

Following a double pulse, the population of the excited state is:

$$P_2 = 4 \sin^2 \theta \sin^2 \frac{\Omega' \tau}{2} \left\{ \cos \frac{\Omega' \tau}{2} \cos \frac{\Delta_0 T}{2} - \cos \theta \sin \frac{\Omega' \tau}{2} \sin \frac{\Delta_0 T}{2} \right\},$$

under the assumption that initially,

$$\begin{bmatrix} C_1(0) \\ C_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The final state vector is:

$$\begin{bmatrix} C_1(2\tau + T) \\ C_2(2\tau + T) \end{bmatrix} = \rho_2 D \rho_1 \begin{bmatrix} C_1(0) \\ C_2(0) \end{bmatrix}$$

where  $|C_1|^2 + |C_2|^2 = 1$  for all value of time, and  $\rho_1$  and  $\rho_2$  are propagators associated with Pulse 1 and Pulse 2 (both with width  $\tau$ ), respectively.  $D$  is a propagator associated with the field-free evolution of duration  $T$ .

Specifically, in the interaction representation:

$$\rho_1 = e^{-i\bar{\Delta}\tau} \begin{bmatrix} e^{-i\frac{\Delta_0}{2}\tau} \left( \cos \frac{\Omega'\tau}{2} + i \cos \theta \sin \frac{\Omega'\tau}{2} \right) & i e^{-i(\frac{\Delta_0}{2}\tau - \phi_0)} \sin \theta \sin \frac{\Omega'\tau}{2} \\ i e^{i(\frac{\Delta_0}{2}\tau + \phi_0)} \sin \theta \sin \frac{\Omega'\tau}{2} & e^{i\frac{\Delta_0}{2}\tau} \left( \cos \frac{\Omega'\tau}{2} - i \cos \theta \sin \frac{\Omega'\tau}{2} \right) \end{bmatrix}$$

$$\rho_2 = e^{-i\bar{\Delta}\tau} \begin{bmatrix} e^{-i\frac{\Delta_0}{2}\tau} \left( \cos \frac{\Omega'\tau}{2} + i \cos \theta \sin \frac{\Omega'\tau}{2} \right) & i e^{-i(\frac{\Delta_0}{2}\tau + \phi_0)} \sin \theta \sin \frac{\Omega'\tau}{2} e^{-i\Delta_0(\tau+T)} \\ i e^{i(\frac{\Delta_0}{2}\tau + \phi_0)} \sin \theta \sin \frac{\Omega'\tau}{2} e^{i\Delta_0(\tau+T)} & e^{i\frac{\Delta_0}{2}\tau} \left( \cos \frac{\Omega'\tau}{2} - i \cos \theta \sin \frac{\Omega'\tau}{2} \right) \end{bmatrix}$$

and

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note that  $D$ , in the interaction representation, is the identity matrix. This is different from Ramsey's original approach in which the state vector does evolve and change during the delay time  $T$ . The angle  $\theta$  is defined as:

$$\sin \theta = \frac{\Omega_0^*}{\Omega'}$$

and

$$\cos \theta = \frac{\Delta_0 + \Delta_d}{\Omega'}$$

where  $\Omega_0^*$  is the complex conjugate of the Rabi rate, and  $\Omega'$  can be defined as the “effective Rabi rate.”

$$\Omega' = \sqrt{|\Omega_0^*| + (\Delta_0 + \Delta_d)^2}$$