Some potentially useful information

- Euler-Lagrange equations for generalized coordinates q_j

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = \sum_\alpha \lambda_\alpha \frac{\partial f_\alpha}{\partial q_j}\ , \qquad \text{or} \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = \sum_\beta \lambda_\beta \frac{\partial g_\beta}{\partial \dot{q}_j}$$

  constraints: holonomic $f_\alpha(q,t) = 0$ or semiholonomic $g_\beta = \sum_j a_{\beta j}(q,t)\dot{q}_j + a_{\beta t}(q,t) = 0$

- Generalized forces: $d/dt(\partial L/\partial \dot{q}_j) - \partial L/\partial q_j = R_j$

  Friction forces: $\vec{f}_i = -h(v_i)\vec{v}_i/v_i$, $\vec{v}_i = \dot{\vec{r}}_i$ gives $R_j = -\partial \mathcal{F}/\partial \dot{q}_j\ ,\ \ \mathcal{F} = \sum_i \int_0^{v_i} dv_i'\, h(v_i')$

- Hamilton's equations for canonical variables (q_j, p_j): $\quad \dot{q}_j = \dfrac{\partial H}{\partial p_j}\ , \quad \dot{p}_j = -\dfrac{\partial H}{\partial q_j}$

- Hamiltonian for a Lagrangian quadratic in velocities

$$L = L_0(q,t) + \dot{\vec{q}}^T \cdot \vec{a} + \frac{1}{2}\dot{\vec{q}}^T \cdot \hat{T} \cdot \dot{\vec{q}} \quad \Rightarrow \quad H = \frac{1}{2}(\vec{p} - \vec{a})^T \cdot \hat{T}^{-1} \cdot (\vec{p} - \vec{a}) - L_0(q,t)$$

- The Moment of Inertia Tensor and its relations:

$$I_{ab} = \int dV\, \rho(\vec{r})[\vec{r}^{\,2}\delta_{ab} - r_a r_b] \qquad \text{or} \qquad I^{ab} = \sum_i m_i[\delta^{ab}\vec{r}_i^{\,2} - r_i^a r_i^b]$$

$$I_{ab}^{(Q)} = M(\delta_{ab}\,\vec{R}^2 - R_a R_b) + I_{ab}^{(\mathrm{CM})}\ , \qquad \hat{I}' = \hat{U}\,\hat{I}\,\hat{U}^T$$

- Euler's Equations:

$$I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = \tau_1$$
$$I_2\dot{\omega}_2 - (I_3 - I_1)\omega_3\omega_1 = \tau_2$$
$$I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 = \tau_3$$

- Vibrations: $L = \frac{1}{2}\dot{\vec{\eta}}^T \cdot \hat{T} \cdot \dot{\vec{\eta}} - \frac{1}{2}\vec{\eta}^T \cdot \hat{V} \cdot \vec{\eta}$ has Normal modes $\vec{\eta}^{(k)} = \vec{a}^{(k)}\exp(-i\omega^{(k)}t)$

$$\det(\hat{V} - \omega^2\hat{T}) = 0\ , \qquad \left(\hat{V} - [\omega^{(k)}]^2\,\hat{T}\right) \cdot \vec{a}^{(k)} = 0\ , \qquad \vec{\eta} = \mathrm{Re}\sum_k C_k\,\vec{\eta}^{(k)}$$

- Generating functions for Canonical Transformations: $K = H + \partial F_i/\partial t$ and

$$F_1(q,Q,t): \quad p_i = \frac{\partial F_1}{\partial q_i}\ ,\ P_i = -\frac{\partial F_1}{\partial Q_i}\ , \qquad F_2(q,P,t): \quad p_i = \frac{\partial F_2}{\partial q_i}\ ,\ Q_i = \frac{\partial F_2}{\partial P_i}$$

- Poisson Brackets: $\quad [u,v]_{q,p} = \displaystyle\sum_j \left[\frac{\partial u}{\partial q_j}\frac{\partial v}{\partial p_j} - \frac{\partial u}{\partial p_j}\frac{\partial v}{\partial q_j}\right]\ , \qquad \frac{du}{dt} = [u,H] + \frac{\partial u}{\partial t}$

- Relations for Hamilton's Principle function, $S = S(q_1,\ldots,q_n;\alpha_1,\ldots,\alpha_n,t)$

$$K = 0\ , \qquad P_i = \alpha_i\ , \qquad Q_i = \beta_i = \frac{\partial S}{\partial \alpha_i}\ , \qquad p_i = \frac{\partial S}{\partial q_i}$$

- Relations for Hamilton's Characteristic function, $W = W(q_1,\ldots,q_n;\alpha_1,\ldots,\alpha_n)$

$$K = H = \alpha_1\ , \qquad P_i = \alpha_i\ , \qquad \beta_1 + t = \frac{\partial W}{\partial \alpha_1}\ , \qquad \beta_{i>1} = \frac{\partial W}{\partial \alpha_i}\ , \qquad p_i = \frac{\partial W}{\partial q_i}$$

- Action Angle Variables: $\quad J = \displaystyle\oint p\,dq\ , \qquad w = \frac{\partial W(q,J)}{\partial J}\ , \qquad \dot{w} = \frac{\partial H(J)}{\partial J} = \nu(J)$