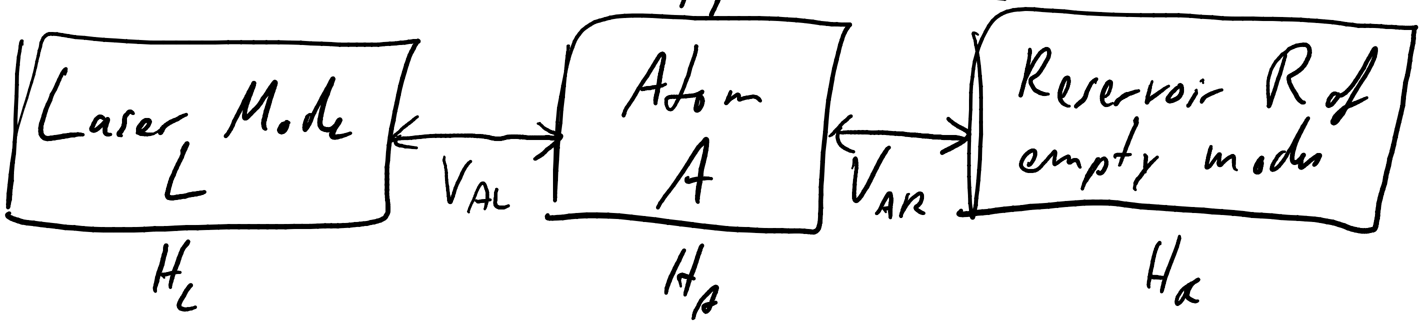
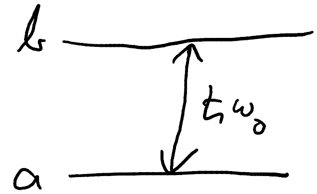
The Dressed Atom Approach   API VI



one atom and one <u>quantum</u> mode of the e.m. field.

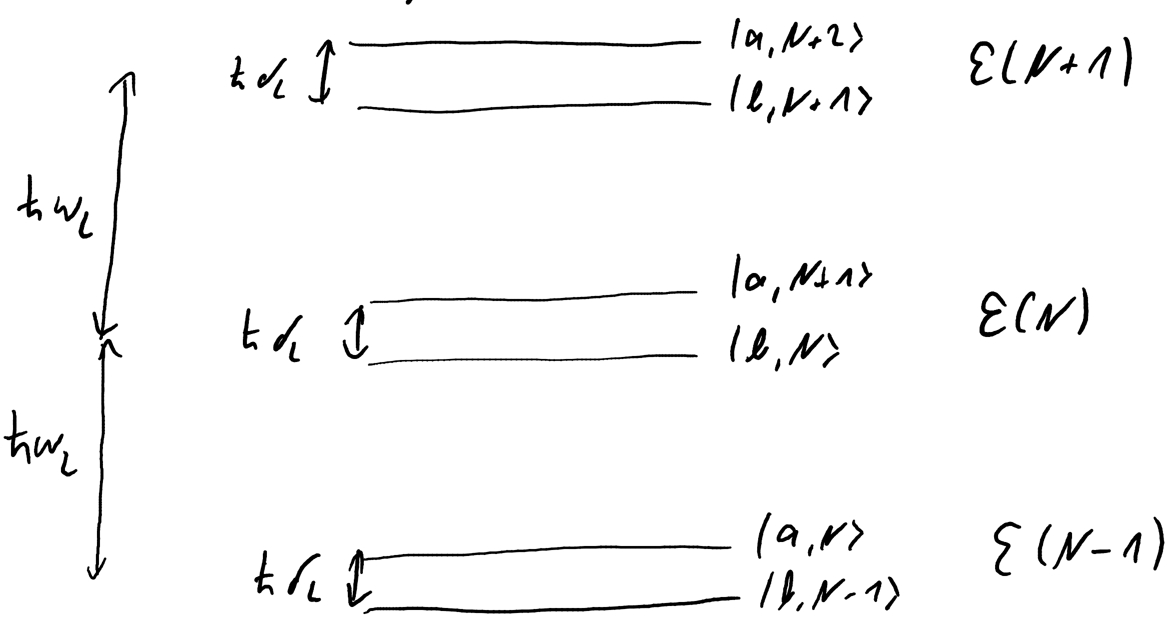
allows <u>time-independent</u> perturbation theory as opposed to the <u>time-dependent</u> hamiltonian for a classical field.

$$H_L = \hbar\omega_L \left(a^+ a + \tfrac{1}{2}\right)$$
$$H_A = \hbar\omega_0 \, |b\rangle\langle b|$$



$$|\delta_L| = |\omega_L - \omega_0| \ll \omega_0$$

Resonant manifold: $\mathcal{E}(N) = \{|a, N+1\rangle, |b, N\rangle\}$



Atom-Laser Coupling

$$V_{AL} = -\vec{d} \cdot \vec{E}_\perp(\vec{R})$$

$$\vec{E}_\perp(\vec{R}) = \sqrt{\frac{\hbar\omega_L}{2\varepsilon_0 V}}\, \vec{\varepsilon}_L (a + a^+)$$

$$\vec{d} = \vec{d}_{ab} (S_+ + S_-) \qquad S_+ = |b\rangle\langle a| \quad S_- = |a\rangle\langle b|$$

$$V_{AL} = g (S_+ + S_-)(a + a^+)$$

$$g = -\vec{\varepsilon}_L \cdot \vec{d}_{ab} \sqrt{\frac{\hbar\omega_L}{2\varepsilon_0 V}}$$

Coupling within each manifold $\mathcal{E}(N)$

$$\langle b, N | V_{AL} | a, N+1 \rangle = g\sqrt{N+1}$$

$|a, N+1\rangle$ also couples to $|b, N+2\rangle$ } off-resonant
$|b, N\rangle$ " " " $|a, N-1\rangle$ } $\pm 2\hbar\omega_L$.

Neglect here. Gives Bloch-Siegert Shift of MR lines

Assume N large, $\Delta N \sim \sqrt{N} \ll N$

$$v_N = g\sqrt{\langle N \rangle} \quad \text{independent of } N$$

Use coherent state: $\langle \alpha e^{-i\omega_L t} | \vec{E}_\perp(\vec{R}) | \alpha e^{-i\omega_L t} \rangle = \vec{\mathcal{E}}_0 \cos(\omega_L t)$

$$\vec{\mathcal{E}}_0 = 2\vec{\varepsilon}_L \sqrt{\frac{\hbar\omega_L}{2\varepsilon_0 V}} \sqrt{\langle N \rangle}$$

Rabi Frequency $\quad \hbar\Omega_1 = -\vec{d}_{ab} \cdot \vec{\mathcal{E}}_0$

$$v_N = \frac{\hbar\Omega_1}{2}$$

Dressed States

Hamiltonian in $\mathcal{E}(N)$: $\frac{\hbar}{2}\begin{pmatrix} -\delta_L & \Omega_1 \\ \Omega_1 & \delta_L \end{pmatrix}$

