2D Turing Patterns

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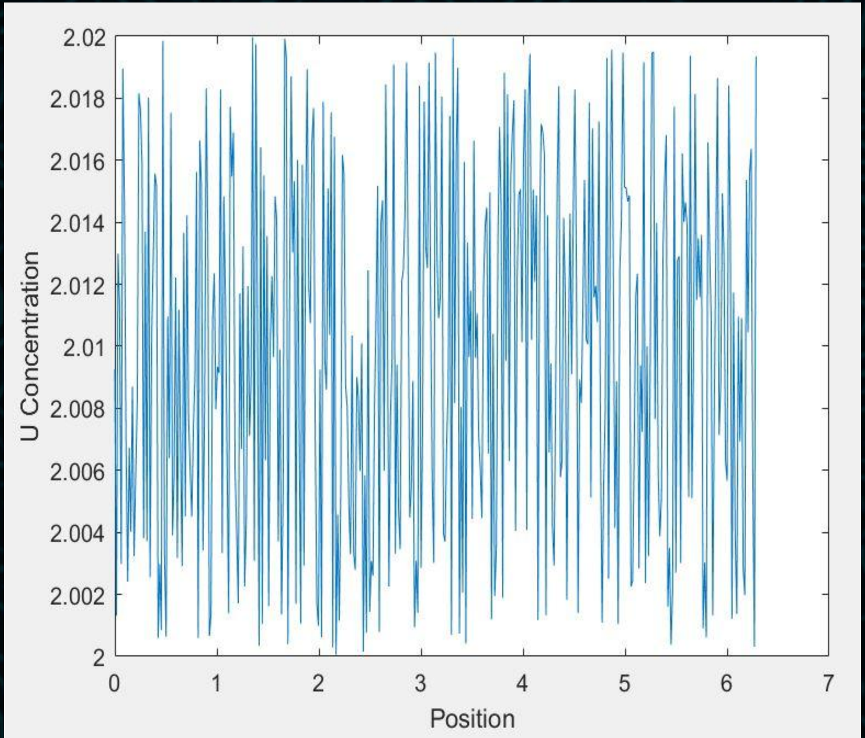
PH333: Experimental Soft Matter Physics

1D Turing Patterns

Simple Reaction-Diffusion

$$\frac{\partial}{\partial t}U = f(U,V) + D_U \frac{\partial^2}{\partial x^2}U,$$
$$\frac{\partial}{\partial t}V = g(U,V) + D_V \frac{\partial^2}{\partial x^2}V.$$

Oscillations in concentration



2D patterns: How to generate?

Recipe:

- The PDEs (Brusselator)

$$\begin{cases} \dot{U} = D_U \nabla^2 U + U^2 V + A - U(1+B) \\ \dot{V} = D_V \nabla^2 V - U^2 V + BU \end{cases}$$

- Uniformly random initial condition: U(x,y) = rand(n), V(x,y) = rand(n)
- Periodic boundary condition (later)

