All Coils Ioffe-Pritchard Magnetic Trap

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1 Introduction

The aim of this research project was to desing and test an alternative component for the permanent magnet, hybrid IP trap currently used in the experimental set up of the *McGuirk Ultra Cold Atom Research Group*, looking to improve features such as flexibility and optical access. After a sensible AMO literature survey, the design introduced in [1] and described further in [2] of an All Coils IP Trap was chosen. Multiple simulations were later developed in MATLAB to test the feasibility of the design. Some preliminary machine drawings were proposed as well, taking into account physical and practical constraints. This document includes and explains the trap design, simulations and results in detail.

Special thanks to the *MITACS Globalink Research Internship Program* for providing funding and support during the development of this project.

2 Electromagnetic Theory

All units should be assumed to be in **CGS**, with $\mu_0 = 0.4\pi$. The current is given in Amp-turns, with $I = NI_0$ with I_0 being the source current in amps.

2.1 Current Coil

The magnetic field for a current loop with radius R, current I, centered at the z axis and displaced from the origin by a vertical distance A (see Figure 1) can be calculated from through the vector potential:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla} \times \frac{\mu_0}{4\pi} \iiint \frac{\boldsymbol{J}}{|\boldsymbol{r}|} \, d^3r \tag{2.1}$$

and it can be shown (see Appendix A) that this leads to the respective component equations[3, 4, 5]:

$$\begin{aligned} B_\rho &= \frac{\mu_0 I}{2\pi\rho} \frac{z-A}{\sqrt{(R+\rho)^2 + (z-A)^2}} \left\{ -\mathbb{K}(\kappa^2) + \frac{R^2 - \rho^2 - (z-A)^2}{(R-\rho)^2 + (z-A)^2} \mathbb{E}(\kappa^2) \right\} \\ B_z &= \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{(R+\rho)^2 + (z-A)^2}} \left\{ \mathbb{K}(\kappa^2) + \frac{R^2 - \rho^2 - (z-A)^2}{(R-\rho)^2 + (z-A)^2} \mathbb{E}(\kappa^2) \right\} \\ B_\phi &= 0 \end{aligned} \tag{2.2}$$

with

$$\kappa^2 = \frac{4R\rho}{(R+\rho)^2 + (z-A)^2}$$

where $\mathbb{K}(\kappa^2)$ and $\mathbb{E}(\kappa^2)$ are the elliptic integrals of first and second kind, respectively.

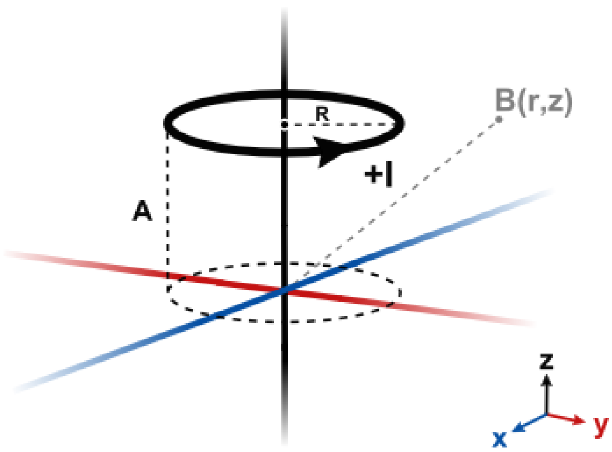


Figure 1: Current Coil with a positive current I, with a radius R and a vertical distance from the origin A

The field equations given in (2.2) can be approximated around the origin (i.e. $z = 0$ and $\rho = 0$) with a series expansion. If we define the purely geometrical factors:

$$\begin{aligned} \mathbb{F} &= \frac{R^2}{(R^2 + A^2)^{3/2}} \qquad \mathbb{G} = \frac{3}{2} \frac{AR^2}{(R^2 + A^2)^{5/2}} \\ \mathbb{H} &= \frac{3R^2(4A^2 - R^2)}{(R^2 + A^2)^{7/2}} \qquad \mathbb{I} = \frac{5}{2} \frac{AR^2(4A^2 - 3R^2)}{(R^2 + A^2)^{9/2}} \end{aligned} \tag{2.3}$$

These geometrical factors have units of inverse length ([cm^{-1}] for $\mathbb{F}$, [cm^{-2}] for $\mathbb{G}$, and so on) so that when multiplied by the factor $\mu_0 I$ they result in units of magnetic field (or its respective spatial derivatives). Up third order, the expansion can be written as (see Appendix B):

$$\begin{aligned} B_z &= \mu_0 I \left\{ \frac{1}{2}\mathbb{F} + \mathbb{G}\, z + \frac{1}{4}\mathbb{H}\,(z^2 - \rho^2/2) + \frac{1}{2}\mathbb{I}\,(z^3 - 3z\rho^2/2) + ... \right\} \\ B_\rho &= \mu_0 I \left\{ + \mathbb{G}\,(-\rho/2) + \frac{1}{4}\mathbb{H}\,(-\rho z) + \frac{1}{2}\mathbb{I}\,(3\rho^3/8 - 3\rho z^2/2) + ... \right\} \end{aligned} \tag{2.4}$$

In the equations above, $\mathbb{F}$ can be associated with an overall bias field. $\mathbb{G}$ and $\mathbb{H}$ are also closely related to the field's gradient and curvature, respectively.

2.2 Helmholtz Coils Pair

A Helmholtz pair is assembled with two current loops with a radius R, aligned along a common axis z and vertically displaced by A and $-A$ respectively; both run a current I in the same direction (see Figure 2). The magnetic field can be exactly calculated by adding the contribution of each coil as given by equation (2.2).

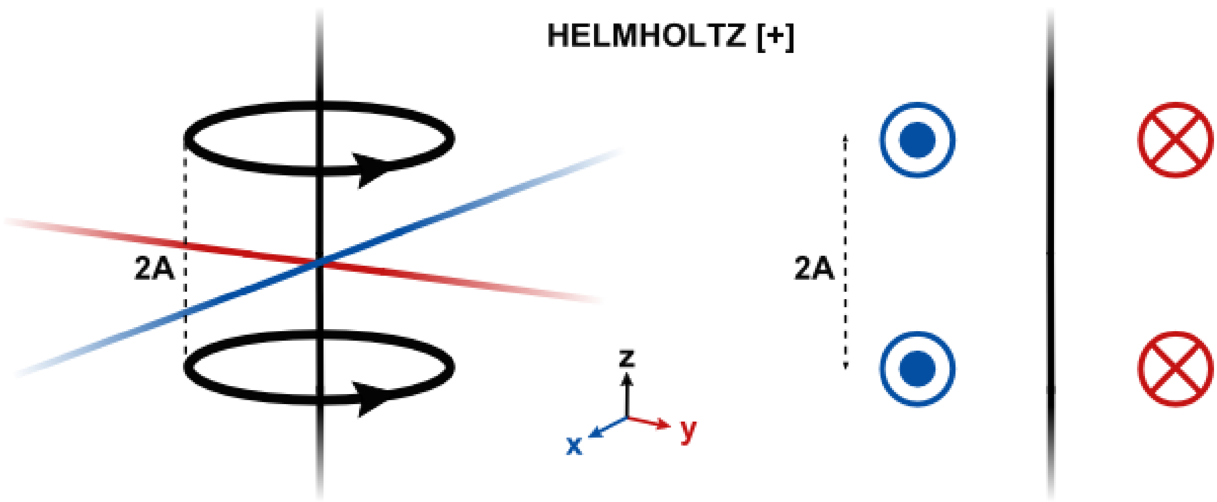


Figure 2: Helmholtz coil pair with a total separation of $2A$, running a positive current I on both coils

The field near the origin, however, can be approximated by using equation (2.4). Given the described configuration, the geometrical factors for each coil remain invariant except for $\mathbb{G}$ and $\mathbb{I}$, which become negative for the lower coil due to being odd functions of A. With both currents running in the same direction, both the $\mathbb{G}$ and $\mathbb{I}$ terms cancel each other while the rest doubles. Thus, the total field can be approximated up to third order by:

$$
\begin{aligned}
B_z &= \mu_0 I \left\{ \quad \mathbb{F} + \frac{1}{2}\mathbb{H}\,(z^2 - \rho^2/2) + \ldots \right\} \\
B_\rho &= \mu_0 I \left\{ \qquad \frac{1}{2}\mathbb{H}\,(-\rho z \qquad) + \ldots \right\}
\end{aligned} \tag{2.5}
$$

It is worth noticing that $\mathbb{H} = 0$ for $R = 2A$, and thus, using such configuration produces a nearly constant field along the common axis of the pair. On the other hand, $\mathbb{H}$ is a maximum with $R = \sqrt{\frac{4}{3}}A$.

2.3 Anti-Helmholtz Coils Pair

An anti-Helmholtz pair is assembled in a closely similar manner; except that in this case the coil's currents I run in opposite directions (see Figure). just as before, the magnetic field can be exactly calculated by adding the contribution of each coil as given by equation (2.2).

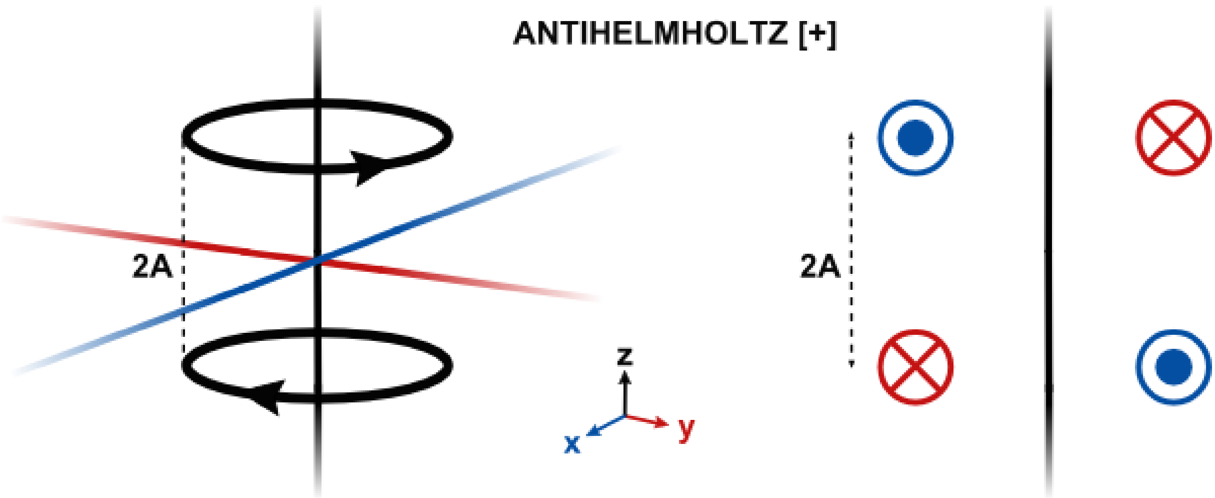


Figure 3: Anti-Helmholtz coil pair with a total separation of $2A$, running a positive current I on the upper coil

With both currents running in the opposite direction, the $\mathbb{F}$ and $\mathbb{H}$ terms cancel each other while the rest doubles. Thus, the total field can be approximated up to third order by:

$$
\begin{aligned}
B_z &= \mu_0 I \left\{ \; 2\,\mathbb{G}\,z \qquad + \; \mathbb{I}\,(\; z^3 \quad - 3z\rho^2/2) + \ldots \right\} \\
B_\rho &= \mu_0 I \left\{ \; 2\,\mathbb{G}\,(-\rho/2) + \; \mathbb{I}\,(3\rho^3/8 - 3\rho z^2/2) + \ldots \right\}
\end{aligned} \tag{2.6}
$$

It is worth noticing that $\mathbb{I} = 0$ for $R = \sqrt{\frac{4}{3}}A$, and thus, using such configuration produces cancels out the higher order terms. On the other hand, $\mathbb{G}$ is a maximum with $R = 2A$. Also, the magnitude of the linear gradient in the axial direction doubles that of the radial one.

3 The Ioffe-Pritchard Trap

3.1 BEC's and Magnetic Trapping

In order to create a Bose condensate, atoms must be cooled and compressed until the thermal de Broglie wavelength is comparable to the average interatomic spacing. The use of conservative traps allows for both accommodating and compressing these atoms. This provides a high collision environment, which in turn allows for an efficient RF evaporative cooling process. Such trapping potentials have been realized with both DC and AC magnetic fields, microwave fields and off-resonant laser beams [6].

Magnetic trapping is particularly effective when working with alkali metals, as magnetic forces interact strongly with the magnetic moment resulting from the unpaired electron[7]. The energy of this interaction is given by:

$$U = -\boldsymbol{\mu} \cdot \boldsymbol{B}(\boldsymbol{r}) \tag{3.1}$$

where $\boldsymbol{\mu} = m_F g_F \mu_B \boldsymbol{F}$ and m_F is the Zeeman state of the atom, g_F is the total momentum g-factor, μ_B is the Bohr magneton and $\boldsymbol{F}$ is the total angular momentum of the atom. Given that the force felt by the atoms is position dependent as well, a magnetic field gradient can be used for confinement if it allows for a magnetic potential energy minimum. Weak field seeking states ($m_F g_F > 0$) can be trapped, for example, with a local minimum in the magnetic field. Strong field seeking states, on the other hand, can't be trapped with static fields, as Wing's theorem prohibits the existence of a local magnetic field maxima on the later.

This situation imposes a strong limiting factor in the shape of the magnetic trap's field, as the presence of zeros on it would crate regions which favor Majorana spin flips in the atoms. The possibility for some of the trapped atoms to transition into strong seekers would then result in an unstable configuration[2]. The lowest order (and therefore tightest) trap which can use an overall bias field to overcome this problem is a harmonic trap[7]. Such a potential is generally achieved with small variations on the magnetic trapping's workhorse, the Ioffe-Pritchard trap.

3.2 Standard IP Trap

The original configuration for the IP trap (see Figure) consists of 4 straight, current-carrying bars which create a confining quadrupole field in the radial direction. A pair of coils known as pinch coils (PI) are also used to add a curvature to the field in the axial direction and therefore to provide the axial confinement. Lastly, an extra pair of coils known as the compensation coils (CO) is also incorporated to compensate for the field offset created by the PI coils and control the trap's bias and depth in a more systematic way. Generally, since the bars and the PI coils are relatively close to the atoms, this configuration is particularly efficient at producing tight trapping potentials[7]. There are multiple configurations which produce an IP geometry, and since its introduction, multiple winding patters such as the baseball, yin-yang, racetrack, QUIC, or cloverleaf ones have been introduced, each one having different degrees of flexibility, efficiency, optical access and ease of construction.

