Sub-Doppler Sysiphus Cooling

The surprise finding:

Optical molasses are much
Colder than expected

Also:
The variation of temperature
with detuning is not as expected

Where to look for ideas?
→ Check your assumptions!
We assumed:
- 2-level system

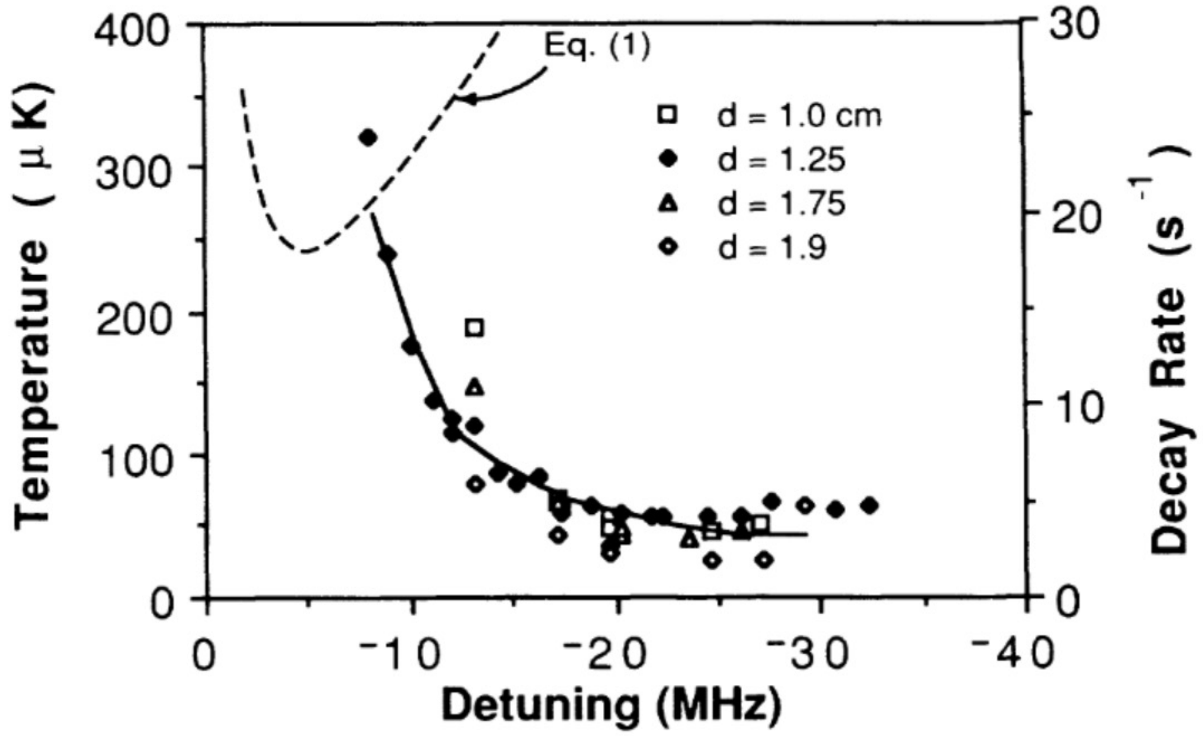
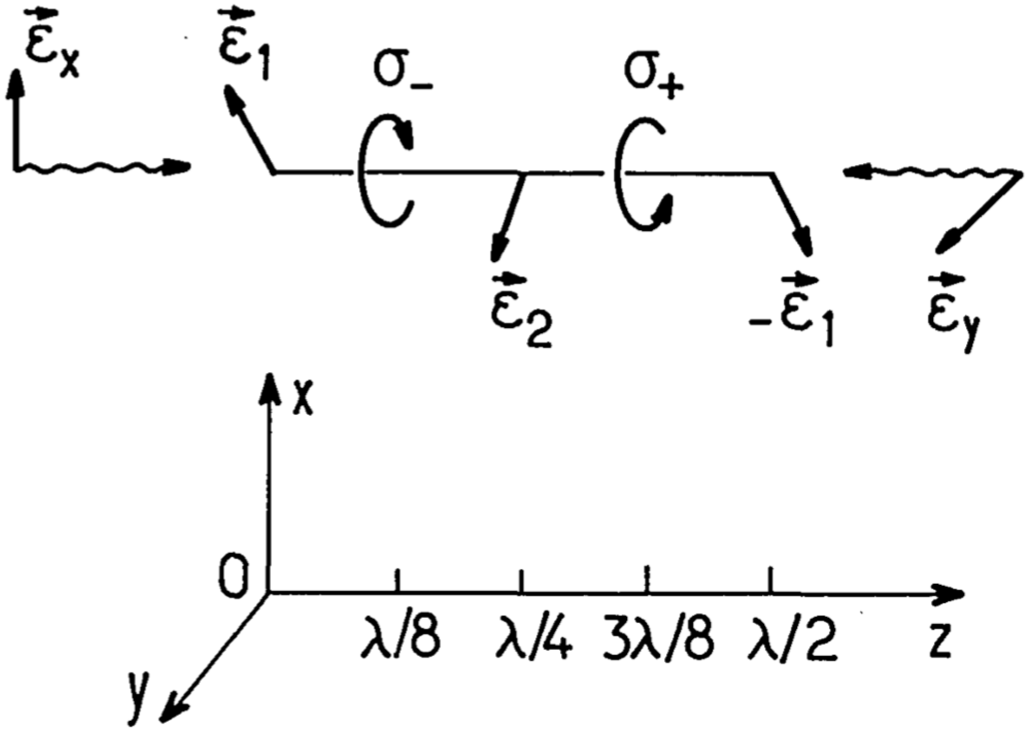


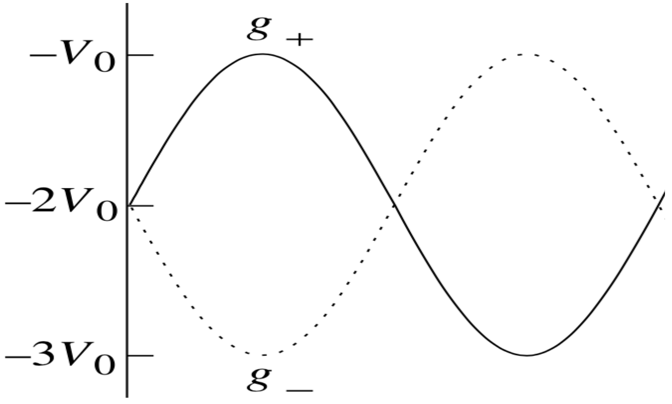
FIG. 2. T vs Δ by TOF for various separations d. The solid curve represents the measured molasses decay rate; it is not a fit to the temperature data, but its scale was chosen to emphasize its proportionality to the temperature data. The dashed line shows the temperature expected on the basis of Eq. (1).

P. Lett, R. Watts, C. Westbrook, W. Philipps,
P. Gould, H. Metcalf, PRL 61, 169 (1988)

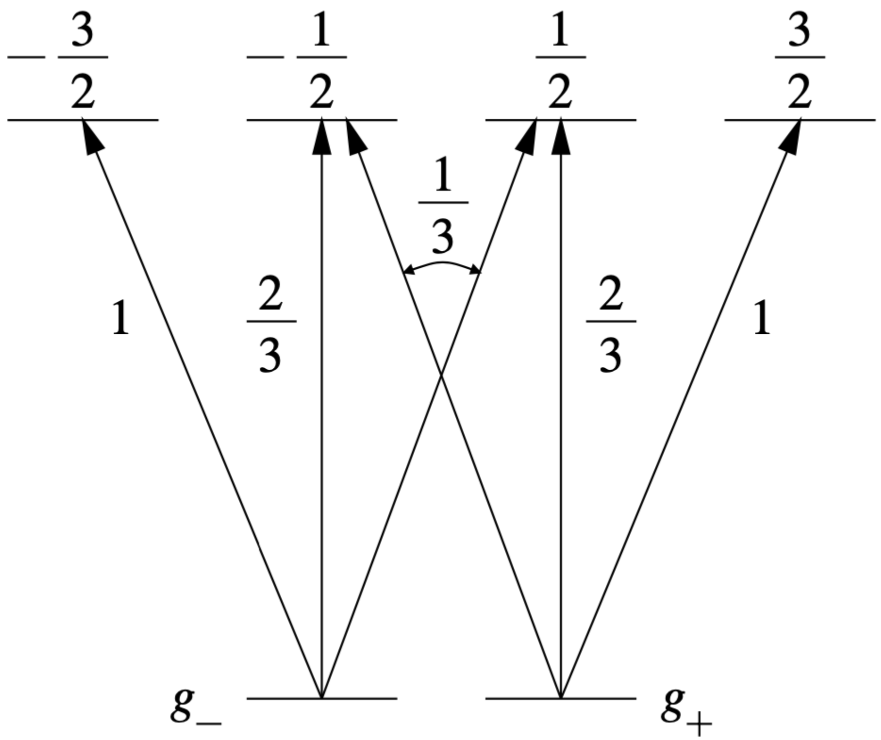
Sub-Doppler Sysiphus Cooling

Polarization gradients





Internal structure



Spatially dependent light shifts

Sub-Doppler Sysiphus Cooling

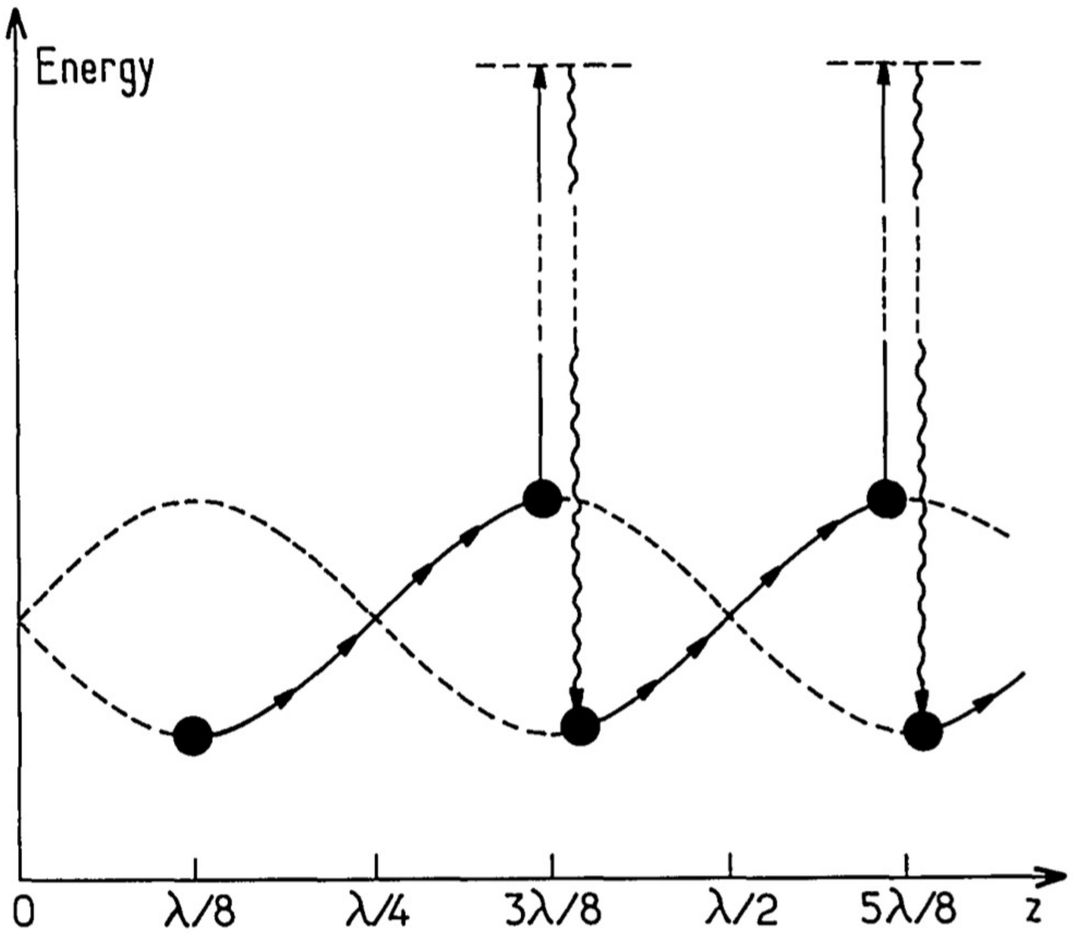


Fig. 4. Atomic Sisyphus effect in the lin ⊥ lin configuration. Because of the time lag τ_p due to optical pumping, the atom sees on the average more uphill parts than downhill ones. The velocity of the atom represented here is such that $v\tau_p \sim \lambda$, in which case the atom travels overs λ in a relaxation time τ_p. The cooling force is then close to its maximal value.

Looks like stimulated blue molasses, but note:
These dressed levels are superpositions of ground-state levels!
The effective scattering rate and light shifts are:

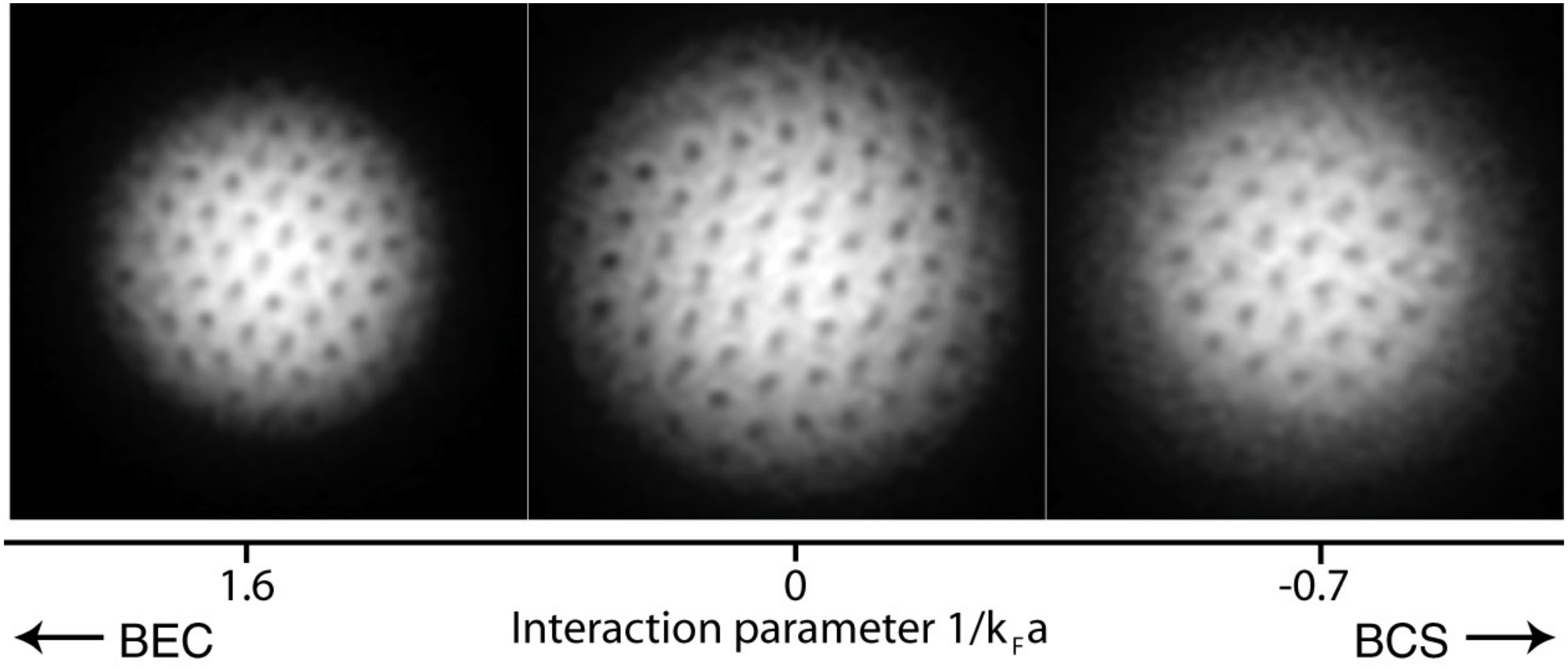
$$\Gamma' \sim \Omega^2\Gamma/\delta^2,$$

$$\Delta' \sim \Omega^2/\delta,$$

Cooling Rate:

$$\frac{\mathrm{d}W}{\mathrm{d}t} \sim \frac{-\hbar\Delta'}{\tau_p} = -\hbar\Delta'\Gamma'.$$

Superfluid Bose and Fermi Gases



Some reviews

Varenna Notes:

on BEC:

Ketterle, Durfee, Stamper-Kurn, *Making, Probing and Understanding BEC*

http://cua.mit.edu/ketterle_group/Projects_1999/Pubs_99/kett99varenna.pdf

on Fermi Gases:

Ketterle, Zwierlein*, Making, Probing and Understanding Ultracold Fermi Gases*

http://arxiv.org/abs/0801.2500

Zwierlein, *Thermodynamics of Strongly Interacting Fermi Gases*,

Varenna School of Physics “Enrico Fermi” 2014, vol 191

Stefano Giorgini, Lev P. Pitaevskii, Sandro Stringari

The theory of Fermi gases

http://arxiv.org/abs/0706.3360

Immanuel Bloch, Jean Dalibard, Wilhelm Zwerger

Many-Body Physics with Ultracold Gases:

http://arxiv.org/abs/0704.3011

Lecture Notes “The BEC-BCS crossover and the Unitary Fermi Gas”

Edited by W. Zwerger, Springer, 2012

How cold is ultracold?

Atoms move at:

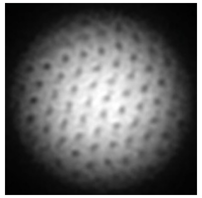
~ 1 mm/s

~ 100 m/s

~ 10^6 m/s

NY – Paris in 10 seconds

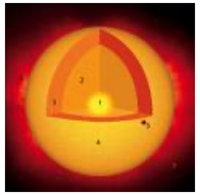
10^{-9} 10^{-6} 10^{-3} 1 10^3 10^6 10^9 T(K)



***Ultracold* atom experiments**

Outer space

Your living room



Center of the sun

Supernova explosion

Particles behave as waves

Louis de Broglie

$$\lambda = \frac{h}{mv}$$

Planck's constant

Mass

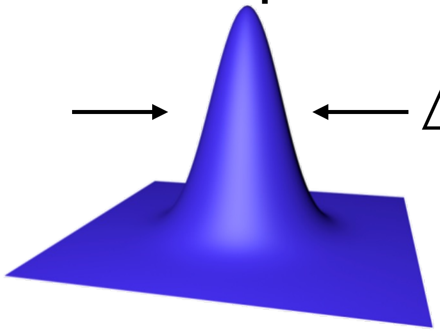
Velocity

10^{-9} 10^{-6} 10^{-3} 1 10^{3} 10^{6} 10^{9} T(K)

Werner Heisenberg

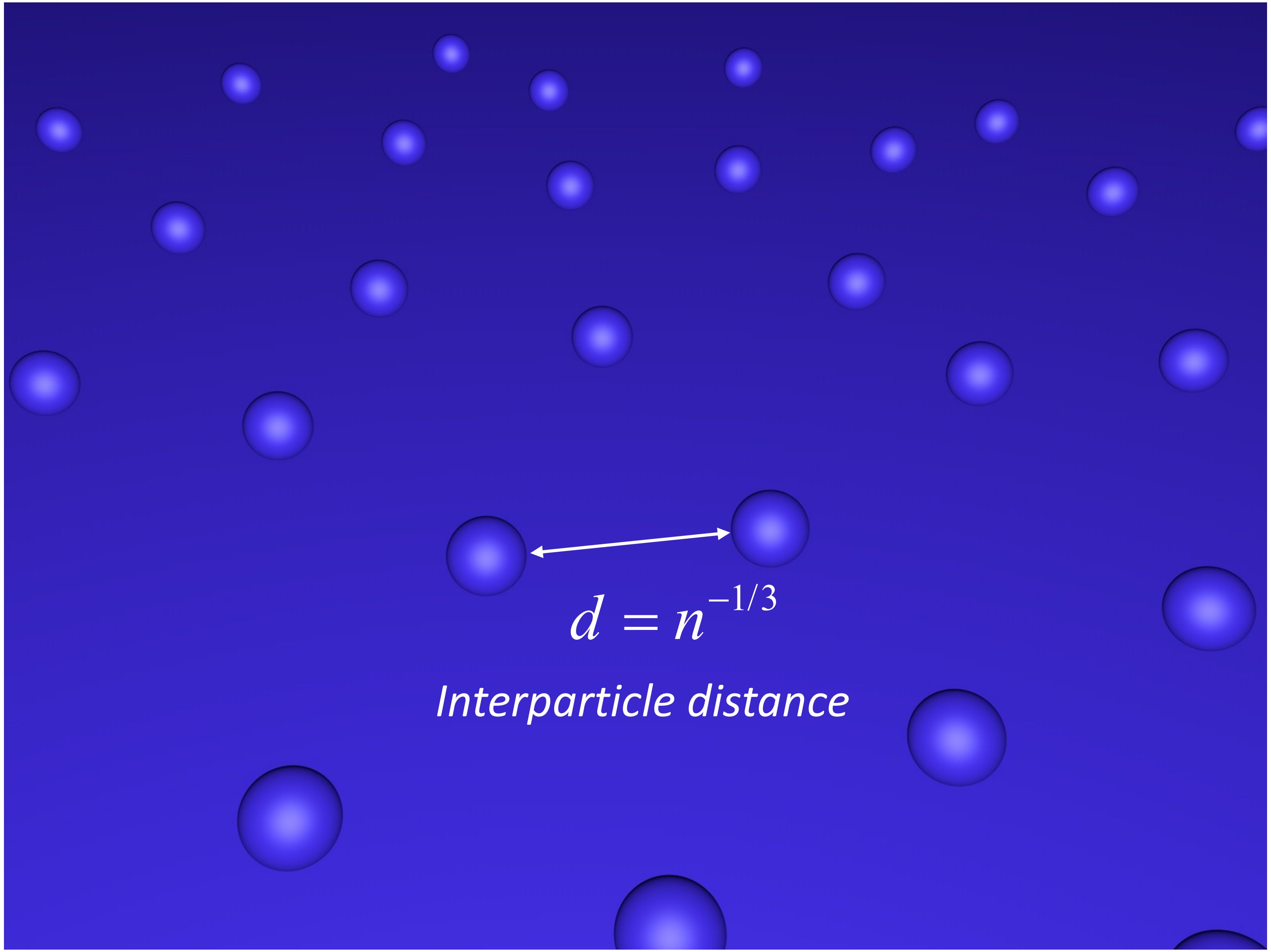
Temperature = Uncertainty of velocity^2

Size of a wave packet = Uncertainty of position

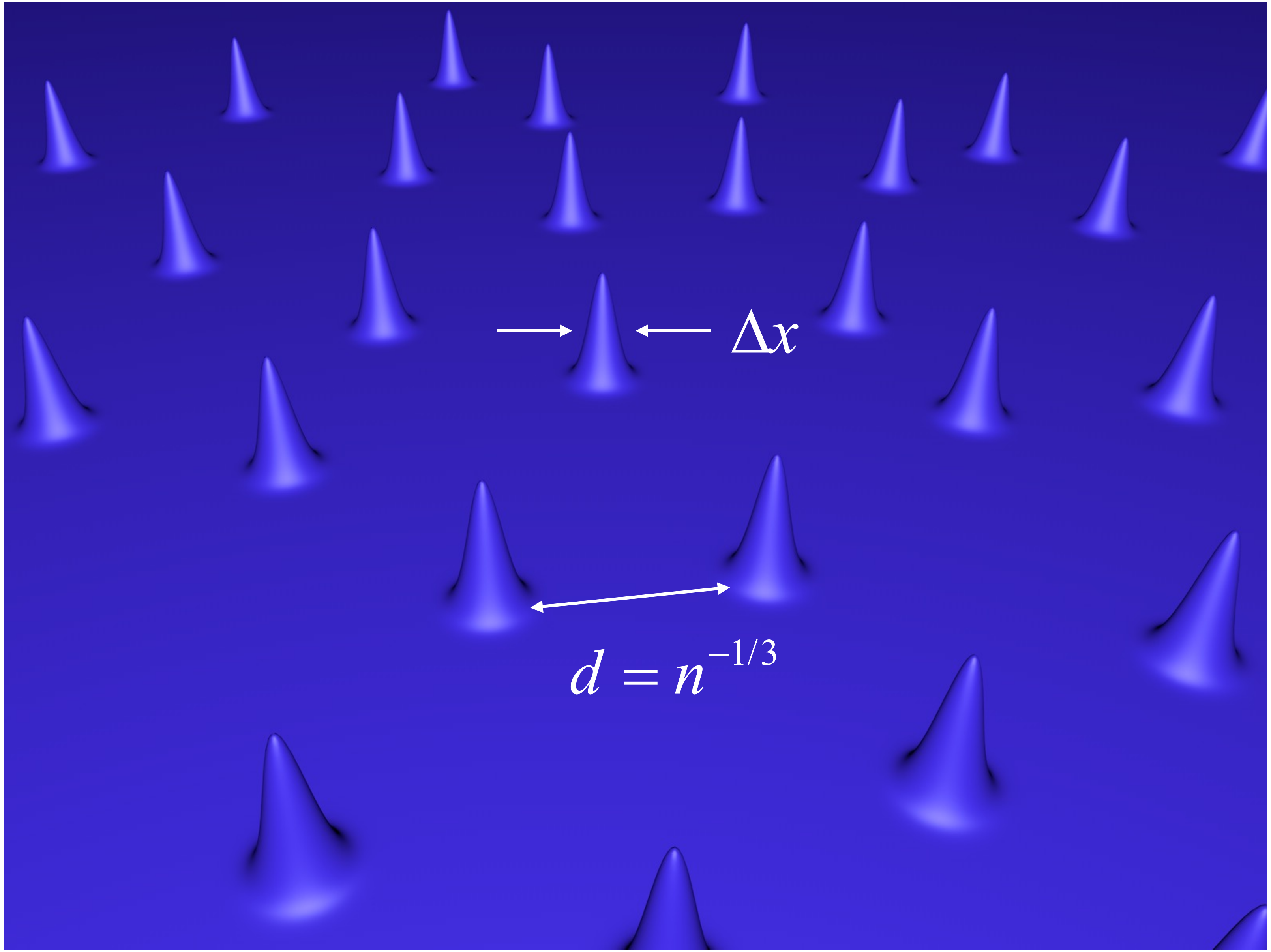


$$\Delta x \approx \frac{\hbar}{\Delta p} \approx \frac{\hbar}{\sqrt{mT}}$$

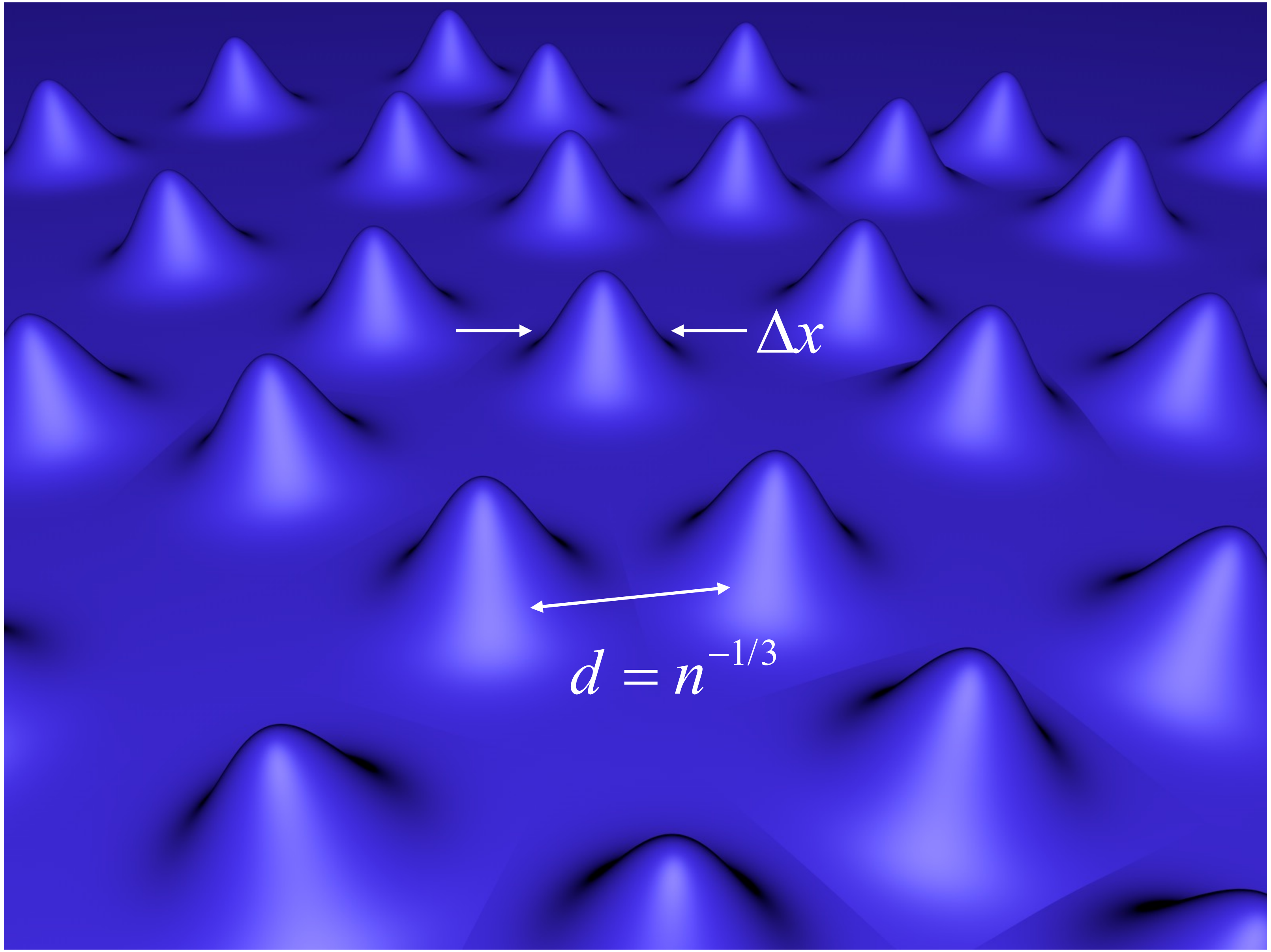
Particles...



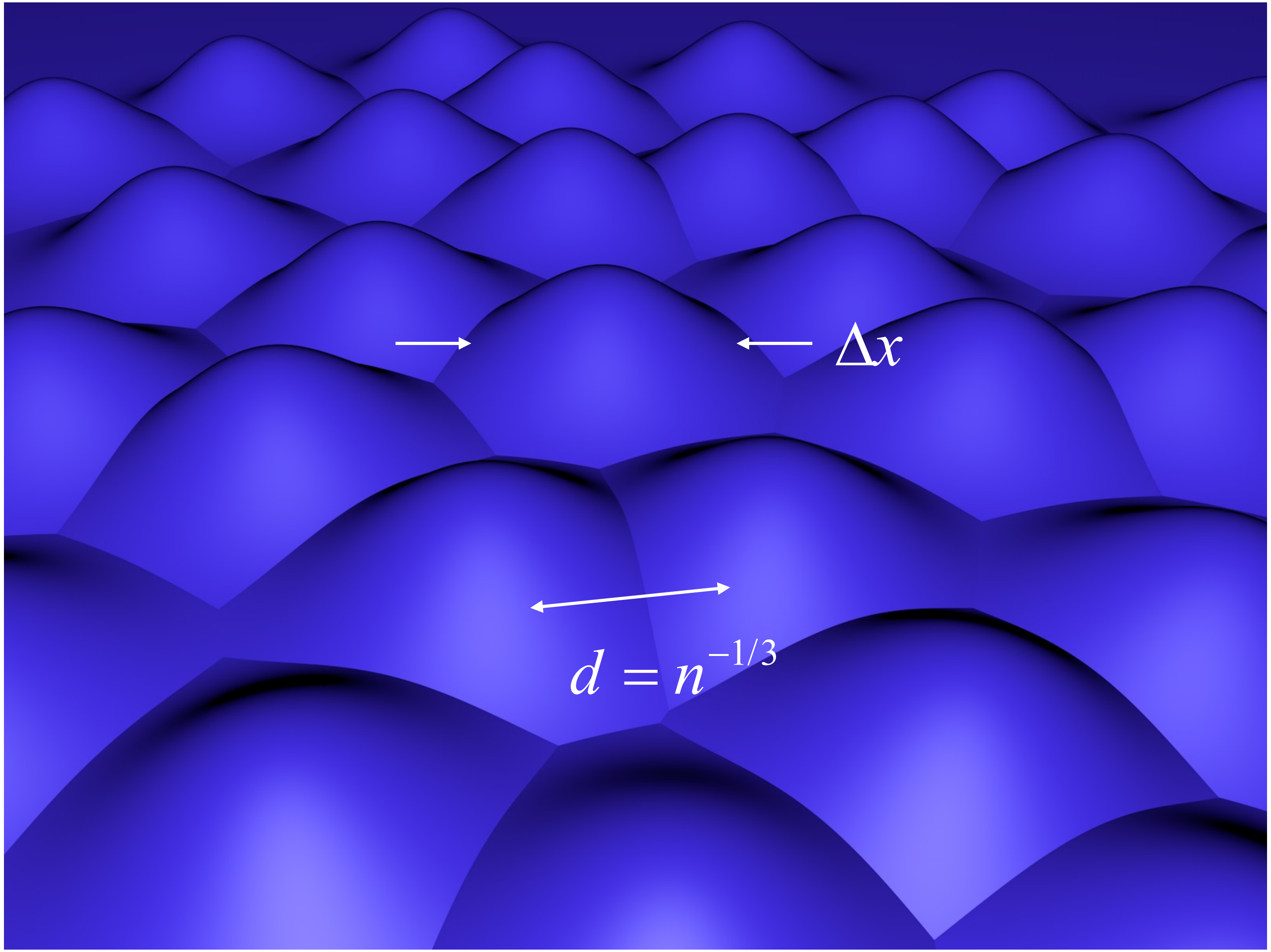
...behave as waves



When does wave mechanics matter?



When does wave mechanics matter?



Bosons versus Fermions

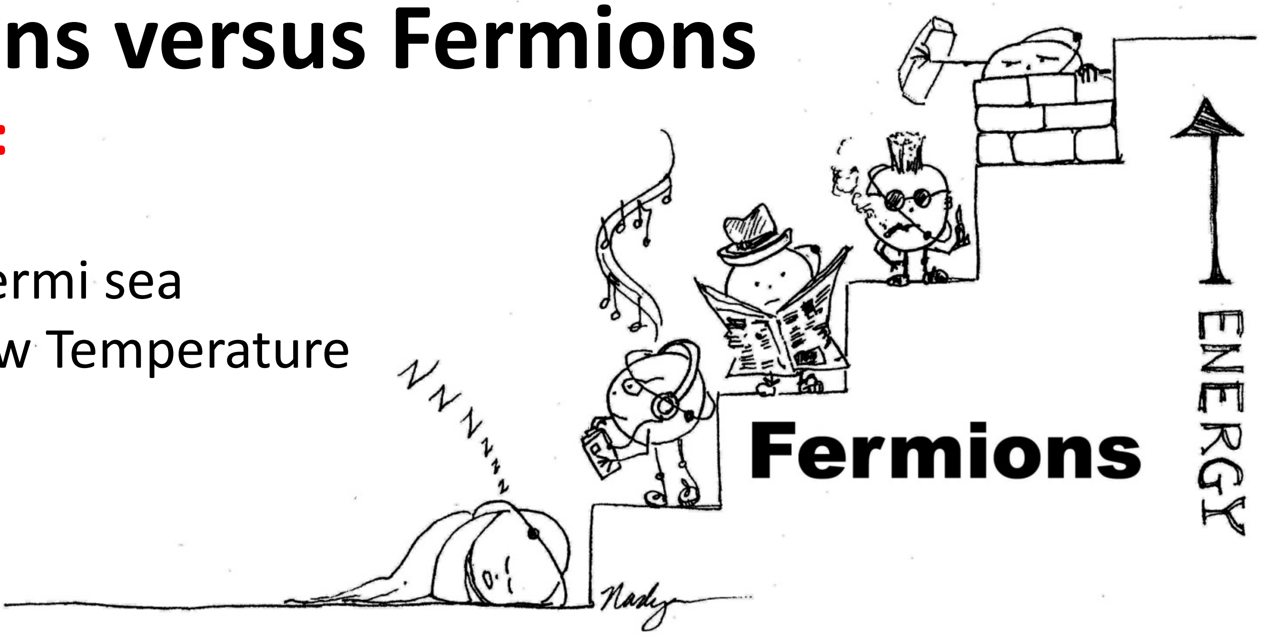
Fermions (unsociable):

Half-Integer Spin

Pauli blocking → Form Fermi sea

No phase transition at low Temperature

Fermi Feb. 1926
Dirac Oct. 1926



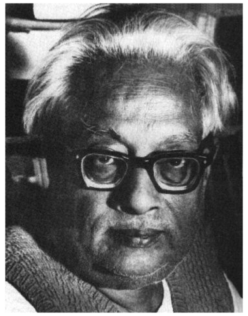
Bosons (sociable):

Integer Spin

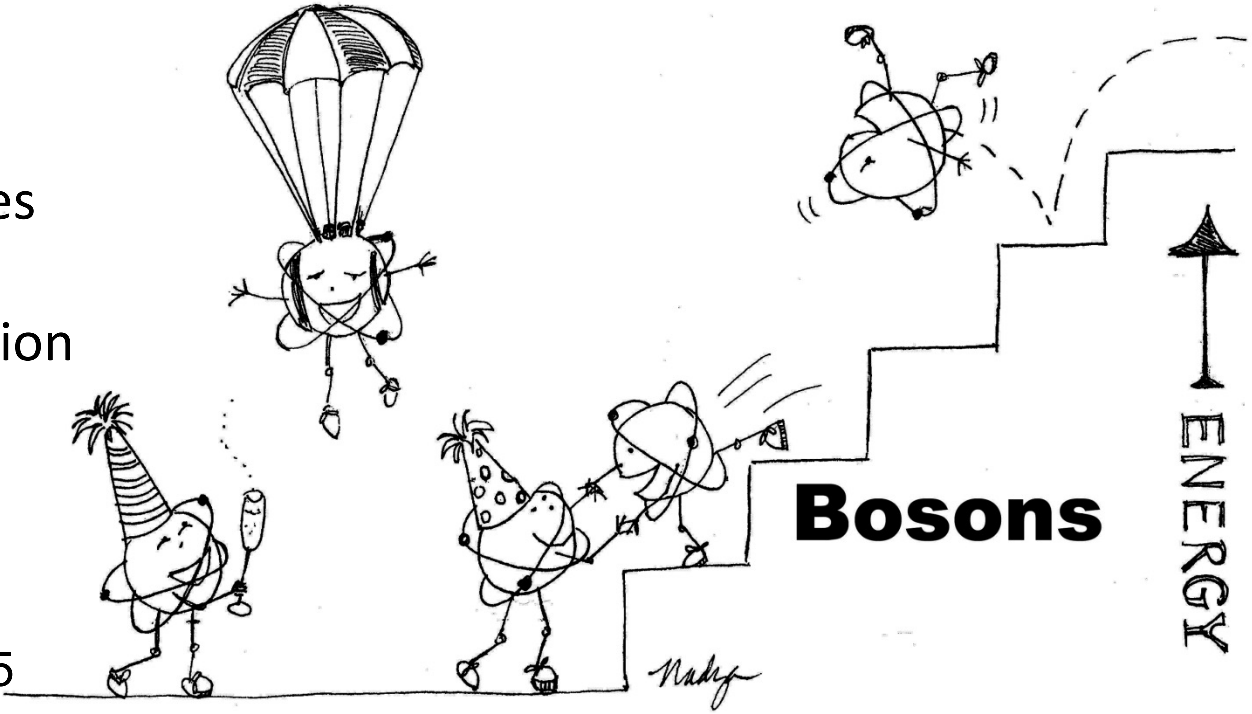
Can share quantum states

At low temperatures:

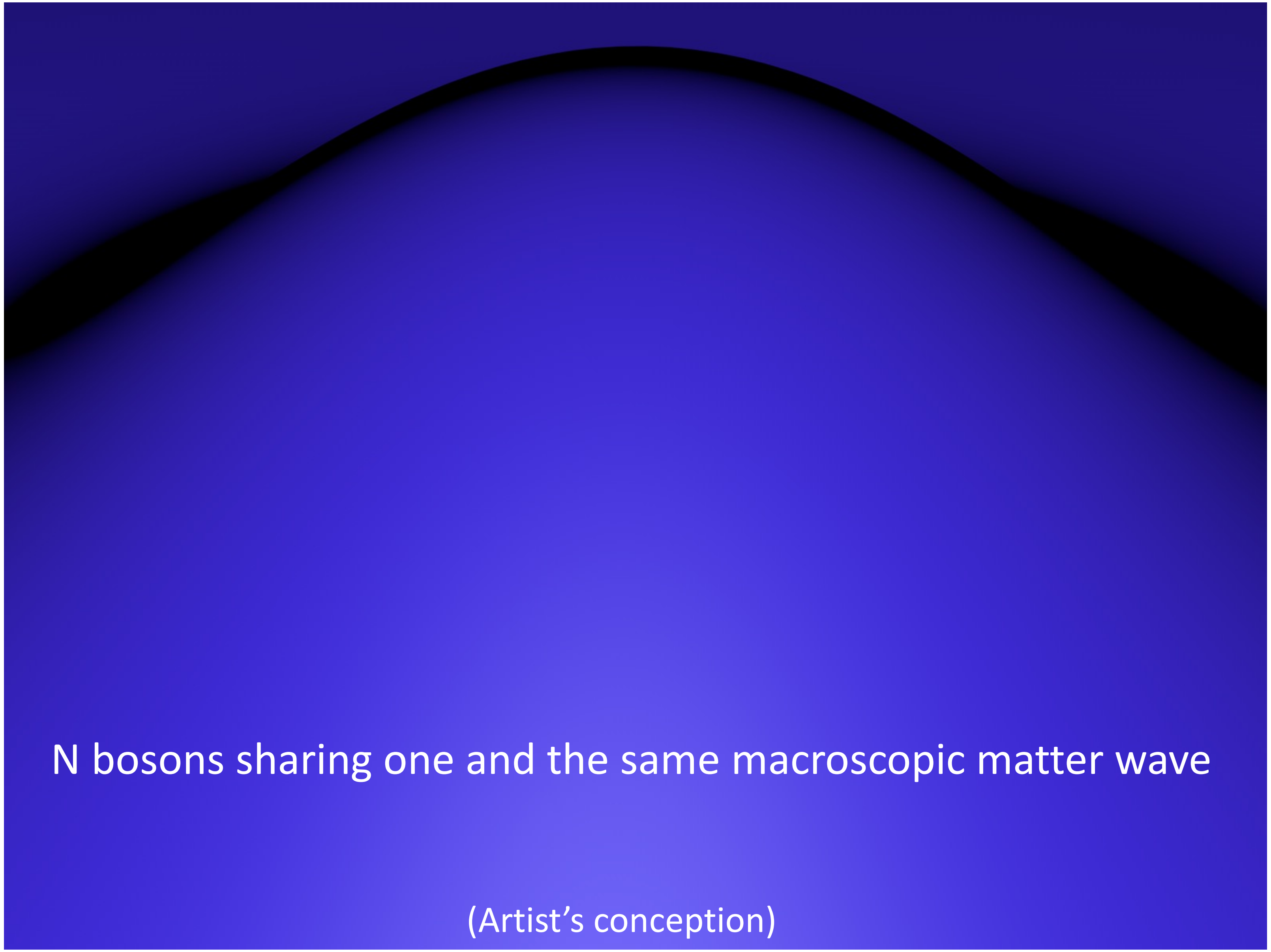
Bose-Einstein condensation



Bose 1924
Einstein 1924/25

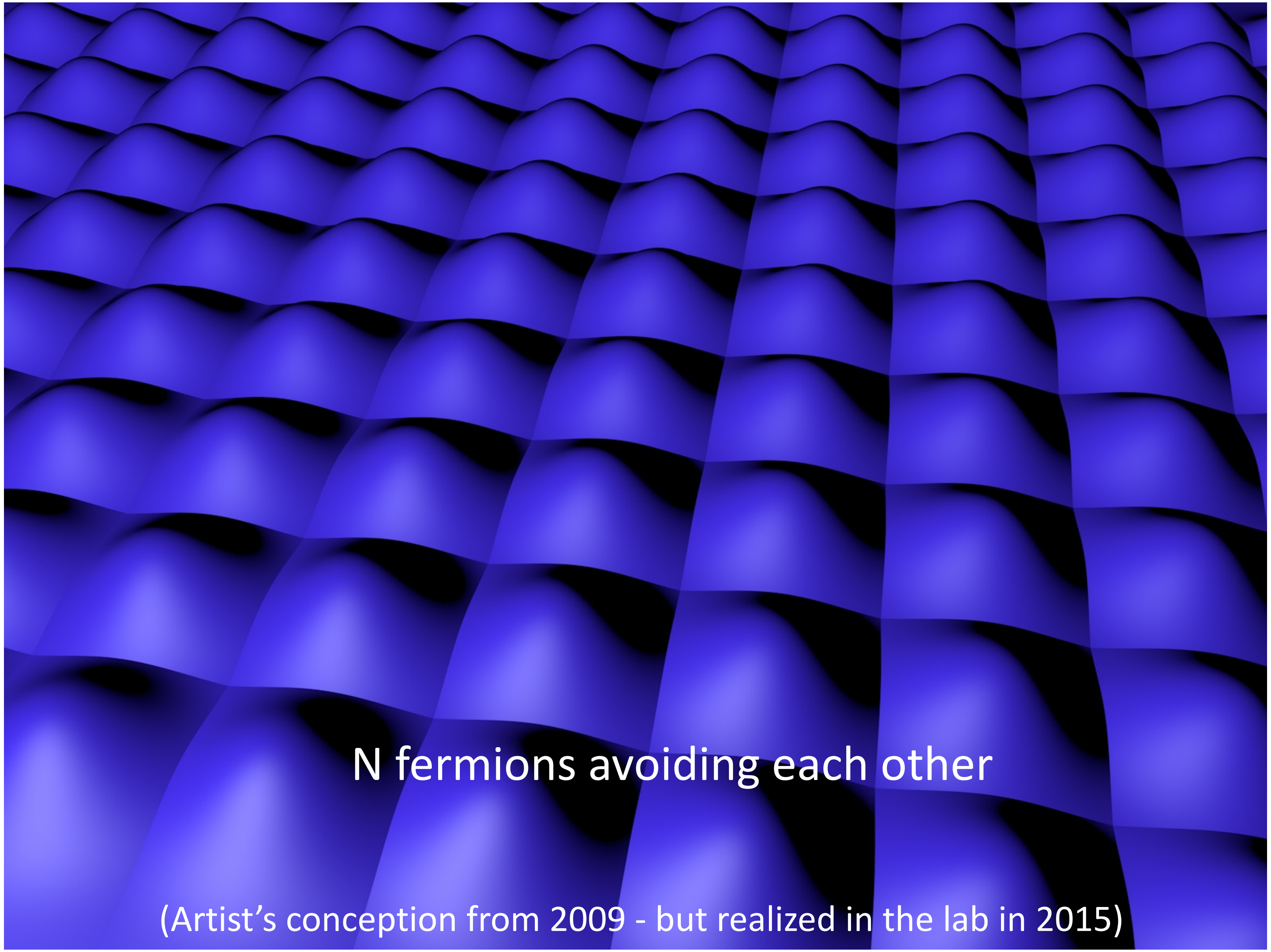


Bosons



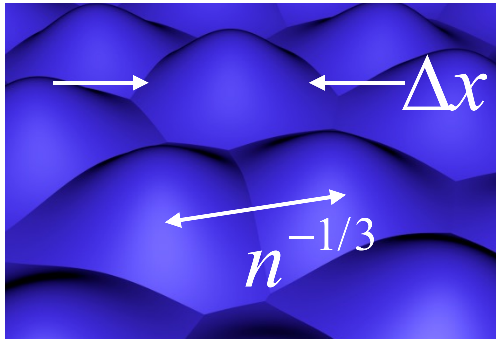
(Artist’s conception)

Fermions

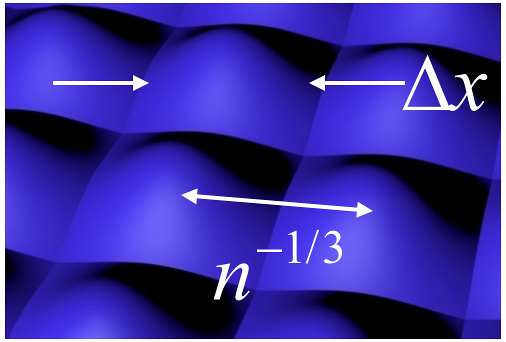

N fermions avoiding each other
(Artist's conception from 2009 - but realized in the lab in 2015)

Condition for quantum degeneracy

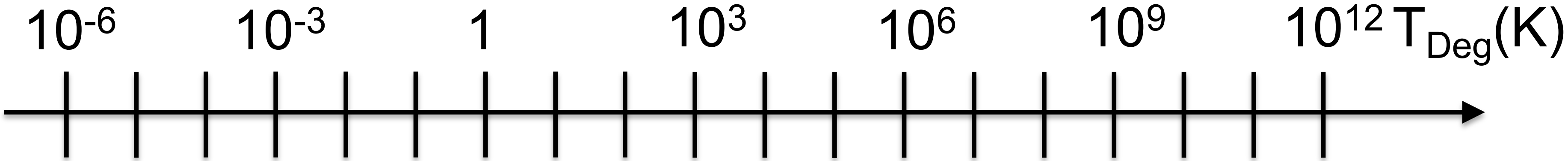
Position uncertainty ~ Interparticle spacing

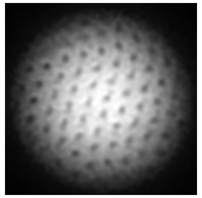


$$\Delta x = \frac{\hbar}{\Delta p} \approx n^{-1/3}$$

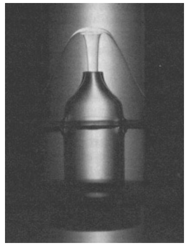


$$k_B T_{\text{Degeneracy}} \approx \frac{\hbar^2}{m} n^{2/3} \approx E_F$$ **Fermi energy**





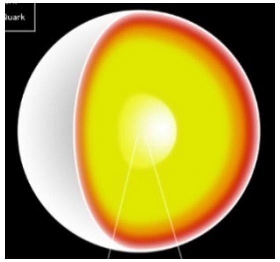
Ultracold **atomic gases**



Liquid Helium

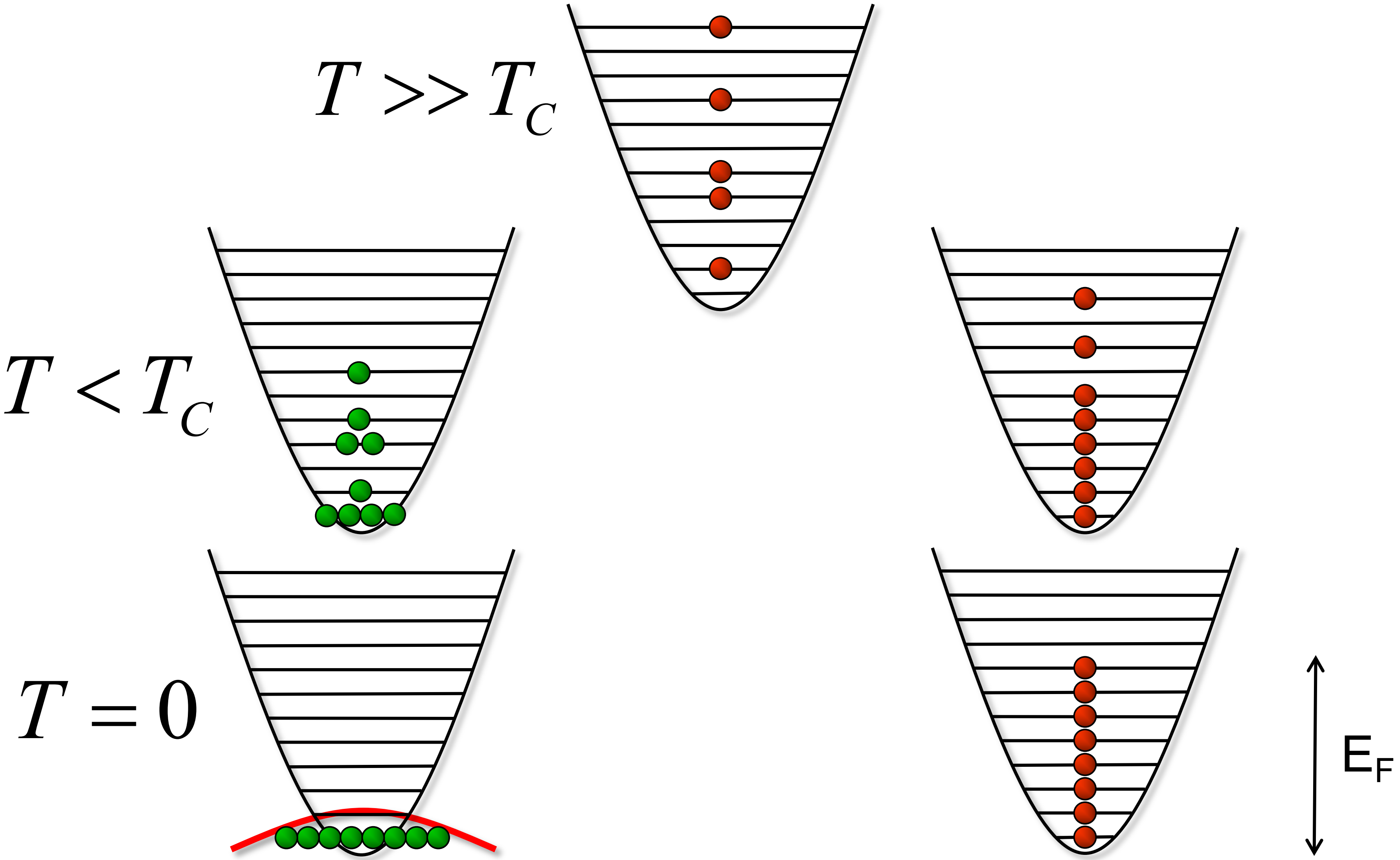
Metals

White dwarf



Neutron star

Bosons vs Fermions



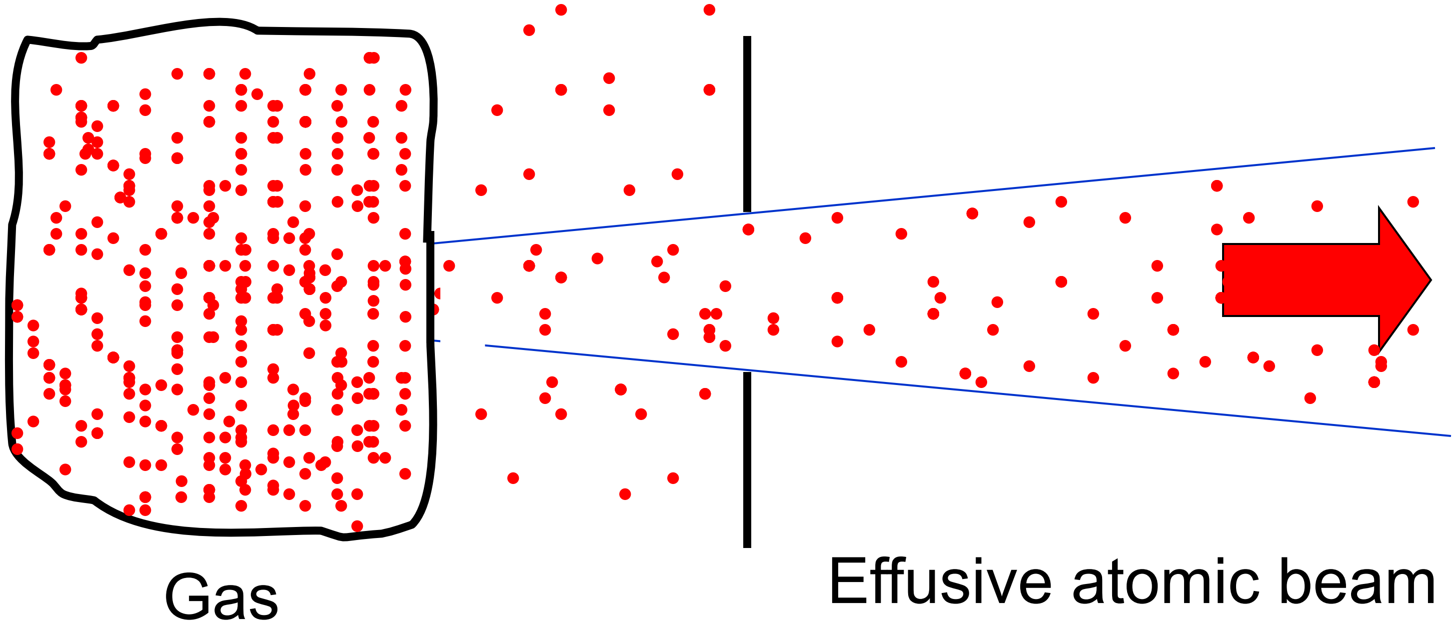
Bosons

e.g.: ^{1}H, ^{23}Na, $^{6}Li_2$

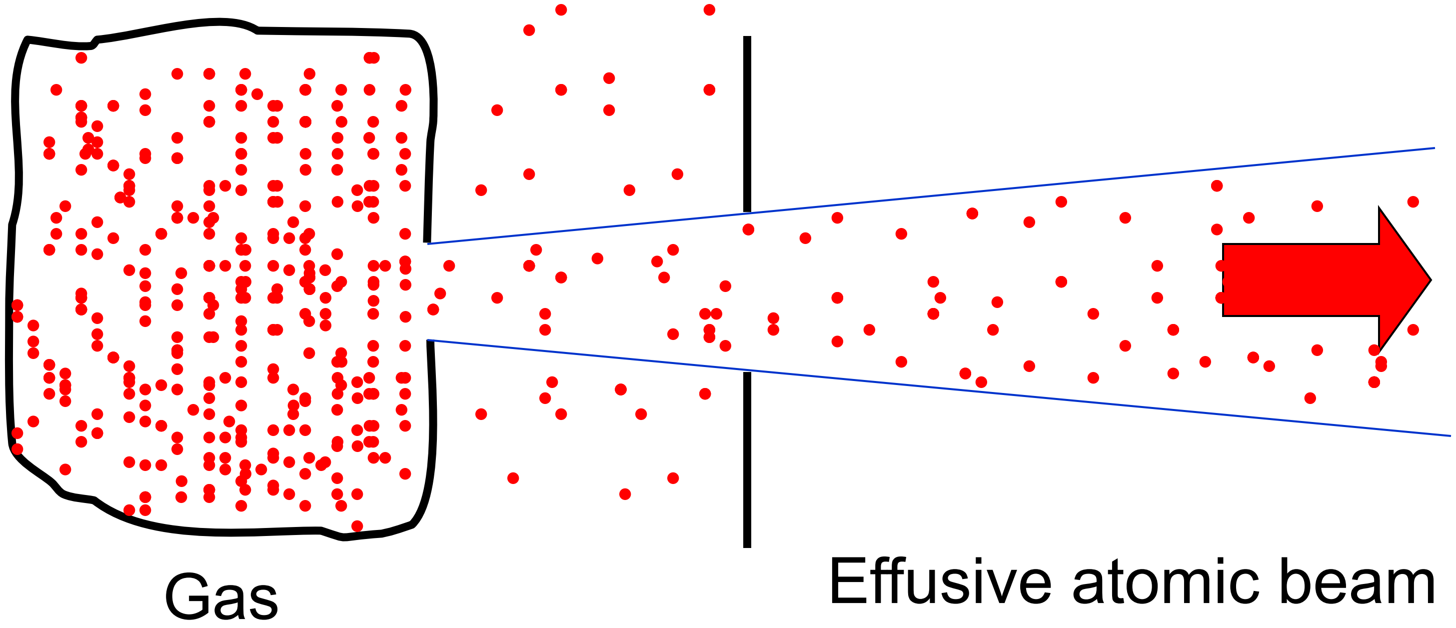
Fermions

e.g.: e^-, ^{3}He, ^{6}Li, ^{40}K

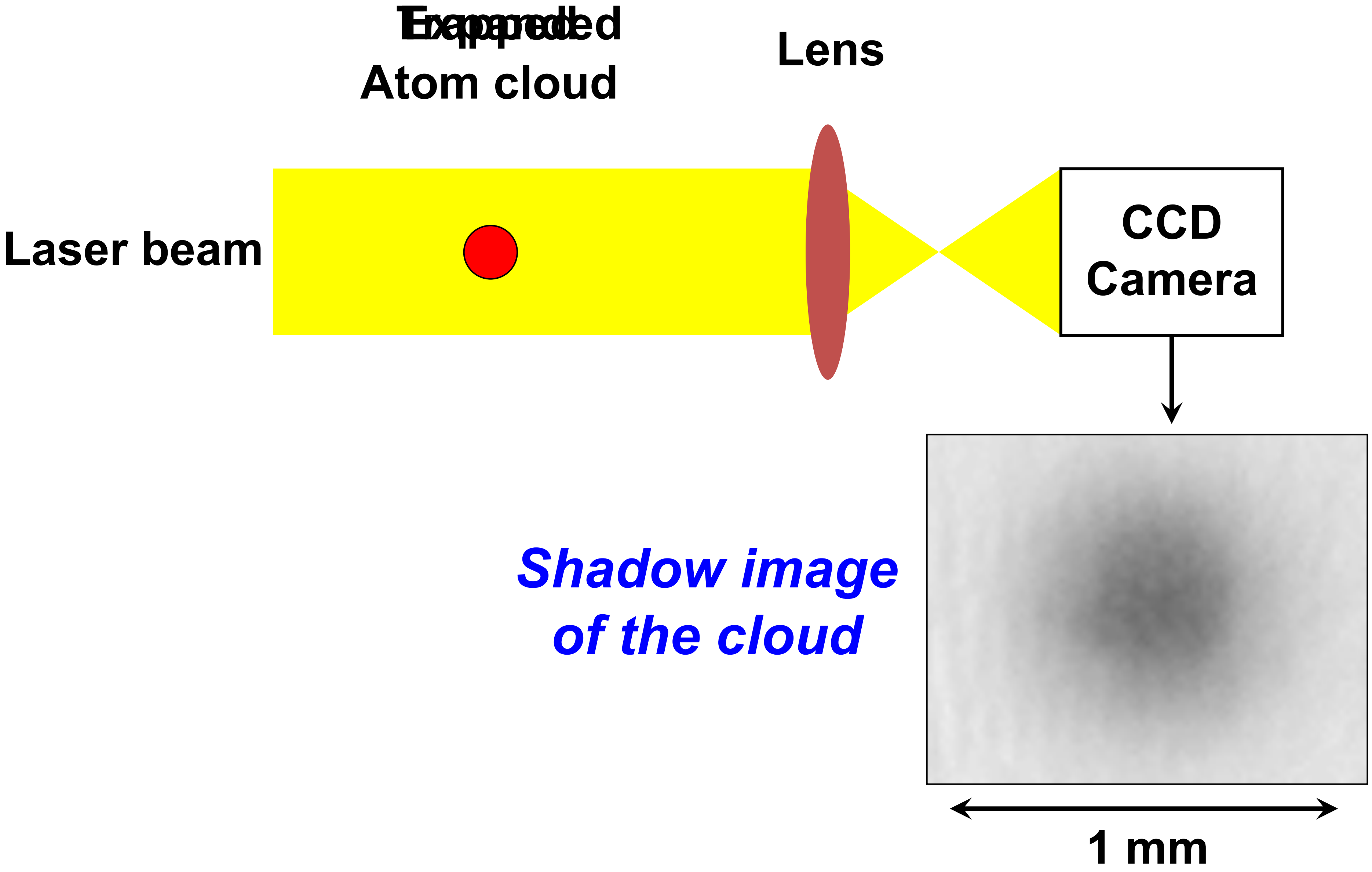
How to measure temperature?

