①

Ramsey i_2 + Stark shift Derivation

Interaction Representation

Jun 25, 2018

$$\boxed{i\begin{pmatrix}\dot a_i(t)\\ \dot a_f(t)\end{pmatrix}=\begin{pmatrix}\Delta_i & -\frac{\Omega_0^*}{2}e^{-i\Delta_0 t}\\ -\frac{\Omega_0}{2}e^{+i\Delta_0 t} & \Delta_f\end{pmatrix}\begin{pmatrix}a_i(t)\\ a_f(t)\end{pmatrix}}$$

$$i\ddot a_i = \Delta_i \dot a_i - \frac{\Omega_0^*}{2}e^{-i\Delta_0 t}\dot a_f + \frac{i\Delta_0}{2}\Omega_0^* e^{-i\Delta_0 t} a_f(t)$$

<u>where</u> $a_f \cdot \left(\frac{\Omega_0^*}{2}e^{-i\Delta_0 t}\right) = \Delta_i a_i(t) - i\dot a_i(t)$

$$\rightarrow a_f = \left[\Delta_i a_i(t) - i\dot a_i(t)\right]\left(\frac{2}{\Omega_0^*}e^{i\Delta_0 t}\right)$$

<u>Also</u>

$$i\dot a_f = -\frac{\Omega_0}{2}e^{i\Delta_0 t}a_i(t) + \Delta_f a_f(t)$$

$$\Rightarrow i\dot a_f = -\frac{\Omega_0}{2}e^{i\Delta_0 t}a_i(t) + \Delta_f\left[\Delta_i a_i(t) - i\dot a_i(t)\right]\left(\frac{2}{\Omega_0^*}e^{i\Delta_0 t}\right)$$

$$\Rightarrow \dot a_f = \frac{i\Omega_0}{2}e^{i\Delta_0 t}a_i - i\Delta_f\frac{2}{\Omega_0^*}e^{i\Delta_0 t}\left[\Delta_i a_i - i\dot a_i\right]$$

$$= \frac{2i}{\Omega_0^*}\left\{\frac{|\Omega_0|^2}{4}a_i - \Delta_f\Delta_i a_i + i\Delta_f \dot a_i\right\}e^{+i\Delta_0 t}$$

So $$\ddot a_i = -i\Delta_i \dot a_i + \frac{i\Omega_0^*}{2}e^{-i\Delta_0 t}\left[\frac{2i}{\Omega_0^*}e^{i\Delta_0 t}\left\{\frac{|\Omega_0|^2}{4}a_i - \Delta_f\Delta_i a_i + i\Delta_f\dot a_i\right\}\right]$$
$$+ \frac{i\Delta_0}{2}\Omega_0^* e^{-i\Delta_0 t}\frac{2}{\Omega_0^*}e^{i\Delta_0 t}\left[\Delta_i a_i(t) - i\dot a_i(t)\right]$$

So $$\ddot a_i = -i\Delta_i\dot a_i - \frac{|\Omega_0|^2}{4}a_i + \Delta_f\Delta_i a_i - i\Delta_f\dot a_i + \Delta_0\Delta_i a_i(t) - i\Delta_0\dot a_i(t)$$

So $$\boxed{\ddot a_i = -i\left[\Delta_0 + \Delta_i + \Delta_f\right]\dot a_i(t) - \left(\frac{|\Omega_0|^2}{4} - \Delta_i(\Delta_f + \Delta_0)\right)a_i}$$

②

$$\ddot{a}_i + i(\Delta_0+\Delta_i+\Delta_f)\dot{a}_i + \frac{1}{4}\left(|\Omega_0|^2 - 4\Delta_i\Delta_f - 4\Delta_i\Delta_0\right)a_i = 0$$

2nd order HMG ODE

Guess solution $a_i(t) = a_0 e^{i\omega t} \rightarrow \dot{a}_i = a_0 i\omega e^{i\omega t}$

$\ddot{a}_i = -\omega^2 e^{i\omega t} a_0$

Characteristic poly

$$-\omega^2 + i(\Delta_0+\Delta_i+\Delta_f)(i\omega) + \frac{1}{4}\left(|\Omega_0|^2 - 4\Delta_i\Delta_0 - 4\Delta_i\Delta_f\right) = 0$$

or $$-\omega^2 - (\Delta_0+\Delta_i+\Delta_f)\omega + \frac{1}{4}\left(|\Omega_0|^2 - 4\Delta_i\Delta_0 - 4\Delta_i\Delta_f\right) = 0$$

$$\boxed{\omega = \frac{-(\Delta_0+\Delta_i+\Delta_f)}{2} \pm \frac{1}{2}\sqrt{(\Delta_0+\Delta_i+\Delta_f)^2 + |\Omega_0|^2 - 4\Delta_i\Delta_0 - 4\Delta_i\Delta_f}}$$

Let $\bar{\Delta} = \frac{\Delta_i+\Delta_f}{2}$

$$(\Delta_0+\Delta_i+\Delta_f)^2 + |\Omega_0|^2 - 4\Delta_i\Delta_0 - 4\Delta_i\Delta_f$$
$$= \Delta_0^2 + \Delta_i^2 + \Delta_f^2 + 2\Delta_0\Delta_i + 2\Delta_i\Delta_f + 2\Delta_0\Delta_f + |\Omega_0|^2$$
$$- 4\Delta_i\Delta_0 - 4\Delta_i\Delta_f$$

$$= \Delta_0^2 + |\Omega_0|^2 + (\Delta_i - \Delta_f)^2 - 2\Delta_0(\Delta_i - \Delta_f)$$

So $$\boxed{\omega = \frac{-(\Delta_0+\Delta_i+\Delta_f)}{2} \pm \frac{1}{2}\sqrt{\Delta_0^2 + |\Omega_0|^2 + (\Delta_i - \Delta_f)^2 - 2\Delta_0(\Delta_i - \Delta_f)}}$$

Define $\Delta_d = \Delta_f - \Delta_i \Rightarrow \omega = \frac{-(\Delta_0+\Delta_i+\Delta_f)}{2} \pm \frac{1}{2}\sqrt{|\Omega_0|^2 + (\Delta_0+\Delta_d)^2}$