The IP configuration which was chosen and simulated was the one firstly introduced in [1] and described further by Vasiliki Bolpasi on her Ph.D. dissertation[2]. The novelty of the design consists on replacing the traditional Ioffe bars by a configuration of 2 pairs of coils which can produce the same confining, quadrupole field with a lower power consumption. The main strength of this coil-based configuration lies on its flexibility, as smart combinations of currents in the different coils can produce a range of aspect ratios and trapping parameters. Also, due to the ready-made shape and geometry, the trap can be used as a QUIC trap or a TOP trap as well by using specific distributions of currents in the coils.

3.3 All Coil Configuration

The spatial configuration of the All-Coils IP trap is shown in Figures 4 and 5. The z direction is assumed to be parallel to the vacuum cell's length. Both the PI coils and the CO coils are running currents in a Helmholtz configuration (i.e. both coils in the same direction), but the former runs a positive current while the later a negative one (for simulation purposes). Both the BI and SI coils are running currents in an Anti-Helmholtz configuration (opposite directions. It is worth noticing that for the SI set, the upper coil is running a positive current, while for the BI set the upper one is running a negative one. This inversion accounts for the cancellation of the z contribution to the magnetic field by the Ioffe set (i.e. BI and SI combined).

3.4 Magnetic Field Derivation

BIG IOFFE COILS (BI)

By setting the constrain $R_{BI} = \sqrt{\frac{4}{3}}A_{BI}$ it follows that $\mathbb{I}_{BI} = 0$, and so the magnetic field $\boldsymbol{B_{BI}}$ can be directly extracted from the field equation for an anti-Helmtholtz pair as described in (2.6). Expressed in cartesian coordinates $\langle x, y, z\rangle$, it would be given by:

$$\boldsymbol{B_{BI}} = \langle\ -\alpha_{BI}\ ,\ -\alpha_{BI}\ ,\ 2\alpha_{BI}\ \rangle$$

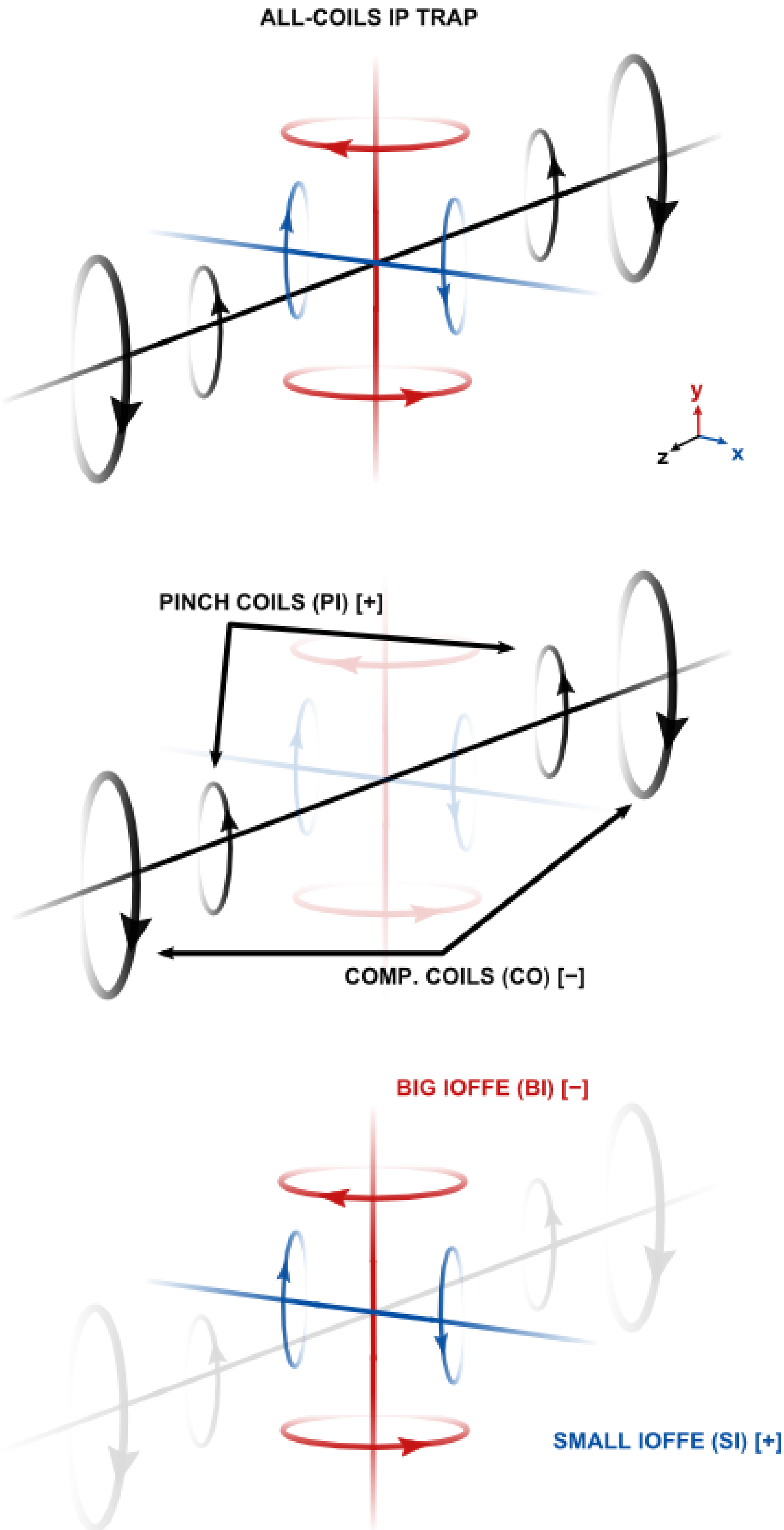


Figure 4: Coil configuration for the All Coils IP Trap. The Pinch (PI) and Compensation (CO) coils are shown in black. The Big Ioffe coils (BI) are shown in red and the small ones (SI) in blue

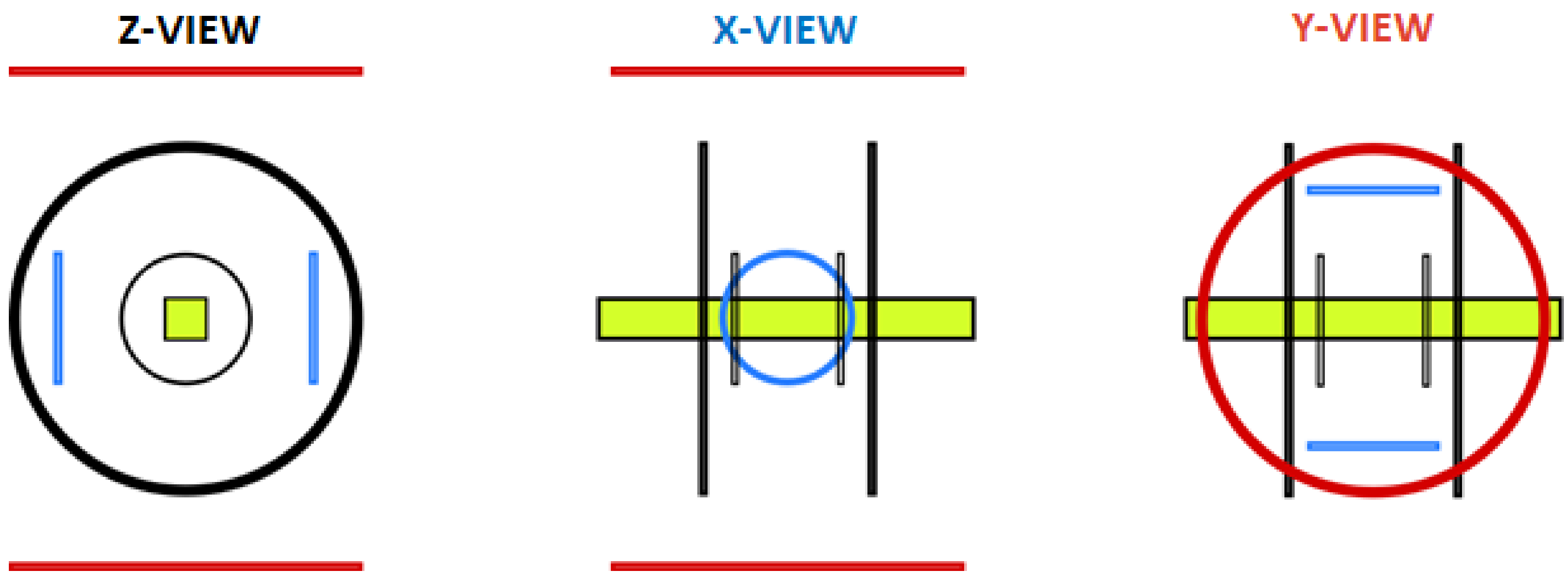


Figure 5: Side view of the IP trap.The Pinch (PI) and Compensation (CO) coils are shown in black. The Big Ioffe coils (BI) are shown in red and the small ones (SI) in blue. The Vacuum cell is shown in yellow.

with

$$\alpha_{BI} = \mu_0 I_{BI} \mathbb{G}_{BI}$$

However, given that the real BI coils are rotated so that y is their symmetry axis ($\mathbb{R}_x(-\pi/2)$), and they are running currents in the opposite direction from that described in (2.6). Thus, the correct field equation would be given by:

$$\boldsymbol{B_{BI}} = \langle\ \alpha_{BI}\ ,\ -2\alpha_{BI}\ ,\ \alpha_{BI}\ \rangle \tag{3.2}$$

SMALL IOFFE COILS (BI) Similarly, $\boldsymbol{B_{SI}}$ can be extracted from (2.6). In this case, however, x is their symmetry axis ($\mathbb{R}_y(\pi/2)$). No reversing of current is needed this time, and so the correct field equation would be given by:

$$\boldsymbol{B_{SI}} = \langle\ 2\alpha_{SI}\ ,\ -\alpha_{SI}\ ,\ -\alpha_{SI}\ \rangle \tag{3.3}$$

PINCH COILS (PI)

In this case the magnetic field $\boldsymbol{B_{PI}}$ can be extracted from the field equation a Helmholtz pair as described in (2.5), which would yield:

$$\boldsymbol{B_{PI}} = \left\langle\ -\frac{1}{2}\beta_{PI}xz\ ,\ -\frac{1}{2}\beta_{PI}yz\ ,\ B_{0,PI} + \frac{1}{2}\beta_{PI}(z^2+\rho^2)\ \right\rangle \tag{3.4}$$

with

$$B_{0,PI} = \mu_0 I_{PI} \mathbb{F}_{PI} \qquad \text{and} \qquad \beta_{PI} = \mu_0 I_{PI} \mathbb{H}_{PI}$$

COMPENSATION COILS (CO)

Similarly, $\boldsymbol{B_{CO}}$ can be extracted from the field equation a Helmholtz pair as described in (2.5). In this case, however, the coils are running currents in the opposite direction from that described in (2.5). Thus, the adjusted field equation would be given by

$$\boldsymbol{B_{CO}} = \left\langle\ \frac{1}{2}\beta_{CO}xz\ ,\ \frac{1}{2}\beta_{CO}yz\ ,\ -B_{0,CO} - \frac{1}{2}\beta_{CO}(z^2+\rho^2)\ \right\rangle \tag{3.5}$$

with

$$B_{0,CO} = \mu_0 I_{CO} \mathbb{F}_{CO} \qquad \text{and} \qquad \beta_{PI} = \mu_0 I_{CO} \mathbb{H}_{CO}$$

FULL FIELD

When adding the contributions from each pair of coils and collecting terms, the total magnetic field is given by:

$$
\begin{aligned}
\boldsymbol{B} &= \boldsymbol{B_{BI}} + \boldsymbol{B_{SI}} + \boldsymbol{B_{PI}} + \boldsymbol{B_{CO}} \\
&= \begin{pmatrix} 0 \\ 0 \\ B_{0,PI} - B_{0,CO} \end{pmatrix} + \begin{pmatrix} (\alpha_{BI} + 2\alpha_{SI})\, x \\ -(2\alpha_{BI} + \alpha_{SI})\, y \\ (\alpha_{BI} - \alpha_{SI})\, z \end{pmatrix} + \frac{1}{2}(\beta_{PI} - \beta_{CO}) \begin{pmatrix} -xz \\ -yz \\ z^2 - \rho^2/2 \end{pmatrix}
\end{aligned}
$$

By imposing the constrain

$$\alpha_{BI} = \alpha_{SI} \quad \Longrightarrow \quad I_{BI}\mathbb{G}_{BI} = I_{SI}\mathbb{G}_{SI} \tag{3.6}$$

to assure that the Ioffe pairs balance each other, and by defining:

$$\boldsymbol{\alpha} = 3\alpha_{BI} = 3\alpha_{BI}, \qquad \boldsymbol{\beta} = \beta_{PI} - \beta_{CO}, \qquad \text{and} \quad \boldsymbol{\delta} = B_{0,PI} - B_{0,CO}$$

as the trap's gradient, curvature and overall bias, the full tramp's magnetic field can be written more concisely as:

$$\boldsymbol{B} = \boldsymbol{\delta} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \boldsymbol{\alpha} \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix} + \frac{1}{2}\boldsymbol{\beta} \begin{pmatrix} -xz \\ -yz \\ z^2 - \rho^2/2 \end{pmatrix} \tag{3.7}$$

3.5 Trap parameters

The trapping frequencies for the trap can be extracted from (3.7) (see Appendix C), thus yielding the shorthand equations:

$$\omega_z \approx \sqrt{\gamma \boldsymbol{\beta}} \tag{3.8}$$

and

$$\omega_\rho \approx \sqrt{\gamma \left(\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\delta}} - \frac{\boldsymbol{\beta}}{2} \right)} \tag{3.9}$$

with

$$\gamma = \frac{g_F m_F \mu_B}{m_{Rb}} \approx 32.1308 \ \left[\mathrm{cm}^2/\mathrm{Gs}^2\right] \qquad \text{for the state } |1, 2\rangle$$

The expressions above can be further expressed in terms of the trap's design parameters given by the geometrical factors and the coil's currents:

$$
\begin{aligned}
\omega_z &= \sqrt{\gamma(\beta_{PI} - \beta_{CO})} \\
&= \sqrt{\gamma\mu_0(I_{PI}\mathbb{H}_{PI} - I_{CO}\mathbb{H}_{CO})}
\end{aligned}
$$

If the restriction $R_{CO} = 2A_{CO}$ is imposed the compensation coil only affects the overall bias field, and with $\mathbb{H}_{CO} = 0$, the axial frequency is then favorably decoupled:

$$\omega_z = \sqrt{\gamma\mu_0 I_{PI}\mathbb{H}_{PI}} \tag{3.10}$$

The field's bias, gradient and curvature also given by:

$$\boldsymbol{\delta} = \mu_0(I_{PI}\mathbb{F}_{PI} - I_{CO}\mathbb{F}_{CO}) \tag{3.11}$$

$$\boldsymbol{\alpha} = 3\mu_0 I_{BI}\mathbb{G}_{BI} = 3\mu_0 I_{SI}\mathbb{G}_{SI} \tag{3.12}$$

$$\boldsymbol{\beta} = \mu_0 I_{PI}\mathbb{H}_{PI} \tag{3.13}$$

$$\omega_z = \sqrt{\gamma\mu_0 I_{PI}\mathbb{H}_{PI}} \tag{3.14}$$

Finally, the radial frequency would then be expressed by:

$$\omega_\rho = \sqrt{\gamma\mu_0 \left\{ \frac{(3I_{BI}\mathbb{G}_{BI})^2}{I_{PI}\mathbb{F}_{PI} - I_{CO}\mathbb{F}_{CO}} - \frac{I_{PI}\mathbb{H}_{PI}}{2} \right\}} \tag{3.15}$$

The aspect ratio is simply defined as:

$$\Lambda = \omega_\rho/\omega_z \tag{3.16}$$

3.6 DOFs, Parameters, and Constraints

Given the amount of input variables and output parameters involved in the equations presented above, it is worth to recount the extent of the degrees of freedom in the all coils magnetic trap. Initially, one could think of 12 DOFs by choosing separate radii R, vertical separation A and current I for each of the coils pairs: BI, SI, PI and CO. Several practical and physical constraints should be considered though.

