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Measurement-based quantum computation

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Quantum computation offers a promising new kind of information processing, where the non-classical features of quantum mechanics are harnessed and exploited. A number of models of quantum computation exist. These models have been shown to be formally equivalent, but their underlying elementary concepts and the requirements for their practical realization can differ significantly. A particularly exciting paradigm is that of measurement-based quantum computation, where the processing of quantum information takes place by rounds of simple measurements on qubits prepared in a highly entangled state. We review recent developments in measurement-based quantum computation with a view to both fundamental and practical issues, in particular the power of quantum computation, the protection against noise (fault tolerance) and steps towards experimental realization. Finally, we highlight a number of connections between this field and other branches of physics and mathematics.

Quantum computation is a promising and fruitful area of research, and impressive theoretical and experimental achievements have been reported in recent years. At the same time, many fundamental questions remain unanswered. Realizing a large-scale computational device with the technology available in the foreseeable future remains a challenge, and the full range of applications for a working quantum computer is still unknown. A number of quantum algorithms are known for particular problems, including factoring and simulation of other quantum systems, but discovering new quantum algorithms that outperform classical ones remains a great challenge. On a more fundamental level, we are still missing a good understanding of where the border in computational power between classical and quantum resources lies.

Even the very notion of what makes a quantum computer and what it should be capable of doing, is not entirely understood. The latter point is highlighted by the existence of different models for quantum computation, including the quantum circuit (or network) model[1,2], adiabatic quantum computation[3], the quantum Turing machine[4,5] and measurement-based models such as teleportation-based approaches[6–9] as well as the one-way quantum computer[10–12]. Indeed, there seem to be many ways to exploit nature for quantum information processing.

As the features of these models differ significantly, some computational schemes may lend themselves more than others providing insight into certain aspects of quantum computation and to overcome challenges in their experimental realization. The new paradigm of 'measurement-based quantum computation[6–19]' (MQC), with the 'one-way quantum computer' and the teleportation-based model as the most prominent examples, is particularly promising in these respects, and provides a new conceptual framework in which these experimental and theoretical challenges can be faced. Whereas in, for example, the circuit model quantum information is processed by coherent unitary evolutions (quantum gates), in MQC the processing of quantum information takes place by carrying out sequences of adaptive measurements. Moreover, whereas in the teleportation-based model joint (that is, entangling) measurements are used, in the one-way quantum computer—which will be the focus of this progress article—universal quantum computation can be achieved with single-qubit measurements alone.

More specifically, in one-way quantum computation, the system is first prepared in a highly entangled quantum state, the two-dimensional (2D)-cluster state[20] (see Box 1), independently of the

Box 1 | Graph states.

The cluster state[20] belongs to a family of highly entangled multi-particle quantum states, which can be efficiently parameterized by mathematical graphs. These are the so-called 'graph states'[21]. A graph $G=(V,E)$ is a set of N vertices (V), together with a set of edges $E\subseteq[V]^2$, which connect the vertices in an arbitrary way. To every such graph, we associate a specific N-qubit quantum state $|G\rangle$. The graph state $|G\rangle$ is obtained by preparing all N qubits in the $+1$ eigenstate of the Pauli operator, σ_x, namely $|+\rangle=(|0\rangle+|1\rangle)/\sqrt{2}$, and by applying two-qubit phase gates $U_{\mathrm{PG}}=\mathrm{diag}(1,1,1,-1)$ between all pairs of qubits connected by an edge: $|G\rangle=\prod_{\{i,j\}\in E}U_{\mathrm{PG}}^{(i,j)}|+\rangle^{\otimes N}$. Equivalently, $|G\rangle$ can be defined as a simultaneous fixed point of the correlation operators $K_j=\sigma_x^{(j)}\bigotimes_{\{i,j\}\in E}\sigma_z^{(i)}$, which are entirely determined by the graph.

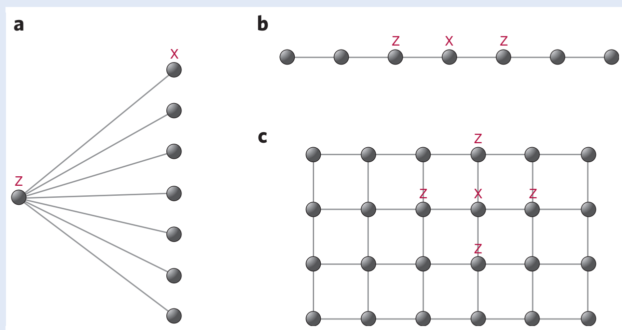

Figure B1 | Three examples of graph states.
a, Greenberger–Horne–Zeilinger state. **b**, 1D cluster state. **c**, 2D cluster state (which is a universal resource for MQC). $X=\sigma_x$ and $Z=\sigma_x$ (Pauli matrices composing one of the K_j).

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quantum algorithm that is to be implemented—one thus calls the cluster state a 'universal resource'. In a second step, the qubits in the system are measured individually, in a certain order and basis, and it is this measurement pattern that specifies the entire algorithm (see Box 2). The quantum algorithm thereby corresponds, in an explicit sense, to a processing of quantum correlations.