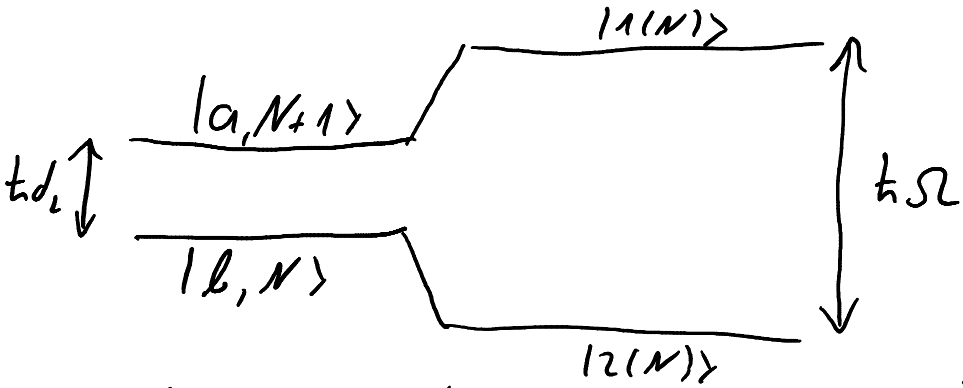
Solution: Eigenstates are called dressed states

$$|1(N)\rangle = \sin\vartheta\,|a, N+1\rangle + \cos\vartheta\,|b, N\rangle$$

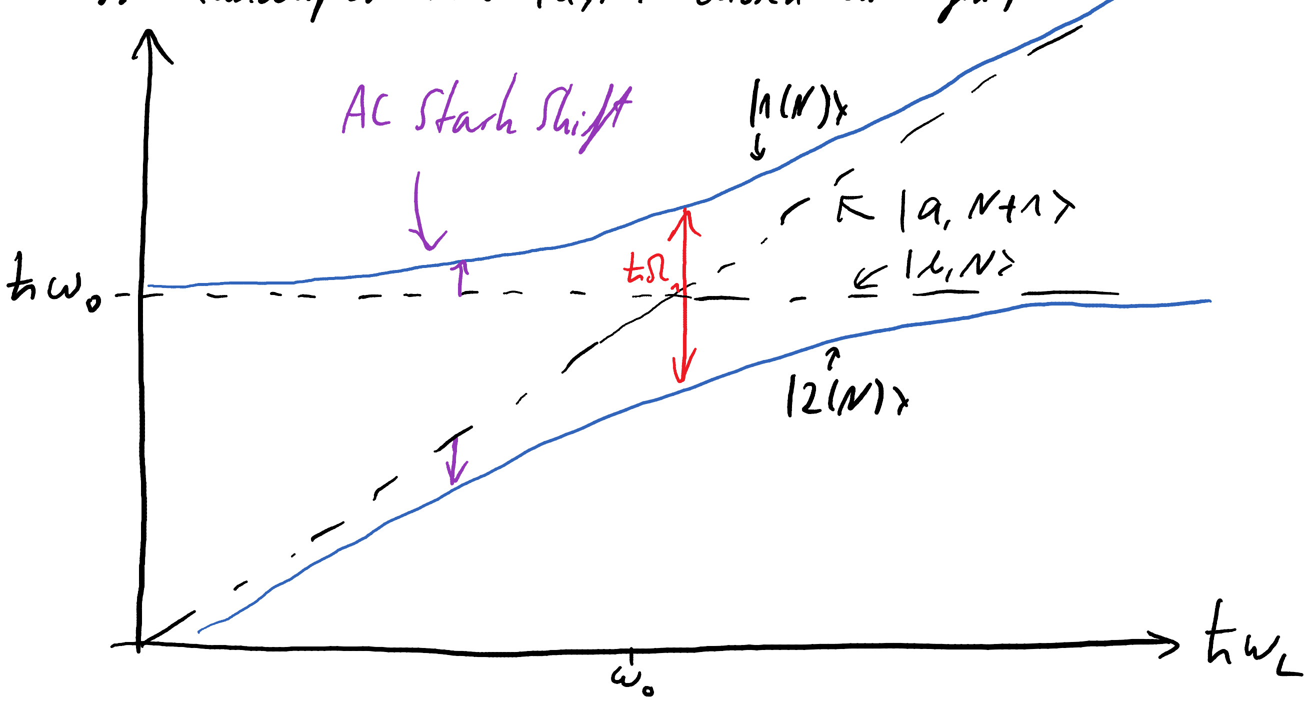
$$|2(N)\rangle = \cos\vartheta\,|a, N+1\rangle - \sin\vartheta\,|b, N\rangle$$

with $\tan 2\vartheta = -\frac{\Omega_1}{\delta_L}$ $\qquad 0 \leq 2\vartheta < \pi$

eigenvalues: $\pm\frac{\hbar\Omega}{2}$, where $\Omega = \sqrt{\Omega_1^2 + \delta_L^2}$



Energy (uncoupled state $|a,N\rangle$ chosen as origin)



Couplings V_r describe absorbtion $|a, N+1\rangle \to |b, N\rangle$
or stimulated emission $|b, N\rangle \to |a, N+1\rangle$

Say $|\psi(0)\rangle = |a, N+1\rangle$ at $t=0$
What is Probability to find atom in $|b, N\rangle$ at t?

$P(t)$ = Rabi oscillation, oscillating at Bohr frequency Ω between the two perturbed levels $|1(N)\rangle$ and $|2(N)\rangle$.

Full oscillation if uncoupled states $|a, N+1\rangle$ and $|b, N\rangle$ contain equal proportions of coupled states $|1(N)\rangle$ and $|2(N)\rangle$

$\to$ Rabi oscillation at frequency Ω_1 on resonance $\delta_L = 0$.

Resonance Fluorescence as Radiative Cascade

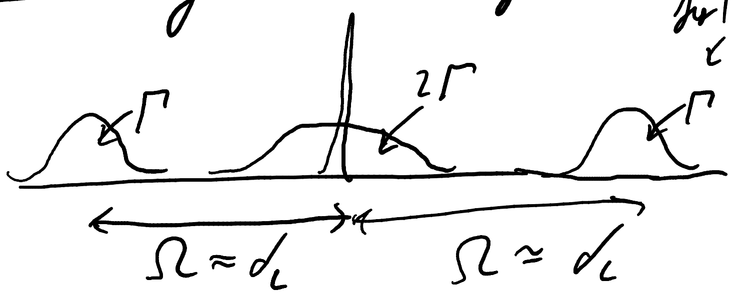
Add in coupling to vacuum.

As before $\tau_c \ll \frac{1}{\Omega}$, i.e. $\omega_0 \gg \Omega_n, |d_L|$.

For temporal aspect, using very wideband detector
→ use bare, uncoupled basis, as for each spont. emission process, laser photons are spectators.