The geometric parameters restricted both by the fixed dimensions of the current experimental set up and the desired roles for each coil pair. In order to decouple $omega_z$ so that it is only influenced by the PI coils, the CO should only yield and overall bias field, thus requiring $R_{CO} = 2A_{CO}$. Technically there is not a particular constraint for the PI coils, but their performance (and therefore axial frequency) is enhanced if $R_{PI} = \sqrt{\frac{4}{3}}A_{PI}$. Both Pi and CO coils should have a radius big enough to let the vacuum chamber through. Also, in an ideal manner $R = \sqrt{\frac{4}{3}}$ could be enforced in both the SI and BI coils to prevent the appearance of higher order terms. Thus, the geometrical DOFs are reduced from 8 down to 4 at the best case. For the sake of the simulations, however, it was assumed that the physical parameters of the trap would be relatively fixed for a functioning trap. All of the geometrical apramesters $\mathbb{F}$, $\mathbb{G}$, $\mathbb{H}$ were ssumed to be fixed *ab initio*. Therefore, only coil currents were initially considered as parameters to manipulate.

The Ioffe pairs (BI and SI) should balance each other so that when combined, they do not introduce a field in the z direction (preserving ω_z to be dependent on the Pi coil pair only). This restriction, as given in (3.6), implies that I_{BI} and I_{SI} are not arbitrarily independent but proportional to a fixed factor given by $\mathbb{G}_{SI}/\mathbb{G}_{BI}$. Added to this, equation (3.11) shows that I_{PI} and I_{CO} are coupled if a fixed bias field δ is chosen. Lastly, equation (3.15) establishes a condition that the trap parameters should comply to avoid the radial confinement from breaking down:

$$I_{BI} > \frac{1}{3\mathbb{G}_{BI}}\sqrt{\frac{I_{PI}\mathbb{H}_{PI}}{2}(I_{PI}\mathbb{F}_{PI} - I_{CO}\mathbb{F}_{CO})} \tag{3.17}$$

With the previous discussion in mind, technically only 3 DOFs are left, and therefore, the simulations only had 3 *free* parameters. The selected ones were I_{PI}, from which ω_z can be tuned by using equation (3.10); δ, which is a particularly interesting trap parameter and directly related with the trap depth; and I_{SI} (I_{BI} could have been chosen as well).

It is worth mentioning, lastly, that all the currents are ultimately constrained by the available current sources and the cooling methods.

4 Numerical Simulations

Several numerical simulations were performed with MATLAB in order to evaluate the different trap parameters which the all coils IP trap could provide under both geometrical constraints such as trap size and physical constraints such as affordable input currents for the coils. The magnetic field was first calculated by applying the equations in (2.2) to 8 ideal coils positioned as described in the previous section, i.e. forming the PI, CO, BI and SI pairs for a total of 8 coils. From the field, trap parameters such as axial and radial frequencies, aspect ratio, linear gradients and curvature were then calculated for a given set of dimensions and desired parameters. The MATLAB routines used for the simulations were developed in a modular manner, and are fully available in Appendix D.

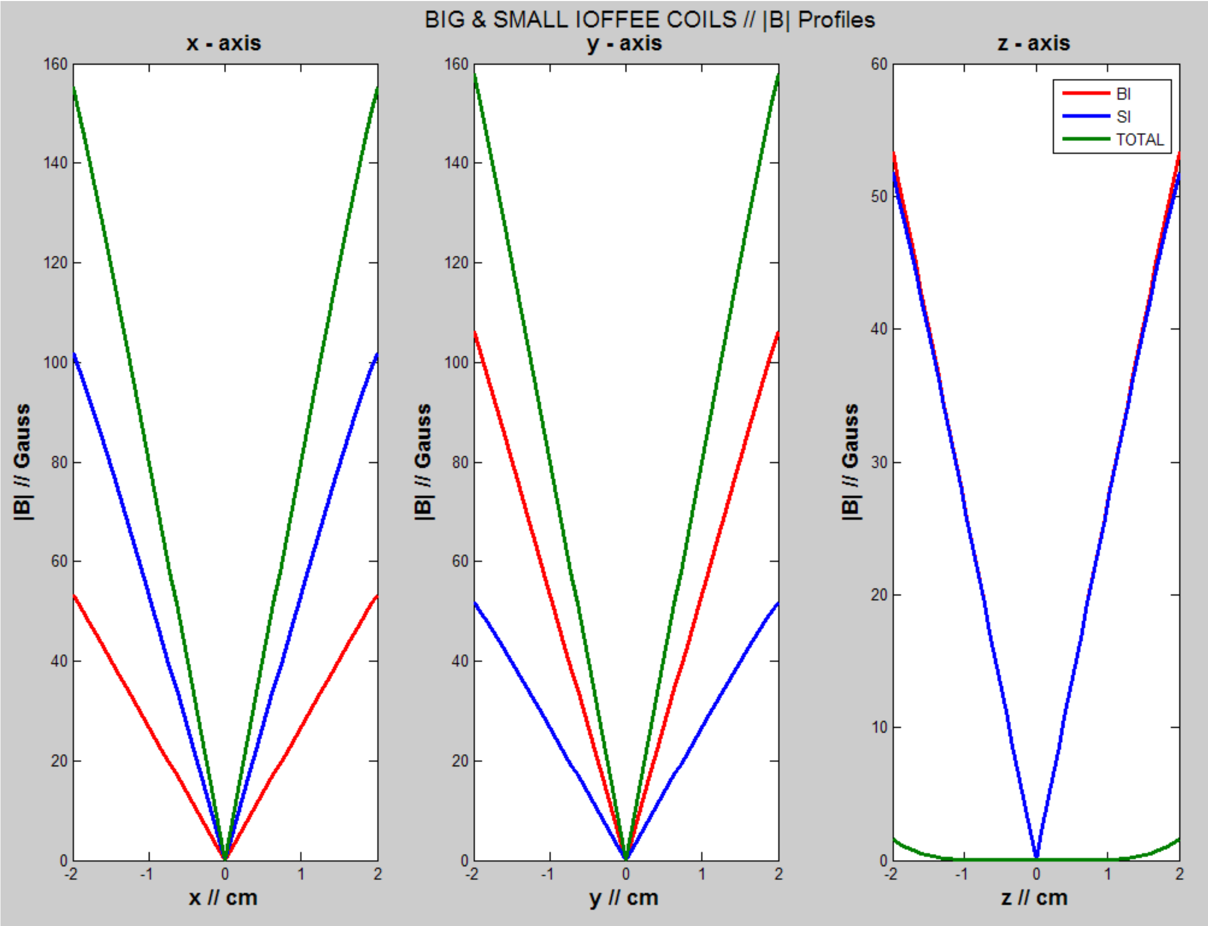


Figure 6: Magnetic Field magnitude profiles for the Ioffe Coils. The selected parameters are $I_{PI} = 12A$, $I_{SI} = 50A$, $\delta = 5G$ and $N = 16$ for every set of coils

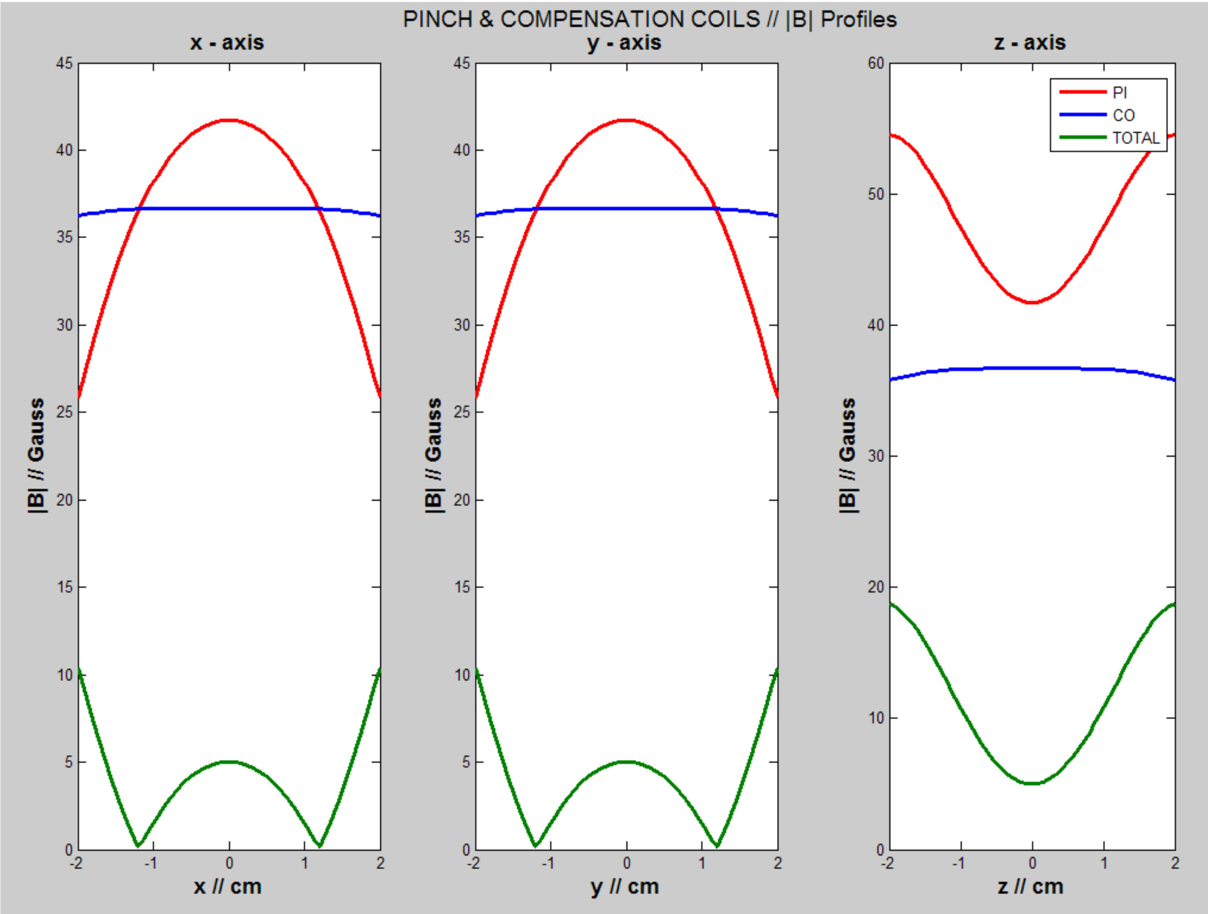


Figure 7: Magnetic Field magnitude profiles for the PI and CO coils. The selected parameters are $I_{PI} = 12A$, $I_{SI} = 50A$, $\delta = 5G$ and $N = 16$ for every set of coils

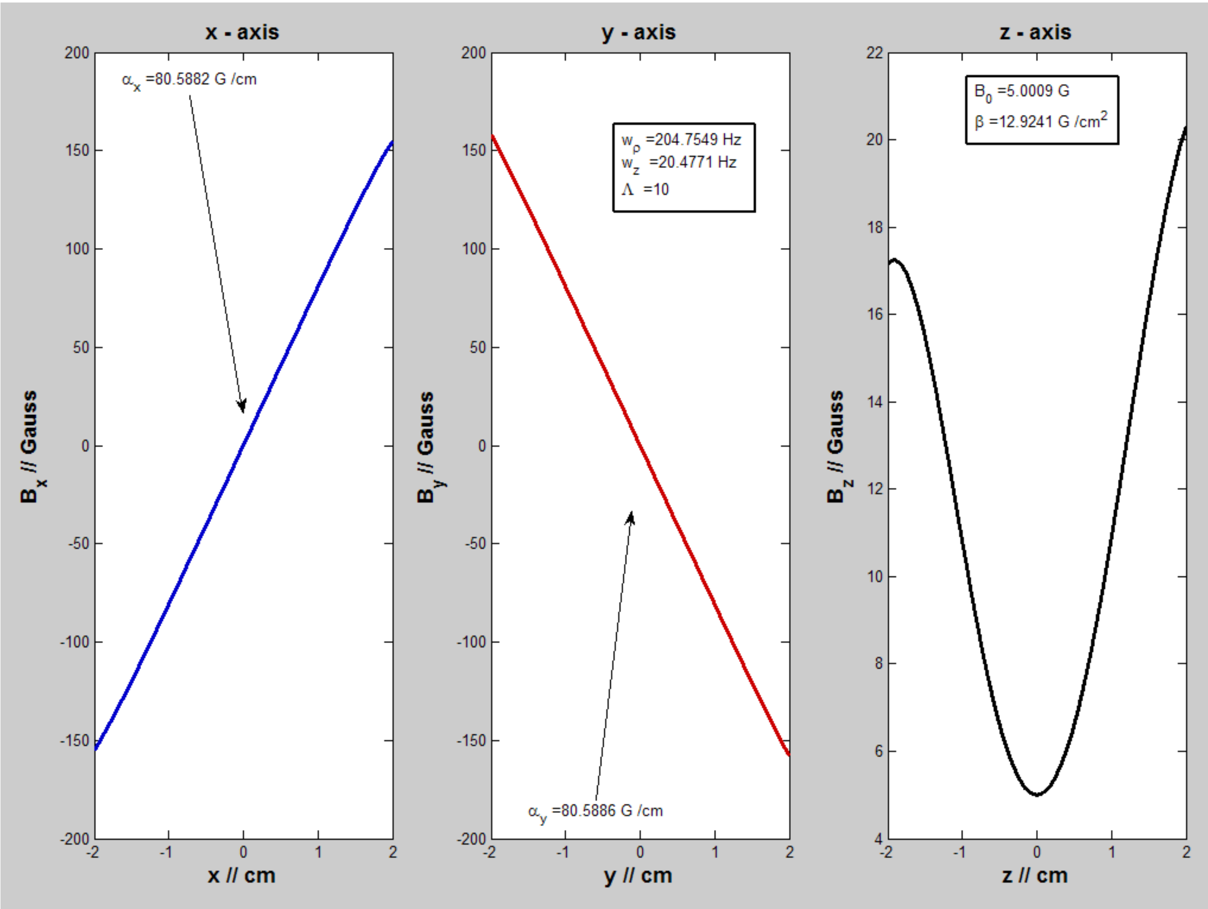


Figure 8: Magnetic Field profiles for the combination of every coil.Some output parameters can be seen displayed as well in the textboxes. The selected parameters are $I_{PI} = 12A$, $I_{SI} = 50A$, $\delta = 5G$ and $N = 16$ for every set of coils

