Measurement-based Quantum Computing & Efficient variational simulation of non-trivial quantum states

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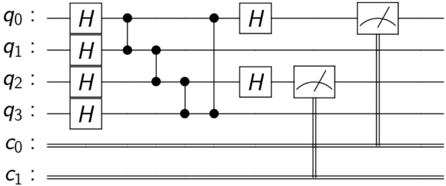
Layout

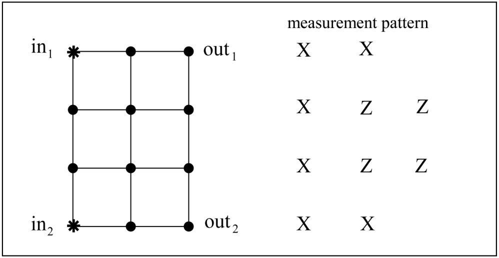
- Measurement-based quantum computing (MBQC)
- Variational simulation of non-trivial quantum state
- Research question: MBQC as an efficient simulation?

MBQC: One-way quantum computer [RB01]

Conventional quantum circuit models:

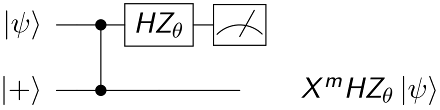


Cluster state: [Joz06]



MBQC: One-way quantum computer

Quantum teleportation = Entanglement + Measurement



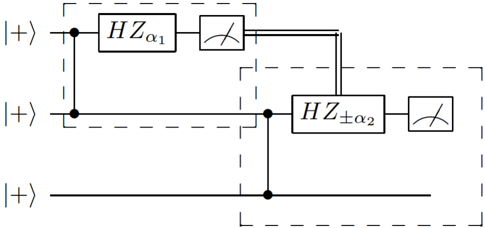
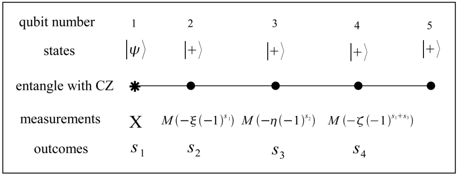


Figure: From [Nie06]

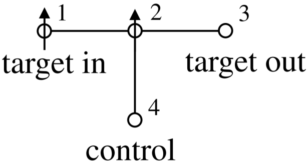
MBQC: One-way quantum computer

Universality: Quantum circuit model $\equiv$ Cluster state formulation

- Transfer of information by teleportation
- Any qubit rotation can be done on a chain of qubits
- The CNOT gate can be implemented in a “T” configuration



(a) From [Joz06]



(b) From [RB01]

Variational simulation of non-trivial quantum states

QAOA [FGG14]: Quantum approximate optimization algorithm

- Principle: Quantum adiabatic theorem on $H = H_2 + H_1$
- Variational ansatz:

$$\boxed{|\psi(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle = \underbrace{e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2}}_{p \text{ layers}} |\psi_1\rangle}$$

  - $(\boldsymbol{\gamma},\boldsymbol{\beta}) = (\gamma_p,\dots,\gamma_1,\beta_p,\dots,\beta_1)$
  - $|\psi_1\rangle$ = ground state of H_1 (easy to prepare)
- Cost function:

Overlap: $|\langle\psi_0|\psi(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle|^2$, or Energy: $\langle\psi(\boldsymbol{\gamma},\boldsymbol{\beta})|\, H \,|\psi(\gamma,\beta)\rangle$.

Variational simulation of non-trivial quantum states

Example: GHZ state $\sim |0\rangle^{\otimes L} + |1\rangle^{\otimes L}$

$$H_{GHZ} = -\sum_{i=1}^{T} Z_i Z_{i+1} = \underbrace{-\sum_{i=1}^{T} Z_i Z_{i+1}}_{H_2} - 0 \underbrace{\sum_{i=1}^{L} X_i}_{H_1}, \qquad |GS_{H_1}\rangle = \bigotimes_{i=1}^{L} |+\rangle$$

$\Longrightarrow$ Perfect fidelity, $p \sim L/2$.

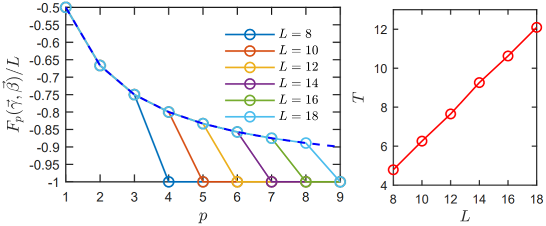


Figure: GHZ state simulation. Fidelity & p vs. L, [HH19]

Variational simulation of TFIM ground state

Example: Transverse field Ising model

$$H := H_2 + H_1 = -\sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=i}^{L} X_i$$

$\implies$ Perfect fidelity, $p \sim L/2$

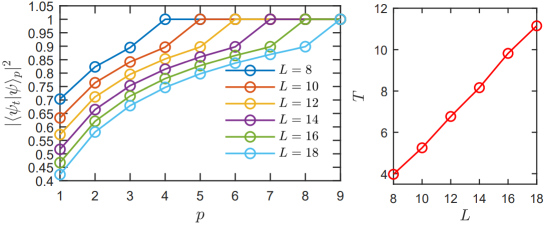


Figure: TFIM state simulation. Fidelity & p vs. L, [HH19]

Variational simulation of TFIM ground state

Limitation of protocol in [HH19]:

- $p \sim L$
- Requires non-local unitaries.
- MERA construction [Vid08]: $p \sim \log(L)$, but non-local unitaries required.

$\implies$ Is there a way out?

Measurement-based QAOA for TFIM

QAOA ansatz:

$$|(\gamma, \boldsymbol{\beta})\rangle =$$

Can we do better?

How robust is QAOA?

Consider the TFIM without translation invariance:

$$\mathcal{H} = \sum_j Z_j Z_{j+1} + \sum_j g_j X_j$$

Summary

References

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- Robert Raussendorf and Hans J. Briegel, *A one-way quantum computer*, Phys. Rev. Lett. **86** (2001), 5188–5191.
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