3

So $$\boxed{\omega = -\frac{\Delta_0}{2} - \bar{\Delta} \pm \frac{1}{2}\sqrt{|\Omega_0|^2 + (\Delta_0 + \Delta_d)^2}}$$

where $\bar{\Delta} = \frac{\Delta_f + \Delta_i}{2}$, $\Delta_d = \frac{\Delta_f - \Delta_i}{2}$

Define $\Omega' = \sqrt{|\Omega_0|^2 + (\Delta_0 + \Delta_d)^2}$

So $$a_i(t) = a_+ e^{i\omega_+ t} + a_- e^{+i\omega_- t}$$

$$= e^{-i\bar{\Delta}t} e^{-i\frac{\Delta_0}{2}t} \left\{ a_+ e^{\frac{i\Omega'}{2}t} + a_- e^{-\frac{i\Omega'}{2}t} \right\}$$

OR $$\boxed{a_i(t) = e^{-i\bar{\Delta}t} e^{-i\frac{\Delta_0}{2}t} \left\{ A\cos\left(\frac{\Omega' t}{2}\right) + B\sin\left(\frac{\Omega' t}{2}\right) \right\}}$$

Next, solve for $a_f(t)$...

$$\boxed{0} \quad a_f(t) = \frac{2}{\Omega_0^*} e^{i\Delta_0 t}\left(\Delta_i a_i - i\dot{a}_i\right)$$

$$= \frac{2}{\Omega_0^*} e^{i\Delta_0 t} \left\{ \Delta_i e^{-i\bar{\Delta}t} e^{-i\frac{\Delta_0}{2}t} \left[A\cos\left(\frac{\Omega' t}{2}\right) + B\sin\left(\frac{\Omega' t}{2}\right) \right] - i\dot{a}_i(t) \right\}$$

where

$$-i\dot{a}_i(t) = -i\left\{ -i\left(\bar{\Delta} + \frac{\Delta_0}{2}\right) e^{-i\bar{\Delta}t} e^{-i\Delta_0 t/2} \left[A\cos\left(\frac{\Omega' t}{2}\right) + B\sin\left(\frac{\Omega' t}{2}\right) \right] + \frac{\Omega'}{2} e^{-i\bar{\Delta}t} e^{-i\frac{\Delta_0}{2}t} \left[-A\sin\left(\frac{\Omega' t}{2}\right) + B\cos\left(\frac{\Omega' t}{2}\right) \right] \right\}$$

$$= e^{-i\bar{\Delta}t} e^{-i\Delta_0 t/2} \left\{ -\left(\bar{\Delta} + \frac{\Delta_0}{2}\right)\left(A\cos\left(\frac{\Omega' t}{2}\right) + B\sin\left(\frac{\Omega' t}{2}\right)\right) + i\frac{\Omega'}{2}\left(A\sin\left(\frac{\Omega' t}{2}\right) - B\cos\left(\frac{\Omega' t}{2}\right)\right) \right\}$$

④

So $a_f(t) = ?\ ,\ a_i(t) = ?$

Let $t=0 \Rightarrow A = a_i(0)$

$$\Rightarrow B = \frac{i\Omega_0^*}{\Omega'} a_f(0) + i\left(\frac{\Delta_d + \Delta_0}{\Omega'}\right) a_i(0)$$

So $a_i(t) = e^{-i\bar{\Delta}t} e^{-i\frac{\Delta_0}{2}t}\left\{ a_i(0)\cos\left(\frac{\Omega' t}{2}\right) + \left(\frac{i(\Delta_0+\Delta_d)}{\Omega'} a_i(0) + \frac{i\Omega_0^*}{\Omega'} a_f(0)\right)\sin\left(\frac{\Omega' t}{2}\right)\right\}$

$$\text{So}\quad \boxed{a_i(t) = e^{-i\bar{\Delta}t} e^{-i\frac{\Delta_0}{2}t}\left\{ a_i(0)\left[\cos\left(\frac{\Omega' t}{2}\right) + i\left(\frac{\Delta_0+\Delta_d}{\Omega'}\right)\sin\left(\frac{\Omega' t}{2}\right)\right] + a_f(0)\cdot\left[\left(\frac{i\Omega_0^*}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\right)\right]\right\}}$$

$$a_f(t) = \frac{2}{\Omega_0^*} e^{-i\bar{\Delta}t} e^{i\frac{\Delta_0}{2}t}\left\{\left(a_i(0)\cos\frac{\Omega' t}{2} + \left(\frac{i\Omega_0^*}{\Omega'} a_f(0) + i\left(\frac{\Delta_d+\Delta_0}{\Omega'}\right) a_i(0)\right)\sin\left(\frac{\Omega' t}{2}\right)\right)\Delta_i\right.$$
$$+ \left(-\bar{\Delta} - \frac{\Delta_0}{2}\right)\left(a_i(0)\cos\frac{\Omega' t}{2} + \left(\frac{i\Omega_0^*}{\Omega'} a_f(0) + \frac{i}{2}\left(\frac{\Delta_d+\Delta_0}{\Omega'}\right) a_i(0)\right)\sin\left(\frac{\Omega' t}{2}\right)\right)$$
$$\left. + i\frac{\Omega'}{2}\left(a_i(0)\sin\left(\frac{\Omega' t}{2}\right) - \left(\frac{i\Omega_0^*}{\Omega'} a_f(0) + \frac{i}{2}\left(\frac{\Delta_d+\Delta_0}{\Omega'}\right) a_i(0)\right)\cos\left(\frac{\Omega' t}{2}\right)\right)\right.$$

$$= \frac{2}{\Omega_0^*} e^{-i\bar{\Delta}t} e^{i\frac{\Delta_0}{2}t}\left\{\left(A\cos\left(\frac{\Omega' t}{2}\right) + B\sin\left(\frac{\Omega' t}{2}\right)\right)\left[\Delta_i - \bar{\Delta} - \frac{\Delta_0}{2}\right] + \frac{i\Omega'}{2}\left(A\sin\left(\frac{\Omega' t}{2}\right) - B\cos\left(\frac{\Omega' t}{2}\right)\right)\right\}$$