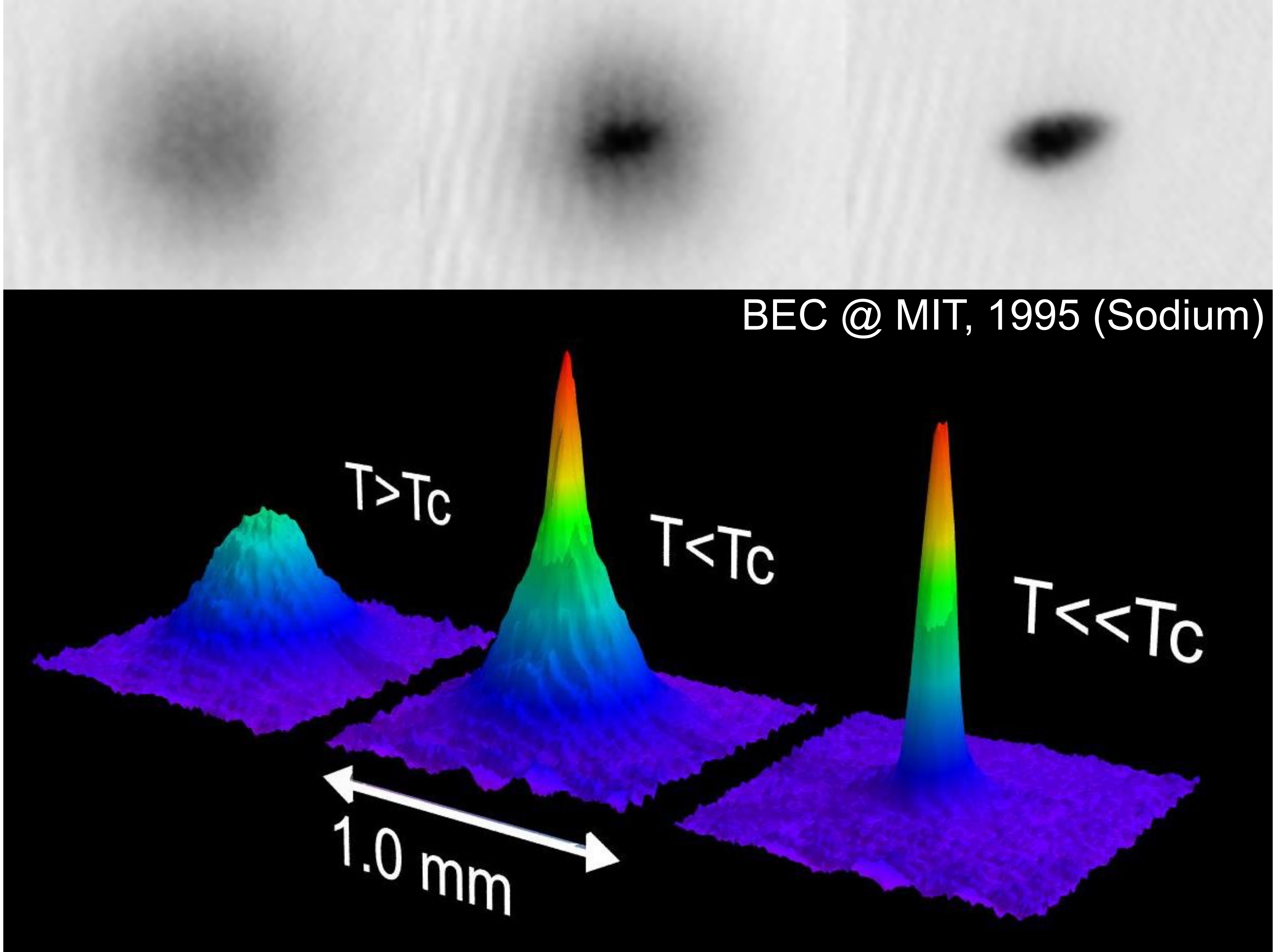


How to measure temperature?



Observation of the atom cloud




BEC @ MIT, 1995 (Sodium)
T>Tc
T<Tc
T<<Tc
1.0 mm

Superfluidity in Bosonic Gases

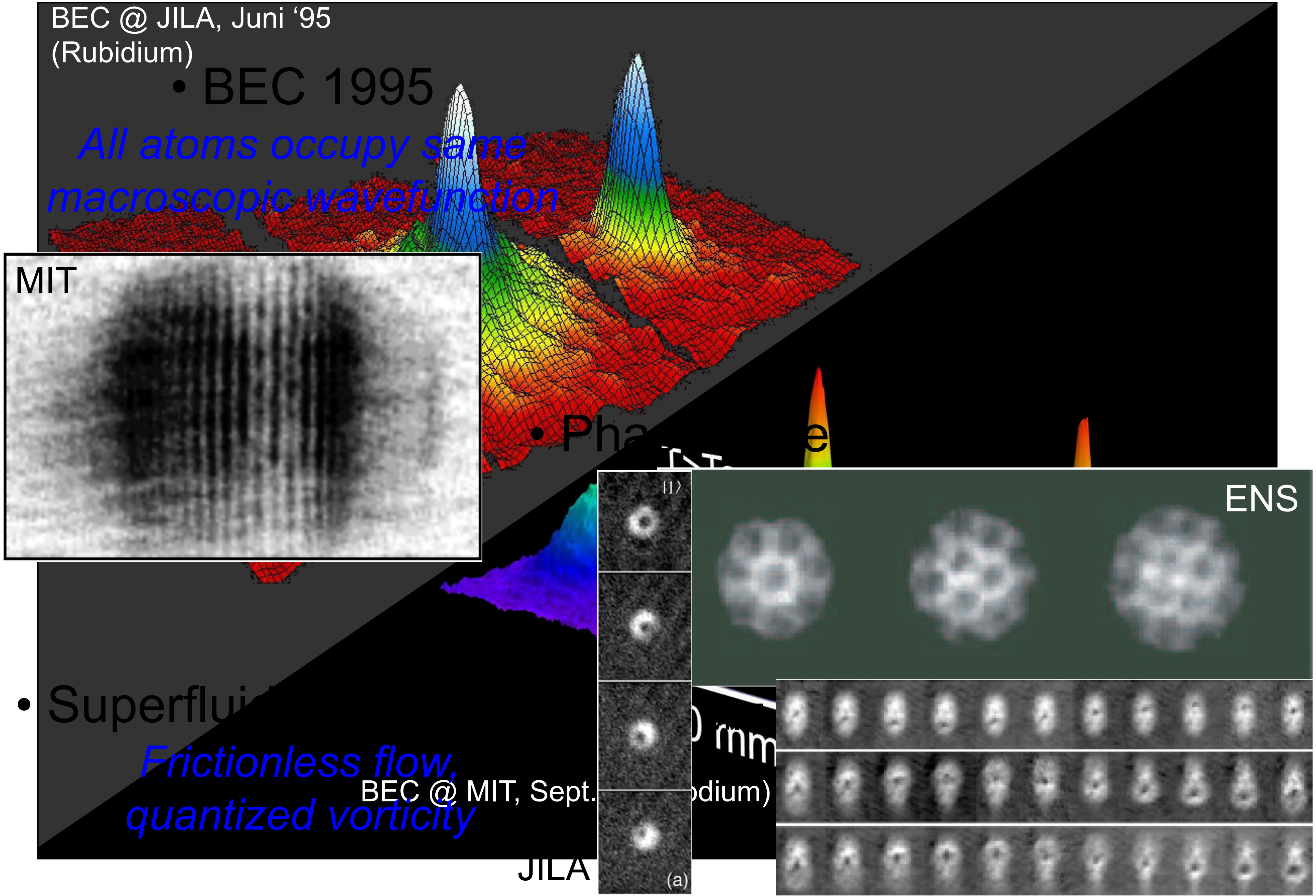
- BEC 1995

All atoms occupy same macroscopic wavefunction

- Pha e

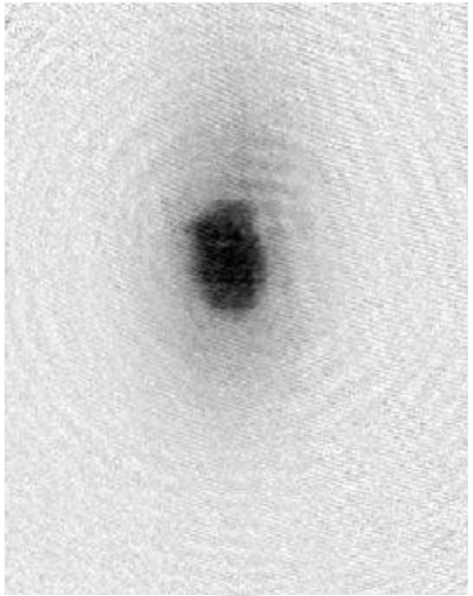
- Superflui

Frictionless flow, quantized vorticity

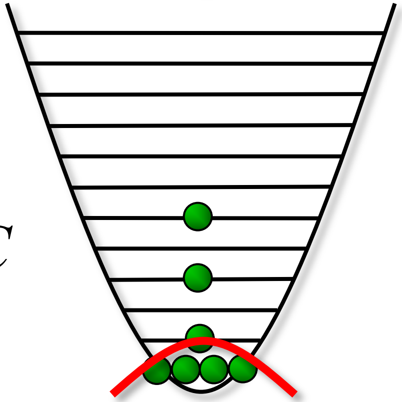


Bosons vs Fermions

^{23}Na



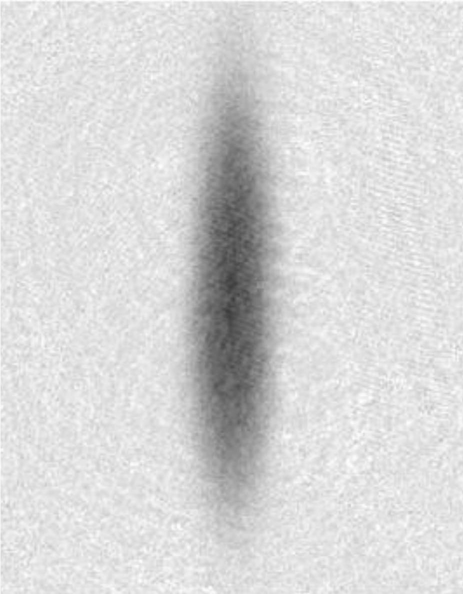
$$T < T_C$$

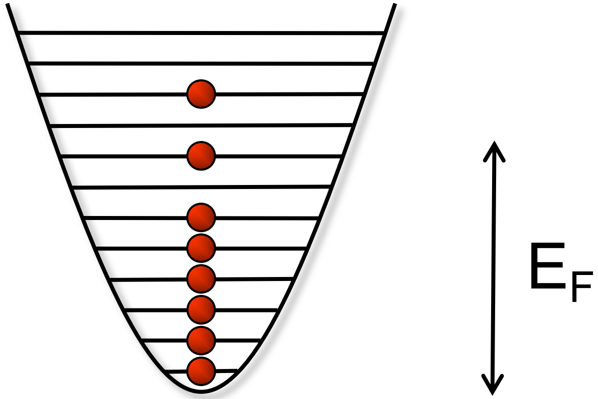


Bosons

e.g.: ^{1}H, ^{23}Na, ^{6}Li$_2$

^{6}Li





Fermions

e.g.: e^{-}, ^{3}He, ^{6}Li, ^{40}K

Non-interacting Fermi gas

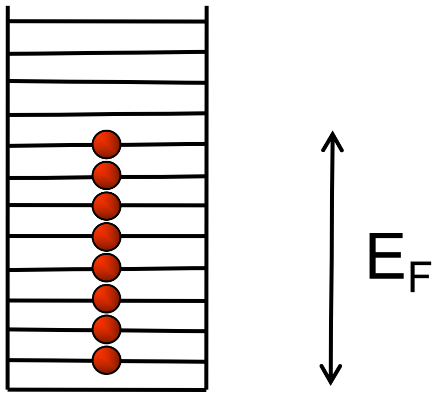
- Fermi-Dirac Statistics:

$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)}+1}$$

At T = 0:

$$f(\epsilon) \to \theta(E_F - \epsilon)$$

- Fermi gas in a box:



$$\epsilon = \frac{\hbar^2 k^2}{2m}$$

$$N = \int d^3x \int \frac{d^3k}{(2\pi)^3} \theta\left(E_F - \frac{\hbar^2 k^2}{2m}\right)$$

$$n = \frac{1}{(2\pi)^3}\text{Volume of sphere in k-space with radius } k_F$$

$$= \frac{1}{6\pi^2} k_F^3$$

$$E_F = \frac{\hbar^2 k_F^2}{2m}$$

$$k_F = \left(6\pi^2 n\right)^{-1/3} \sim 1/\text{Interparticle spacing}$$

Non-interacting Fermi gas

- Fermi-Dirac Statistics:

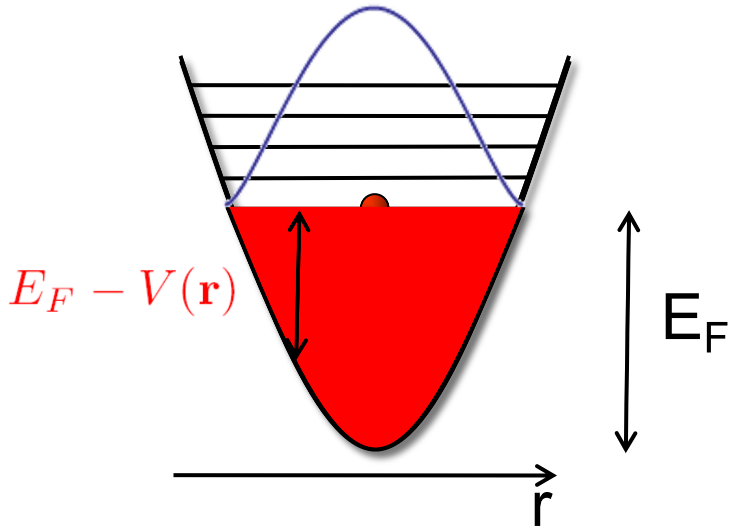
$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$

At T = 0:

$$f(\epsilon) \to \theta(E_F - \epsilon)$$

- Fermi gas in a trap:

$$\epsilon_F(\mathbf{r}) = E_F - V(\mathbf{r})$$

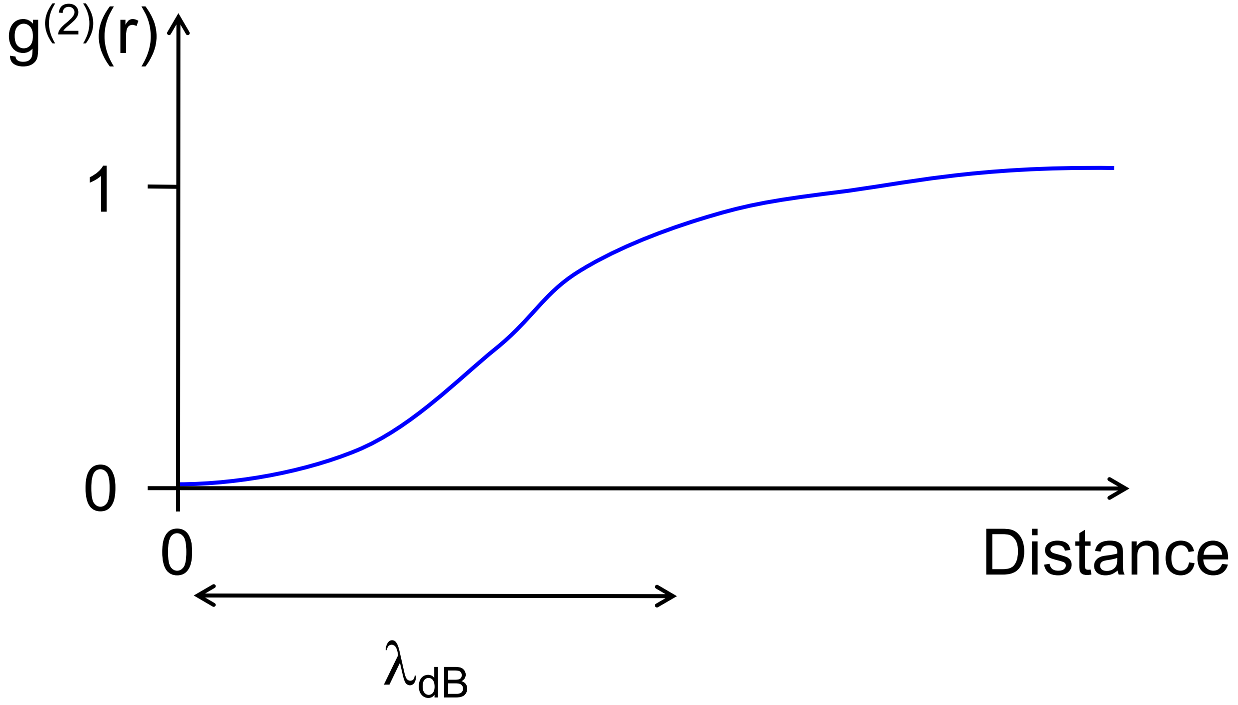


$$n(\mathbf{r}) = \frac{1}{6\pi^2} k_F(\mathbf{r})^3 = \frac{1}{6\pi^2\hbar^3} \left(2m(E_F - V(\mathbf{r}))\right)^{3/2}$$

(Local density approximation)

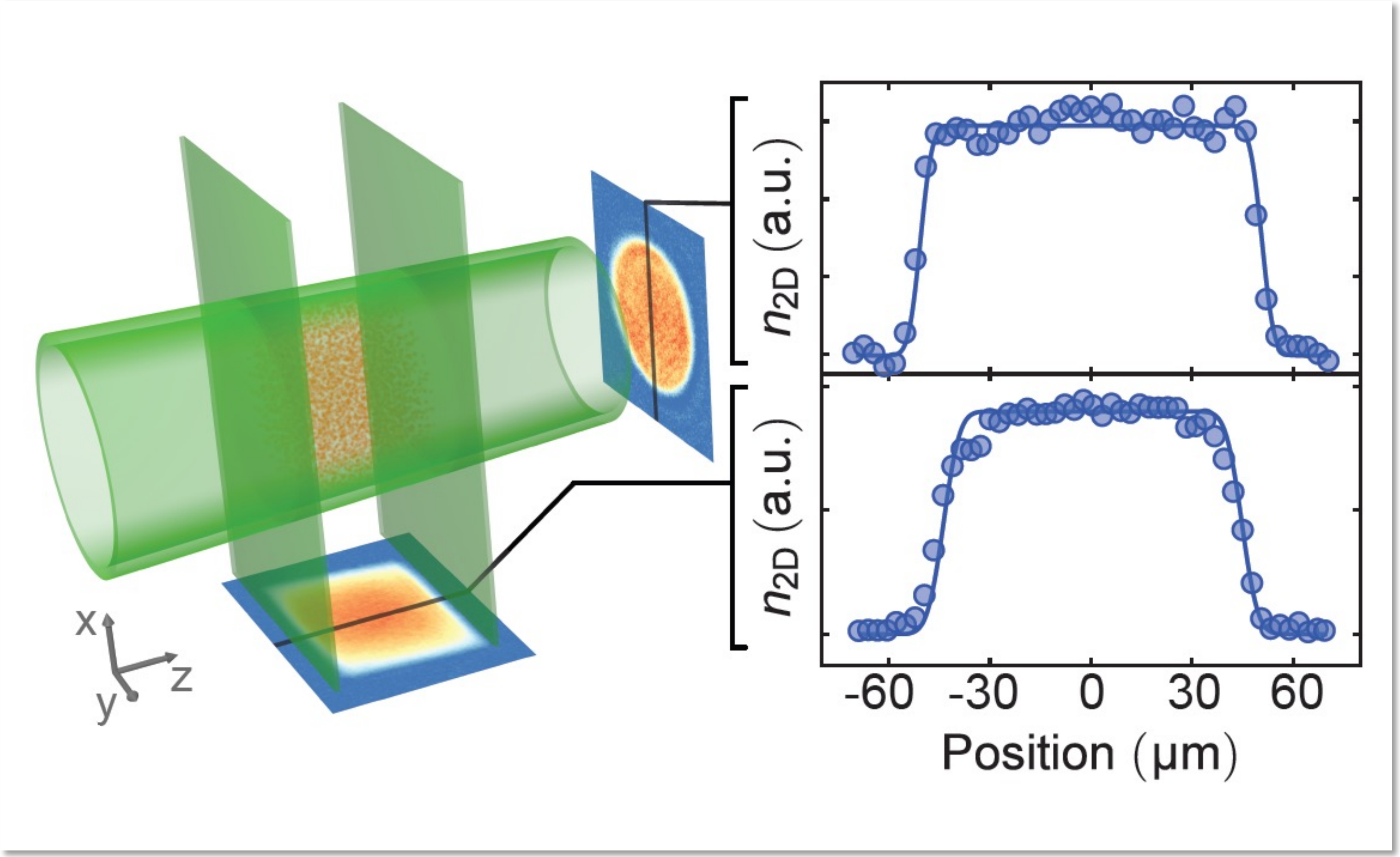
Freezing out of collisions

Pair correlations in a Fermi gas:



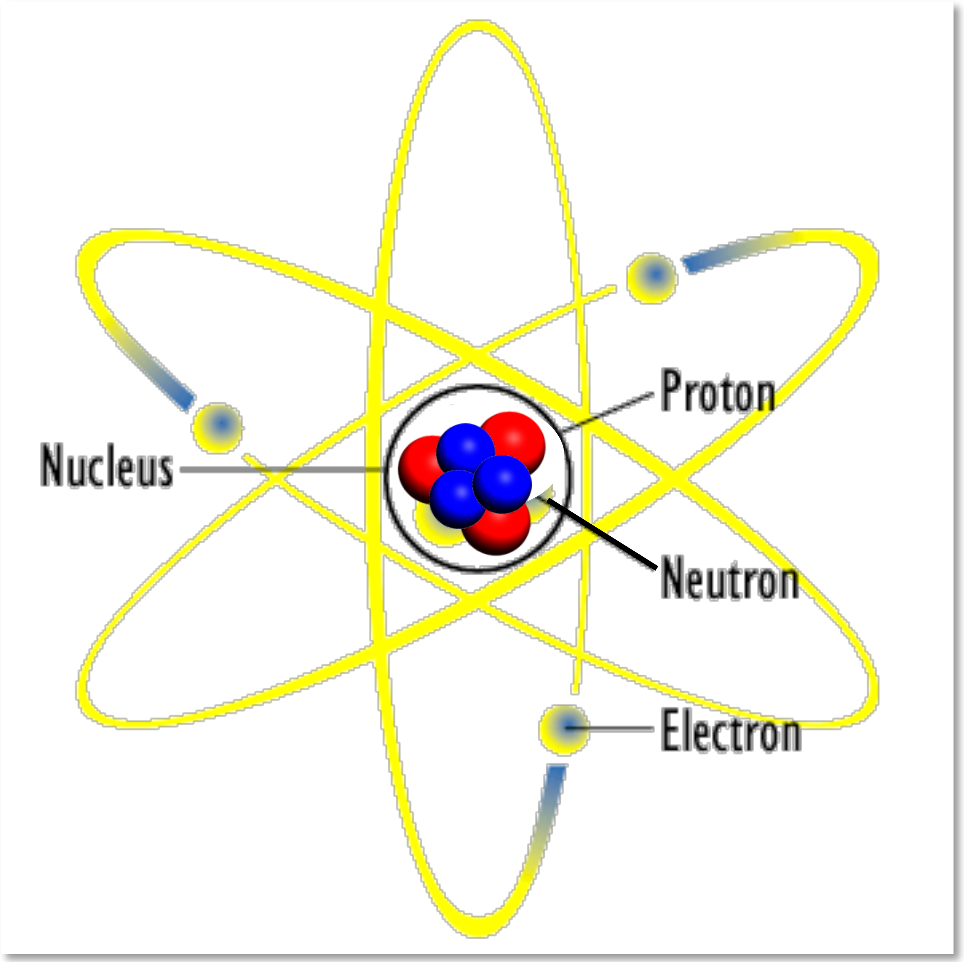
No interactions if range of potential is < λ_{dB}

Fermions in a Box

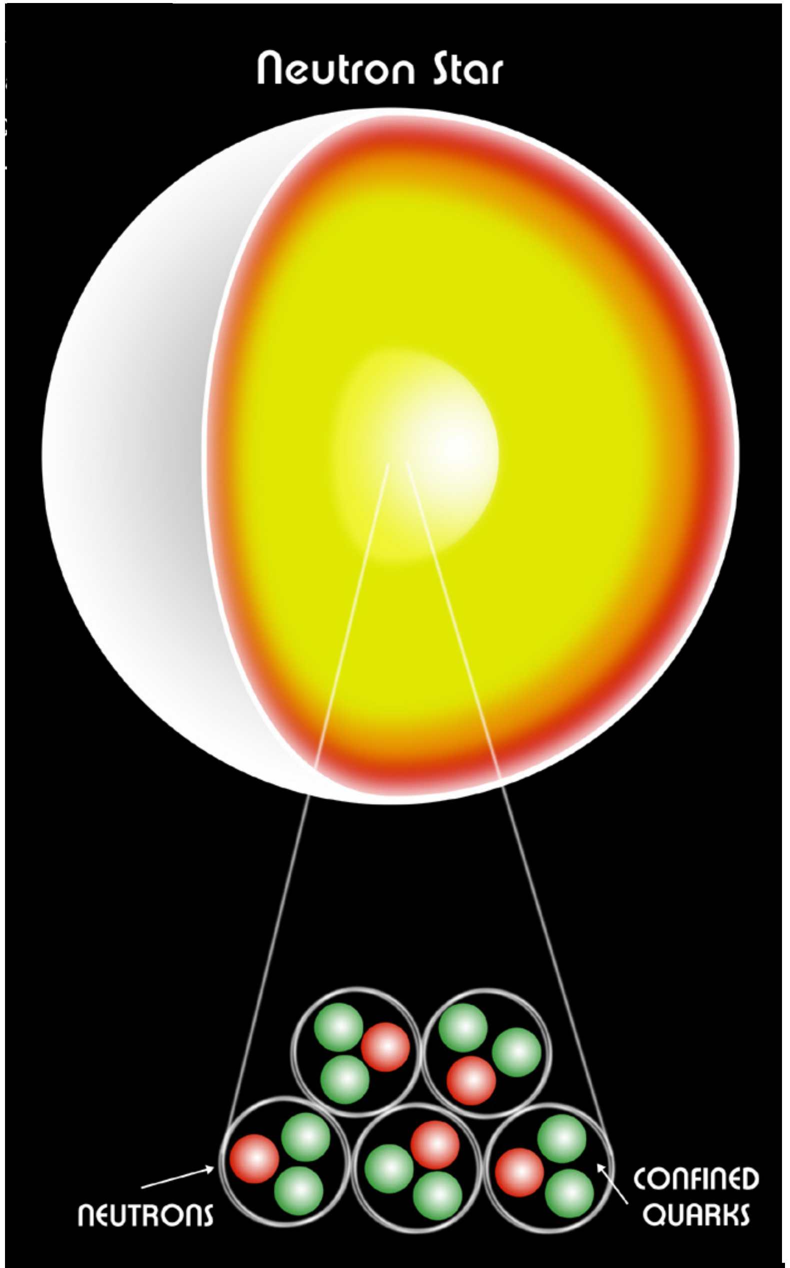


Z. Yan, P. Patel, B. Mukherjee, Z. Hadzibabic, T. Yefsah, J. Struck, MWZ, PRL 2017

Fermions – The Building Blocks of Matter



Lithium-6



Harvard-Smithsonian Center for Astrophysics

Scattering Theory

Incoming plane wave

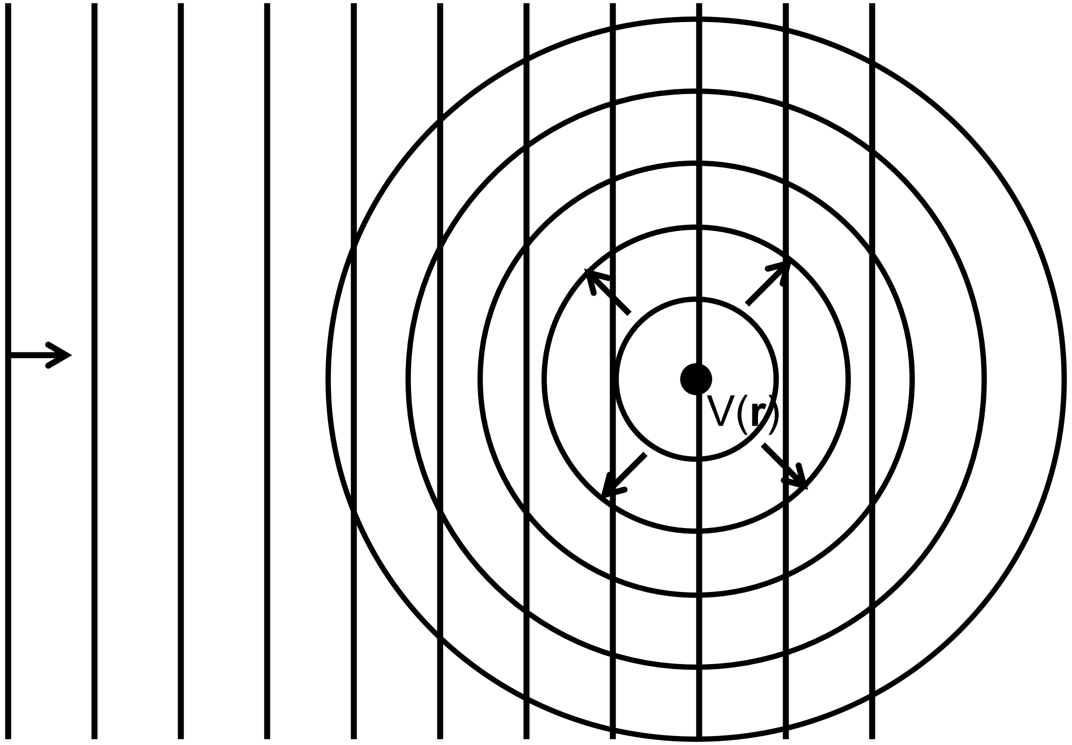
$e^{ikz-i\omega t}$

Radially outgoing wave

$$f_k(\vartheta,\varphi)\frac{1}{r}e^{ikr-i\omega t}$$

For s-wave:

$$e^{ikz} \approx \frac{1}{kr}\sin(kr)$$



For s-wave:

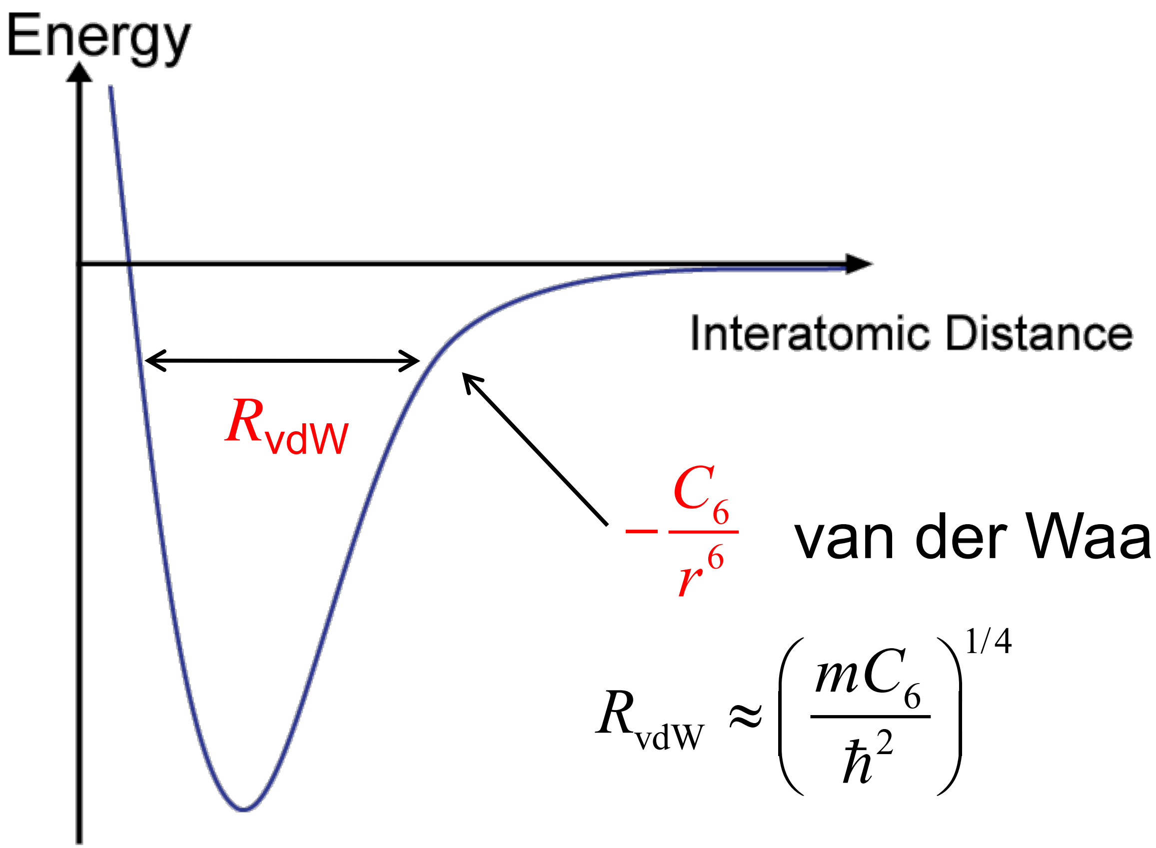
$$f_k\frac{1}{r}e^{ikr-i\omega t}$$

$$e^{ikz} + f_k\frac{1}{r}e^{ikr} \to \frac{1}{kr}e^{i\delta(k)}\sin(kr+\delta(k))$$

$$f_k = \frac{1}{k}e^{i\delta(k)}\sin(\delta(k))$$

In words: All that happens at low momenta is that one gets a phase shift

Interatomic interactions



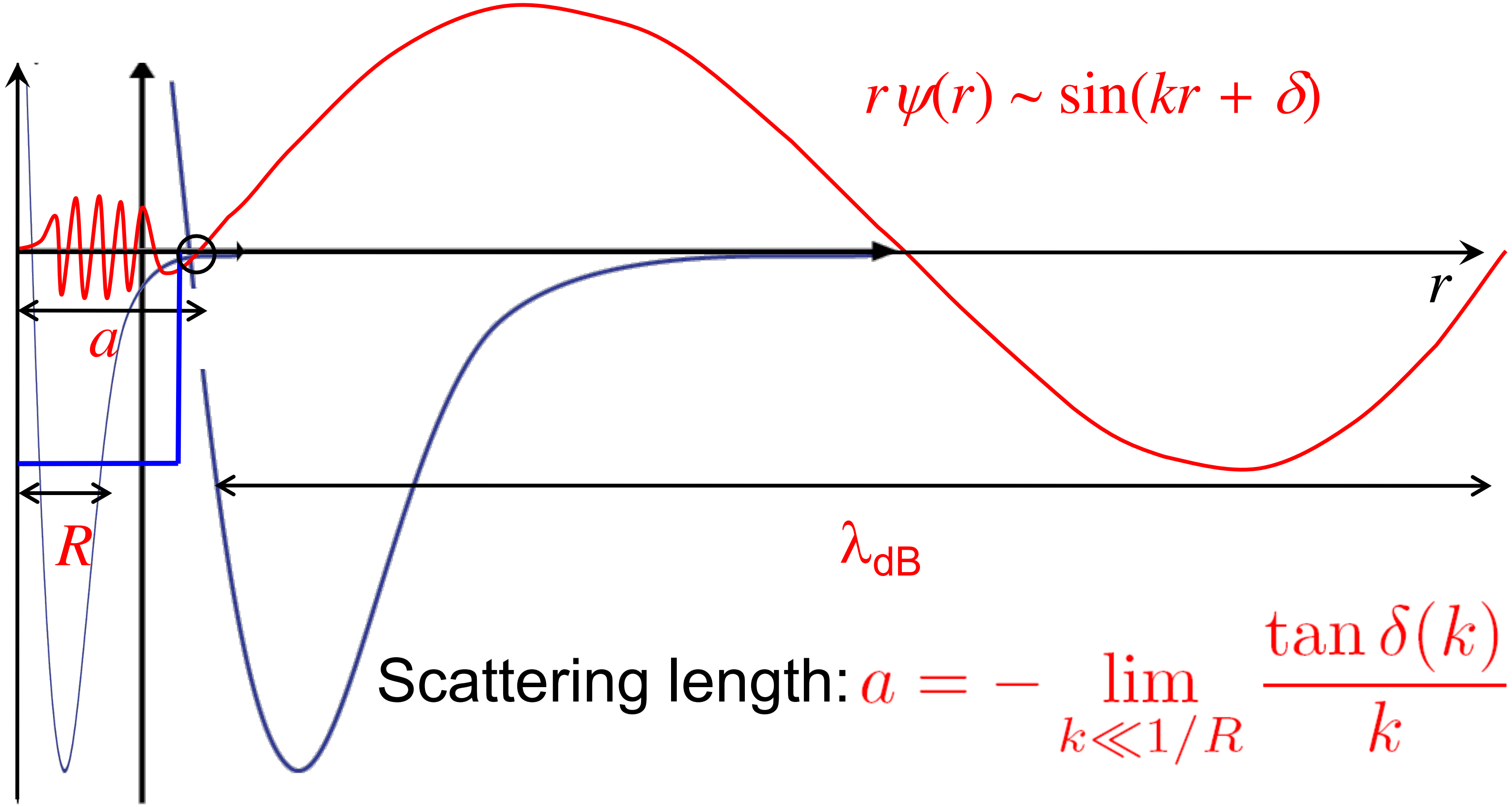
$-\dfrac{C_6}{r^6}$ van der Waals potential

$$R_{\mathrm{vdW}} \approx \left(\frac{mC_6}{\hbar^2}\right)^{1/4}$$

- For Alkali atoms: $R_{\mathrm{vdW}} \sim 50\text{-}200\ a_0$
- Ultracold collisions: $\lambda_{dB} \approx 1\mu m >> R$

→ atoms do not probe the details of the potential

Interatomic interactions

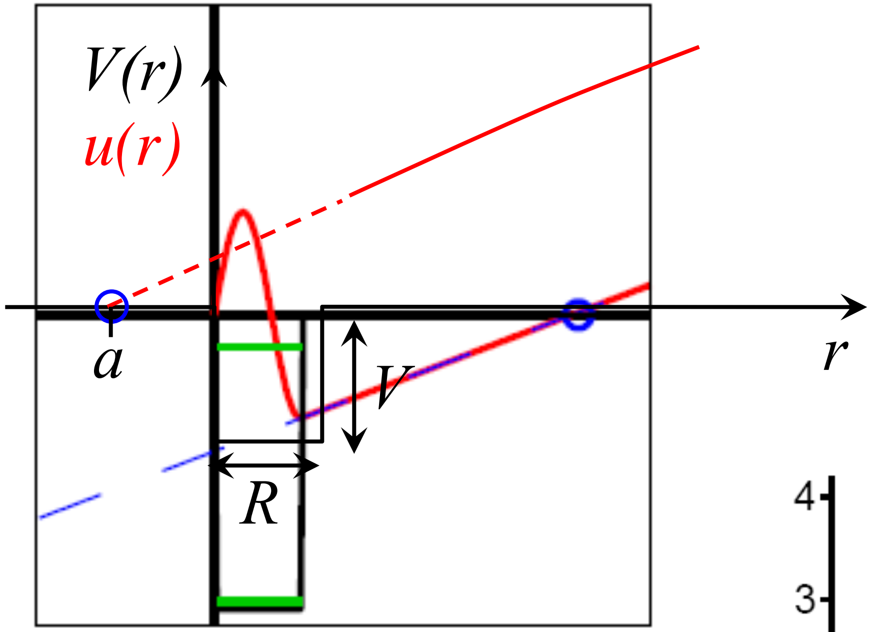


Scattering length: $a = -\lim_{k \ll 1/R} \frac{\tan\delta(k)}{k}$

- For Alkali atoms: $R_{\mathrm{vdW}} \sim$ 50-200 a_0
- Ultracold collisions: $\lambda_{dB} \approx 1 \mu m >> R$

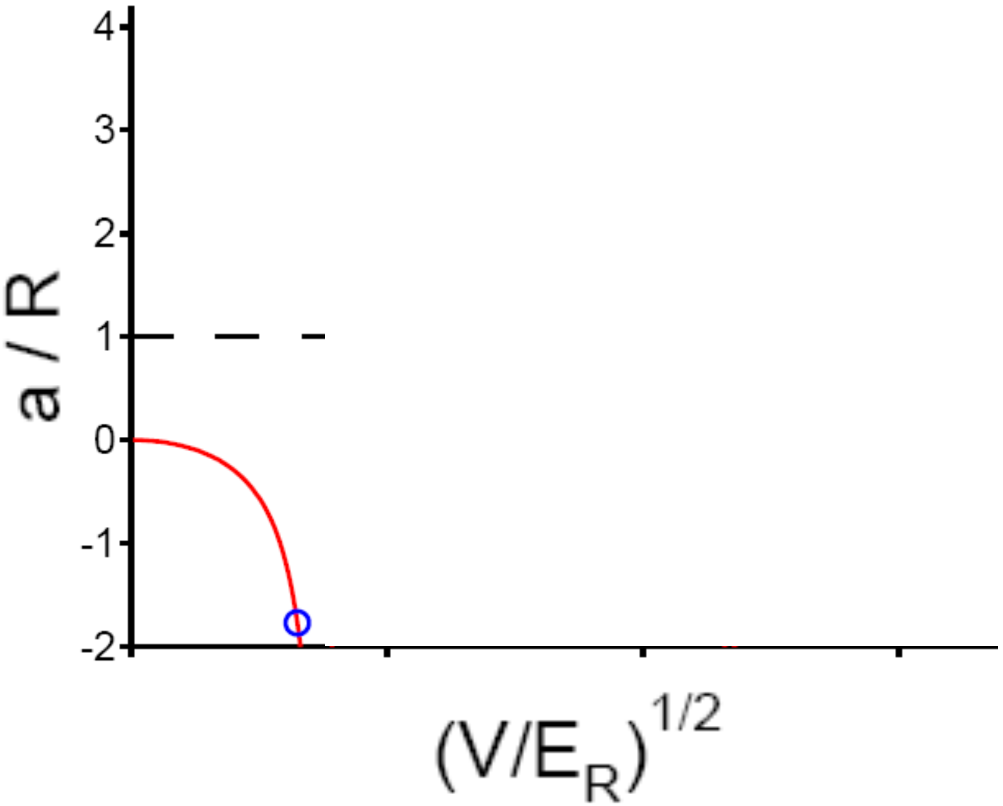
→ atoms do not probe the details of the potential

Scattering Resonances



$$u(r) \overset{r>R}{=} \sin(k(r-a))$$

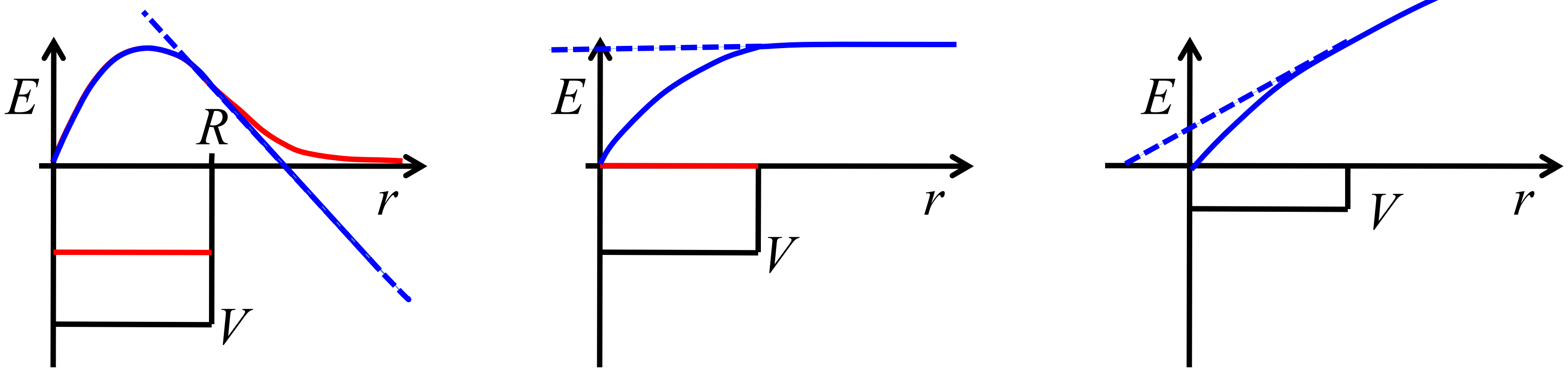
(s-wave) scattering length



Tunable Interactions

Vary interaction strength between spin up and spin down

Example: tunable square well (with $k_F R << 1$):



strong attraction	Resonance	weak attraction
deep bound state	*bound state appears*	*no bound state*

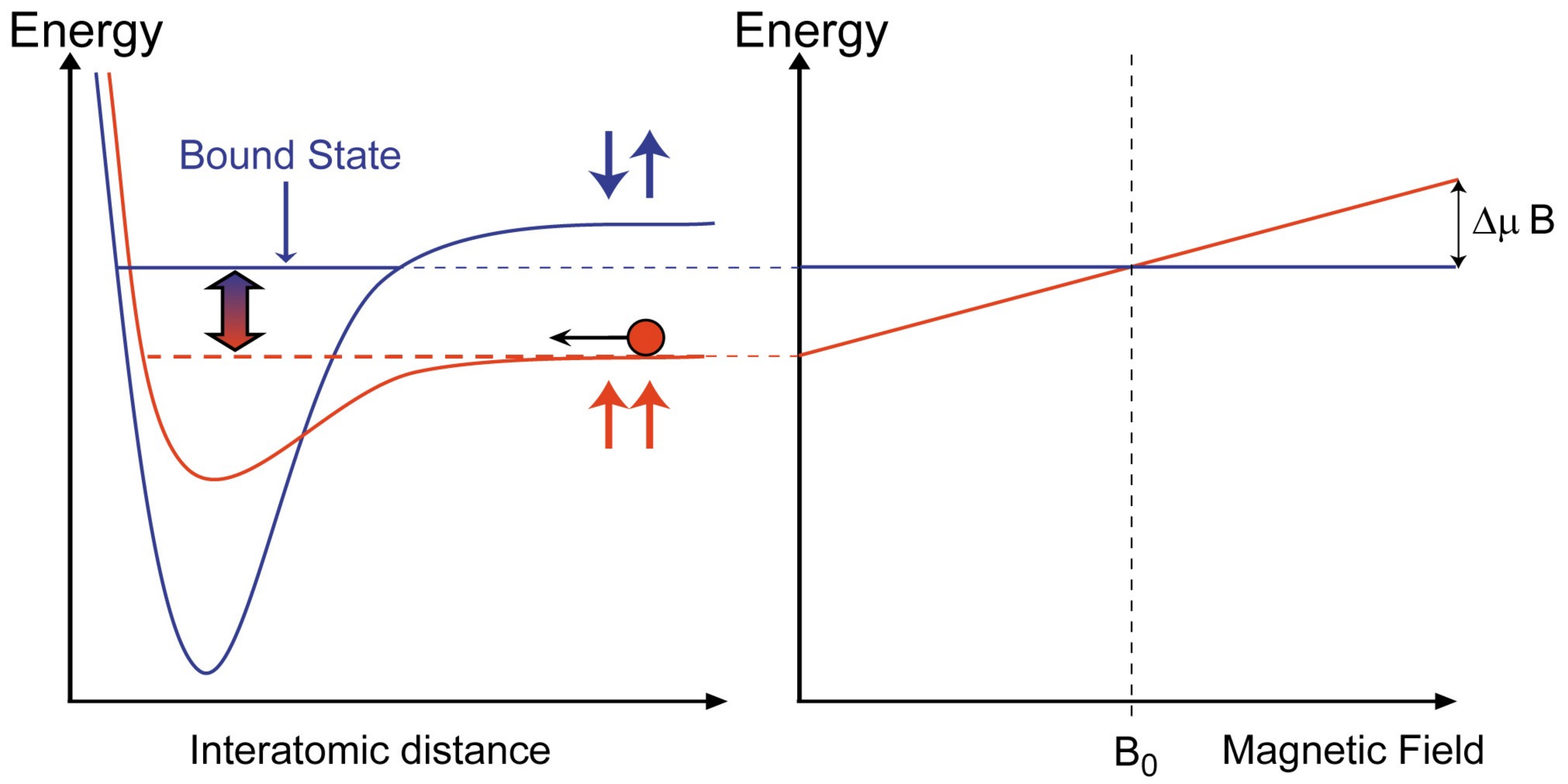
scattering length

$a > 0$	$a \to \pm\infty$	$a < 0$

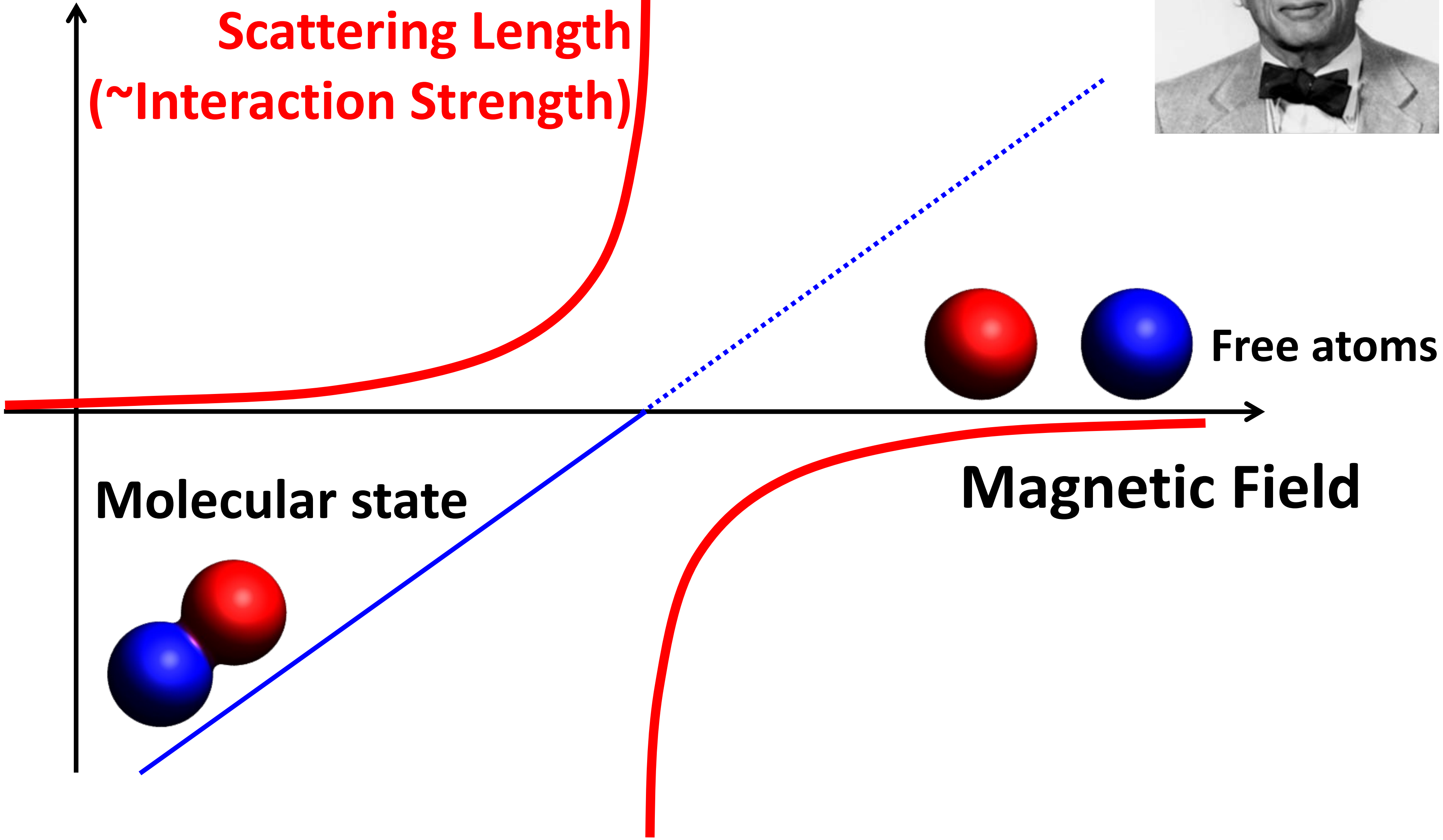
Important for Many-Body Physics:

$$\boxed{\frac{\text{Interparticle distance}}{\text{Scattering length}} = \frac{1}{k_F a}}$$