2D patterns…

Idea from image processing: **To diffuse U(x,y) = To blur the "image" U(x,y)**

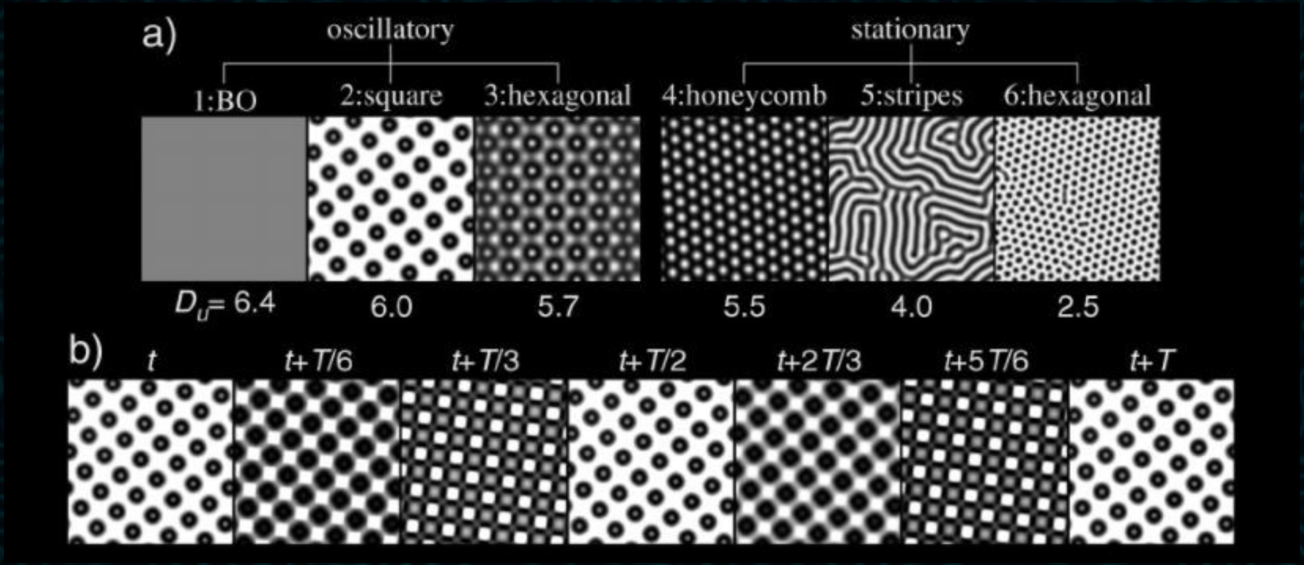
→ Replace the Laplacian with a simple, mathematically equivalent matrix computation

→ Easy to implement periodic boundary condition

Example:

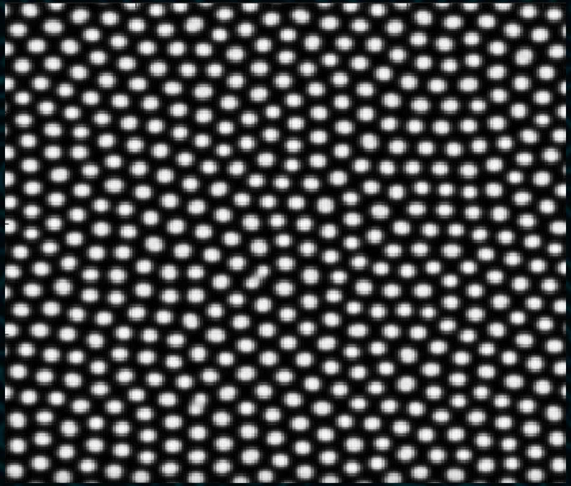
Known Turing patterns from the Brusselator

- Yang, Zhabotinsky, Epstein,
  Stable Squares and Other Oscillatory Turing Patterns in a Reaction-Diffusion Model

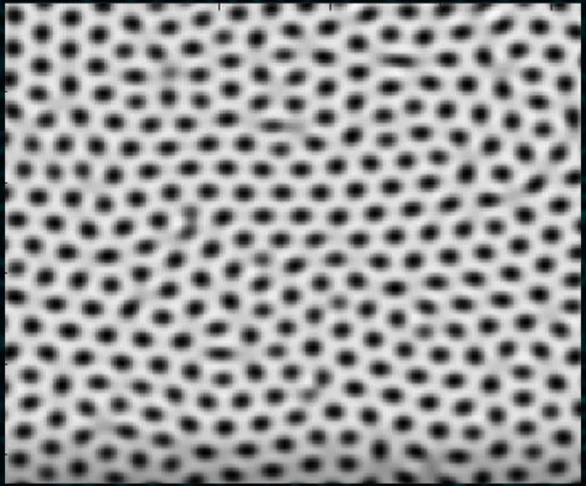


What we generated and found

Stationary Turing patterns:



Honeycomb

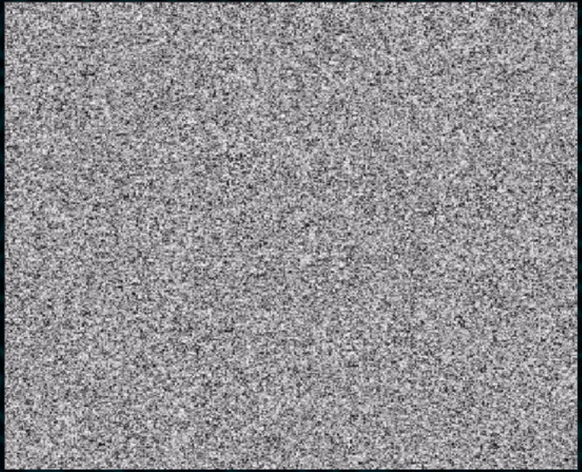


Hexagonal

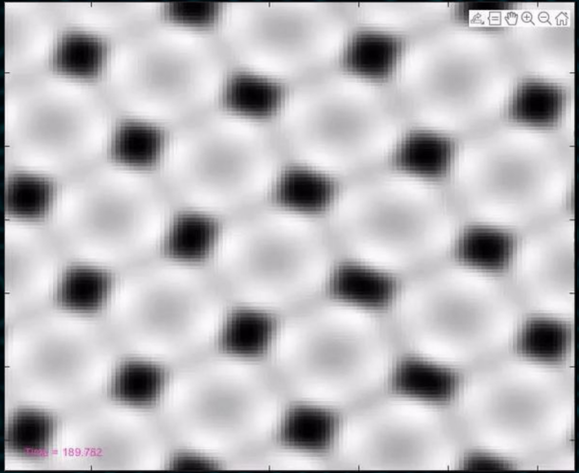
Stripes

What we generated and found

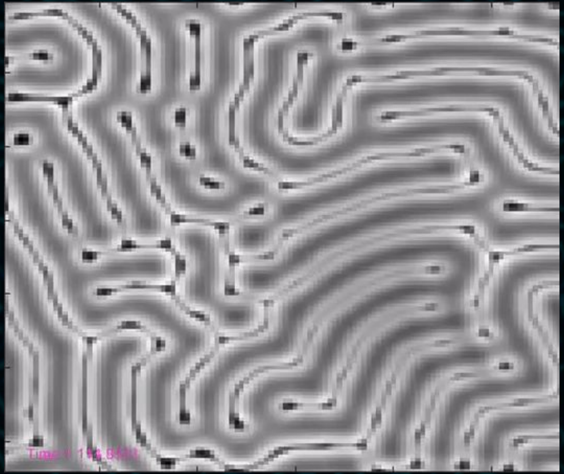
Oscillatory Turing patterns



Osc. Hexagonal



Osc. Squares



Osc. Stripes

Gray-Scott Model

