Problem Set 9

Due: Friday 5pm, April 15th, via Canvas upload or in envelope outside 26-255
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1 Transition Lifetimes and Blackbody Radiation

a) Do you have to worry about blackbody radiation when you trap a Bose-Einstein condensate in a magnetic trap? Any transition to another state will spin-flip the atoms and/or give the atoms recoil energy, causing the atom to be ejected from the trap. Assume that after each absorption event, it will lose one atom in the trap, and you want to ensure a trapping time of at least one minute in a closed, cubic box. Consider sodium, which has a dominant electronic excitation at a wavelength of 590 nm and a lifetime of 16 ns.

i. What is the average number of photons per mode from blackbody radiation at this transition which leads to an absorption rate of 1 photon per minute? Hint: You might find Einstein's coefficients useful for this problem.

ii. What is the corresponding blackbody temperature? Do you need to shield the vacuum system from room temperature radiation or cool the vacuum system to cryogenic temperatures (*e.g.* 4 K)?

b) Here we estimate the lifetime of the hydrogen in the $F = 1$ hyperfine level of the $1S$ state. The decay of $F = 1$ to $F = 0$ gives rise to the famous 21 cm line of radio astronomy.

i. What is the dominant coupling mechanism for this process, *e.g.* is it an electric dipole transition, or a magnetic dipole transition? or it is actually an electric octupole transition?

ii. What is the lifetime of the $F = 1$ state? Assume that the matrix element is ea_0 for electric dipole transition, μ_B for magnetic dipole transition, and ea_0^2 for electric quadrupole transition, where a_0 is the Bohr radius, and μ_B is the Bohr magneton.

c) A hydrogen Bose-Einstein condensate has been created in the $F = 1$ state at MIT in the group of Dan Kleppner and Tom Greytak (Fried *et al.*, *Phys. Rev. Lett.* **81**, 3811 (1998)).

i. What is the average number of photons per mode from blackbody radiation at the 21 cm line at 300 K and 4 K?

ii. For blackbody radiation at 300 K and 4 K, at what rate does it induce transitions to the $F = 0$ state respectively, which cannot be magnetically trapped?

iii. Should you be concerned about blackbody radiation from the environment limiting your experiment with hydrogen in the $F = 1$ state, if you need a trapping time of 1 minute?

d) You probably noticed from parts *a*) and *c*) that the average number of photons per mode at their respective relevant frequencies and temperatures are rather different. In particular, there are many more photons per mode for one of them than the other. Yet, the lifetime is much longer for the one with more photons per mode, for these two examples. Why is that?

2 Saturation Intensity

We define the saturation intensity of a laser for an optical transition as the intensity (power/area) at which a monochromatic beam excites the transition at a rate equal to one half of its natural line width. In this problem, we compute the saturation intensity for the principal transition in sodium, 590 nm.

a) Express the Einstein A coefficient by the oscillator strength f, the fine structure constant α and the transition frequency ω. Estimate the lifetime of sodium by assuming an oscillator strength of unity.

b) Find the saturation intensity for the principal transition in sodium. Treat the atom as a two-level system, neglecting fine and hyperfine structure.

3 Saturation of Atomic Transitions

We discussed excitation of atoms via weak radiation. In this limit the atom scatters incident radiation at a rate proportional to the light intensity, corresponding to a fixed cross-section.

We also discussed the excitation of atoms via strong radiation and showed that in this limit the atom performs Bloch oscillations between the ground and excited states. Since during these oscillations the mean excited state population population is at most 1/2 and the excited state decays with rate Γ, the atom can scatter at most $\Gamma/2$ photons per unit time. To obtain a fixed scattering rate, as the radiation intensity increases, the photon-scattering cross-section decreases, becoming very low at high light intensities.

This problem will motivate this saturation of atomic transitions by considering broadband excitation. The obtained results can be exactly extended to narrowband transitions.

a) In the case of broadband excitation, the atom dynamics is correctly described by the Einstein rate equations. Consider a two-state atom with $R_{ge} = R_{eg}$ the stimulated absorption/emission rate and $A = \Gamma$ the spontaneous emission rate. Define the saturation parameter s as $s = 2R_{ge}/\Gamma$. Show that in equilibrium the ratio of the excited state to the ground state populations is $N_b/N_a = s/(s+2)$.

b) Express the equilibrium spontaneous emission rate per atom AN_b in terms of Γ and s. Show that the cross-section for photon absorption bleaches out as $\sigma(s) = \sigma(s=0)/(1+s)$.

c) Find the energy density $\langle w \rangle_{SAT}$ per unit frequency corresponding to $s = 1$. Explain why $\langle w \rangle$ is independent of the atomic dipole matrix element $\langle g| er |e\rangle$.

d) Use the relationship between Einstein's A and B coefficients to obtain an expression for $\langle w \rangle_{SAT}$ independent of the atomic dipole. For $s = 1$, what is the mean occupation number n per photon mode?

e) Suppose that the light is provided by a laser beam of intensity I_0 and Lorentzian lineshape centered at the atomic transition frequency ω_0 and of FWHM $\Gamma' \gg \Gamma$. What is the energy density of this beam per frequency interval at ω_0? What beam intensity I_s corresponds to $s = 1$?

f) Let ω_R be the Rabi frequency corresponding to a monochromatic beam with the same intensity I_0 as the broadband beam. Show that the stimulated broadband absorption rate can be written as $R = \omega_R^2/\Gamma'$. What is ω_R^2 corresponding to $s = 1$?

g) If you set $\Gamma' = \Gamma$, you get exactly the saturation intensity of a monochromatic laser beam and the Rabi frequency at saturation. Argue why.