MA439: Functional Analysis
Tychonoff Spaces: 1, 2, 5, 7 pg. 51, Ben Mathes

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Due: Wed, Oct 28, 2020

Exercise 1. *$C(X) = \{f : X \to \mathbb{C} : f$ unif. cont., bdd$\}$ and uniform norm $||f|| = \sup_{x\in X} |f(x)|$. Consider $B(X) = \{f : X \to \mathbb{C},$ bdd$\}$ Show that $C(X) \subseteq B(X)$ is closed, i.e. a uniform limit of unif. cont. fn is unif. cont.*

Proof. This is the full generality. To make this easier, prove this example: consider a metric space (X, d) and $f_n : X \to \mathbb{C}$ bdd, unif. cont. fns. and $||f_n - f|| \to 0$ uniformly where $||h|| = \sup_{x\in\mathcal{X}} |h(x)|$. This implies that f is unif. cont. $\square$