Physics 8.321, Fall 2021
Midterm

You have **75 minutes** to complete the exam and a grace period of an additional 15 minutes to get your solutions uploaded to gradescope. You may use your books and notes including the notes on canvas from the course, and you may use symbolic manipulation tools like mathematica and matlab, but you may not consult other online resources, and you may not communicate with other people in any way while doing the midterm. You also may not communicate any information about the exam to anyone after you have completed it until the exam period is over at the end of the day on 10/28/21.

Note: you are expected to upload your completed exam to gradescope immediately after completing the exam and within 75 minutes or less of downloading it. You have a few extra minutes in case of technical complications. The system will log your download and upload times. If for some reason you have difficulty uploading your exam after completion, please email it immediately to one of the course staff.

1. (35 points)

Consider a system of three spin-1/2 particles. The system is initialized to an eigenstate of each of the operators $S_x^{(i)}$ with eigenvalue $+\hbar/2$. For each of the following observables defined as Hermitian operators give the possible results of a measurement of this operator and the probability of each possible result. You need list only results with a nonzero probability.

(a) $S_x^{(1)} S_x^{(2)} S_x^{(3)}$

(b) $S_z^{(1)} S_z^{(2)} S_z^{(3)}$

(c) $S_z^{(1)} S_z^{(2)} S_y^{(3)}$

(d) $S_x^{(1)} S_x^{(2)} S_z^{(3)}$

(e) $iS_x^{(1)} S_y^{(2)} S_z^{(2)}$ (hint: why is this operator Hermitian?)

2. (30 points)

A particle of mass M is in a 3D box with sides of length $A \times 2A \times 3A$. The potential is infinite outside the box and vanishes in the box.

(a) Write the full energy spectrum for the system

(b) Give the lowest 5 energy eigenvalues explicitly

(c) Identify an energy where the spectrum is degenerate

3. (35 points)

In quantum computing, a "two qubit operation" is a unitary transformation acting on a pair of qubits (spin-1/2 degrees of freedom). Such an operation is implemented by a general 4×4 unitary matrix acting on the degrees of freedom $| \pm_i \pm_j \rangle$ of qubits i, j in an N-qubit system.

(a) How many *physically inequivalent* independent degrees of freedom are there in such a choice of two qubit operation?

(b) Find a minimal sequence of two qubit operations that implement the three qubit transformation (in the z spin bases).

$$|s_1 s_2 s_3\rangle \to |s_2 s_3 s_1\rangle \, .$$

(c) Give a lower bound on the number of distinct two qubit operations that would need to be performed to get a completely general three qubit operation.

(d) Give a lower bound on the number of distinct two qubit operations that would need to be performed to get a completely general N qubit operation. How does this scale as N gets large? Is this practical when the number of qubits is greater than e.g. $N = 100$?

(e) Do you think that this lower bound (or the order of magnitude thereof) can be realized in principle for any N qubit operation? How might you try to prove this? [a brief qualitative answer here is sufficient; don't spend much time on this during the exam but if you want to think more about this after the exam, extra credit for a proof or counterexample for $N = 3$ or higher.]