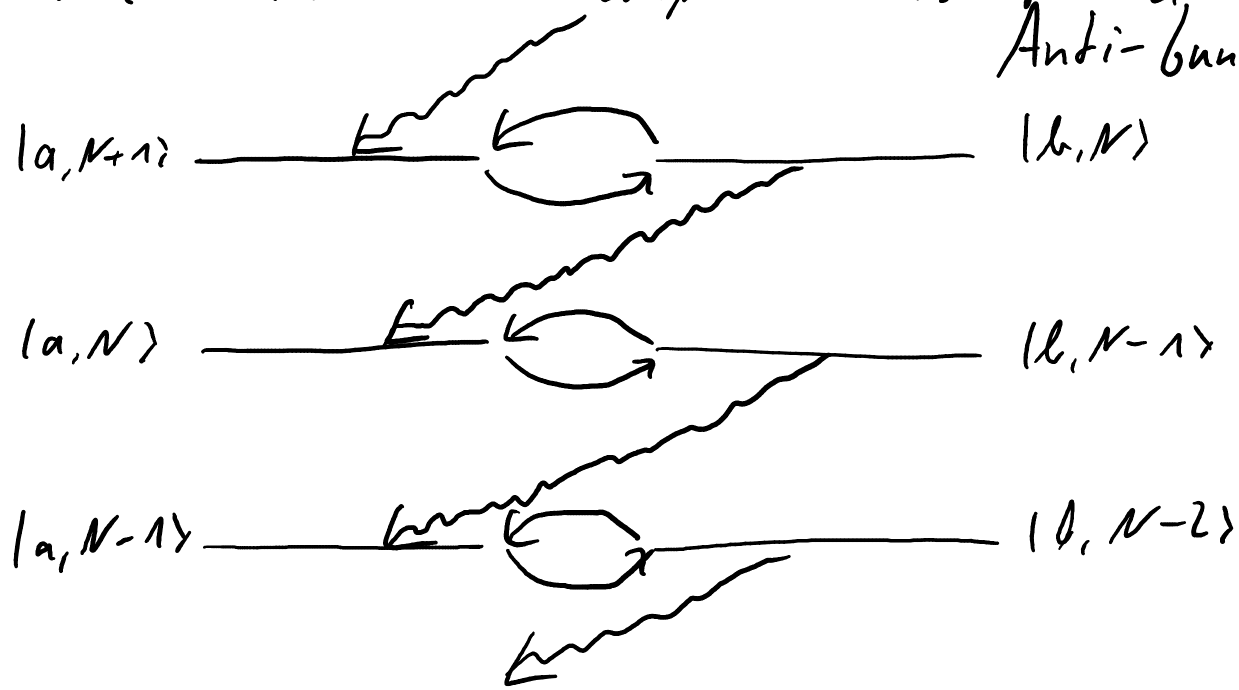
For energy aspect, using narrowband detector
→ use dressed atom picture.

<u>Spectrum of emitted light:</u>



for $|d_L| \gg |\Omega|$: Carrier + two sidebands
Mollow triplet

Radiative Cascade in uncoupled basis: Photon Anti-bunching



Radiative Cascade in the Dressed Atom Basis

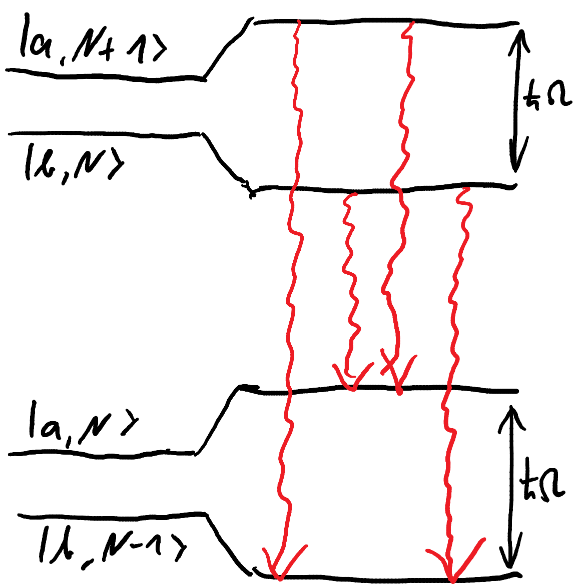
$|1(N)\rangle$ and $|2(N)\rangle$ both are "contaminated" by $|b,N\rangle$.

$\underset{\text{in } \mathcal{E}(N)}{|b,N\rangle}$ can decay to $\underset{\text{in } \mathcal{E}(N-1)}{|a,N\rangle}$ via spontaneous emission.

$|1(N-1)\rangle$ and $|2(N-1)\rangle$ both contaminated by $|a,N\rangle$

$\longrightarrow$ four transitions are allowed.

Transition matrix elements $\propto \langle i(N)|J_+|j(N-1)\rangle$



$\sin\vartheta\,|a,N+1\rangle + \cos\vartheta\,|b,N\rangle = |1(N)\rangle$

$\cos\vartheta\,|a,N+1\rangle - \sin\vartheta\,|b,N\rangle = |2(N)\rangle$

$\sin\vartheta\,|a,N\rangle + \cos\vartheta\,|b,N-1\rangle = |1(N-1)\rangle$

$\cos\vartheta\,|a,N\rangle - \sin\vartheta\,|b,N-1\rangle = |2(N-1)\rangle$

Matrix elements:

$$\langle 1(N)|J_+|1(N-1)\rangle = (J_+)_{11} = \sin\vartheta\cos\vartheta$$
$$\langle 2(N)|J_+|2(N-1)\rangle = (J_+)_{22} = -\sin\vartheta\cos\vartheta$$
$$\langle 1(N)|J_+|2(N-1)\rangle = (J_+)_{12} = \cos^2\vartheta$$
$$\langle 2(N)|J_+|1(N-1)\rangle = (J_+)_{21} = -\sin^2\vartheta$$

Fluorescence Triplet

$|1(N)\rangle \rightarrow |2(N-1)\rangle$: frequency $\omega_L + \Omega$

$|2(N)\rangle \rightarrow |1(N-1)\rangle$: $\omega_L - \Omega$

$|i(N)\rangle \rightarrow |i(N-1)\rangle$: ω_L $(i=1,2)$

Order: After a "large" photon emission, dressed atom is in $|2(N)\rangle$, can only emit a "small" or "medium" photon.

$\rightarrow$ Between two subsequent emissions of a large photon, there must necessarily be the emission of a small photon.

Such correlations have been observed
A. Aspect et al., PRL 45, 617 (1980)

Master Equation for Dressed Atom

Recall: $$\frac{d}{dt}\sigma_{AL} = -\frac{i}{\hbar}[H_{AL},\sigma_{AL}] - \frac{\Gamma}{2}(S_+S_-\sigma_{AL} + \sigma_{AL}S_+S_-) + \Gamma S_-\sigma_{AL}S_+$$

$$\dot\sigma_{bb} = -\Gamma\sigma_{bb}$$
$$\dot\sigma_{aa} = \Gamma\sigma_{bb}$$
$$\dot\sigma_{ab} = i\omega_0\sigma_{ab} - \frac{\Gamma}{2}\sigma_{ab}$$

still same! after projecting over uncoupled basis.
$\rightarrow$ OBE recovered.