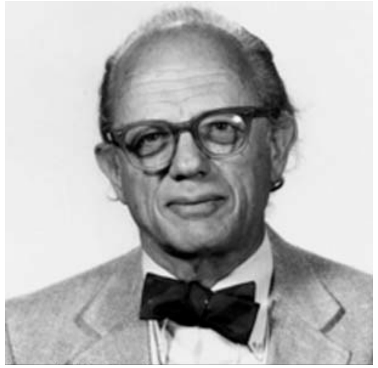
Feshbach resonances: Tuning the interactions



Feshbach resonances: Tuning the interactions

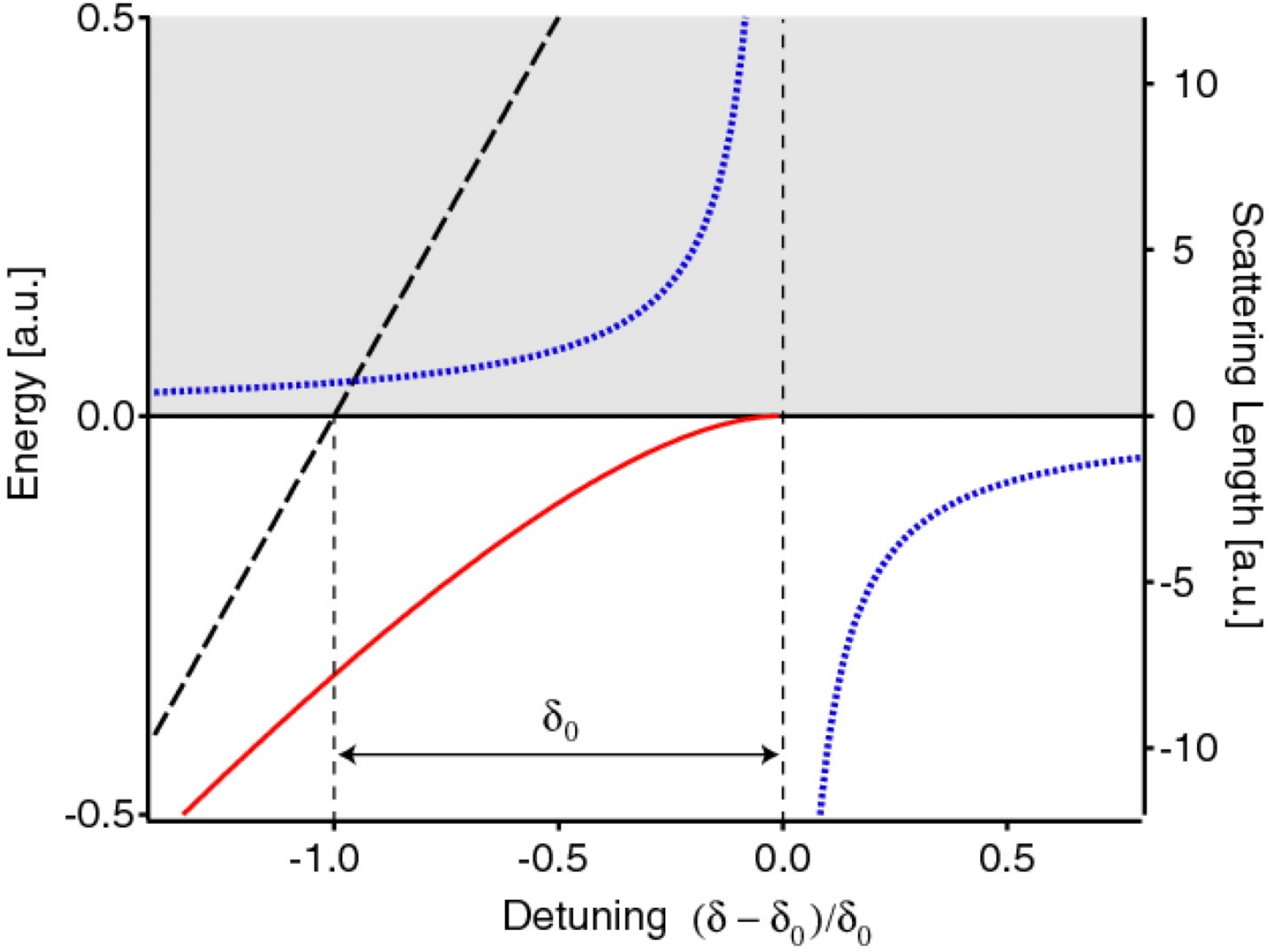


Herman Feshbach



Feshbach Resonances: Tuning the interactions

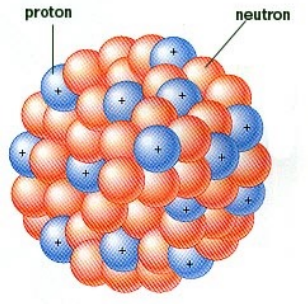
Recall HW7



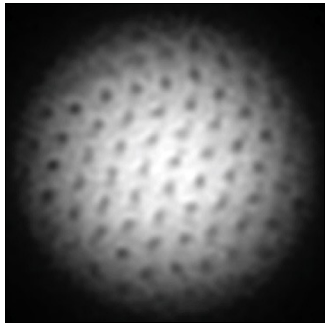
Strongly Interacting Fermi Systems

Length scales

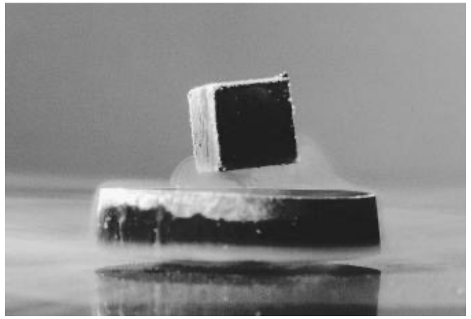
10^{-15} m	1 mm	1 m	10^4 m	10^7 m



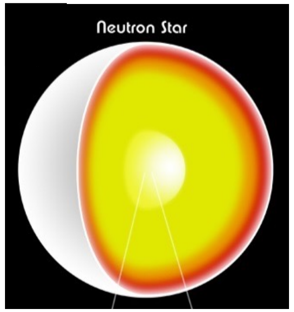
Nuclei



Ultracold Gases

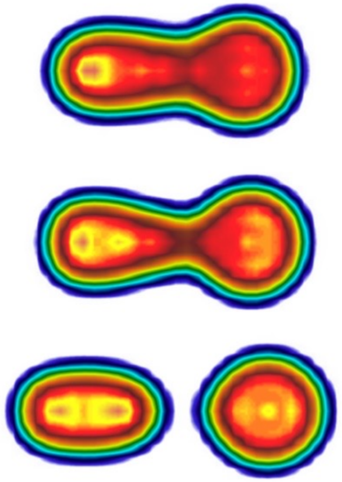


High-T_c Superconductors



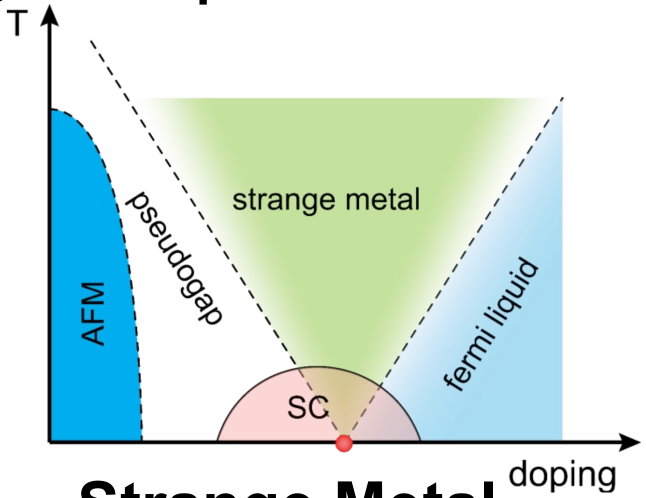
Neutron Star

White dwarf

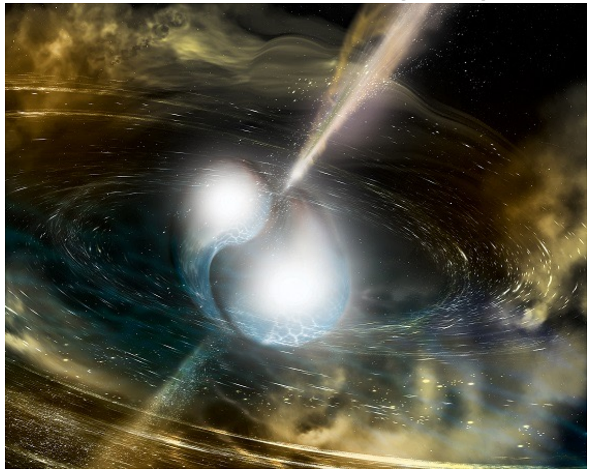


Nuclear Fission

Bulgac et al., PRL 116, 122504 (2016)



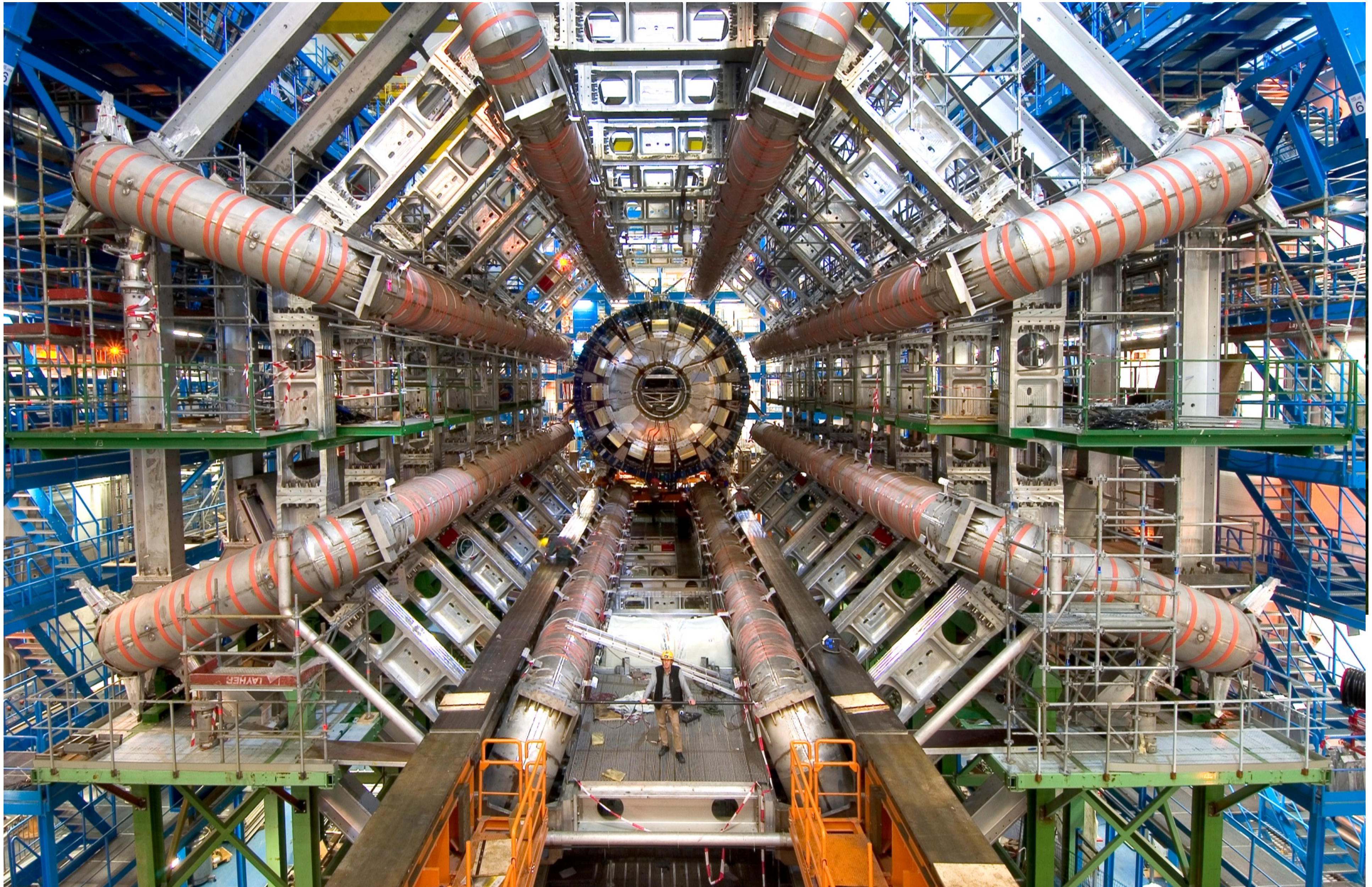
Strange Metal

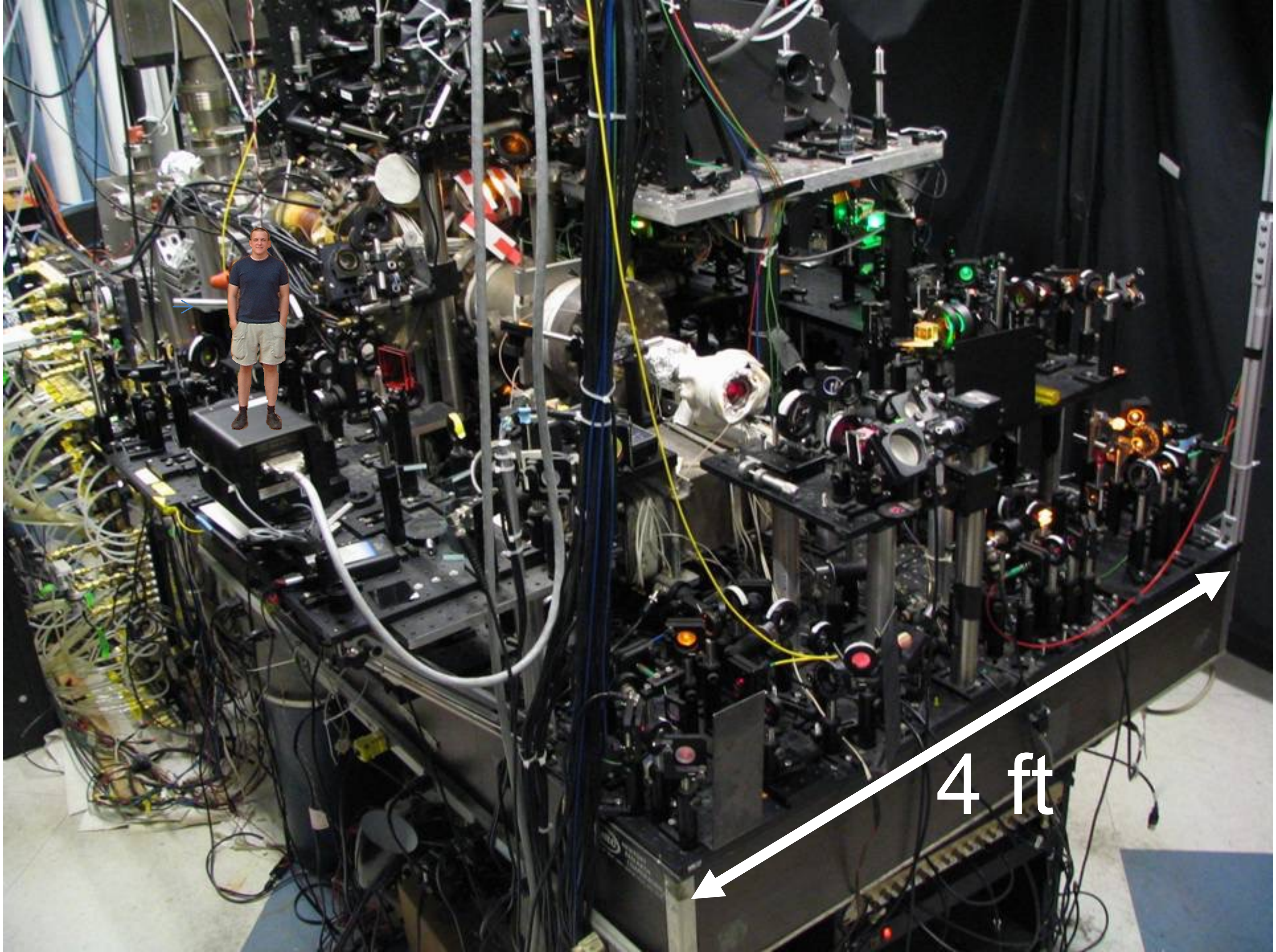


Neutron Star Merger

NSF/LIGO/Somona/A. Simonnet

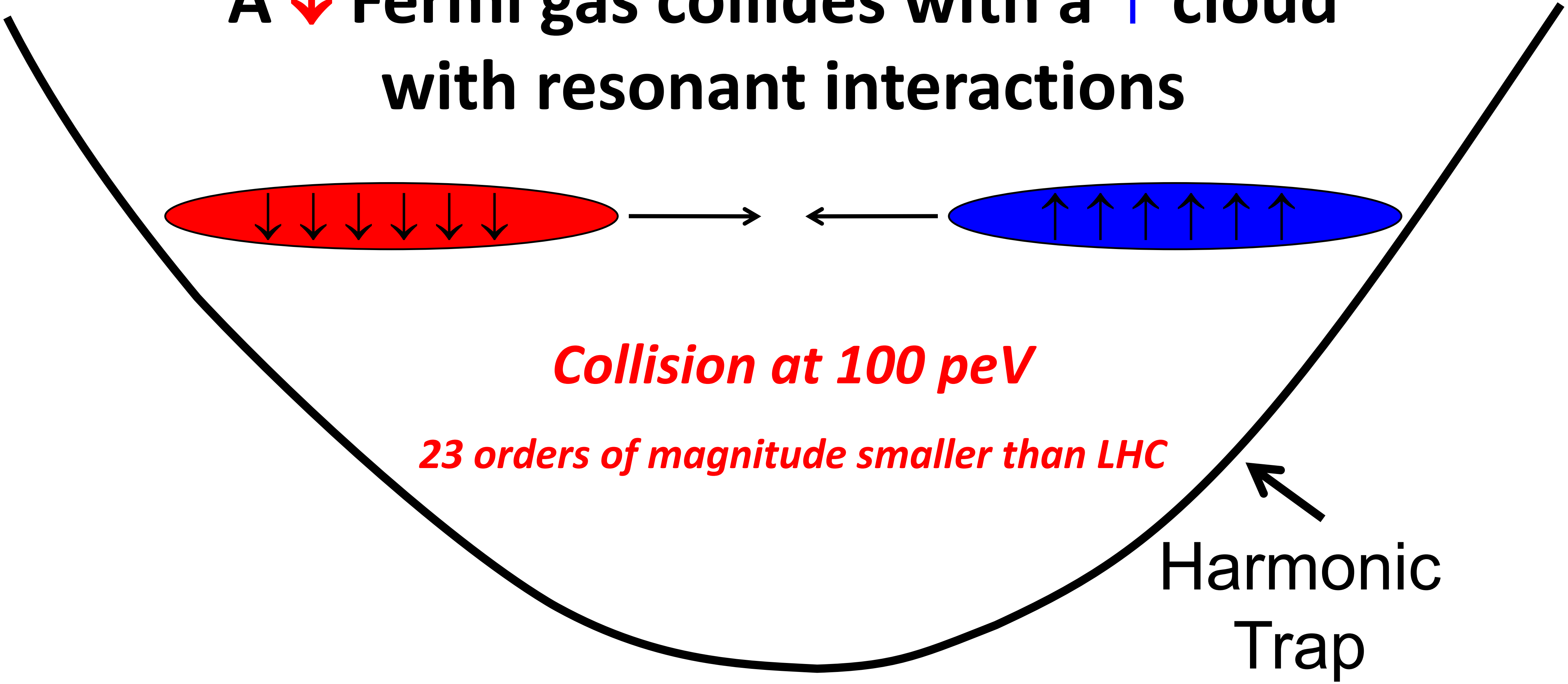
Large Hadron Collider (LHC)




4 ft

Little Fermi Collider (LFC)

A ↓ Fermi gas collides with a ↑ cloud with resonant interactions



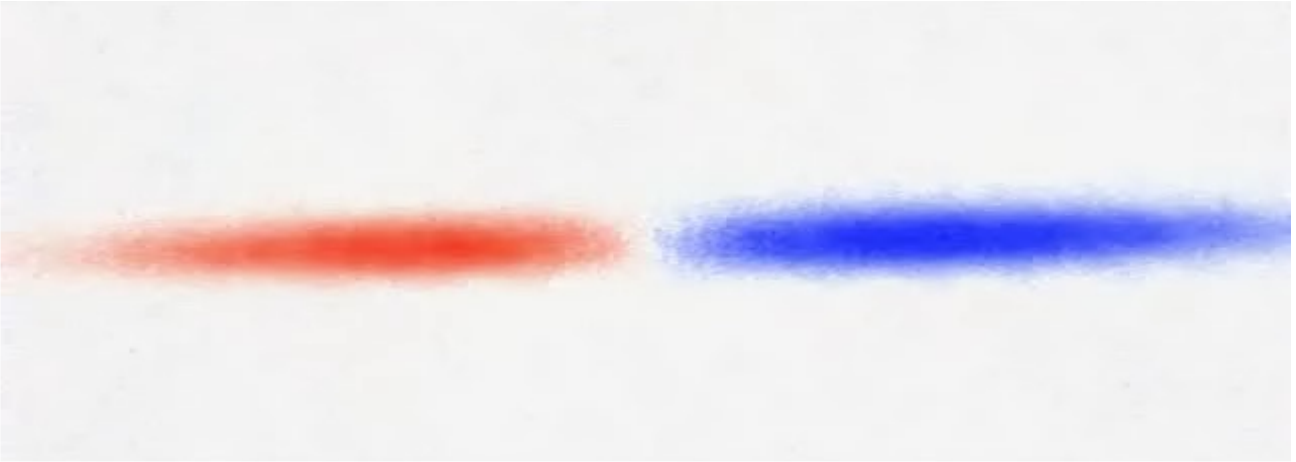
Collision at 100 peV

23 orders of magnitude smaller than LHC

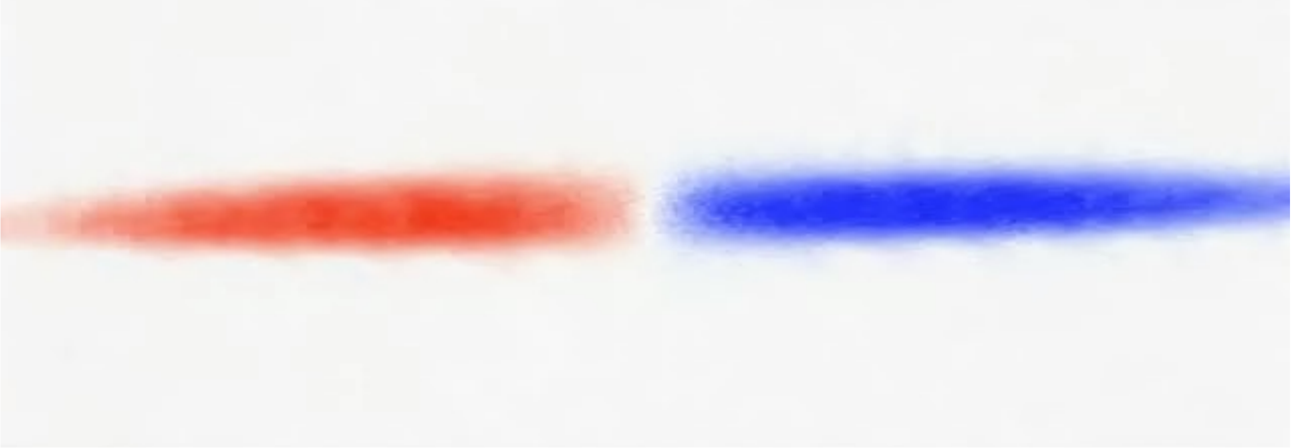
A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Little Fermi Collider (LFC)

Without Interactions



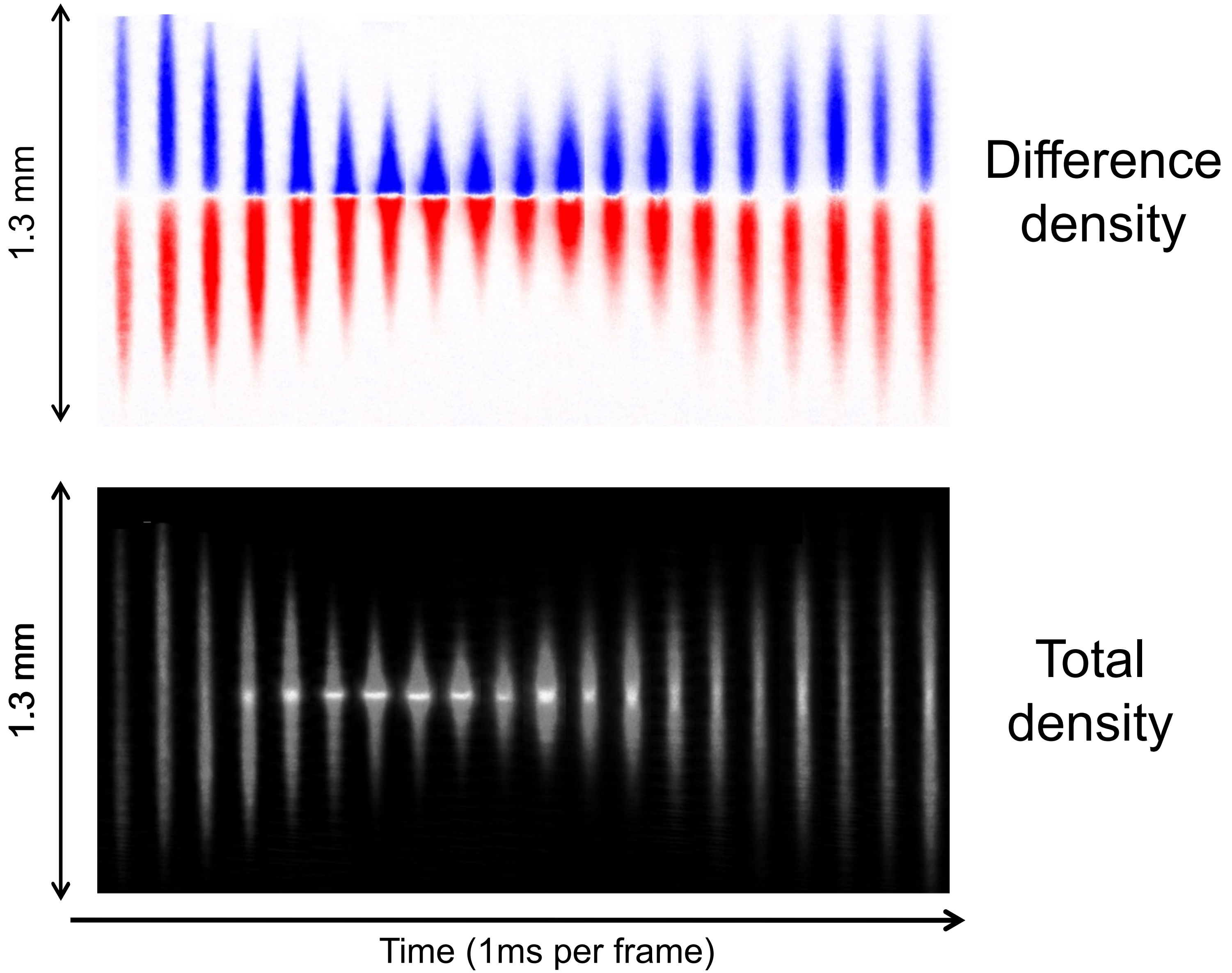
Resonant Interactions



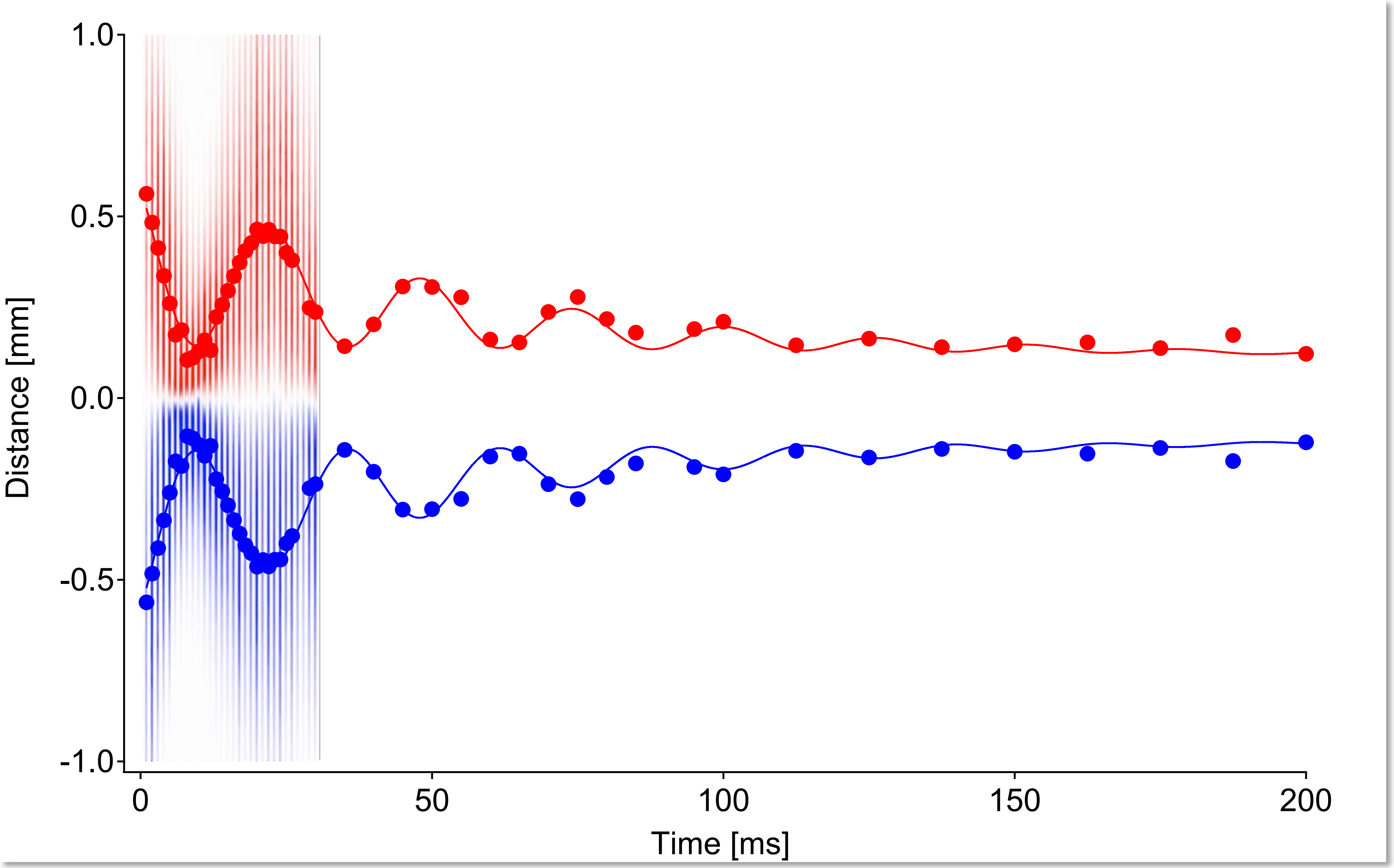
A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

The bouncing gas

First collision

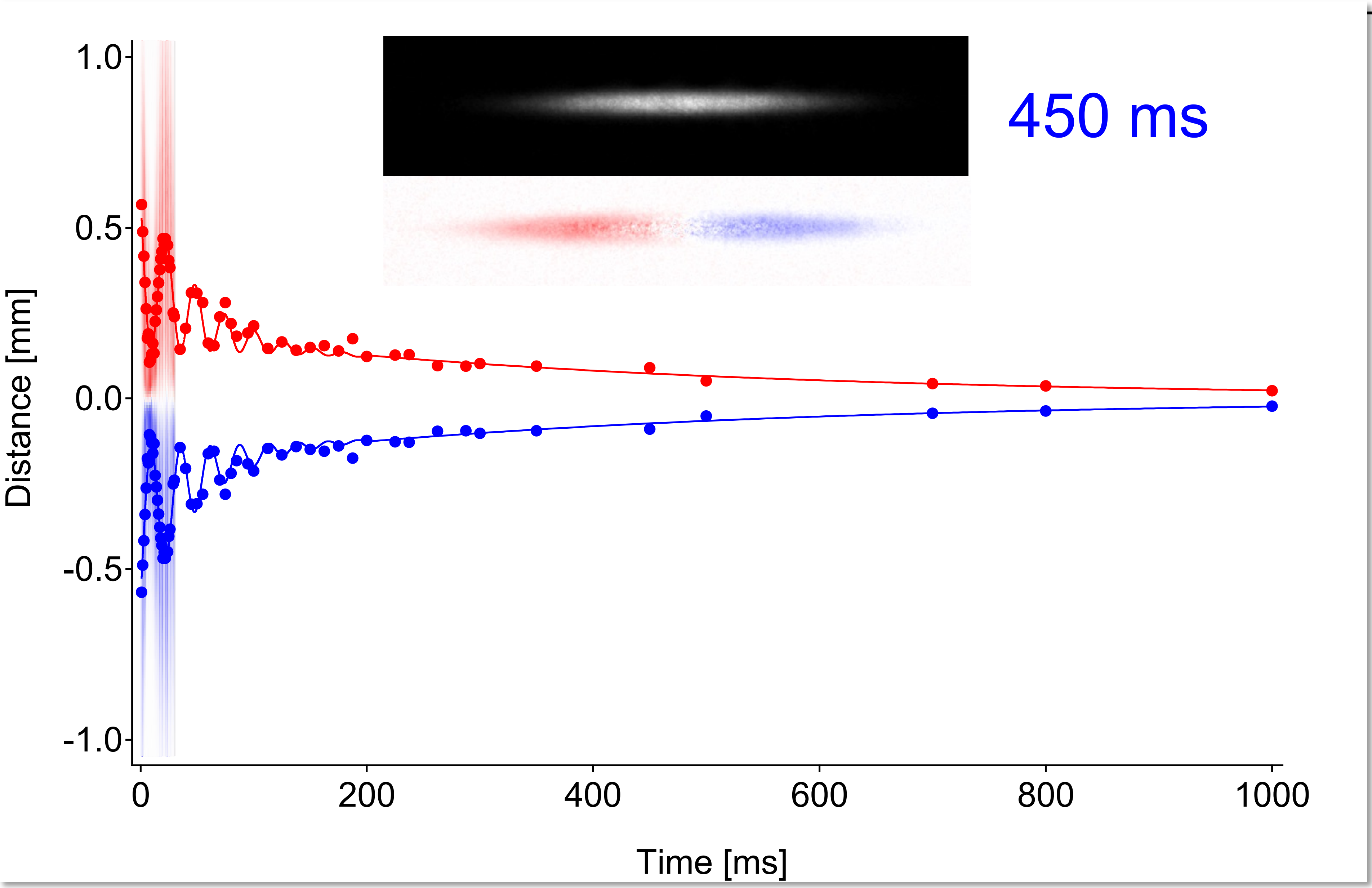


Later times



A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Much later times



A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Quantum limit of spin diffusion

Mean free path ~ Interparticle spacing d

Diffusion constant:

$$D \sim \text{mean free path} \times \text{average velocity}$$

$$\cancel{d} \times \frac{\hbar}{m\cancel{d}}$$

$$D \sim \frac{\hbar}{m} = \frac{\text{Planck's constant}}{\text{Particle mass}} = \frac{(0.1\ \text{mm})^2}{1\text{s}}$$

→ Quantum Limit of Diffusion

In a hot relativistic fluid (e.g. Quark-Gluon Plasma): $D \sim \frac{\hbar c^2}{T}$

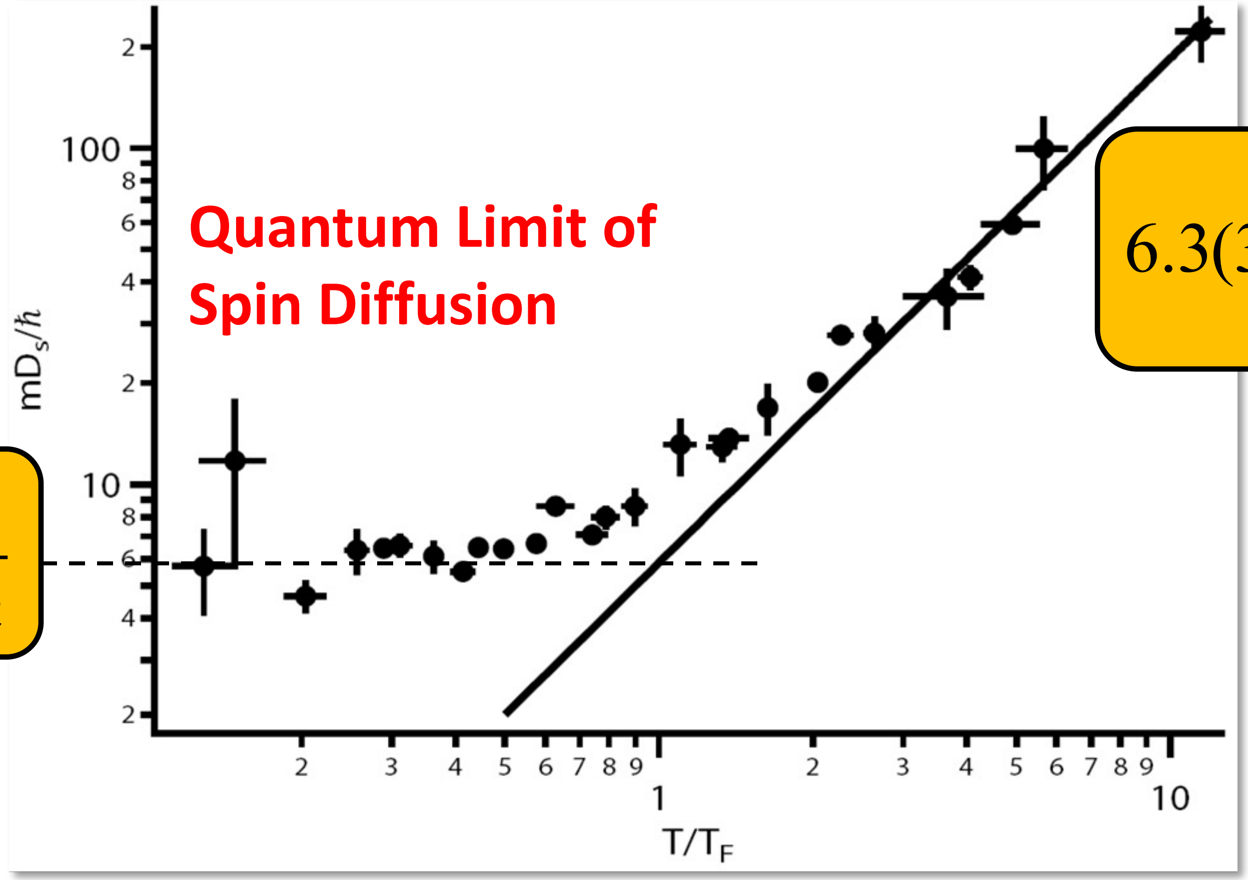
$$mc^2 \to T$$

A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Spin Diffusion vs Temperature

Spin current = -D $\cdot$ Spin density gradient

Universal high-T behavior:

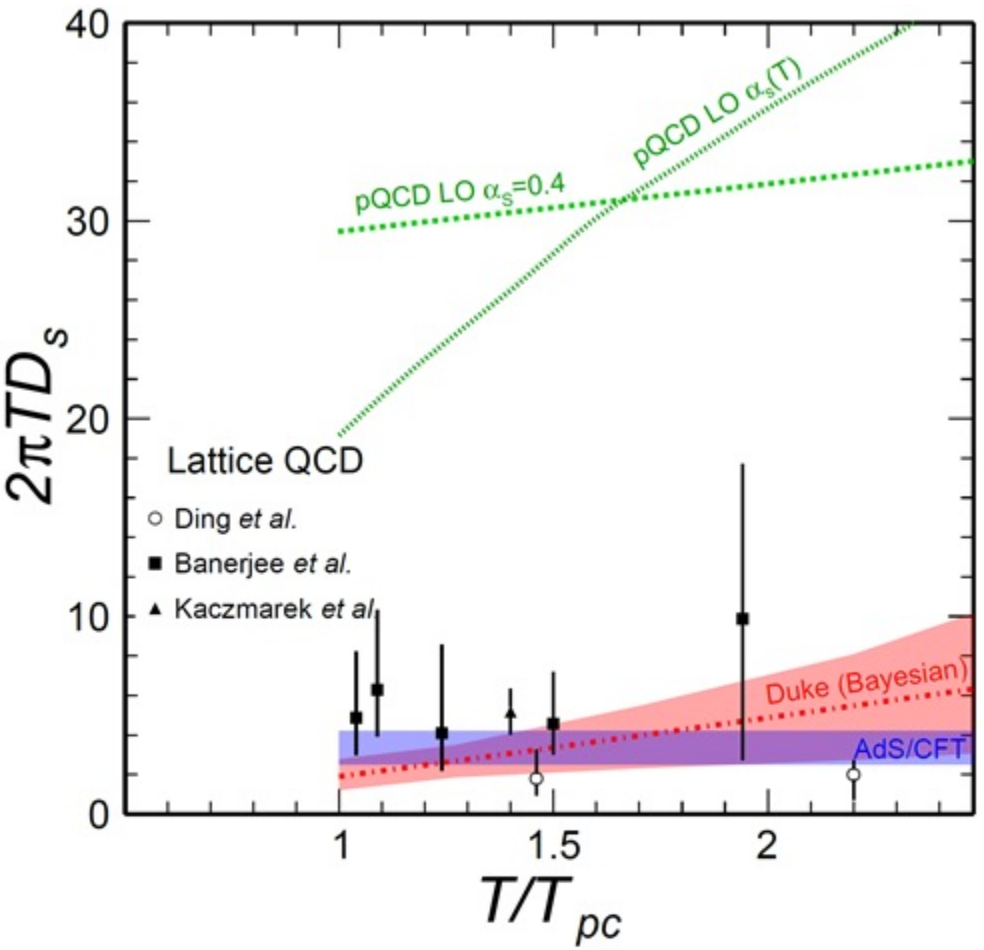


$$6.3(3)\,\frac{\hbar}{m}\left(\frac{T}{T_F}\right)^{3/2}$$

$$5.8\,\frac{\hbar}{m}$$

A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Diffusivity in the Quark-Gluon Plasma

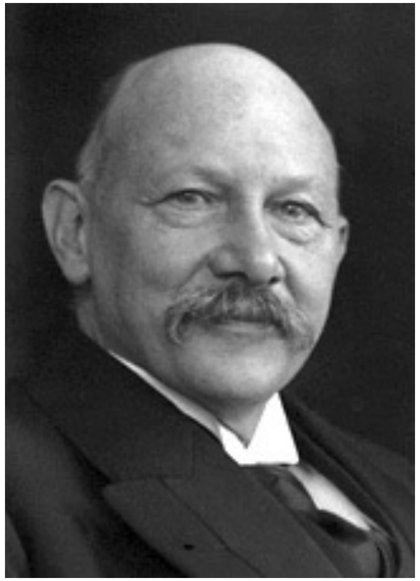


Can Fermi Gases become superfluid?

Superconductivity

Electrons are Fermions

Discovery of superconductivity 1911



Heike
Kamerlingh-Onnes
1911

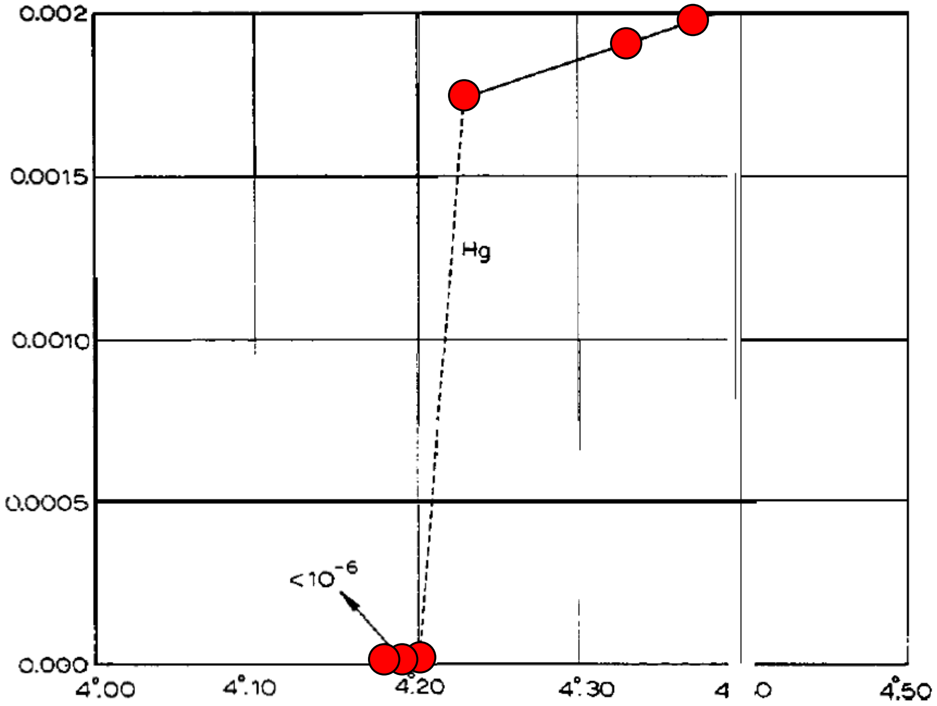


Fig. 17.

Fermionic Superfluidity

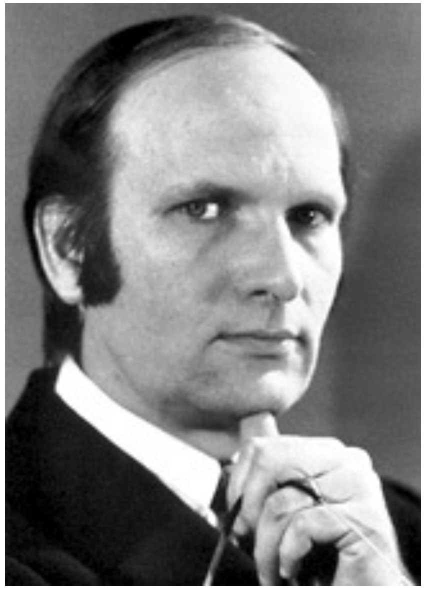
Condensation of Fermion Pairs

- Helium-3 (Lee, Osheroff, Richardson 1971)
- Superconductors: *Charged* superfluids of electron pairs

Frictionless flow ⇔ Resistance-less current

John Bardeen

Leon N. Cooper

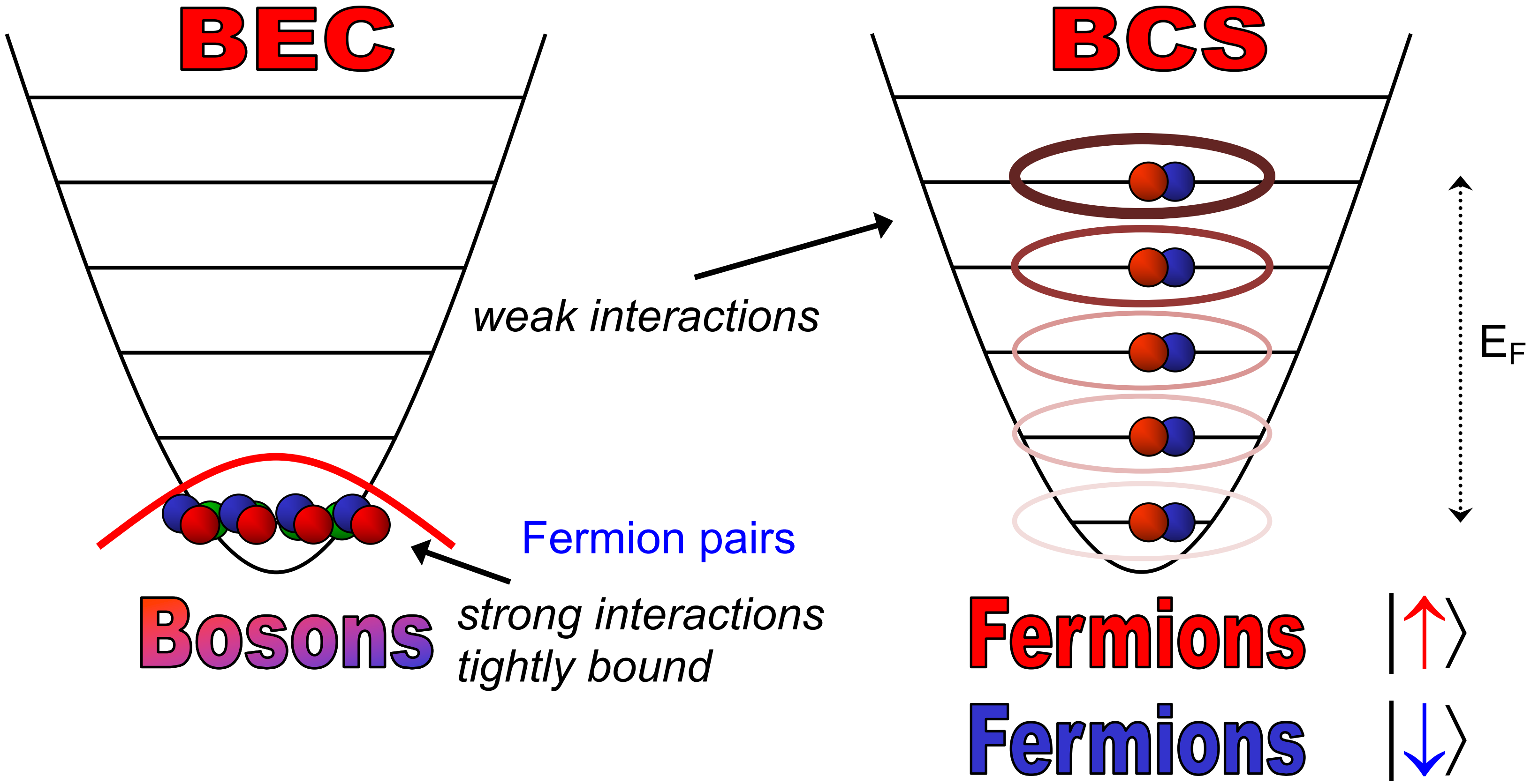


John R. Schrieffer

- Neutron stars

In the core: Quark superfluid

Bosons vs Fermions



e.g.: $e^- + p$, ^1H, ^{23}Na; $^6Li_2$ e.g.: $^{23}Na + e^-$, $^6Li + {}^6Li$

$$T_C \approx T_F$$

e.g.: e^-, ^3He, ^6Li, ^{40}K

$$T_C \approx T_F e^{-\pi/2k_F|a|}$$

Interatomic interactions

- Mean-field interaction energy: $E_{\mathrm{MF}} = \dfrac{4\pi\hbar^2 n a}{m}$
- Weak or strong interaction? $\dfrac{E_{\mathrm{MF}}}{E_F} \simeq k_F a$

$$k_F = (6\pi^2 n)^{1/3} \sim 1/2000a_0$$

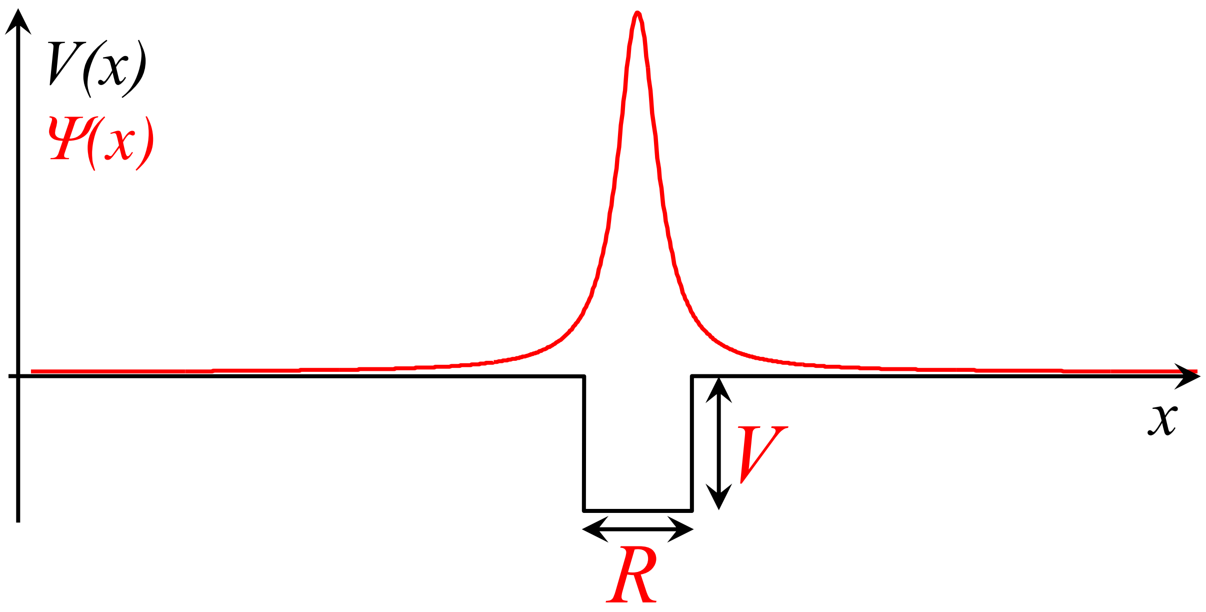
$$a \sim 50 - 100a_0$$

$$k_F a \lesssim 5\%$$

- Typically weak interaction
- Superconductors: Electron-Phonon interaction also weak: $\dfrac{\hbar\omega_D}{E_F} = \dfrac{100\ \mathrm{K}}{10\,000\ \mathrm{K}} \sim 1\%$

How can we have atom pairs with arbitrarily weak interaction?