The one-way quantum computer is equipped with a remarkable feature, namely that the entire resource for the computation is provided by the entangled cluster state in which the system is initialized. This implies, in particular, that the computational power of such a quantum computer can be traced back entirely to the properties of its entangled resource state, thereby offering a focused way of thinking about the nature and strength of quantum computation. Moreover, the problem of an experimental realization of a quantum computer is now reduced to the preparation of a specific multi-particle state and the ability to carry out single-qubit measurements, offering practical advantages for certain physical set-ups. Finally, a fruitful marriage of ideas from MQC and topological error correction was recently achieved, paving the ground for a scalable computational device that operates in a noisy environment.

The computational scheme of the one-way quantum computer was introduced in ref. 10. This work has inspired numerous studies into MQC, both theoretical and experimental in nature. Apart from offering an alternative approach towards realizing quantum computation, today MQC has become an interdisciplinary field of research, relating entanglement theory, graph theory, topology, computational complexity, logic and statistical physics. Here, we discuss a selection of recent results in MQC, to illustrate the vigour and diversity of research in this field. We will consider MQC in the sense of the one-way quantum computer, but we emphasize that there are other measurement-based approaches to quantum computation—in particular teleportation-based models—as cited above.

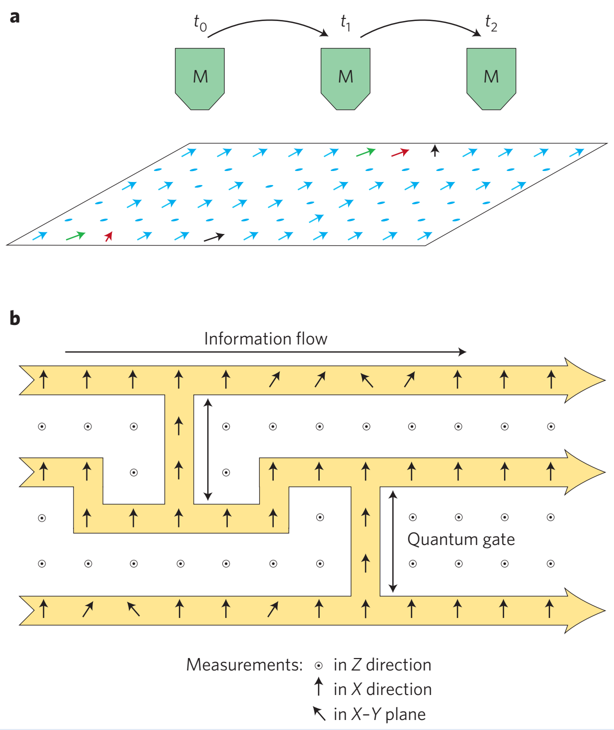
Experimental proposals and achievements

Apart from its useful conceptual status as an alternative model of quantum computation, MQC can have practical advantages over the standard circuit model in a variety of different physical settings, from optical lattices and single photons to spatially separated matter qubits. In an optical lattice, cold atoms are kept in a standing-wave potential created by counter-propagating laser fields. The potential minima create a lattice of sites in which individual atoms can be trapped, storing quantum information in their long-lived internal states (see Fig. 1a). Tuning the polarization of the trapping lasers can induce entangling interactions[22,23] between neighbouring atoms across the array and create a cluster state across the whole lattice[20,24,25]. In recent years, there has been substantial experimental progress in the trapping, cooling and manipulation of ultracold atomic gases in optical lattices in one, two and three dimensions (see, for example ref. 26). In particular, the creation of a Mott insulator state with a crystal-like arrangement of single atoms in the lattice[27,28], and the realization of controlled entangling collisions[22,25] have been key achievements for the coherent control of matter on the atomic level in these systems. Recent experiments use exchange interactions in double-well potentials to create arrays of robust Bell pairs, which could be used as an alternative way to create cluster states in the lattice[29,30,36].

A remaining obstacle to the implementation of MQC in these systems is that the lattice spacing is typically of the order of the wavelength of the trapping light, too small for individual atoms to be addressed and measured. However, recent progress in the creation of lattices with wider spacing[31], sorting atoms in periodic potentials[32], proposed methods for single-site addressing in tighter lattices[33], as well as new methods achieving subwavelength resolution[34,35], are promising. Combining single-site addressing and lattice-wide entangling operations would enable large-scale one-way quantum computation to be realized in an optical lattice system.

Proof-of-principle experiments of a one-way quantum computer with few qubits have already been carried out in

Box 2 | The one-way quantum computer.