Model for Glycolysis

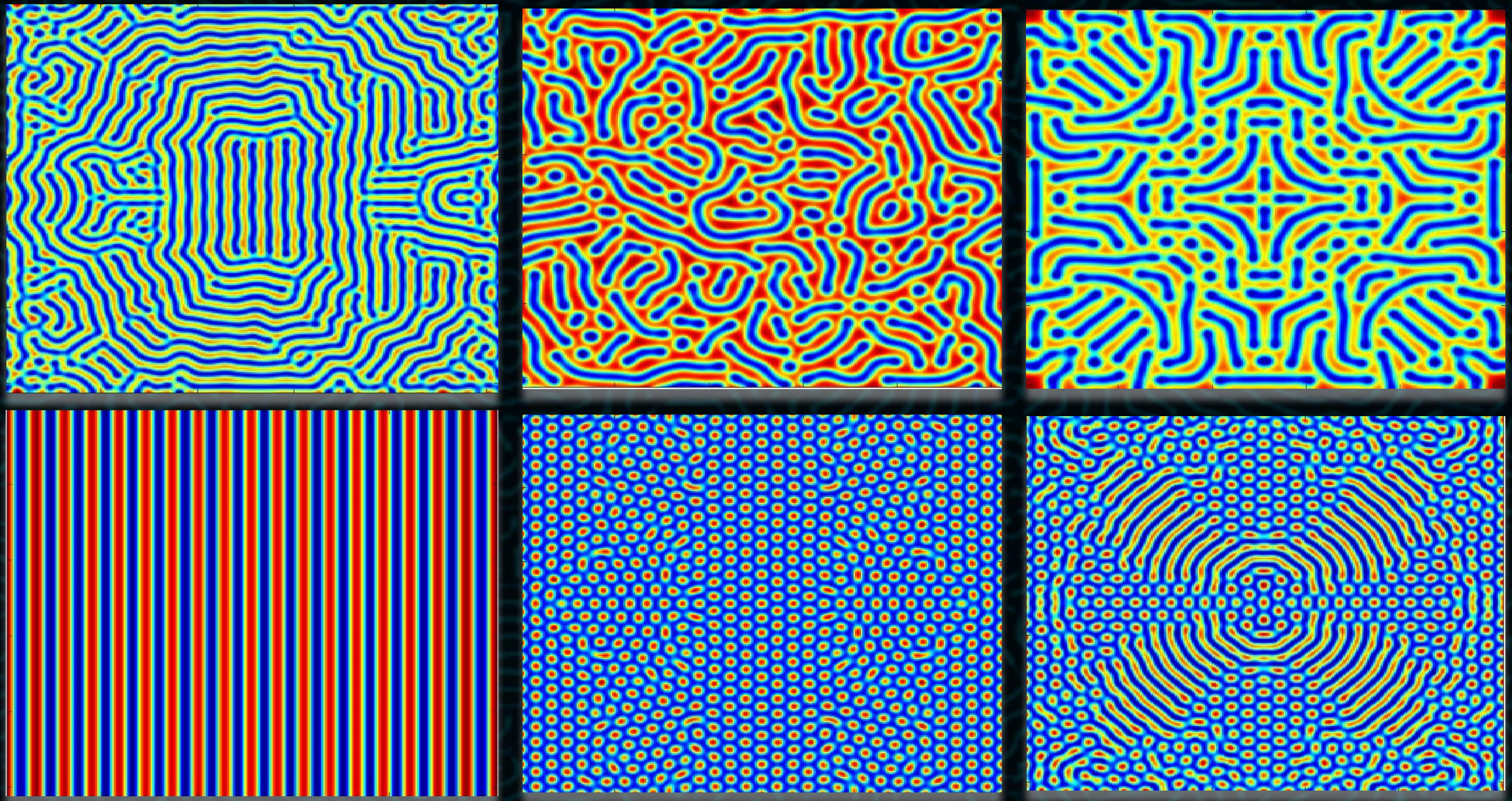
k controlled rate of killing U

F controls rate of feeding U

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u - uv^2 + F(1-u),$$
$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + uv^2 - (F+k)v.$$