Two particle bound states in 1D, 2D, 3D



Localizing a wavefunction to within R

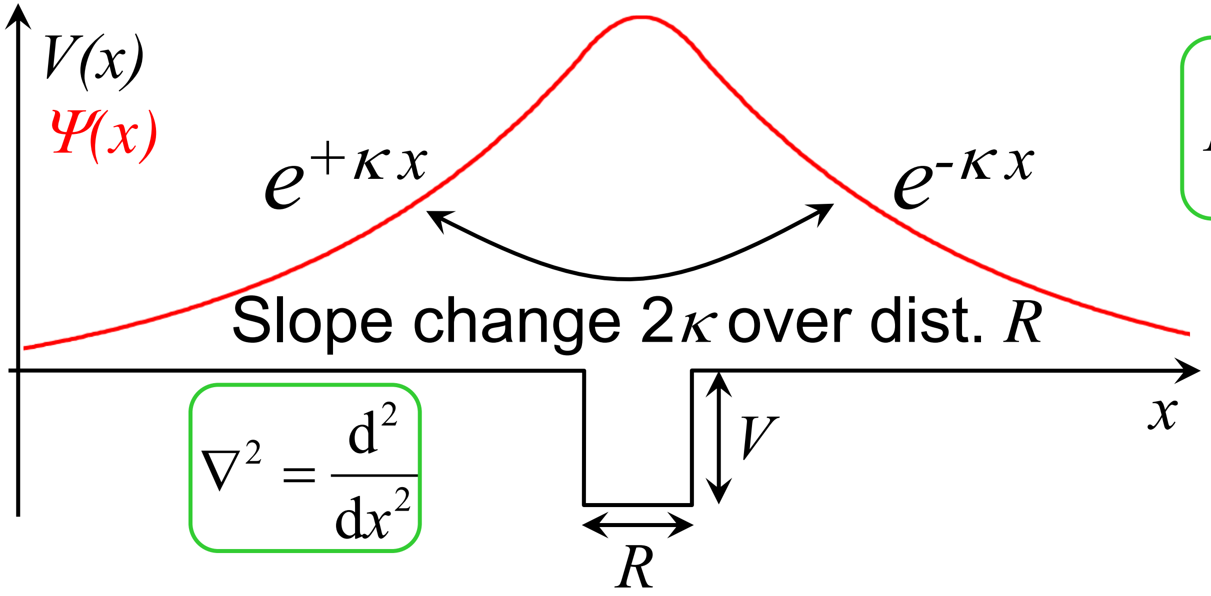
→ momentum uncertainty $\Delta p = \frac{\hbar}{R}$

→ kinetic energy cost: $E_{Kin} = \frac{\hbar^2}{mR^2} \equiv E_R$

Requires potential energy $V \approx \frac{\hbar^2}{mR^2} = E_R$

Two particle bound states in 1D, 2D, 3D

1D:



$$E_B = -\frac{\hbar^2\kappa^2}{2m}$$

$$\nabla^2 = \frac{d^2}{dx^2}$$

Question: ***Is there a bound state for arbitrarily small V ?***

Kinetic energy cost: E_{kin} = Curvature of $\Psi \sim \kappa/R$

Potential energy $V \approx E_{\mathrm{kin}}$, hence: $\kappa \sim V$

Answer: ***Yes!***

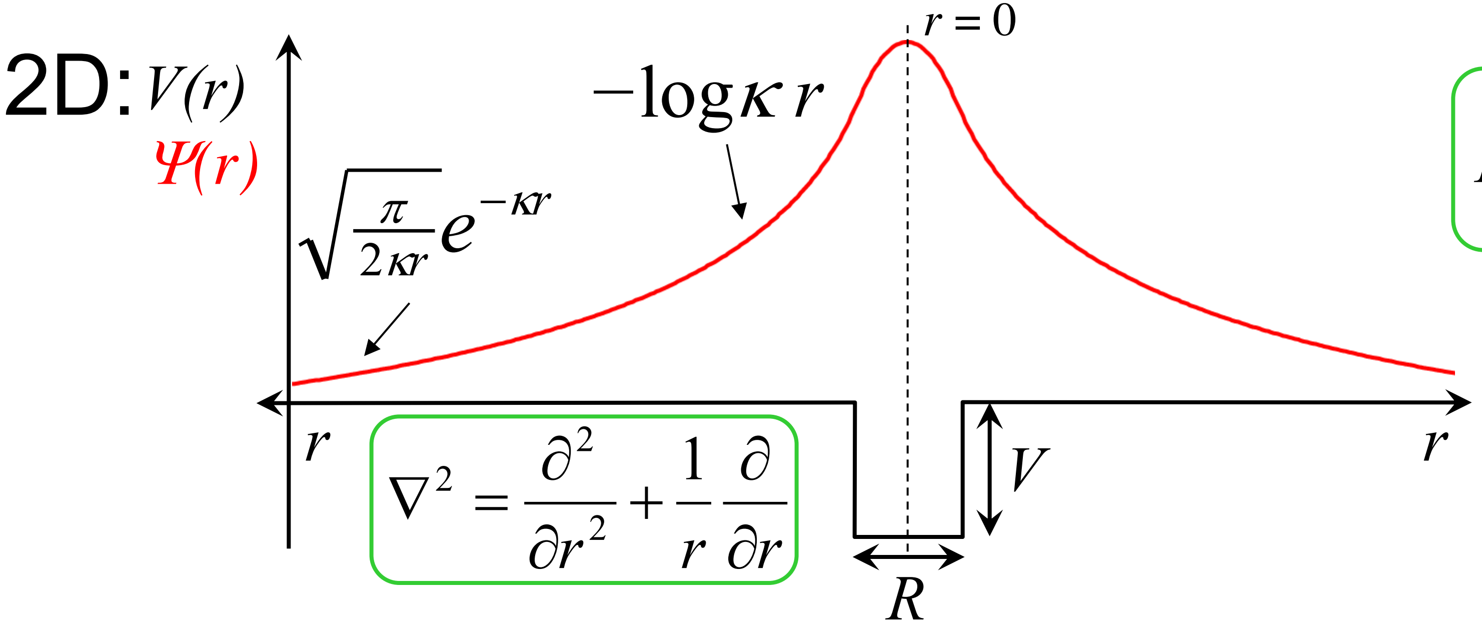
Bound state energy:

$$\boldsymbol{E_B \approx -V^2/E_R}$$

quadratic in V

Particle localized to within $R\,E_R/V$

Two particle bound states in 1D, 2D, 3D



$$E_B = -\frac{\hbar^2\kappa^2}{2m}$$

Question: ***Is there a bound state for arbitrarily small V ?***

Kinetic energy cost: $E_{\mathrm{kin}} = E_R/|\log(\kappa R)|$

Potential energy $V \approx E_{\mathrm{kin}}$, hence: $\kappa = \frac{1}{R}e^{-cE_R/V}$

Answer: ***Yes!***

Bound state energy:

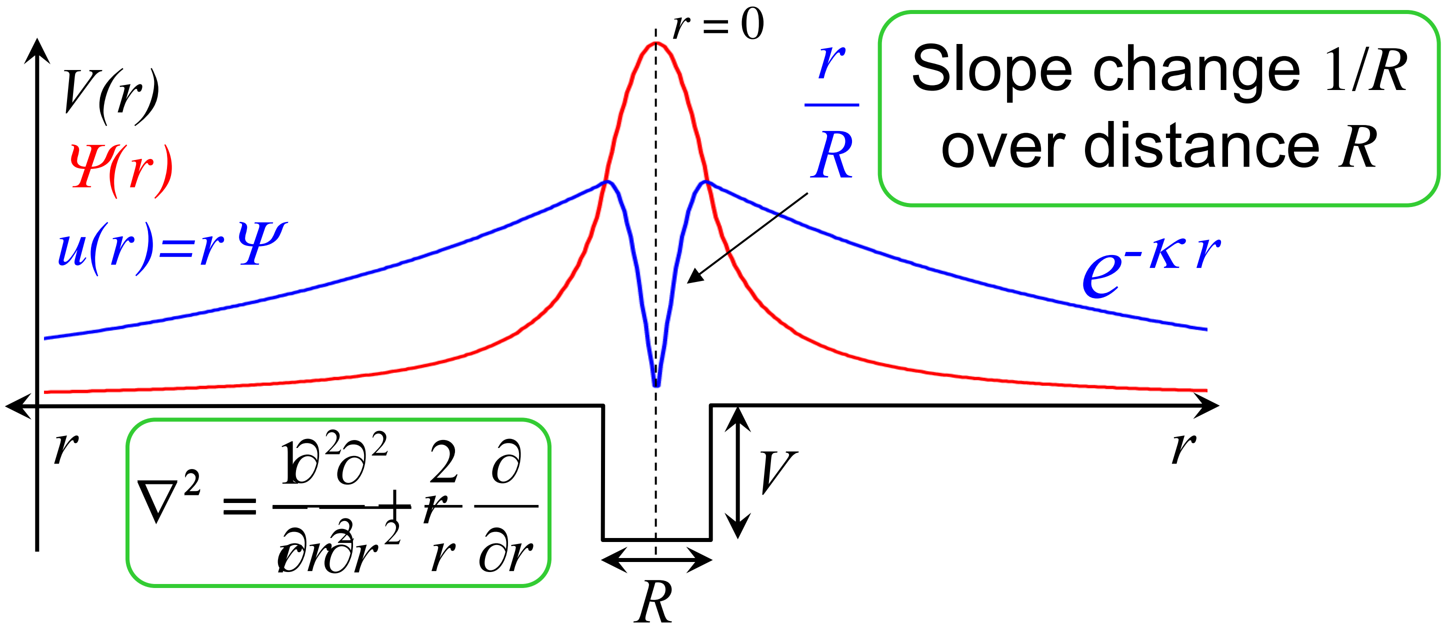
$$E_B = E_R e^{-2cE_R/V}$$

exponential in 1/V

Particle localized to within $R\, e^{cE_R/V}$

Two particle bound states in 1D, 2D, 3D

3D:



Question: ***Is there a bound state for arbitrarily small V ?***

Kinetic energy cost: E_{kin} = Curvature of u = $\boldsymbol{E_R}$

Potential energy $V \approx E_{\mathrm{kin}}$ must be at least ~E_R

Answer: ***No!***

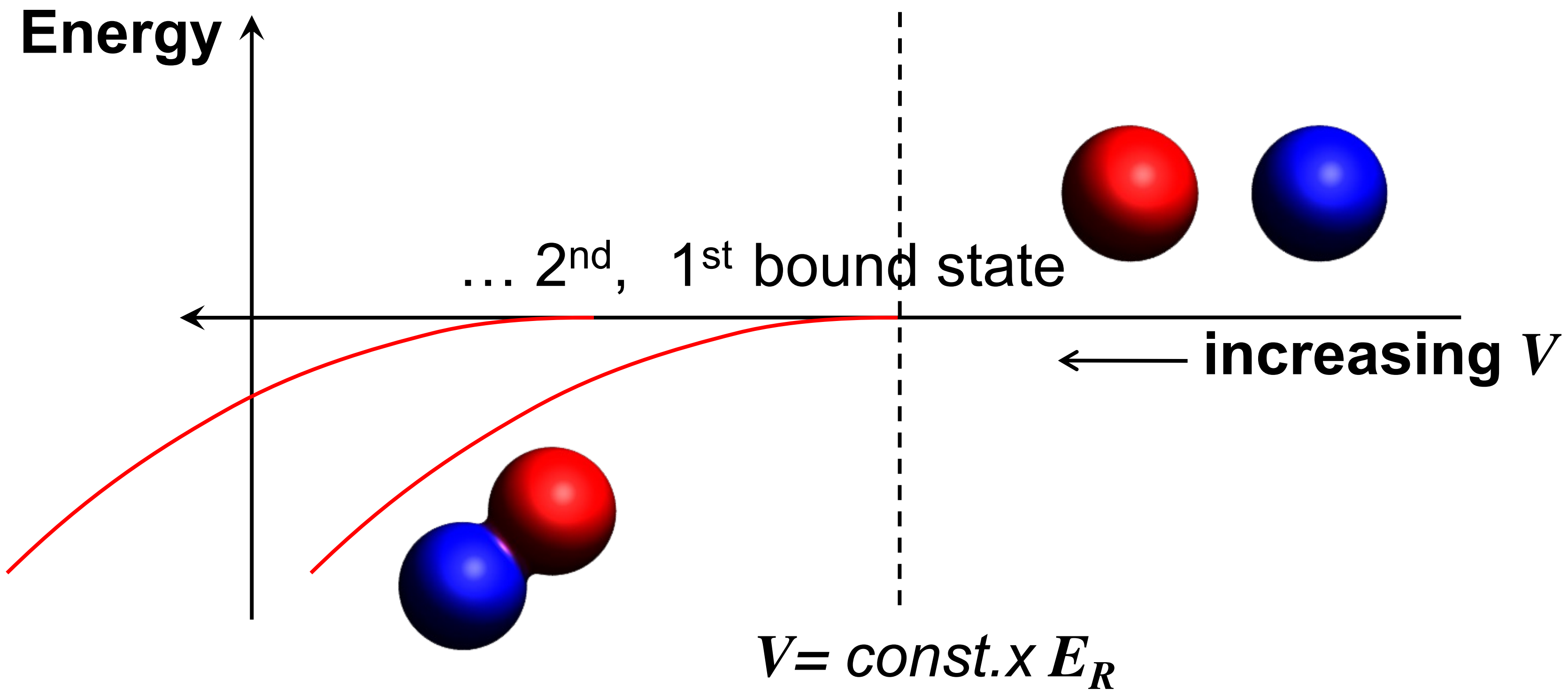
Bound state energy:

$$\boldsymbol{E_B \approx -\,(V\text{-}V_{th})^2\,/E_R}$$

Particle localized to within $R/(V\text{-}V_{th})$

quadratic in $V\text{-}V_{th}$

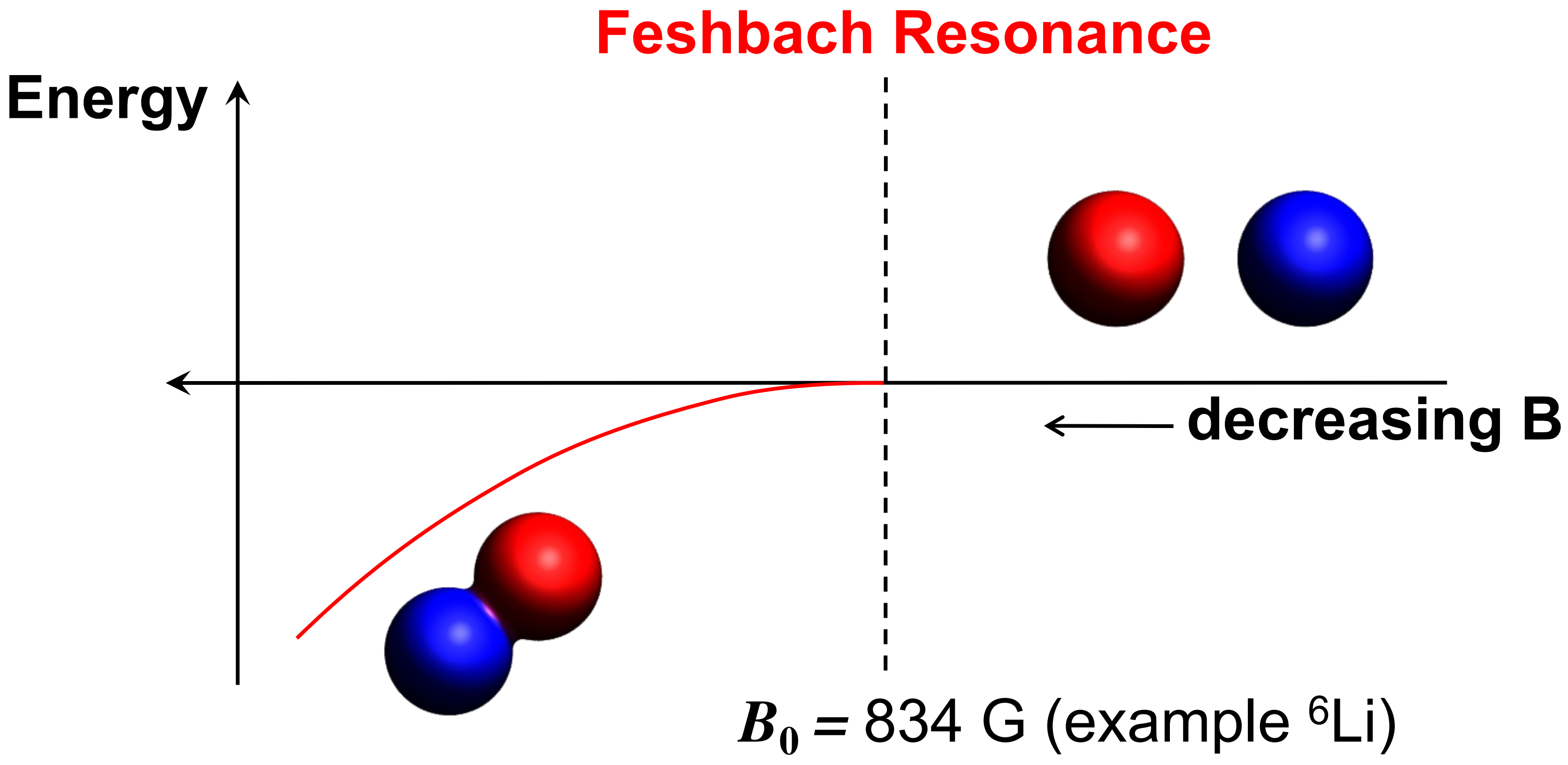
Two-particle bound states in 3D



Atoms form stable molecules

Isolated atom pairs are unstable

Two-particle bound states in 3D



Atoms form stable molecules

Isolated atom pairs are unstable

Relation to the density of states

Schrödinger equation for two interacting particles:

$$\Big(-\frac{\hbar^2}{\imath}\frac{\hbar^2}{m}(\nabla^2-\kappa^2)\psi(\mathbf{r}) \quad = \quad V(\mathbf{r})\psi(\mathbf{r})$$

In momentum space:

$$\Big(E=-\frac{\hbar^2\kappa^2}{m}\Big)$$

$$\frac{\hbar^2}{m}(-q^2-\kappa^2)\psi(\mathbf{q}) \quad = \quad -\frac{m}{\hbar^2}\frac{1}{q^2+\kappa^2}\int\frac{d^n q'}{(2\pi)^n}V(\mathbf{q}-\mathbf{q}')\psi(\mathbf{q}')$$

Short range potential: $\big(R \ll 1/\kappa\big)$ $\quad V(\mathbf{q}) = -VR^n \text{ for } q<1/R$

$$\frac{1}{VR^n} = \int_{q\lesssim\frac{1}{R}}\frac{d^n q}{(2\pi)^n}\quad\frac{m}{\hbar^2}\frac{1}{q^2+\kappa^2}$$

Integrate over $\boldsymbol{q}$, divide by common factor $\int_{q'\lesssim\frac{1}{R}}\frac{d^n q'}{(2\pi)^n}\psi(\mathbf{q}')$, introduce

$$\boxed{\frac{\Omega}{VR^n} = \int_{\epsilon<E_R} d\epsilon\,\frac{\rho_n(\epsilon)}{2\epsilon+|E|}}$$

$$\varepsilon=\frac{\hbar^2 q^2}{2m}$$

$\rho_n(\varepsilon)$ – d.o.s.

Ω – Volume

Connection with the density of states

Bound state equation:

$$\frac{\Omega}{V R^n} = \int_{\epsilon < E_R} d\epsilon \, \frac{\rho_n(\epsilon)}{2\epsilon + |E|}$$

Note: This is simply $\left\langle \frac{1}{\text{potential energy}} \right\rangle = \left\langle \frac{1}{\text{kinetic energy}} \right\rangle$

Left hand side $\to \infty$ for $|V| \to 0$

Right hand side $\to \infty$ only if integral diverges for $E \to 0$

1D:	$\rho_1(\epsilon) \sim \frac{1}{\sqrt{\epsilon}}$	$\frac{1}{V} \sim \frac{1}{\sqrt{\lvert E\rvert}}$	$E_{1D} \sim -V^2/E_R$	There is always a bound state
2D:	$\rho_2(\epsilon) \sim \text{const.}$	$\frac{1}{V} \sim \log(\lvert E\rvert)$	$E_{2D} \sim -2E_R\, e^{-\frac{2\Omega/R^2}{[illegible] V}}$	Logarithmic divergence, exponentially weakly bound state
3D:	$\rho_3(\epsilon) \sim \sqrt{\epsilon}$	$\frac{1}{V} [illegible] \frac{1}{E_R}\left([illegible] \sqrt{\frac{\lvert E\rvert}{E_R}}\right)$	$E_{3D} \sim -(V - V_{\text{th}})^2/E_R$	No bound state if interaction is too weak

Cooper Pairing

- In 3D and for too weak an attraction, there is no bound state for two isolated particles
- But: For a pair condensate we need pairing…
- Idea: The presence of nearby atoms might modify the density of states! This could lead to pairing!

Cooper Pairing

Bound Electron Pairs in a Degenerate Fermi Gas*

LEON N. COOPER

Physics Department, University of Illinois, Urbana, Illinois

(Received September 21, 1956)

Cooper Pairing

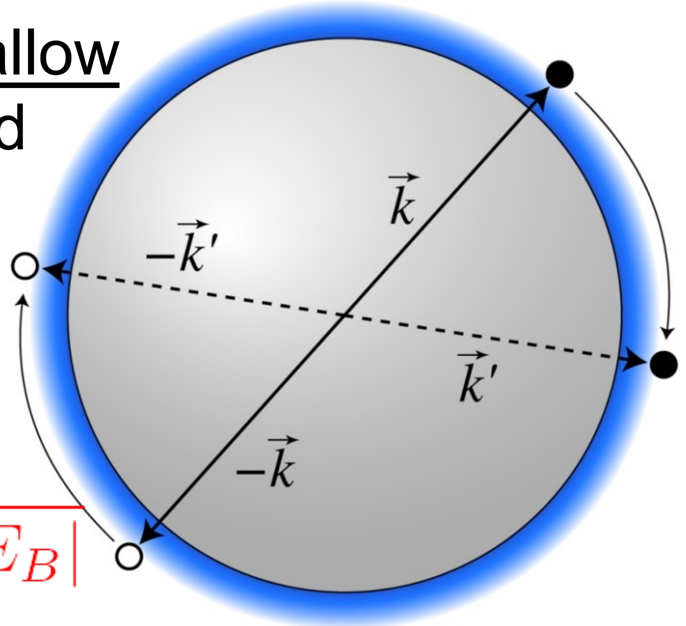
Consider two particles ↑, ↓, on top of a filled, "inert" Fermi sea

- Proceed exactly as before, but now disallow momenta below k_F that are Pauli blocked
- Search for a small binding energy

$$E_B = E - 2E_F < 0$$

- Bound-state equation:

$$\frac{\Omega}{VR^3} = \int_{E_F<\epsilon<E_F+E_R} d\epsilon \frac{\rho_{3\mathrm{D}}(\epsilon)}{2(\epsilon - E_F) + |E_B|}$$



- For superconductors: $E_R = \hbar\omega_{Debye} << E_F$ scattering confined to Fermi surface → a constant like in 2D! → Bound state!

$$\rho_{3D}(\varepsilon) \approx \rho_{3D}(E_F) = \text{const.}$$

$$E_B = -2\hbar\omega_D e^{-\frac{2\Omega R^3}{\rho_{3D}(E_F)V}}$$

The Fermi sea is unstable towards pairing

Cooper pairing in atomic gases

- For atomic gases: $E_R = \frac{\hbar^2}{mR^2} >> E_F$, integral diverges as $E_R \to \infty$
- This divergence, clearly due to short range physics, is already encountered for two-particle scattering
- Low-energy particles will never probe the high-energy scale of *V*, the well depth. The relevant physical parameter is the scattering phase shift / the scattering length *a*

→ via renormalization one can replace *V* with scattering length a

$$\frac{\Omega}{VR^3} = \frac{m\Omega}{4\pi\hbar^2 a} - \int d\epsilon \rho_3(\epsilon) \frac{1}{2\epsilon} \qquad \left(VR^3 \to \frac{4\pi\hbar^2}{m} a \right)$$

- Intuitively, as all energies are < E_F, the Fermi energy itself will provide the natural cut-off.
- Bound state:

$$E_B = -\frac{8}{e^2} E_F e^{-\frac{\pi}{k_F |a|}}$$

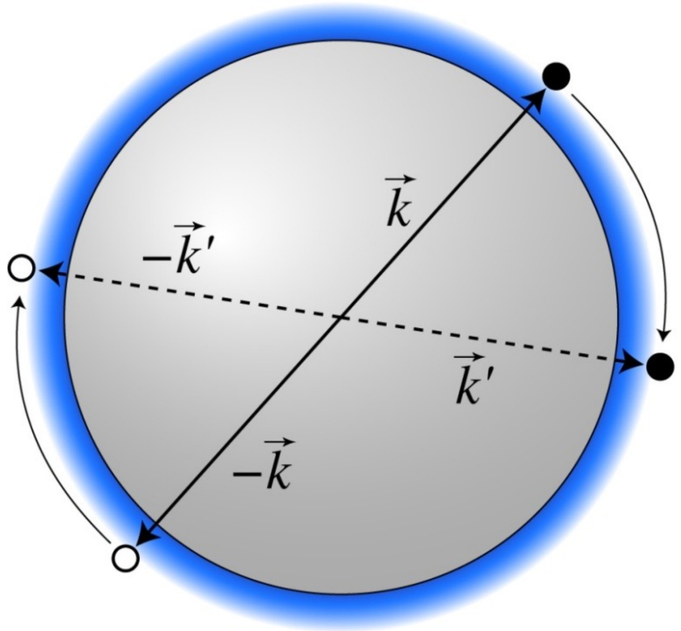
Typical: |a| ~ 5 nm
$1/k_F$ ~ 100 nm

$$\approx E_F 10^{-28}$$

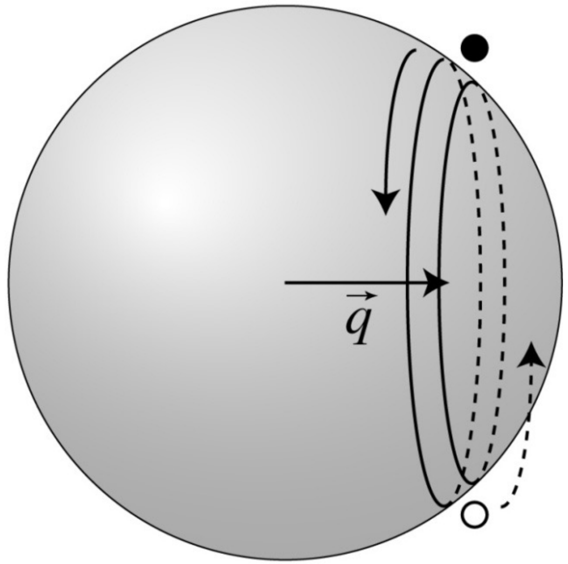
VERY small energy

Cooper Pairing

Consider two particles ↑, ↓, on top of a filled, "inert" Fermi sea



Total momentum zero



Total momentum non-zero

- Reduced density of states
- Much smaller binding energy

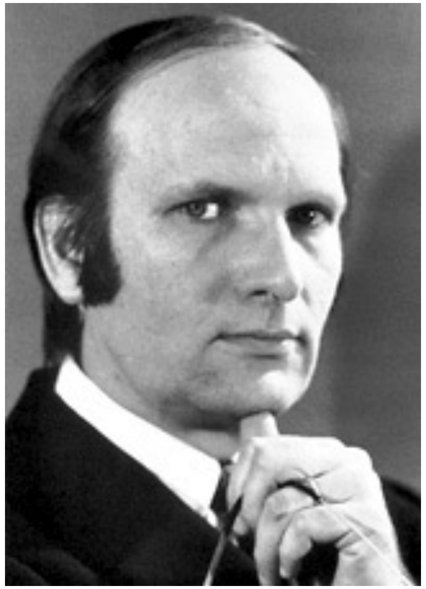
The important pairs are those with zero momentum

BCS Wavefunction

How can we find a state in which all fermions are paired in a self-consistent way?

John Bardeen

Leon N. Cooper



John R. Schrieffer

BCS Wavefunction

- Many-body wavefunction for a condensate of Fermion Pairs:

$$\Psi\left(\mathbf{r}_1,\ldots,\mathbf{r}_N\right) = \varphi(|\mathbf{r}_1-\mathbf{r}_2|)\chi_{12}\ldots\varphi(|\mathbf{r}_{N-1}-\mathbf{r}_N|)\chi_{N-1,N}$$

Spatial pair wavefunction

Spin wavefunction

$$\chi_{ij} = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_i\left|\downarrow\right\rangle_j - \left|\downarrow\right\rangle_i\left|\uparrow\right\rangle_j\right)$$

- Second quantization:

$$|\Psi\rangle_N = \int\prod_i d^3r_i\,\varphi(\mathbf{r}_1-\mathbf{r}_2)\Psi_\uparrow^\dagger(\mathbf{r}_1)\Psi_\downarrow^\dagger(\mathbf{r}_2)\ldots\varphi(\mathbf{r}_{N-1}-\mathbf{r}_N)\Psi_\uparrow^\dagger(\mathbf{r}_{N-1})\Psi_\downarrow^\dagger(\mathbf{r}_N)\,|0\rangle$$

- Fourier transform: Pair wavefunction: $\varphi(\mathbf{r}) = \sum_k \varphi_k \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$

  Operators: $\Psi_\sigma^\dagger(\mathbf{r}) = \sum_k c_{k\sigma}^\dagger \frac{e^{-i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$

- Pair creation operator: $b^\dagger = \sum \varphi_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger$

- Many-body wavefunction: $|\Psi\rangle_N = b^{\dagger\,N/2}\,|0\rangle$

  a fermion pair condensate

$|\Psi\rangle_N$ is not a Bose condensate

$$|\Psi\rangle_N = {b^\dagger}^{N/2}|0\rangle$$

- Commutation relations for pair creation/annihilation operators

$$[b^\dagger, b^\dagger]_- = \sum_{kk'} \varphi_k \varphi_{k'} \left[c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}, c^\dagger_{k'\uparrow} c^\dagger_{-k'\downarrow}\right]_- = 0 \quad ✓$$

$$[b, b]_- = \cdots = 0 \quad ✓$$

$$[b, b^\dagger]_- = \cdots = \sum_k |\varphi_k|^2 (1 - n_{k\uparrow} - n_{k\downarrow}) \neq 1 \quad ✘$$

Occupation of momentum k

- pairs do not obey Bose commutation relations, *unless* $n_k \ll 1$

$$[b, b^\dagger]_- \approx \sum_k |\varphi_k|^2 = 1$$

BEC limit of tightly bound molecules

BCS Wavefunction

- Introduce coherent state / switch to grand-canonical description:

$$\mathcal{N}\,|\Psi\rangle = \sum_{J_{\text{even}}} \frac{N_p^{J/4}}{(J/2)!}\,|\Psi\rangle_J = \sum_M \frac{1}{M!} N_p^{M/2}\, b^{\dagger\,M}\,|0\rangle$$

$$= e^{\sqrt{N_p}\, b^\dagger}\,|0\rangle$$

$$= \prod_k e^{\sqrt{N_p}\,\varphi_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}}\,|0\rangle \qquad c^\dagger_k \text{ and } c^\dagger_{k'} \text{ commute}$$

$$= \prod_k (1 + \sqrt{N_p}\,\varphi_k\, c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})\,|0\rangle \quad \text{because } c_k^{\dagger 2} = 0$$

- Normalization: $\mathcal{N} = \prod_k \frac{1}{u_k} = \prod_k \sqrt{1 + N_p|\varphi_k|^2}$

- BCS wavefunction: $\boxed{|\Psi_{\text{BCS}}\rangle = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})\,|0\rangle}$

with $v_k = \sqrt{N_p}\,\varphi_k u_k$ and $|u_k|^2 + |v_k|^2 = 1$

Many-Body Hamiltonian

- Second quantized Hamiltonian for interacting fermions:

$$\hat{H} = \sum_\sigma \int \mathrm{d}^3 r \hat{\Psi}^\dagger_\sigma(\mathbf{r}) \left(-\frac{\hbar^2\nabla^2}{2m}\right) \hat{\Psi}_\sigma(\mathbf{r}) + \int \mathrm{d}^3 r \int \mathrm{d}^3 r' \hat{\Psi}^\dagger_\uparrow(\mathbf{r}) \hat{\Psi}^\dagger_\downarrow(\mathbf{r}') V(\mathbf{r}-\mathbf{r}') \hat{\Psi}_\downarrow(\mathbf{r}') \hat{\Psi}_\uparrow(\mathbf{r})$$

- Contact interaction: $V(\mathbf{r}) = V_0 \delta(\mathbf{r})$
- Fourier transform via $\Psi^\dagger_\sigma(\mathbf{r}) = \sum_k c^\dagger_{k\sigma} \frac{e^{-i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}$

$$\hat{H} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{V_0}{\Omega} \sum_{k,k',q} c^\dagger_{k+\frac{q}{2}\uparrow} c^\dagger_{-k+\frac{q}{2}\downarrow} c_{k'+\frac{q}{2}\downarrow} c_{-k'+\frac{q}{2}\uparrow}$$

- BCS Approximation:

  Only include scattering between zero-momentum pairs

$$\hat{H} = \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{V_0}{\Omega} \sum_{k,k'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{k'\downarrow} c_{-k'\uparrow}$$

- Solve via 1) Variational Ansatz, 2) via Bogoliubov transformation

Variational Ansatz:

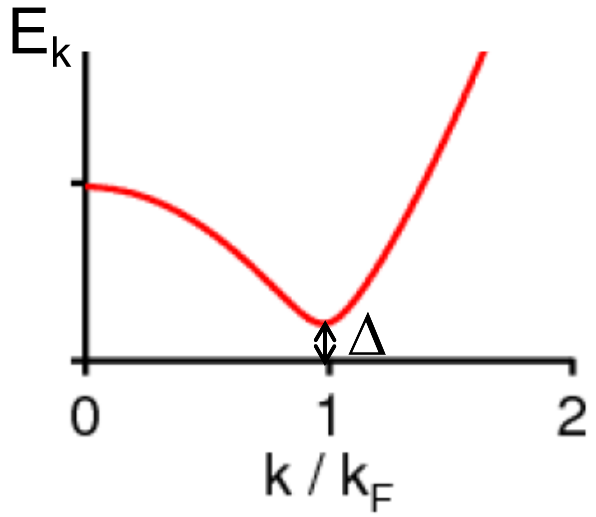
- Insert BCS wavefunction into Many-Body Hamiltonian.
- Minimize Free Energy:

$$\mathcal{F} = \left\langle \hat{H} - \mu\hat{N} \right\rangle = \sum_k 2\xi_k v_k^2 + \frac{V_0}{\Omega}\sum_{k,k'} u_k v_k u_{k'} v_{k'}$$

$$\text{with} \;\; \xi_k = \epsilon_k - \mu$$

- Result:

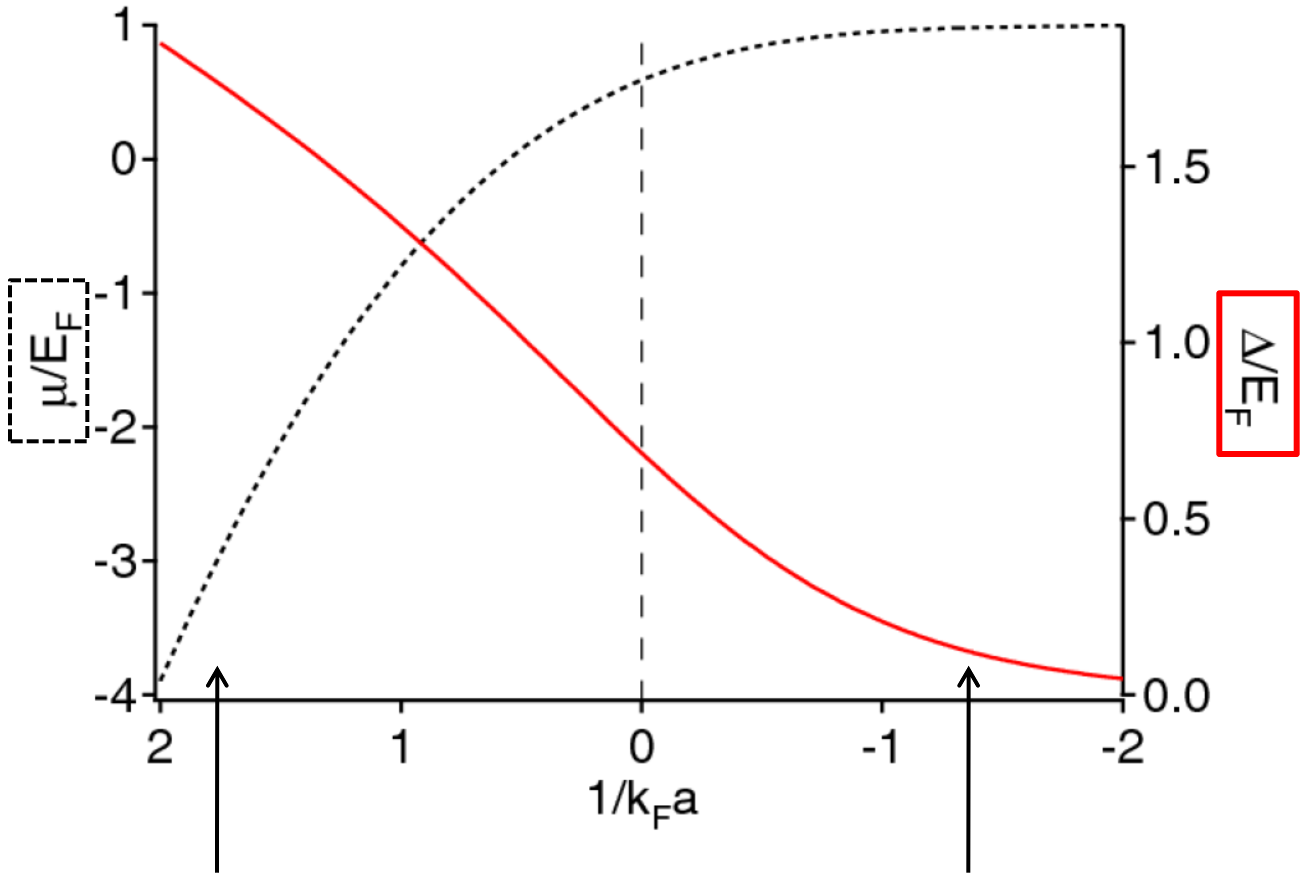
$$\begin{aligned} v_k^2 &= \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) \\ u_k^2 &= \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) \\ \text{with} \;\; E_k &= \sqrt{\xi_k^2 + \Delta^2} \end{aligned}$$



- Gap equation:

$$\Delta = -\frac{V_0}{\Omega}\sum_k u_k v_k = -\frac{V_0}{\Omega}\sum_k \frac{\Delta}{2E_k}$$

Solution of the gap equation



BEC-side: Molecules

$$\mu \approx -\frac{\hbar^2}{2ma^2} = \frac{1}{2}E_B$$

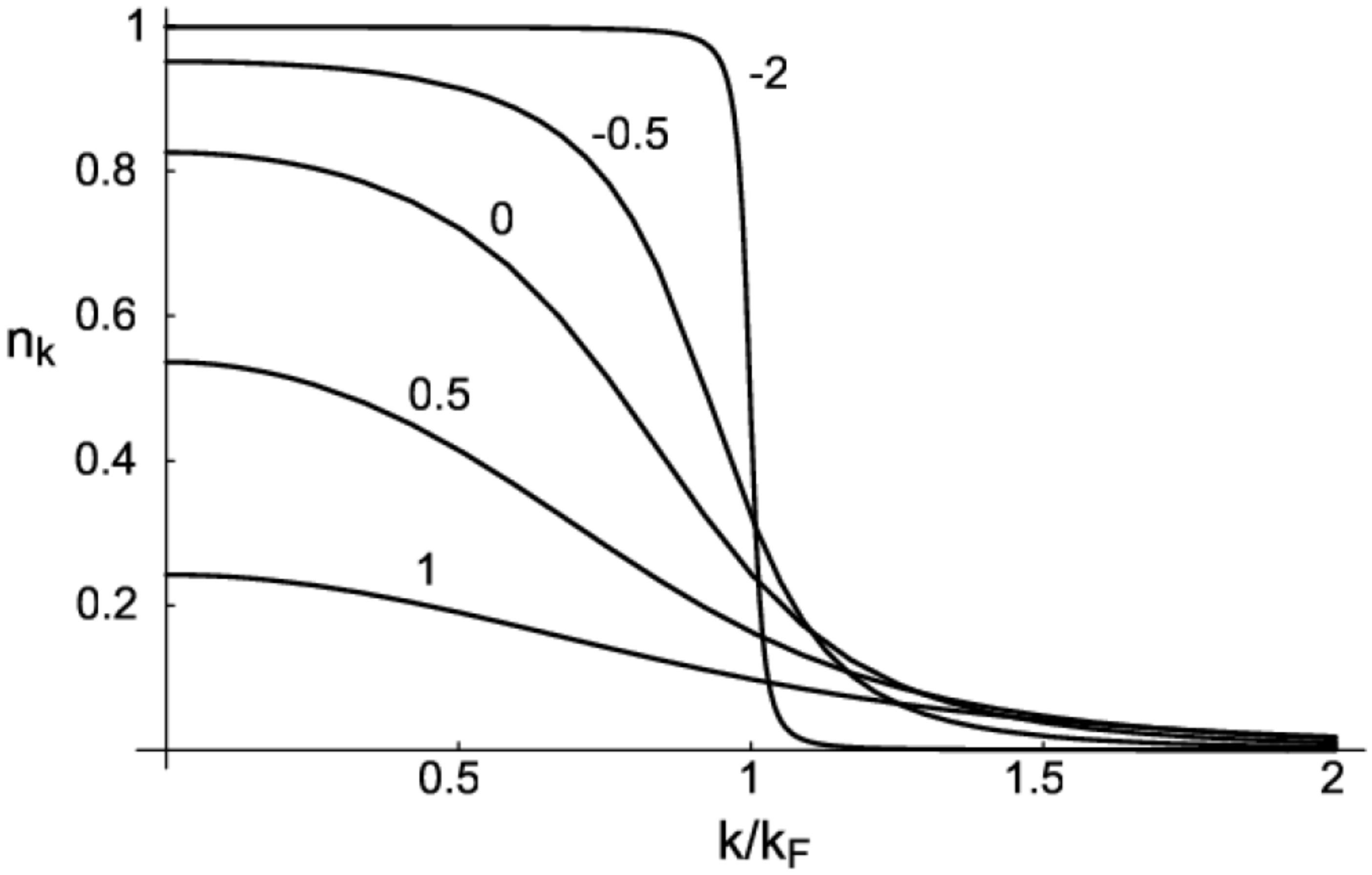
BCS-side: Gap exponentially small

$$\begin{aligned} \mu &\approx E_F \\ \Delta &\approx \frac{8}{e^2} e^{-\pi/2k_F|a|} \end{aligned}$$

Comparison of bound state energies

Bound state equation	Binding energy (after renormalization)
• Molecules:	
$\frac{\Omega}{VR^3} = \int d\epsilon \frac{\rho_3(\epsilon)}{2\epsilon + \lvert E \rvert}$	$E = -\frac{\hbar^2}{ma^2}$ only for $a > 0$
• Cooper pairs (simple model):	$a < 0$
$\frac{\Omega}{VR^3} = \int_{\epsilon > E_F} d\epsilon \frac{\rho_{3\mathrm{D}}(\epsilon)}{2(\epsilon - E_F) + \lvert E_B \rvert}$	$E_B = -\frac{8}{e^2} E_F e^{-\frac{\pi}{k_F \lvert a \rvert}}$
• Cooper pairs (BCS solution):	
$\frac{\Omega}{VR^3} = \int d\epsilon \frac{\rho_3(\epsilon)}{2\sqrt{(\epsilon - \mu)^2 + \Delta^2}}$	$\Delta = -\frac{8}{e^2} E_F e^{-\frac{\pi}{2k_F \lvert a \rvert}}$
	$k_F \lvert a \rvert \ll 1$

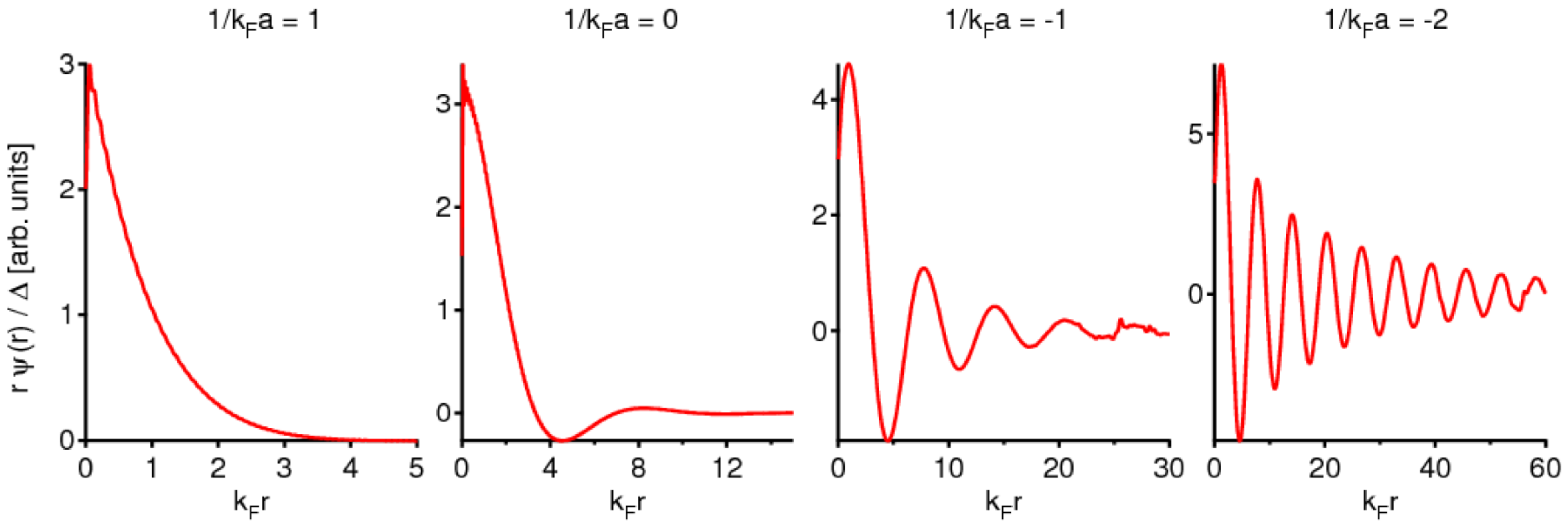
Momentum distribution



Evolution of the Pair wavefunction

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \langle \Psi_{BCS} | \Psi_\uparrow^\dagger(\mathbf{r}_1) \Psi_\downarrow^\dagger(\mathbf{r}_2) | \Psi_{BCS} \rangle = \frac{1}{\Omega} \sum_k u_k v_k e^{-i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2)}$$

$$= \frac{1}{\Omega} \sum_k \frac{\Delta}{2E_k} e^{-i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2)}$$

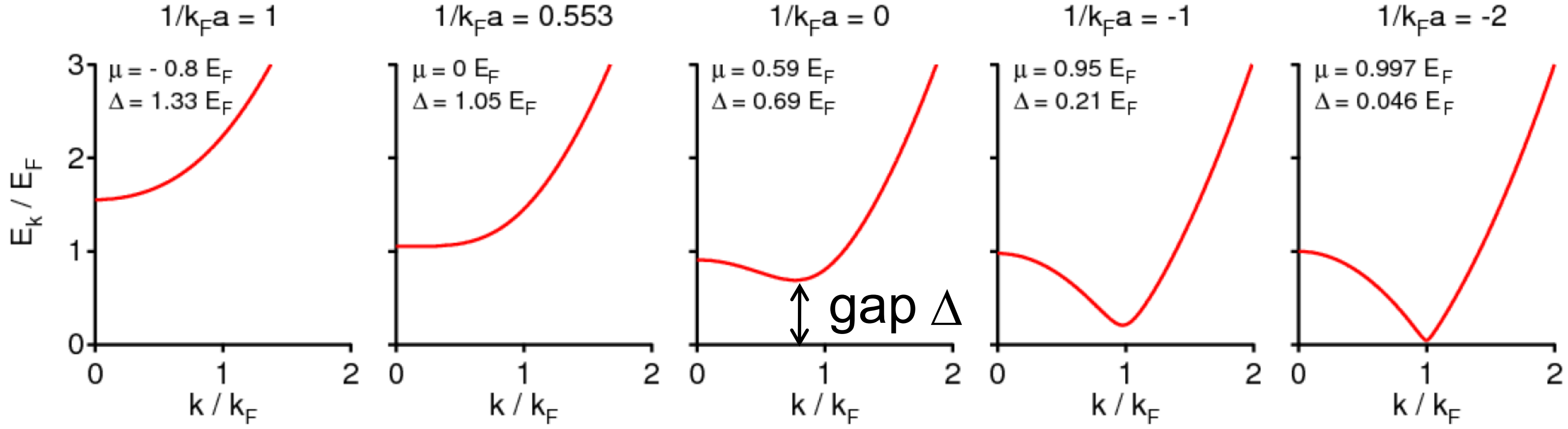


Molecules

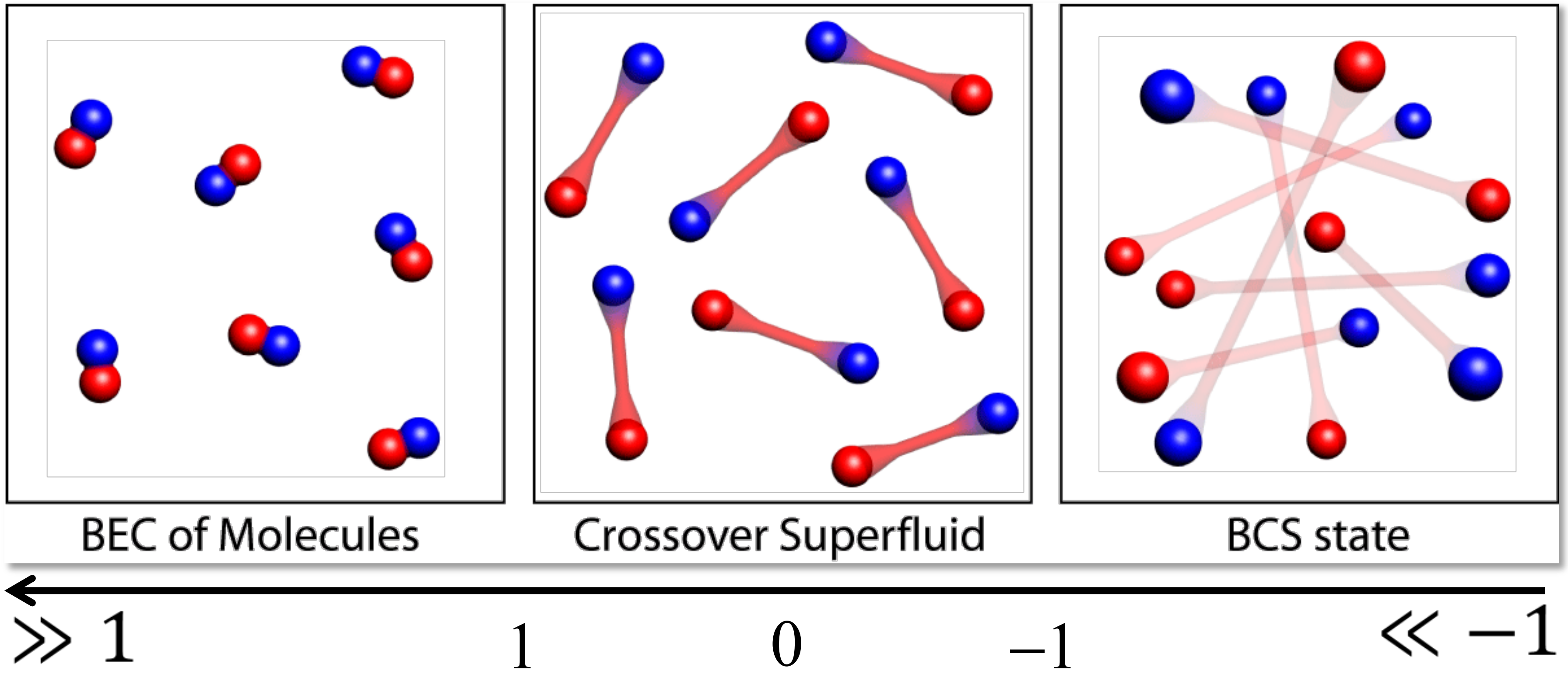
Cooper pairs

Quasi-particle Spectrum

$$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}$$



From BEC to BCS



$$(k_F a)^{-1} = \frac{\text{Interparticle Distance}}{\text{Scattering Length}}$$

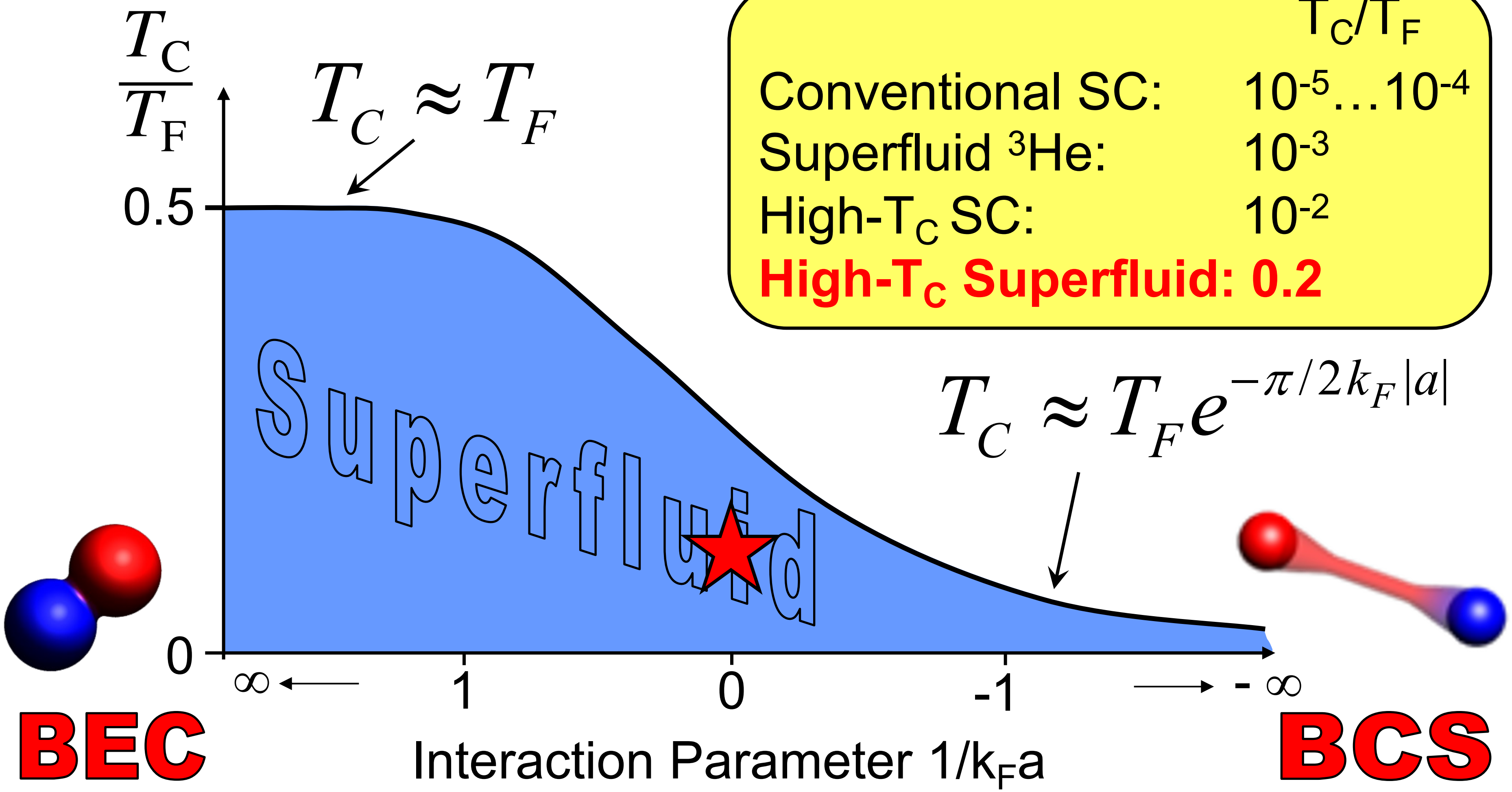
Weakly Interacting Bosons

→ Strongly Interacting Bosons

→ Strongly Interacting Fermions

→ Weakly Interacting Fermions

Critical Temperature for Fermionic Superfluidity

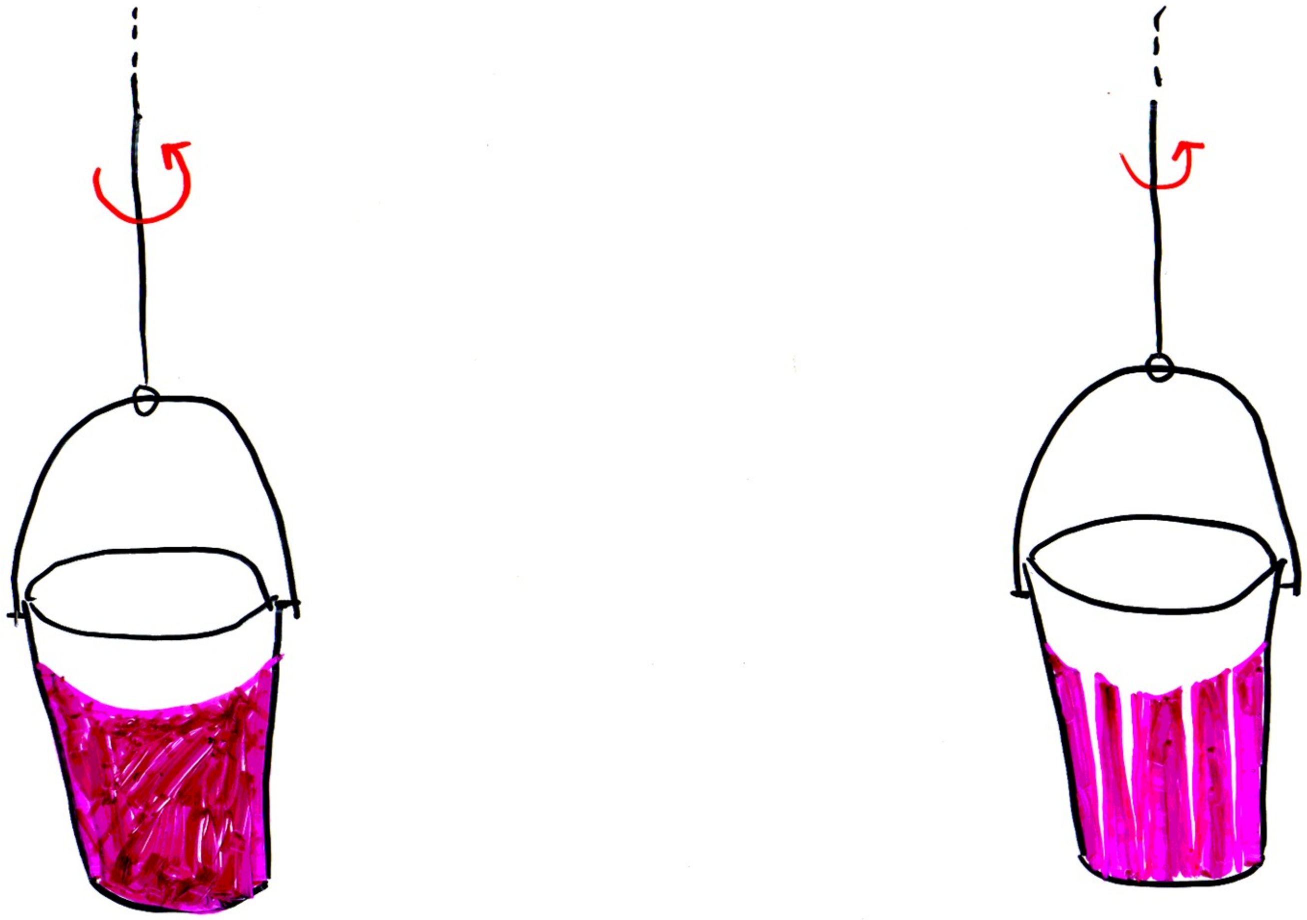


	T_C/T_F
Conventional SC:	$10^{-5}\ldots10^{-4}$
Superfluid ^{3}He:	10^{-3}
High-T_C SC:	10^{-2}
High-T_C Superfluid:	**0.2**

Scaled to the density of electrons in solids:

Superconductivity far above room temperature!