In contrast to the quantum circuit model, where quantum computations are implemented by unitary operations, in the one-way quantum computer, information is processed by sequences of single-qubit measurements[10]. These measurements are carried out on a universal resource state—the 2D-cluster state[20]—which does not depend on the algorithm to be implemented. A one-way quantum computation proceeds as follows (see **a** and **b**). (1) A classical input is provided, which specifies the data and the program. (2) A 2D-cluster state $|\mathcal{C}\rangle$ of sufficiently large size is prepared. The cluster state serves as the resource for the computation. (3) A sequence of adaptive one-qubit measurements M (see **a**) is implemented on certain qubits in the cluster. In each step of the computation, the measurement bases (see **b**) depend on the program and on the outcomes of previous measurements. A simple classical computer is used to compute which measurement directions have to be chosen in every step. (4) After the measurements, the state of the system has the form $|\xi^\alpha\rangle|\psi^\alpha_{\rm out}\rangle$, where α indexes the collection of measurement outcomes of the different branches of the computation. The states $|\psi^\alpha_{\rm out}\rangle$ in all branches are equal to the desired output state up to a local (Pauli) operation; the measured qubits are in a product state $|\xi^\alpha\rangle$, which also depends on the measurement outcomes. The one-way quantum computer is universal: even though the results of the measurements in every step of the computation are random, any quantum computation can deterministically be realized. The temporal ordering of the measurements has an important role and has been formalized, for example in refs 11,15,17. For different perspectives and recent reviews on MQC, see refs 14,18,104–106. Part **b** reprinted with permission from ref. 10 © 2001 APS.

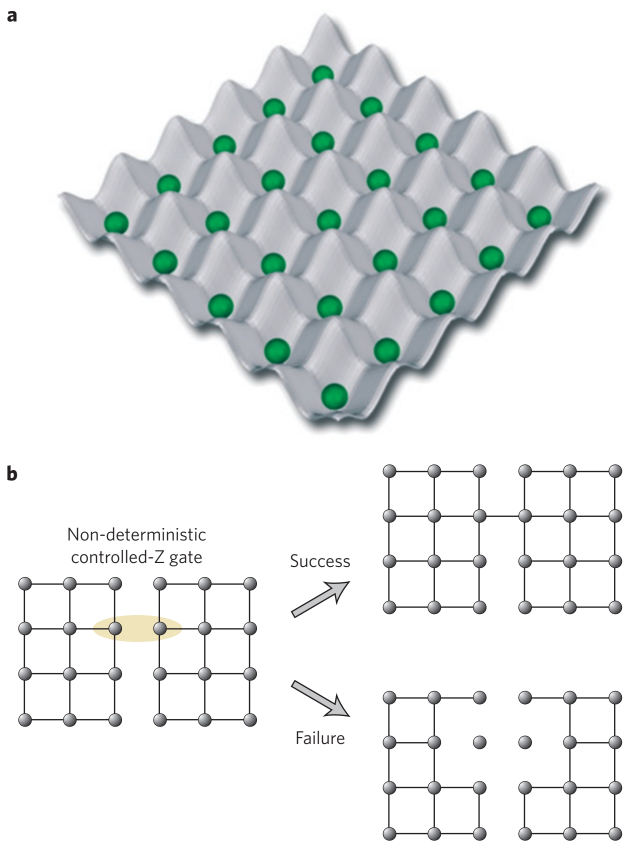


Figure 1 | Practical implementations One-way quantum computation can be implemented in a variety of physical systems, including systems that offer massively parallel operations, such as atoms in optical lattices, and systems where two-qubit quantum gates are not deterministic, such as photons with linear optics. **a**, Cold neutral atoms are trapped in an optical lattice, a standing-wave potential formed from counter-propagating laser fields that holds atoms in a three-dimensional array (reproduced from ref. 26). State-dependent entangling operations can be realized in parallel across the lattice, by tuning the trapping fields[22]. As was experimentally demonstrated in ref. 25, this can be used to generate, in few steps, a cluster state over the entire lattice (see also ref. 24). If the problem of addressing single lattice sites could be solved (see discussion in the main text), this would open the way for a large-scale implementation of one-way quantum computation. **b**, One-way quantum computation enables scalable quantum computation also with non-deterministic entangling logic gates. With probability $p < 1$, the gate is achieved as required, but otherwise the participating qubits are measured. In a circuit-based approach, this would destroy the coherence of the computation state, disrupting the computation. Cluster states can, however, be efficiently built with such operations, because a successful entangling operation increases the number of cluster-state qubits. A failure, while reducing the number of entangled qubits, leaves the remaining qubits in an intact cluster state. Strategies can be adopted to enable the efficient generation of cluster states of any size.

which qubits are represented by photons. A photon can encode a qubit, for example, in its polarization, or in spatial degree of freedom. The generation and detection of single photons is making great advances and single-qubit operations can be achieved with precision through interferometers or wave plates. However, the deterministic two-qubit gates, which would enable universal quantum computation, cannot be achieved with interferometric techniques (that is, with linear optics) alone. By adding photon counters to interferometric networks, non-deterministic entangling gates can, however, be achieved[7]. For some measurement outcomes, the gate is successful; for other 'failure outcomes', the photon states are measured. In a standard quantum circuit, these failure outcomes would be very damaging, because the measurement would destroy the coherence of the state, disrupting the computation.

