:



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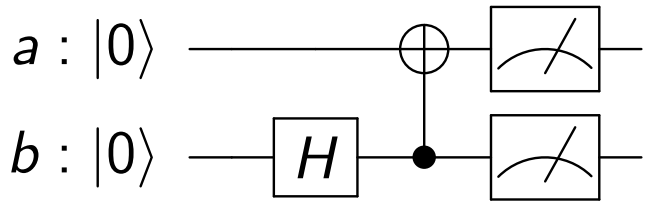

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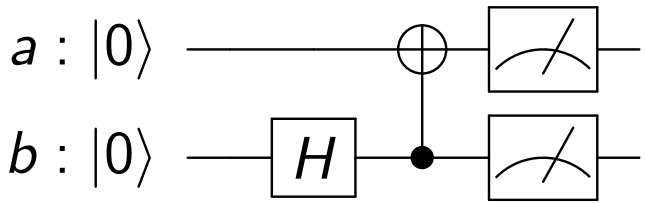
Also called "entangled."

Entanglement Circuit

Time to decode:



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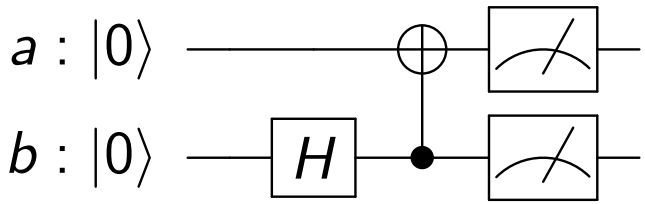

1. Step 1:

Entanglement Circuit

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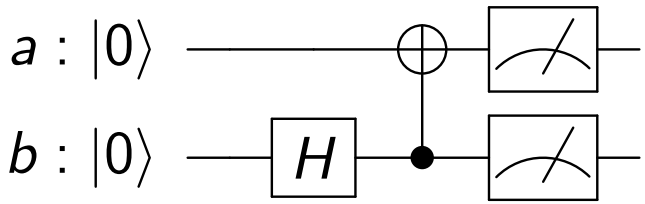

1. Step 1:

$$a : |0\rangle \to |0\rangle$$

$$b : |0\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|a'b'\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}^\top$$

Entanglement Circuit

2 Step 2:

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$$CNOT_b \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

Entanglement Circuit

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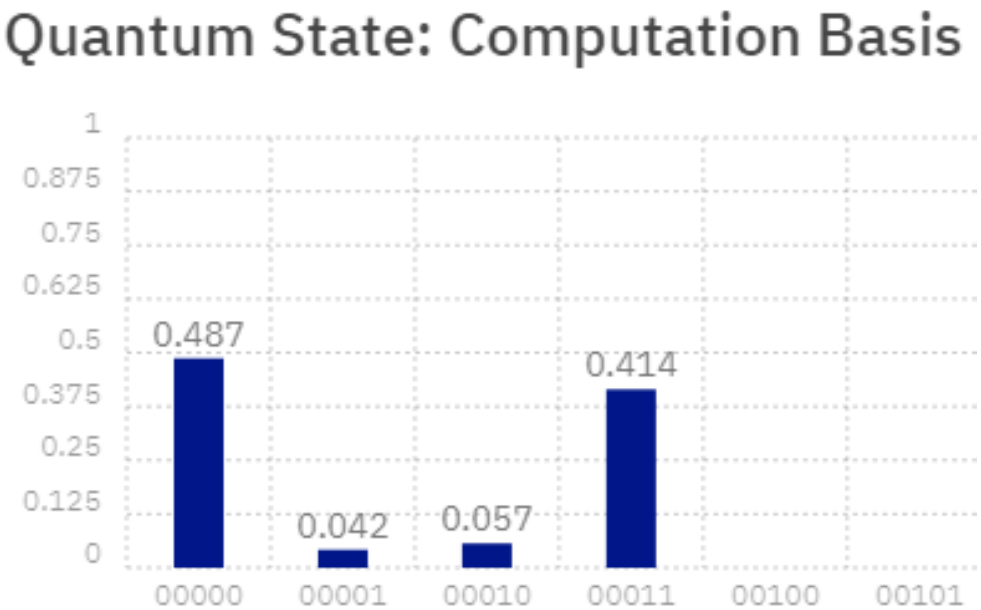
which is:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle \leftarrow \textbf{Entangled}$$

Simulation on IBM-Q



Tensor Product

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Why tensor product?

