QM Review Notes

HqB

TACTICS FOR QM 1

important details to know

- **Perturbation Th**
  - non degenerate < 1st order / 2nd order
  - degenerate
  - Stark shift, corrections in $\mathcal{H}$
- **QHO**
  - regular stuff, with ladder operators
  - in a magnetic + electric field
    ↳ gauges → Landau levels
  - energy levels
  - spherical harmonics ?

- Addition of angular momentum

↳ change of basis.

Clebsch-Jordan coeff

- degeneracy, state counting
- symmetries
- orbital angular momentum
- Formulas for various operators

L^2, L_z, L_x, L_y, L_+, L_-

...

- $L \cdot S$ coupling
- Zeeman splitting etc
- spherical harmonics, Y_ℓ^m,

decomposition in spherical harmonics to extract eigenvalues....

- Spin chains,

  Pauli matrices, measurements,

  Projection operators, probability,

  etc.

- WKB & variational principle

  - ground state energy estimation

    for certain problems

  - Gaussian family

- Quantum dynamics

  - Time evolution op

  - Schrödinger vs Heisenberg picture

  - Heisenberg EOM

  - [Tricky] time derivations ...

- <u>Special Topics</u>

    - ⊕ Coherent States
    - ⊕ Squeezed states
    - ⊕ Landau levels
    - ⊕ Feynmann Path Integrals (formulation)
    - ⊕ Interpretation stuff (probability density, flux, phase, ...) flow, ...
    - ⊕ Euler Angles (midterm)
    - ⊕ Gauge transformation stuff (E&M mostly)

$\boxed{\text{New, content + references}}$

1 Perturbation Theory (time-independent)

2 cases

Non degenerate

$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{pert}$

$$\boxed{E_n^{(k)} = \langle n^{(0)} | \mathcal{H}_{pert} | n^{(k-1)} \rangle}$$

in particular First-order

$$\boxed{E_n^{1} = \langle n^{(0)} | \mathcal{H}_{pert} | n^{(0)} \rangle}$$

unperturbed wfn

1^{st} order correction to E_n of n^{th} unperturbed wfn eigV

First-order correction to wfn

$$\boxed{\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | \mathcal{H}_{pert} | \psi_n^0 \rangle}{(E_n^0 - E_m^0)} \psi_m^0}$$

only works if energy spectrum is nondegenerate.

Second order correction (if first-order finds 0)

$$\boxed{E_n^2 = \sum_{m \neq n} \frac{|\langle \psi_m^0 | \mathcal{H}_{pert} | \psi_n^0 \rangle|^2}{E_n^0 - E_m^0}}$$

don't care about 2^{nd} order correction to wfn ...

Degenerate pert. theory

Technique here is to calculate the perturbation matrix W_{ij} where

$$\boxed{W_{ij} = \langle \psi_i^0 | \mathcal{H}_{pert} | \psi_j^0 \rangle}$$

required when $E_i^0 - E_j^0 = 0$

Then, diagonalize & find eigenvalues of W

→ 1st energy corrections ...

Remember to keep track of the basis ...

$$\boxed{\text{Brief summary of perturbation Theory applied to Hydrogen}}$$

(time-independent)

Recall wfn

$$\boxed{\psi_{n\ell m_\ell} = R_{n\ell}(r)\, Y_\ell^{m_\ell}(\theta,\phi)}$$

(details in <u>Notes 12-6+8-21</u>)

E_n has n^2 degeneracy

($2n^2$ if count spin $1/2$ of electron)

<u>Examples of perturbation theory</u> 7

- Relativistic correction (fine structure)
- Stark effect (quad)
- Zeeman effect
- $\vec{L}\cdot\vec{S}$ coupling
- Stark effect (linear)
- Hyperfine
- Van der Waals

Of these,

nondegenerate	Degenerate
Relativistic (FS) — fine structure	VdW
Quadratic Stark (n=1)	Zeeman
	Hyperfine (Hfs)
	$\vec{L}\cdot\vec{S}$
	Lin. Stark (n=2)