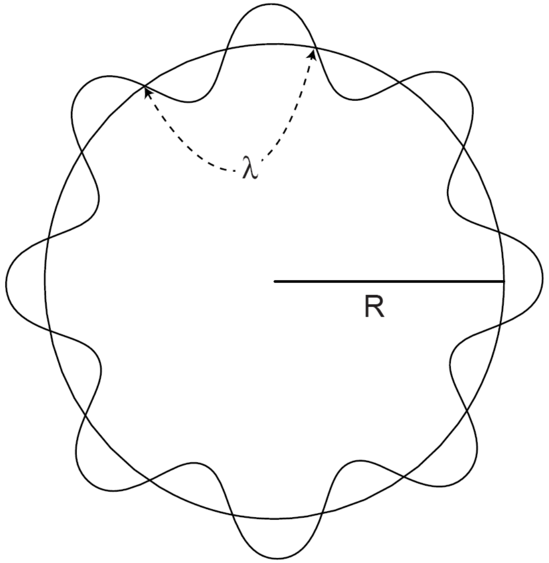
Rotating Fluids



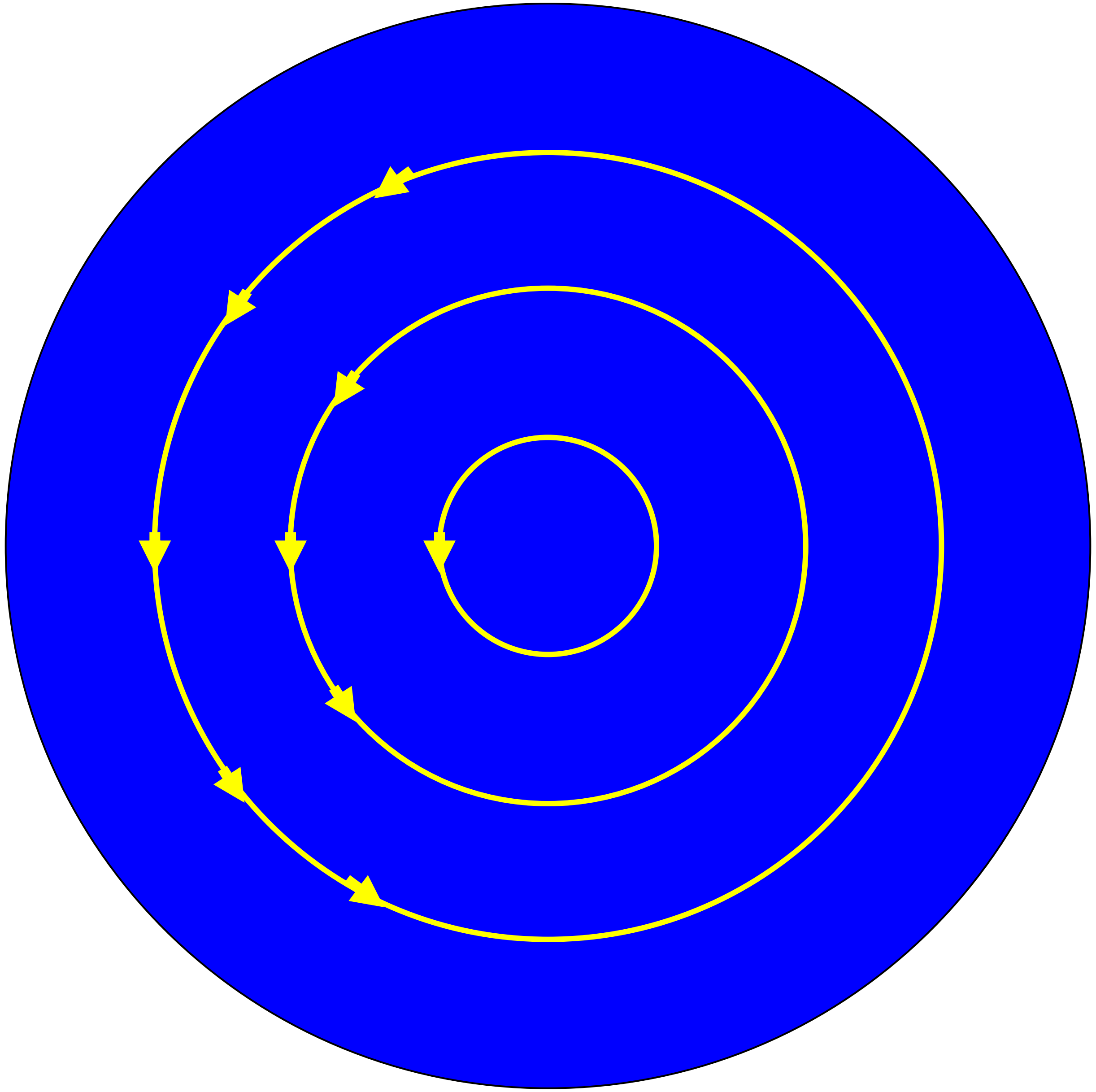
Normal Quantum

Fluid

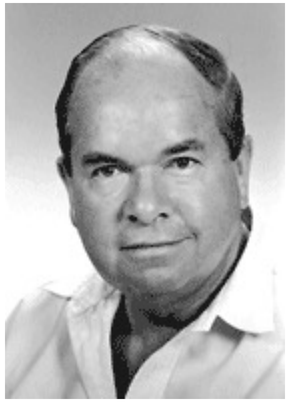
Rotating superfluid



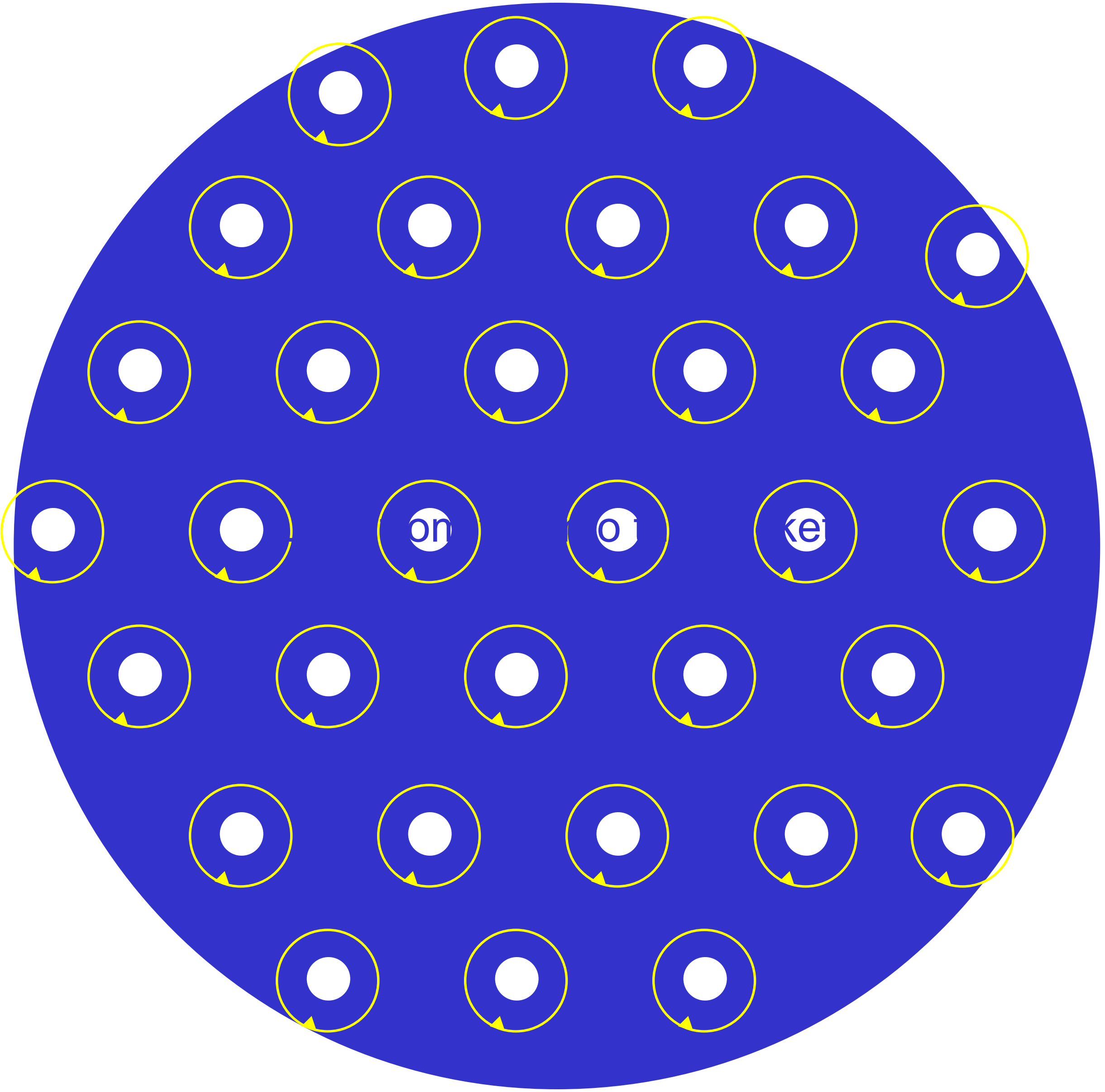
Does it rotate like this?



Like this!

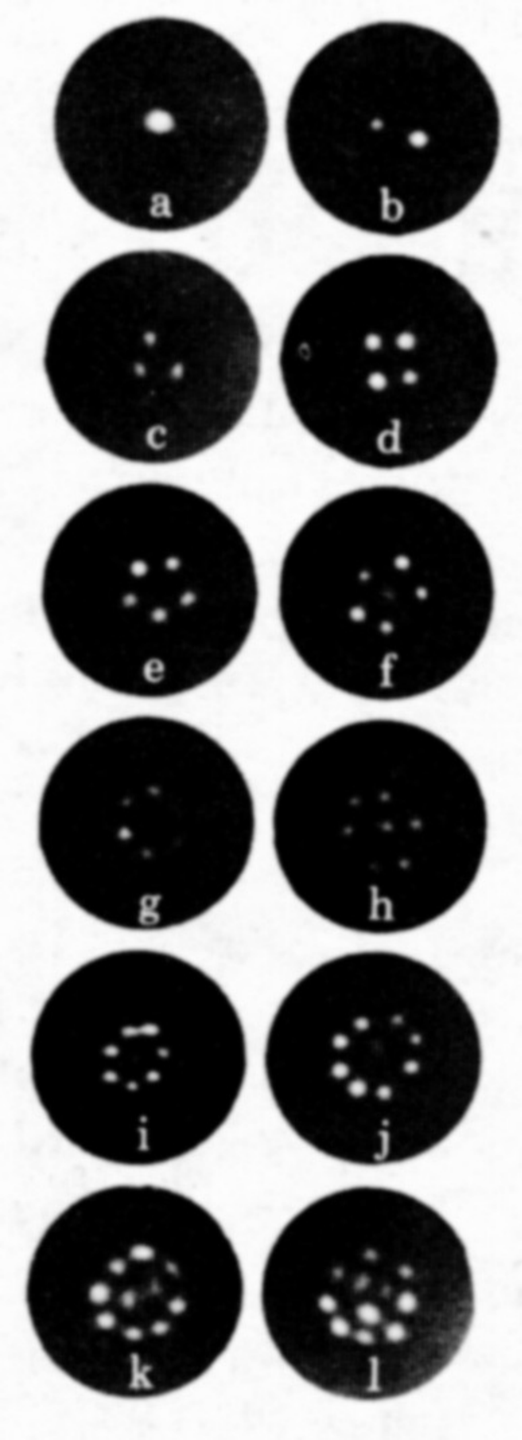


Aleksei A.
Abrikosov



Abrikosov lattice (triangular lattice)

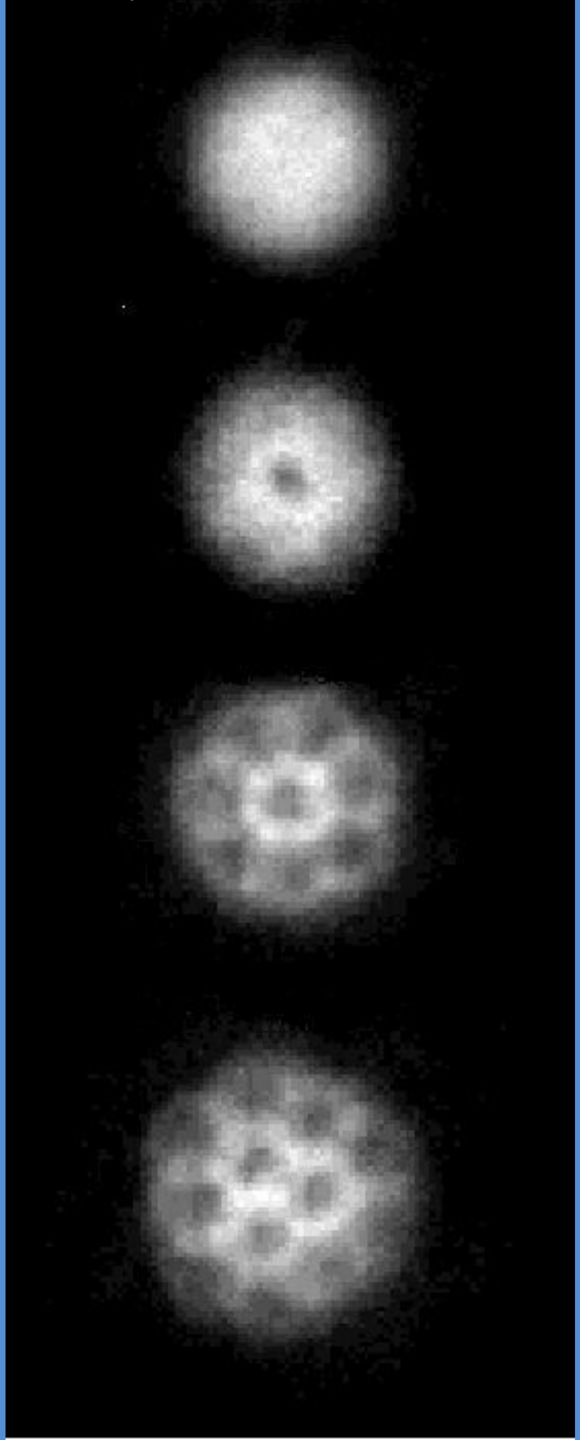
Vortex Arrays
in Bosonic Gases / Fluids



Berkeley
(R.E. Packard, 1979)
Helium-4

ENS
(J. Dalibard, 2000)
Rubidium BEC

Also: Phase engineering of single vortices in BEC: JILA (1999)



THE DIRECT OBSERVATION OF INDIVIDUAL FLUX LINES IN TYPE II SUPERCONDUCTORS

U. ESSMANN and H. TRÄUBLE

Institut für Physik am Max-Planck-Institut für Metallforschung, Stuttgart and Institut für theoretische und angewandte Physik der Technischen Hochschule Stuttgart

Received 4 April 1967

Neutral superfluids under rotation

$$\vec{F} = 2m\vec{v} \times \vec{\omega}$$

Coriolis force in rotating frame

$$\Leftrightarrow$$

Superconductors in magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

Lorentz Force

U. Essmann and H. Träuble, Physics Letters A, **24**, 526 (1967)

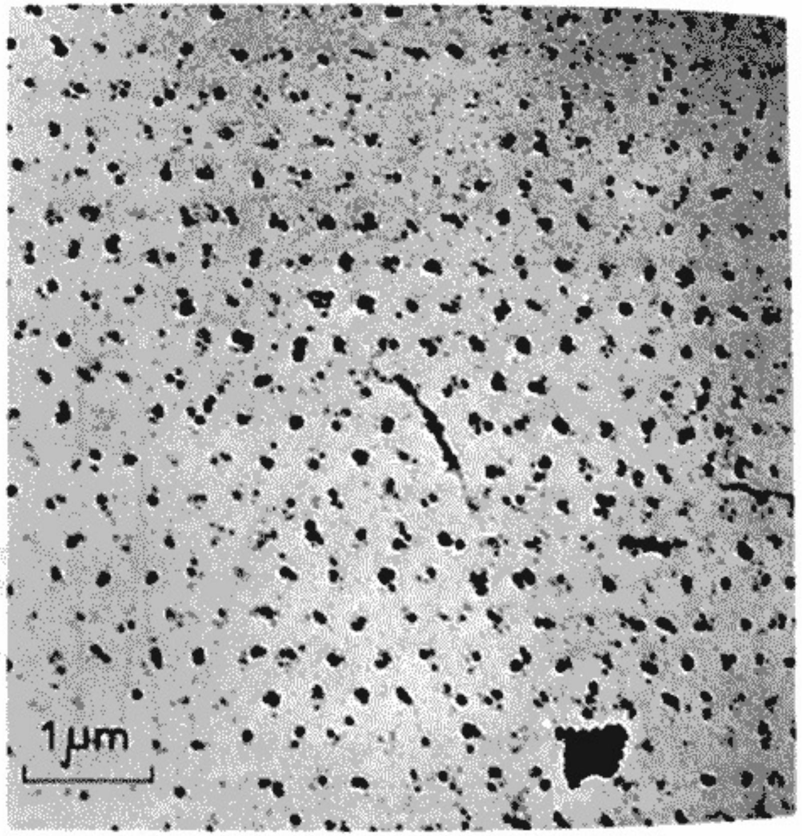
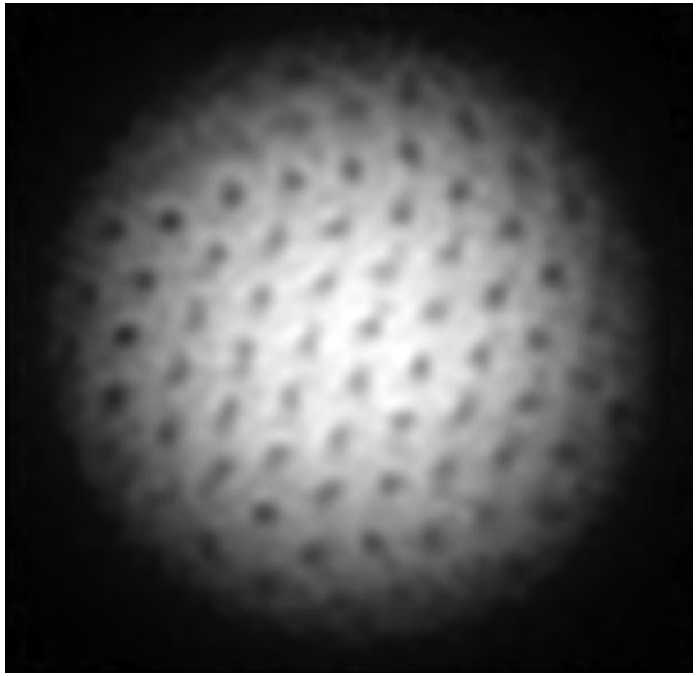
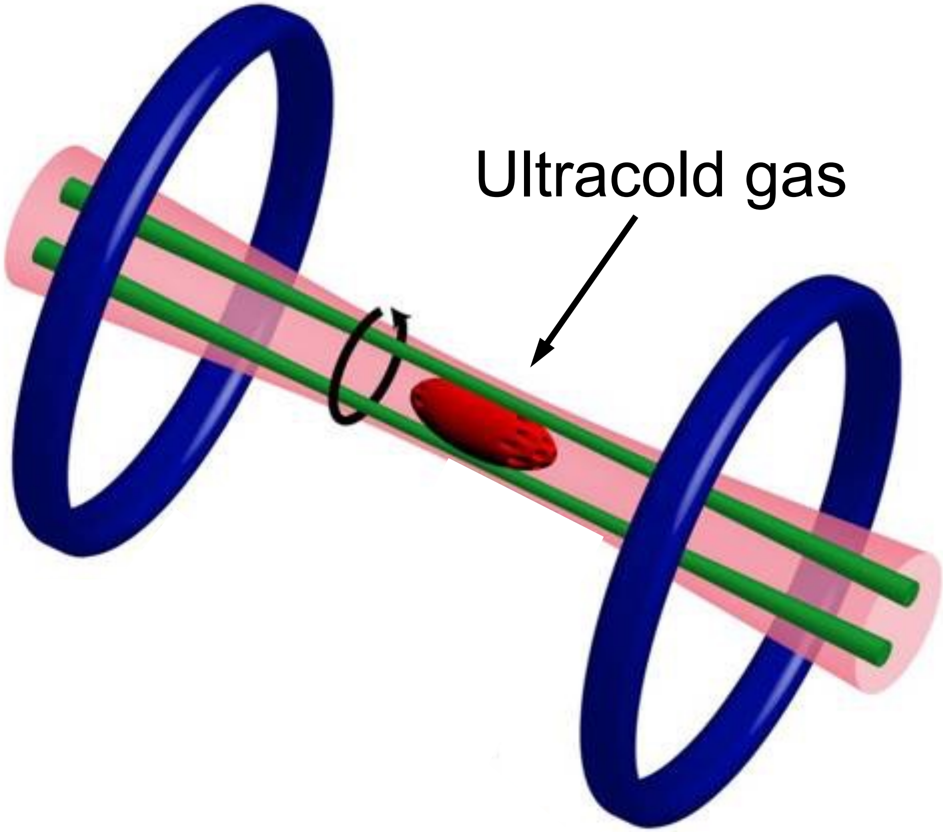


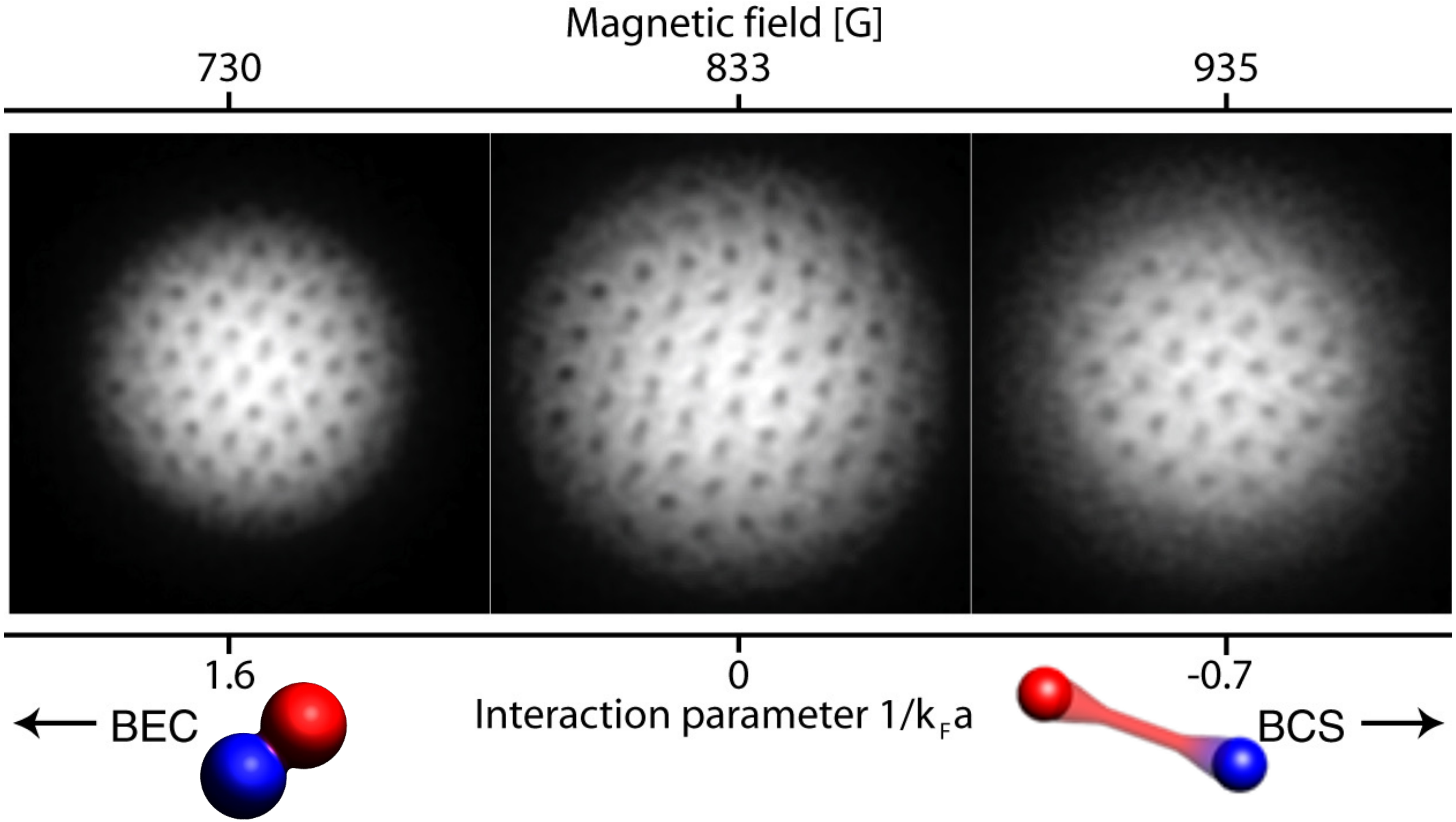
Fig. 1. "Perfect" triangular lattice of flux lines on the surface of a lead-4at% indium rod at 1.1°K. The black dots consist of small cobalt particles which have been stripped from the surface with a carbon replica.

Demonstration of superfluidity in a Fermi gas





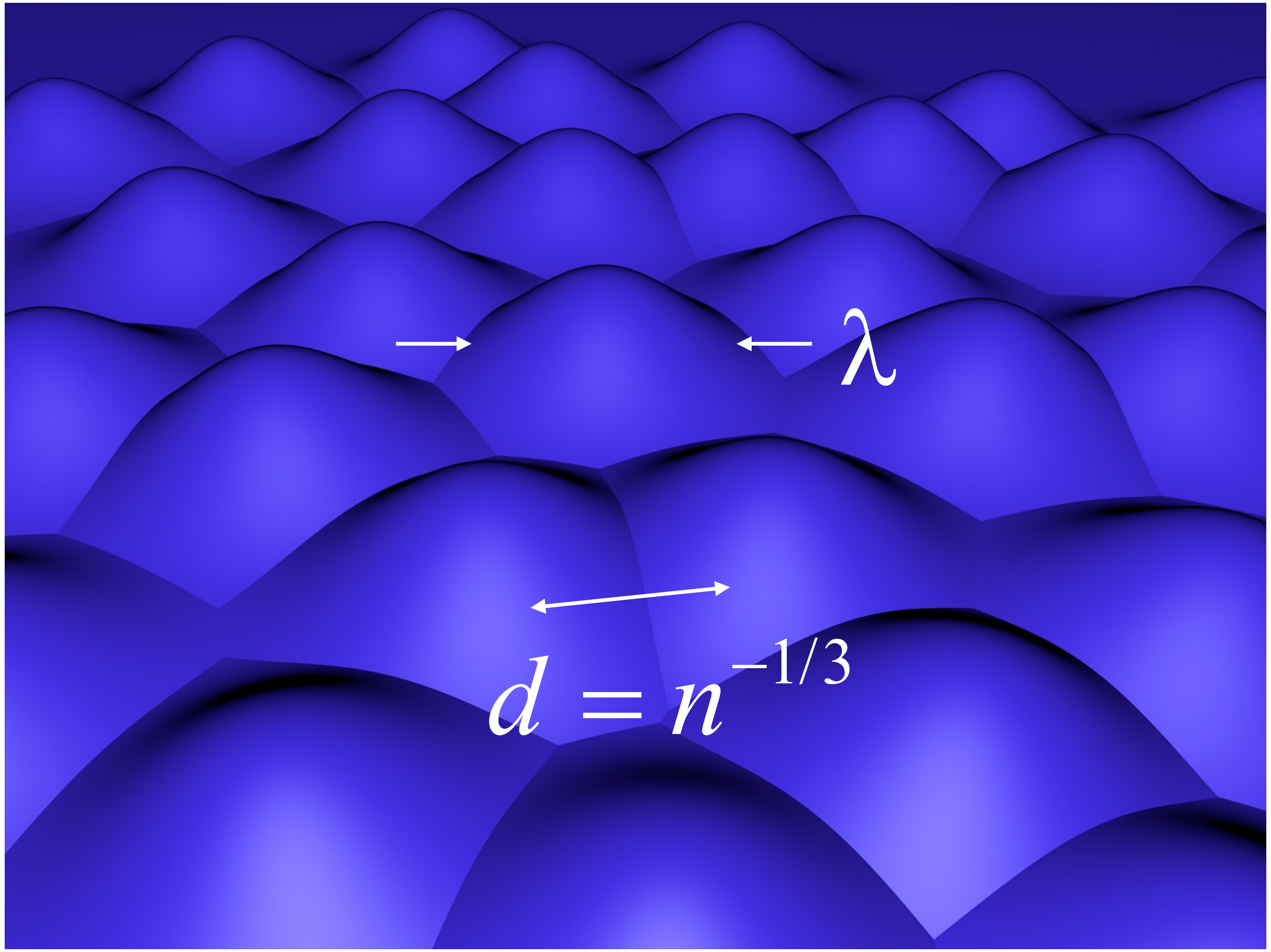
Vortex lattices in the BEC-BCS crossover



M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature 435, 1047-1051 (2005)

The Unitary Fermi Gas

Only two length scales:		Only two corresp. energy scales:	
Interparticle spacing	$n^{-1/3}$	Fermi energy	E_F
De Broglie wavelength	λ_{dB}	Temperature	$k_{\mathrm{B}}T$



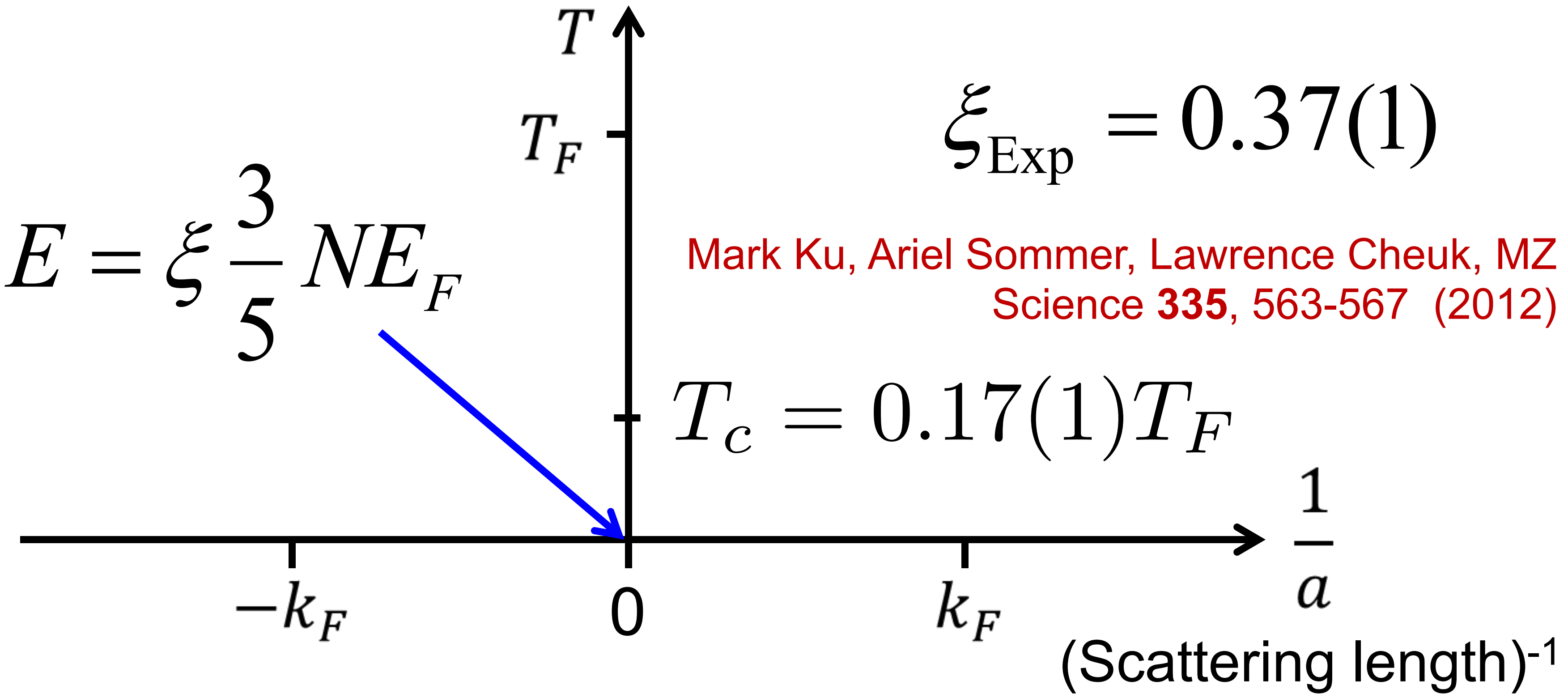
The Unitary Fermi Gas

Only two length scales:
Interparticle spacing $n^{-1/3}$
De Broglie wavelength λ_{dB}

Only two corresp. energy scales:
Fermi energy E_F
Temperature $k_{\mathrm{B}}T$

→ Universal equation of state

Duke/NC State, ENS, JILA, Innsbruck, Swinburne, MIT



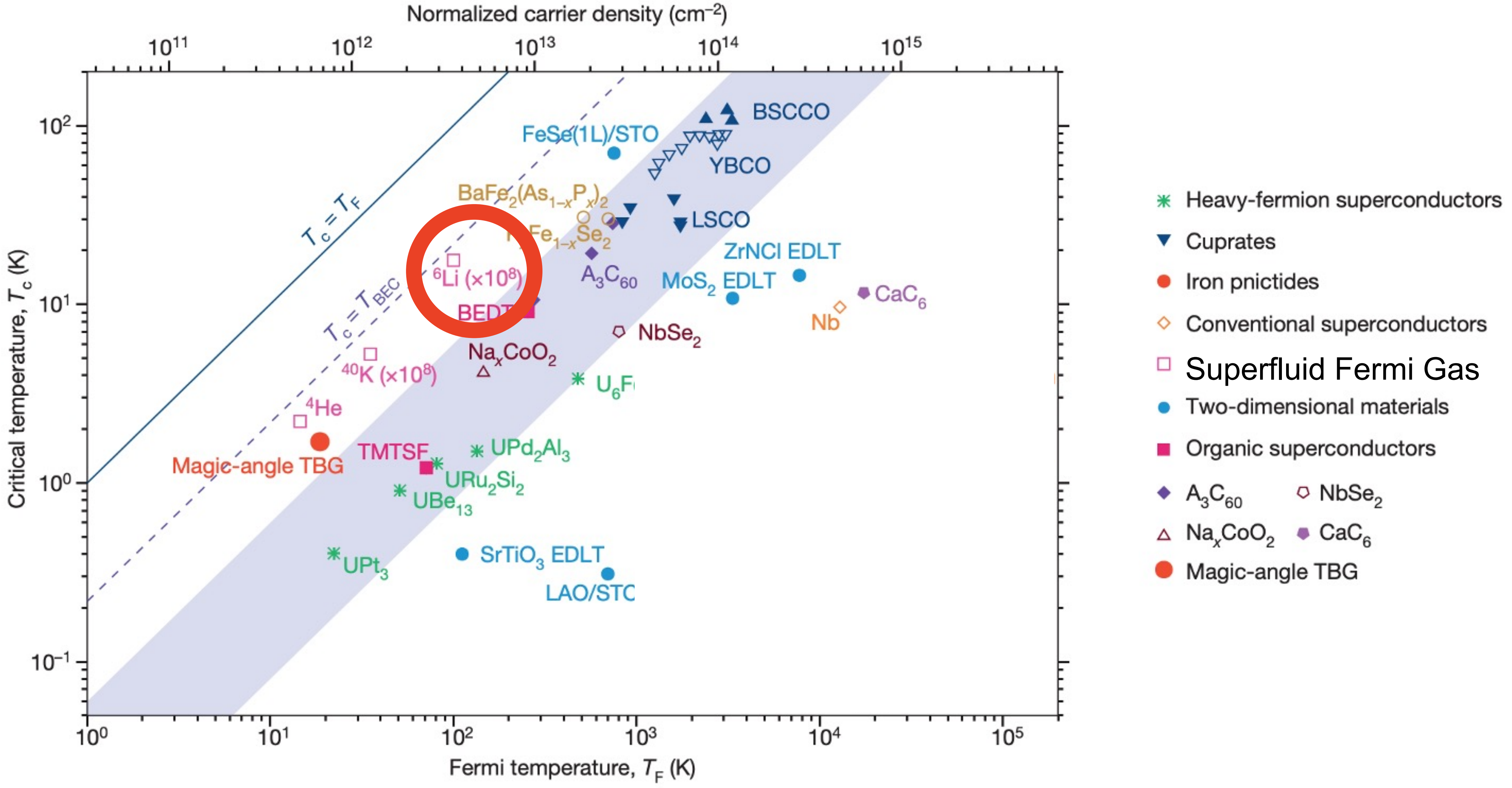
$$E = \xi \frac{3}{5} N E_F$$

$$\xi_{\mathrm{Exp}} = 0.37(1)$$

Mark Ku, Ariel Sommer, Lawrence Cheuk, MZ
Science **335**, 563-567 (2012)

$$T_c = 0.17(1) T_F$$

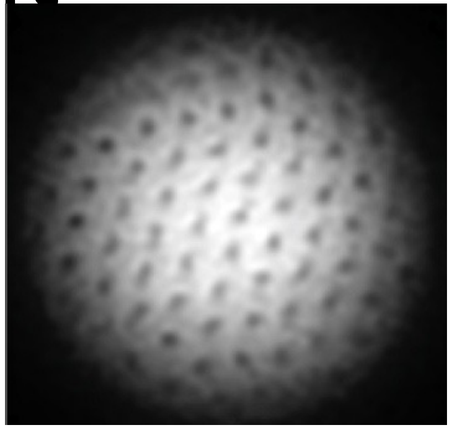
Fermi Gas on the Uemura Plot



From Cao et al., Nature 2018

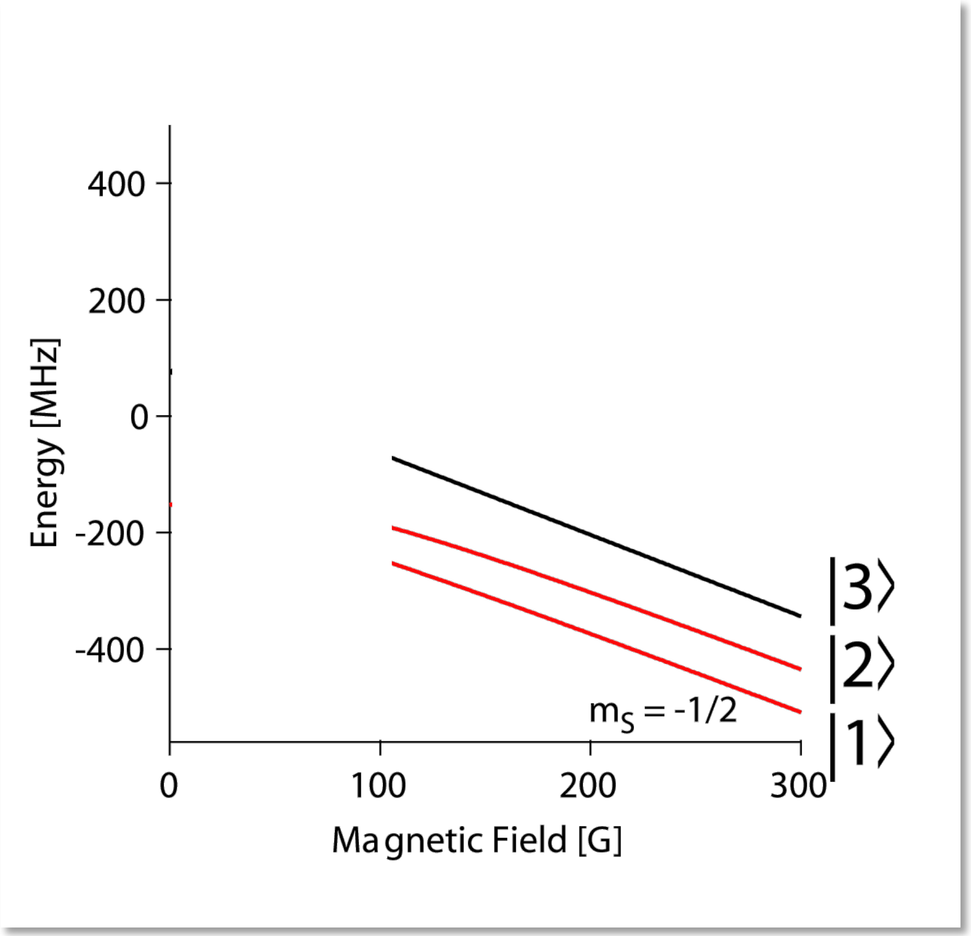
Binding Energy of Pairs

A Gedanken experiment



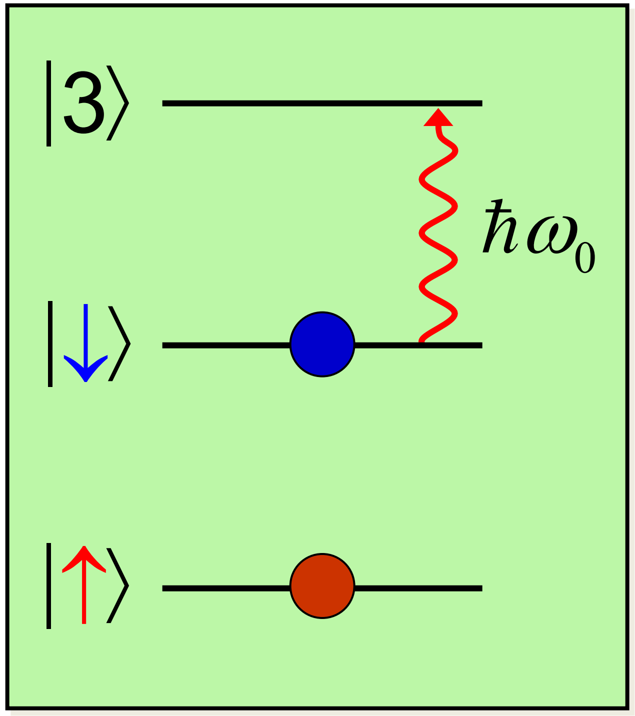
What is the energy cost of removing one fermion from the superfluid?

Radiofrequency spectroscopy

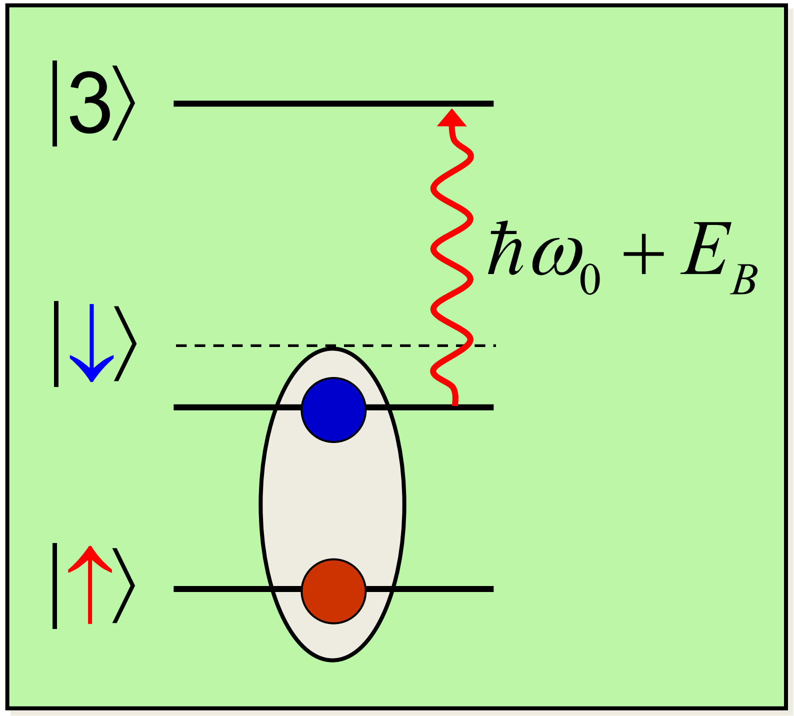


Radiofrequency spectroscopy

No interactions



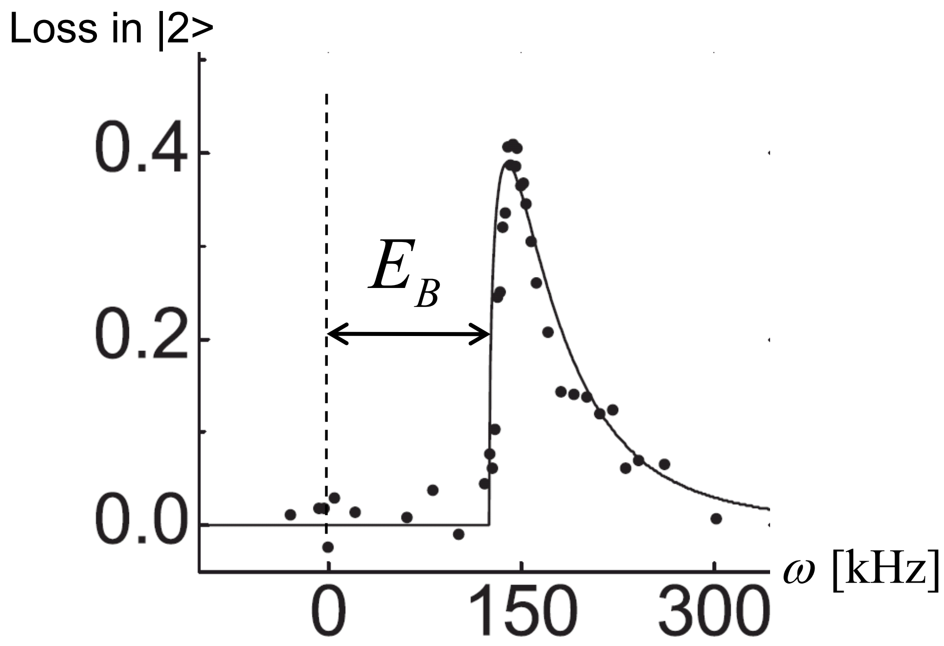
Molecular Pairing



Photon energy = Zeeman + Binding + Kinetic energy

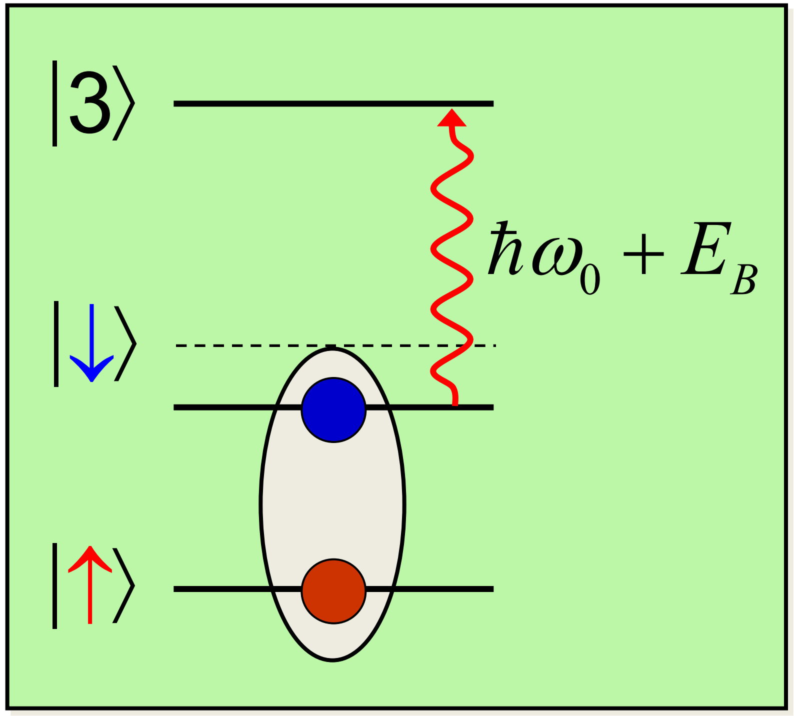
$$\hbar\omega = \hbar\omega_0 + E_B + 2\varepsilon_k$$

Radiofrequency spectroscopy



C.Chin et al. Science, **305**, 1128 (2004)

Molecular Pairing



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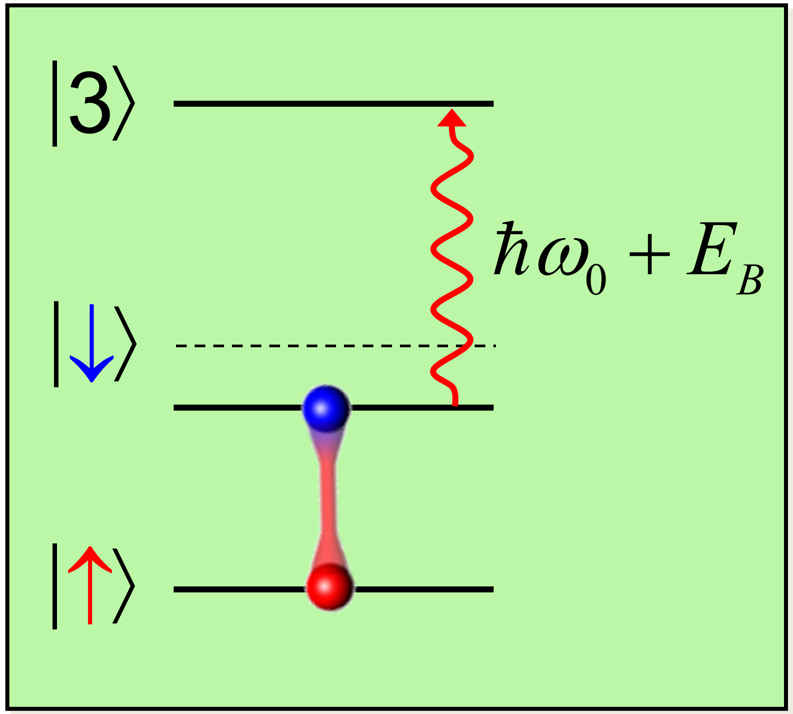
Radiofrequency spectroscopy

BCS pairing is a many-body affair. Does the picture still hold?



Energy gain due to pairing (BCS):

$$\delta E(N) = E_{SF}(N) - E_{Normal}(N) = -\frac{3}{8} N \frac{\Delta^2}{E_F}$$

Binding energy per particle:

$$E_B = \frac{\delta E(N+2) - \delta E(N)}{2} = -\frac{\Delta^2}{2E_F}$$

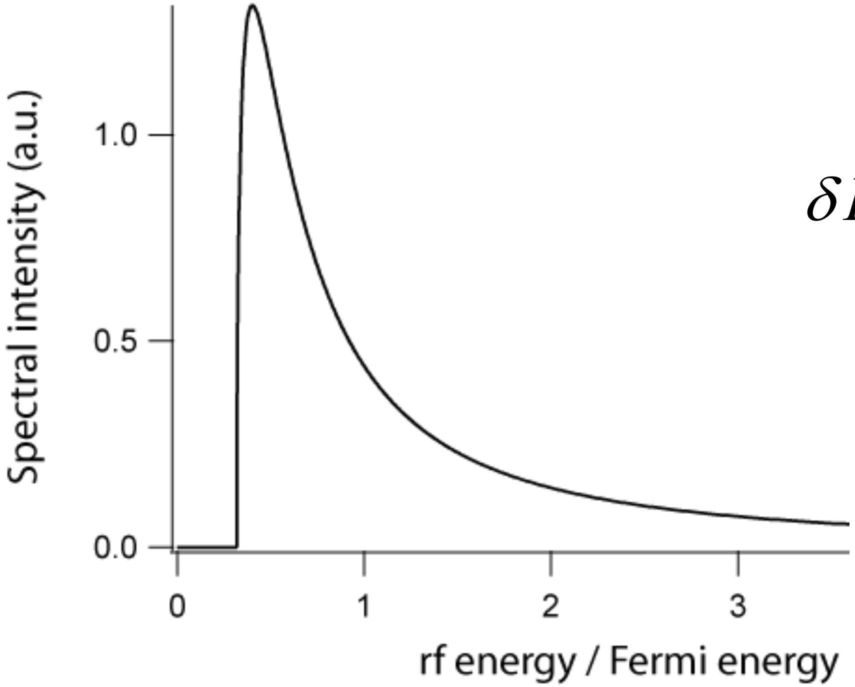
Photon energy = Zeeman + Quasiparticle + Kinetic energy

$$\hbar\omega = \hbar\omega_0 + E_k - \mu + \varepsilon_k$$

Onset at $$\hbar\omega = \hbar\omega_0 + \frac{\Delta^2}{2E_F}$$

BCS pairing is a many-body affair. Does the picture still hold?

Radiofrequency spectroscopy



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$$\hbar\omega = \hbar\omega_0 + E_k - \mu + \varepsilon_k$$

Onset at $$\hbar\omega = \hbar\omega_0 + \frac{\Delta^2}{2E_F}$$

Radiofrequency spectroscopy

Fermi's Golden Rule:

$$\Gamma(\omega) \equiv \frac{2\pi}{\hbar} \sum_f \left| \left\langle f \left| \hat{V} \right| \Psi_{\mathrm{BCS}} \right\rangle \right|^2 \delta\left(\hbar\omega - E_f\right)$$

Interaction: $\hat{V} = V_0 \sum_k c^\dagger_{k3} c_{k\uparrow} + c^\dagger_{k\uparrow} c_{k3}$

Insert quasiparticle operators: $c_{k\uparrow} = u_k \gamma_{k\uparrow} + v_k \gamma^\dagger_{-k\downarrow}$

Act on BCS-state: $c^\dagger_{k3} c_{k\uparrow} \left|\Psi_{\mathrm{BCS}}\right\rangle = v_k c^\dagger_{k3} \gamma^\dagger_{-k\downarrow} \left|\Psi_{\mathrm{BCS}}\right\rangle$

And thus: $\hat{V}\left|\Psi_{\mathrm{BCS}}\right\rangle = V_0 \sum_k v_k c^\dagger_{k3} \gamma^\dagger_{-k\downarrow} \left|\Psi_{\mathrm{BCS}}\right\rangle$

Radiofrequency spectroscopy

Fermi's Golden Rule:

$$\Gamma(\omega) \equiv \frac{2\pi}{\hbar} \sum_f \left| \left\langle f \left| \hat{V} \right| \Psi_{\mathrm{BCS}} \right\rangle \right|^2 \delta\left(\hbar\omega - E_f\right)$$

Possible final states: $|k\rangle \equiv c^\dagger_{k3}\gamma_{-k\uparrow}|\Psi_{\mathrm{BCS}}\rangle$

RF Photon provides: $\hbar\Omega(k) = \hbar\omega_{\uparrow 3} + E_k + \epsilon_k - \mu$

Invert: $\hbar\Omega(k) = \hbar\omega_{\uparrow 3} + E_k + \epsilon_k - \mu$ to get ϵ_k in terms of ω.

