Midterm 1 QUANTUM THEORY II (8.321)

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Date: Oct 27 - Oct 28, 2021

7 State start with $|+_x\rangle|+_x\rangle|+_x\rangle$

(a) Measure $S_x^{(1)} S_x^{(1)} S_x^{(1)}$ gives $\left(\frac{\hbar}{2}, \frac{\hbar}{2}, \frac{\hbar}{2}\right)$ with probability $Pr\left(\frac{\hbar}{2}, \frac{\hbar}{2}, \frac{\hbar}{2}\right) = \boxed{1}$

State after measurement $|+_x\rangle|+_x\rangle|+_x\rangle$

(b) Since $|+_x\rangle = \frac{1}{\sqrt{2}}(|+_z\rangle + |-_z\rangle)$. Measuring $S_z^{(1)} S_z^{(2)} S_z^{(3)}$

we have 8 possibilities with the same probability:

$\left(\pm\frac{\hbar}{2}, \pm\frac{\hbar}{2}, \pm\frac{\hbar}{2}\right)$ with probability of each is $\boxed{\frac{1}{8}}$

(c) $S_z^{(1)} S_z^{(2)} S_y^{(3)} \to \left(\pm\frac{\hbar}{2}, \pm\frac{\hbar}{2}, \pm\frac{\hbar}{2}\right)$ each again with probability $\boxed{\frac{1}{8}}$

(d) $S_x^{(1)} S_x^{(2)} S_z^{(3)} \to \left(\frac{\hbar}{2}, \frac{\hbar}{2}, \pm\frac{\hbar}{2}\right)$

Two possibilities, each with probability $\boxed{\frac{1}{2}}$

(e) $i S_x^{(1)} S_y^{(2)} S_z^{(2)}$ we don't measure (3), so only get results for the first two.

. 1^{st} particle returns $+\hbar/2$ with probability 1.

. 2^{nd} particle ... $i S_y^{(2)} S_z^{(2)} = \cancel{\ldots} -\frac{\hbar^2}{4}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \cancel{\ldots}$

$= -\hbar/2\, S_x^{(2)}$

so get $-\hbar/2$ on (2) with $Pr = 1$

$\to$ outcome $= \left(\hbar/2, -\frac{\hbar^2}{4}, \cancel{\ }\right)$ with $Pr = \boxed{1}$

2 (a)

$$E = \overset{x}{\overset{\uparrow}{E_A}} + \overset{y}{\overset{\uparrow}{E_{2A}}} + \overset{z}{\overset{\uparrow}{E_{3A}}}$$

$$= \frac{\hbar^2\pi^2 n_x^2}{2mA^2} + \frac{\hbar^2\pi^2 n_y^2}{2m(2A)^2} + \frac{\hbar^2\pi^2 n_z^2}{2m(3A)^2}$$

$$\boxed{E = \frac{\hbar^2\pi^2}{2mA^2}\left(n_x^2 + \frac{n_y^2}{4} + \frac{n_z^2}{9}\right)}$$

$$n_i = 1, 2, 3, \ldots$$

(b) lowest 5 eigennumbers -- ~~obtained by the [illegible]~~

1. $E(1,1,1) = \left(\hbar^2\pi^2/2mA^2\right) \cdot 49/36$
2. $E(1,1,2) = \left(\hbar^2\pi^2/2mA^2\right) \cdot 61/36$

$\cancel{E(1,1,3) = \left(\hbar^2\pi^2/2mA^2\right) \cdot 81/36}$

$\cancel{E(1,1,4) = \left(\hbar^2\pi^2/2mA^2\right) \cdot 109/36}$

3. $E(1,2,1) = \left(\hbar^2\pi^2/2mA^2\right) \cdot 76/36 \rightsquigarrow 19/9$
4. $E(1,1,3) = \left(\hbar^2\pi^2/2mA^2\right) \cdot 81/36 \rightsquigarrow 9/4$
5. $E(1,2,3) = \left(\hbar^2\pi^2/2mA^2\right) \cdot 3$

(c) Energy where spectrum is degenerate...

$$\boxed{E = \frac{\hbar^2\pi^2}{2mA^2} \cdot 6} \quad \text{since} \quad \boxed{E(2,2,3) = E(1,4,3)}$$

"2"

(3)

(a) Any 2 qubit can be written as

$$e^{i\phi_1} a|++\rangle + e^{i\phi_2} b|+-\rangle + e^{i\phi_3} c|-+\rangle + e^{i\phi_4} d|--\rangle$$

in the z basis ⇒ need $4+4=8$ parameters.

But need normalization & phase doesn't matter (overall) ⇒ $8-2=\boxed{6}$ degrees of freedom.

(b) Swap $s_1 \leftrightarrow s_2$ $\quad |s_1 s_2 s_3\rangle \longrightarrow |s_2 s_1 s_3\rangle$

Swap $s_1 \leftrightarrow s_3$ $\quad |s_2 s_1 s_3\rangle \longrightarrow |s_2 s_3 s_1\rangle$

Full operation:

$$\boxed{(I \otimes SWAP)\,(SWAP \otimes I)}$$

where $SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ in z-basis.

(c) 3 qubit ... the unitary needs $\overset{2^N}{8} \times 2 - 2 = 14$ degrees of freedom.

Since each 2-qubit unitary has 6 degrees of freedom.

⇒ we'll need at least $\boxed{3}$ to get a general 3-qubit operation.

(d) again, N-qubit will need $2\times 2^N - 2$ dofs

phase + probabilities

normalization overall phase

(e) ⇒ need at ~~most~~ least $\boxed{\left\lceil \frac{2\times 2^N - 2}{6} \right\rceil}$ → round up 2-qubit operations

scales exponentially in N → not practical when $N \sim 100$

(f)

(e) I think it is possible to realize this lower bound, in principle, for any N qubit operation.

May prove this by <u>induction</u>. Prove the case for N = 3. Then we'll need to assume true for N. Then look at case where ~~N+1~~ N' = N+1.

What we'll need to do is "connect" this extra qubit to the rest of the system ~~to get a general N+1~~ by <u>2-qubit operations</u>.

The combined unitary (on N and then on the extra qubit) ~~should be a~~ could be decomposed into 2-qubit operations. Then we can count the total ...