The one-way quantum computation model provides a way to scalable quantum computation with such non-deterministic gates[37–41], too. The non-deterministic gates are used to create the cluster state, which can be done offline and stochastically (see Fig. 1b). Once the cluster state has been created, one can then proceed deterministically with the one-way quantum computation. Furthermore, a polarizing beam splitter provides a simple non-deterministic entangling gate for efficient cluster state generation[42].

For the above linear optical schemes to be truly scalable, single-photon detectors and generators with extremely high efficiency are required, beyond the capabilities of current experiments. However, demonstration experiments of the key components have been achieved, using post-selection[43–49]. Data are kept only when every detector fires, enabling loss or inefficiency errors to be discounted. These achievements have demonstrated the basic principles of one-way quantum computation, with recent experiments including simple algorithms and active feedforward of measurement results[46,49].

In addition to all-optical approaches, hybrid optical–matter schemes[38] seem increasingly promising. In these schemes, matter qubits—such as atoms, quantum dots or diamond nitrogen–vacancy centres—are kept isolated from one another, for instance in separate cavities. Entangling operations can be achieved non-deterministically, through the emission of photons, entangled with the qubits, and linear optical entangling gates[50,51]. Owing to the non-deterministic nature of the entangling gates, the one-way model seems again a natural approach to scalable quantum computation either through non-deterministic strategies[39,53], repeat-until-success strategies[52] or so-called 'broker–client' approaches[54]. Such approaches carry significant advantages: individual qubits are isolated from one another, reducing correlated error, and the modularity of the approach facilitates scaling up to many qubits.

Scaling up these approaches would seem to require optical networks of increasingly complicated switching circuits—a potential barrier to their implementation. However, by exploiting a phenomenon known as percolation, scalable quantum computation with non-deterministic gates is possible even with simple non-switching optical circuits[41]. Also in these implementations, the simplicity and uniformity of the entanglement structure of cluster/graph states[21] (see Fig. B1) is essential for this approach to succeed.

In addition to these studies, there has been a series of proposals on ways to implement one-way quantum computation (or to create cluster states) in solid-state systems, using superconducting circuits[55] and quantum dots[56,57].

Topological protection and fault-tolerant computation

In real physical systems, decoherence tends to make quantum systems behave more classically. One could therefore expect that decoherence would threaten any computational advantage possessed by a quantum computer. However, the effects of decoherence can be counteracted by quantum error correction[58]. In fact, arbitrarily large quantum computations can be carried out with arbitrary accuracy, provided the error level of the elementary components of the quantum computer is below a certain threshold. This important result is called the threshold theorem of quantum computation[59–62].

Recent work has been dedicated to bringing fault tolerance closer to experimental reality. Proofs of fault tolerance have been derived for error models showing the characteristic features of realistic physical systems such as long-range correlated errors[63]. Moreover, a very high threshold of 3% (that is, on average, one gate in thirty