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Postulate (QM):

The state space of a composite physical system is the *tensor product* of the state spaces of the component physical systems.

Tensor Product

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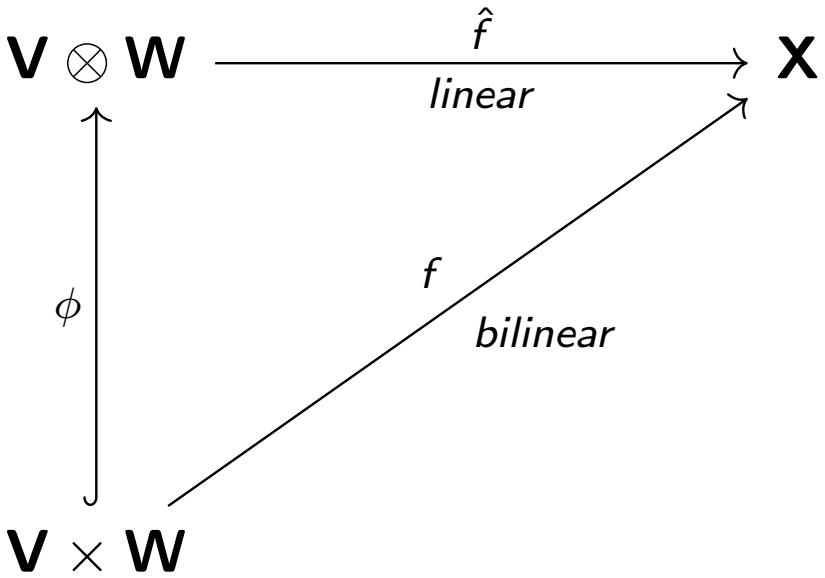

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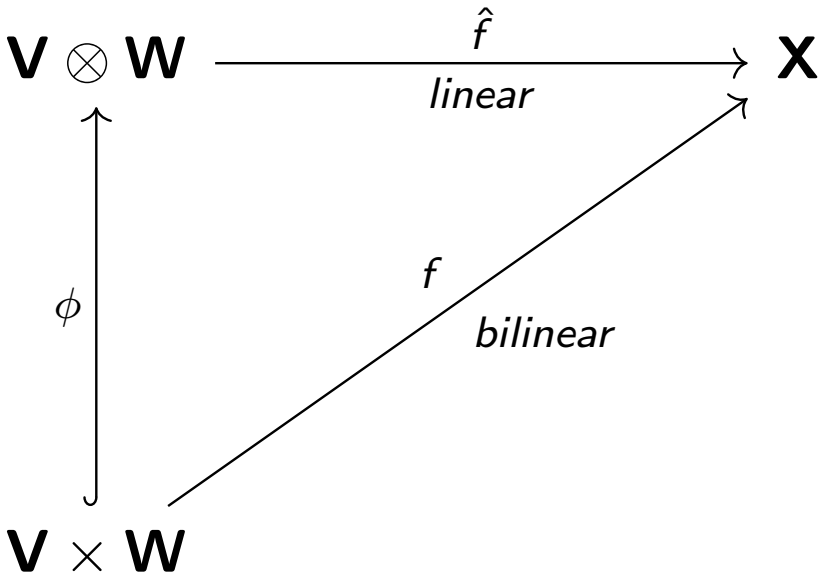

Roughly speaking...