Then: $$\delta(\hbar\omega - \hbar\Omega(k)) = \frac{1}{\hbar}\frac{d\epsilon_k}{d\Omega}\,\delta(\epsilon_k - \epsilon(\omega))$$

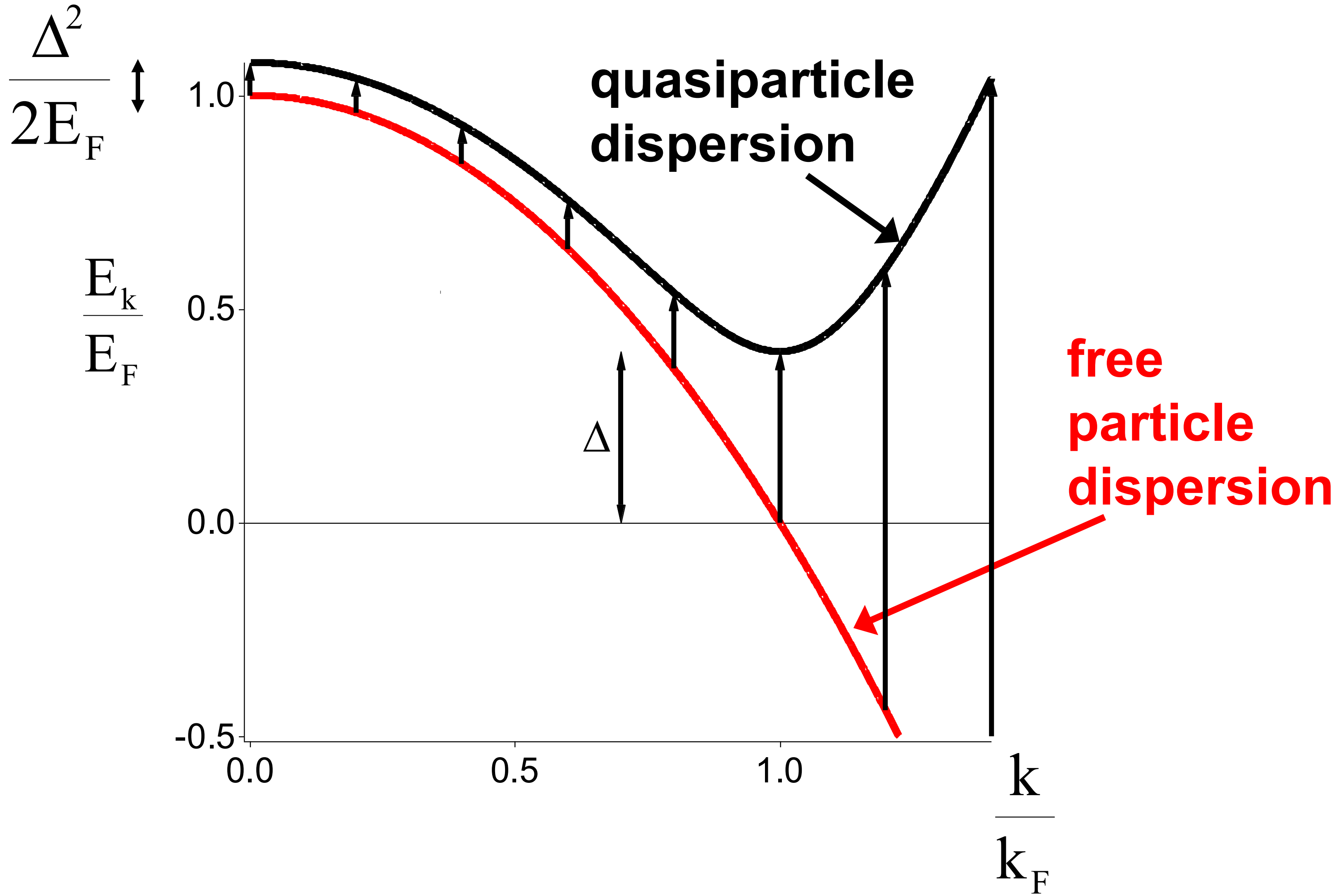
But: $$\frac{d\Omega}{d\epsilon_k} = \frac{\xi_k}{E_k} + 1 = 2u_k^2$$

And the final RF spectrum becomes:

$$\boxed{\Gamma(\omega) = \frac{\pi}{\hbar} V_0^2\, \rho(\epsilon_k)\, \left.\frac{v_k^2}{u_k^2}\right|_{\epsilon_k = \epsilon(\omega)} = \pi N_p\, V_0^2\, \rho(\epsilon_k)\, \left.|\varphi_k|^2\right|_{\epsilon_k = \epsilon(\omega)}}$$

Radiofrequency spectroscopy

$$\hbar\Omega = \hbar\omega_{\uparrow 3} + E_k \;\; -(\mu - \varepsilon_k)$$



$$\boxed{\Gamma(\omega) = \pi N_p \, V_0^2 \, \rho(\epsilon_k) \ \ |\varphi_k|^2 \ \Big|_{\epsilon_k = \epsilon(\omega)}}$$

$$\hbar\omega_{\mathrm{th}} = \sqrt{\mu^2 + \Delta^2} - \mu \rightarrow \begin{cases} \dfrac{\Delta^2}{2E_F} & \text{in the BCS-limit} \\ 0.31 E_F & \text{on resonance} \\ |E_B| = \dfrac{\hbar^2}{ma^2} & \text{in the BEC-limit} \end{cases}$$

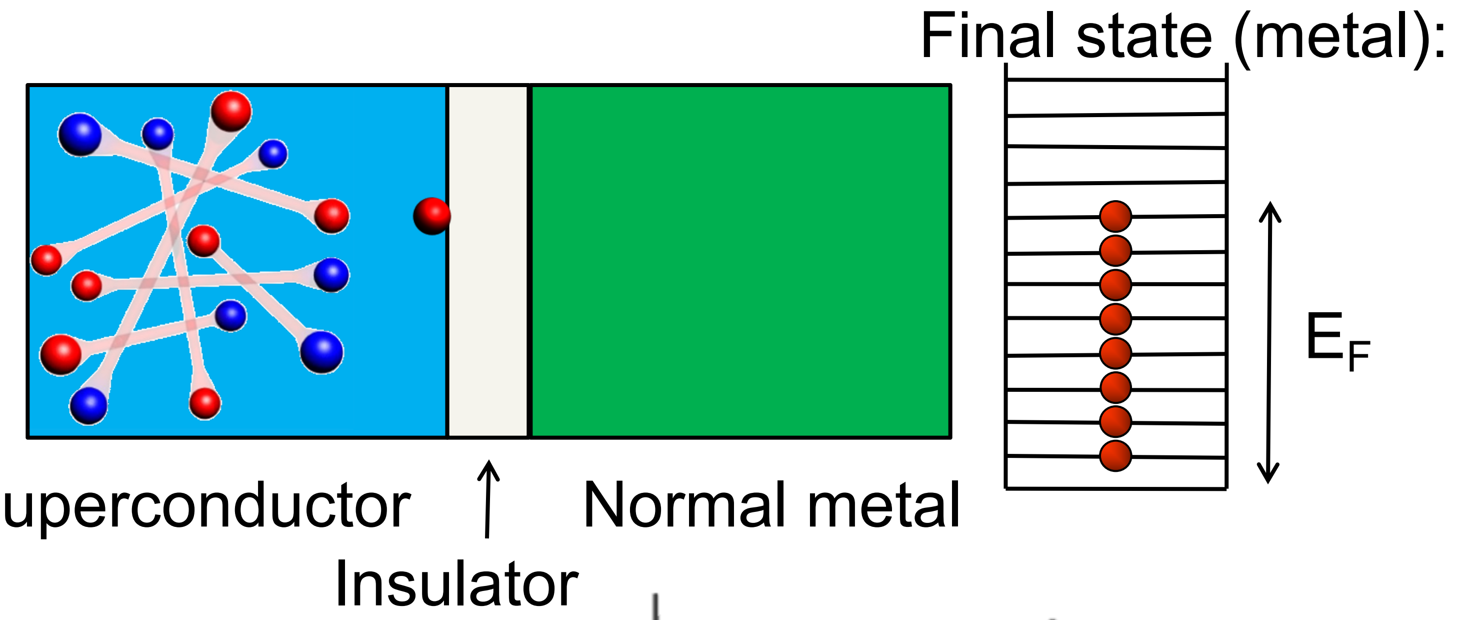
Explicitely:

$$\Gamma(\omega) = \frac{3\pi}{4\sqrt{2}\hbar} \, \frac{N \, V_0^2 \Delta^2}{E_F^{3/2}} \, \frac{\sqrt{\hbar\omega - \hbar\omega_{\mathrm{th}}}}{\hbar^2 \omega^2} \sqrt{1 + \frac{\omega_{\mathrm{th}}}{\omega} + \frac{2\mu}{\hbar\omega}}$$

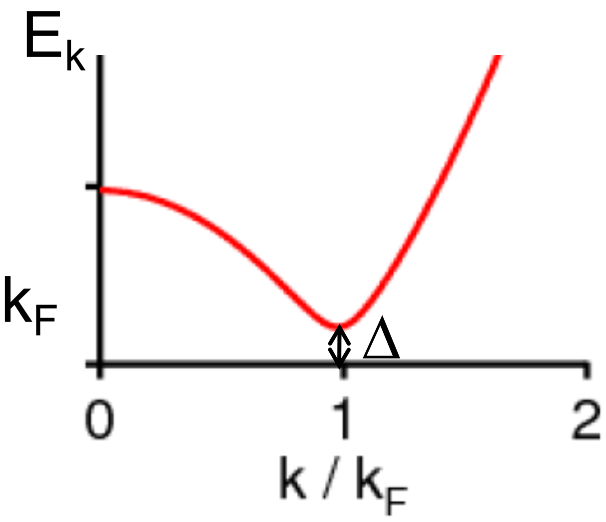
BEC-limit:

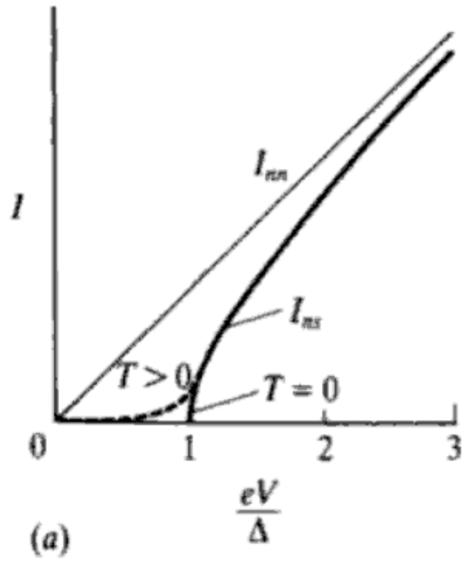
$$\Gamma_{\mathrm{BEC}}(\omega) = \frac{4}{\hbar} N_M \, V_0^2 \, \sqrt{|E_B|} \frac{\sqrt{\hbar\omega - |E_B|}}{\hbar^2 \omega^2}$$

Connection to tunneling experiments



Transfer at Fermi surface $k=k_F$



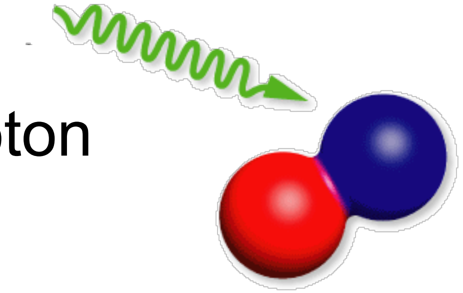


Onset at: $$eV = \Delta$$

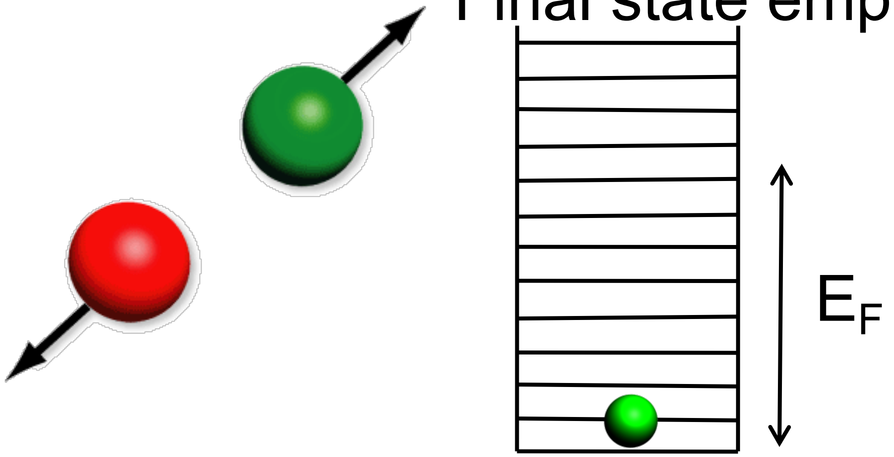
Source: Tinkham, Introduction to Superconductivity

Connection to tunneling experiments

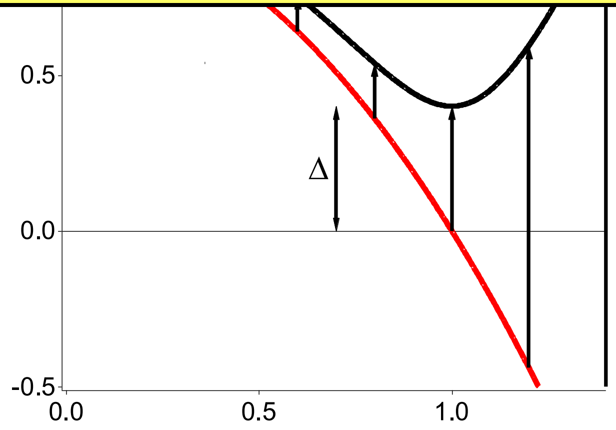
RF Photon

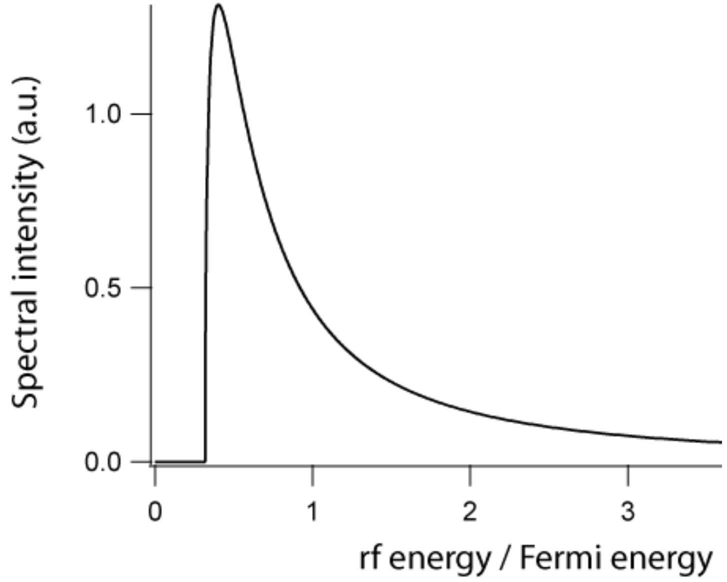


Final state empty:



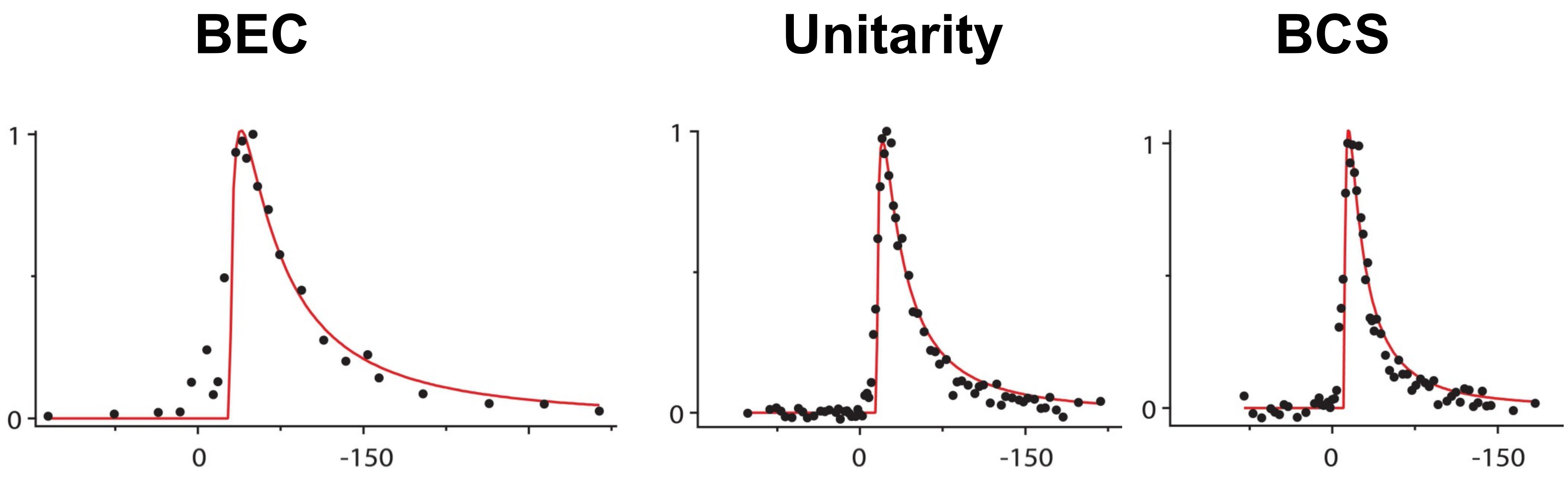
RF spectroscopy directly measures the binding energy of fermion pairs (not the gap)





BCS limit, onset at: $$\hbar\omega - \hbar\omega_0 = \frac{\Delta^2}{2E_F}$$

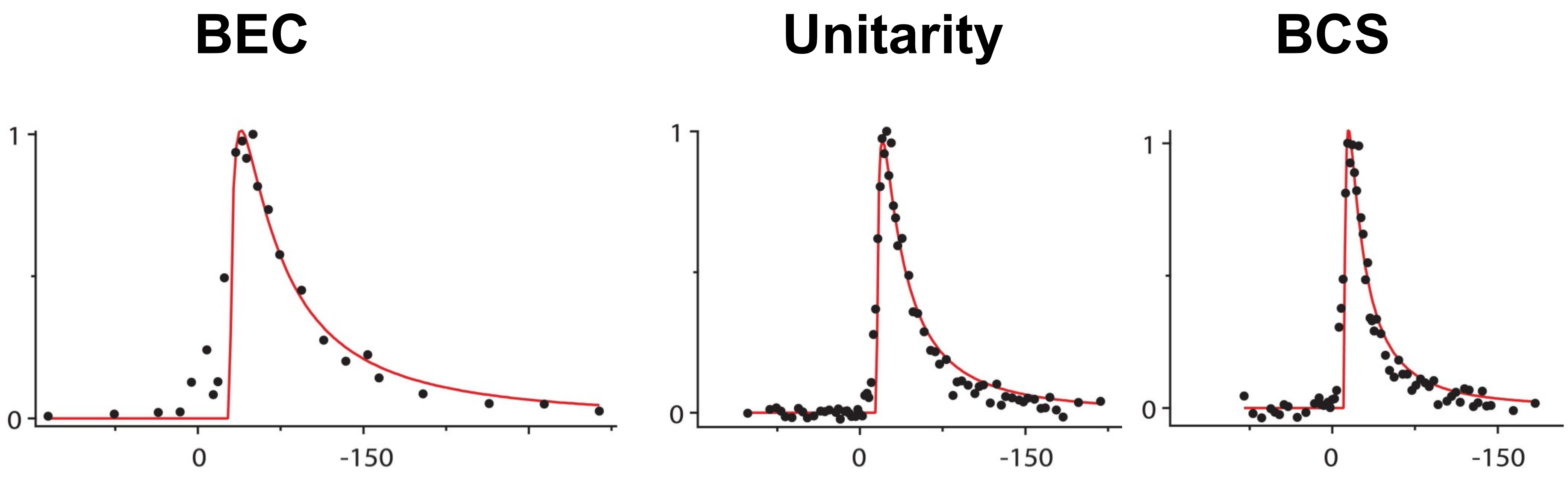
Rf Spectra in the BEC-BCS Crossover



- Determine binding energy spectroscopically
- Infer size of the pairs at unitarity:
  about half the interparticle spacing

Christian H. Schunck, Yong-il Shin, Andre Schirotzek,
Wolfgang Ketterle, Nature **454**, 739 (2008)

Rf Spectra in the BEC-BCS Crossover



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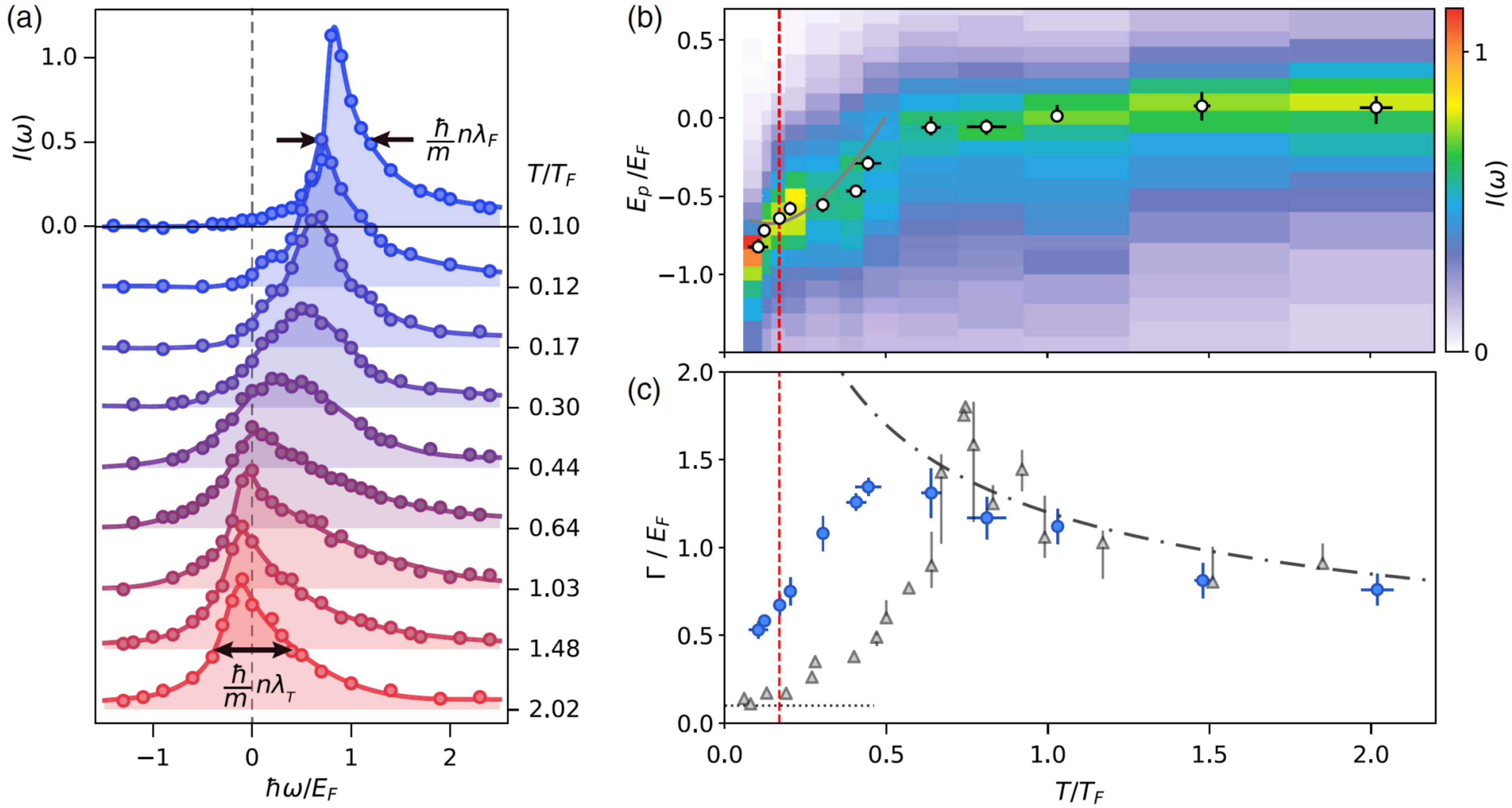
Wolfgang Ketterle, Nature **454**, 739 (2008)

Homogeneous spectra of balanced gas



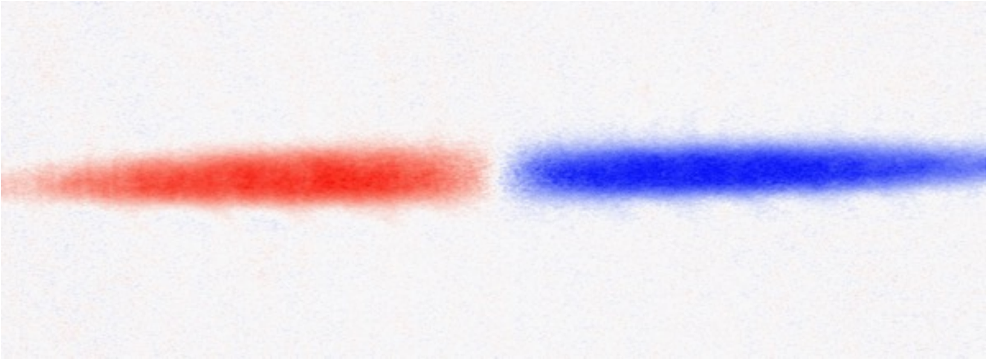
A single peak at all temperatures

B. Mukherjee, P. Patel, Z. Yan, R. Fletcher, J. Struck, MZ, PRL 122, 203402 (2019)

Trapped spectra show double peaks: C. Chin et al., Grimm group, Science 2005

Transport in Strongly Interacting Systems

e.g. Unitary Gas @ Feshbach Resonance



Mean free path ~ Interparticle spacing d

Average velocity $v \sim \dfrac{\hbar}{md}$

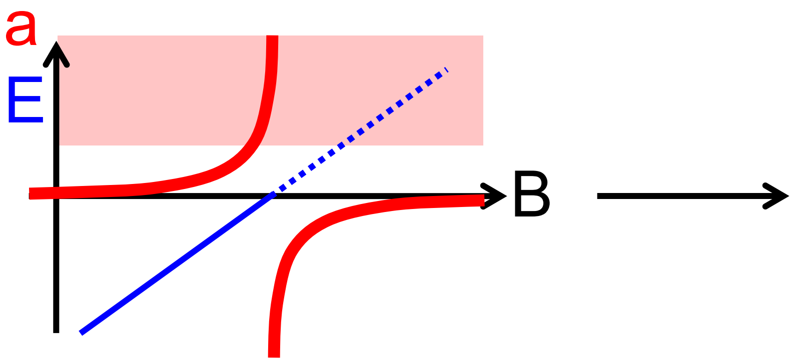
Quantum Limit of Diffusion (charge, spin, momentum, thermal)

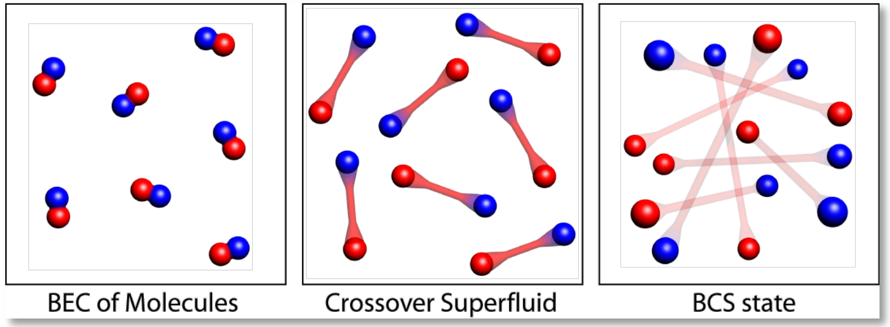
$$D \sim vl \sim \frac{\hbar}{m}$$

A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Sound in strongly interacting Fermi Gases

Feshbach Resonances → e.g. BEC-BCS Crossover





Quantum Gases → Quantum Fluids


Parth Patel


Zhenjie Yan

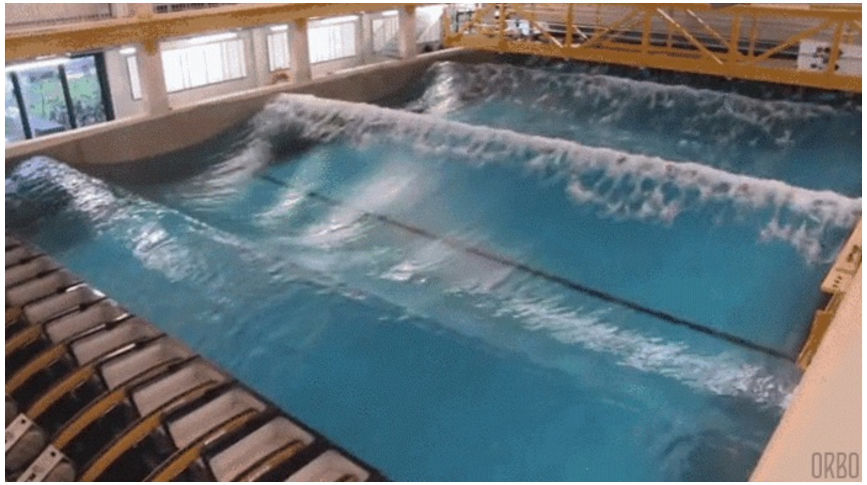

Biswaroop Mukherjee

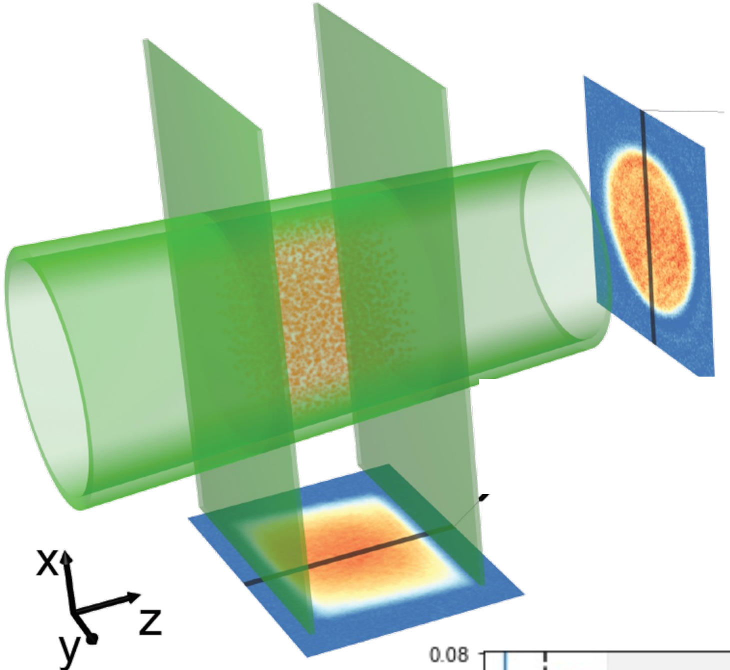

Dr. Richard Fletcher

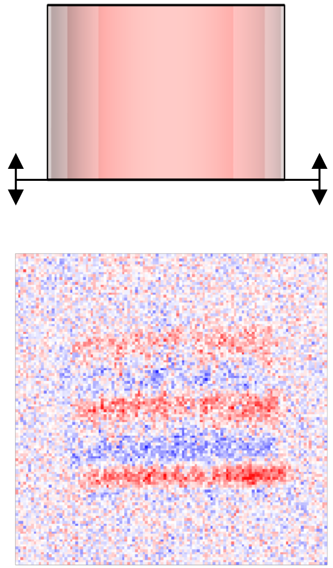

Dr. Julian Struck

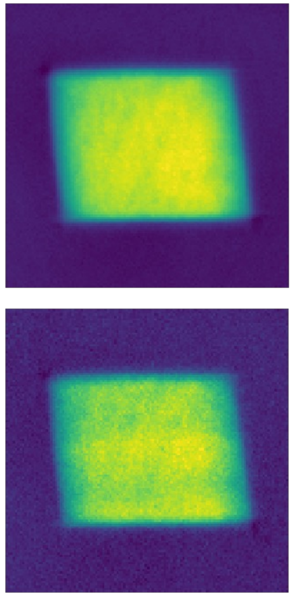
P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

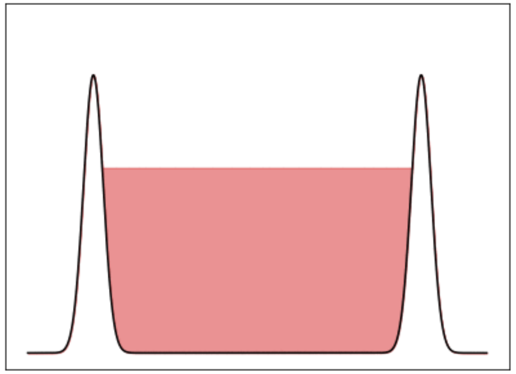
Creating Sound Waves in a Box

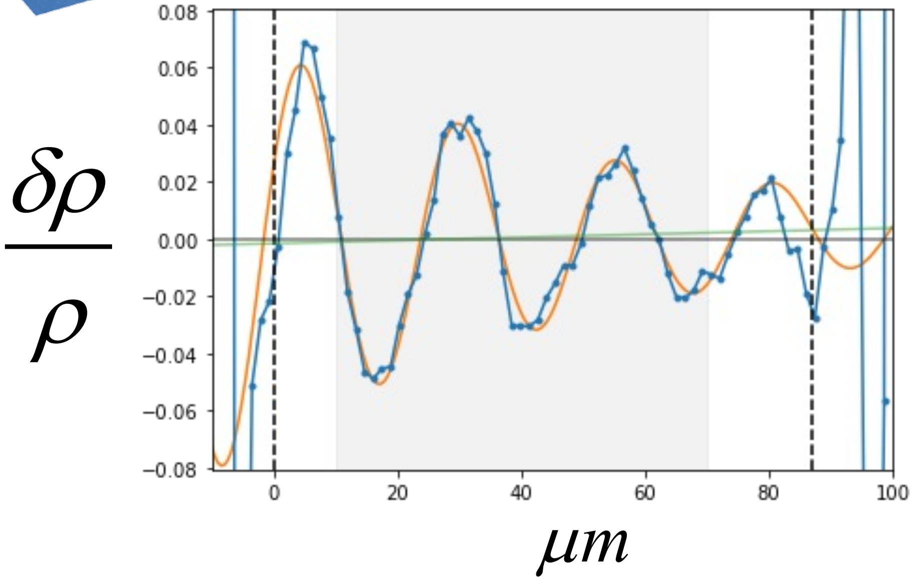






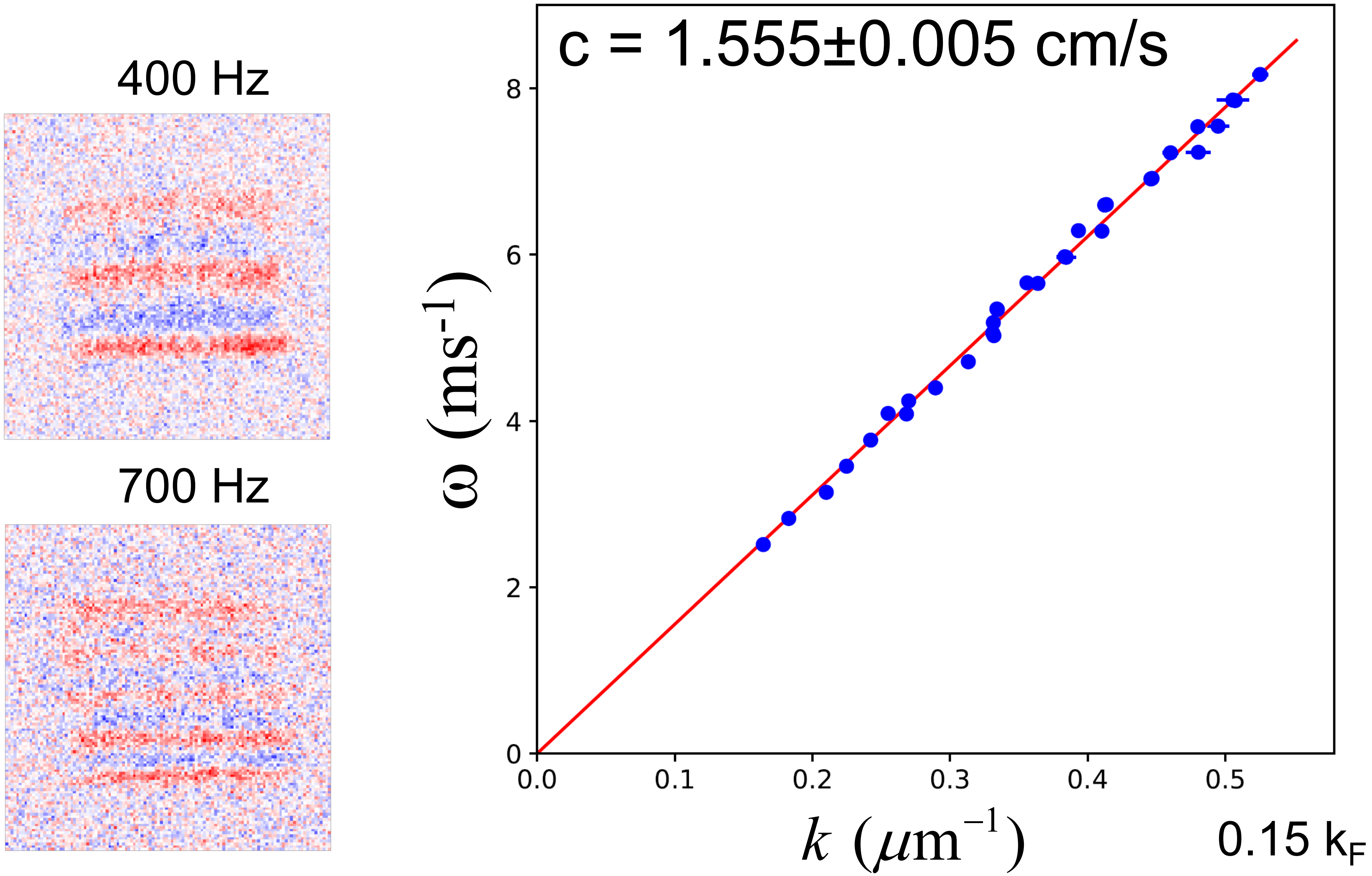






See also: 2D Bose gas, ENS (Dalibard),
3D Bose gas, Cambridge (Hadzibabic), Fermi gas, NCState (Thomas), 2D Fermi Gas, Hamburg (Moritz)

Dispersion relation



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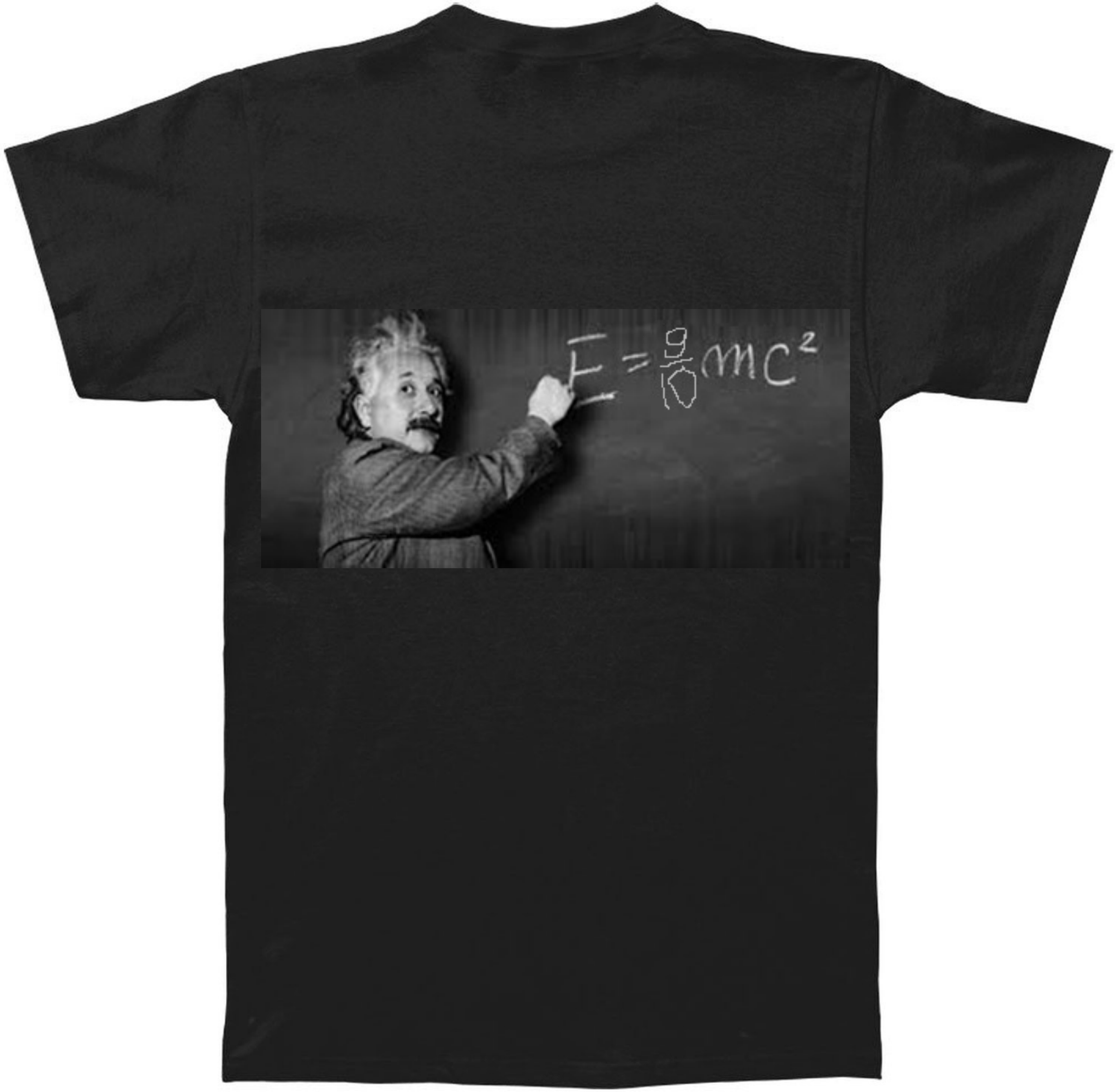
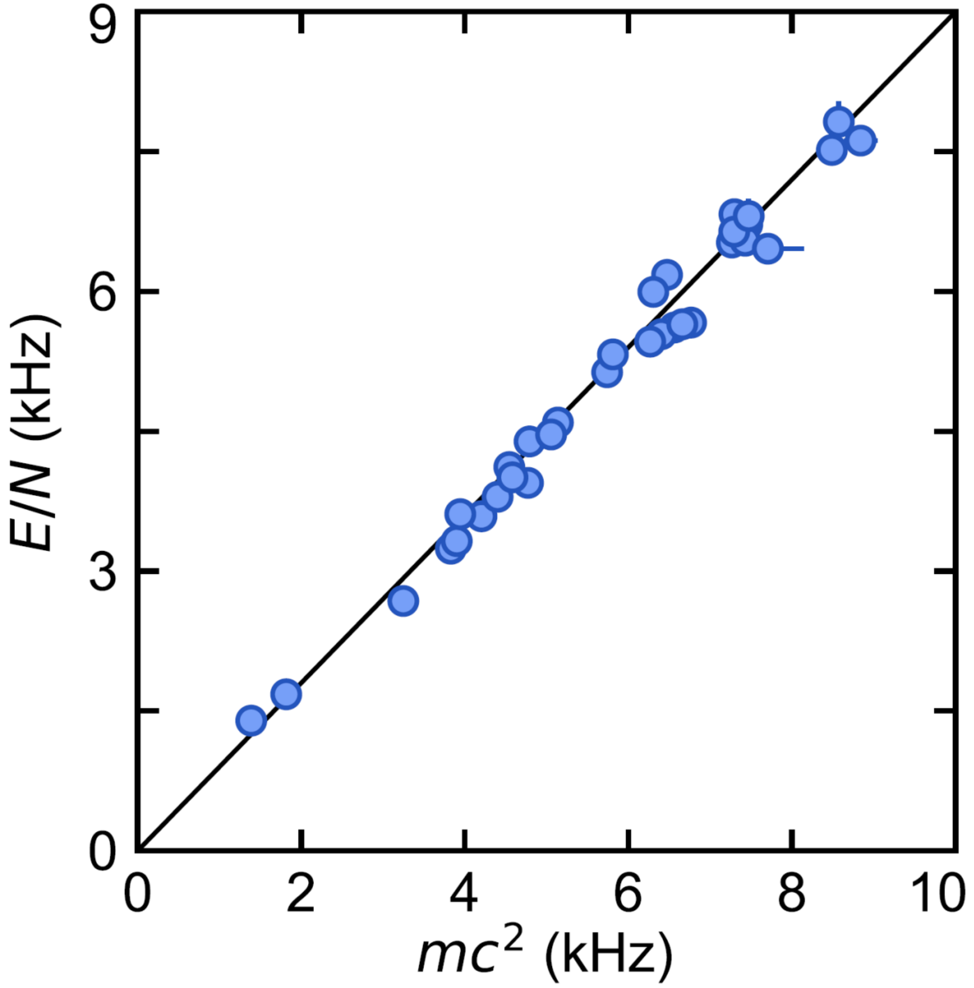
$$E = \frac{9}{10} m c^2$$

?


$E = \frac{9}{10} mc^2$

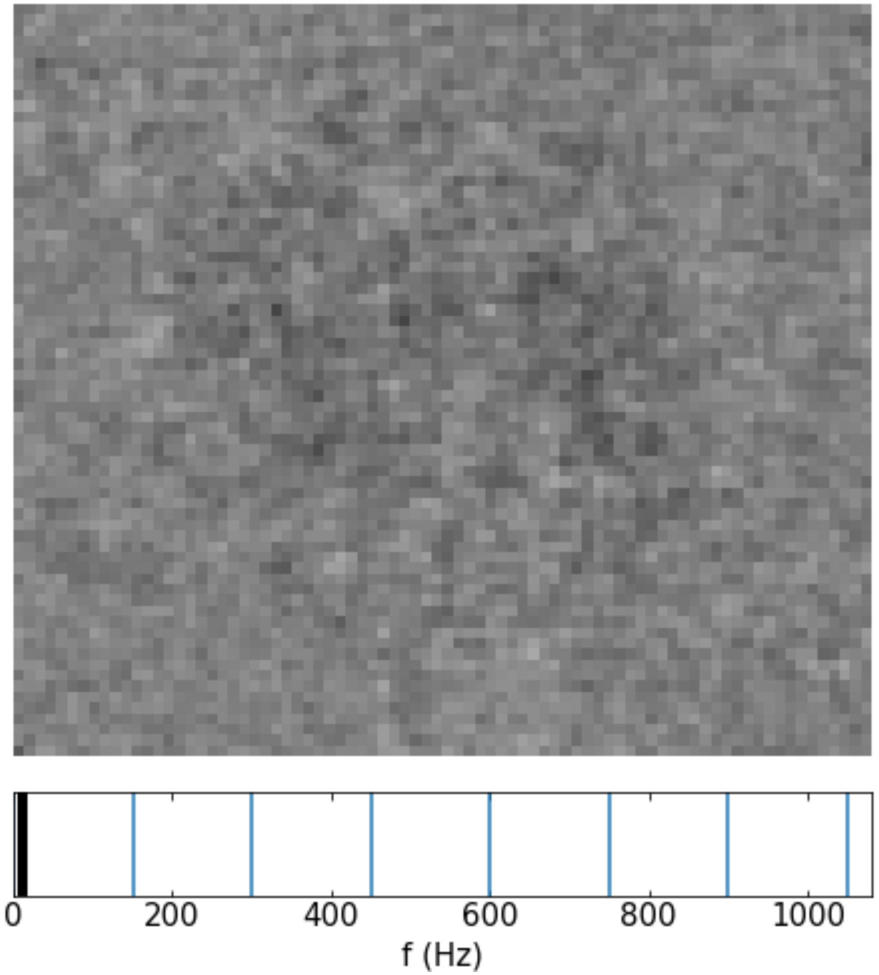
$E = \frac{9}{10} m c^2$

At unitarity: $$\boxed{mc^2 = \left.\frac{\partial p}{\partial n}\right|_s = f(s)\left.\frac{\partial p_0}{\partial n}\right|_s = \frac{5}{3}\frac{p}{n} = \frac{10}{9}\frac{E}{N}}$$



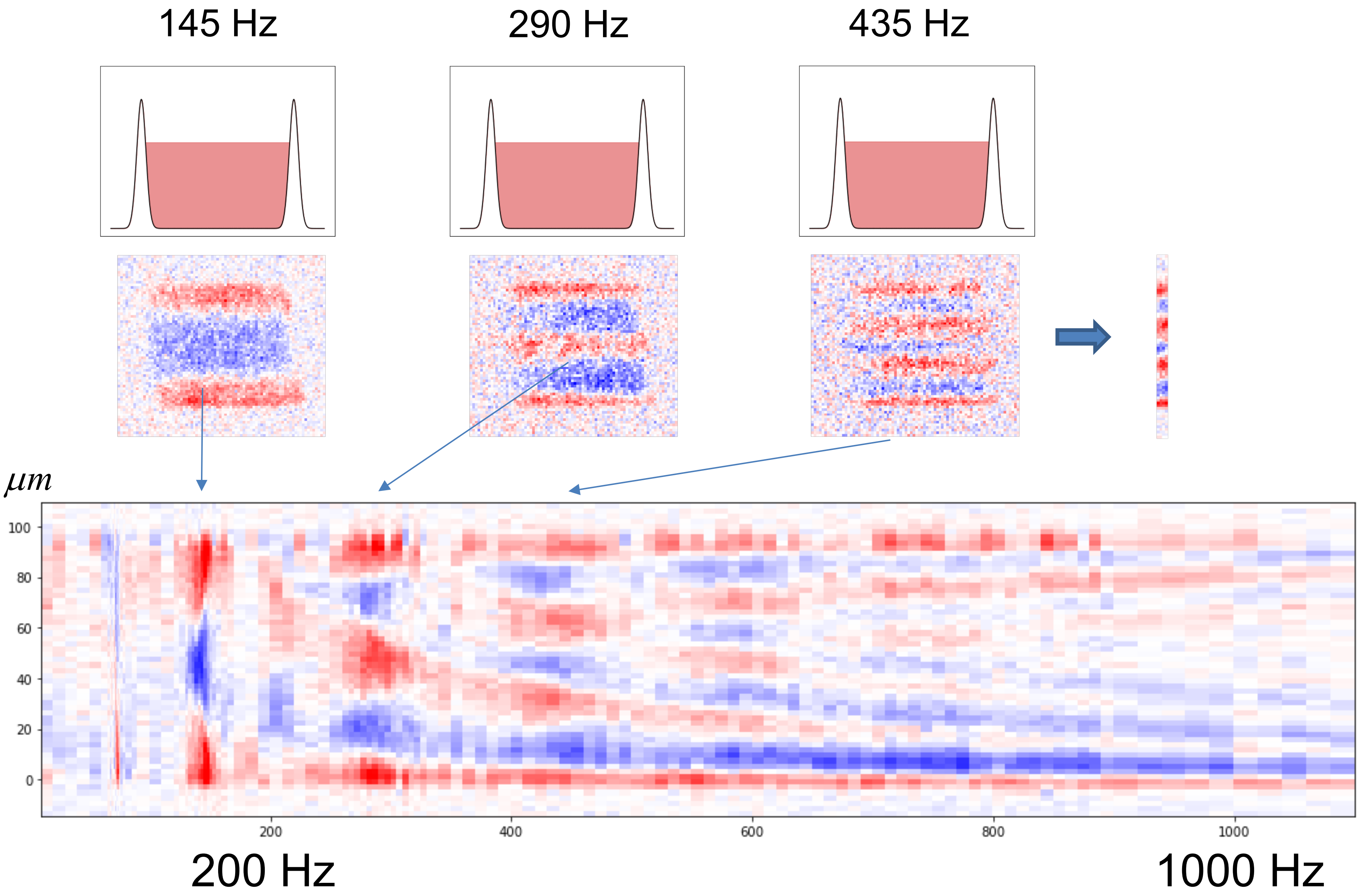
P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Resonant Modes

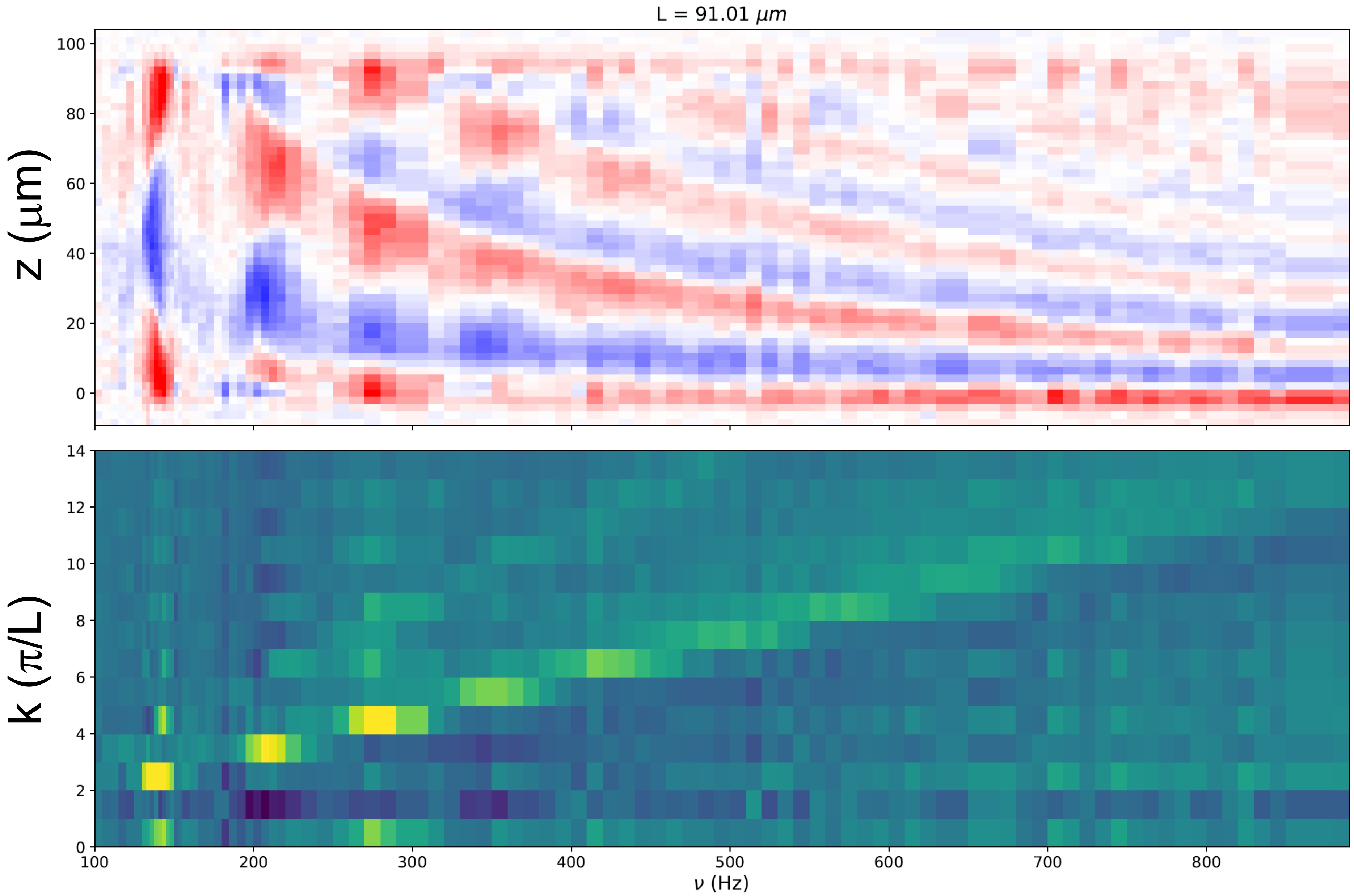


P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Resonant Modes



One-sided shaking: Even and odd



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Unitary Hydrodynamics

Mass $$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial z} = 0$$ Continuity equation

Momentum $$\frac{\partial j}{\partial t} + \frac{\partial p}{\partial z} = \frac{4}{3}\eta \frac{\partial^2 v}{\partial^2 z}$$ Navier-Stokes equation

viscosity damps fluid

No bulk viscosity (ζ) due to scale invariance

Heat $$\frac{\partial s}{\partial t} + \frac{\partial (vs)}{\partial z} = \frac{\kappa}{T}\frac{\partial^2 T}{\partial^2 z}$$ thermal conduction increases entropy

Sound Diffusivity

due to viscosity & thermal conductivity

$$\boxed{D = \frac{4}{3}\frac{\eta}{\rho} + \frac{4}{15}\frac{\kappa T}{P}}$$

Dispersion relation

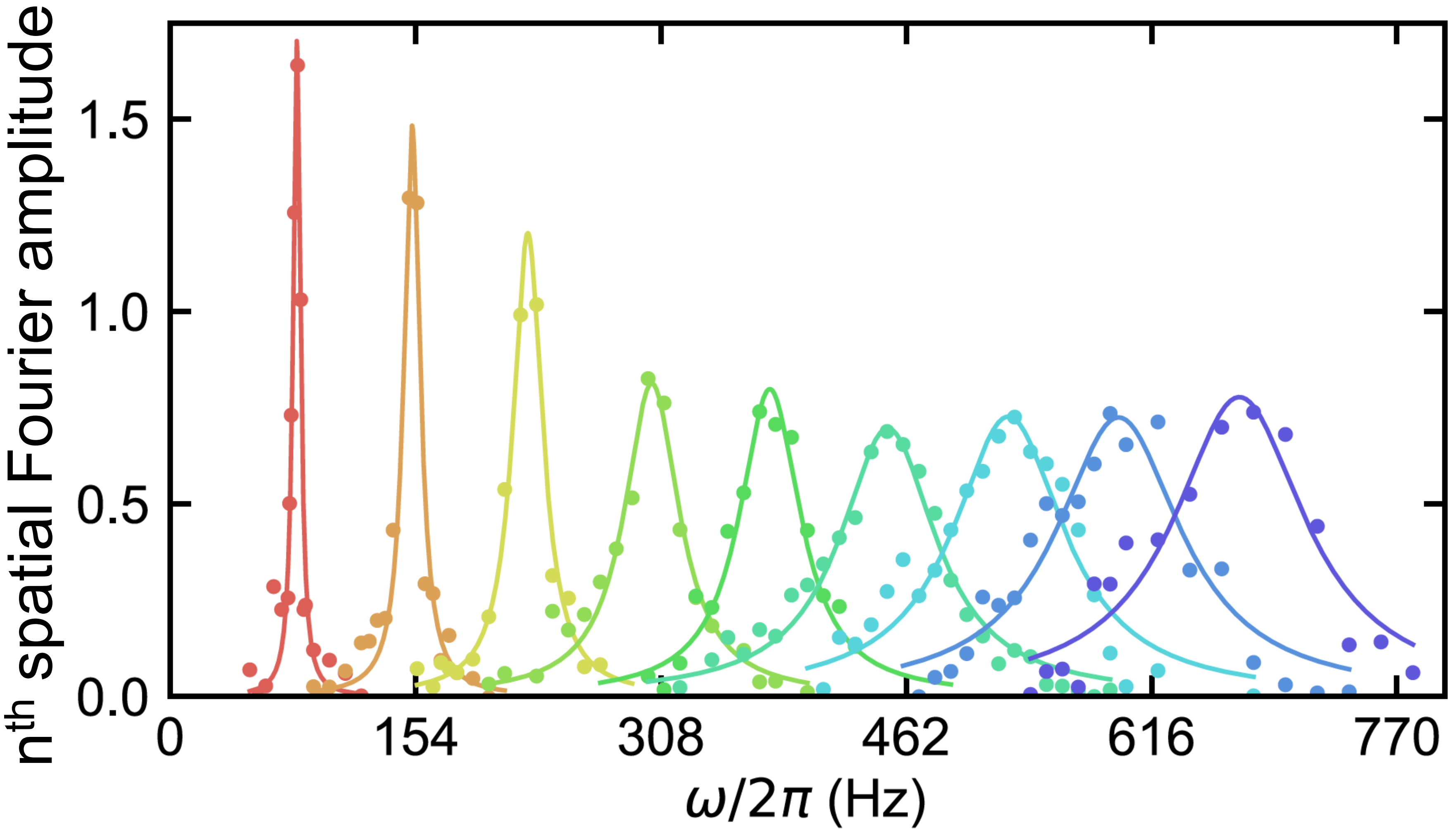
$$\boxed{\omega^2 = c^2 k^2 + i\omega D k^2}$$

Damping rate:

$$\boxed{\Gamma = Dk^2}$$

Sound resonances

A direct measurement of the density response function



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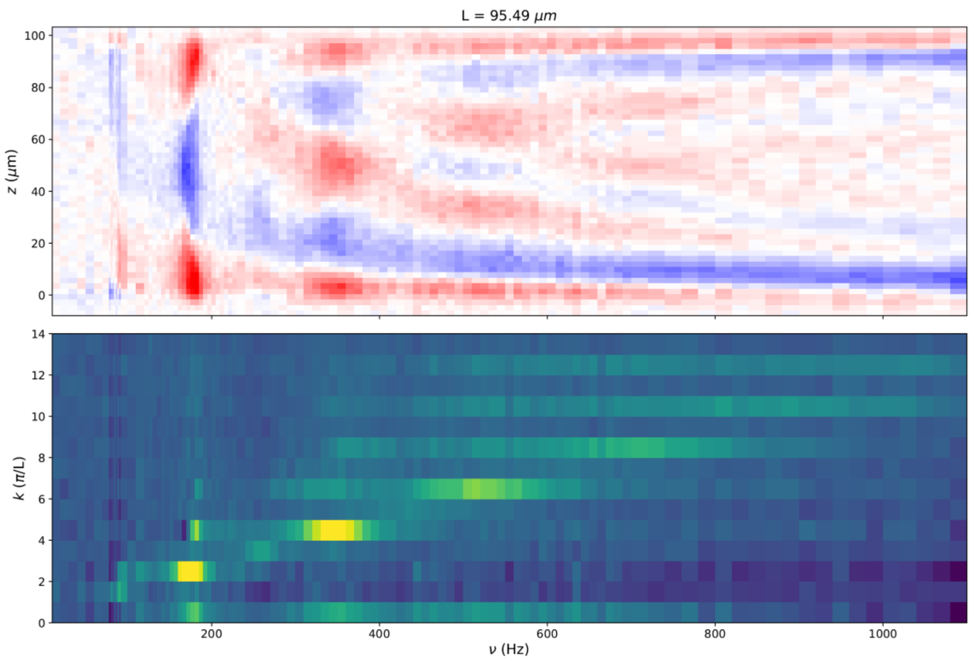
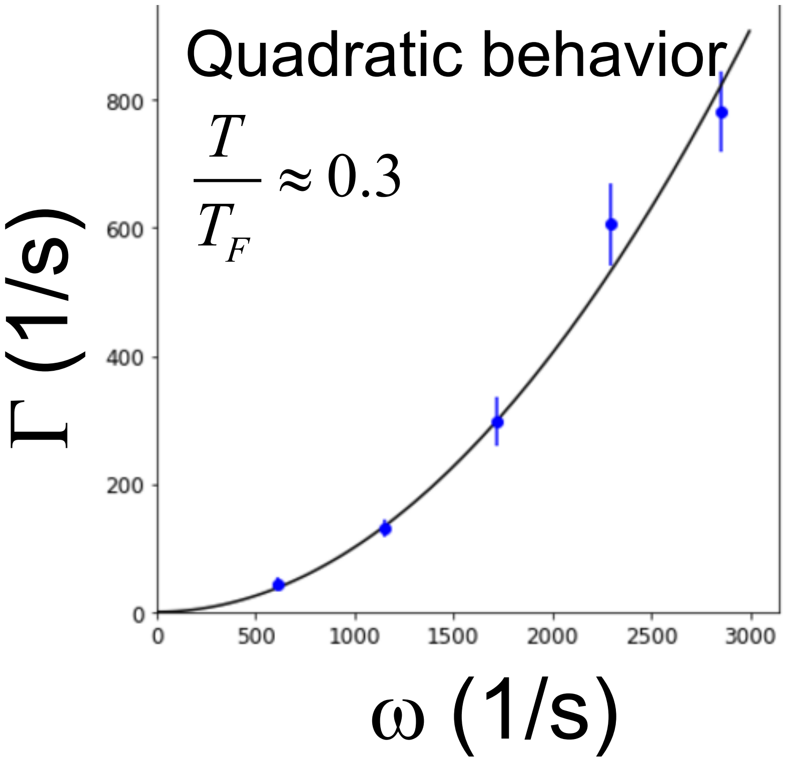
Sound Diffusivity from Sonogram Peaks





$$\boxed{D = 2.1\frac{\hbar}{m}}$$

Quantum limited sound diffusion

Spin diffusion: Sommer et al., MIT 2011

Viscosity: Schaefer, Thomas (e.g. Science 2011)

Theory: Enss, Haussmann, Zwerger, Taylor, Randeria et al.