Now $\Delta_i - \bar{\Delta} = \Delta_i - \frac{\Delta_i + \Delta_f}{2} = \frac{-\Delta_d}{2}$

So $$a_f(t) = e^{-i\bar{\Delta}t + i\frac{\Delta_0}{2}t}\left\{ A\left(\frac{-(\Delta_d+\Delta_0)}{\Omega_0^*}\cos\left(\frac{\Omega' t}{2}\right) + \frac{i\Omega'}{\Omega_0^*}\sin\left(\frac{\Omega' t}{2}\right)\right) - B\left(\frac{+(\Delta_d+\Delta_0)}{\Omega_0^*}\sin\left(\frac{\Omega' t}{2}\right) + \frac{i\Omega'}{\Omega_0^*}\cos\left(\frac{\Omega' t}{2}\right)\right)\right\}$$

So

$$a_f(t) = e^{-i\bar{\Delta}t} e^{i\frac{\Delta_0}{2}t} \Big\{ a_i(0)\left[-\frac{(\Delta_d+\Delta_0)}{\Omega_0^*}\cos\left(\frac{\Omega' t}{2}\right) + \frac{i\Omega'}{\Omega_0^*}\sin\left(\frac{\Omega' t}{2}\right)\right]$$

$$- \left(\frac{i\Omega_0^*}{\Omega'} a_f(0) + \frac{i(\Delta_d+\Delta_0)}{\Omega'} a_i(0)\right)\cdot\left(\frac{(\Delta_d+\Delta_0)}{\Omega_0^*}\sin\left(\frac{\Omega' t}{2}\right) + \frac{i\Omega'}{\Omega_0^*}\cos\left(\frac{\Omega' t}{2}\right)\right)\Big\}$$

$$= e^{-i\bar{\Delta}t} e^{i\frac{\Delta_0}{2}t} \Big\{ a_i(0)\left[\sin\left(\frac{\Omega' t}{2}\right)\left(\frac{i\Omega'^2}{\Omega_0^*\Omega'} - \frac{i(\Delta_d+\Delta_0)^2}{\Omega_0^*\Omega'}\right)\right]$$

$$+ a_f(0)\left[\cos\left(\frac{\Omega' t}{2}\right) - i\left(\frac{\Delta_d+\Delta_0}{\Omega'}\right)\sin\left(\frac{\Omega' t}{2}\right)\right]\Big\}$$

Note $\Omega'^2 = (\Delta_0+\Delta_d)^2 + |\Omega_0|^2 \Rightarrow \Omega'^2 - (\Delta_d+\Delta_0)^2 = |\Omega_0|^2 = \Omega_0\Omega_0^*$

So

$$\boxed{a_f(t) = e^{-i\bar{\Delta}t} e^{i\frac{\Delta_0}{2}t}\left\{ a_i(0)\sin\left(\frac{\Omega' t}{2}\right)\cdot\left(\frac{i\Omega_0}{\Omega'}\right) + \underbrace{\left(\cos\left(\frac{\Omega' t}{2}\right) - \frac{i(\Delta_d+\Delta_0)}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\right)}_{a_f(0)}\right\}}$$

So finally...

$e^{i\omega t_1}$?

$$\boxed{\begin{pmatrix} a_i(t) \\ a_f(t)\end{pmatrix} = e^{-i\bar{\Delta}t}\begin{pmatrix} e^{-i\frac{\Delta_0}{2}t}\left[\cos\left(\frac{\Omega' t}{2}\right) + i\left(\frac{\Delta_0+\Delta_d}{\Omega'}\right)\sin\left(\frac{\Omega' t}{2}\right)\right] & e^{-i\frac{\Delta_0}{2}t}\frac{i\Omega_0^*}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right) \\ e^{i\frac{\Delta_0}{2}t}\frac{i\Omega_0}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right) & e^{i\frac{\Delta_0}{2}t}\left[\cos\left(\frac{\Omega' t}{2}\right) - \frac{i(\Delta_d+\Delta_0)}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\right]\end{pmatrix}\begin{pmatrix} a_i(0) \\ a_f(0)\end{pmatrix}}$$

Consistent with Ramsey? Yes! (Weird...)

$$\boxed{\begin{pmatrix} C_p \\ C_q \end{pmatrix} = \begin{pmatrix} \left(i\cos\theta\sin\frac{aT}{2} + \cos\frac{aT}{2}\right)e^{iT(w/2-(W_q+W_p)/2\hbar)} & \left(i\sin\theta\sin\frac{aT}{2}\right)e^{iT\left(\frac{1}{2}w - \frac{W_q+W_p}{2\hbar}\right)} \\ \left(i\sin\theta\sin\frac{aT}{2}\right)e^{\left(\frac{-1}{2}w - \frac{W_q+W_p}{2\hbar}\right)iT} & \left(-i\cos\theta\sin\frac{aT}{2} + \cos\left(\frac{aT}{2}\right)\right)e^{iT\left(\frac{-w}{2} - \frac{W_p+W_q}{2\hbar}\right)}\end{pmatrix}\begin{pmatrix} C_p(0) \\ C_q(0)\end{pmatrix}}$$

Correspondences

$$\bar{\Delta} \equiv \frac{W_q + W_p}{2\hbar} \qquad a = \sqrt{(\omega_0-\omega)^2 + (2b)^2} \equiv \Omega' = \sqrt{(\Delta_0+\Delta_d)^2 + |\Omega_0|^2}$$

$$\omega \equiv -\Delta_0 \qquad \omega_0 - \omega \equiv \Delta_0 + \Delta_d \quad \underline{so} \quad \omega_0 \equiv \Delta_d = (\Delta_f - \Delta_i)/2$$

$\hookrightarrow \left(\frac{W_q - W_p}{\hbar}\right)$

So corresponding Hamiltonians

Schrödinger representation

Prof. Conover — Interaction rep.