Might very well appear on exam:

Zeeman effect @ weak / strong / intermediate B

$$\boxed{\mathcal{H}_Z' = \frac{e}{2m}(\vec{L}+2\vec{S})\cdot\vec{B}_{ext}}$$

Fine structure dominates

Weak B $\Rightarrow$ good quantum numbers are

n, ℓ, j, m_j

details in Griffiths pg 245

$$\boxed{E_Z' = \langle n \ell j m_j | \mathcal{H}_Z' | n \ell j m_j \rangle}$$

$= \mu_B g_J B_{ext} m_j$

Strong B ⟶ good quantum numbers are

$\hookrightarrow B_z \gg B_{int}$ (fs)

$$\boxed{n,\ \ell,\ m_\ell,\ m_s}$$

In fact, in exact calculation, use this basis, and everything ok.

$$\boxed{H_Z' = \frac{e}{2m} B_{ext} (L_z + 2S_z)}$$

↳ get

$$\boxed{E_{n m_\ell m_s} = \frac{-13.6\,eV}{n^2} + \mu_B B_{ext} (m_\ell + 2m_s)}$$

⟶ need fs correction in this

$$\boxed{E_{fs}' = \frac{13.6\,eV}{n^3} \alpha^2 \left\{ \frac{3}{4n} - \left(\frac{\ell(\ell+1) - m_\ell m_s}{\ell(\ell+1/2)(\ell+1)} \right) \right\}}$$

Intermediate B

Then

$$\boxed{\mathcal{H}' = \mathcal{H}_Z' + \mathcal{H}_{fs}'}$$

Can work in basis

$\{n, \ell, j, m_j\}$ <u>OR</u> $\{n, \ell, m_\ell, m_s\}$

Doesn't really matter ...

Preferred by Griffiths

$\rightarrow$ give same result anyway.

Nugget

$\boxed{\text{Feynmann - Hellmann Thm}}$

$$\frac{\partial E_n}{\partial \lambda} = \langle \psi_n | \frac{\partial \mathcal{H}}{\partial \lambda} | \psi_n \rangle$$

Other stuff look at

Lecture notes $\boxed{12-6+8-21}$

or Griffiths.

Things to know regarding

- Angular momentum
- Addition of angular momentum
- Symmetries in QM
- Spins

First, well-known stuff

- $\vec{L} = \vec{r} \times \vec{p}$
- $L_x = y p_z - z p_y = \frac{\hbar}{i}(y\partial_z - z\partial_y)$
- $L_y = z p_x - x p_z = \frac{\hbar}{i}(z\partial_x - x\partial_z)$
- $L_z = x p_y - y p_x = \frac{\hbar}{i}(x\partial_y - y\partial_x)$

- $[L_x, L_y] = i\hbar L_z$
- $[L_y, L_z] = i\hbar L_x$
- $[L_z, L_x] = i\hbar L_y$

- $[L^2, L_i] = 0 \quad \forall\, i = x, y, z$ $\quad [L^2, \vec{L}] = 0$
- $\sigma_{L_x} \sigma_{L_y} \geq \frac{\hbar}{2} |\langle L_z \rangle|$
- $L^2 = L_x^2 + L_y^2 + L_z^2 \quad (\text{rfc})$
- $L_\pm = L_x \pm i L_y$
- $[L_z, L_\pm] = \pm \hbar L_\pm$
- $[L^2, L_\pm] = 0$
- $L_\pm$ are raising / lowering ops
  $\pm$ by $\hbar$
- $L^2 |\ell, m\rangle = \ell(\ell+1)\hbar^2 |\ell, m\rangle$
- $L_z |\ell, m\rangle = m\hbar |\ell, m\rangle$