Strong analogy between dressed states $|1(N)\rangle$ and $|2(N)\rangle$ and eigenstates of fictitious spin along effective field $\vec{B}_e$.
Angle 2ϑ : angle between $\vec B_e$ and $\vec b_0$!

Master equation in secular limit, $\Omega \gg \Gamma$.

$$\Pi_{i(N)} = \langle i(N) | \sigma | i(N) \rangle$$

$$\dot{\Pi}_{i(N)} = -\left(\sum_{j=1,2} \Gamma_{i\to j} \right) \pi_{i(N)} + \sum_{\ell=1,2} \Gamma_{\ell \to i} \, \pi_{\ell(N+1)}$$

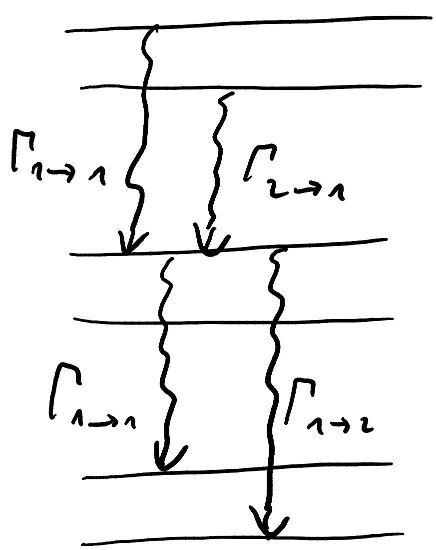
$$\Gamma_{i\to j} = \Gamma \left| \langle i(N) | S_+ | j(N-1) \rangle \right|^2 = \Gamma \left| (\theta_+)_{ij} \right|^2$$

$$\Gamma_{1\to 1} = \Gamma_{2\to 2} = \Gamma \cos^2\vartheta \sin^2\vartheta$$

$$\Gamma_{2\to 1} = \Gamma \sin^4\vartheta$$

$$\Gamma_{1\to 2} = \Gamma \cos^4\vartheta$$

$$\dot{\Pi}_{1(N)} = -\pi_{1(N)} \left(\Gamma_{1\to 1} + \Gamma_{1\to 2} \right) + \pi_{2(N+1)} \Gamma_{2\to 1} + \pi_{1(N+1)} \Gamma_{1\to 1}$$



Transfer of coherences

$$\frac{d}{dt}\langle 1(N)|\sigma|2(N)\rangle = -i\Omega\langle 1(N)|\sigma|2(N)\rangle - $$
$$-\Gamma_{12}\langle 1(N)|\sigma|2(N)\rangle + K_{12}\langle 1(N+1)|\sigma|2(N+1)\rangle$$

$$\Gamma_{12} = \frac{1}{2}\left[\Gamma_{1\to 1} + \Gamma_{1\to 2} + \Gamma_{2\to 1} + \Gamma_{2\to 2}\right] = \frac{\Gamma}{2}.$$

$$K_{12} = \Gamma\langle 1(N)|S_-|1(N+1)\rangle\langle 2(N+1)|S_+|2(N)\rangle$$
$$= \Gamma(S_+)^*_{11}(S_+)_{22} = -\Gamma\sin^2\vartheta\cos^2\vartheta$$

Reduced populations and coherences

$$\Pi_i = \sum_N \pi_{i(N)}$$
$$\sigma_{12} = \sum_N \langle 1(N)|\sigma|2(N)\rangle$$

$$\dot{\Pi}_1 = -\pi_1\Gamma_{1\to 2} + \pi_2\Gamma_{2\to 1}$$
$$\dot{\Pi}_2 = -\pi_2\Gamma_{2\to 1} + \pi_1\Gamma_{1\to 2}$$

Steady-state:

$$\Pi_1^{st} = \frac{\Gamma_{2\to 1}}{\Gamma_{1\to 2} + \Gamma_{2\to 1}} = \frac{\sin^4\vartheta}{\cos^4\vartheta + \sin^4\vartheta}$$

$$\Pi_2^{st} = \frac{\Gamma_{1\to 2}}{\Gamma_{1\to 2} + \Gamma_{2\to 1}} = \frac{\cos^4\vartheta}{\sin^4\vartheta + \cos^4\vartheta}$$

$$\Pi_1^{st}\Gamma_{1\to 2} = \pi_2^{st}\Gamma_{2\to 1}.$$ detailed balance.

$$\Pi_1^{st} + \Pi_2^{st} = 1$$

Transient regime: $\pi_i(t) = \pi_i^{st} + [\pi_i(0) - \pi_i^{st}]\, e^{-t/\tau_{pop}}$

$$\tau_{pop}^{-1} = \Gamma_{pop} = \Gamma_{1\to 2} + \Gamma_{2\to 1} = \Gamma(\cos^4\vartheta + \sin^4\vartheta)$$

Transient regime: $\pi_i(\tau) = \pi_i^{st} + (\pi_i(0) - \pi_i^{st})\, e^{-\tau/\tau_{pop}}$

$$\Gamma_{pop} = \Gamma_{1\to 2} + \Gamma_{2\to 1} = \Gamma(\cos^4\vartheta + \sin^4\vartheta)$$

Coherences:

$$\dot{\sigma}_{12} = -(i\Omega + \Gamma_{coh})\,\sigma_{12}$$

$$\Gamma_{coh} = \Gamma_{12} - \underset{\substack{\uparrow \\ \text{transfer term}}}{K_{12}} = \Gamma\left(\tfrac{1}{2} + \cos^2\vartheta \sin^2\vartheta\right)$$

transfer term, $\Gamma_{coh} \neq \frac{\Gamma}{2}$!

Widths and weights of the Mollow triplet.

Recall: $\mathcal{I}(\omega) = \frac{\Gamma}{\pi} \operatorname{Re} \int_0^\infty d\tau\; e^{-i\omega\tau} \langle S_+(\tau) S_-(0) \rangle$

Evolution of mean dipole moment $\langle S_+(\tau) \rangle$ ... see API.

Result ($\omega_L + \Omega$ sideband) $\Pi_1^{st} \Gamma_{1\to 2} \quad \frac{1}{\pi} \frac{\Gamma_{coh}}{(\omega - \omega_L - \Omega)^2 + \Gamma_{coh}^2}$

$\Rightarrow$ weight $\Pi_1^{st} \Gamma_{1\to 2}$: population $\times$ rate

FWHM $\quad 2\Gamma_{coh} = \Gamma(1 + 2\cos^2\vartheta \sin^2\vartheta)$

On resonance: $\vartheta = 45°$ $\Gamma\left(1 + 2\,\frac{1}{2}\cdot\frac{1}{2}\right) = \Gamma\cdot\frac{3}{2}$ ✓

$\omega_L - \Omega$ sideband: $\Pi_2^{st}\cdot\Gamma_{2\to 1}$.