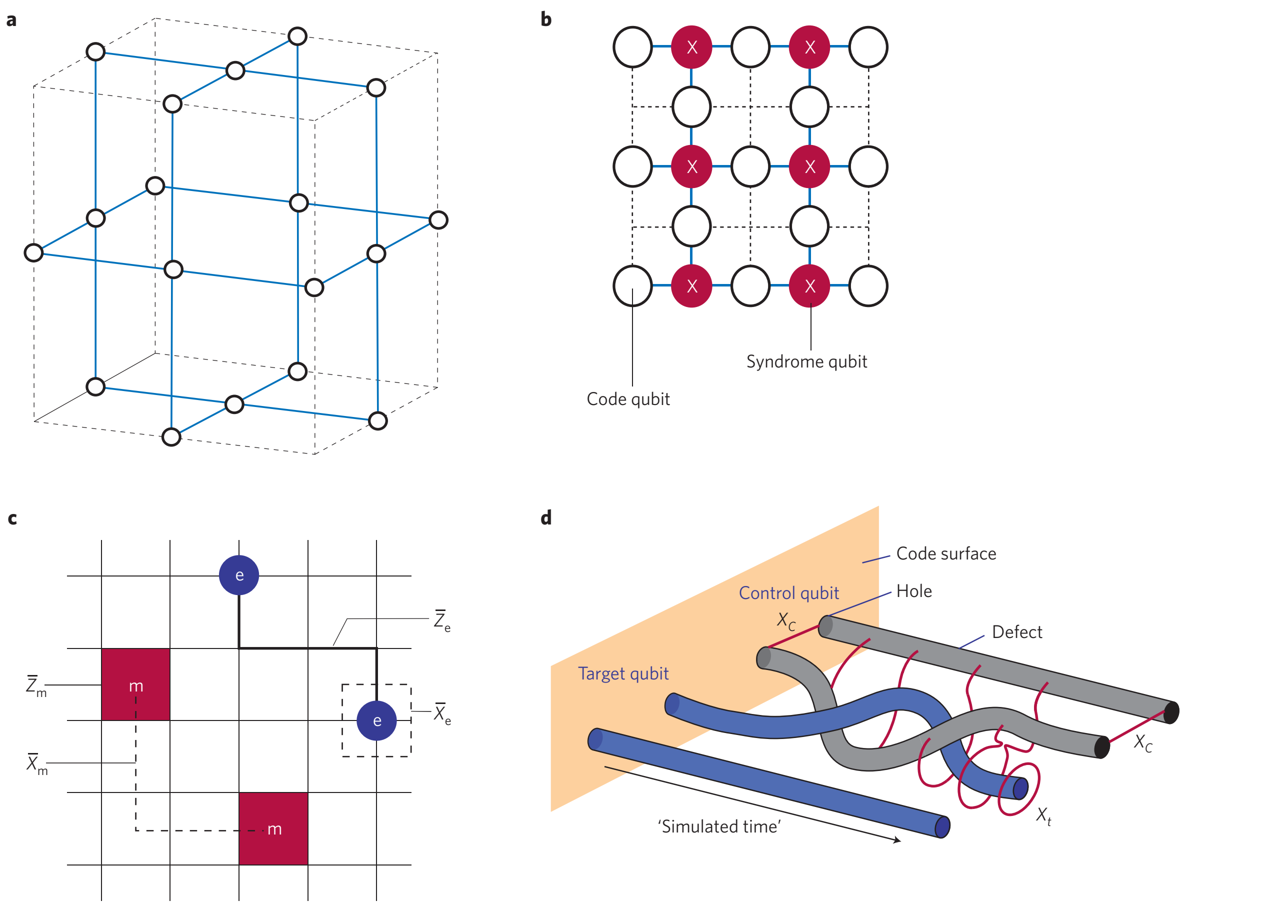


Figure 2 | Topological fault tolerance from 3D cluster states. **a**, Elementary cell of the cluster lattice (reprinted from ref. 73 © 2006 Elsevier). **b**, A single 2D layer of the cluster. If the syndrome qubits are measured in the X-basis, the code qubits are projected into a surface code[76] (code lattice indicated by dashed lines). **c**, A surface code with electric and magnetic holes, pairwise forming encoded electric and magnetic qubits, respectively. Strings supporting the encoded Pauli operators are also shown (parts **b**, **c** reprinted from ref. 75 © 2007 APS). **d**, 3D cluster for a CNOT gate between two encoded qubits. The gate is implemented by a monodromy between world lines of holes in the code surface. The holes evolve in 'simulated time' (the third cluster dimension). Also shown is the string corresponding to an encoded Pauli operator $\overline{X}$ on the control qubit and its evolution from the initial to the final code surface.

is allowed to fail) has been obtained for a method using offline preparation and post-selection within the circuit model[64].

What is the status of fault tolerance in MQC? Results have so far been obtained for the one-way quantum computer. The existence of a non-zero threshold was first proven by reduction to the circuit model[65–67]. Subsequent developments evolved along two lines. First, after it was realized[37,42,68] that the one-way quantum computer may be advantageously combined with the Knill–Laflamme–Milburn (KLM) scheme of optical quantum computation[7], fault-tolerant schemes using photons were developed[69–71]. The dominant sources of error in this setting are photon loss and gate inaccuracies. The constraint of short-range interaction and arrangement of qubits in a 2D lattice—a characteristic feature of the initial one-way quantum computer—is not relevant for photons. In ref. 69, both photon loss and gate inaccuracies were taken into account, yielding a trade-off curve between the two respective thresholds. Fault-tolerant optical computation is possible, for example, for a gate error rate of 10^{-4} and photon loss rate of 3×10^{-3} (the stability against the main error source of photon loss is discussed in ref. 71). With non-unit efficiencies η_S and η_D of photon creation and detection as the only imperfections, the very high threshold of $\eta_S\,\eta_D > 2/3$ was established[72].

A second line of research[73] kept the geometric constraint of nearest-neighbour interaction, which is a realistic scenario for stationary qubits. To achieve fault tolerance, it is then advantageous to increase the lattice dimension from two to three. A 3D cluster state combines the universality already found in the 2D counterpart with the topological error-correction capabilities of the toric code[74]. Error correction is directly built into the cluster lattice and yields a threshold of 6.7×10^{-3} (ref. 75), for a model with probabilistic gate errors, including imperfect preparation of the cluster state.

Topologically protected quantum gates are carried out by measuring some regions of qubits in the Z-basis, which effectively removes the qubits from the state. The remaining cluster, the qubits of which are measured in the X- and $X\pm Y$-basis, thereby attains a non-trivial topology in which fault-tolerant quantum gates can be encoded. A topological method of fault tolerance can then be achieved[77]. We shall explain this method in some detail. First, we map the 3D cluster state to a surface code[76] propagating in time. We consider a 3D cluster with an elementary cell as in Fig. 2a, and single out one spatial direction on the cluster as 'simulated time'. As a first step, we consider a perpendicular 2D slice of this cluster, as shown in Fig. 2b. The qubits are subdivided into code and syndrome qubits. As can be easily verified, measurement of the syndrome qubits in the X-basis projects the code qubits into a surface code state. In a 3D cluster consisting of many linked 2D slices, measurement of the code qubits results in teleportation of the encoded state from one slice to the next (plus local Hadamard gates), and measurement of the syndrome qubits amounts to measurement of the surface code stabilizer.