$$\begin{pmatrix} \Delta_i & -\frac{\Omega_0^*}{2} e^{-i\Delta_0 t} \\ -\frac{\Omega_0}{2} e^{+i\Delta_0 t} & \Delta_f \end{pmatrix}$$

Ramsey

$$\begin{pmatrix} W_p & \hbar b e^{i\omega t} \\ \hbar b e^{-i\omega t} & W_q \end{pmatrix}$$

Further correspondences

$$\begin{cases} \cos\theta = \frac{\omega_0 - \omega}{a} \equiv \frac{\Delta_0 + \Delta_d}{\Omega'} \\ \sin\theta = \frac{2b}{a} \equiv \frac{\Omega_0^*}{\Omega'} \end{cases}$$

where $\Delta_0 = \omega_{21} - 2\omega_0$

$\Delta_d = (\Delta_f - \Delta_i)/2$

$\bar{\Delta} = \frac{\Delta_f + \Delta_i}{2}$

Next step, delayed Pulse

But before this, let's revisit $\Omega_0, \Omega_0^* \ldots$

$\hookrightarrow$ Introduce phase $\Omega_0 = |\Omega_0| e^{i\varphi_0} \rightarrow$ constant...

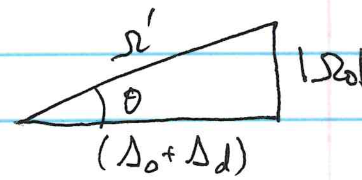
$\underline{so}$ ...

The matrix we're using is:

$$
e^{-i\bar{\Delta}t}\begin{pmatrix} e^{-i\frac{\Delta_0}{2}t}\left(\cos\left(\frac{\Omega' t}{2}\right)+i\left(\frac{\Delta_0+\Delta_d}{\Omega'}\right)\sin\left(\frac{\Omega' t}{2}\right)\right) & e^{-i\frac{\Delta_0}{2}t}\,\frac{i\Omega_0^*}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right) \\ e^{i\frac{\Delta_0}{2}t}\,\frac{i\Omega_0}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right) & e^{i\frac{\Delta_0}{2}t}\left[\cos\left(\frac{\Omega' t}{2}\right)-i\left(\frac{\Delta_0+\Delta_d}{\Omega'}\right)\sin\left(\frac{\Omega' t}{2}\right)\right] \end{pmatrix}
$$

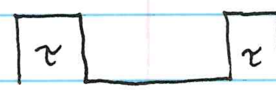
Let's define... $\Omega_0 = |\Omega_0| e^{i\varphi_0}$

$\to \Omega_0^* = |\Omega_0| e^{-i\varphi_0}$



$$\cos\theta = \frac{\Delta_0+\Delta_d}{\Omega'}$$

$$\sin\theta = \frac{|\Omega_0^*|}{\Omega'} \Rightarrow \frac{\Omega_0^*}{\Omega'} = \sin\theta\cdot e^{-i\varphi_0}$$

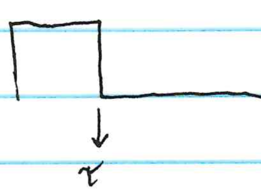


So our matrix, after time τ, becomes...

$$
e^{-i\bar{\Delta}\tau}\begin{pmatrix} e^{-i\frac{\Delta_0}{2}\tau}\left[\cos\frac{\Omega'\tau}{2}+i\cos\theta\sin\frac{\Omega'\tau}{2}\right] & e^{-i\frac{\Delta_0\tau}{2}}(i)\sin\theta\, e^{-i\varphi_0}\sin\left(\frac{\Omega'\tau}{2}\right) \\ e^{i\frac{\Delta_0}{2}\tau}(i)\sin\theta\, e^{+i\varphi_0}\sin\left(\frac{\Omega'\tau}{2}\right) & e^{i\frac{\Delta_0\tau}{2}}\left(\cos\left(\frac{\Omega'\tau}{2}\right)-i\cos\theta\sin\frac{\Omega'\tau}{2}\right) \end{pmatrix}
$$

Further simplification gives

$$
e^{-i\bar{\Delta}\tau}\begin{pmatrix} e^{-i\frac{\Delta_0}{2}\tau}\left(\cos\frac{\Omega'\tau}{2}+i\cos\theta\sin\frac{\Omega'\tau}{2}\right) & i e^{-i\left(\frac{\Delta_0}{2}\tau+\varphi_0\right)}\sin\theta\sin\left(\frac{\Omega'\tau}{2}\right) \\ i e^{i\left(\frac{\Delta_0}{2}\tau+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2} & e^{i\frac{\Delta_0\tau}{2}}\left(\cos\left(\frac{\Omega'\tau}{2}\right)-i\cos\theta\sin\frac{\Omega'\tau}{2}\right) \end{pmatrix}
$$



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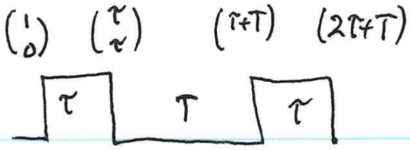


$\boxed{\text{Ramsey's approach}}$

ONLY WORKS FOR INITIAL PULSE (delay $t=0$)

Call the "τ" matrix η. The state following τ pulse is $\eta\begin{pmatrix}1\\0\end{pmatrix}$

Now: $\eta\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}c_i(\tau)\\c_f(\tau)\end{pmatrix}$

→ No driving field ⇒ $b=0$

In the region T, assume energy is constant, then, from S.E.

$$\boxed{i\begin{pmatrix}\dot a_i(\tau+t)\\ \dot a_f(\tau+t)\end{pmatrix} = \begin{pmatrix}\Delta_i & 0\\ 0 & \Delta_f\end{pmatrix}\begin{pmatrix}a_i(\tau+t)\\ a_f(\tau+t)\end{pmatrix}}$$

So
$$\begin{cases} i\dot a_i(t) = \Delta_i a_i(t)\\ i\dot a_f(t) = \Delta_f a_f(t)\end{cases} \rightarrow \begin{cases} a_i(\tau+T) = a_i(\tau)e^{-i\Delta_i T}\\ a_f(\tau+T) = a_f(\tau)e^{-i\Delta_f T}\end{cases}$$

So
$$\boxed{\begin{pmatrix}a_i(\tau+T)\\ a_f(\tau+T)\end{pmatrix} = \begin{pmatrix}e^{-i\Delta_i T} & 0\\ 0 & e^{-i\Delta_f T}\end{pmatrix}\begin{pmatrix}a_i(\tau)\\ a_f(\tau)\end{pmatrix}}$$

Well actually this should ↑ work!