Note: $\Pi_2^{st}\Gamma_{2\to 1} = \Pi_1^{st}\Gamma_{1\to 2}$ due to detailed balance

Central line: Correlation

$$\langle [J_+^{11}(\tau) + J_+^{22}(\tau)]\, J_-(0)\rangle$$

$$\sim A\, e^{(i\omega_L - \Gamma_{pop})\tau} + B\, e^{i\omega_L \tau}$$

$\Rightarrow$ coherent component $I_{coh}(\omega) = B\Gamma\,\delta(\omega-\omega_L)$

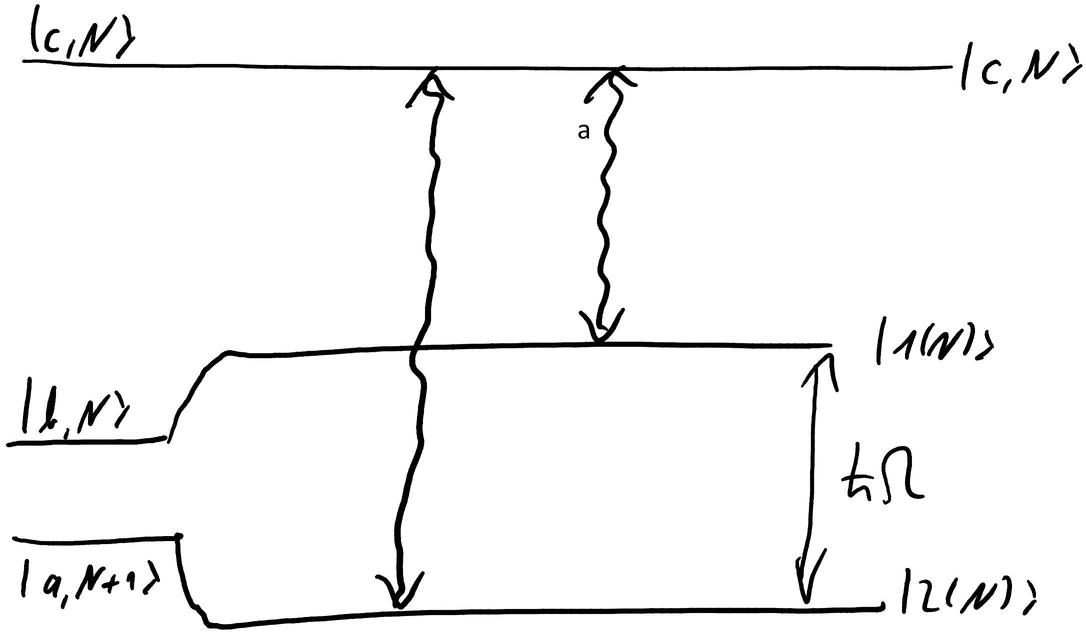
central inelastic component, weight ΓA, FWHM $2\Gamma_{pop}$.

$$2\Gamma_{pop} = 2\Gamma(\cos^4\vartheta + \sin^4\vartheta) = \Gamma \text{ on resonance}$$

$$\Gamma(A+B) = \Pi_1^{st}\Gamma_{1\to1} + \Pi_2^{st}\Gamma_{2\to2}$$

$$I_{coh} = \Gamma B = \Gamma_{1\to1}\left(\Pi_1^{st} - \Pi_2^{st}\right)^2$$

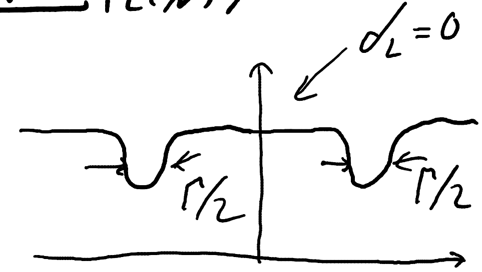
Autler-Townes Splitting



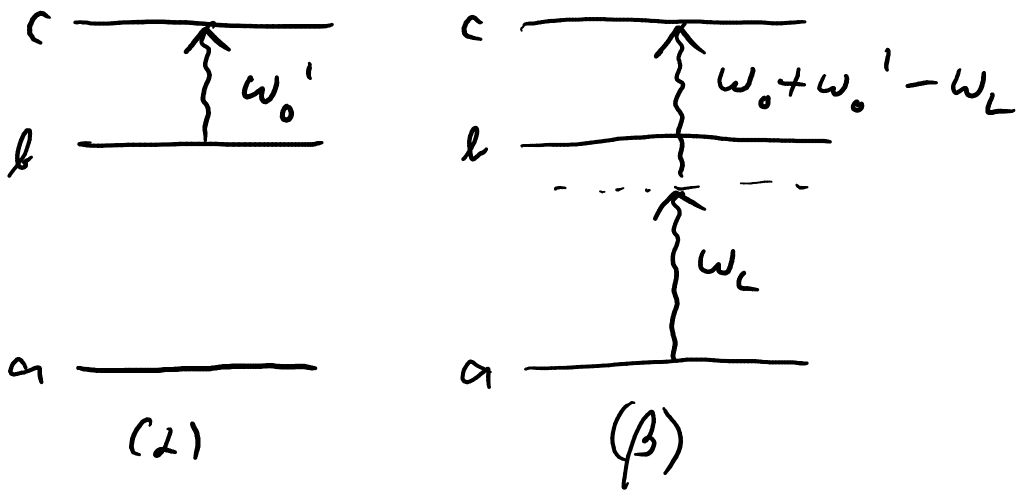
transition rate $\propto |\langle i(N)|b,N\rangle|^2$

weight: $\Pi_i^{st}\cdot$ rate.