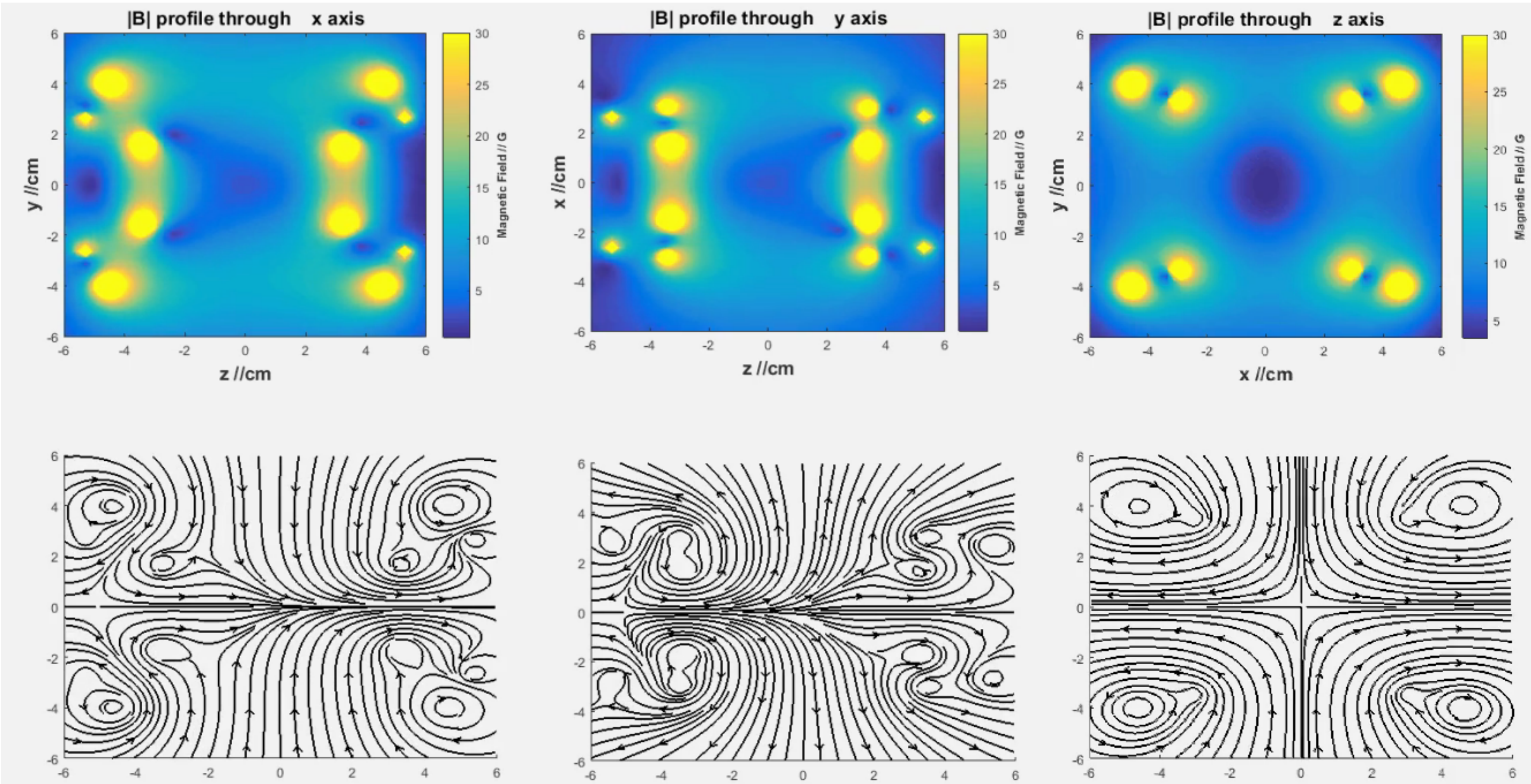


Figure 9: Magnetic Field Profiles as simulated in MATLAB. Animated versions of the profiles can be found here (https://goo.gl/SyCuoz)

$w_\rho = 259Hz$	$I_{BI} = 35.5A$	$I_{PI} = 5.4A$
$w_z = 9Hz$	$I_{SI} = 20A$	$I_{CO} = 8A$

$\Lambda = 29$ $\quad \alpha = 32.2\ G/cm$ $\quad \beta = 2.53\ G/cm^2$ $\quad B_0 = 0.5\ G$

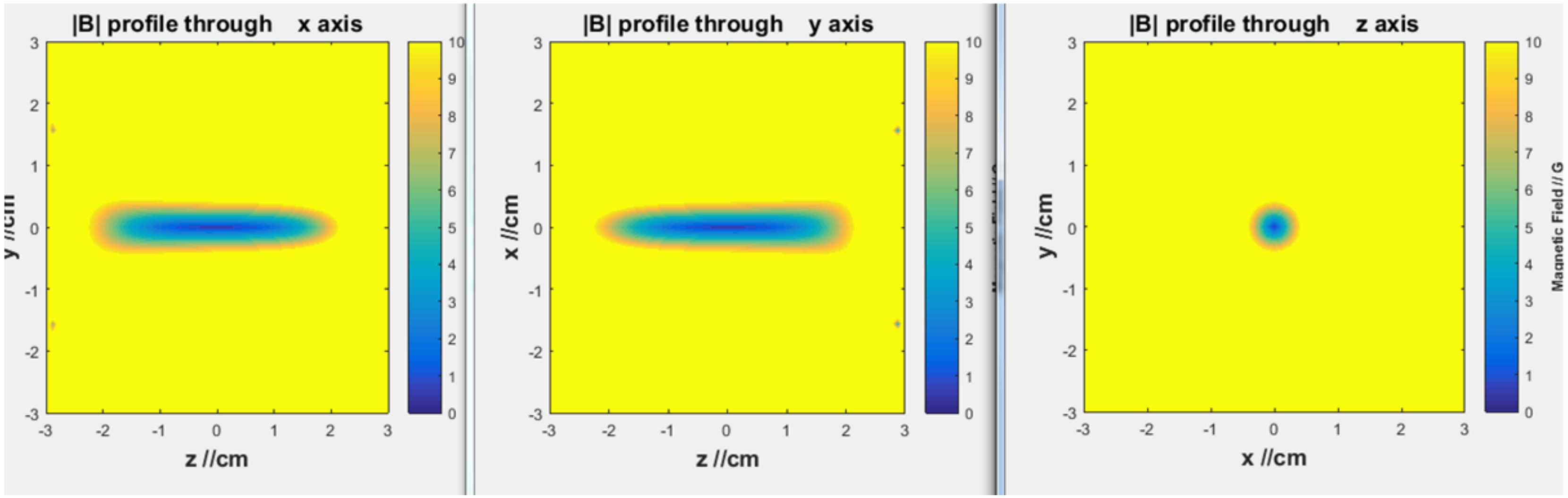
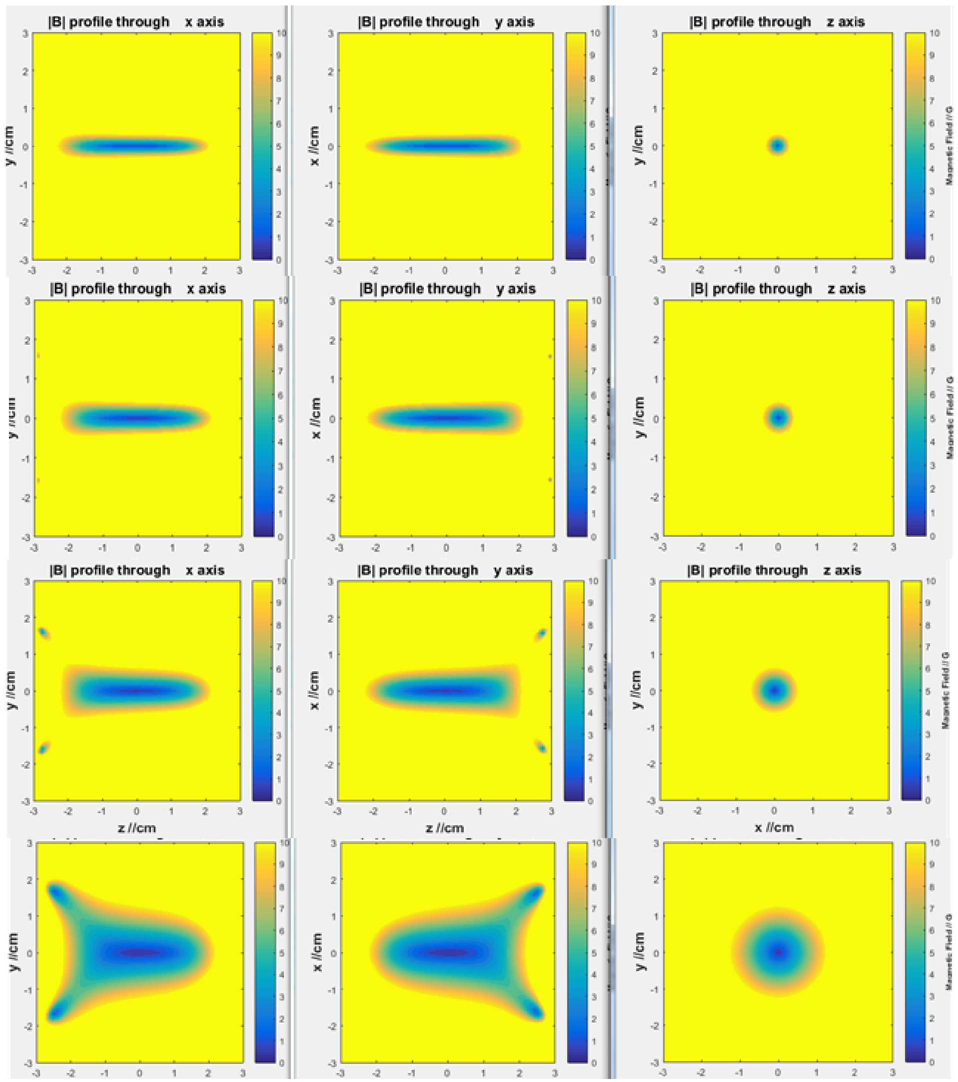


Figure 10: Magnetic Field Profiles as simulated in MATLAB. Parameters selected to show a *pencil* like aspect ratio



$w_\rho = 259Hz$ $I_{BI} = 35.5A$ $I_{PI} = 5.4A$

$w_z = 9Hz$ $I_{SI} = 20A$ $I_{CO} = 8A$

$\Lambda = 29$ $\alpha = 32.2\,G/cm$ $\beta = 2.53\,G/cm^2$ $B_0 = 0.5\,G$

$w_\rho = 194Hz$ $I_{BI} = 26.6A$

$w_z = 9Hz$ $I_{SI} = 15A$

$\Lambda = 21$

$w_\rho = 129Hz$ $I_{BI} = 17.7A$

$w_z = 9Hz$ $I_{SI} = 10A$

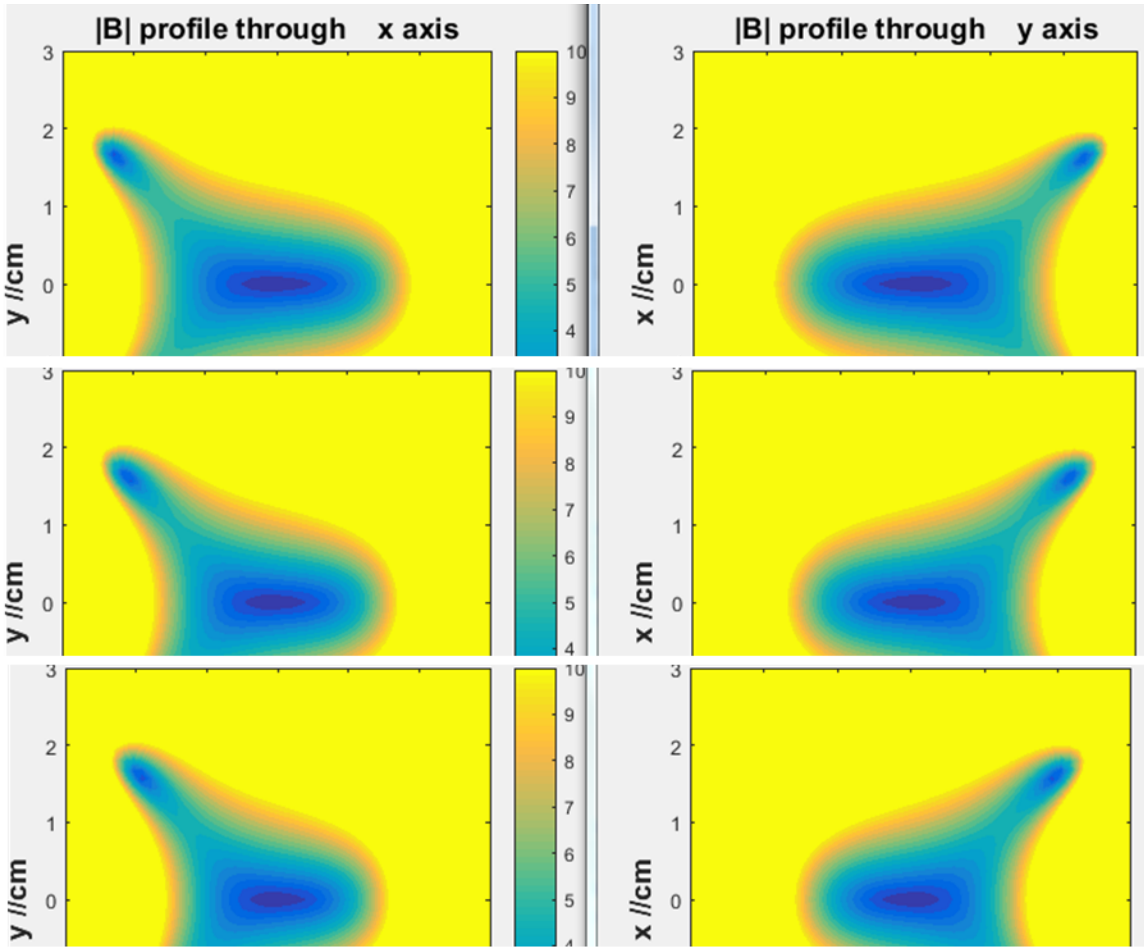
$\Lambda = 14$

$w_\rho = 64Hz$ $I_{BI} = 8.8A$

$w_z = 9Hz$ $I_{SI} = 5A$

$\Lambda = 7$

Figure 11: Magnetic Field Profiles as simulated in MATLAB, showing the variability of the aspect ratio when the current on the Ioffe coil pairs is reduced



