Amnon Yariv, Optical Electronics in Modern Communications, 5th Edition, Oxford University Press 1997

9 Electrooptic Modulation of Laser Beams

9.0 INTRODUCTION

In Chapter 1 we treated the propagation of electromagnetic waves in anisotropic crystal media. It was shown how the properties of the propagating wave can be determined from the index ellipsoid surface. (see Eq. 9.1-1)

In this chapter we consider the problem of propagation of optical radiation in crystals in the presence of an applied electric field. We find that in certain types of crystals it is possible to effect a change in the index of refraction that is proportional to the field. This is the linear electrooptic effect. It affords a convenient and widely used means of controlling the intensity or phase of the propagating radiation. This modulation is used in an ever expanding number of applications including: the impression of information onto optical beams, Q-switching of lasers (Section 6.9) for generation of giant optical pulses, mode locking, and optical beam deflection. Some of these applications will be discussed further in this chapter. Modulation and deflection of laser beams by acoustic beams are considered in Chapter 12.

9.1 ELECTROOPTIC EFFECT

In Chapter 1 we found that, given a direction in a crystal, in general two possible linearly polarized modes exist: the so-called rays of propagation. Each mode possesses a unique direction of polarization (that is, direction of **D**) and a corresponding

index of refraction (that is, a velocity of propagation). The mutually orthogonal polarization directions and the indices of the two rays are found most easily by using the index ellipsoid

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \qquad (9.1\text{-}1)$$

where the directions x, y, and z are the principal dielectric axes—that is, the directions in the crystal along which **D** and **E** are parallel. The existence of two rays (one "ordinary"; the other "extraordinary") with different indices of refraction is called *birefringence.*

The linear electrooptic effect is the change in the indices of the ordinary and extraordinary rays that is caused by and is proportional to an applied electric field. This effect exists only in crystals that do not possess inversion symmetry.[1] This statement can be justified as follows: Assume that in a crystal possessing an inversion symmetry, the application of an electric field E along some direction causes a change $\Delta n_1 = sE$ in the index, where s is a constant characterizing the linear electrooptic effect. If the direction of the field is reversed, the change in the index is given by $\Delta n_2 = s(-E)$, but because of the inversion symmetry the two directions are physically equivalent, so $\Delta n_1 = \Delta n_2$. This requires that $s = -s$, which is possible only for $s = 0$, so no linear electrooptic effect can exist. The division of all crystal classes into those that do and those that do not possess an inversion symmetry is an elementary consideration in crystallography and this information is widely tabulated [1].

Since the propagation characteristics in crystals are fully described by means of the index ellipsoid (9.1-1), the effect of an electric field on the propagation is expressed most conveniently by giving the changes in the constants $1/n_x^2$, $1/n_y^2$, $1/n_z^2$ of the index ellipsoid.

Following convention [1–2], we take the equation of the index ellipsoid in the presence of an electric field as

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1 \qquad (9.1\text{-}2)$$

If we choose x, y, and z to be parallel to the principal dielectric axes of the crystal, then with zero applied field, Equation (9.1-2) must reduce to (9.1-1); therefore,

[1] If a crystal contains points (one in each unit cell) such that inversion (replacing each atom at **r** by one at $-$**r**, with **r** being the position vector relative to the point) about any one of these points leaves the crystal structure invariant, the crystal is said to possess inversion symmetry.

$$\left(\frac{1}{n^2}\right)_1\bigg|_{E=0} = \frac{1}{n_x^2} \qquad \left(\frac{1}{n^2}\right)_2\bigg|_{E=0} = \frac{1}{n_y^2}$$

$$\left(\frac{1}{n^2}\right)_3\bigg|_{E=0} = \frac{1}{n_z^2} \qquad \left(\frac{1}{n^2}\right)_4\bigg|_{E=0} = \left(\frac{1}{n^2}\right)_5\bigg|_{E=0} = \left(\frac{1}{n^2}\right)_6\bigg|_{E=0} = 0$$