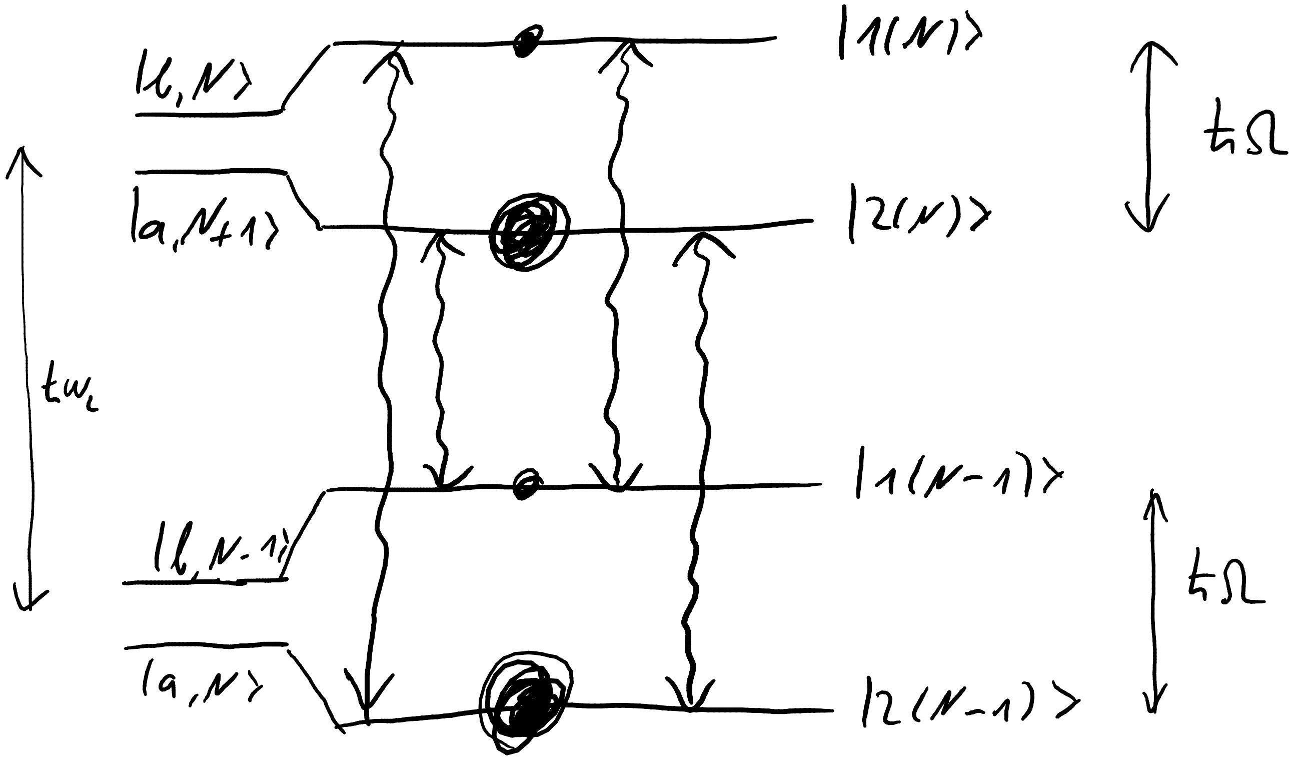
width: width of c + $\sum_j \Gamma_{i\to j}$



Perturbative interpretation for $|\delta_L| \gg \Omega_1, \Gamma$

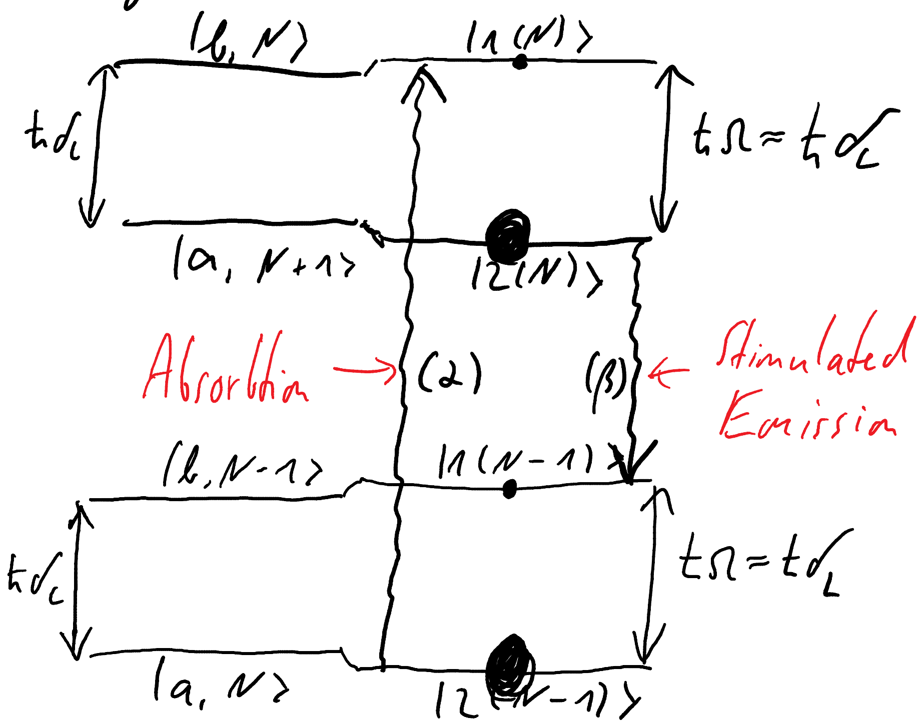


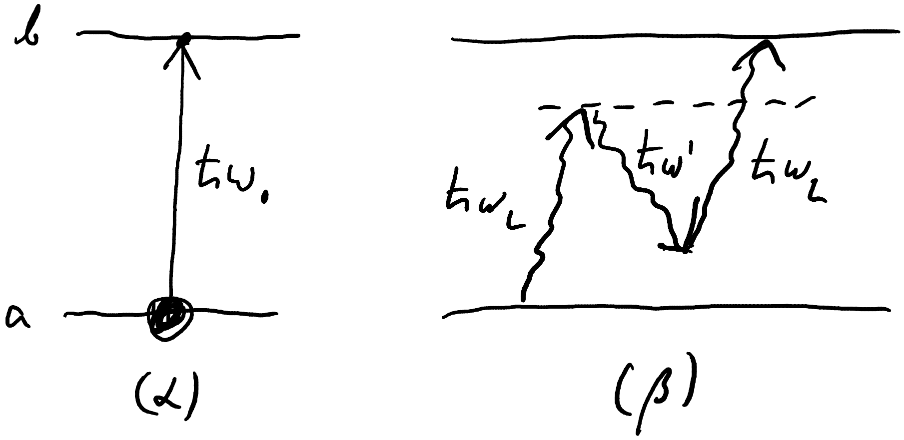
Absorbtion of weak probe beam

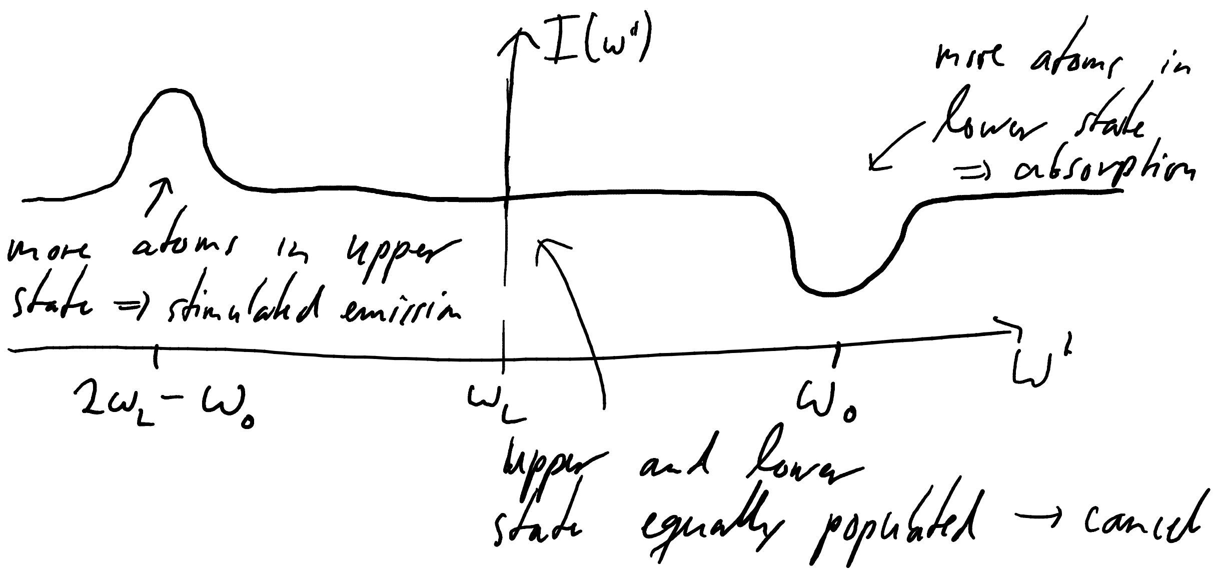


$|1(N)\rangle \to |2(N-1)\rangle$, $\omega_L + \Omega$ is absorbing
$|2(N)\rangle \to |1(N-1)\rangle$ $\omega_L - \Omega$ is amplifying
other transitions: No effect as levels equally populated.