Now note that we can modify the code surface of Fig. 2a by changing measured observables from X to Z. For example, if we measure a syndrome qubit on plaquette p in the Z-basis, we choose to not read out the surface code stabilizer $B_p = \bigotimes_{f\in\partial p} Z_f$ of the corresponding plaquette p. If we measure a code qubit on edge e in the Z-basis, then we destroy syndrome information. Specifically, the eigenvalues of the surface code stabilizers $A_s = \bigotimes_{f|s\in\partial f} X_f$ on sites adjacent to e will become undefined. These elementary operations lead to techniques for manipulating the code surface. First, we can punch holes into it. Holes come in two types, electric and magnetic. An electric hole is a site s where the condition $\langle A_s\rangle = 1$ is not imposed on the code space. Two electric holes support an electric encoded qubit. Similarly, a magnetic hole is a plaquette p with $\langle B_p\rangle = 1$ not imposed on the code space. Two magnetic holes support a magnetic qubit. Now, the encoded Pauli operators $\overline{Z}_e$, $\overline{X}_e$ and $\overline{Z}_m$, $\overline{X}_m$ for these encoded qubits can be described in geometric terms. Namely, (E) for the electric qubit on $\{s, s'\}$, $\overline{Z}_e$ is a tensor product of Pauli operators Z along a string stretching from s to s'. $\overline{X}_e$ is a tensor product of Pauli operators X along a string looping around either s or s'. (M) for the magnetic qubit on $\{p, p'\}$ is just the same, with the roles of Z and X interchanged; see Fig. 2c. To protect these encoded qubits against harmful errors, the holes are enlarged from one site or plaquette to extended connected sets of sites or plaquettes.

By slightly shifting the locations of Z-measurements from one cluster slice to the next, the holes in the code surface can be moved and fused. This gives rise to encoded unitary gates and measurements, respectively. As an example, consider the topological encoded CNOT gate shown in Fig. 2d, between a magnetic control and electric target qubit. It is realized by 'moving' one of the electric holes around one of the magnetic holes. Again, by 'moving', we mean slightly shifting the locations of Z-measurements in consecutive cluster slices. It is important to note that when the holes are moved, the above rules (E), (M) continue to apply. Thus, the strings corresponding to the encoded Pauli operators are dragged along. We can now easily verify the four conjugation relations for the CNOT gate in a topological manner, by sliding the operator strings forward along the hole world lines. One example, $X_c \longrightarrow X_c \otimes X_t$ (t: target, electric; c: control, magnetic) is shown in Fig. 2d; the other three are similar. Note that fault-tolerant CNOT gates between two electric or two magnetic encoded qubits can also be carried out but are more complicated; they require fusion of holes[75].

Finally, by way of the described mapping between the 3D cluster state and a 2D surface code changing in time, there exists a circuit variant of fault-tolerant cluster-state computation that requires only a 2D lattice of qubits. In addition, this 2D variant requires only translation-invariant nearest-neighbour interaction. It yields a threshold value of 7.5×10^{-3} (ref. 77), which is the highest known threshold for a 2D local architecture (see also ref. 78). This scenario is suited for realization in, for example, optical lattices, but also arrays of superconducting qubits and ion traps.

Entanglement as a resource for computational power

In MQC, universal quantum computation is realized by carrying out sequences of single-qubit measurements on a system that has initially been prepared in a 2D-cluster state. As individual measurements can only destroy entanglement, the entire computational power of the one-way quantum computer is carried by the entanglement structure of its resource state. Can we understand and quantify what are the essential features that make the 2D-cluster state, and possibly other states, 'universal resources', which are capable of enabling arbitrary measurement-based quantum computations? There has been recent progress on this issue, but some important questions remain.

A first feature that is to be emphasized, is that there exist several natural notions of 'universality'[79]. In its strongest form, universality is defined as the capability of generating every possible quantum state from the resource by means of single-qubit operations. As an important example, the 2D-cluster state is a universal resource in this sense. A universal measurement-based quantum computer is then identified as a device that enables universal state preparation by local operations alone[79,80]. This implies that, whenever any given type of entanglement—as quantified by an appropriate entanglement measure[81]—is to be generated from the resource state, it must already be present in the resource itself, as single-qubit operations cannot add entanglement to the system. Using this intuition, it can be shown that every such universal resource must exhibit maximal (scaling of) entanglement with respect to essentially all types of entanglement[79,80]. The 2D-cluster state provides a key example of such a maximally entangled resource state, but we emphasize that the entanglement criteria hold for every possible universal state preparator.

