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Based on Chapter 8 of Arfken, Weber, and Harris (7th Edition)

## Recitation on Sturm-Liouville Theory

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### Why?

- I like studying real physical systems
  - Laplace's Solutions, Poisson's Solutions
    - Not very often used (by me). Not very intuitive (to me)
      - I hardly stick charges inside conductive boxes.
      - Besides electronic circuits, and the one thing I'm concerned with is how electric fields leak in and out of the system, as then they can effect other electronic circuits.
        - Shield E fields,
        - Partially Shield B fields at high frequencies?
    - For me, Green's Function's are enough.
      - Intuitive, and easy to understand.
      - Fully solves E&M
        - E&M is done once we covered these.
        - much like eigenfunctions solve QM, ideally, but not practically.
  - So Why Laplace/Poisson?
    - Practice.
      - Fortune favors the prepared mind.
      - Life is like a box of chocolates, you never know what you're going to get - Forrest Gump
    - Preparation for Quantum Theory
      - Most Fascinating Theory in the World
      - Why? Because there's a lot of confusion surrounding it

- Can show that there is no need for quantum woo in many cases, or hand waviness to the physics. SL Theory shows why this is true for the claim "Nature is Discretized".
- Two Possible Motivations for Sturm-Liouville Theory (SL)
  - Tells you why quantum mechanics has the structure it has,
    - Intellectual
  - Prepares you for physical problems as yet unseen.
    - Practical.

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## What?

- Tells you when our magic tricks work.
  - Physicists love magic tricks.
    - Orthonormal Basis
    - Separation of Variables.
  - SL Theory tells you when they work, that it is not merely happen stance, restricted to the special problems shown in class.

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## Tell me more about these magic tricks

- Separation of Variables
  - Bad Guess
    - Almost never a real solution.
- Orthonormal Basis
  - Saving Grace of these bad guesses.
    - These bad guesses form an orthonormal basis.
      - Let's you write any solution as a sum of bad guesses!!!
      - BUT WHYY???? Why did this bad guess work?

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## Sturm-Liouville Theory and It's Conditions

- Self Adjoint Operator
  - This random idea is the secret sauce to so many physics magic tricks.
    - Going to focus on the Differential Form, not a matrix form
      - common in E&M
      - Common in Schrodinger Form of Quantum Theory
  - Defined with respect to an inner product
    - Weighted Inner Products
    - Unweighted Inner Products
- Self Adjoint Lemma
  - See when  $\langle v(x), \hat{L}u(x) \rangle = ? \langle \hat{L}v(x), u(x) \rangle$ 
    - Integration by Parts
      - gives a Boundary Term
    - Depends on Boundary Conditions!!!
    - "Bad Guesses" only work when the modes have a very special constraint!!!
      - "Bad Guesses" equals solution to Separation of Variables form of PDE.
- What Boundary Conditions give Self Adjoint
  - Vanish at the Boundary
  - Derivative Vanish at the Boundary
  - Periodic at the Boundary
  - More?
    - $p_0(x)$  vanishes at the boundary
- Self Adjoint Lemma on Eigen Functions
  - Let's stick in Eigenfunctions into this Condition of Self Adjoint.
  - Found a Constraint on the Inner Product!!!

- **Orthonormality if and only if the eigenvalues are different and the boundary terms vanish.**
  - Our secret sauce depends heavily on the random physical constraints we put on the system! Interesting...
- What if the system is not Self Adjoint?
  - Can make it self adjoint by a weighting factor  $w(x)$
  - Changes the inner product! and boundary term!