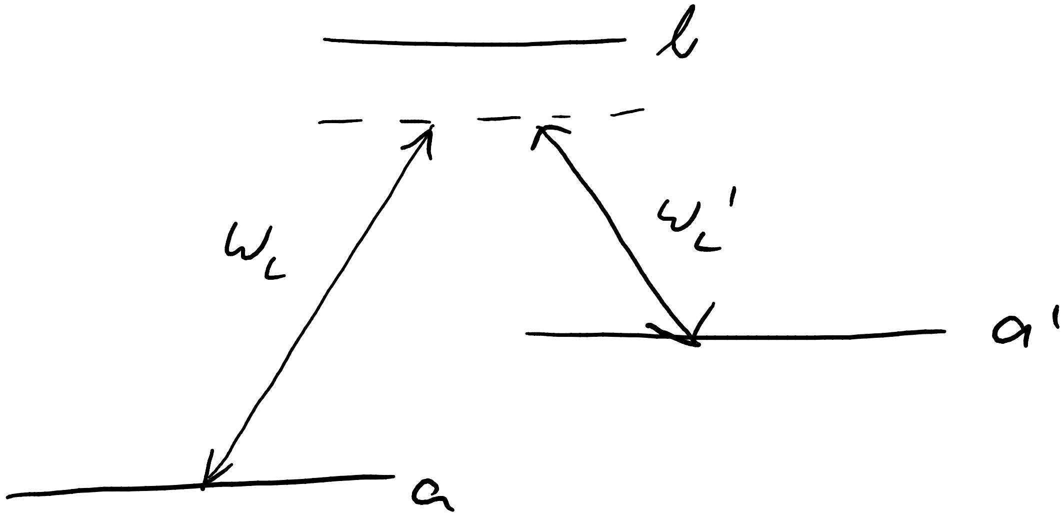
Diagrams in far-detuned case:







Dark States



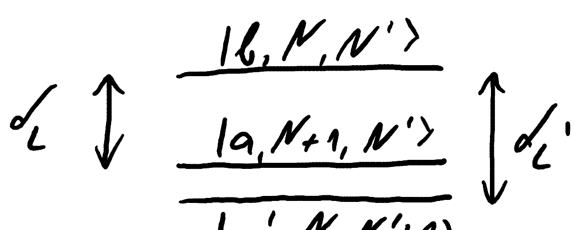
$\hbar(\omega_L - \omega_L') = E_{a'} - E_a$ dark resonance

$\rightarrow$ complete disappearance of Fluorescence.

Explanation in Dressed Atom picture:

Resonant substates are

$$\mathcal{E}(N,N') = \{|b,N,N'\rangle, |a,N+1,N'\rangle, |a',N,N'+1\rangle\}$$



$$\delta_L = \omega_L - \omega_{ab}$$
$$\delta_L' = \omega_L' - \omega_{a'b}$$

Non-hermitian effective hamiltonian within $\mathcal{E}(N,N')$ describes evolution of

$$|\Psi(t)\rangle = c_a(t)\,|a,N+1,N'\rangle + c_{a'}(t)\,|a',N,N'+1\rangle + c_b(t)\,|b,N,N'\rangle$$

valid in between spontaneous emission events

$$i\dot{c}_b = \frac{\Omega_1}{2} c_a + \frac{\Omega_2}{2} c_{a'} - i\frac{\Gamma}{2} c_b$$

$$i\dot{c}_a = \delta_L c_a + \frac{\Omega_1}{2} c_b$$

$$i\dot{c}_{a'} = \delta_L' c_{a'} + \frac{\Omega_2}{2} c_b$$

$$i\begin{pmatrix} \dot{c}_b \\ \dot{c}_a \\ \dot{c}_{a'} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -i\Gamma & \Omega_1 & \Omega_2 \\ \Omega_1 & \delta_L & 0 \\ \Omega_2 & 0 & \delta_L' \end{pmatrix}\begin{pmatrix} c_b \\ c_a \\ c_{a'} \end{pmatrix}$$

Resonant case: $\delta_L = \delta_L' = 0$

Steady state: $c_b = 0$ (!) No fluorescence!

and $\Omega_1 c_a + \Omega_2 c_{a'} = 0$

So $c_a \propto \Omega_2$ and $c_{a'} \propto -\Omega_1$

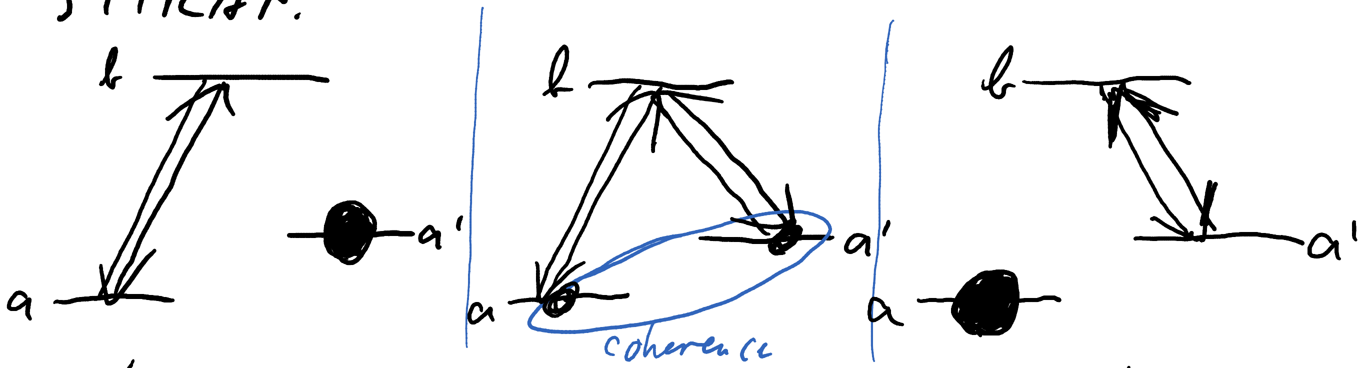
$$\rightarrow \quad |\Psi(t)\rangle = \frac{\Omega_2 |a, N+1, N'\rangle - \Omega_1 |a', N, N'+1\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}}$$

= constant in time

Dark state!