$w_\rho = 64Hz$ $\quad I_{PI} = 7.9A$

$w_z = 10Hz$ $\quad I_{CO} = 12A$

$\Lambda = 6$ $\quad \beta = 3.63\ G/cm^2$

$w_\rho = 64Hz$ $\quad I_{PI} = 10.4A$

$w_z = 12Hz$ $\quad I_{CO} = 16A$

$\Lambda = 5$ $\quad \beta = 4.73\ G/cm^2$

$w_\rho = 64Hz$ $\quad I_{PI} = 12.9\ A$

$w_z = 13Hz$ $\quad I_{CO} = 20A$

$\Lambda = 5$ $\quad \beta = 5.83\ G/cm^2$

Figure 12: Magnetic Field Profiles as simulated in MATLAB, showing the variability of the aspect ratio when the current on the PI coils is increased

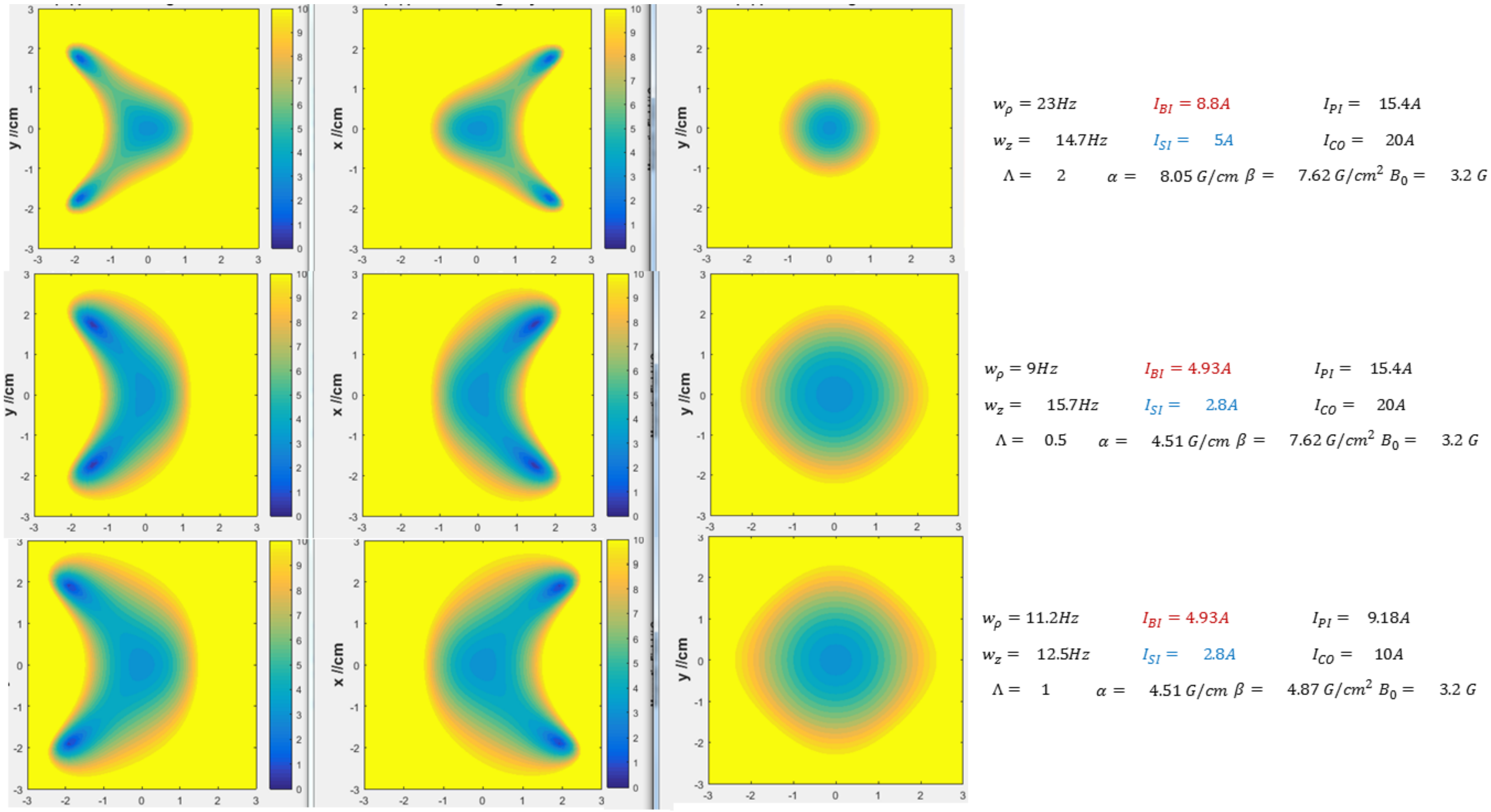


Figure 13: Magnetic Field Profiles as simulated in MATLAB, showing the variability of the aspect ratio when the current on the Ioffe coil pairs is reduced while I_{PI} is still high, leading to a nearly spherical aspect ratio

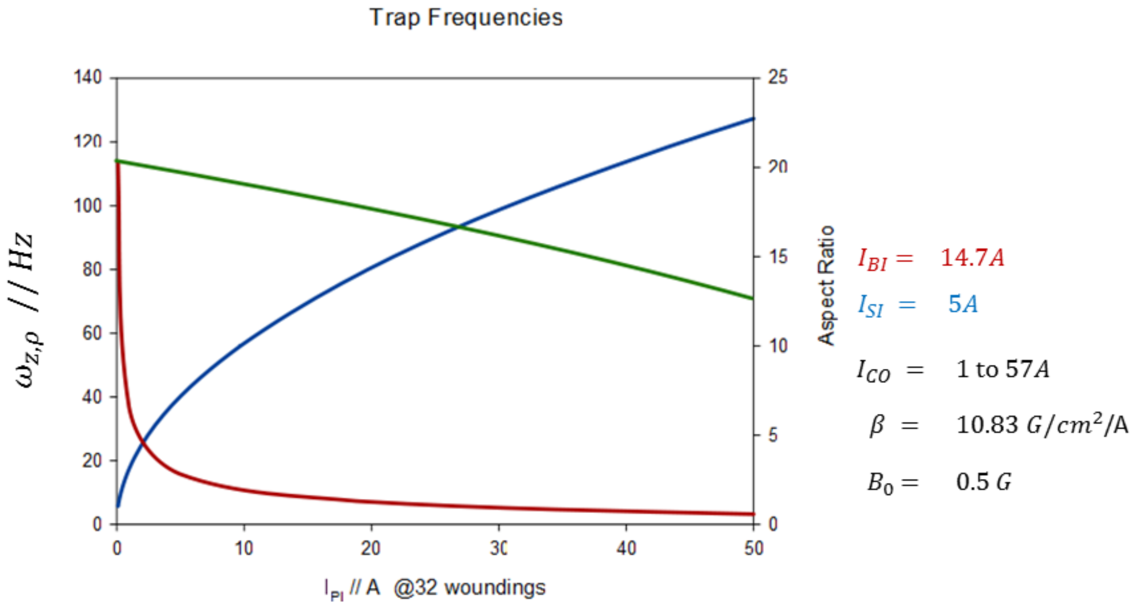


Figure 14: Trap frequencies(left axis, green line for radial and blue for axial) and aspect ratio (right axis, red line) profiles as a function of I_{PI}

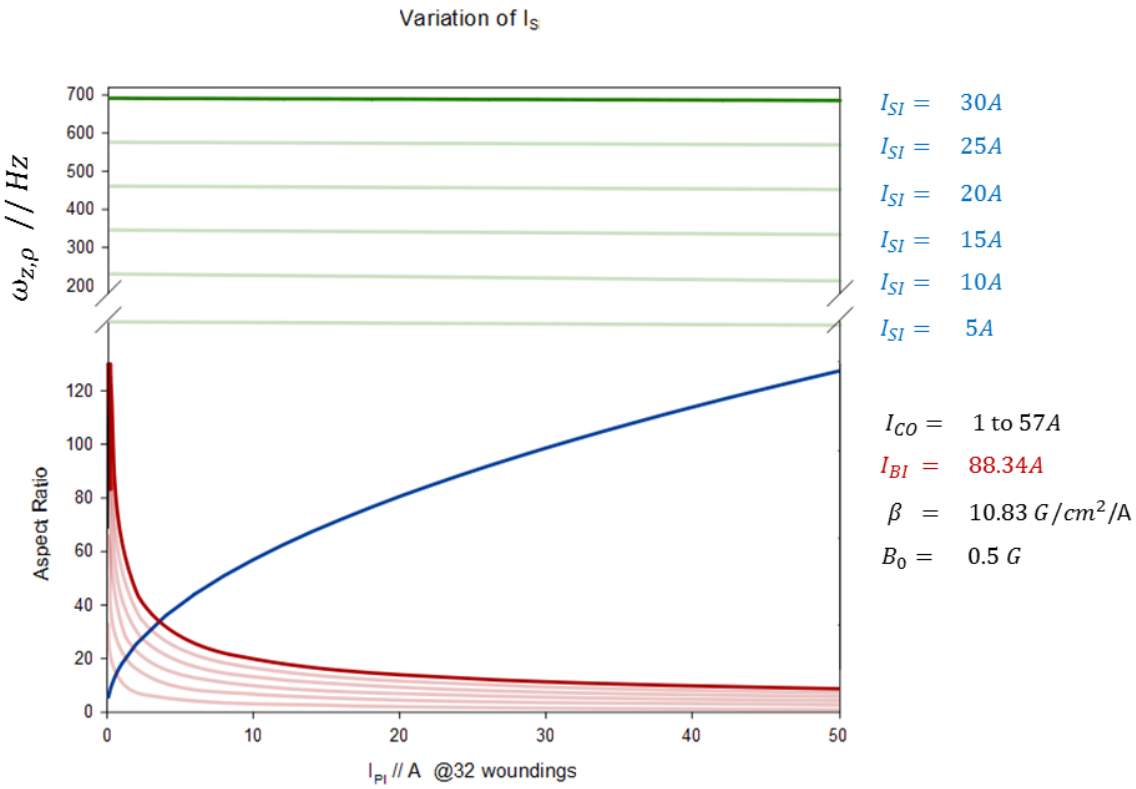


Figure 15: Variability in trap frequencies(left axis, green line for radial frequency) and aspect ratio (red line) profiles as I_{SI} ranges from 30A (upper graph) down to 5A(lower graph).

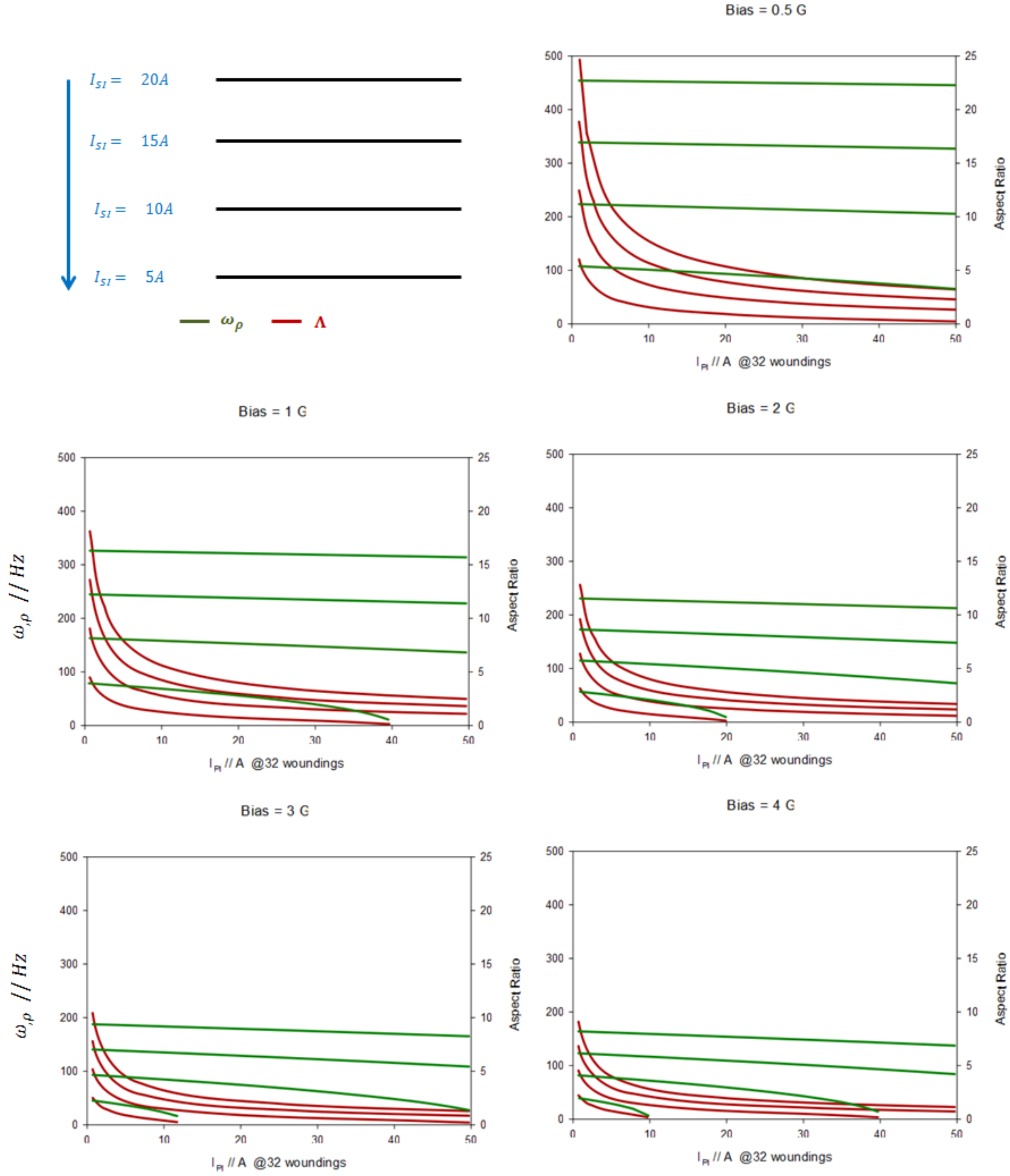


Figure 16: Variability in trap radial frequency(left axis, green line for radial frequency) and aspect ratio (right axis, red line) profiles as δ ranges from 0.5G up to 4G(lower graph) and I_{SI} ranges from 20A (upper graph) down to 5A(lower graph).