Giving the $\hat{f} : \mathbf{V} \otimes \mathbf{W} \overset{\text{linear}}{\longrightarrow} \mathbf{X}$ is the same as giving $f : \mathbf{V} \times \mathbf{W} \overset{\text{bilinear}}{\longrightarrow} \mathbf{X}$.
$f = \hat{f} \circ \phi$

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If the target space $\mathbf{X}$ is $\mathbf{V} \otimes \mathbf{W}$. $\mathcal{L}$ is an operator on $\mathbf{V}$, $\mathcal{M}$ on $\mathbf{W}$,

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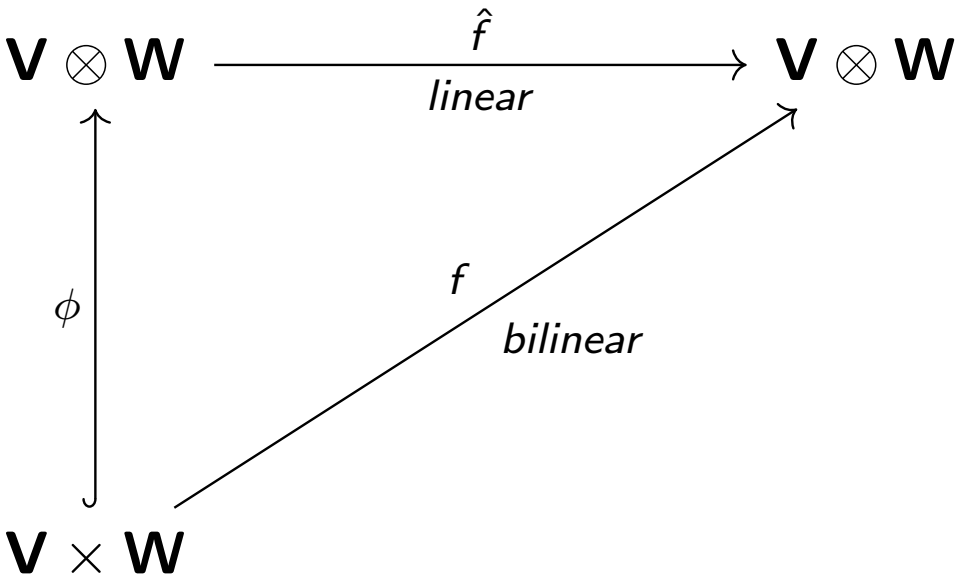

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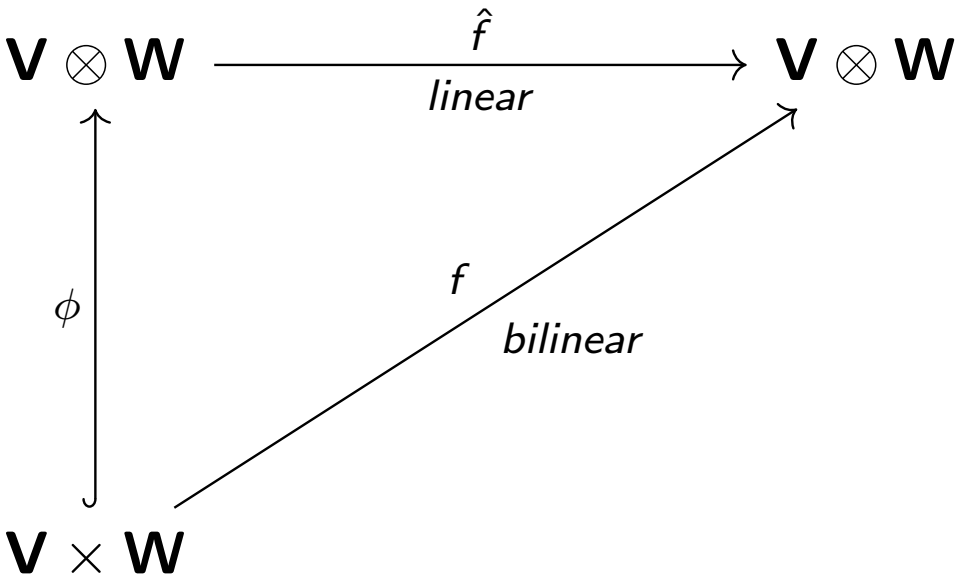