This insight can be used to develop a systematic framework to investigate which states are universal state preparator resources for MQC, and in particular to obtain no-go results. Following this approach, it can be shown that, for example, n-particle 1D-cluster states, Greenberger–Horne–Zeilinger states, W-states, Dicke states and certain ground states of strongly correlated 1D spin systems, are not universal resources[79]. Note that many of the above states are considered to be highly entangled. However, in each case there is at least one type of entanglement that is non-maximal, implying that these states cannot be universal state preparator resources.

States that do not violate any of the entanglement criteria include the graph states[21] associated with various types of regular 2D lattice (triangular, hexagonal, kagome), as well as lattices with a high degree of defects (up to about 41% corresponding to the classical site-percolation threshold in 2D); in fact, it can be proven that such states are universal in the same way as the 2D-cluster states[79,82].

The notion of 'universality' in MQC used in the preceding discussion is the strongest one possible, and can be relaxed in several ways. Most importantly, one may study an altogether different concept of universality, where the goal is not to prepare arbitrary quantum states as outputs. In contrast, it is only required that a universal measurement-based quantum computer is capable of (efficiently) reproducing the classical output of any quantum computation implemented on a standard gate array quantum computer. Even though it seems difficult to formulate entanglement-based criteria for this form of universality, one can study it from a constructive perspective: in refs 19,85 it is shown how such universal resources for MQC—beyond the 2D-cluster states—can systematically be constructed. The underlying structure of this approach can be described mathematically using the language of matrix-product states or projected entangled pairs[18,86,87]. The main conclusion of this investigation is that a number of extremal entanglement features (such as maximal localizable entanglement) exhibited by 2D-cluster states no longer have to be present in universal resources if only classical outputs are considered[19,85]. At present, it is not clear whether the latter form of universality is fundamentally distinct from universal state preparation under relaxed conditions, such as encodings[36,79,83,84]. This issue is now under investigation.

When a state is identified as not being a universal resource, this does not necessarily mean that it could not be used for a specific quantum computational task for which it may still outperform classical computers. This naturally leads to a study of classical simulation of MQC, where we ask which resource states do not offer any computational speed-up with respect to classical computation. This question is closely related to major investigations in condensed-matter theory, where one studies under which conditions quantum systems can efficiently be described and simulated.

Recently, several techniques have been developed to tackle classical simulatability of MQC (refs 88–91). For example, for most states that have been identified above as not being universal, it has been shown that efficient classical simulation is possible. More precisely, for such states it is possible to efficiently and exactly compute the outcome probabilities of any sequence of single-qubit measurements. Further examples of simulatable states include the toric code states[92] and the ground states of several 1D spin systems[93]. The techniques invoked to obtain these results are again centred around entanglement; for example, the entanglement measure 'entanglement width' can be used to identify efficiently simulatable states[91,94].

Here, we have described two—in some sense complementary—investigations regarding the origin of the quantum computational power. Even though these investigations were carried out within the specific framework of the one-way computer, the insights are of general relevance, because the different quantum computational models can simulate one another efficiently and are thus, in this complexity-theoretic sense, equivalent.

Although significant progress has been obtained in the issues of universality and simulatability, these matters are far from being fully understood. Most importantly, it is at present not known whether a universal quantum computer disallows efficient classical simulation—in other words, whether quantum computers are truly (exponentially) 'more powerful' than classical ones. Nevertheless, we believe that the recent studies provide the first steps to understanding this important but difficult question.

MQC and classical statistical mechanics

We have already seen that the study of the principles of MQC turns out to be connected to different fields, such as entanglement theory, topology and graph theory. We discuss now how some of the central questions raised in the study of MQC are related to notions of the statistical physics of classical spin systems, such as the Ising model and the Potts model[97]. Such spin models were introduced in the context of (anti-)ferromagnetism, but they seem to have widespread applications not only in physics, but also, for example, in optimization theory and biology[98].

Let us illustrate these connections by considering the example of the Ising model in the presence of an external field. In the Ising model, one considers a large lattice $\mathcal{L}$ (for example, a 2D square lattice) of (classical) spins $s_a = \pm 1$. The spins interact according to the Hamiltonian function

$$H_{\mathcal{L}}(\{s_a\}) = -\sum_{\langle a,b\rangle} J_{ab} s_a s_b - \sum_a h_a s_a.$$

The couplings J_{ab} and h_a determine the strength of the pairwise interaction and the external field, respectively. The lattice $\mathcal{L}$ may be arbitrary, in the sense that lattices of arbitrary dimension—and in fact arbitrary graphs—are possible. The partition function $Z_{\mathcal{L}} = \sum_{\{s_a\}} \mathrm{e}^{-H_{\mathcal{L}}(\{s_a\})/(k_{\mathrm{B}}T)}$, where k_{B} is the Boltzmann constant and T is the temperature, is a central quantity in this context, and from it other relevant system properties such as free energy or magnetization can be derived.

