QUANTUM MECHANICS
A Quick Guide

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Preface

Greetings,

Quantum Mechanics, A Quick Guide to... is my reading notes from Shankar's *Principles of Quantum Mechanics, Second Edition.* Additional material will come from my class notes and my comments/interpretations/solutions.

Enjoy!

Contents

1 Mathematical Introduction

1.1 Linear Vector Spaces

We should familiar with defining characteristics of linear vector spaces at this point. Here are some important definitions/theorems again:

Definition 1.1. A linear vector space $\mathbf{V}$ is a collection of objects called *vectors* for which there exists

1. A definite rule for summing, and
2. A definite rule for scaling, with the following features:
   - Closed under addition: for $x, y \in \mathbf{V}$, $x + y \in \mathbf{V}$.
   - Closed under scalar multiplication: $x \in \mathbf{V}$, then $ax \in \mathbf{V}$ for some scalar a.
   - Scalar multiplication is distributive.
   - Scalar multiplication is associative.
   - Addition is commutative.
   - Addition is associative.
   - There exists a (unique) null element in $\mathbf{V}$.
   - There exists a (unique) additive inverse.

Vector spaces are defined over some field. The field can be real numbers, complex numbers, or it can also be finite. As for good practice, we will begin to label vectors with Dirac bra-ket notation. So, for instance, $|v\rangle \in \mathbf{V}$ denotes vector $v \in \mathbf{V}$. Basic manipulations of these vectors are intuitive:

1. $|0\rangle$ is unique, and is the null element.
2. $0\,|V\rangle = |0\rangle$.
3. $|-V\rangle = -\,|V\rangle$.
4. $|-V\rangle$ is a unique additive inverse of $|V\rangle$.

The reasons for choosing to use the Dirac notation will become clear later on. Another important basic concept is *linear (in)dependence*. Of course, there are a number of equivalent statement for linear independence. We shall just give one here:

Definition 1.2. A set of vectors is said to be linearly independent if the only linear relation

$$\sum_{i=1}^{n} a_i\,|i\rangle = |0\rangle \tag{1}$$

is the trivial one where the components $a_i = 0$ for any i.

The next two basic concepts are *dimension* and *basis*.

Definition 1.3. A vector space $\mathbf{V}$ has dimension n if it can accommodate a maximum of n linearly independent vectors. We denote this n-dimensional vector space as $\mathbf{V}^n$.

We can show that

Theorem 1.1. Any vector $|v\rangle \in \mathbf{V}^n$ can be written (uniquely) as a linear combination of any n linearly independent vectors.

Definition 1.4. A set of n linearly independent vectors in a n-dimensional space is called a *basis*. So if $|1\rangle, \ldots, |n\rangle$ form a basis for $\mathbf{V}^n$, then any $|v\rangle \in \mathbf{V}$ can be written uniquely as

$$|v\rangle = \sum_{i=1}^{n} a_i |i\rangle . \tag{2}$$

It is nice to remember the following:

$$\boxed{\text{Linear Independence} = \text{Basis} + \text{Span}} \tag{3}$$

When a collection of vectors span a vector space $\mathbf{V}$, it just means that any $|v\rangle \in \mathbf{V}$ can be written as a linear combination of (some of) these vectors.

The algebra of linear combinations is quite intuitive. If $|v\rangle = \sum_i a_i |i\rangle$ and $|w\rangle = \sum_i b_i |i\rangle$ then

1. $|v + w\rangle = \sum_i (a_i + b_i) |i\rangle$.
2. $c |v\rangle = c \sum_i a_i |i\rangle = \sum_i c a_i |i\rangle$.

A linear algebra text will of course provide a much better coverage of these topics.

1.2 Inner Product Spaces

A generalization of the familiar dot product is the *inner product* or the *scalar product*. An inner product between two vectors $|v\rangle$ and $|w\rangle$ is denoted $\langle v|w|v|w\rangle$. An inner product has to satisfy the following properties:

1. Conjugate symmetry (or skew-symmetry):$\langle v|w\rangle = \langle w|v\rangle^*$.
2. Positive semi-definiteness: $\langle v|v\rangle \geq 0$.
3. Linearity in ket: $\langle v|aw + bz\rangle = a \langle v|w\rangle + b \langle v|z\rangle$.
4. Conjugate-linearity in bra: $\langle av + bz|w\rangle = \bar{a} \langle v|w\rangle + \bar{b} \langle z|w\rangle$.

Definition 1.5. An inner product space is a vector space with an inner product.

Definition 1.6. $\langle v|w\rangle = 0 \iff |v\rangle \perp |w\rangle$.

Definition 1.7. The *norm* (or length) of $|v\rangle$ is defined as

$$\|v\| = \sqrt{\langle v|v\rangle}. \tag{4}$$

Unit vectors have unit norm. Unit vectors are said to be *normalized*.

Definition 1.8. A set of basis vectors all of unit norm, which are pairwise orthogonal will be called an *orthonormal basis* or ONB.

Let $|v\rangle = \sum_i a_i |i\rangle$ and $|w\rangle = \sum_i b_i |j\rangle$, then

$$\langle v|w\rangle = \sum_i a_i^* b_i \langle i|j\rangle . \tag{5}$$

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