Rigid Body Rotation

Q: How are Euler eqns derived?

Q: In what frame are ω_i's?

Q: How to get back to lab frame?

Q: What is intermediate axis thm?

Q: Difference when $\tau = 0$, $\dot{\phi} = 0$ and $\tau \neq 0$ symmetric top?

Q: What are the unusual quantities of a symmetric top with $\vec{J}$?

Hint: (3)

Q: How do I change under
rotation by π?
translation?

Vibration & Oscillation

Q: what are normal coords,
mathematically?

Q: Simult. diag of $\hat{T}, \hat{V}$

Q: what are $\hat{T}$? $\hat{V}$?

Canonical transformation

HJE

Action-angle vars

Q: Why canonical transform?

Q: What are E-L eqns invariant?

Q: What makes a transformation canonical?

Q: What are generating fns?

Q: easy way to check for canonicity?

Q: Poisson bracket fundamental relation?

Q: two facts of Poisson bracket?

→ invariant canonically

→ phase space volume?

Q: How are time evolution written in terms of { }?

$$\frac{dH}{dt} = \{H, \mathcal{H}\} + \frac{\partial H}{\partial t}$$

If $H = H(q, p, t)$:

$$\partial_t H + \frac{\partial H}{\partial q}\frac{\partial q}{\partial t} + \frac{\partial H}{\partial p}\frac{\partial p}{\partial t} = \partial_t H + \{H, \mathcal{H}\}$$

$$\frac{\partial q}{\partial t} \to \frac{\partial \mathcal{H}}{\partial p}, \qquad \frac{\partial p}{\partial t} \to -\frac{\partial \mathcal{H}}{\partial q}$$

Q: when is a quantity conserved?

↓

L) "commutes" with $\mathcal{H}$

& is time-indep

$\dot{n} = 0$

$\ast$

$\partial_t n = 0$

Q: (T/F) n, v conserved

$\Rightarrow \{n, v\}$ conserved ?

T: use Jacobi identity to prove

or compute explicitly ...

Q: What are infinitesimal symmetries?

L) Noether's approach:

what generates what?

$(p, L_z, \mathcal{H}) \dashrightarrow (?, ?, ?)$

Q: What is the key to HJE?

Look for F_2 s.t.

$$K = H + \frac{\partial F_2}{\partial t} = 0$$

and Q, P cyclic in $\mathcal{H}$

Q What is Jacobi-Hamilton's principle fn?

Q: How do q, p vary w/ t?

$$\mathcal{H}(p, q, t) + \frac{\partial F_2}{\partial t}(q, P, t) = 0 = K$$

$$p = \frac{\partial F_2}{\partial q}$$

$$\mathcal{H}\left(q, \frac{\partial F_2}{\partial q}, t\right)$$

Q: How are S & F_2 related? What are their arguments?

$$F_2 \equiv S = S(\tilde{q}, \tilde{\alpha}, t)$$

constants

α_i

$\because$ $F_2 = F_2(q, P)$

(constant, by force)

by canonical transformation

constant

constant since P, Q cyclic

$$\boxed{P_i = \alpha_i}$$

$$\boxed{Q_i = \beta_i = \frac{\partial S}{\partial \alpha_i}}$$

(actually $\frac{\partial F_2}{\partial P}$)

α

$\because$ α is const

of S.

Q How are p, q related to S?

$$\begin{cases} p_i = \dfrac{\partial}{\partial q_i}(S(q, \alpha, t)) \Rightarrow q = q(\alpha, \beta, t) \\ p = \partial S / \partial q = p(\alpha, \beta, t) \end{cases}$$

(7) How to get q, p from S ?

→ first $\beta_i = \frac{\partial S}{\partial \alpha_i}$ to get q_i

then use

$p_i = \frac{\partial S}{\partial q_i}$ to get p_i.

Q: What is the difference between

$S \sim W$?

time dependent

Shift

$\alpha_i = \alpha_i$

time independent

Shift

$Q_i = t + \beta_i = \frac{\partial W}{\partial \alpha_i}$

(Some const. shift)

Q: action-angle variables.

(J, ω)

J: action, ω: angle.

Q: Why are A.A vars useful?

$p = p(q, E)$

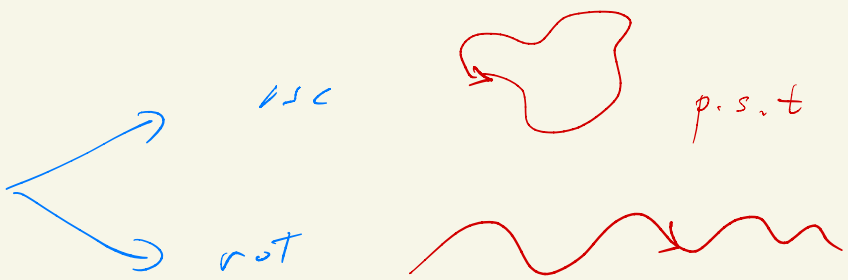
$J = \oint p\,dq \Rightarrow$ const

$J = J(E)$

$\rightarrow$ Let $\frac{\partial J}{\partial E}$

$\left(\frac{\partial E}{\partial J}\right) = \nu$ freq.

Q: What are the 2 types of periodic motion?



Q: Condition for 2D osc to be periodic in time?

$\frac{\nu_1}{\nu_2}$ rational

Q How is J and W related

$$J = \oint p\, dq$$

$$= \oint \left(\frac{\partial W}{\partial q}\right) dq = \cdots$$

↓

know W solve ....

$$J_i = \oint p_i\, dq_i$$

↳ height J_r

J_θ

J_φ