So the state after the 2nd pulse is can be calculated by

NOT ??? QUITE! → careful

$$\boxed{\eta\begin{pmatrix}e^{-i\Delta_i T} & 0\\ 0 & e^{-i\Delta_f T}\end{pmatrix}\eta\begin{pmatrix}a_i(0)\\ a_f(0)\end{pmatrix}}$$

where

$$\eta = e^{-i\bar\Delta\tau}\begin{pmatrix} e^{-\frac{i\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right) & ie^{-i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\left(\frac{\Omega'\tau}{2}\right)\\ ie^{i\left(\frac{\Delta_0}{2}\tau+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2} & e^{i\frac{\Delta_0}{2}\tau}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\end{pmatrix}$$

$\boxed{\text{Beware of initial conditions...}}$

$$\boxed{\text{Fully Solve... Assume } a_i(0) = 1,\ a_f(0) = 0}$$

After time τ

$$\boxed{\tau}$$

$0 \quad \tau$

$$a_i(\tau) = e^{-i\bar{\Delta}\tau}\left\{ e^{-i\frac{\Delta_0}{2}\tau}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\right\}$$

$$a_f(\tau) = e^{-i\bar{\Delta}\tau}\left\{ ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}\right\}$$

After time $T+\tau$

$$\boxed{\tau}\ \ T$$

$$a_i(\tau+T) = a_i(\tau)e^{-i\Delta_i T} = \left\{e^{-i\frac{\Delta_0}{2}\tau}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\right\}e^{-i(\bar{\Delta}\tau+\Delta_i T)}$$

$$a_f(\tau+T) = a_f(\tau)e^{-i\Delta_f T} = \left\{ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}\right\}e^{-i(\bar{\Delta}\tau+\Delta_f T)}$$

After time $T+2\tau$

$$a_i(\tau+T+\tau) = e^{-i\bar{\Delta}\tau}\left\{e^{-i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\cdot a_i(\tau+T) + ie^{-i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\left(\frac{\Omega'\tau}{2}\right)\cdot a_f(\tau+T)\right\}$$

$$a_f(T+2\tau) = e^{-i\bar{\Delta}\tau}\left\{ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}\cdot a_i(\tau+T) + e^{i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)a_f(\tau+T)\right\}$$

BEWARE

The FULL $|f\rangle$ amplitude is (without $e^{\pm i\Delta_0 T}$ term)

$$a_f(T+2\tau) = e^{-i\bar{\Delta}\tau}\Big\{ ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)} \sin\theta \sin\left(\frac{\Omega'\tau}{2}\right) \cdot e^{-i(\bar{\Delta}\tau+\Delta_i T)} \cdot \left[e^{-i\frac{\Delta_0\tau}{2}}\left(\cos\left(\frac{\Omega'\tau}{2}\right) + i\cos\theta \sin\frac{\Omega'\tau}{2}\right)\right]$$

$$+ e^{i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right) \cdot e^{-i(\bar{\Delta}\tau+\Delta_f T)} \cdot \left[ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)} \sin\theta\sin\frac{\Omega'\tau}{2}\right]\Big\}$$

Simplify $a_f(T+2\tau)$

$$= ie^{-i2\bar{\Delta}\tau} \cdot e^{i(\Delta_0\tau/2+\varphi_0)} \sin\theta\sin\left(\frac{\Omega'\tau}{2}\right) \times \Big\{ e^{-i\Delta_0\tau/2 - i\Delta_i T} \cdot \left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right)$$

$$+ e^{i\frac{\Delta_0\tau}{2} - i\Delta_f T}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\Big\}$$

How to take c.c of this?

Consider this term: $e^{-i\frac{\Delta_0\tau}{2} - i\Delta_i T}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right) + e^{i\frac{\Delta_0\tau}{2} - i\Delta_f T} \cdot \left(c\frac{\Omega'\tau}{2} - ic\theta s\frac{\Omega'\tau}{2}\right)$

Let $a = \cos\frac{\Omega'\tau}{2}$, $b = \cos\theta\sin\frac{\Omega'\tau}{2}$

$$= \left[\cos\left(\frac{\Delta_0\tau}{2} + \Delta_i T\right) - i\sin\left(\frac{\Delta_0\tau}{2}+\Delta_i T\right)\right](a+ib)$$

$$+ \left[\cos\left(\frac{\Delta_0\tau}{2} - \Delta_f T\right) + i\sin\left(\frac{\Delta_0\tau}{2} - \Delta_f T\right)\right](a-ib)$$

Call this $(R+iI)$

(11)

__Real__

$$\bar{\Delta} = \frac{\Delta_f + \Delta_i}{2}$$

$$\Delta_d = \frac{\Delta_f - \Delta_i}{2}$$

$$R = a\cos\left(\frac{\Delta_0\tau}{2} + \Delta_i T\right) + b\sin\left(\frac{\Delta_0\tau}{2} + \Delta_i T\right)$$

$$+ a\cos\left(\frac{\Delta_0\tau}{2} - \Delta_f T\right) + b\sin\left(\frac{\Delta_0\tau}{2} - \Delta_f T\right)$$

$$= 2a\cos\left(\Delta_0\tau/2 - \Delta_d T/2\right)\cos\left(\bar{\Delta}T\right)$$

$$+ 2b\sin\left(\Delta_0\tau/2 - \Delta_d T/2\right)\cos\left(\bar{\Delta}T\right)$$

$$\boxed{R = 2\cos(\bar{\Delta}T)\left[\cos\frac{\Omega'\tau}{2}\cos\left(\Delta_0\tau/2 - \Delta_d T/2\right) + \cos\theta\sin\left(\frac{\Omega'\tau}{2}\right)\sin\left(\Delta_0\tau/2 - \Delta_d T/2\right)\right]}$$