Viscosity of Superfluid Helium-4

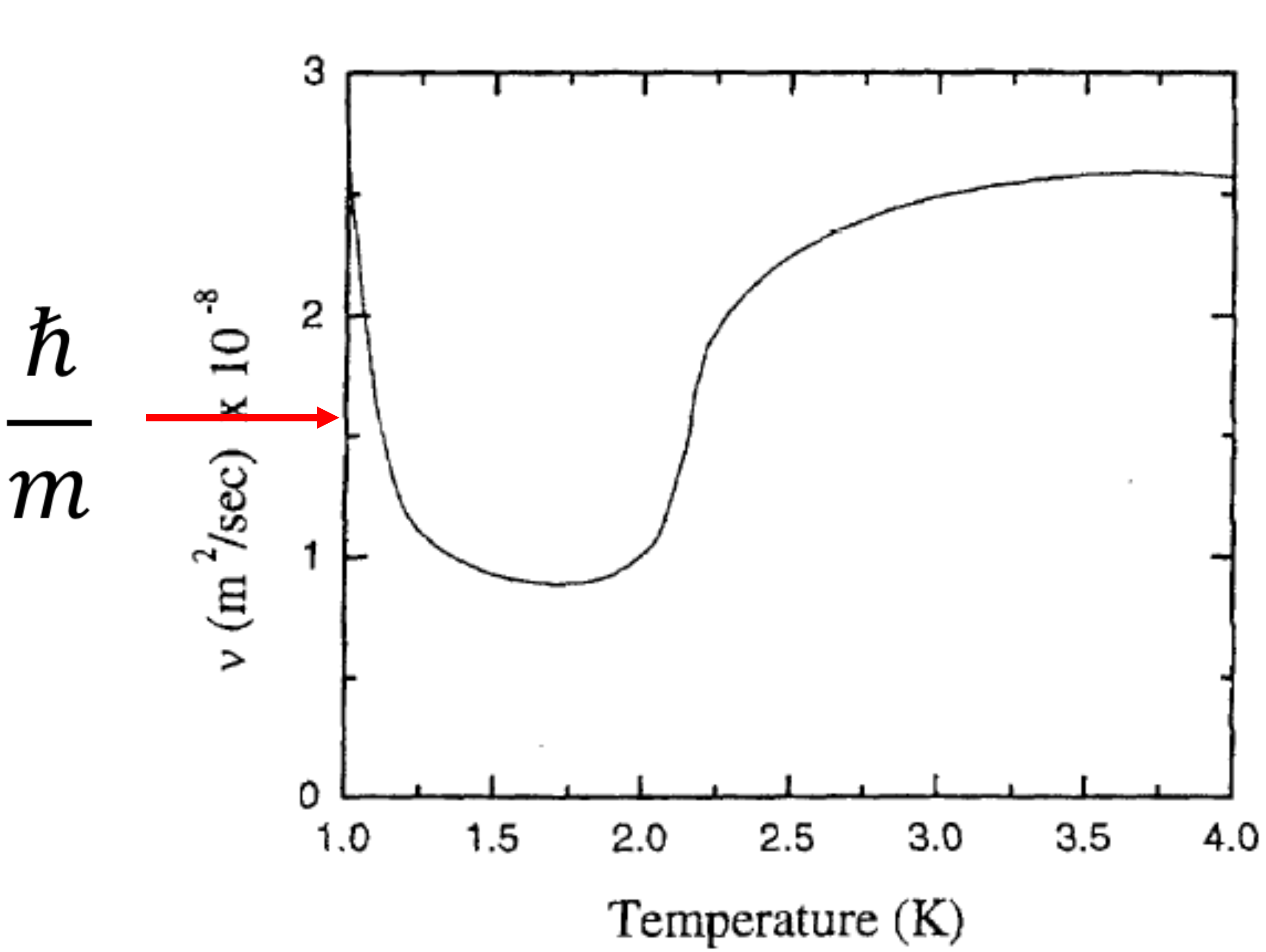
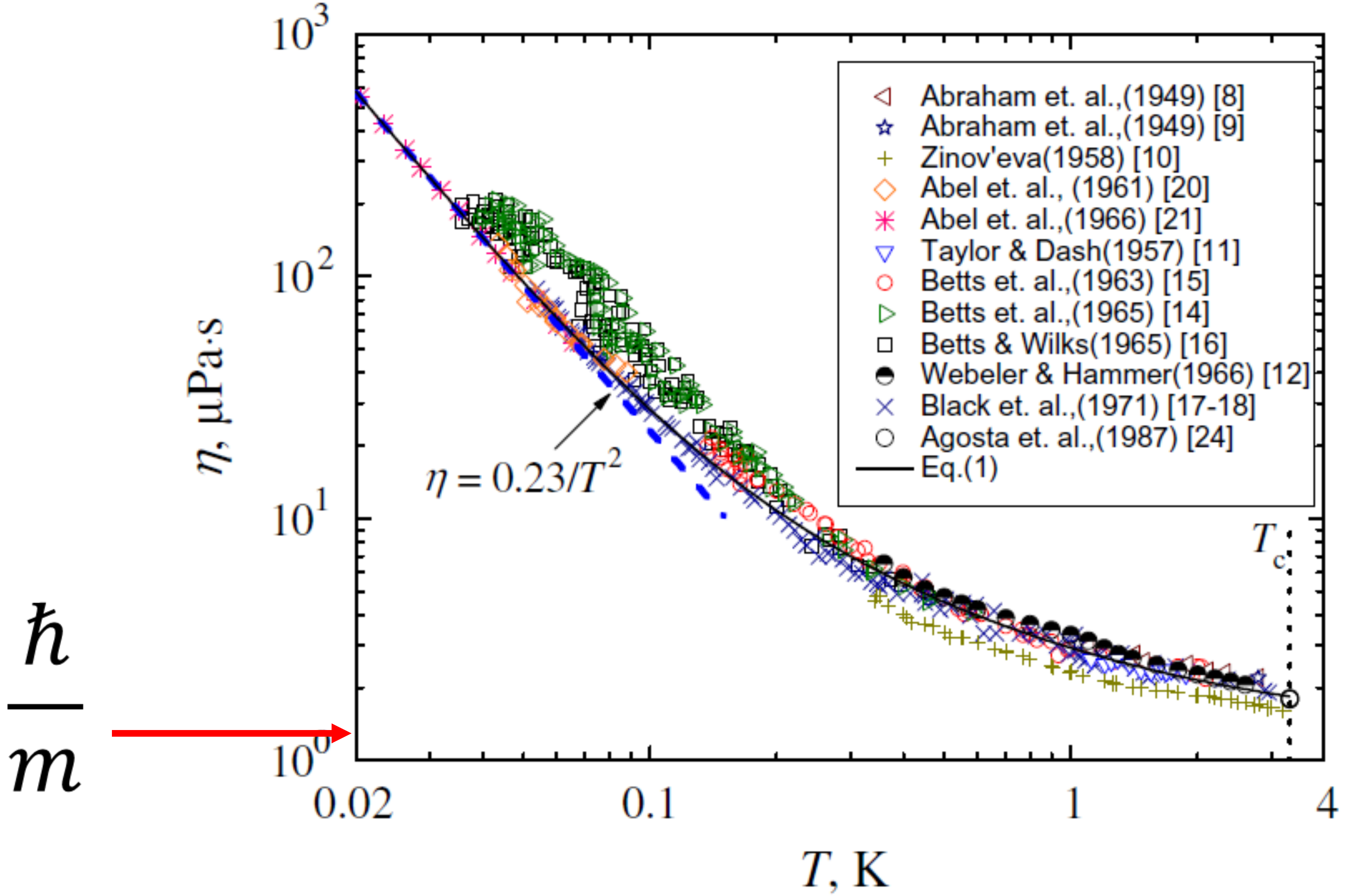


FIG. 12.3. The recommended values of the kinematic viscosity of liquid ^{4}He, $\nu = \eta/\rho$, as a function of temperature at the saturated vapor pressure.

From: Donnelly, Barenghi, JPC Reference Data, 1998

Viscosity of Superfluid Helium-3

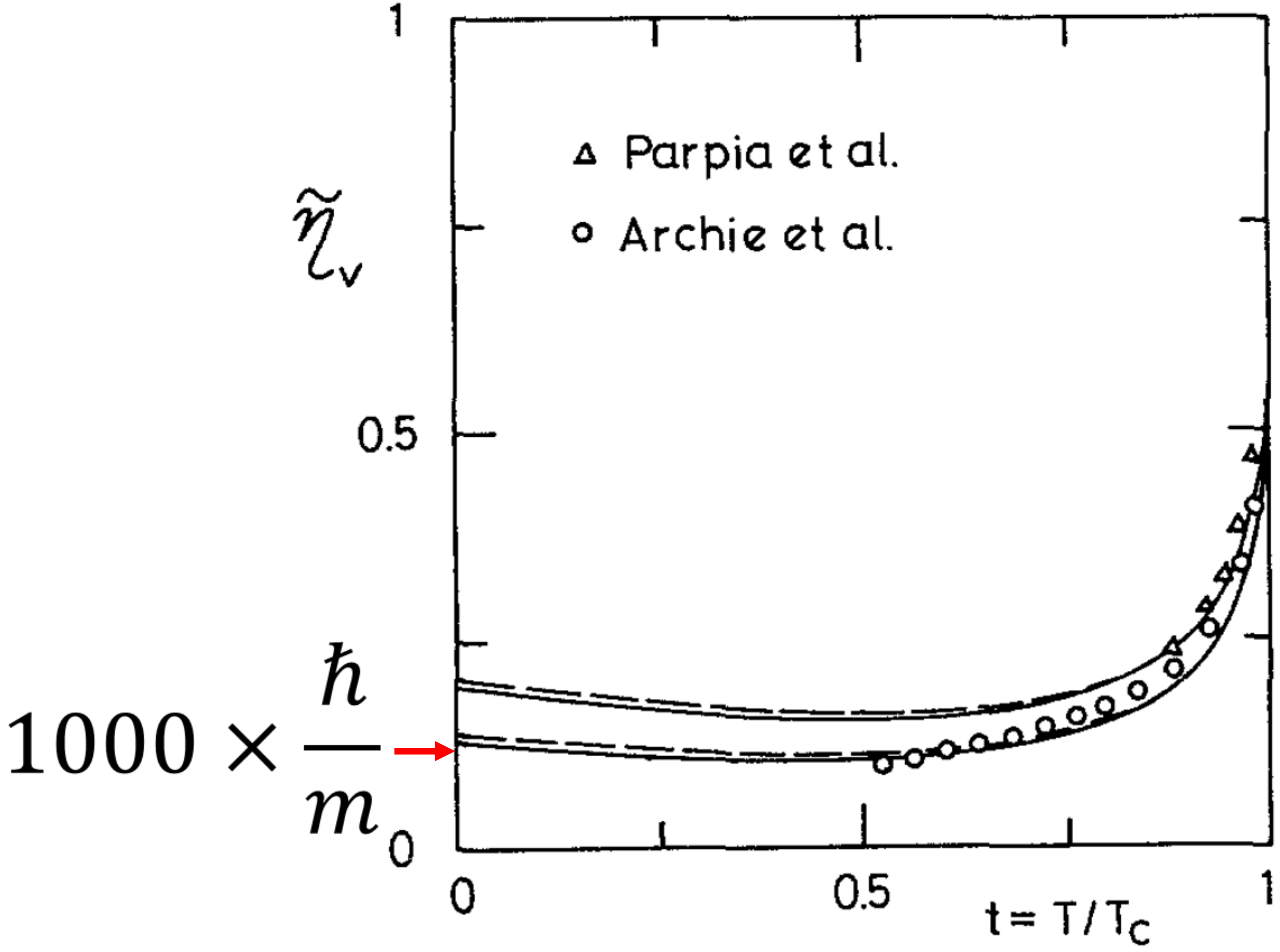
Helium-3 above T_c: Fermi Liquid behavior



Huang et al., Cryogenics 52, 538 (2012)

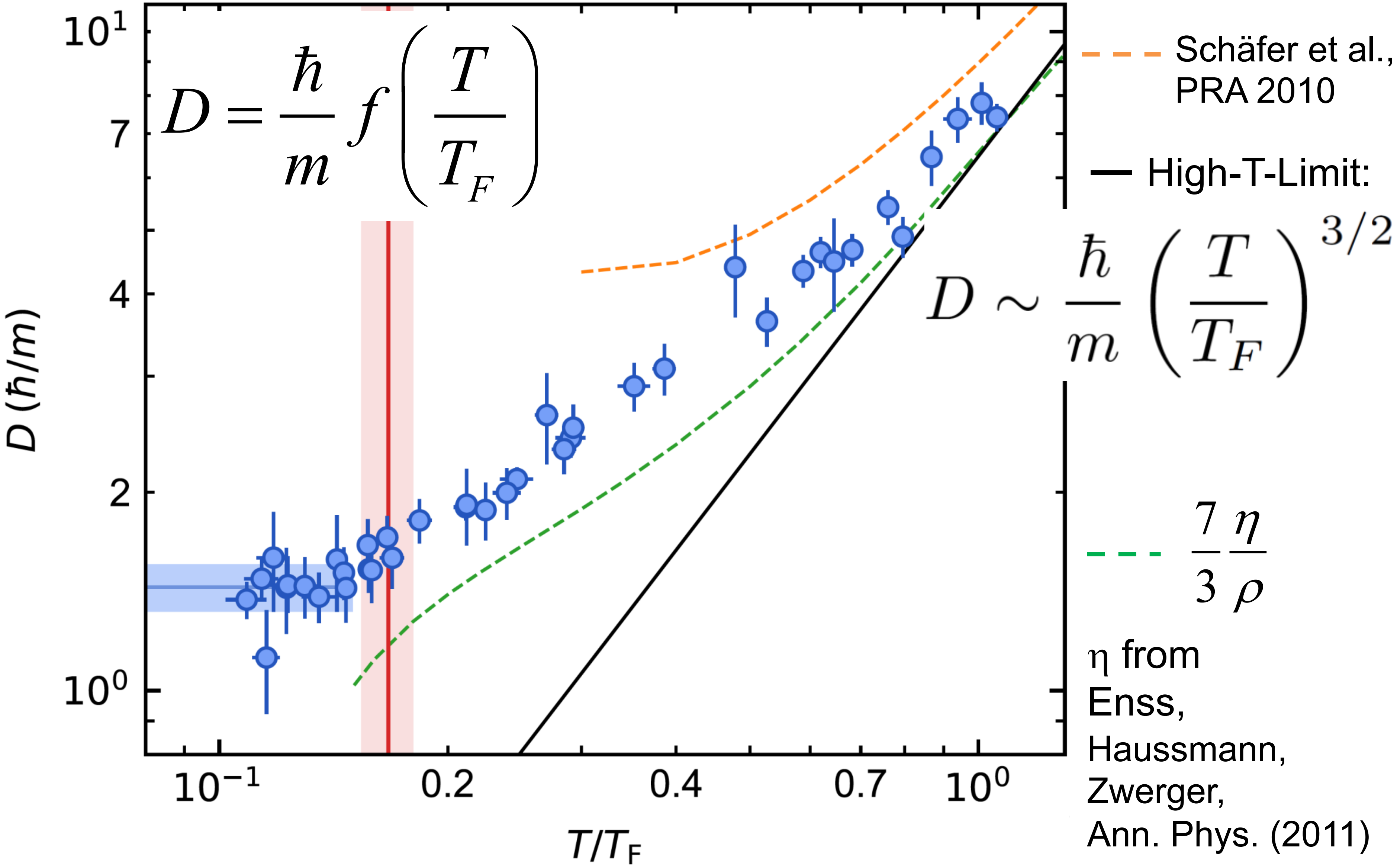
Viscosity of Superfluid Helium-3

Helium-3 below T_c: Sudden decrease, then constant



Ono et al.,
JLTP 48, 167
1982

Quantum Limited Sound Diffusion



P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Unitary Two-Fluid Hydrodynamics

For 1D flow, linearized:

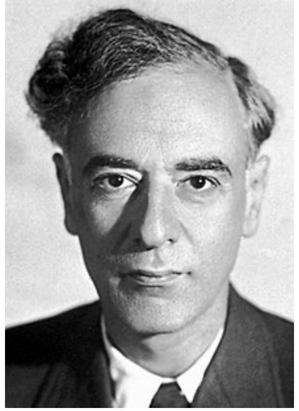
Laszlo Tisza

$$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial z} = 0$$

Continuity equation

$$\frac{\partial j}{\partial t} + \frac{\partial p}{\partial z} = \frac{4}{3}\eta \frac{\partial^2 v_n}{\partial^2 z}$$

Navier-Stokes eq.
viscosity
damps normal fluid



Lev Landau

$$\frac{\partial s}{\partial t} + \frac{\partial (v_n s)}{\partial z} = \frac{\kappa}{T}\frac{\partial^2 T}{\partial^2 z}$$

Entropy moving
with normal fluid,
thermal conduction
increases entropy

Superfluid flow:

$$\frac{\partial v_s}{\partial t} + \frac{\partial \mu / m}{\partial z} = \rho_s \zeta_3 \frac{\partial^2 (v_s - v_n)}{\partial^2 z}$$

bulk viscosity
normal-superfluid
interconversion

+Thermodynamics relating p, ρ, T, s, μ

No other transport coefficients ($\zeta_1, \zeta_2, \zeta_4$)
due to scale invariance (Son, Stringari&Pitaevskii)

First and Second Sound

Two fluids → Two sound modes

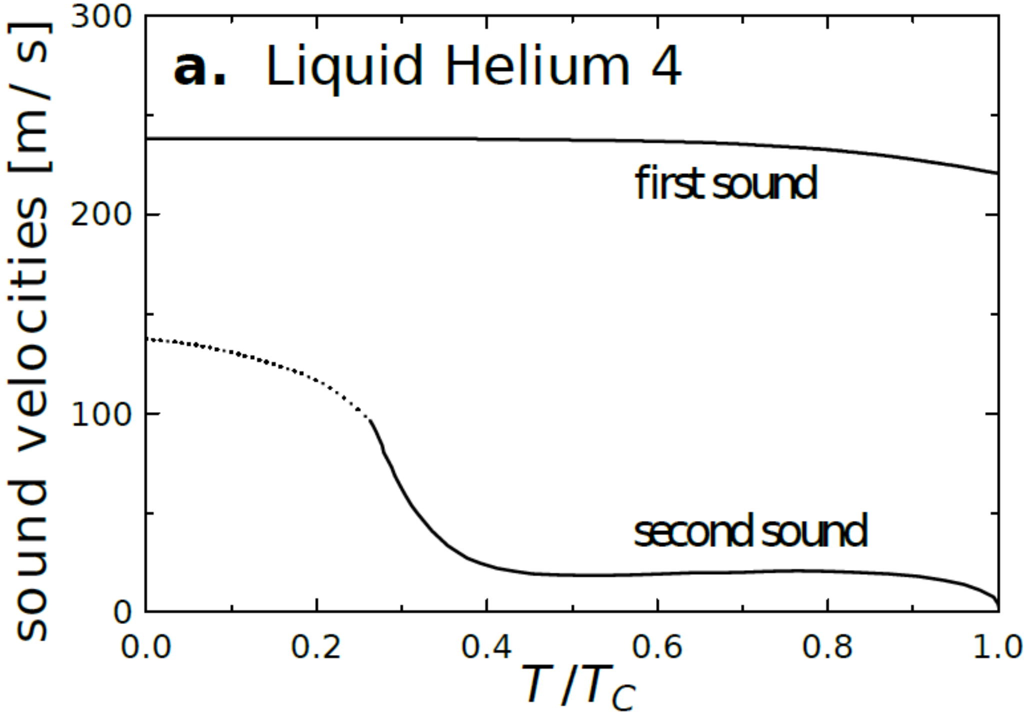
Review: Stringari&Pitaevskii arXiv:1510.01306

For small expansivity: Density and Heat

$$c_{10}^2 = \left.\frac{\partial p}{\partial \rho}\right|_{S,N} = \frac{1}{\rho \kappa_s}$$

First sound, a density wave normal and superfluid oscillate in phase

$$c_{20}^2 = \frac{\rho_s}{\rho_n} \left.\frac{\partial T}{\partial \sigma}\right|_{\rho} \frac{\sigma^2}{\gamma} = \frac{\rho_s}{\rho_n} \frac{\sigma^2 T}{c_p}$$



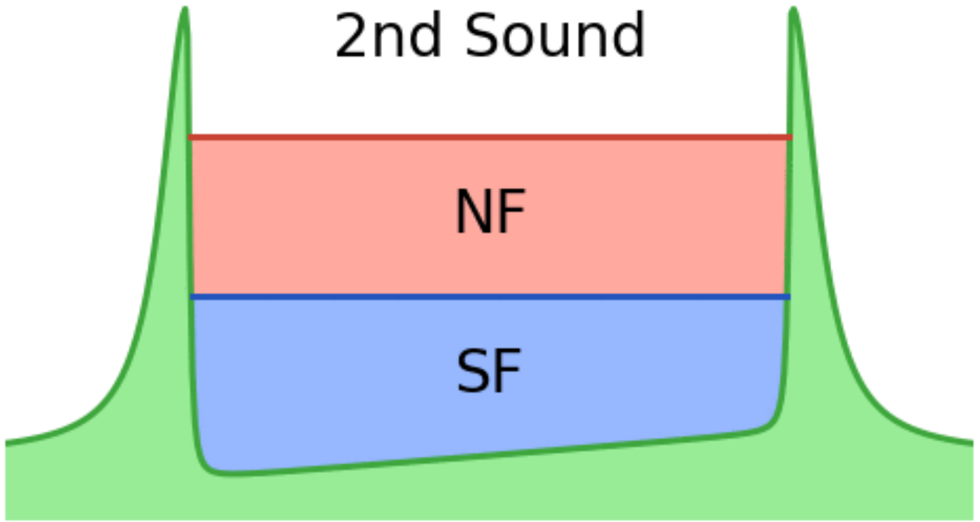
Second sound, a heat wave, normal and superfluid oscillate out of phase

Second sound in a box

(Isentropic) expansivity at unitarity $-\frac{1}{\rho}\frac{\partial\rho}{\partial T}\Big|_S = -\frac{3}{2}\frac{1}{T}$ always non-zero

→ Can drive temperature wave using potential acting on density

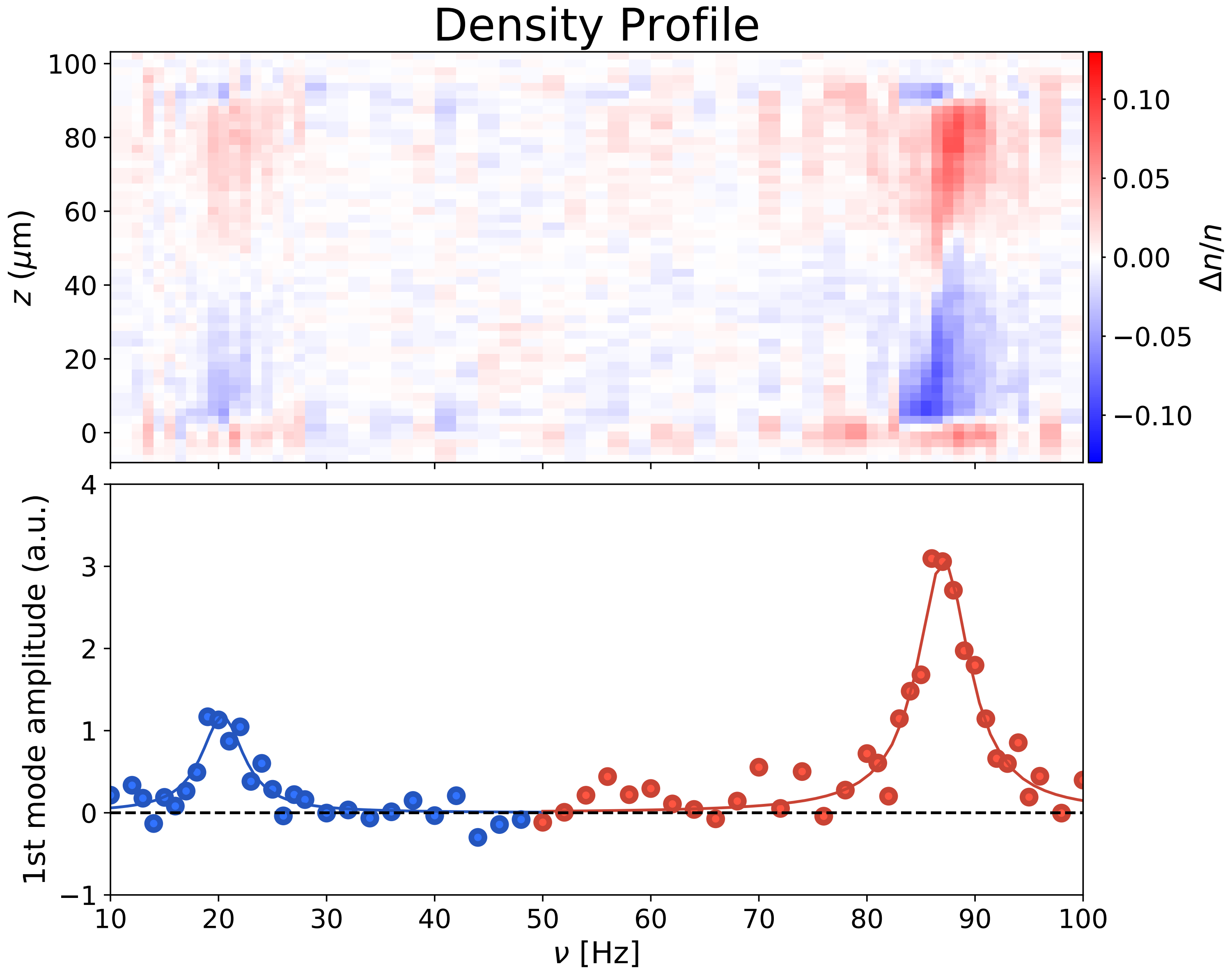
Apply oscillating gradient to the box



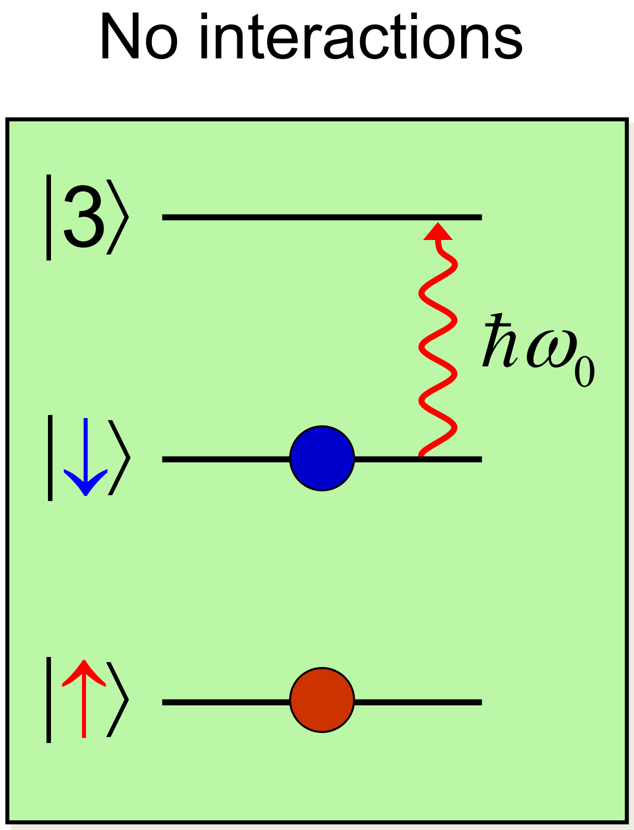
Observation of Second Sound in Quasi-1D geometry:
Sidorenkov et al., Grimm, Stringari&Piatevskii, Nature 2013

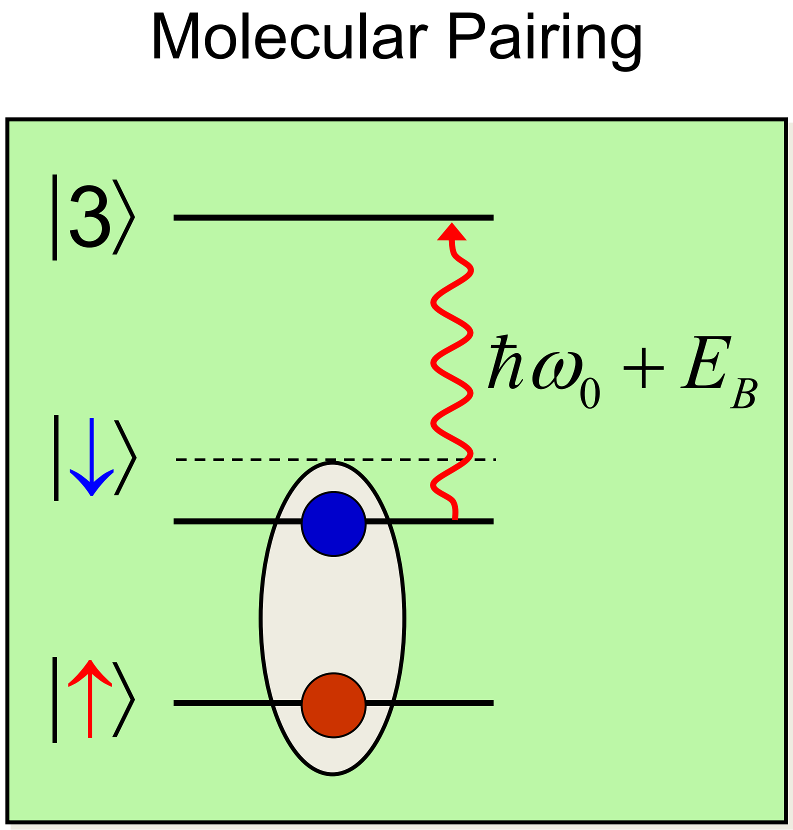
On bosons: weakly interacting situation: JILA (Debbie Jin 1996), MIT
Hydrodynamic gas: van der Straten

Second Sound seen in Density



Radiofrequency spectroscopy

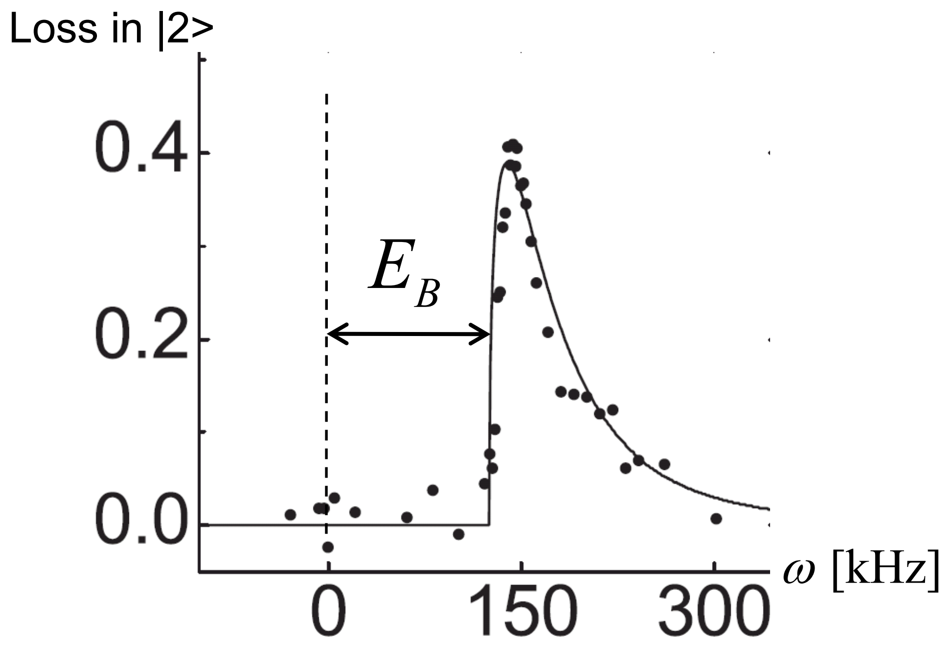




Photon energy = Zeeman + Binding + Kinetic energy

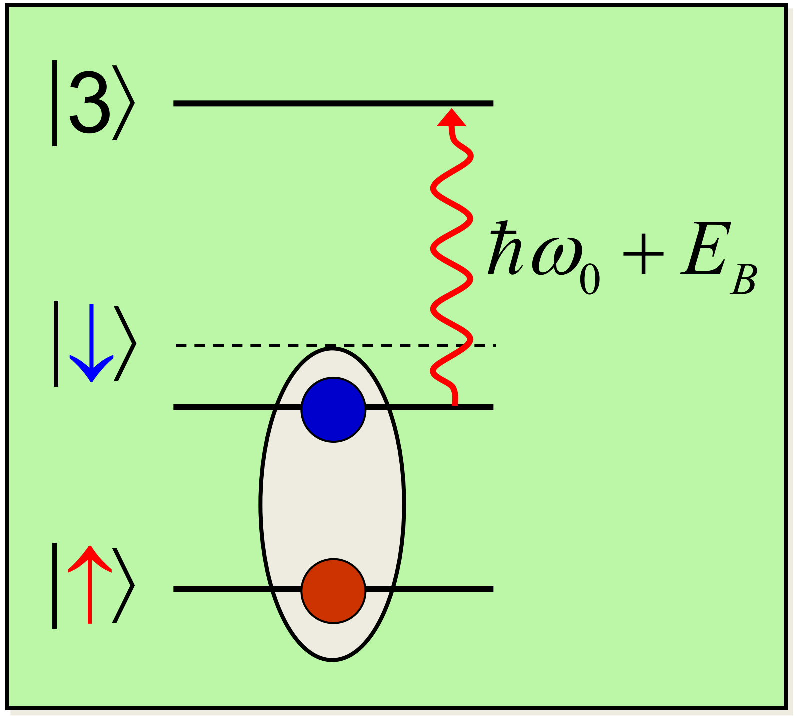
$$\hbar\omega = \hbar\omega_0 + E_B + 2\varepsilon_k$$

Radiofrequency spectroscopy



C.Chin et al. Science, **305**, 1128 (2004)

Molecular Pairing

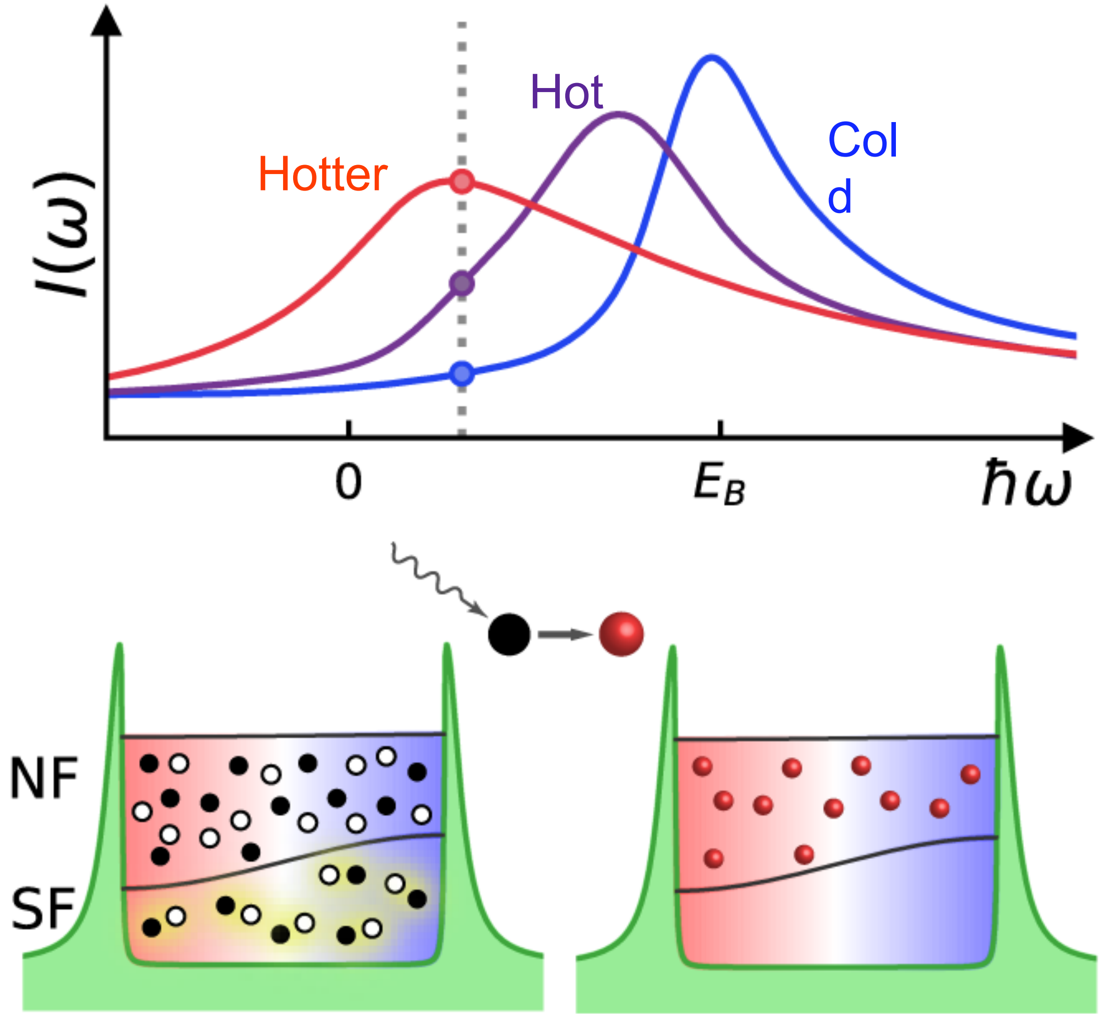


Photon energy = Zeeman + Binding + Kinetic energy

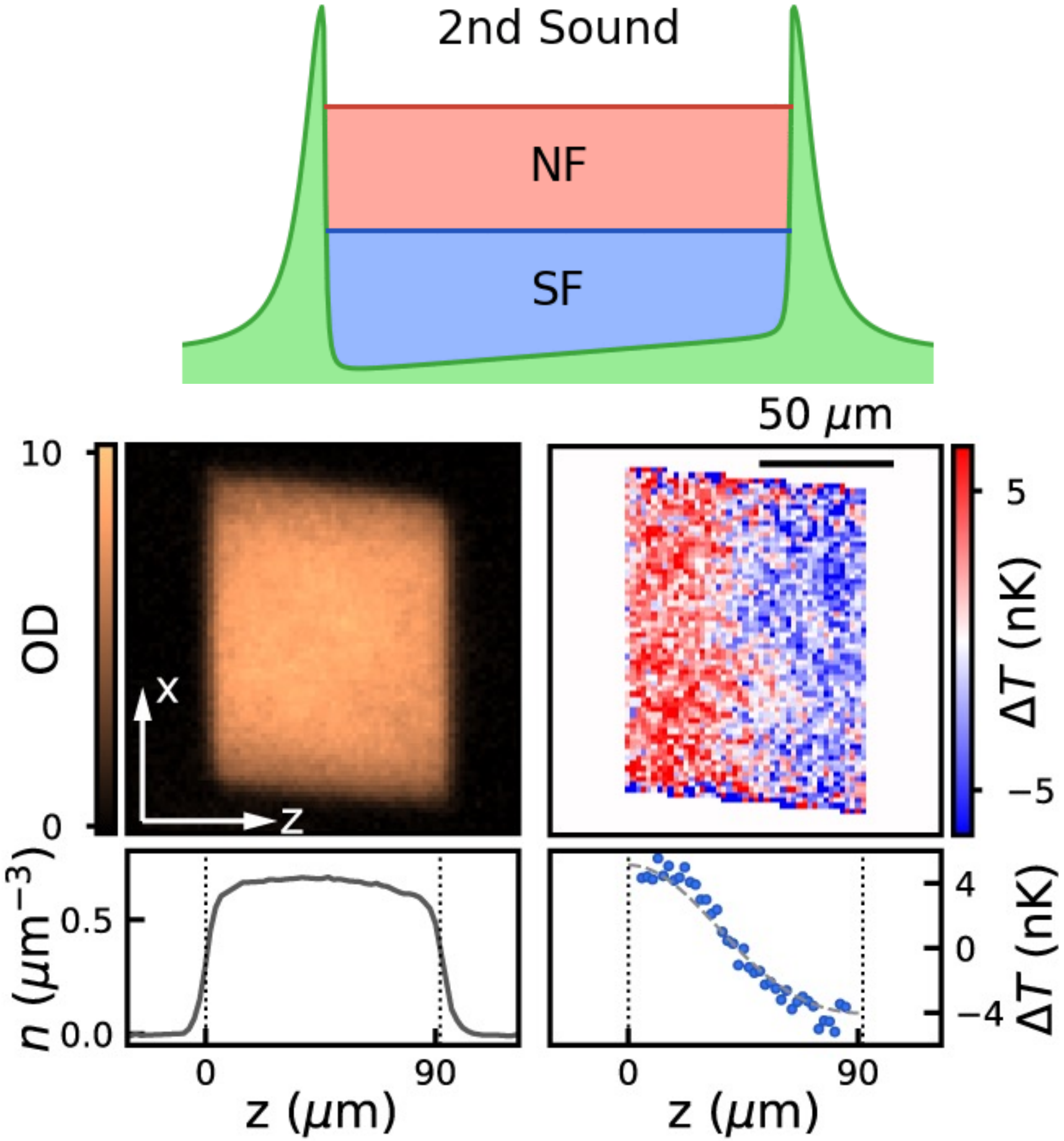
$$\hbar\omega = \hbar\omega_0 + E_B + 2\varepsilon_k$$

A local thermometer for heat transport

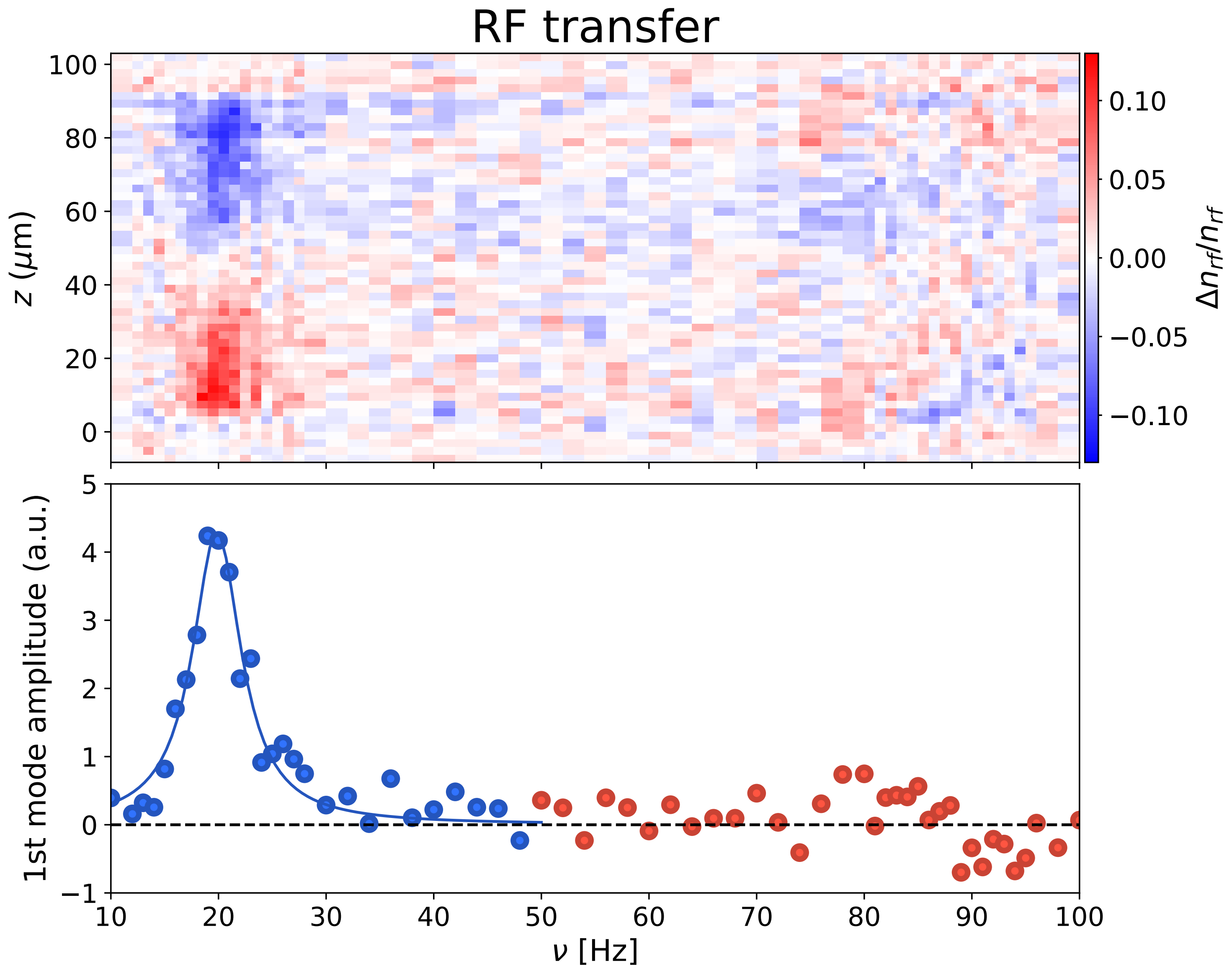
RF absorption spectra



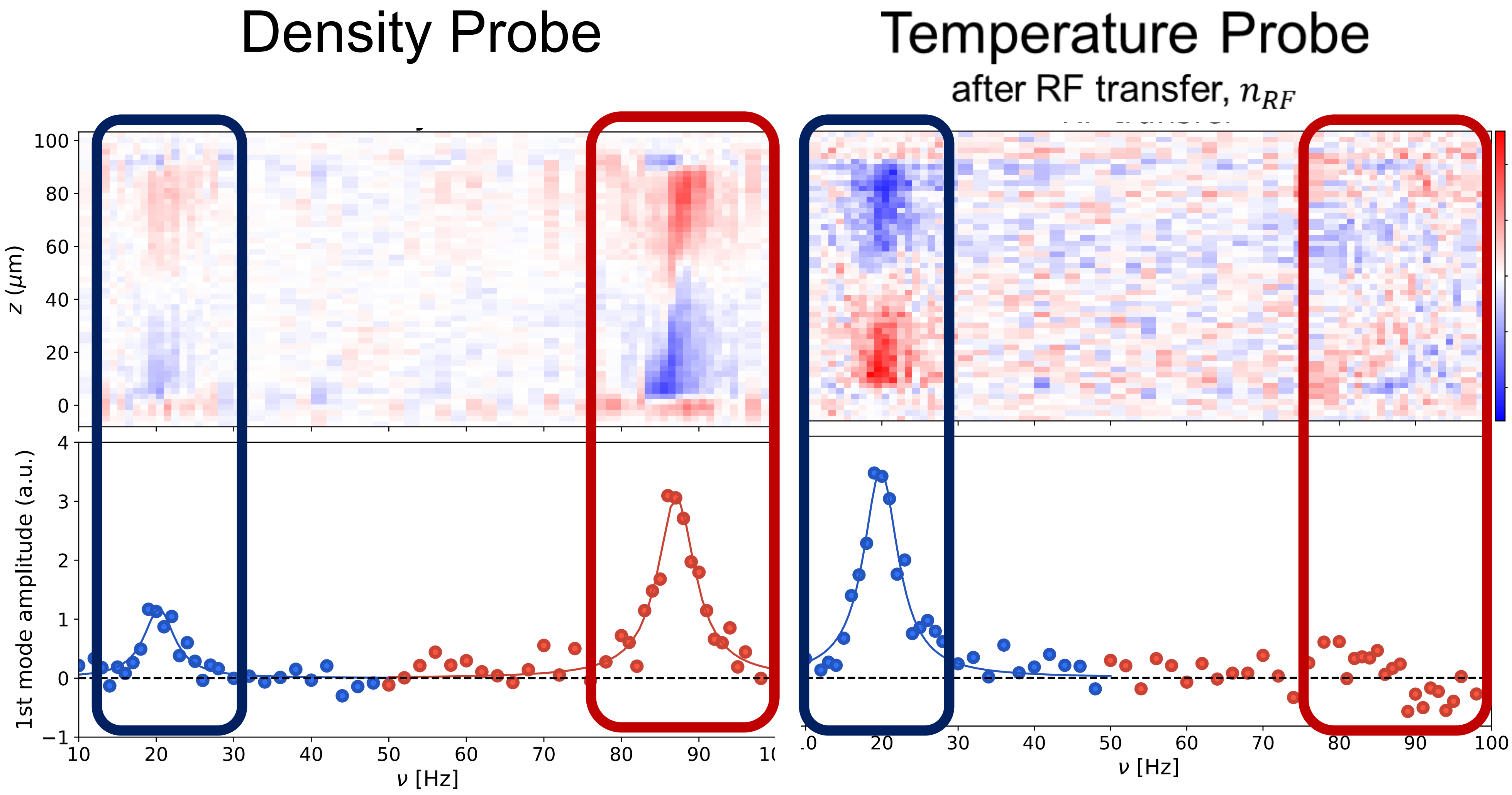
2nd Sound Creation



Second Sound with local thermometer

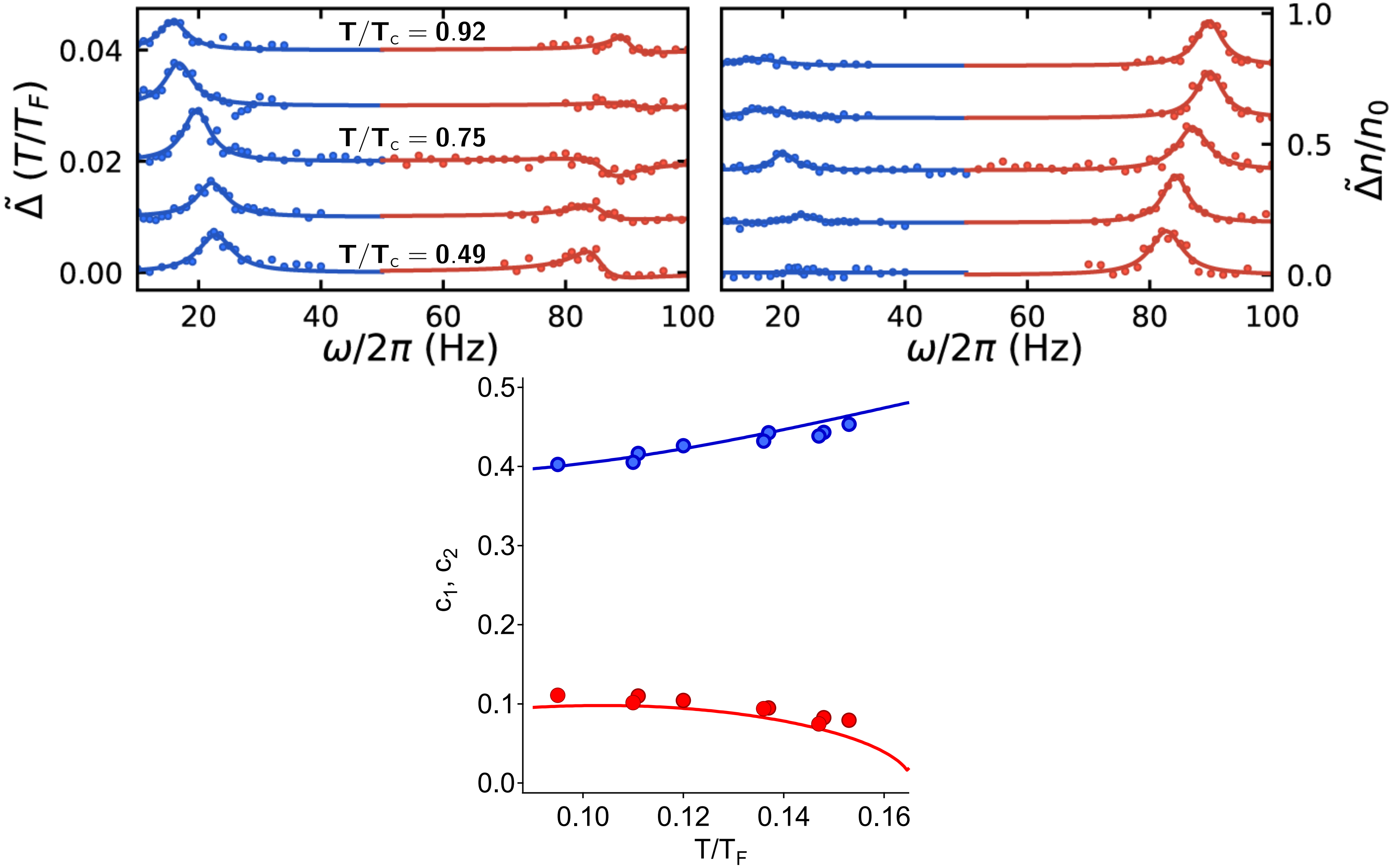


First and Second Sound



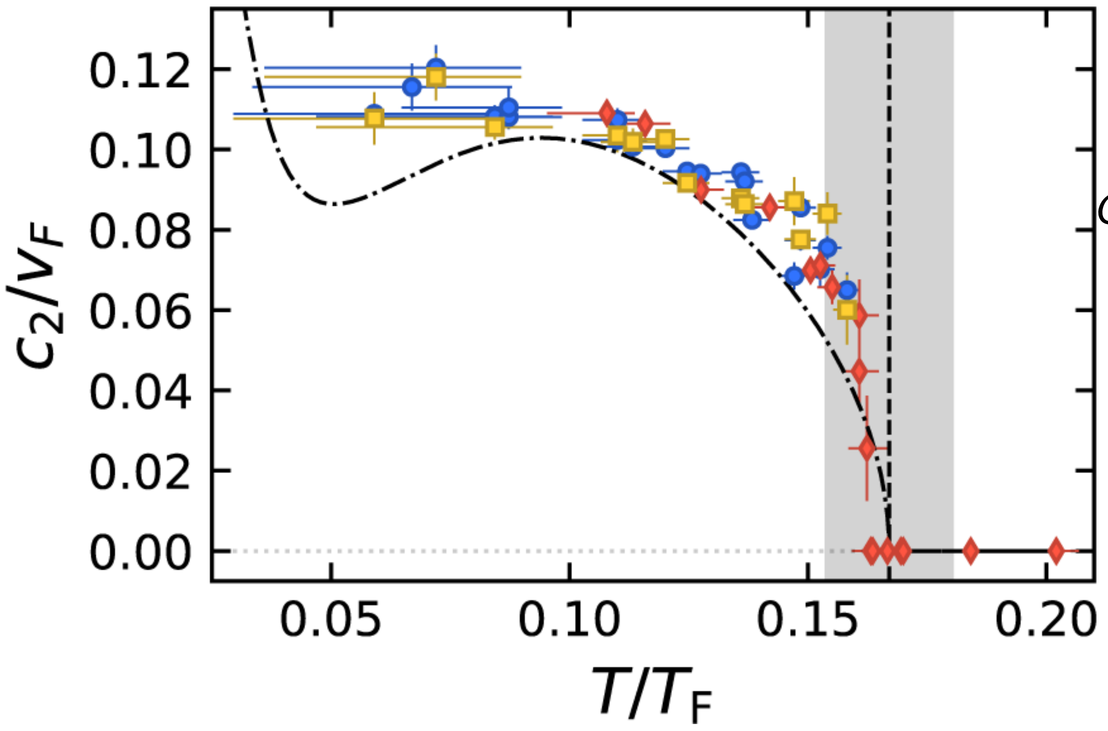
→ Speeds and Decay rates of First and Second sound