$\Rightarrow$ Atoms will be pumped into dark state.

Clever transfer scheme from a' to a: STIRAP.



Step I Step II Step III

Dark state:

$$|a'\rangle \longrightarrow \Omega_2|a\rangle - \Omega_1|a'\rangle \longrightarrow |a\rangle$$

"Adiabatic evolution" of dark state from $|a'\rangle$ to $|a\rangle$.

Is it ever perfect? Not if done in finite time.

$$i\dot{C}_a = \frac{\Omega_1}{2} C_b \Rightarrow$$ can never increase population in $|a\rangle$ without having $C_b \neq 0$.

Let's say I take time T. C_a goes from 0 to 1.

$$\Rightarrow |C_b| \approx \frac{1}{\Omega_1 T}$$

success requires $\Omega_1^2 T \gg \Gamma$

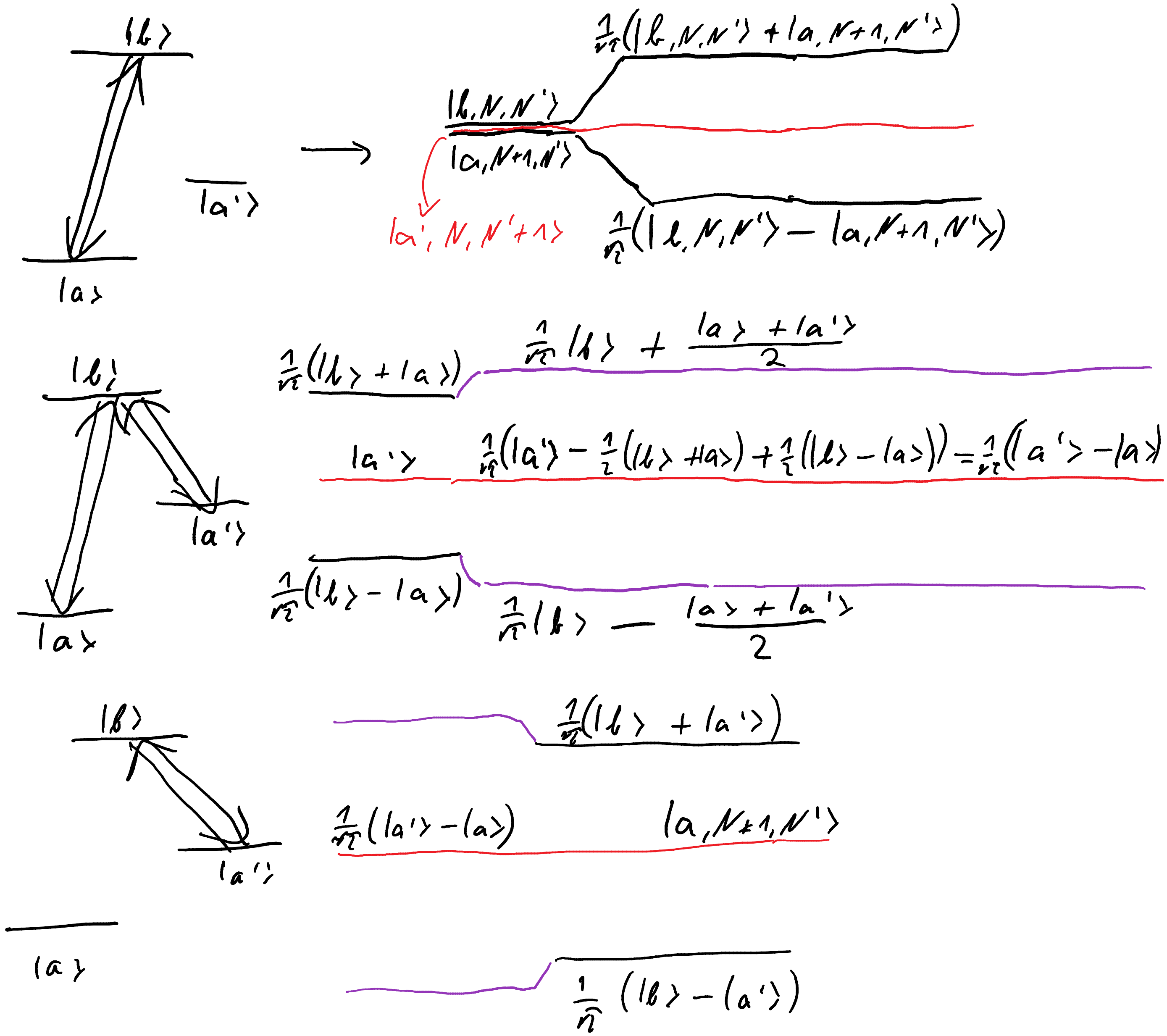
# of scattered photons: $\Gamma |C_b|^2 \cdot T = \dfrac{\Gamma}{\Omega_1^2 T}$.

Try two successive π-pulses instead.

Best to do this off-resonantly. I still want to be done after time T.

Two-photon Rabi Frequency: $\omega_{2ph} = \dfrac{\Omega_1^2}{\delta} \approx \dfrac{1}{T} \Rightarrow \delta = \Omega_1^2 T$

# of scattered photons: $\Gamma \cdot \dfrac{\Omega_1^2}{\delta^2} \cdot T = \dfrac{\Gamma}{\Omega_1^2 T}$. $\Rightarrow$ Same!


|b⟩
|a'⟩
|a⟩
$\frac{1}{\sqrt{2}}(|b,N,N'\rangle + |a,N+1,N'\rangle)$
$|b,N,N'\rangle$
$|a,N+1,N'\rangle$
$|a',N,N'+1\rangle$
$\frac{1}{\sqrt{2}}(|b,N,N'\rangle - |a,N+1,N'\rangle)$
|b⟩
|a'⟩
|a⟩
$\frac{1}{\sqrt{2}}|b\rangle + \frac{|a\rangle + |a'\rangle}{2}$
$\frac{1}{\sqrt{2}}(|b\rangle + |a\rangle)$
$|a'\rangle$
$\frac{1}{\sqrt{2}}\left(|a'\rangle - \frac{1}{2}(|b\rangle + |a\rangle) + \frac{1}{2}(|b\rangle - |a\rangle)\right) = \frac{1}{\sqrt{2}}(|a'\rangle - |a\rangle)$
$\frac{1}{\sqrt{2}}(|b\rangle - |a\rangle)$
$\frac{1}{\sqrt{2}}|b\rangle - \frac{|a\rangle + |a'\rangle}{2}$
|b⟩
|a'⟩
|a⟩
$\frac{1}{\sqrt{2}}(|b\rangle + |a'\rangle)$
$\frac{1}{\sqrt{2}}(|a'\rangle - |a\rangle)$
$|a,N+1,N'\rangle$
$\frac{1}{\sqrt{2}}(|b\rangle - |a'\rangle)$