The connection between MQC and the Ising model is given by a mapping of the Ising model on an arbitrary lattice $\mathcal{L}$ to an MQC model where the entangled resource is determined by the geometry of $\mathcal{L}$,

$$Z_{\mathcal{L}} \cong \langle \psi_{\mathcal{L}} | \bigotimes_i |\alpha_i\rangle. \tag{1}$$

This expression states that the partition function $Z_{\mathcal{L}}$ is identified with a quantum mechanical amplitude that is obtained as an overlap between two quantum states[92,95,96,99]. The multi-particle entangled state $|\psi_{\mathcal{L}}\rangle$ (ref. 95) encodes the interaction pattern and is a graph state[21] (see Fig. B1). The product state $\bigotimes_i |\alpha_i\rangle$ contains no entanglement and specifies interaction strengths and local magnetic fields as well as the temperature of the model.

How does the expression (1) enable us to connect this model with MQC? To see this, simply consider an MQC with the state $|\psi_{\mathcal{L}}\rangle$ as a resource state. Then, according to (1), the (computation of the) partition function corresponds to a specific measurement pattern on the resource state $|\psi_{\mathcal{L}}\rangle$. In this way, a connection is drawn between concepts from MQC and statistical physics.

This simple connection opens the possibility to obtain a cross-fertilization between statistical mechanics and MQC. For example, one finds a notable relation between the solvability of the Ising model on a lattice $\mathcal{L}$ and the computational power of an MQC operating on a resource state $|\psi_{\mathcal{L}}\rangle$. This brings us back to the issue of the power of quantum computation, and the possibility (or impossibility) of an efficient classical simulation. The central quantities to be considered in this context are overlaps between the resource state and product states: these quantities need to be computed to determine with which probabilities the outcomes of local measurements occur, and exactly those overlaps are identified with the partition function $Z_{\mathcal{L}}$. Equation (1) now implies that any model where the partition function can efficiently be computed, leads to a corresponding MQC that offers no computational advantage over classical devices, and vice versa. For example, the solvability of the 2D Ising model without magnetic fields implies that MQC on the toric code state[74] can be efficiently simulated[92]. In turn, the efficient classical simulation of MQC on stabilizer states with bounded tree width[94], as demonstrated in ref. 91, yields a novel classical algorithm to efficiently calculate the partition function on (inhomogeneous) q-state spin models on tree-like-graphs with or without magnetic fields[95,96].

The relation (1) can also be exploited in a different sense. For example, in ref. 95, the universality of the 2D-cluster states was used to prove that the 2D Ising model with magnetic fields is 'complete' in the sense that the partition function of any q-state spin model on an arbitrary lattice can be expressed as a special instance of the partition function of the 2D Ising model (in a complex parameter regime). We believe that several other interesting applications can be found. For one, these connections enable one to phrase statistical physics problems naturally in a quantum mechanical setting[87,92,95,96,99–102]. This may open a new path towards, for example, quantum algorithms for problems in this area.

Finally, further connections between MQC and other fields have been established, for example to decidability of formal languages in mathematical logic[103].

Outlook

The discovery of the one-way quantum computer has opened new experimental avenues towards the realization of quantum computation in the laboratory. At the same time it has challenged the traditional view of the very nature of quantum computation itself.

For the future, we see several open problems and challenges. On the experimental side, one of the main challenges is to realize large-scale quantum computation in the laboratory, beyond proof-of-principle demonstrations. For measurement-based schemes, we believe that optical lattices are still one of the most suitable candidates that enable us to create large-scale cluster states with high efficiency. However, the problem of addressing single sites in the lattice—and thus to fully implement one-way computation—remains unsolved so far, even though there are new and encouraging developments. Recent progress with photonic one-way quantum computation is exciting, and few-qubit applications, such as in the context of the quantum repeater, are conceivable; however, for a scalable set-up that could go much beyond proof-of-principle experiments, new single-photon sources with higher efficiency are needed. In the meantime, a variety of new and promising proposals for one-way quantum computation using hybrid systems have been put forward, which combine the advantages of different physical implementations. It remains to be seen which system will become practical in the long run. Apart from engineering issues, the capabilities of the system to naturally accommodate quantum error correction (that is, on the hardware level), and therefore to facilitate fault-tolerant operation, will have an important role.

On the theoretical side, the study of fault-tolerant schemes and the search for new quantum algorithms will remain central issues, related to the fundamental issue of universality, classical simulation and the role of entanglement. In the context of measurement-based computation, a deeper understanding of universality (that is, the properties that make an entangled state a universal resource) will help us to find resource states that are tailored to specific physical systems. It will also provide a basis to improve schemes for fault-tolerant computation, for example by choosing more robust states or by reducing the physical overhead. A deeper understanding of efficient classical simulation, on the other hand, will narrow down the set of interesting quantum algorithms. A promising strategy to find new quantum algorithms is to connect quantum computation with other fields. An example of such a connection (with classical statistical mechanics) has been given here, but there seem to be many more.

In conclusion, it seems that the conceptual framework of the one-way model continues to be an attractive alternative platform for experimental and theoretical investigations of quantum computation and its ramifications.

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