The linear change in the coefficients

$$\left(\frac{1}{n^2}\right)_i \qquad i = 1, \ldots, 6$$

due to an arbitrary dc electric field $\mathbf{E}(E_x, E_y, E_z)$ is defined by

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^{3} r_{ij}E_j \tag{9.1-3}$$

where in the summation over j we use the convention $1 = x$, $2 = y$, $3 = z$. Equation (9.1-3) can be expressed in a matrix form as

$$\begin{vmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^3}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{vmatrix} = \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{vmatrix} \begin{vmatrix} E_1 \\ E_2 \\ E_3 \end{vmatrix} \tag{9.1-4}$$

where, using the rules for matrix multiplication, we have, for example,

$$\Delta\left(\frac{1}{n^2}\right)_6 = r_{61}E_1 + r_{62}E_2 + r_{63}E_3$$

The 6×3 matrix with elements r_{ij} is called the electrooptic tensor. We have shown above that in crystals possessing an inversion symmetry (centrosymmetric), $r_{ij} = 0$. The form, but not the magnitude, of the tensor r_{ij} can be derived from symmetry considerations [1], which dictate which of the 18 r_{ij} coefficients are zero, as well as the relationships that exist between the remaining coefficients. In Table 9-1 we give the form of the electrooptic tensor for all the noncentrosymmetric crystal classes. The electrooptic coefficients of some crystals are given in Table 9-2.

Table 9-1 The Form of the Electrooptic Tensor for all Crystal Symmetry Classes

Symbols:

• zero element

● nonzero element

●—● equal nonzero elements

●—○ equal nonzero elements, but opposite in sign

The symbol at the upper left corner of each tensor is the conventional symmetry group designation.

Centrosymmetric—All elements zero

Triclinic

Monoclinic

2 (parallel to x_2)

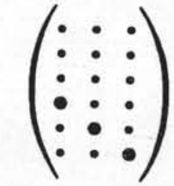
(parallel to x_3)

m (perpendicular to x_2)

(perpendicular to x_3)

Orthorhombic

222



$mm2$

Table 9-1 *(continued)*

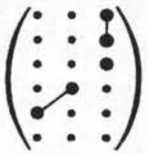
Tetragonal

4

$\bar{4}$

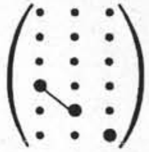
422

$4mm$



Example: ($BaTiO_3$)

$\bar{4}2m$ (2 parallel to x_1)



Example: KH_2PO_4(KDP)

Cubic

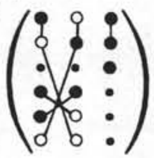
$\bar{4}3m$, 23

Examples: (Crystals of the zinc blende class: GaAs, InAs, CdTe)

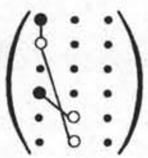
432

Trigonal

3

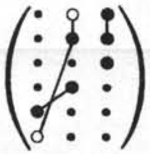


32

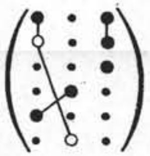


Examples: (Te, quartz)

$3m$ (m perpendicular to x_1) standard orientation



$3m$(m perpendicular to x_2)



Example: ($LiNbO_3$ $LiTaO_3$)

Hexagonal

6

$6mm$

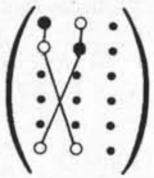
Example: (CdS)

(same as $4mm$)

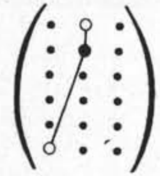
622

Table 9-1 *(continued)*

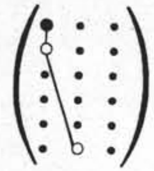
$\bar{6}$



$\bar{6}m2$ (m perpendicular to x_1 standard orientation)



(m perpendicular to x_2)



Example: The Electrooptic Effect in KH_2PO_4

Consider the specific example of a crystal of potassium dihydrogen phosphate (KH_2PO_4), also known as KDP. The crystal has a fourfold axis of symmetry,[2] which by strict convention is taken as the z (optic) axis, as well as two mutually orthogonal twofold axes of symmetry that lie in the plane normal to z. These are designated as the x and y axes. The symmetry group of this crystal is $\bar{4}2m$.[3] Using Table 9-1, we take the electrooptic tensor in the form of

$$r_{ij} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{vmatrix} \tag{9.1-5}$$

so the only nonvanishing elements are $r_{41} = r_{52}$ and r_{63}. Using (9.1-2), (9.1-4), and (9.1-5), we obtain the equation of the index ellipsoid in the presence of a field $\mathbf{E}(E_x, E_y, E_z)$ as

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_xyz + 2r_{41}E_yxz + 2r_{63}E_zxy = 1 \tag{9.1-6}$$

[2]That is, a rotation by $2\pi/4$ about this axis leaves the crystal structure invariant.