__Imaginary__

$$I = i\left[b\cos\left(\frac{\Delta_0\tau}{2} + \Delta_i T\right) - a\sin\left(\frac{\Delta_0\tau}{2} - \Delta_f T\right)\right.$$

$$\left. + a\sin\left(\frac{\Delta_0\tau}{2} - \Delta_f T\right) - b\cos\left(\frac{\Delta_0\tau}{2} - \Delta_f T\right)\right]$$

$$= i\left[b(-2)\sin\left(\Delta_0\tau/2 - \Delta_d T/2\right)\sin(\bar{\Delta}T) - 2a\cos\left(\Delta_0\tau/2 - \Delta_d T/2\right)\sin(\bar{\Delta}T)\right]$$

$$\boxed{I = -2i\sin(\bar{\Delta}T)\left[\cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\Delta_0\tau/2 - \Delta_d T/2\right) + \cos\frac{\Omega'\tau}{2}\cos\left(\Delta_0\tau/2 - \Delta_d T/2\right)\right]}$$

__So__ Real + Im

$$= 2\left(\cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\Delta_0\tau/2 - \Delta_d T/2\right) + \cos\frac{\Omega'\tau}{2}\cos\left(\Delta_0\tau/2 - \Delta_d T/2\right)\right) \times \overbrace{\left[\cos\bar{\Delta}T - i\sin\bar{\Delta}T\right]}^{e^{-i\bar{\Delta}T}}$$

Therefore

$$a_f^*(T+2\tau) = -ie^{+i2\bar{\Delta}\tau} \cdot e^{i\left(\Delta_0\frac{\tau}{2}+\beta_0\right)} \sin\theta \sin\frac{\Omega'\tau}{2} \cdot 2e^{+i\bar{\Delta}T}\left[\cos\frac{\Omega'\tau}{2}\cos\left(\Delta_0\frac{\tau}{2} - \Delta_d\frac{T}{2}\right) + \cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\Delta_0\frac{\tau}{2} - \Delta_i\frac{T}{2}\right)\right]$$

Hence

$$\boxed{P_2 = |a_f|^2 = 4\sin^2\theta\sin^2\frac{\Omega'\tau}{2}\left[\cos\frac{\Omega'\tau}{2}\cos\left(\Delta_0\frac{\tau}{2} - \Delta_d\frac{T}{2}\right) + \cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\Delta_0\frac{\tau}{2} - \Delta_d\frac{T}{2}\right)\right]^2}$$

* Ramsey's version

$$\boxed{P_2 = 4\sin^2\theta\sin^2\frac{\Omega'\tau}{2}\left[\cos\frac{\Omega'\tau}{2}\cos\left(\frac{1}{2}\lambda T\right) - \cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\frac{1}{2}\lambda T\right)\right]^2}$$

where $\lambda = \frac{\overline{W}_q - \overline{W}_p}{\hbar} - \omega \simeq \frac{W_q - W_p}{\hbar} - \omega \simeq \omega_0 - \omega$

which should correspond to $\Delta_0 + \Delta_d$

Which is False, since we've defined... $\omega \equiv -\Delta_0 = \frac{W_q - W_p}{\hbar}$

$\omega_0 \equiv \Delta d = \Delta_f - \Delta_i$

$\boxed{\text{What about the } T = (\tau \text{ and } T) \text{ difference?}}$

But we're getting $\frac{\Delta_0\tau - \Delta_d T}{2}$ instead of $\frac{\Delta_0 + \Delta_d}{2}T$

What's going on wrong here? Factor of $e^{-i\Delta_0 T}$?

(13)

add this term

what if?

$$\eta(t_1) = \eta = \boxed{e^{-i\bar{\Delta}\tau}\begin{pmatrix} e^{-i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right) & ie^{-i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\left(\frac{\Omega'\tau}{2}\right)e^{-i\Delta_0 t_1} \\ ie^{i\left(\frac{\Delta_0}{2}\tau+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}e^{+i\Delta_0 t_1} & e^{i\frac{\Delta_0\tau}{2}}\left(\cos\left(\frac{\Omega'\tau}{2}\right) - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\end{pmatrix}} \cdot C(t_1)$$

calculates $C(\tau + t_1)$

we define $\sin\theta = \frac{\Omega_0^*}{\Omega} = \frac{-2b}{a}$, unlike in Ramsey's paper

Then $C(\tau)$, $C(\tau+T)$ still the same, but $C(\tau\cdot 2+T)$ different...

Jun 26 2018

$\boxed{\text{Solution: Delayed Pulse Matrix}}$ → additional term $e^{-i\Delta_0 T}$ where T is the wait time...

So

$$\begin{pmatrix} a_i(t+\tau) \\ a_f(t+\tau)\end{pmatrix} = e^{-i\bar{\Delta}\tau}\begin{pmatrix} e^{-i\frac{\Delta_0}{2}\tau}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right) & ie^{-i\left(\frac{\Delta_0}{2}\tau+\varphi_0\right)}\sin\theta\sin\left(\frac{\Omega'\tau}{2}\right)e^{-i\Delta_0 t} \\ ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}e^{i\Delta_0 t} & e^{i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\end{pmatrix}\begin{pmatrix} a_i(t) \\ a_f(t)\end{pmatrix}$$

Using state following $\tau + T$ → find state @ $2\tau + T$ → $t = \tau + T$

$$a_i(2\tau+T) = e^{-i\bar{\Delta}\tau}\left\{e^{-i\Delta_0\tau/2}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right)a_i(\tau+T) + ie^{-i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\cdot\sin\theta\sin\frac{\Omega'\tau}{2}e^{-i\Delta_0(\tau+T)}a_f(\tau+T)\right\}$$

$$a_f(2\tau+T) = e^{-i\bar{\Delta}\tau}\left\{ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}e^{i\Delta_0(\tau+T)}\cdot a_i(\tau+T) + e^{i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\cdot a_f(\tau+T)\right\}$$

(14)