$$\mathcal{L}[v] \otimes \mathcal{M}[w]$$

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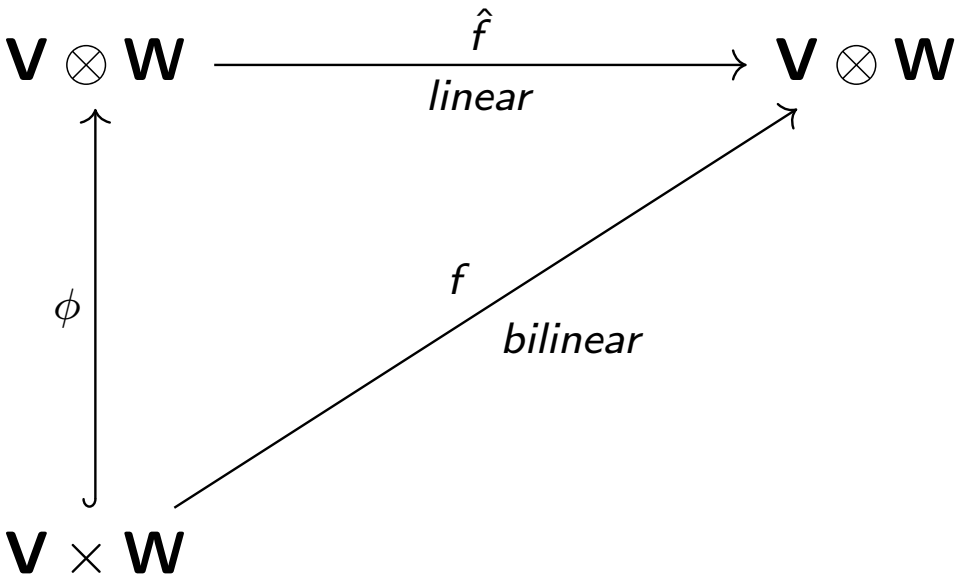

$$(\mathcal{L} \otimes \mathcal{M})(v \otimes w) \qquad \mathcal{L}[v] \otimes \mathcal{M}[w]$$

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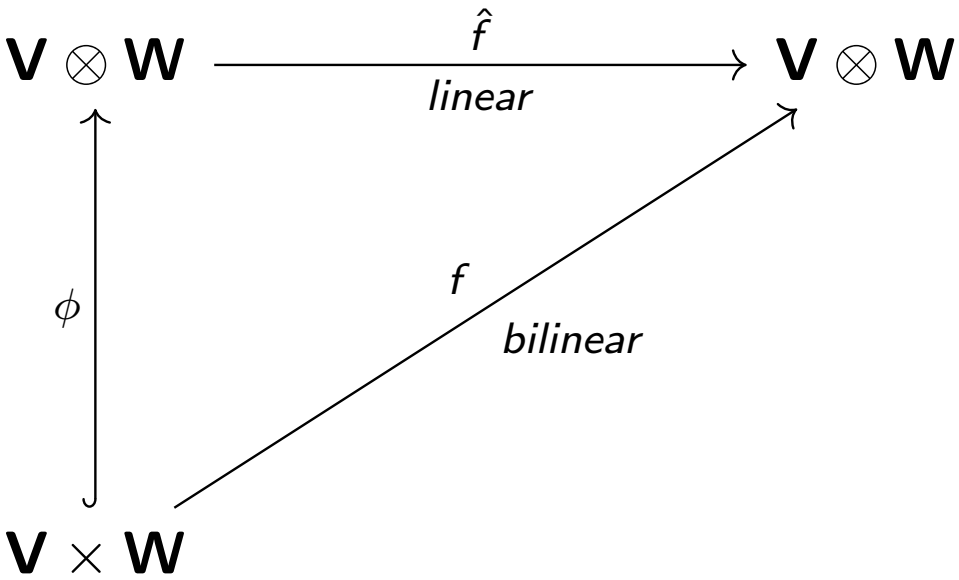

$\rightarrow$ by uniqueness

$$(\mathcal{L} \otimes \mathcal{M})(v \otimes w) = \mathcal{L}[v] \otimes \mathcal{M}[w]$$