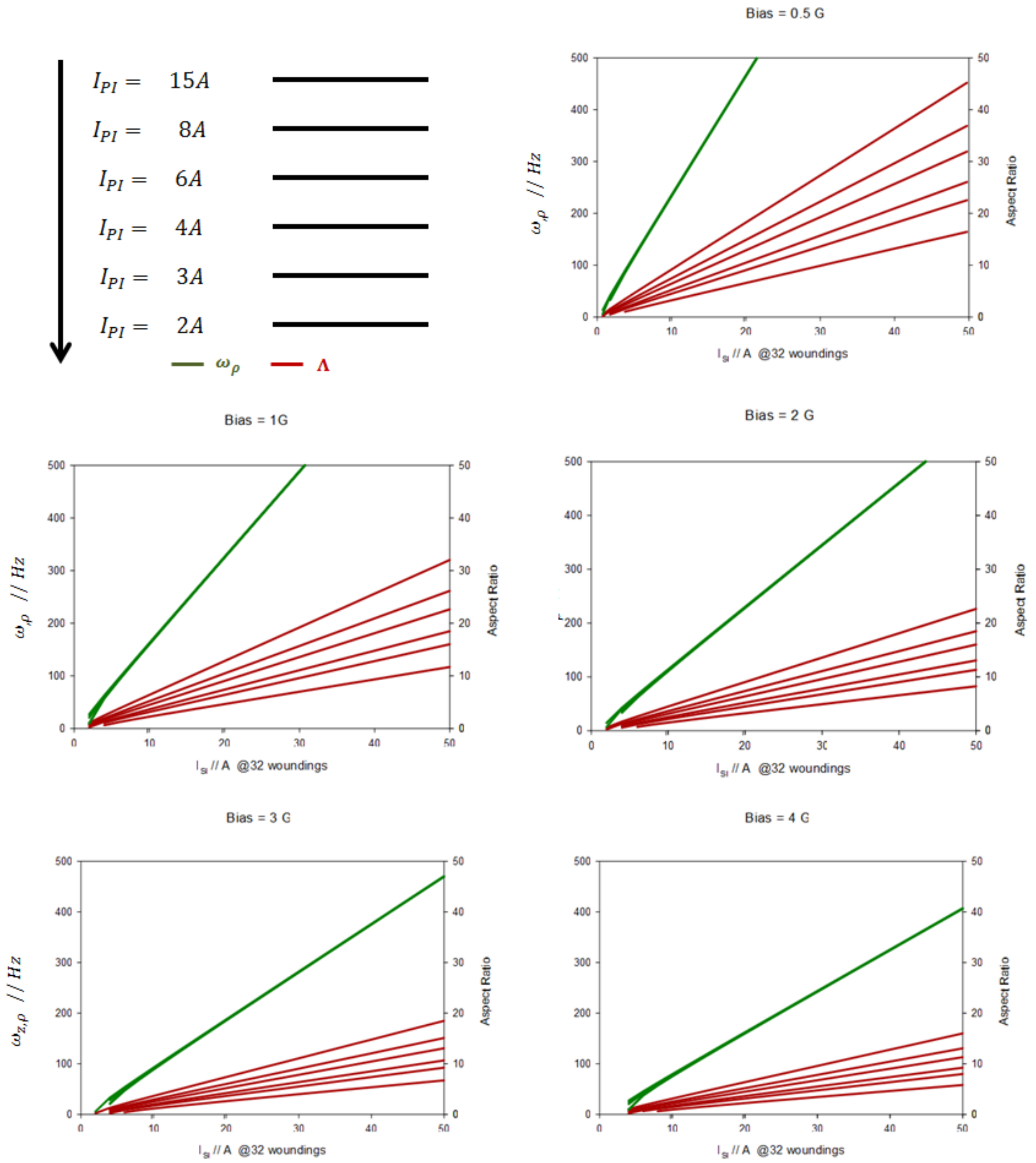


Figure 17: Variability in trap radial frequency(left axis, green line for radial and blue for axial) and aspect ratio (right axis, red line) profiles as δ ranges from 0.5G up to 4G(lower graph) and I_{PI} ranges from 15A (upper graph) down to 2A(lower graph).

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Appendix A Magnetic Field for an ideal coil

This derivation can be also found elsewhere [3]. As stated, the magnetic field for an ideal current coil will be calculated from the general expression

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla} \times \frac{\mu_0}{4\pi} \iiint \frac{\boldsymbol{J}}{|\boldsymbol{r}|}\, d^3r$$

We first need to calculate $\boldsymbol{A}$. From the circular symmetry of the problem, the vector potential should be independent of the polar angle φ, so for simplicity, we can calculate it for a point located at $\varphi = 0$ (see Figure). Pairing the arc length contributions from $+\varphi$ and $-\varphi$ is also enlightening, as the $\boldsymbol{ds}$ components in the radial direction ds_ρ cancel out while the polar ones ds_φ add up. Then, $\boldsymbol{A}$ only has a polar component, which would be given by:

$$\begin{aligned}
\boldsymbol{A} &= \frac{\mu_0 I}{4\pi} \int \frac{\boldsymbol{ds}}{|\boldsymbol{r}|} \\
\implies \boldsymbol{A}_\varphi &= \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{\boldsymbol{ds}_\varphi}{|\boldsymbol{r}|} \\
&= \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R\,cos(\varphi)\,d\varphi}{\sqrt{R^2+\rho^2+z^2-2R\rho cos(\varphi)}} \\
&= \frac{\mu_0 I}{2\pi} \int_0^{\pi} \frac{R\,cos(\varphi)\,d\varphi}{\sqrt{R^2+\rho^2+z^2-2R\rho cos(\varphi)}} \\
&! \quad \text{Introduce change of variable } \varphi = \pi + 2\theta \\
&\quad\ \ \text{with } cos(\varphi) = 2sin^2(\theta) - 1 \text{ and } d\varphi = 2d\theta \\
\text{so that} &= \frac{\mu_0 R I}{\pi} \int_0^{\pi/2} \frac{(2sin^2(\theta)-1)\ \ d\theta}{\sqrt{(R+\rho)^2+z^2-4R\rho sin^2(\theta)}} \\
&! \quad \text{Introduce change of variable } \kappa^2 = \frac{4R\rho}{(R+\rho)^2+(z)^2} \\
\text{so that} &= \frac{\kappa\mu_0 I}{2\pi}\sqrt{\frac{R}{\rho}} \left\{ \left(\frac{2}{\kappa^2}-1\right) \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-\kappa^2\,sin^2(\theta)}} - \frac{2}{\kappa^2}\int_0^{\pi/2} \sqrt{1-\kappa^2\,sin^2(\theta)}\,d\theta \right\} \\
\boldsymbol{A}_\varphi &= \frac{\mu_0 I}{\pi\kappa}\sqrt{\frac{R}{\rho}} \left\{ \left(1-\frac{1}{2}\kappa^2\right)\mathbb{K}(\kappa^2) - \mathbb{E}(\kappa^2) \right\}
\end{aligned}$$

From this, $\boldsymbol{B}$ can be evaluated by using:

$$
\begin{aligned}
\boldsymbol{B_\rho} &= -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho \boldsymbol{A_\varphi}\right) = -\frac{\partial \boldsymbol{A_\varphi}}{\partial z} \\
\boldsymbol{B_\varphi} &= \frac{\partial \boldsymbol{A_\rho}}{\partial z} = 0 \\
\boldsymbol{B_z} &= -\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \boldsymbol{A_\varphi}\right)
\end{aligned}
$$

and the following relations:

$$
\begin{gathered}
\frac{\partial \mathbb{K}}{\partial \kappa} = \frac{\mathbb{E}}{\kappa(1-\kappa^2)} - \frac{\mathbb{K}}{\kappa} \\
\frac{\partial \mathbb{E}}{\partial \kappa} = \frac{\mathbb{E}-\mathbb{K}}{\kappa} \\
\frac{\partial \kappa}{\partial z} = -\frac{z\kappa^3}{4R\rho} \\
\frac{\partial \kappa}{\partial \rho} = \frac{\kappa}{2\rho} - \frac{\kappa^3}{4\rho} - \frac{\kappa^3}{4R}
\end{gathered}
$$

After simplifying and collecting terms, one obtains:

$$
\begin{aligned}
B_\rho &= \frac{\mu_0 I}{2\pi\rho}\frac{z-A}{\sqrt{(R+\rho)^2+z^2}}\left\{-\mathbb{K}(\kappa^2) + \frac{R^2-\rho^2-z^2}{(R-\rho)^2+z^2}\,\mathbb{E}(\kappa^2)\right\} \\
B_z &= \frac{\mu_0 I}{2\pi}\frac{1}{\sqrt{(R+\rho)^2+z^2}}\left\{\quad \mathbb{K}(\kappa^2) + \frac{R^2-\rho^2-z^2}{(R-\rho)^2+z^2}\,\mathbb{E}(\kappa^2)\right\} \\
B_\phi &= 0
\end{aligned}
$$

The translation in z when the coil has a vertical displacement from the origin A follows rather naturally.

Appendix B Power Series Derivation

A deeper treatment of the series expansion for magnetic fields can be found elsewhere[4]. Maxwell's equations for a magnetostatic field allow for the existence of a scalar potential Ψ which satisfies:

$$\boldsymbol{B} = \boldsymbol{\nabla\Psi} = \frac{\partial\Psi}{\partial\rho}\boldsymbol{\rho} + \frac{1}{\rho}\frac{\partial\Psi}{\partial\varphi}\boldsymbol{\varphi} + \frac{\partial\Psi}{\partial z}\boldsymbol{z} \tag{B.1}$$

Motivated in a multipole expansion, we then seek to express both $\boldsymbol{B}$ and $\boldsymbol{\Psi}$ as an expansion of Legendre polynomials. We are trying to find a relatively simple series expansion for 2.2, something in the shape of:

$$\boldsymbol{B_z(z, \rho = 0)} = \sum_{\boldsymbol{n=0}} b_n\, z^n \tag{B.2}$$

which would be a basic Taylor expansion. With this in mind, and taking into account that the order of each term will decrease when going from $\boldsymbol{\Psi}$ to $\boldsymbol{B}$, one can propose the expansion of the form:

$$\boldsymbol{\Psi} = \sum_{\boldsymbol{n=1}} \frac{b_{n-1}}{n}\, p_n(\rho, z) \tag{B.3}$$

where the $p_n(\rho, z)$ terms are the Legendre polynomials expressed in ρ and z. By applying (B.3) to (B.1), one obtains:

$$\begin{aligned} \boldsymbol{B_z} &= \frac{\partial\boldsymbol{\Psi}}{\partial z} = \sum_{\boldsymbol{n=1}} \frac{b_{n-1}}{n}\,\frac{\partial p_n(\rho, z)}{\partial z} \\ &= b_0\frac{\partial p_1}{\partial z} + \frac{b_1}{2}\frac{\partial p_2}{\partial z} + \frac{b_2}{3}\frac{\partial p_3}{\partial z} + + \frac{b_3}{4}\frac{\partial p_4}{\partial z} + ... \\ &= b_0\,(1) + \frac{b_1}{2}\,(2z) + \frac{b_2}{3}\left(3z^2 - 3\rho^2/2\right) + \frac{b_3}{4}\left(4z^3 - 6z\rho^2\right) + ... \\ &= b_0(1) + b_1(z) + b_2(z^2 - \rho^2/2) + b_3(z^3 - 3z\rho^2/2) + ... \end{aligned} \tag{B.4}$$

and

$$
\begin{aligned}
\boldsymbol{B_\rho} &= \frac{\partial \boldsymbol{\Psi}}{\partial \rho} = \sum_{\boldsymbol{n=1}} \frac{b_{n-1}}{n} \frac{\partial p_n(\rho, z)}{\partial \rho} \\
&= b_0 \frac{\partial p_1}{\partial \rho} + \frac{b_1}{2} \frac{\partial p_2}{\partial \rho} + \frac{b_2}{3} \frac{\partial p_3}{\partial \rho} + + \frac{b_3}{4} \frac{\partial p_4}{\partial \rho} + ... \\
&= b_0\,(0) + \frac{b_1}{2}\,(-\rho) + \frac{b_2}{3}\,(-3\rho z) + \frac{b_3}{4}\left(3\rho^3/2 - 6\rho z^2\right) + ... \\
&= b_0(0) + b_1(-\rho/2) + b_2(-\rho z) + b_3(3\rho^3/8 - 3\rho z^2/2) + ... \qquad \text{(B.5)}
\end{aligned}
$$

From the equations above, it is worth noticing that the same coefficients b_n directly appear for both the radial and axial expansions. In (2.4), it is easily seen that the coefficients match for each of the expansion's orders, and the geometrical factors arise directly from this b_n coefficients.

