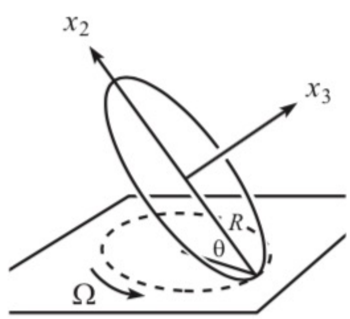
8.(3)09 Section 5

October 8, 2021

1 Wobbling Coin (Morin Problem 9.24)

Consider spinning a coin around a vertical diameter on a table. It will slowly lose energy and begin a wobbling motion. The angle between the coin and the table gradually decreases, eventually coming to rest. Assume that this process is slow, and consider motion when the coin makes an angle θ with the table. Assume that the CM is essentially motionless. Assume the coin rolls without slipping. R is the radius of the coin and Ω the frequency of the contact point on the table tracing out a circle.



(a)

Show that the angular velocity of the coin is $\vec{\omega} = \Omega \sin\theta\, \hat{x}_2$, where $\hat{x}_2$ points upward along the coin, directly away from the contact point.

(b)

Show that

$$\Omega = 2\sqrt{\frac{g}{R\sin\theta}}$$

(c)

Show that Abe (or Tom, Franklin, George, John, Dwight, Susan, or Sacagawea) appears to rotate, when viewed from above, with frequency $2(1-\cos\theta)\sqrt{\frac{g}{R\sin\theta}}$.

2 Corrections to the pendulum (Morin 4.23)

(a)

For small oscillations, the period of a pendulum is approximately $T \approx 2\pi\sqrt{l/g}$, independent of the amplitude, θ_0. For finite oscillations, use $dt = dx/v$ to show that the exact expression for T is

$$T = \sqrt{\frac{8l}{g}} \int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}.$$

(b)

Find an approximation to the expression for T in (a), up to second order in θ_0, in the following way: make use of the identity $\cos\phi = 1 - 2\sin^2(\phi/2)$ to write T in terms of sines. Then make the change of variables $\sin x = \sin(\theta/2)/\sin(\theta_0/2)$. Finally, expand the integrand in powers of θ_0 and perform the integrals to show that

$$T \approx 2\pi\sqrt{\frac{l}{g}} \left(1 + \frac{\theta_0^2}{16} + \left[\frac{11}{3072}\theta_0^4\right] + \cdots\right),$$

where the term in square brackets is optional (just try to get the leading order correction).