Tensor Product & Kronecker Product

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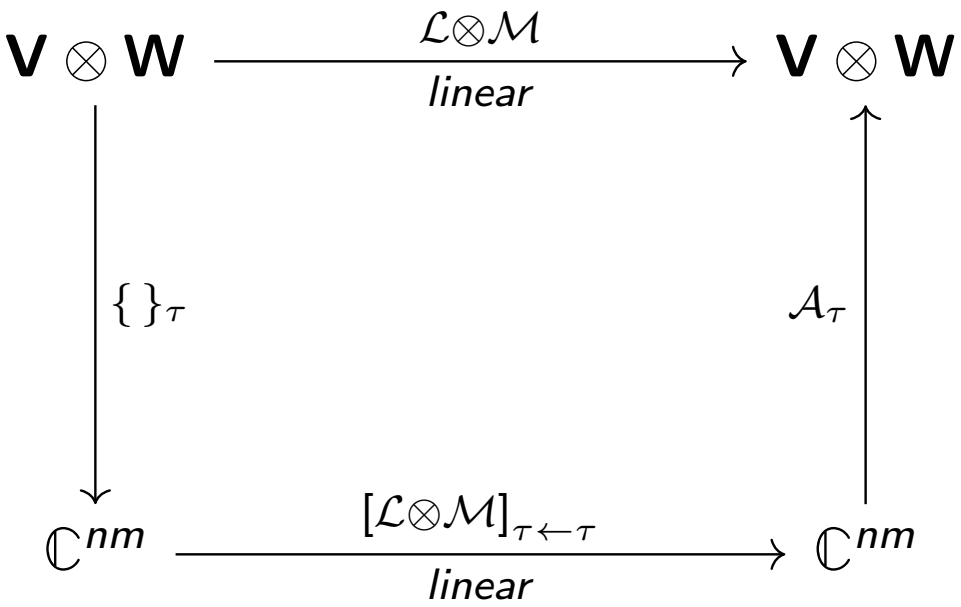

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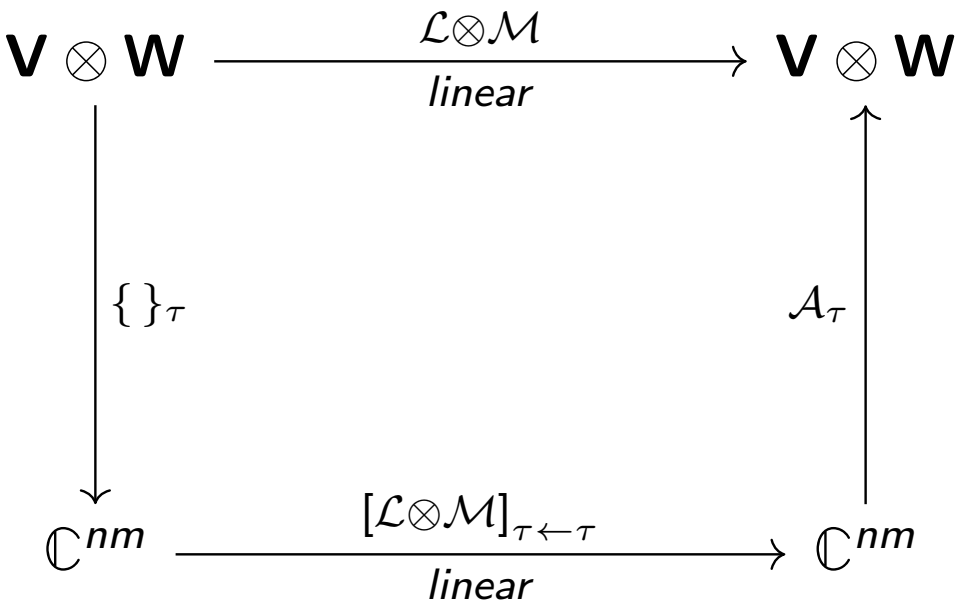

$$\boxed{[\mathcal{L} \otimes \mathcal{M}]_{\tau \leftarrow \tau} = [\mathcal{L}]_{\nu \leftarrow \nu} \otimes [\mathcal{M}]_{\omega \leftarrow \omega}}$$

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References

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