[3]The significance of the symmetry group symbols and a listing of most known crystals and their symmetry groups is to be found in any basic book on crystallography.

where the constants involved in the first three terms do not depend on the field and, since the crystal is uniaxial, are taken as $n_x = n_y = n_o$, $n_z = n_e$. We thus find that the application of an electric field causes the appearance of "mixed" terms in the equation of the index ellipsoid. These are the terms with xy, xz, and yz. This means that the major axes of the ellipsoid, with a field applied, are no longer parallel to the x, y, and z axes. It becomes necessary, then, to find the directions and magnitudes of the new axes, in the presence of **E**, so that we may determine the effect of the field on the propagation. To be specific we choose the direction of the applied field parallel to the z axis, so (9.1-6) becomes

$$\frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1 \tag{9.1-7}$$

The problem is one of finding a new coordinate system—x', y', z'—in which the equation of the ellipsoid (9.1-7) contains no mixed terms; that is, it is of the form

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1 \tag{9.1-8}$$

x', y', and z' are then the directions of the major axes of the ellipsoid in the presence of an external field applied parallel to z. The length of the major axes of the ellipsoid is, according to (9.1-8), $2n_{x'}$, $2n_{y'}$, and $2n_{z'}$, and these will, in general, depend on the applied field.

In the case of (9.1-7) it is clear from inspection that in order to put the equation in a diagonal form we need to choose a coordinate system x', y', z' where z' is parallel to z, and because of the symmetry of (9.1-7) in x and y, x' and y' are related to x and y by a 45° rotation, as shown in Figure 9-1. The transformation relations from x, y to x', y' are thus

$$x = x' \cos 45^\circ + y' \sin 45^\circ$$

$$y = -x' \sin 45^\circ + y' \cos 45^\circ$$

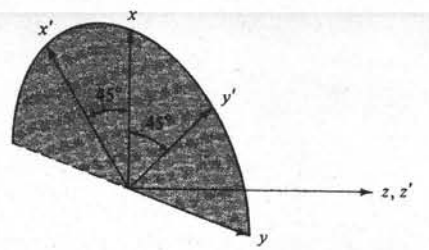


Figure 9-1 The x, y, and z axes of $\bar{4}2m$ crystals (such as KH_2PO_4) and the x', y', and z' axes, where z is the fourfold optic axis and x and y are the twofold axes of crystals with $\bar{4}2m$ symmetry.

which, upon substitution in (9.1-7), yield

$$\left(\frac{1}{n_o^2} - r_{63}E_z\right) x'^2 + \left(\frac{1}{n_o^2} + r_{63}E_z\right) y'^2 + \frac{z^2}{n_e^2} = 1 \tag{9.1-9}$$

Equation (9.1-9) shows that x', y', and z are indeed the principal axes of the ellipsoid when a field is applied along the z direction. According to (9.1-9), the length of the x' axis of the ellipsoid is $2n_{x'}$, where

$$\frac{1}{n_{x'}^2} = \frac{1}{n_o^2} - r_{63}E_z$$

which, assuming $r_{63}E_z \ll n_o^{-2}$ and using the differential relation $dn = -(n^3/2)\,d(1/n^2)$, gives for the change in $n_{x'}$, $dn_{x'} = -(n_o^3/2)r_{63}E_z$ so that

$$n_{x'} = n_o + \frac{n_o^3}{2} r_{63}E_z \tag{9.1-10}$$

and, similarly,

$$n_{y'} = n_o - \frac{n_o^3}{2} r_{63}E_z \tag{9.1-11}$$

$$n_z = n_e \tag{9.1-12}$$

The electrooptic effect in the practical imortant $\bar{4}3m$ crystal class (GaAs, InP, ZnS) is treated in detail in Appendix B.

The General Solution

We now consider the problem of optical propagation in a crystal in the presence of an external dc field along an arbitrary direction.

The index ellipsoid with the dc field on is given by (9.1-2), which we reexpress in the quadratic form

$$S_{ij}x_ix_j = 1 \tag{9.1-13}$$

so that $S_{11} = (1/n^2)_1$, $S_{32} = S_{23} = (1/n^2)_4$, and so on. We also use the convention of summation over repeated indices. Our problem consists of finding the directions and magnitudes of the principal axes of the ellipsoid (9.1-13).

Before proceeding we need remind ourselves of one basic result of vector calculus. If the vector from the origin to a point (x_1, x_2, x_3) on the ellipsoid (9.1-13) is denoted by $\mathbf{R}(x_1, x_2, x_3)$, then the vector **N** with components

$$N_i = S_{ij}x_j \tag{9.1-14}$$

is normal to the ellipsoid at **R**.