$\ell = 0, 1/2, 1, 3/2, \ldots$

$m = -\ell, -\ell+1, \ldots, \ell-1, \ell \quad (\# = 2\ell+1)$

- $L_\pm |\ell, m\rangle = \hbar\sqrt{\ell(\ell+1) - m(m\pm1)}\,|\ell, m\pm1\rangle$

- (Spherical Harmonics now)

- $L_x = \frac{\hbar}{i}\left(-\sin\phi\,\partial_\theta - \cos\phi\,\cot\theta\,\partial_\phi\right)$

- $L_y = \frac{\hbar}{i}\left(\cos\phi\,\partial_\theta - \sin\phi\,\cot\theta\,\partial_\phi\right)$

- $L_z = \frac{\hbar}{i}\,\partial_\phi$

- $L_\pm = \pm\hbar e^{\pm i\phi}\left(\partial_\theta \pm i\cot\theta\,\partial_\phi\right)$

- Eigenfns: $f_\ell^m(\theta, \phi) = Y_\ell^m(\theta, \phi)$

$\rightsquigarrow$ use Mathematica to generate $Y_\ell^m(\theta, \phi)$

- For spin (general), replace L by S and that's it (not related to spherical harmonics)

- Clebsch-Gordan coeff

These are the coefs that let us transform from

$$\boxed{|j, m_j\rangle \longleftrightarrow |j_1, m_{j_1}, j_2, m_{j_2}\rangle}$$

In general

$$\boxed{|j\, m\rangle = \sum_{m_1+m_2=m} C^{j_1 j_2 j}_{m_1 m_2 m} |j_1\, m_1\rangle |j_2\, m_2\rangle}$$

$$|j_1 - j_2| < j < |j_1 + j_2|$$

$$m_j = -j, -j+1, \dots, j-1, j$$

How to use the Clebsch-Gordan table

(Griffiths pg. 168)

Ex $\boxed{s_1 = 2,\ s_2 = 1 \text{ with } s = 2,\ m = 1}$

→ go to the 2×1 table ...

now, since we want $|2\,1\rangle$

→ go to col $\begin{array}{|c|} 2 \\ +1 \end{array}$ ~> $\begin{pmatrix} 1/3 \\ 1/6 \\ -1/2 \end{pmatrix}$

Rule add $\sqrt{}$

(−) goes outside

So

$$|2\,1\rangle = \sqrt{\frac{1}{3}}\,(\underset{m_{s_1}}{+2}\ \underset{m_{s_2}}{-1}) + \sqrt{\frac{1}{6}}\,(\underset{m_{s_1}}{+1}\ \underset{m_{s_2}}{0}) - \sqrt{\frac{1}{2}}\,(\underset{m_{s_1}}{0}\ \underset{m_{s_2}}{+1})$$

$$= \sqrt{\frac{1}{3}}\,|\underset{s_1}{2}\,2\rangle|\underset{s_2}{1}\,{-1}\rangle + \sqrt{\frac{1}{6}}\,|2\,1\rangle|1\,0\rangle - \sqrt{\frac{1}{2}}\,|2\,0\rangle|1\,1\rangle$$

s_1, s_2 are fixed

Also look at inverse

$$|s_1 m_1\rangle |s_2 m_2\rangle = \sum_s C^{s_1 s_2 s}_{m_1 m_2 m} |s m\rangle$$

Ex

$$\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle |1\ 0\rangle = \boxed{?}$$

s_1 m_1 s_2 m_2

go to $\left(\tfrac{3}{2} \times 1\right)$ table $\rightarrow$ look at row!

$\boxed{+1/2 \quad 0} \rightarrow \left(3/5 \quad 1/15 \quad -1/3\right)$

So

$$\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle |1\ 0\rangle = \sqrt{\tfrac{3}{5}} \left|\tfrac{5}{2}\ \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{15}} \left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle - \sqrt{\tfrac{1}{3}} \left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

✓

More general theory related to these ...

$\boxed{\text{Symmetry in QM}}$

probably don't have to worry about this ...

Griffiths suffice

Coherent States maybe