MA439: Functional Analysis
Tychonoff Spaces: Exercises 5, 6, 12, 13, 14 on p.31, Ben Mathes

Huan Q. Bui

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Exercise 1 (Ex 5, p.31)**.** *In an arbitrary topological space $\mathcal{X}$, we say that a sequence (x_i) converges to x (and write $x_i \to x$) if, for every open set G containing x, the sequence (x_i) is eventually in G. Prove that $x_i \to x$ if and only if, for every subbasic open set S, (x_i) is eventually in S.*

Proof. $\square$

Exercise 2 (Ex 6, p.31)**.** *In arbitrary topological spaces, the neighborhood filter $\mathcal{F}_x$ of a point x is defined to be the collection of all subsets that contain an open set containing x, and we again define $\mathcal{F} \to x$ to mean $\mathcal{F}_x \subseteq \mathcal{F}$. Prove that $F \to x$ if and only if every subbasic open set containing x is in $\mathcal{F}$.*

Proof. $\square$

Exercise 3 (Ex 12, p.31)**.** *Assume that $S = p_k^1(G)$ is a subbasic open set in a product space $\prod_i \mathcal{X}_i$. Prove that $S = p_k^1(p_k(S))$, and if $p_k(E) \subseteq p_k(S)$, then $E \subseteq S$.*

Proof. $\square$

Exercise 4 (Ex 13, p.31)**.** *Prove that a topological space is compact if and only if every open covering by basic open sets has a finite subcover.*

Proof. $\square$

Exercise 5 (Ex 14, p.31)**.** *Prove that a topological space is compact if and only if every open covering by subbasic open sets has a finite subcover. (This requires the axiom of choice.)*

Proof. $\square$