FULL $|f\rangle$ state amplitude, with $e^{\pm i\Delta_0(\tau+T)}$ term

$$\underline{a_f(2\tau+T)} = e^{-i\bar{\Delta}\tau}\Big\{ ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)} \sin\theta \sin\frac{\Omega'\tau}{2}\, e^{i\Delta_0(\tau+T)} \cdot e^{-i(\bar{\Delta}\tau+\Delta_i T)} \cdot$$

$$\times\left[e^{-i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\right]$$

$$+ e^{i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\cdot e^{-i(\bar{\Delta}\tau+\Delta_f T)} \cdot$$

$$\times\left[ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)} \sin\theta\sin\frac{\Omega'\tau}{2}\right]\Big\}$$

Simplify

$$\underline{a_f(2\tau+T)} = \left(ie^{-i\bar{\Delta}\tau} e^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)} e^{-i\bar{\Delta}\tau} \sin\theta\sin\frac{\Omega'\tau}{2}\right)\times\Big\{ e^{i\frac{\Delta_0\tau}{2}} \cdot e^{i\Delta_0 T} \cdot e^{-i\Delta_i T} \times$$

$$\times\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right) + e^{i\frac{\Delta_0\tau}{2}} \cdot e^{-i\Delta_f T}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\Big\}$$

$$= \left(ie^{-i2\bar{\Delta}\tau} \cdot e^{i(\Delta_0\tau+\varphi_0)} \cdot \sin\theta\sin\frac{\Omega'\tau}{2}\right)\times\Big\{ e^{iT(\Delta_0-\Delta_i)} \cdot \left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right)$$

$$+ e^{-i\Delta_f T}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)\Big\}$$

<u>In order to calculate $a_f^* a_f$ → need to take c.c. of</u> a_f.

<u>Consider this term:</u> $e^{iT(\Delta_0-\Delta_i)} \cdot \left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right) + e^{-i\Delta_f T}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right)$

Let $a = \cos\frac{\Omega'\tau}{2}$, $b = \cos\theta\sin\frac{\Omega'\tau}{2}$

$$= \left[\cos\left((\Delta_0-\Delta_i)T\right) + i\sin\left((\Delta_0-\Delta_i)T\right)\right](a+ib)$$

$$+ \left[\cos(+\Delta_f T) - i\sin(\Delta_f T)\right](a-ib)$$

Call this <u>$R + iI$</u>

Real

$$R = a\cos\left(T(\Delta_0-\Delta_i)\right) - b\sin\left(T(\Delta_0-\Delta_i)\right) + a\cos(\Delta_f T) - b\sin(\Delta_f T)$$

$$= 2a\cos\left(\frac{T(\Delta_0-\Delta_i+\Delta_f)}{2}\right)\cos\left(\frac{T(\Delta_0-\Delta_i-\Delta_f)}{2}\right) - 2b\sin\left(\frac{T(\Delta_0-\Delta_i+\Delta_f)}{2}\right)\cos\left(\frac{T(\Delta_0-\Delta_i-\Delta_f)}{2}\right)$$

$$\Delta_i + \Delta_f = 2\bar{\Delta}$$
$$-\Delta_i + \Delta_f = \Delta_d$$

So $\boxed{R = 2\cos\left(T\left(\frac{\Delta_0}{2}-\bar{\Delta}\right)\right)\left[\cos\frac{\Omega'\tau}{2}\cos\left(\frac{T(\Delta_0+\Delta_d)}{2}\right) - \cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\frac{T(\Delta_0+\Delta_d)}{2}\right)\right]}$

Imaginary...

$$I = i\left[b\cos\left(T(\Delta_0-\Delta_i)\right) + a\sin\left(T(\Delta_0-\Delta_i)\right) + (-b)\cos(\Delta_f T) + (-a)\sin(\Delta_f T)\right]$$

$$= i\left[-2b\sin\left(\frac{T(\Delta_0-\Delta_i+\Delta_f)}{2}\right)\sin\left(\frac{T(\Delta_0-\Delta_i-\Delta_f)}{2}\right) + 2a\,\cancel{\cos}\left(\frac{T(\Delta_0-\Delta_i+\Delta_f)}{2}\right)\sin\left(\frac{T(\Delta_0-\Delta_i-\Delta_f)}{2}\right)\right]$$

So $\boxed{I = i(+2)\sin\left(T\left(\frac{\Delta_0}{2}-\bar{\Delta}\right)\right)\left[-\cancel{\cos}\,\theta\sin\frac{\Omega'\tau}{2}\sin\left(\frac{T(\Delta_0+\Delta_d)}{2}\right) + \cos\frac{\Omega'\tau}{2}\cos\left(\frac{T(\Delta_0+\Delta_d)}{2}\right)\right]}$

So Real + i Im = ?

Welll ...

$$a_f^*(2\tau+T) = \left(-ie^{+i\bar{\Delta}\tau} e^{-i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)} e^{+i\bar{\Delta}\tau} e^{-i\frac{\Delta_0\tau}{2}}\right)\left(\sin\theta\sin\frac{\Omega'\tau}{2}\right)\times\left\{2\cos\left(T\left(\frac{\Delta_0}{2}-\bar{\Delta}\right)\right)\cdot\right.$$

$$\left[\cos\frac{\Omega'\tau}{2}\cos\left(\frac{T(\Delta_0+\Delta_d)}{2}\right) - \cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\frac{T}{2}(\Delta_0+\Delta_d)\right)\right] + (-2i)\sin\left(T\left(\frac{\Delta_0}{2}+\Delta_d\right)\right)\times\Big[$$

$$\times\left[\cos\frac{\Omega'\tau}{2}\cos\left(\frac{T}{2}(\Delta_0+\Delta_d)\right)-\cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\frac{T(\Delta_0+\Delta_d)}{2}\right)\right]$$

So

$$a_f^\dagger(2\tau+T)\cdot a_f(2\tau+T)=\sin^2\theta\sin^2\frac{\Omega'\tau}{2}\cdot(4)\cdot\left[e^{iT\left(\frac{\Delta_0}{2}+\Delta_d\right)}e^{-iT\left(\frac{\Delta_0}{2}+\Delta_d\right)}\right]$$

$$\times\left[\cos\frac{\Omega'\tau}{2}\cos\left(\frac{T(\Delta_0+\Delta_d)}{2}\right)-\cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\frac{T(\Delta_0+\Delta_d)}{2}\right)\right]^2$$