We next apply the last result to determine the directions and magnitudes of the principal axes of the ellipsoid (9.1-13). Since the principal axes are normal to the

Table 9-2 Linear Electrooptic Coefficients of Some Commonly Used Crystals

Substance	Symmetry	Wavelength λ (μm)	Electrooptic Coefficients r_{lk} (10^{-12} m/V)		Index of Refraction n_i	n^3r (10^{-12} m/V)	Dielectric Constant* $\epsilon_i(\epsilon_0)$
CdTe (See App. B)	$\bar{4}3m$	1.0	$(T)\ r_{41} = 4.5$		$n = 2.84$	103	$(S)\ \epsilon = 9.4$
		3.39	$(T)\ r_{41} = 6.8$				
		10.6	$(T)\ r_{41} = 6.8$		$n = 2.60$	120	
		23.35	$(T)\ r_{41} = 5.47$		$n = 2.58$	94	
		27.95	$(T)\ r_{41} = 5.04$		$n = 2.53$	82	
GaAs (See App. B)	$\bar{4}3m$	0.9	$r_{41} = 1.1$		$n = 3.60$	51	$(S)\ \epsilon = 13.2$
		1.15	$(T)\ r_{41} = 1.43$		$n = 3.43$	58	$(T)\ \epsilon = 12.3$
		3.39	$(T)\ r_{41} = 1.24$		$n = 3.3$	45	
		10.6	$(T)\ r_{41} = 1.51$		$n = 3.3$	54	
GaP (See App. B)	$\bar{4}3m$	0.55–1.3	$(T)\ r_{41} = -1.0$		$n = 3.66–3.08$		$(S)\ \epsilon = 10$
		0.633	$(S)\ r_{41} = -0.97$		$n = 3.32$	35	
		1.15	$(S)\ r_{41} = -1.10$		$n = 3.10$	33	
		3.39	$(S)\ r_{41} = -0.97$		$n = 3.02$	27	
β-ZnS (sphalerite) (See App. B)	$\bar{4}3m$	0.4	$(T)\ r_{41} = 1.1$		$n = 2.52$	18	$(T)\ \epsilon = 16$
		0.5	$(T)\ r_{41} = 1.81$		$n = 2.42$		$(S)\ \epsilon = 12.5$
		0.6	$(T)\ r_{41} = 2.1$		$n = 2.36$		
		0.633	$(S)\ r_{41} = -1.6$		$n = 2.35$		
		3.39	$(S)\ r_{41} = -1.4$				
ZnSe (See App. B)	$\bar{4}3m$	0.548	$(T)\ r_{41} = 2.0$		$n = 2.66$		$(T)\ \epsilon = 9.1$
		0.633	$(S)\ r_{41} = 2.0$		$n = 2.60$	35	$(S)\ \epsilon = 9.1$
		10.6	$(T)\ r_{41} = 2.2$		$n = 2.39$		
ZnTe (See App. B)	$\bar{4}3m$	0.589	$(T)\ r_{41} = 4.51$		$n = 3.06$		$(T)\ \epsilon = 10.1$
		0.616	$(T)\ r_{41} = 4.27$		$n = 3.01$		$(S)\ \epsilon = 10.1$
		0.633	$(T)\ r_{41} = 4.04$		$n = 2.99$	108	
			$(S)\ r_{41} = 4.3$				
		0.690	$(T)\ r_{41} = 3.97$		$n = 2.93$		
		3.41	$(T)\ r_{41} = 4.2$		$n = 2.70$	83	
		10.6	$(T)\ r_{41} = 3.9$		$n = 2.70$	77	
$Bi_{12}SiO_{20}$	23	0.633	$r_{41} = 5.0$		$n = 2.54$	82	
CdSe	6 mm	3.39	$(S)\ r_{13} = 1.8$		$n_o = 2.452$		$(T)\epsilon_1 = 9.70$
							$(T)\ \epsilon_3 = 10.65$
			$(T)\ r_{33} = 4.3$		$n_e = 2.471$		$(S)\ \epsilon_1 = 9.33$
							$(S)\ \epsilon_3 = 10.20$
α-ZnS (wurtzite)	6 mm	0.633	$(S)\ r_{13} = 0.9$		$n_o = 2.347$		$(T)\ \epsilon_1 = \epsilon_2 = 8.7$
			$(S)\ r_{33} = 1.8$		$n_e = 2.360$		$(S)\ \epsilon_1 = 8.7$
$Pb_{0.814}La_{0.214}$-$(Ti_{0.6}Zr_{0.4})O_3$ (PLZT)	$\infty\ m$	0.546	$n_e^3r_{33} - n_o^3r_{13} = 2320$		$n_o = 2.55$		
$LiIO_3$	6	0.633	$(S)\ r_{13} = 4.1$	$(S)\ r_{33} = 6.4$	$n_o = 1.8830$		
			$(S)\ r_{41} = 1.4$	$(S)\ r_{51} = 3.3$	$n_o = 1.7367$		
Ag_3AsS_3	$3m$	0.633	$(S)\ n_e^3r_e = 70$		$n_o = 3.019$		
			$(S)\ n_o^3r_{22} = 29$		$n_e = 2.739$		
$LiNbO_3$ ($T_c = 1230°C$)	$3m$	0.633	$(T_4)\ r_{13} = 9.6$	$(S)\ r_{13} = 8.6$	$n_o = {}_42.286$		$(T)\ \epsilon_1 = \epsilon_2 = 78$
			$(T)\ r_{22} = 6.8$	$(S)\ r_{22} = 3.4$	$n_e = 2.200$		$(T)\ \epsilon_2 = 32$
			$(T)\ r_{33} = 30.9$	$(S)\ r_{33} = 30.8$			$(S)\ \epsilon_1 = \epsilon_2 = 43$
			$(T)\ r_{51} = 32.6$	$(S)\ r_{51} = 28$			$(S)\ \epsilon_3 = 28$
			$(T)\ r_c = 21.1$				
		1.15	$(T)\ r_{22} = 5.4$		$n_o = 2.229$		
			$(T)\ r_c = 19$		$n_e = 2.150$		
		3.39	$(T)\ r_{22} = 3.1$	$(S)\ r_{33} = 28$	$n_o = 2.136$		
			$(T)\ r_c = 18$	$(S)\ r_{22} = 3.1$	$n_e = 2.073$		
				$(S)\ r_{13} = 6.5$			
				$(S)\ r_{51} = 23$			