D_u and D_v are diffusion constants, just like Turing

Cool patterns from Gray-Scott



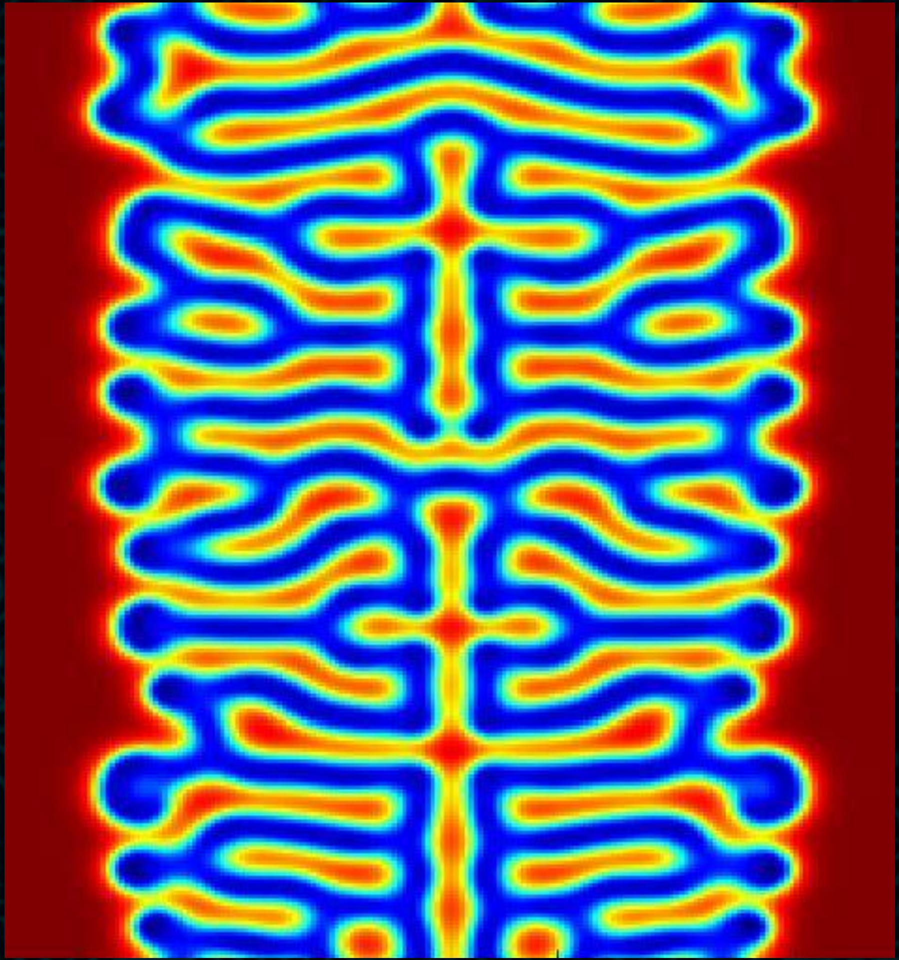
In Nature

“The Chemical Basis of Morphogenesis”

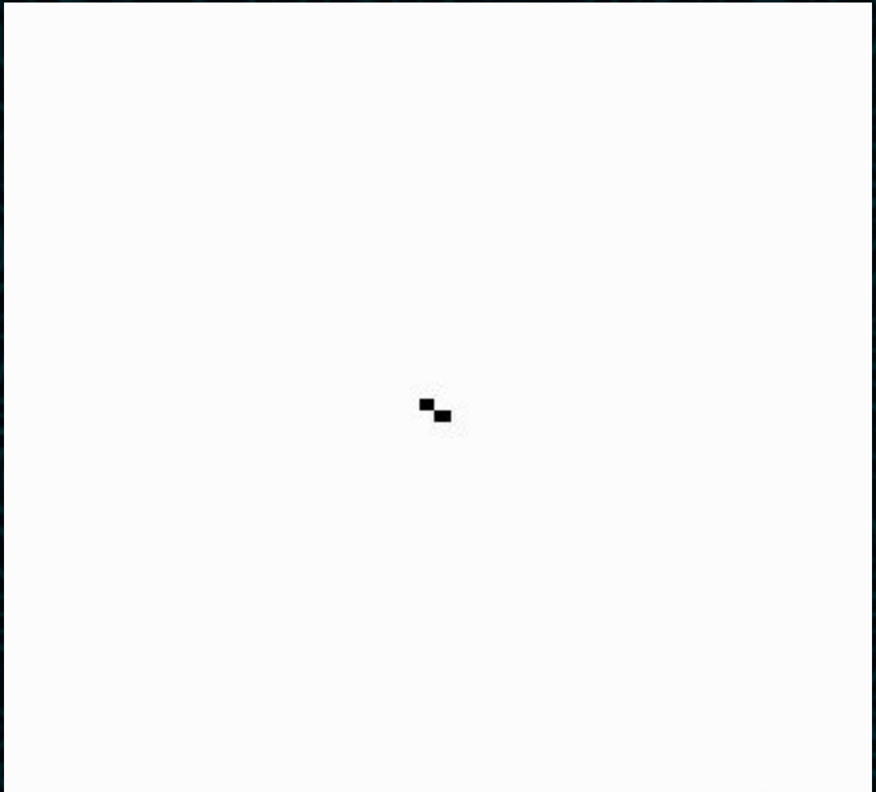
Spots and stripes

Based off of initial conditions and

parameters.



Mitosis!



Bonus slide

Let's quickly revisit the Brusselator.

The Brusselator has a special initial condition which we call the JHM initial condition.


Thermal Physics

References

- L. Yang, A.M. Zhabotinsky, I. R. Epstein *Stable Squares and Other Oscillatory Turing Patterns in a Reaction-Diffusion Model* June 2004 Physical Review Letters, vol. 92, 10.1103/PhysRevLett.92.198303
- L. Zheng, *Pattern formation in reaction-diffusion systems using the Gray-Scott model,* June 2020, https://itp.uni-frankfurt.de/~gros/StudentProjects/Projects_2020/projekt_lichuan_zheng/
- How the Tiger got its Stripes » Mike on MATLAB Graphics - MATLAB & Simulink (mathworks.com)