8.514 Strongly Correlated Systems in Condensed Matter Physics
Spring 2023

Introductory remarks

In this course we will describe the physics of quantum systems made up of many strongly interacting degrees of freedom. Such systems arise in many different contexts.

- Condensed matter physics: electrons in solids, liquid/solid He-4, He-3.
- Nuclear physics: Protons/neutrons in nuclei, quark matter
- Atomic physics: Many electron atoms, cold atomic gases
- Particle physics: Any Quantum Field Theory (QFT) has many degrees of freedom; if these interact strongly (e.g QCD) we have a 'strongly coupled QFT'.

Most of this course will focus on condensed matter physics.

At low temperature (T), most large collections of particles organize themselves into **phases** with some kind of order.

Example: Take a large ($\approx 10^{23}$) collection of inert gas atoms. The equilibrium state at high-T is a gas. The equilibrium state at low-T, on the other hand, is a crystalline solid. The solid is a much more ordered configuration than the gas (or a liquid) : the atoms occupy fixed, well defined, positions. The ground state of a large collection of inert atoms is an ordered solid.

Key question 1: How do we describe the order, if any, in a given phase of a quantum many body system?

Useful concept: Broken symmetry: ground state wavefunction (or at non-zero T, the equilibrium state) does not share the symmetries of the microscopic Hamiltonian.

Many distinct phases of matter can be described by the "broken symmetry" paradigm. Associated with a broken symmetry is the concept of the Landau order parameter - a thermodynamic variable that quantifies the amount of broken symmetry.

The concept of broken symmetry yields a powerful framework for thinking about the properties of many phases in a unified way.

"Recent" discoveries show that "ordered" phases exist where the order is not captured by the broken symmetry paradigm. Examples are the fractional quantum Hall fluids, quantum spin liquid phases in magnetic systems, etc. New concepts like "topological order" (and other order for which we do not have good names) are need to describe these phases.

(Topological order is a global property of the ground state wavefunction with many unique and interesting consequences - see later in the course).

Key question 2: How do we describe the low energy excitation spectrum of a given phase of a quantum many body system?

Interesting point of view: Imagine a micro-organism living inside the phase; it can only do relatively "low energy" experiments. What is the theory of it's universe?

Useful concept: Elementary excitations ("quasiparticles")

Typically low energy excitation spectrum admits a particle spectrum, *i.e*, the micro-organism will identify "elementary" particles inits universe. A general low energy excited state can be described as a collection of these "elementary" excitations. Many "universal" properties of these quasiparticles are determined by some gross features of the ground state wavefunction.

Quasiparticle excitations will carry quantum numbers under the symmetries of the underlying Hamiltonian that are preserved by the ground state. In conventional phases of matter the quasiparticle quantum numbers are the "obvious" ones: they are the same as those of the underlying microscopic particles or those of composites of the microscopic particles. However there also are phases of matter where the quasiparticles have sharp quantum numbers that are fractions of those of the underlying particles (eg: fractional quantum Hall fluid).

A more dramatic departure from conventional physics occurs in phases where the excitation spectrum admits no particle-like description. Such phases have no 'elementary' excitations and the quasiparticle concept itself has broken down. Examples include non-fermi liquid metals, certain quantum spin liquid states, etc.

Describing phenomena in phases of matter with no quasiparticles is a current frontier in quantum many body physics.

Key question 3: How do we describe phase transitions between different states of quantum many body systems?

The simplest situation is when the transition is simply associated with the continuous onset of a broken symmetry. In this case, the singularities at the phase transition are due to long wavelength, long-time fluctuations of the Landau order parameter. The physics is then described by a suitable QFT in terms of the Landau order parameter field. This is known as the "Landau-Ginzburg-Wilson" (LGW) paradigm and is very successful in describing this class of phase transition.

Many other interesting situations exist which fall beyond the LGW paradigm. Some examples are

1. Phase transitions where one or both phases have topological or other non-Landau order

2. Continuous Landau-forbidden phase transitions between Landau allowed phases

3. Phase transitions involving a metal with a Fermi surface.

   A particularly dramatic example is the Mott metal-insulator transition of fermions: must lose the entire Fermi surface of the metal when it transitions into an insulator.

In this course we will explore these questions and their answers. There are many amazing developments in this area of research in the last 3 decades continuing to the present day. My hope is that you learn about some of the excitement and that you get inspired and empowered to contribute to future developments!

Other remarks

Preparation: It is absolutely necessary that students attending this class are totally at ease with what is covered in (at least the first semester of) basic graduate level quantum mechanics and statistical mechanics. I will also assume some exposure to elementary solid state physics at the level of a good undergraduate course. Some exposure to quantum field theory will help but is not strictly necessary.

How to get most out of the class? As this is an advanced graduate class, there is no final exam. You should of course solve the homework problems, and write the required term paper on whatever topic related to the course material interests you. In addition, there is a lot to be learnt by simply showing up to lecture and participating in in-class discussions. I would also urge you to spend a lot of time thinking through (and possibly discussing with fellow students) what you hear about in lecture. Make up your own homework problems just for fun and try to solve them. Attend research seminars in this area, and perhaps read a recent research paper. You should see if you can at least understand the questions that are being asked and form an opinion on why (and whether) they are important and interesting.

I have not recommended any textbooks as there is no single book I plan to follow. Some of what I say will be discussed very well in some books and I might refer to them as appropriate. I find the following books useful in general.

1. Condensed Matter Field Theory, Altland and Simmons
2. Quantum Field Theory of Many-body Systems: From the Origin of Sound to an Origin of Light and Electrons, Xiao-gang Wen.
3. Quantum phase transitions, Subir Sachdev
4. Methods of Quantum Field Theory in Statistical Physics, Abrikosov, Gorkov, Dzyaloshinski
5. Basic Notions of Condensed Matter Physics, P.W. Anderson
6. Many Body Physics, Piers Coleman