Therefore,

our version $\longrightarrow$

$$\boxed{P_2=a_f^\dagger a_f(2\tau+T)=4\sin^2\theta\sin^2\frac{\Omega'\tau}{2}\left[\cos\frac{\Omega'\tau}{2}\cos\frac{T(\Delta_0+\Delta_d)}{2}-\cos\theta\sin\frac{\Omega'\tau}{2}\sin\frac{T(\Delta_0+\Delta_d)}{2}\right]^2}$$

Compared to Ramsey's equation...

$$\boxed{P_2=4\sin^2\theta\sin^2\frac{\Omega'\tau}{2}\left(\cos\frac{\Omega'\tau}{2}\cos\left(\frac{1}{2}\lambda T\right)-\cos\theta\sin\frac{\Omega'\tau}{2}\sin\left(\frac{1}{2}\lambda T\right)\right)^2}$$

where $\lambda=\omega_0-\omega$, which we associate to $\Delta_0+\Delta_d$

$$\Delta_0\equiv-\omega,\quad \omega_0=\Delta_d$$

$\boxed{\text{CORRECT!}}$

$\boxed{\text{Summary}}$

$$\begin{pmatrix}a_i(0)\\a_f(0)\end{pmatrix}=\begin{pmatrix}a_i(0)\\a_f(0)\end{pmatrix}\xrightarrow[\boxed{\tau}]{\eta}\begin{pmatrix}a_i(\tau)\\a_f(\tau)\end{pmatrix}\xrightarrow[T]{T}\begin{pmatrix}a_i(\tau+T)\\a_f(\tau+T)\end{pmatrix}\xrightarrow{\eta^*}\begin{pmatrix}a_i(2\tau+T)\\a_f(2\tau+T)\end{pmatrix}$$

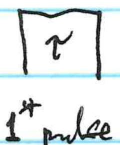
The transformation can be written in matrix form

Next page

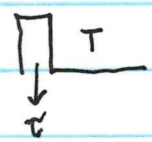
17

$$\boxed{A = \eta^* T \eta A_0}$$

- where A_0 is the initial state vector. $\begin{pmatrix} a_i(0) \\ a_f(0) \end{pmatrix}$

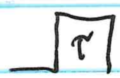


$$\boxed{\eta = e^{-i\bar{\Delta}\tau} \begin{pmatrix} e^{-i\frac{\Delta_0}{2}\tau}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta \sin\frac{\Omega'\tau}{2}\right) & ie^{-i\frac{\Delta_0}{2}\tau - i\varphi_0}\sin\theta \sin\frac{\Omega'\tau}{2} \\ ie^{i\frac{\Delta_0}{2}\tau}\sin\theta\, e^{i\varphi_0}\sin\frac{\Omega'\tau}{2} & e^{i\frac{\Delta_0}{2}\tau}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right) \end{pmatrix}}$$



$$\boxed{T = \begin{pmatrix} e^{-i\Delta_i T} & 0 \\ 0 & e^{-i\Delta_f T} \end{pmatrix}}$$ → Unperturbed evolution in time of state $\begin{pmatrix} a_i(\tau+T) \\ a_f(\tau+T) \end{pmatrix}$

And



$$\boxed{\eta^* = e^{-i\bar{\Delta}\tau} \begin{pmatrix} e^{-i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} + i\cos\theta\sin\frac{\Omega'\tau}{2}\right) & ie^{-i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}\, e^{-i\Delta_0(\tau+T)} \\ ie^{i\left(\frac{\Delta_0\tau}{2}+\varphi_0\right)}\sin\theta\sin\frac{\Omega'\tau}{2}\, e^{i\Delta_0(\tau+T)} & e^{i\frac{\Delta_0\tau}{2}}\left(\cos\frac{\Omega'\tau}{2} - i\cos\theta\sin\frac{\Omega'\tau}{2}\right) \end{pmatrix}}$$

In our case $\begin{pmatrix} a_i(0) \\ a_f(0) \end{pmatrix} \xrightarrow{\eta} \begin{pmatrix} a_i(\tau) \\ a_f(\tau) \end{pmatrix} \xrightarrow{T} \begin{pmatrix} a_i(\tau+T) \\ a_f(\tau+T) \end{pmatrix} \xrightarrow{\eta^*} \begin{pmatrix} a_i(2\tau+T) \\ a_f(2\tau+T) \end{pmatrix}$

- In association w/ Ramsey's parameters...

- $\sin\theta = \frac{\Omega_0^{\pm}}{\Omega'} \equiv \frac{+2b}{a}$, $\cos\theta = \frac{\omega_0 - \omega}{a} \equiv \frac{\Delta_0 + \Delta_d}{\Omega'}$, $2b \equiv \Omega_0^*$

- $\Omega' = \sqrt{|\Omega_0|^2 + (\Delta_0 + \Delta_d)^2} \equiv a = \sqrt{(\omega_0 - \omega)^2 + (2b)^2}$

- $\omega \equiv -\Delta_0$ (osc freq of field), $+\omega_0 \equiv \Delta_d = \Delta_f - \Delta_i$ ~ transition freq

- $\bar{\Delta} = \frac{\Delta_f + \Delta_i}{2} \equiv \frac{W_p + W_q}{2\hbar}$

Note
Ramsey: Schrödinger representation
Pr. Conover: Interaction representation.

Jun 29, 2018

Important Modification

In the interaction rep, in 0 field,

$$T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ since there's no AC STARK SHIFT

So

$$\boxed{P_2 = 4\sin^2\theta \sin^2\frac{\Omega'\tau}{2}\left\{\cos\frac{\Omega'\tau}{2}\cos\frac{\Delta_0 T}{2} - \cos\theta \sin\frac{\Omega'\tau}{2}\sin\frac{\Delta_0 T}{2}\right\}^2}$$

(−) works!