In order to calculate the b_n coefficients, one can Taylor expand the axial coefficient in (2.2) evaluated at $\rho = 0$, as that would result in a series in the form of (B.2). The process for obtaining the first orders of the expansion are presented bellow.

$$
\begin{aligned}
\boldsymbol{B_z(z, \rho = 0)} &= \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{R^2 + (z-A)^2}} \left\{ \quad \mathbb{K}(0) + \frac{R^2 - (z-A)^2}{R^2 + (z-A)^2}\,\mathbb{E}(0) \right\} \\
&= \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{R^2 + (z-A)^2}} \frac{\pi}{2} \left\{ \quad 1 + \frac{R^2 - (z-A)^2}{R^2 + (z-A)^2} \right\} \\
&= \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{R^2 + (z-A)^2}} \frac{\pi}{2} \left\{ \frac{2R^2}{R^2 + (z-A)^2} \right\} \\
&= \frac{\mu_0 I}{2} \frac{1}{\sqrt{R^2 + (z-A)^2}} \left\{ \frac{R^2}{R^2 + (z-A)^2} \right\} \\
&= \frac{\mu_0 I R^2}{2} \frac{1}{(R^2 + (z-A)^2)^{3/2}} \\
&= \frac{\mu_0 I R^2}{2(R^2 + A^2)^{3/2}} \quad \sum_{\boldsymbol{n=0}} g_n \left(\frac{z}{R^2 + A^2}\right)^n = \sum_{\boldsymbol{n=0}} b_n\, z^n \qquad \text{(B.6)}
\end{aligned}
$$

where g_n are the polynomial terms in the series expansion of:

$$
\frac{(R^2 + A^2)^{3/2}}{[R^2 + (z-A)^2]^{3/2}} = \sum_{\boldsymbol{n=0}} g_n \left(\frac{z}{R^2 + A^2}\right)^n
$$

The first terms can be easily shown to be: $g_0 = 1$, $g_1 = 3A$, $g_2 = 3(4A^2 - R^2)/2$, $g_3 = 5A(4A^2 - 3R^2)/2$ and so on.

From the last equation in (B.6), one directly extracts:

$$b_n = \mu_0 I \frac{R^2}{2(R^2+A^2)^{n+3/2}}\, g_n \tag{B.7}$$

and so the first b_n terms would be given by:

$$\begin{aligned} b_0 &= \mu_0 I \frac{R^2}{2(R^2+A^2)^{3/2}} \qquad & b_1 = \mu_0 I \frac{3AR^2}{2(R^2+A^2)^{5/2}} \\ b_2 &= \mu_0 I \frac{3R^2(4A^2-R^2)}{4(R^2+A^2)^{7/2}} \qquad & b_3 = \mu_0 I \frac{5AR^2(4A^2-3R^2)}{4(R^2+A^2)^{9/2}} \end{aligned} \tag{B.8}$$

One can then extract the purely geometrical factors (as defined in (2.3)) from the $\mu_0 I$ part of each b_n term to obtain:

$$\begin{aligned} b_0 &= \mu_0 I \cdot \frac{1}{2}\mathbb{F} \qquad & b_1 = \mu_0 I \cdot \mathbb{G} \\ b_2 &= \mu_0 I \cdot \frac{1}{4}\mathbb{H} \qquad & b_3 = \mu_0 I \cdot \frac{1}{2}\mathbb{I} \end{aligned} \tag{B.9}$$

It is worth noticing that the fractions accompaning some of the prefactors are rather arbitrary, but useful to simplify the involved algebra in further calculations.

By substituing equation (B.9) into (B.4) and (B.5), one then obtains (2.4), as stated before.

Appendix C Trap Frequencies Derivations

AXIAL FREQUENCY (ω_z)

From (3.7), evaluated at $\rho = 0$:

$$\boldsymbol{B(\rho = 0)} = \boldsymbol{\delta} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \frac{1}{2}\boldsymbol{\beta} \begin{pmatrix} 0 \\ 0 \\ z^2 \end{pmatrix}$$

so that

$$\boldsymbol{B_x} = \boldsymbol{B_x} = 0 \quad \text{and} \quad \boldsymbol{B_z} = \boldsymbol{\delta} + \frac{1}{2}\boldsymbol{\beta} z^2$$

and

$$|\boldsymbol{B}| = |\boldsymbol{B_z}| = \boldsymbol{\delta} + \frac{1}{2}\boldsymbol{\beta} z^2$$

We also have

$$U \propto \boldsymbol{\mu} \cdot \boldsymbol{B} = g_F m_F \mu_B |\boldsymbol{B}| = \frac{1}{2} m_{Rb}\, \omega_z^2 z^2$$

so that

$$\frac{\partial^2 U}{\partial z^2} = g_F m_F \mu_B \frac{\partial^2 |\boldsymbol{B}|}{\partial z^2} = g_F m_F \mu_B \boldsymbol{\beta} = m_{Rb}\, \omega_z^2$$

and we obtain

$$\omega_z = \sqrt{\frac{g_F m_F \mu_B}{m_{Rb}} \boldsymbol{\beta}}$$

i.e.

$$\boxed{\omega_z \approx \sqrt{\gamma \boldsymbol{\beta}}}$$

RADIAL FREQUENCY (ω_ρ)

From (3.7), evaluated at $z = 0$:

$$\boldsymbol{B(z=0)} = \boldsymbol{\delta}\begin{pmatrix}0\\0\\1\end{pmatrix} + \boldsymbol{\alpha}\begin{pmatrix}x\\-y\\0\end{pmatrix} + \frac{1}{2}\boldsymbol{\beta}\begin{pmatrix}0\\0\\-\rho^2/2\end{pmatrix}$$

so that

$$\begin{aligned}
|\boldsymbol{B}| &= \sqrt{\boldsymbol{\alpha}^2x^2 + \boldsymbol{\alpha}^2y^2 + \boldsymbol{\delta}^2 - \frac{1}{2}\boldsymbol{\delta\beta}\rho^2 + \frac{1}{16}\boldsymbol{\beta}^2\rho^4} \\
&\approx \sqrt{\boldsymbol{\alpha}^2x^2 + \boldsymbol{\alpha}^2y^2 + \boldsymbol{\delta}^2 - \frac{1}{2}\boldsymbol{\delta\beta}\rho^2} \\
&= \sqrt{\boldsymbol{\delta}^2 + \left(\boldsymbol{\alpha}^2 - \frac{1}{2}\boldsymbol{\delta\beta}\right)\rho^2} \\
&= \boldsymbol{\delta}\sqrt{1 + \left(\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\delta}^2} - \frac{\boldsymbol{\beta}}{2\boldsymbol{\delta}}\right)\rho^2} \\
&\approx \boldsymbol{\delta}\left\{1 + \frac{1}{2}\left(\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\delta}^2} - \frac{\boldsymbol{\beta}}{2\boldsymbol{\delta}}\right)\rho^2\right\} \\
|\boldsymbol{B}| &\approx \boldsymbol{\delta} + \frac{1}{2}\left(\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\delta}} - \frac{\boldsymbol{\beta}}{2}\right)\rho^2
\end{aligned}$$

as before

$$\frac{\partial^2 U}{\partial \rho^2} = m_{Rb}\,\omega_\rho^2 = g_F m_F \mu_B \left(\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\delta}} - \frac{\boldsymbol{\beta}}{2}\right)$$

so that

$$\omega_\rho \approx \sqrt{\frac{g_F m_F \mu_B}{m_{Rb}}\left(\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\delta}} - \frac{\boldsymbol{\beta}}{2}\right)}$$

and

$$\boxed{\omega_\rho \approx \sqrt{\gamma\left(\frac{\boldsymbol{\alpha}^2}{\boldsymbol{\delta}} - \frac{\boldsymbol{\beta}}{2}\right)}}$$