Speed of first and second sound

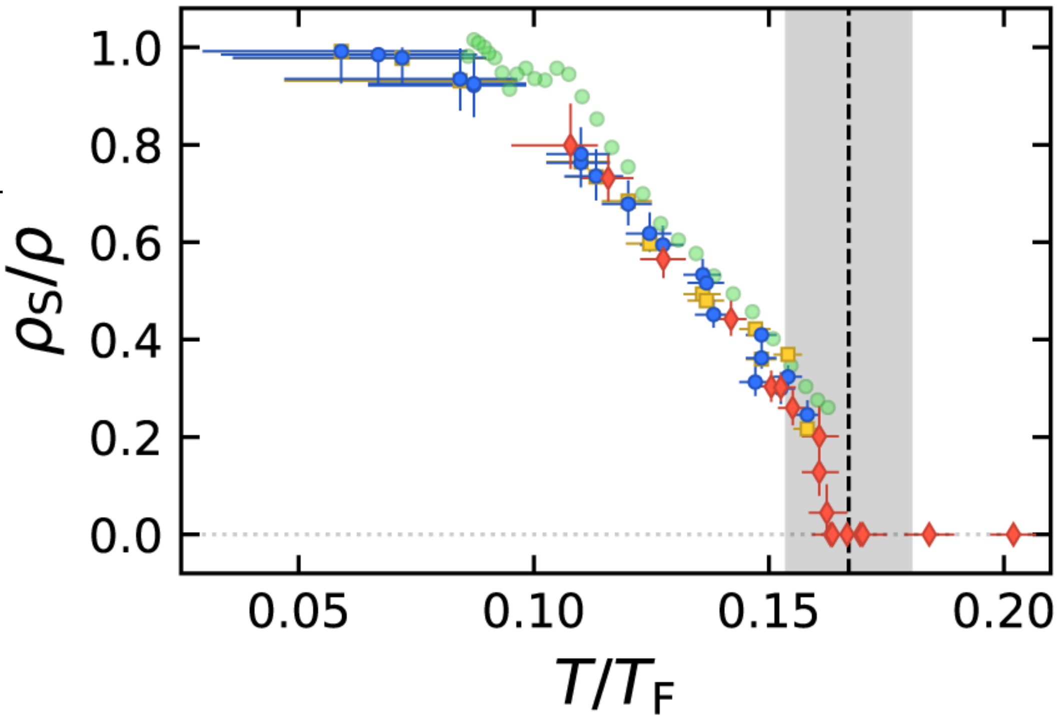


Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

Superfluid fraction



$$c_2^2 = \frac{\rho_S}{\rho_N}\frac{TS^2}{C_p}$$

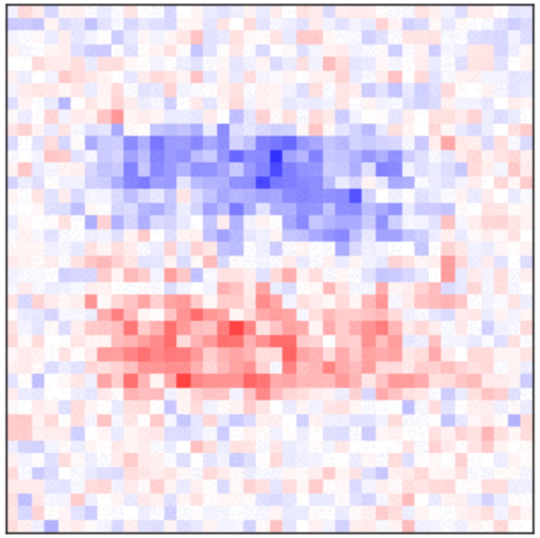


- Steady state response
- Free evolution, resonant shaking
- Free evolution, local heater

Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

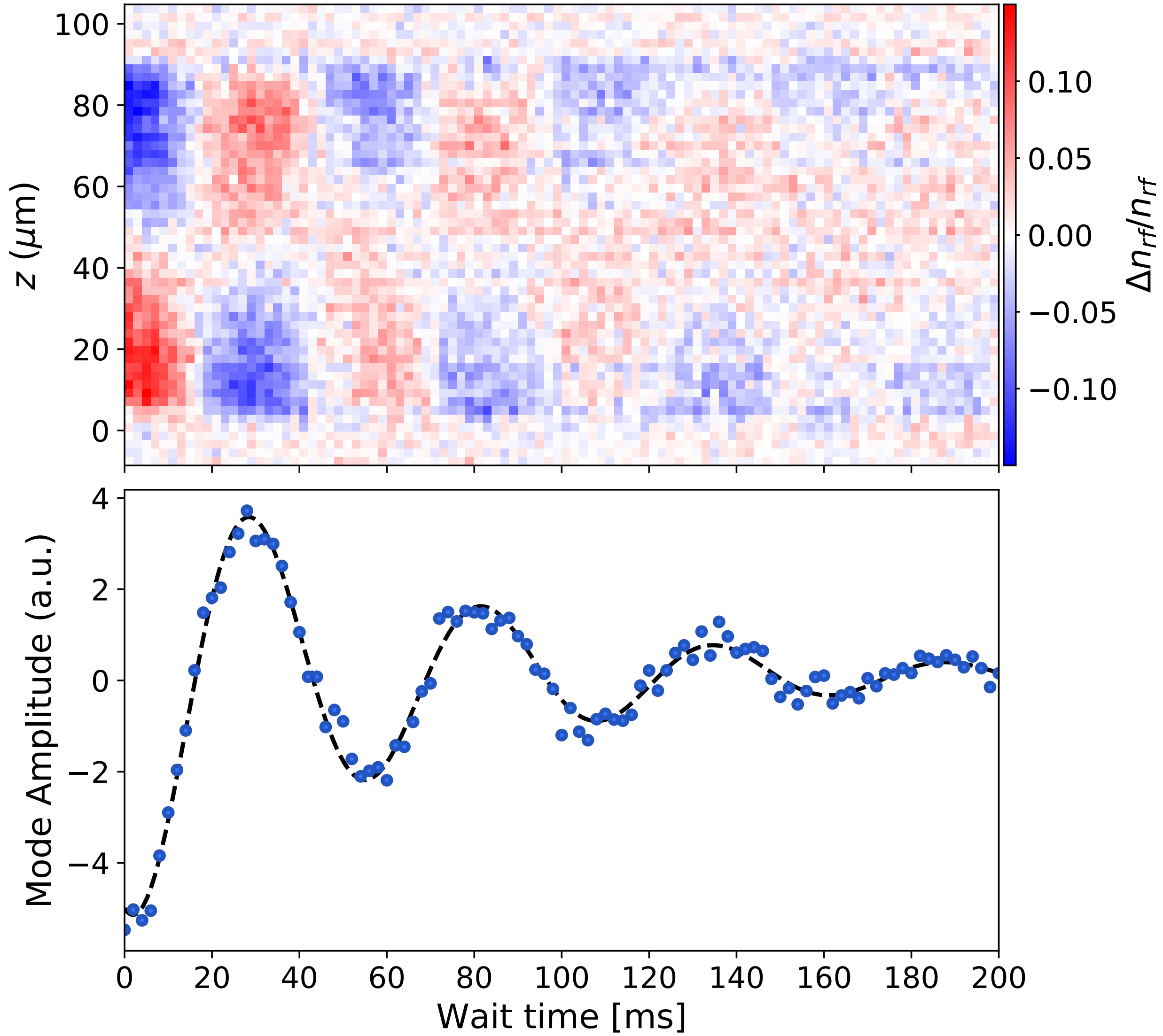
Damping of Second Sound

2nd sound
Rf transfer



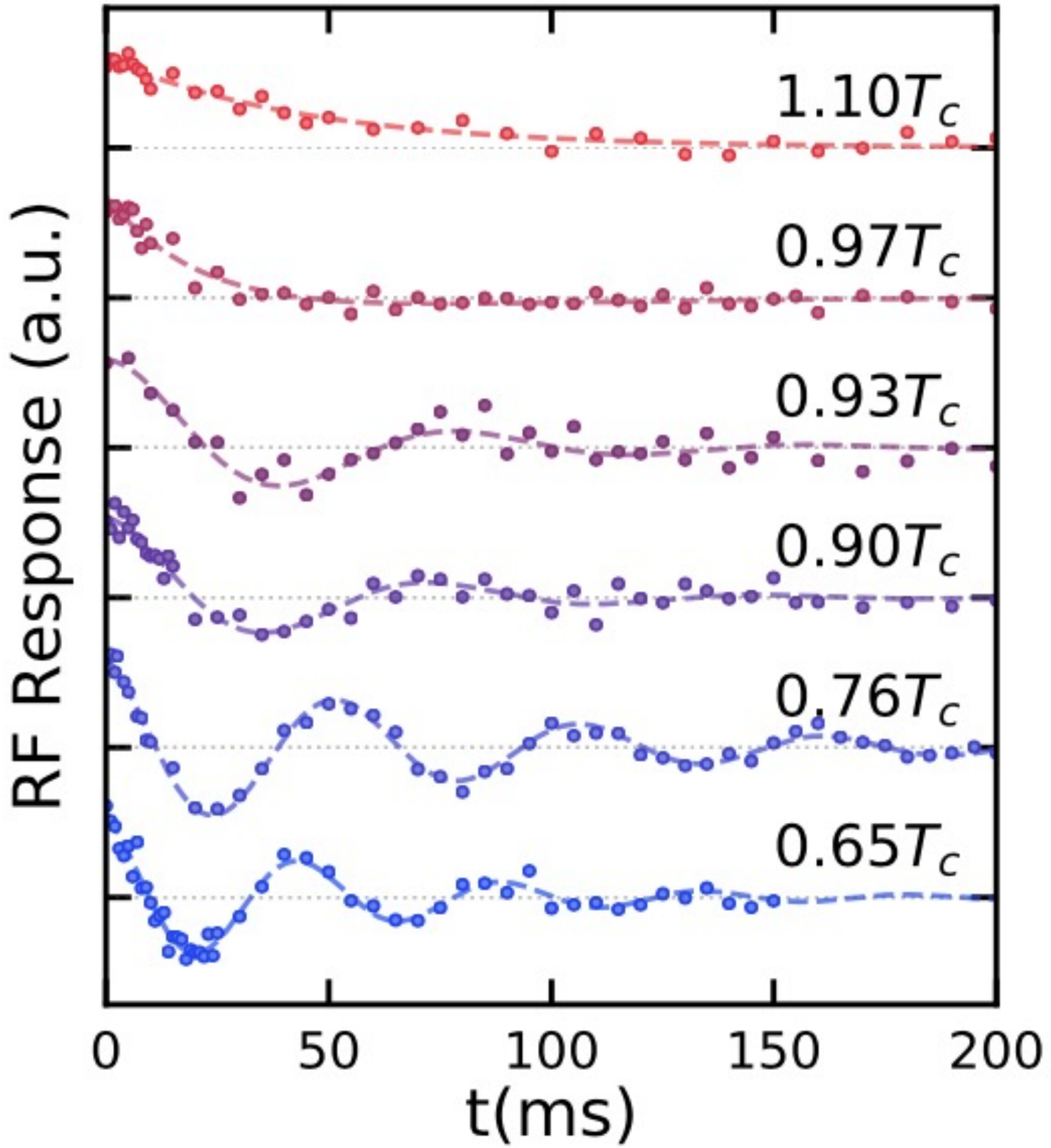
t= 0 ms

Temperature Probe



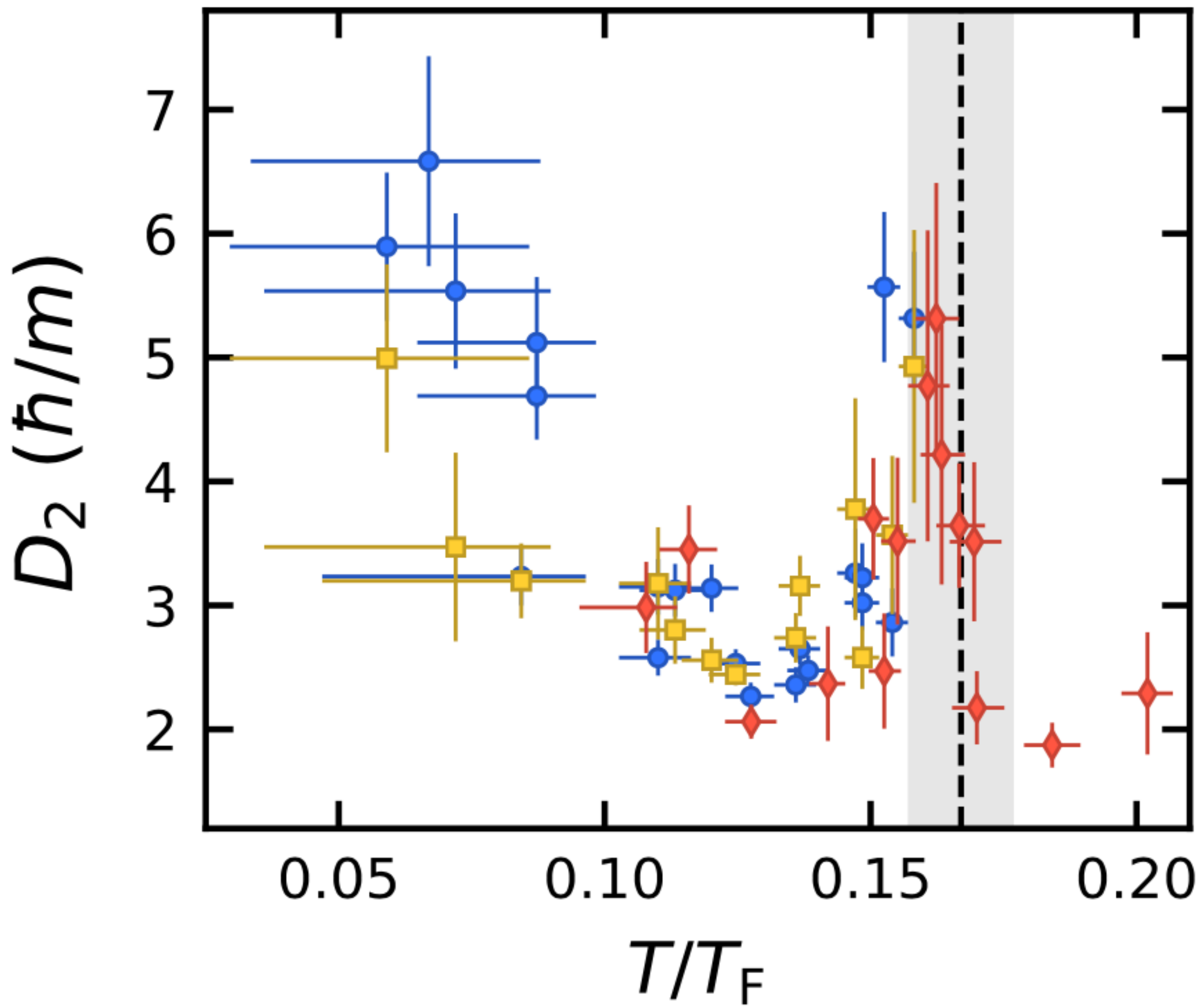
Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

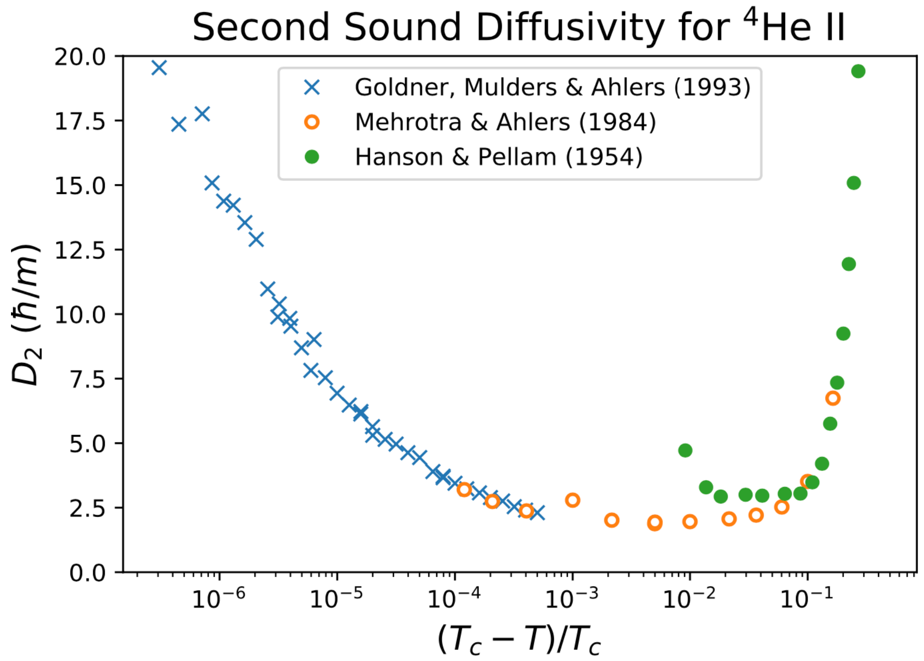
Transition from normal to superfluid



Diffusivity of Second Sound

$$D_2 = \Gamma_2 / k^2$$



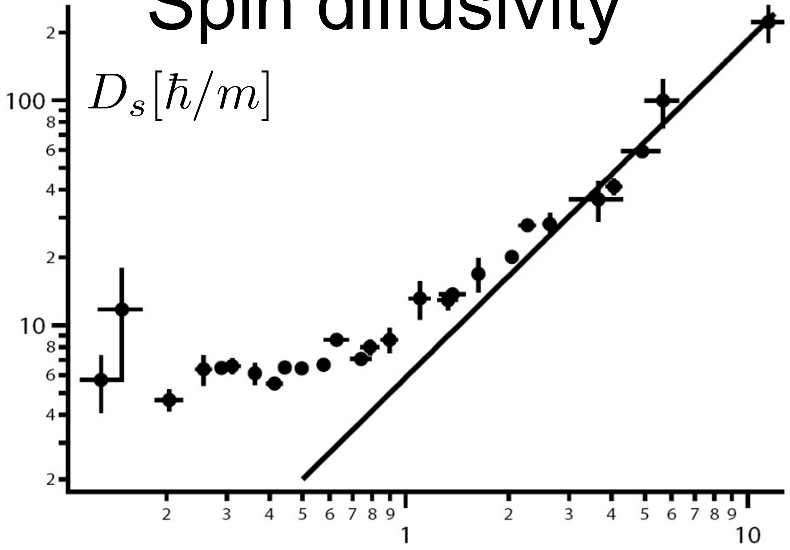


$$D_2 = \frac{\kappa}{c_p} + \frac{\rho_s}{\rho_n}\left(\xi_3 \rho + \frac{4}{3}\frac{\eta}{\rho}\right)$$

Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

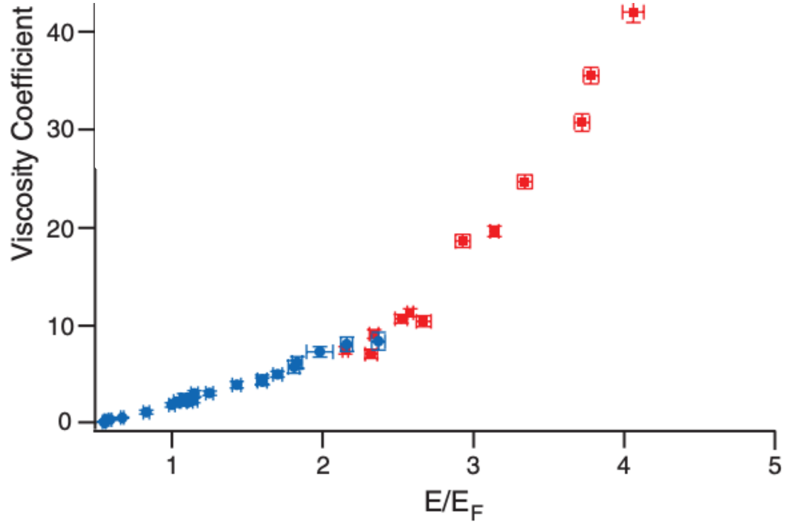
Diffusivities in the Unitary Gas

Spin diffusivity



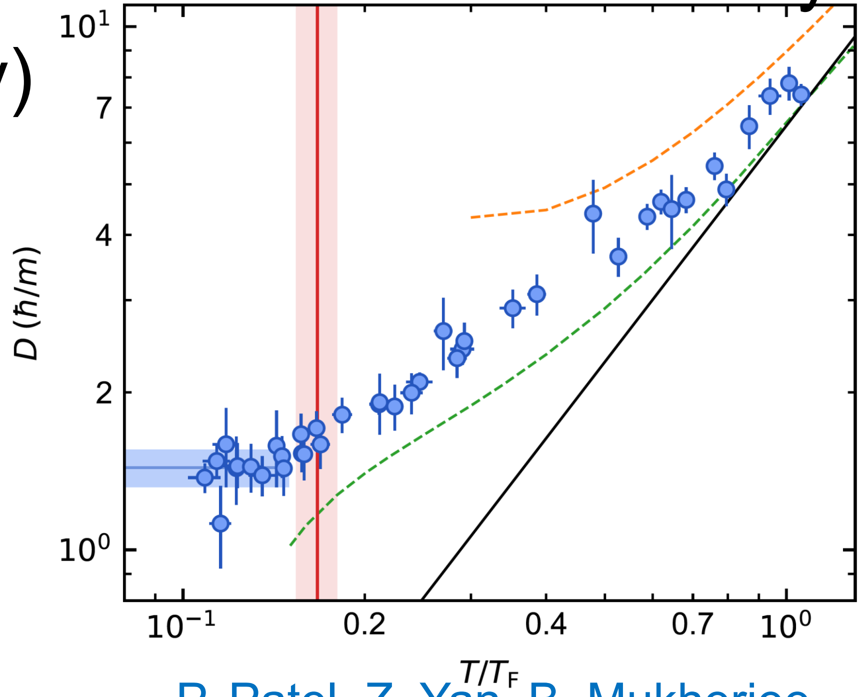
A.T. Sommer, M.J.H. Ku, G. Roati, MZ, Nature 472, 201 (2011)

Kinematic viscosity (= momentum diffusivity)



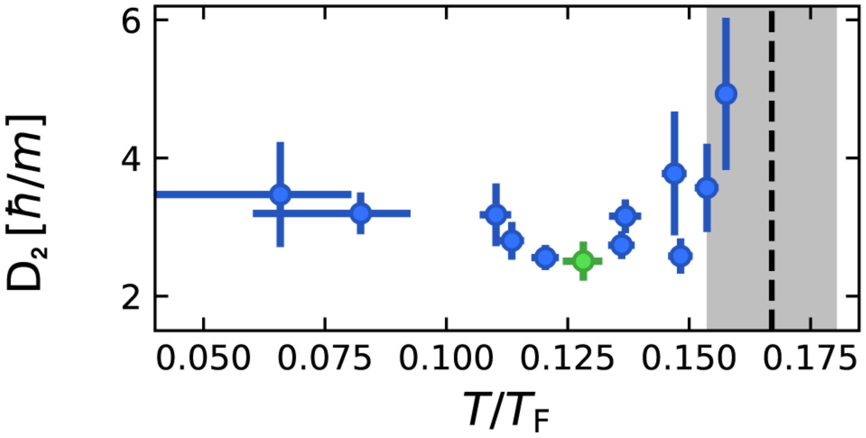
Cao et al., John Thomas group, Science 331, 58 (2011)

1st Sound diffusivity



P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science 370, 1222 (2020)

2nd Sound diffusivity



Diffusivity (charge, spin, momentum, thermal)

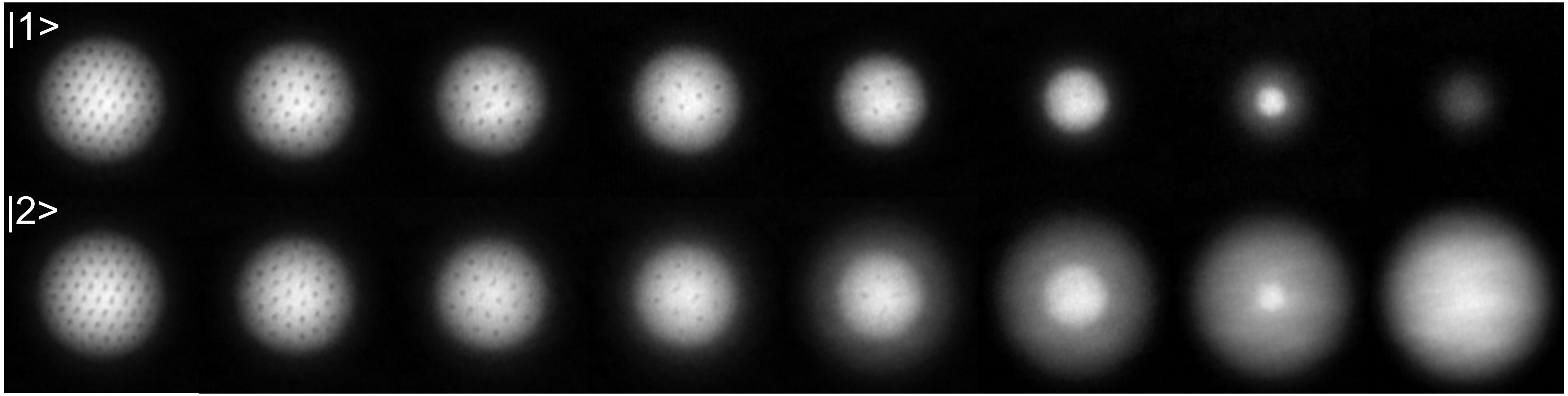
$$D \sim vl \sim \frac{\hbar}{m}$$

Transport in Strongly Correlated Quantum Gases

Unifying themes of strongly interacting Fermi systems:
Loss of quasi-particle description: $\tau^{-1} \approx E_F / \hbar$
Quantum-Limited diffusivities: $D \approx \frac{\hbar}{m}$

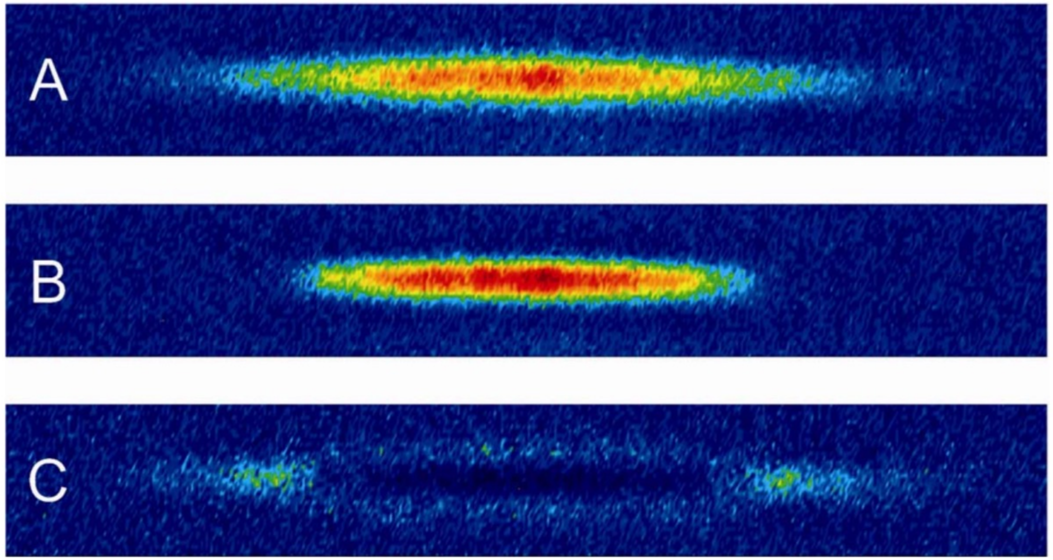
- Universal Sound Diffusion in a Strongly Interacting Fermi Gas
  Parth B. Patel, Zhenjie Yan, Biswaroop Mukherjee, Richard J. Fletcher, Julian Struck, MZ
  Science, 370, 1222 (2020)
- Spin Transport in a Mott Insulator of Ultracold Fermions
  Matthew Nichols, Lawrence Cheuk, Melih Okan, Thomas Hartke, Enrique Mendez, T. Senthil, Ehsan Khatami, Hao Zhang, MZ, Science 363, 383 (2019)
- Doublon-Hole Correlations and Fluctuation Thermometry in a Fermi-Hubbard Gas
  T. Hartke, B. Oreg, N. Jia, MZ, PRL 125, 113601 (2020)
- Bose polarons near quantum criticality
  Zoe Z. Yan, Yiqi Ni, Carsten Robens, MZ, Science, 368, 190-194 (2020)
- Geometric squeezing into the lowest Landau level
  Richard J. Fletcher, Airlia Shaffer, Cedric C. Wilson, Parth B. Patel, Zhenjie Yan, Valentin Crépel, Biswaroop Mukherjee, MZ, arXiv:1911.12347 (Science 2021, to be published)

Spin Imbalanced Fermi Mixtures

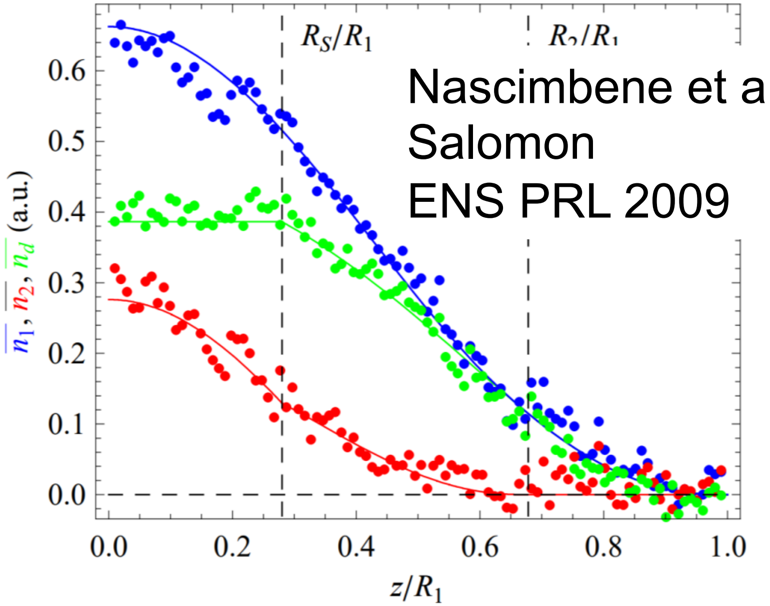


0% 6% 12% 22% 30% 56% 90% 94%

M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle,
Science 311, 492 (2006)

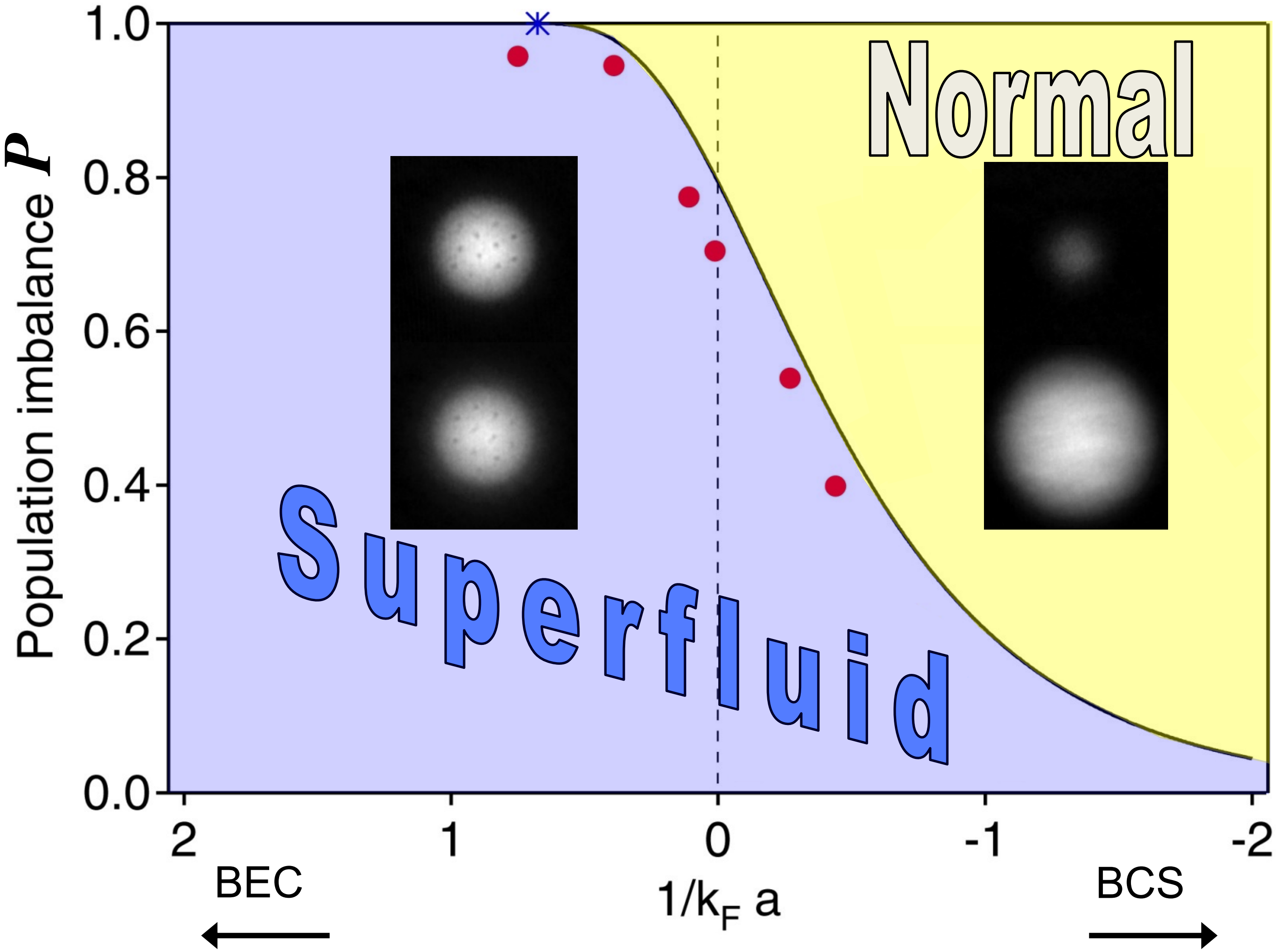


Partridge et al., Hulet group,Rice, Sceince 2006



Nascimbene et al.,
Salomon
ENS PRL 2009

Phase Diagram for Unequal Mixtures



Breakdown: Critical polarization $P_c \propto$ Gap Δ

M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle,
Science 311, 492 (2006)

Swimming in the Fermi sea

© F. Chevy

What is the fate of a single impurity in a Fermi sea?

Relevant for

- electron transport in lattices
- Kondo problem (single magnetic impurity)
- mobility of ^{3}He in ^{4}He
- motion of polaritons in semiconductors…
- determines a system's properties at low temperature
- represents the few-particle limit for N-body EOS

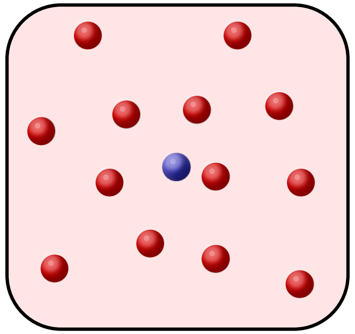
Quasi-particles for strong interactions?

N+1 body problem with resonant interactions

High temperature: Scattering cross section $\sigma \propto \lambda^2$

Scattering rate:

$$\Gamma = n\sigma v = n\lambda^2 \frac{\hbar}{m\lambda} = \frac{\hbar n^{2/3}}{m}\lambda n^{1/3} \sim \frac{E_F}{\hbar}\sqrt{\frac{T_F}{T}}$$



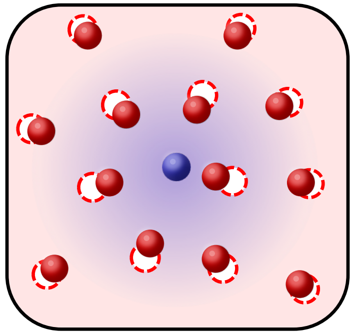
→ Energy uncertainty: $\Delta E = \hbar\Gamma \ll E \sim k_B \mathrm{T}$

Quasi-particles for strong interactions?

N+1 body problem with resonant interactions

Low temperature: Pauli blocking of collisions

Scattering rate: $$\Gamma = n\sigma_F v_F \left(\frac{T}{T_F}\right)^2$$



→ Energy uncertainty: $\Delta E = \hbar\Gamma \ll E \sim E_F$

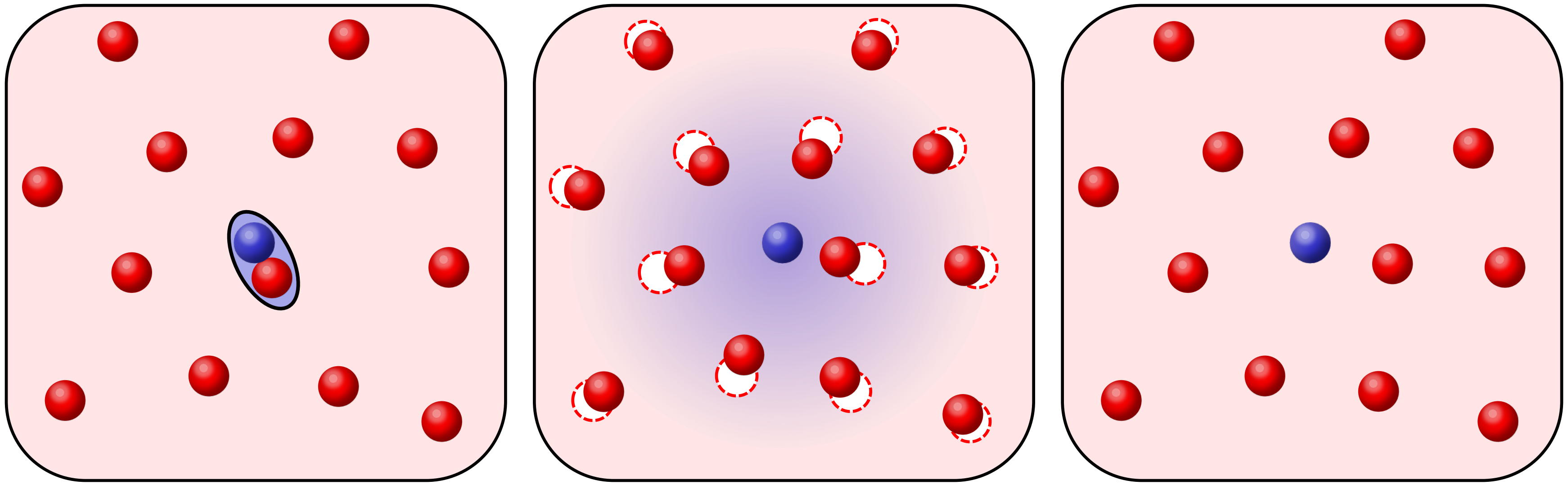
The **Fermi Polaron** emerges

$$E_P = -0.6 E_F$$

Chevy, Lobo, Recati, Giorgini, Stringari, Prokof'ev, Svistunov, Forbes, Bulgac, Combescot, …
Experiments: MIT, ENS, Innsbruck, Cambridge, LENS, MPQ, Technion, …

Some slides from 2009: Swimming in the Fermi Sea

Molecule | Polaron | Mean Field



strong attraction

$$\frac{\hbar^2}{ma^2}$$

?

Energy

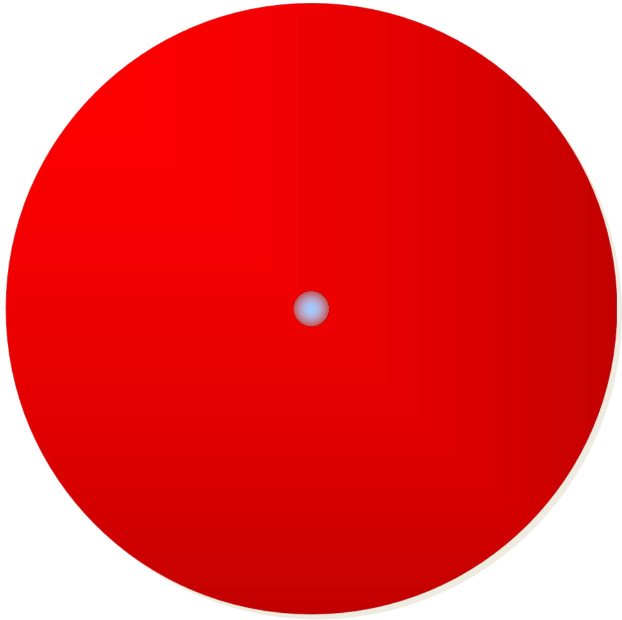
weak attraction

$$\frac{4\pi\hbar^2 a}{m} n_\uparrow$$

Theory: Chevy, Forbes, Bulgac, Lobo, Giorgini, Stringari, Prokof’ev, Svistunov, Sachdev, Sheehy, Radzihovsky, Lamacraft, Combescot, Sa de Melo

Swimming in the Fermi sea

A single $|\downarrow\rangle$ atom immersed in a $|\uparrow\rangle$ cloud
with unitarity limited interactions



Binding energy must be universal

$$E_{\downarrow} = -\mathrm{A} E_{\mathrm{F}\uparrow} \qquad \boxed{\mathrm{A} = 0.6}$$

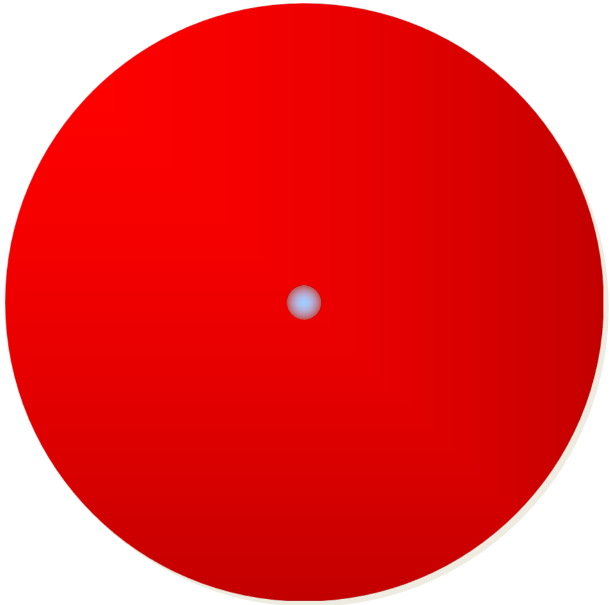
F. Chevy PRA **74**, 063628 (2006), Variational Cooper pair Ansatz

C. Lobo, A. Recati, S. Giorgini, S. Stringari, PRL **97**, 200403 (2006), Monte-Carlo

Swimming in the Fermi sea

Chevy's Ansatz:

$$|\Psi\rangle = \phi_0 |\mathbf{0}\rangle_\downarrow |FS\rangle_\uparrow$$

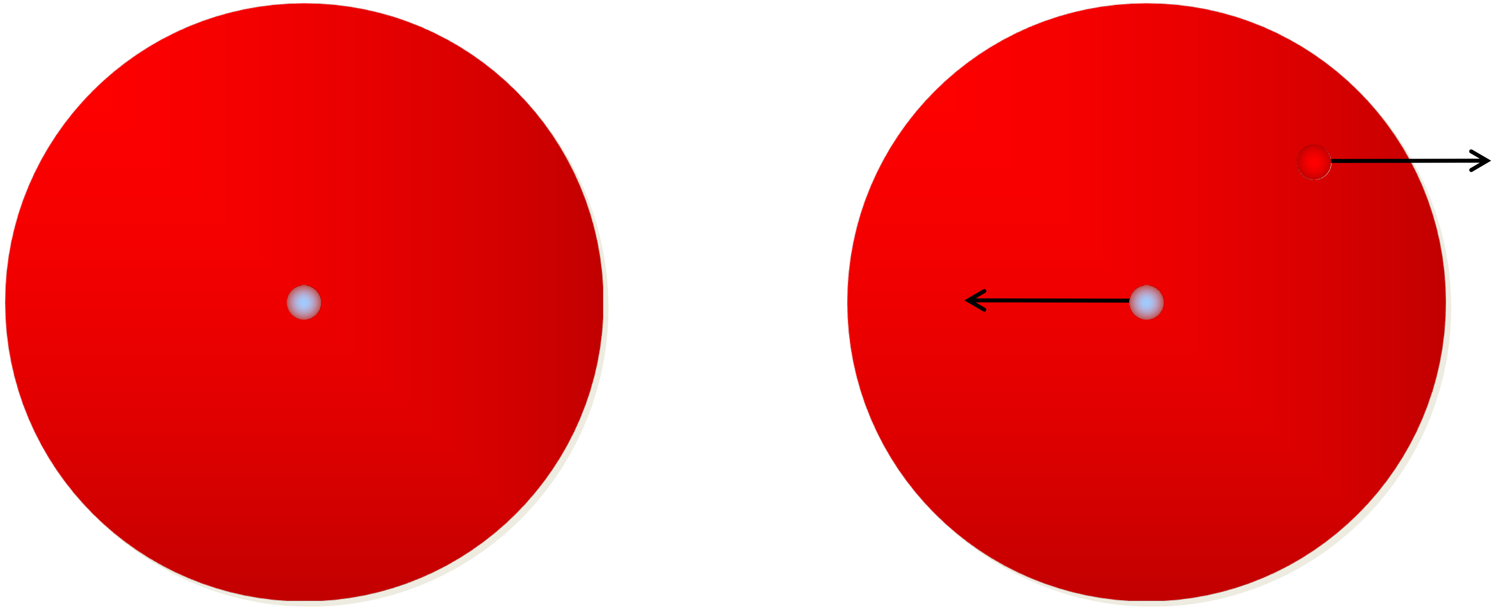


F. Chevy PRA **74**, 063628 (2006), Variational Cooper pair Ansatz
C. Lobo, A. Recati, S. Giorgini, S. Stringari, PRL **97**, 200403 (2006), Monte-Carlo

Swimming in the Fermi sea

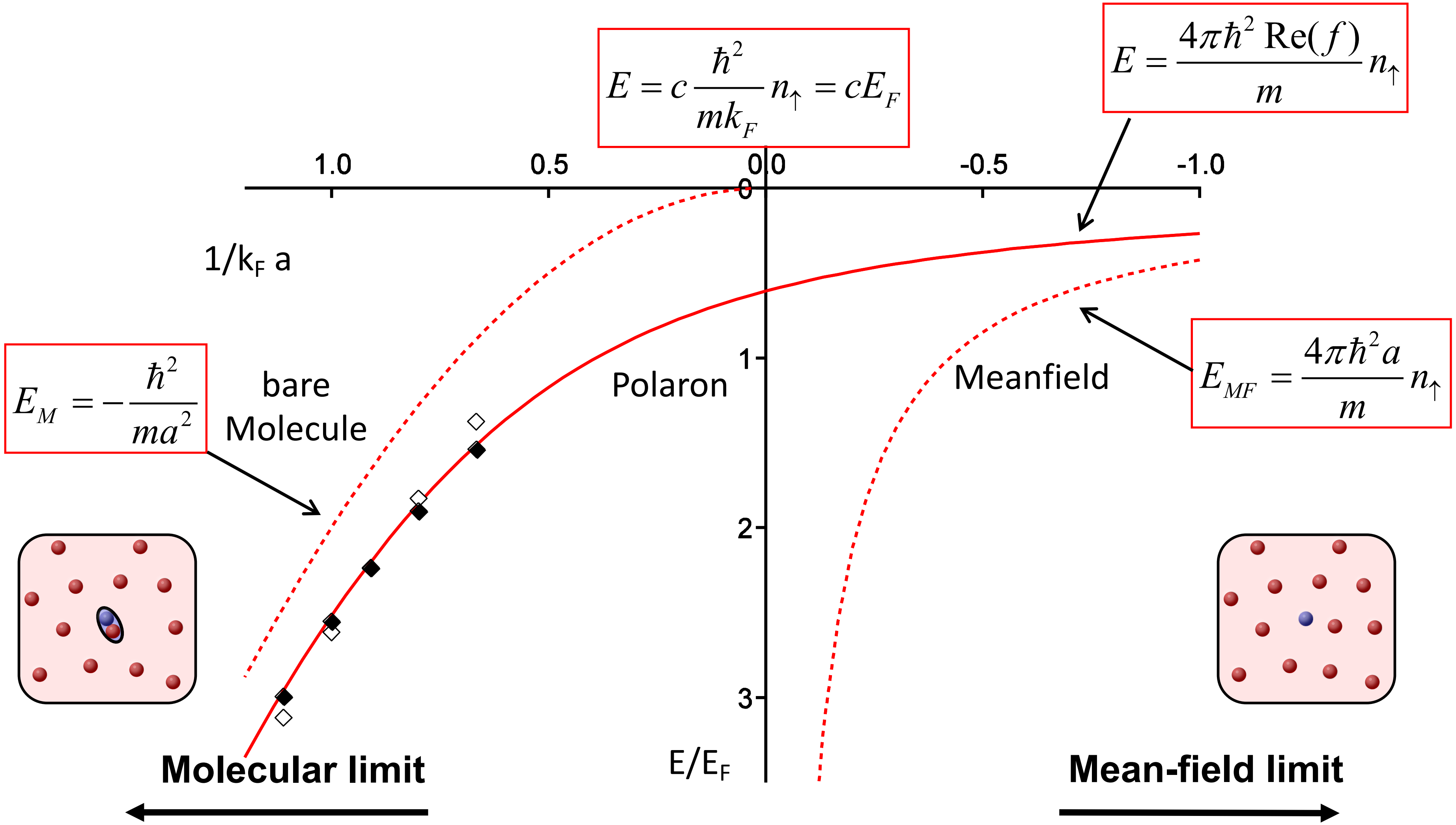
Chevy's Ansatz:

$$|\Psi\rangle = \phi_0|\mathbf{0}\rangle_\downarrow |FS\rangle_\uparrow + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}}|\mathbf{q}-\mathbf{k}\rangle_\downarrow c^\dagger_{\mathbf{k}_\uparrow} c_{\mathbf{q}_\uparrow} |FS\rangle_\uparrow$$



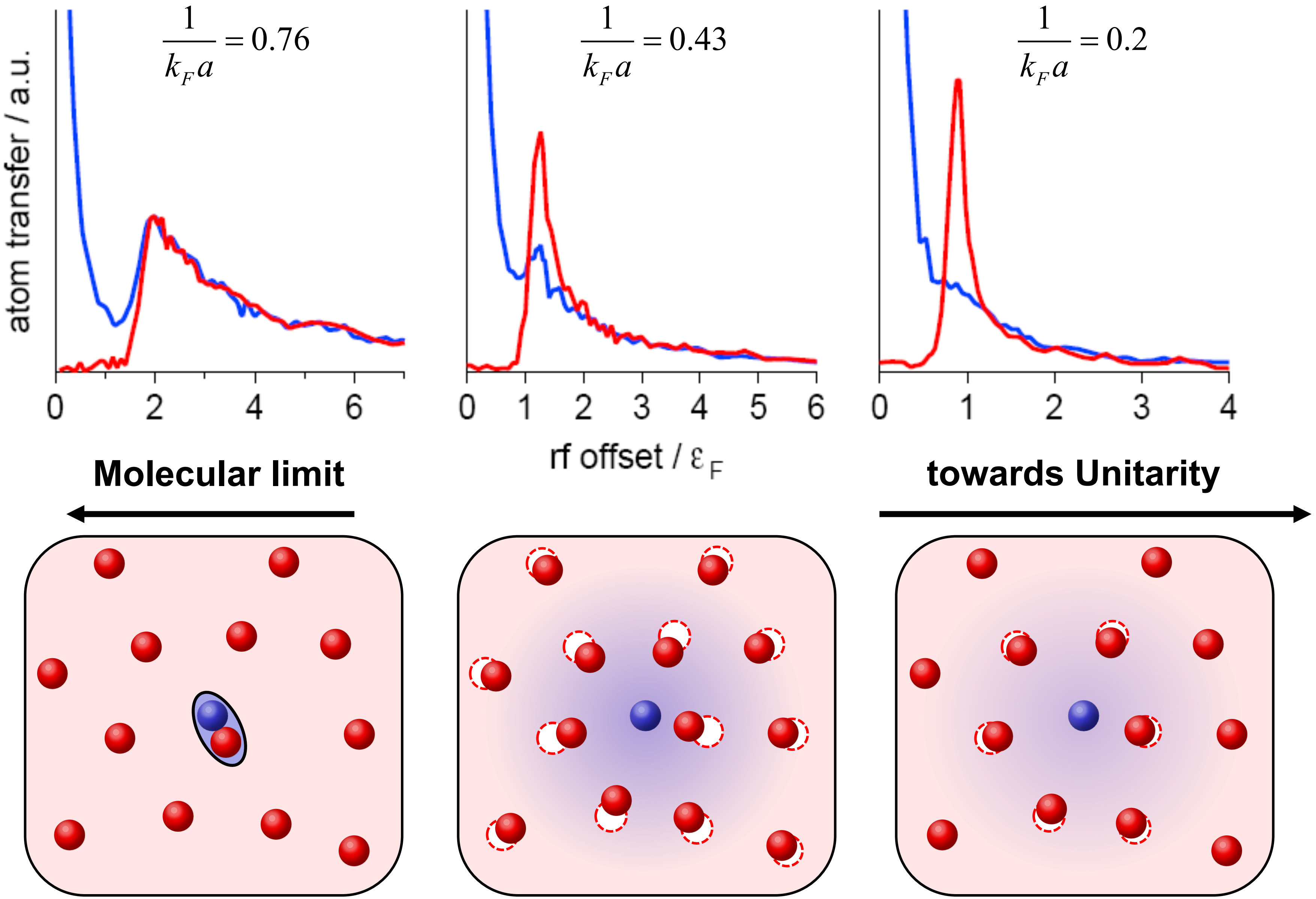
F. Chevy PRA **74**, 063628 (2006), Variational Cooper pair Ansatz
C. Lobo, A. Recati, S. Giorgini, S. Stringari, PRL **97**, 200403 (2006), Monte-Carlo

Polaron Energy



Variational approach/T-Matrix: F. Chevy PRA 74, 063628 (2006), Combescot, Giraud (2008)
Monte Carlo: C. Lobo, A. Recati, S. Giorgini, S. Stringari, PRL 97, 200403 (2006)
Diagrammatic Monte-Carlo: N. V. Prokof'ev and B. V. Svistunov, PRB 77, 125101 (2008)
T-matrix/ladder approximation: P. Massignan, G. Bruun and H. Stoof, PRA 78, 031602 (2008)

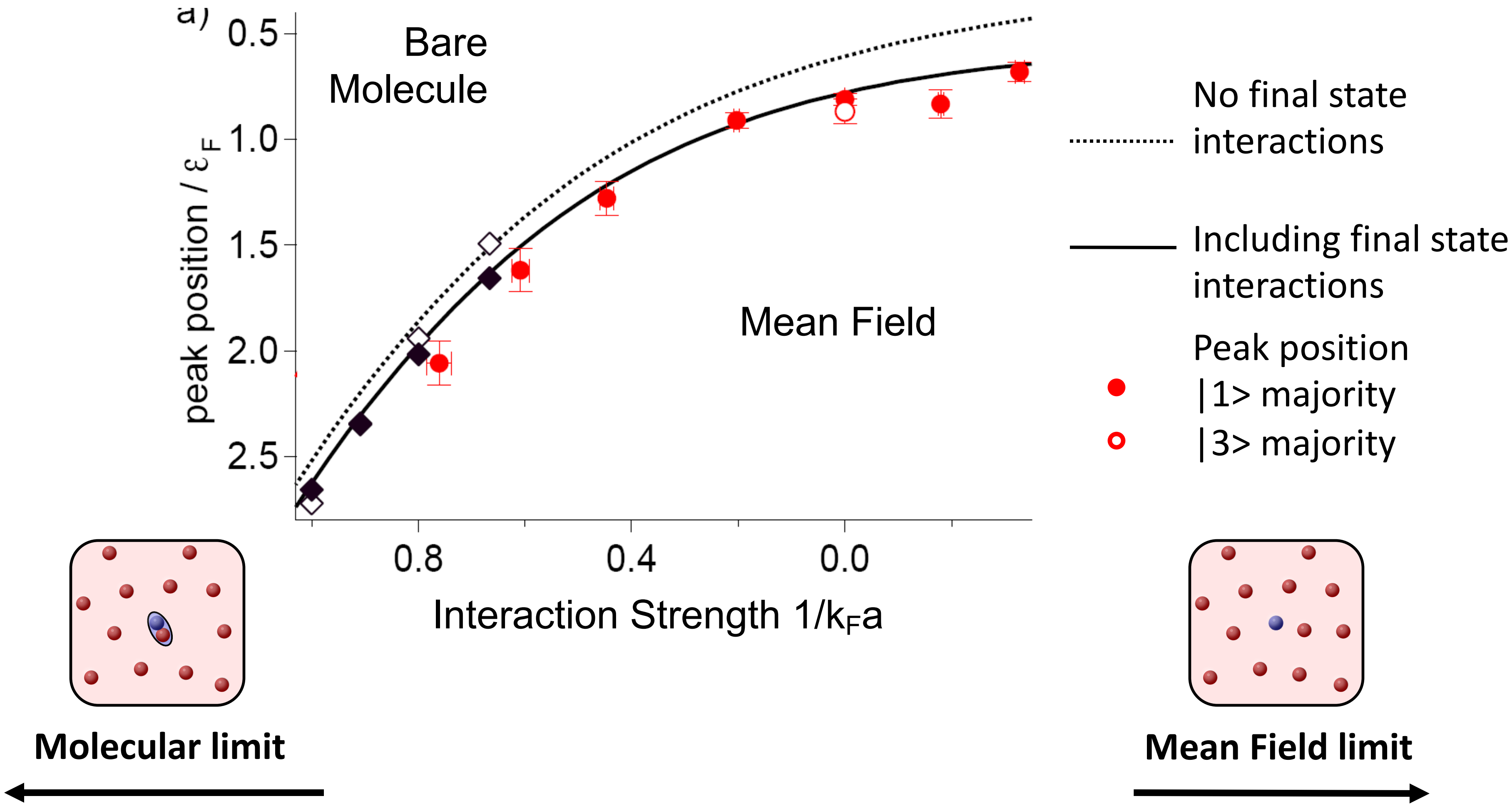
Emergence of the Fermi Polaron



A. Schirotzek, C. Wu, A. Sommer, MWZ, PRL 102, 230402 (2009)

Polaron Energy vs Interaction Strength

A. Schirotzek, C. Wu, A. Sommer, MWZ, PRL 102, 230402 (2009)



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