Appendix D MATLAB routines

```matlab
function [Bz,Brho] = BcoilExact(R,A,z,rho,s)
%BcoilExact Calculates the 3D magnetic field vector of a coil with
%particular radius R and vertical distance A, this at the coordinate
%(rho,z). The 3D is given in cylindrical coordinates, and only rho
%and z are necessary due to the involved symmetry. The factor s
%controls the current.
%
% See "Magnetostatic trapping fields for neutral atoms" by Bergeman
% and W.R. Smythe, "Static and Dynamic Electricity"  (McGraw-Hill), NY
% 1950, pp. 291 :
%
% Available @: https://goo.gl/HA0Igh
%
% For further info on these equations

mu = 0.4*pi; %Mixed unites, i.e. amps,cm and gauss 1T = 10,000G
I = 1*s; %Change 1 to change current units, e.g. 1000 for kA.

dp = (R+rho).^2+(z-A).^2;
dm = (R-rho).^2+(z-A).^2;

k2 = 4*R*rho ./ dp;
[K,E] = ellipke(k2);

Bz = mu.*I/(2*pi).*1./ sqrt(dp).*(K + E .* (R.^2-rho.^2 - (z-A).^2)./dm);
Brho = mu.*I./(2*pi.*rho).*(z-A)./sqrt(dp).*(-K+E.*(R.^2+rho.^2+(z-A).^2)./dm);

end
```

```
function [Bxx,Byy,Bzz] = BCoilPair(Z,RHO,PHI,R1,A1,I1,R2,A2,I2)
%BCOILDPAIR calculates the 3D magnetic field vector at the point  in space
%defined by (rho, phi, z). Phi is technically not necessary for the
%calculation but it is included in order to return the vector in cartesian
%coordinates. The dimensions and currents for each coil are defined, but
%generally a Helmholtz or anti-Helmholzt configurations are used.
%
% See also BcoilExact

[B1_z,B1_rho] = BcoilExact(R1,A1,Z,RHO+1e-6,I1);
[B2_z,B2_rho] = BcoilExact(R2,A2,Z,RHO+1e-6,I2);

    Brho = B1_rho+B2_rho;

    Bxx  = Brho.*cos(PHI);
    Byy  = Brho.*sin(PHI);
    Bzz  = B1_z+B2_z;

end
```

```
function [] = BFieldsBolpasi()
%BFIELDSBOLPASI Simulates the field of an IP, all coils traps as
%described by Bolpasi et. al.. Geometrical parameters such as coil half
%distances and radii are manually defined. Half of the currents are
%manually defined as well, and the rest is calculated to fit other
%the configuration or the bias field parameter
%
% See also BCoilPair, BcoilExact, BProfilesGraph , BLayersGraph

gamma = 32.1308947; %gamma = g_f * mu_f * mu_B / m_Rb87 @ CGS
mu0 = 0.4*pi; %@ CGS

i=1; %% Counter for images

BoxMin = -2; %Box dimensions
BoxMax = 2;

N = 51;           %Number of Samples, i.e. mesh resolution,
n = ceil(N/2);    %index of the box center
n2 = ceil(N/15); %short range for linear/ quadratic regression

if (mod(N,2) == 0)
    msgbox('Even discretization, migh have problems at center!');
end

%% Mesh preparation
x = linspace(BoxMin,BoxMax,N);
y = linspace(BoxMin,BoxMax,N);
z = linspace(BoxMin,BoxMax,N);

[Y,X,Z] = meshgrid(y,x,z);
PHI = atan2(Y,X);
RHO = sqrt(X.^2+Y.^2);

%% PARAMETERS SELECTION

% Number of windings for each coil
N_BI =16;
N_SI =16;
N_PI =16;
N_CO =16;

%Geometrical parameters
%I_PI & I_SI are also manually defined
%I_Co & I_BI are calculated to fit  parameter
[R_PI,A_PI,I_PI] = deal(2.5,2.5*sqrt(3/4),N_PI*12);
[R_CO,A_CO] = deal(2.5*2,2.5) ;
[R_BI,A_BI] = deal(5.2,4.5);
[R_SI,A_SI,I_SI] = deal(3.463999,3,N_SI*50);

%Bias Field
delta = 5;
%}


%%CALCULATING GEOMETRICAL PARAMETERS

% F, Geometrical factor in Helmholtz coils (even), zeroth order
```

```
% F ~ [m]^-1
F_PI = R_PI^2 / (R_PI^2+A_PI^2)^1.5;
F_CO = R_CO^2 / (R_CO^2+A_CO^2)^1.5;

% G, Geometrical factor in anti-Helmholtz coils (odd), first order
% G ~ [m]^-2
G_BI = 1.5*A_BI * R_BI^2 / (R_BI^2+A_BI^2)^2.5;
G_SI = 1.5*A_SI * R_SI^2 / (R_SI^2+A_SI^2)^2.5;

% H, Geometrical factor in Helmholtz coils (even), second order
% H ~ [m]^-3
H_PI = 3*R_PI^2 * (4*A_PI^2-R_PI^2) / (R_PI^2+A_PI^2)^3.5;
H_CO = 3*R_CO^2 * (4*A_CO^2-R_CO^2) / (R_CO^2+A_CO^2)^3.5;

%%CALCULATE remaining currents according to parameters

    % PI_I defined the curvature of the trap,
    % CO_I adjusted to fit the bias field

    I_CO = (F_PI/F_CO)*I_PI - delta/(mu0*F_CO);
    disp(strcat('I_CO=',num2str(round(I_CO/N_CO*1000)/1000),' A'));
    I_BI =  I_SI * (G_SI / G_BI);
    disp(strcat('I_BI=',num2str(round(I_BI/N_BI*1000)/1000),' A'));


%%------------------------------------------------------------
%% Coils on z axis,

%% PINCH Coils (PI)

    [PI_Bxx,PI_Byy,PI_Bzz]= BCoilPair(Z,RHO,PHI,...
                                    R_PI,A_PI,1*I_PI,...
                                    R_PI,-A_PI,1*I_PI);

    PI_BB = sqrt(PI_Bxx.^2 + PI_Byy.^2 + PI_Bzz.^2);


%% COMPENSATION Coils

%  (CO)

    [CO_Bxx,CO_Byy,CO_Bzz]= BCoilPair(Z,RHO,PHI,...
                                    R_CO,A_CO,-1*I_CO,...
                                    R_CO,-A_CO,-1*I_CO);

    CO_BB = sqrt(CO_Bxx.^2 + CO_Byy.^2 + CO_Bzz.^2);


%% FULL Z AXIS

%%  (FZ)

    FZ_Bxx  = PI_Bxx + CO_Bxx;
    FZ_Byy  = PI_Byy + CO_Byy;
    FZ_Bzz  = PI_Bzz + CO_Bzz;


    FZ_BB = sqrt(FZ_Bxx.^2 + FZ_Byy.^2 + FZ_Bzz.^2);
```

```


    figure(i);
    i = BProfilesGraph(x,y,z,PI_BB,CO_BB,FZ_BB,n,...
        'PINCH & COMPENSATION COILS // |B| Profiles',...
        'PI','CO','TOTAL',...
        [BoxMin BoxMax],i,0);


%%------------------------------------------------------------
%% Coils on radial plane,
%% BIG IOFFE Coils

% (BI)

    [BI_Bxx,BI_Byy,BI_Bzz]= BCoilPair(Z,RHO,PHI,...
                                R_BI,A_BI,-1*I_BI,...
                                R_BI,-A_BI,1*I_BI);

    [BI_Bxx,BI_Byy,BI_Bzz] = Rx(BI_Bxx,BI_Byy,BI_Bzz);

    BI_BB = sqrt(BI_Bxx.^2 + BI_Byy.^2 + BI_Bzz.^2);


%% SMALL IOFFE Coils

% (SI)

    [SI_Bxx,SI_Byy,SI_Bzz]= BCoilPair(Z,RHO,PHI,...
                                R_SI,A_SI,1*I_SI,...
                                R_SI,-A_SI,-1*I_SI);

    [SI_Bxx,SI_Byy,SI_Bzz] = Ry(SI_Bxx,SI_Byy,SI_Bzz);

    SI_BB = sqrt(SI_Bxx.^2 + SI_Byy.^2 + SI_Bzz.^2);



%% FULL AROUND

%% (FA)

    FA_Bxx   = BI_Bxx + SI_Bxx;
    FA_Byy   = BI_Byy + SI_Byy;
    FA_Bzz   = BI_Bzz + SI_Bzz;


    FA_BB = sqrt(FA_Bxx.^2 + FA_Byy.^2 + FA_Bzz.^2);

    figure(i);
    i = BProfilesGraph(x,y,z,BI_BB,SI_BB,FA_BB,n,...
        'BIG & SMALL IOFFEE COILS // |B| Profiles',...
        'BI','SI','TOTAL',...
        [BoxMin BoxMax],i,0);


%%% ALL
%% (AA)
```

```
    AA_Bxx   = FA_Bxx + FZ_Bxx;
    AA_Byy   = FA_Byy + FZ_Byy;
    AA_Bzz   = FA_Bzz + FZ_Bzz;


    AA_BB = sqrt(AA_Bxx.^2 + AA_Byy.^2 + AA_Bzz.^2);

    figure(i);
    i = BProfilesGraph(x,y,z,AA_BB,AA_BB,AA_BB,n,...
        'ALL COILS // |B| Profiles',...
        'BI','SI','TOTAL',...
        [BoxMin BoxMax],i,1);

%%B fields

%Linear regression on Bxx & Byy to obtain linear gradient (alpha)
[ax] = polyfit(squeeze(x((n-n2):(n+n2))),...
    squeeze(AA_Bxx((n-n2):(n+n2),n,n))',1);
[ay] = polyfit(squeeze(y((n-n2):(n+n2))),...
    squeeze(AA_Byy(n,(n-n2):(n+n2),n)),1);

%Quadratic regression on Bzz to obtain curvaturee(beta)
[bz] = polyfit(squeeze(z((n-n2):(n+n2))),...
    squeeze(AA_Bzz(n,n,(n-n2):(n+n2)))',2);

%Sanity check
%[ax2] = polyfit(squeeze(x((n-n2):(n+n2))),...
%squeeze(AA_Bzz((n-n2):(n+n2),n,n))',2);
%[ax3] = polyfit(squeeze(y((n-n2):(n+n2))),...
%squeeze(AA_Bzz(n,(n-n2):(n+n2),n)),2);
%beta2= ax2(1)*-4
%beta3= ax3(1)*-4
%betas should be consistent

%Calculate frequencies
wz = sqrt(32.44421*2*bz(1));
wr = sqrt(32.44421*(ax(1)^2/bz(3)-bz(1)));

%% Parameters plot
figure(i)
clf
subplot(1,3,1);

%%
plot(x,squeeze(AA_Bxx(:,n,n))','color',[0 0 0.8],'linewidth',2.5);
xlabel('x // cm','fontsize',14, 'fontweight','bold');
ylabel('B_{x} // Gauss','fontsize',14, 'fontweight','bold');
title('x - axis','fontsize',14, 'fontweight','bold');
xlim([BoxMin BoxMax]);

annotation('textarrow',[0.3/4,0.45/4]+0.125,[0.88,0.55],...
    'String',strcat('\alpha_x = ',num2str(ax(1)),' G /cm'));

subplot(1,3,2);

plot(y,squeeze(AA_Byy(n,:,n))','color',[0.8 0 0],'linewidth',2.5);
xlabel('y // cm','fontsize',14, 'fontweight','bold');
```

```
ylabel('B_{y} // Gauss','fontsize',14, 'fontweight','bold');
title('y - axis','fontsize',14, 'fontweight','bold');
xlim([BoxMin BoxMax]);
annotation('textarrow',[0.45/4,0.55/4]+0.375,[0.15,0.45],...
    'String',strcat('\alpha_y = ',num2str(-ay(1)),' G /cm'));

subplot(1,3,3);
plot(z,squeeze(AA_Bzz(n,n,:))','color','k','linewidth',2.5);
xlabel('z // cm','fontsize',14, 'fontweight','bold');
ylabel('B_{z} // Gauss','fontsize',14, 'fontweight','bold');
title('z - axis','fontsize',14, 'fontweight','bold');
xlim([BoxMin BoxMax]);
annotation('textbox', [0.75,0.8,0.1,0.1],...
           'String',{strcat('B_0 = ',num2str(bz(3)),' G'),...
           strcat( '\beta = ',num2str(2*bz(1)),' G /cm^2')},...
           'Linewidth', 1.5);

annotation('textbox', [0.5,0.75,0.1,0.1],...
           'String',{strcat('w_{\rho} = ',num2str(wr),' Hz'),...
           strcat( 'w_z  = ',num2str(wz),' Hz'),...
           strcat( '\Lambda  = ',num2str(round(wr/wz)))},...
           'Linewidth', 1.5);

%Plot the field profiles
i=i+1;
figure(i)
i = BLayersGraph(z,y,permute(AA_BB,[1 3 2]),...
    permute(AA_Bzz,[1 3 2]),permute(AA_Byy,[1 3 2]),...
    N,[BoxMin BoxMax],'z //cm','y //cm',' x axis',1,i);

figure(i)
i = BLayersGraph(z,x,permute(AA_BB,[3 2 1]),....
    permute(AA_Bzz,[3 2 1]),permute(AA_Bxx,[3 2 1]),...
    N,[BoxMin BoxMax],'z //cm','x //cm',' y axis',2,i);

figure(i)
i = BLayersGraph(x,y,AA_BB,AA_Bxx,AA_Byy,N,[BoxMin BoxMax],...
    'x //cm','y //cm',' z axis',3,i);

%2nd Sanity check, using only  third order approximation,
%should be consistent
%omegaz = sqrt(gamma*mu0*I_PI*H_PI)
%betaa = mu0*I_PI*H_PI
%alfax = 3*mu0 * I_BI*G_BI
%deltaa = mu0*(I_PI*F_PI-I_CO*F_CO)
%omegar = sqrt(gamma * mu0 * (9*I_BI^2*G_BI^2/(I_PI*F_PI - I_CO*F_CO)...
%- 0.5*I_PI*H_PI))

%%Print geometrical parameters
coils = {'PI'; 'CO'; 'BI'; 'SI'}'
Fs = {F_PI;F_CO;'-';'-'}'
Gs = {'-';'-';G_BI;G_SI}'
Hs = {H_PI;H_CO;'-';'-'}'

end
```

```
function [ip] = BLayersGraph(x,y,BB,Bxx,Byy,N,limits,xlabelx,ylabelx,barlabel,d,i)
%BLayersGraph plots a 3D field with both a contour and the vector field. It
%plots at a particular plane (i.e. z=0) but it creates a slider which
%allows to transverse the whole set of z values.
%
%Creating the slider is particularly tricky, and it required a series of
%trics to re-plot everything with every change, and a LONG time of google
%trobule shooting. Hopefully one day MATLAB will be as functional as
%MATHEMATICA in this manner.
%
%N is the number of samples per dimension
%
%The lables for the x, y and the color bar are defined
%
%d is the number of the "free" dimension. For example, if plotting x vs y ,
%then the "free" dimension is z (d=3). Conversely, d=1 for x and d=2 for y.
%This allows for a great flexibility, as any pair of coordinates can be
%plotted easily %by just swapping the columns on BB. It is a little tricky,
%however, as some very annoying discrepancies might appear
%due to the order of dimensions in matrixes (firs row,then column) is
%opposite from the cartesian one(first horizontal, then vertical),
%so beware! For example, sometimes it is necessary to transpose some
% of the vectors that one wants to plot.
%
%i is the image index
%
%onw can modify the aspect of the contour by modifying CRange below
%
%See also BProfilesGraph

ini = round(N/2);

figure(i)
clf

%Slider's legend, lower limit
uicontrol('style','text','Units','normalized', ...
         'Position',[0.10 ,0.03, 0.20, 0.025], ...
         'String', horzcat(barlabel,'    ', num2str(limits(1))),...
         'BackgroundColor', [0.8 0.8 0.8],...
         'FontSize', 14);

%Slider's legend, hupper limit
uicontrol('style','text','Units','normalized', ...
'Position',[0.55 ,0.03, 0.20, 0.025], ...
'String',  num2str(limits(2)), 'BackgroundColor', [0.8 0.8 0.8],...
'FontSize', 14  );

%Slider
SHandle = uicontrol('Style', 'slider','Min', 1,'Max',...
        N, 'Value',round(N/2),'SliderStep',[1/N 10/N],'Units',...
         'normalized','Position',[0.32 ,0.03, 0.3, 0.025 ]);


subplot(2,1,1); %Contour

%VERY IMPORTANT, modify the range to change the way the contour coloring
%looks
```

```
cRange = 0:0.5:30; %Change upper limit for a better view of the trap's depth.
                   %Change  step for a coarser or smoother level curves.

[zzz,cHandle]=contour(x,y,squeeze(selecdim(d,ini,BB))',15,'LevelList',...
    cRange,'fill','on');
colormap(paruly()) %Custom colormap, jsut to make it look nicer
xlabel(xlabelx,'fontsize',14, 'fontweight','bold');
ylabel(ylabelx,'fontsize',14, 'fontweight','bold');
title(horzcat('|B| profile through ',' ',barlabel),'fontsize',14,...
    'fontweight','bold');
hold on
hc=colorbar;
ylabel(hc, 'Magnetic Field // G','fontweight','bold');

set(SHandle,'Callback', {@modify,SHandle}); %Add sensibility to change
                                            %in the slider

pp= subplot(2,1,2); %%vector field
cla(pp)
set(pp,'Color','none');

h=streamslice(x,y,squeeze(selecdim(d,ini,Bxx))',...
    squeeze(selecdim(d,ini,Byy))',1.5,'cubic');
set(h,'LineWidth',1.5,'Color','k')
ylim([limits(1) limits(2)]);
xlim([limits(1) limits(2)]);
fakecolorbar;

ip = i+1;

function BB = selecdim(d,k,B)
%Inner function, takes the 3x3 matrix and extracts a 2x2 matrix, d is the
%"free" dimension, k is the particular plane at the free dimension which
%should be extracted

    switch(d)
        case 1
            BB = B(k,:,:);
        case 2
            BB = B(:,k,:);
        case 3
            BB = B(:,:,k);
    end


end

function modify(src,evt,SHandle)
%Inner function, a trick to redraw the figure with a change in the
%slider
%
   r=round(get(SHandle,'value')); %get the NEW value from the slider
   set(cHandle,'ZData',squeeze(selecdim(d,r,BB))'); %update contour data

   p=subplot(2,1,2); %update the data for streamslice is too complicated,
   cla(p)            % so it is easier to simply redraw it completely.
   set(p,'Color','none');
   alpha(0.5);
```

```
    hold on

    p=streamslice(x,y,squeeze(selecdim(d,r,Bxx))',...
        squeeze(selecdim(d,r,Byy))',1.5,'cubic');
    set(p,'LineWidth',1.5,'Color','k')
    ylim([limits(1) limits(2)]);
    fakecolorbar;
end


end
```

```
function [ip] = BProfilesGraph(x,y,z,B1,B2,B12,n,titlex,l1,l2,l3,limits,i,s)
%BProfilesGraph makes x,y and z profiles for multiple fields, it can either
%plot B1,B2 and their combination or simply the later.
%
%n selects the plane (i.e. other coordinates) for plotting. For example, if
%n is selected to be half of the number of samples per dimension, then
%while plotting B on the x direction, both y and z are zero.
%
%titlex is the overall title of the plot
%
%l1,l2 and l3 are the labels for the legend
%
%i is the image index
%
%s is a boolan which defines if the plots will include 3 lines (B1,B2 and
%B1+B2) [s=0] or just (B1+B2)[s!=0]
%
% See also BLayersGraph

clf

alpha(0.5)

subplot(1,3,1);
if( s ==0)
plot(x,squeeze(B1(:,n,n))','r','linewidth',2.5);
hold on
plot(x,squeeze(B2(:,n,n))','b','linewidth',2.5);
hold on
end
plot(x,squeeze(B12(:,n,n))','color',[0 .5 0],'linewidth',2.5);
xlabel('x // cm','fontsize',14, 'fontweight','bold');
ylabel('|B| // Gauss','fontsize',14, 'fontweight','bold');
title('x - axis','fontsize',14, 'fontweight','bold');
xlim([limits(1) limits(2)]);

MasterTitle(titlex,...
          'fontsize',14,'color','k',...
          'xoff',1.4,'yoff',.025);

subplot(1,3,2);
if (s==0)
plot(y,squeeze(B1(n,:,n))','r','linewidth',2.5);
hold on
plot(y,squeeze(B2(n,:,n))','b','linewidth',2.5);
hold on
end
plot(y,squeeze(B12(n,:,n))','color',[0 .5 0],'linewidth',2.5);
xlabel('y // cm','fontsize',14, 'fontweight','bold');
ylabel('|B| // Gauss','fontsize',14, 'fontweight','bold');
title('y - axis','fontsize',14, 'fontweight','bold');
xlim([limits(1) limits(2)]);

subplot(1,3,3);
if(s==0)
plot(z,squeeze(B1(n,n,:))','r','linewidth',2.5);
hold on
plot(z,squeeze(B2(n,n,:))','b','linewidth',2.5);
```

```
hold on
end
plot(z,squeeze(B12(n,n,:))','color',[0 .5 0],'linewidth',2.5);
xlabel('z // cm','fontsize',14, 'fontweight','bold');
ylabel('|B| // Gauss','fontsize',14, 'fontweight','bold');
title('z - axis','fontsize',14, 'fontweight','bold');
xlim([limits(1) limits(2)]);
if (s==0)
legend(l1,l2,l3);
else
legend(l3);
end
ip = i+1;

end
```

This code snippets are used to rotate the BI and SI coils into their proper positions. it is worth mentioning that both the mesh and the coordinate axes should be adjusted. The former is done with the aid of the function *rot90_3D*, while the later are simply adjusted as required on each case. The function *rot90_3D* works with counterclockwise (positive angles) rotations only.

```
function [xx,yy,zz] = Rx(x,y,z)

x = rot90_3D(x, 1, 3);
y = rot90_3D(y, 1, 3);
z = rot90_3D(z, 1, 3);

xx = x;
yy = z;
zz = -y;

end
```

```
function [xx,yy,zz] = Ry(x,y,z)

x = rot90_3D(x, 2, 1);
y = rot90_3D(y, 2, 1);
z = rot90_3D(z, 2, 1);

xx = -z;
yy = y;
zz = x;

end
```