Table 9-2 (*continued*)

Substance	Symmetry	Wavelength λ (μm)	Electrooptic Coefficients r_{lk} (10^{-12} m/V)		Index of Refraction n_i	n^3r (10^{-12} m/V)	Dielectric Constant* $\epsilon_i(\epsilon_0)$
$LiTaO_3$	$3m$	0.633	$(T)\ r_{13} = 8.4$	$(S)\ r_{13} = 7.5$	$n_o = 2.176$		$(T)\ \epsilon_1 = \epsilon_2 = 51$
			$(T)\ r_{33} = 30.5$	$(S)\ r_{33} = 33$	$n_e = 2.180$		$(T)\ \epsilon_3 = 45$
			$(T)\ r_{22} = -0.2$	$(S)\ r_{51} = 20$			$(S)\ \epsilon_1 = \epsilon_2 = 41$
			$(T)\ r_c = 22$	$(S)\ r_{22} = 1$			$(S)\ \epsilon_3 = 43$
		3.39	$(S)\ r_{33} = 27$		$n_o = 2.060$		
			$(S)\ r_{13} = 4.5$		$n_e = 2.065$		
			$(S)\ r_{51} = 15$				
			$(S)\ r_{22} = 0.3$				
$AgGaS_2$	$\bar{4}2m$	0.633	$(T)\ r_{41} = 4.0$		$n_o = 2.553$		
			$(T)\ r_{63} = 3.0$		$n_e = 2.507$		
CsH_2AsO_4	$\bar{4}2m$	0.55	$(T)\ r_{41} = 14.8$		$n_o = 1.572$		
(CDA)			$(T)\ r_{63} = 18.2$		$n_e = 1.550$		
KH_2PO_4	$\bar{4}2m$	0.546	$(T)\ r_{41} = 8.77$		$n_o = 1.5115$		$(T)\ \epsilon_1 = \epsilon_2 = 42$
(KDP)			$(T)\ r_{63} = 10.3$		$n_e = 1.4698$		$(T)\ \epsilon_3 = 21$
		0.633	$(T)\ r_{41} = 8$		$n_o = 1.5074$		$(S)\ \epsilon_1 = \epsilon_2 = 44$
			$(T)\ r_{63} = 11$		$n_e = 1.4669$		$(S)\ \epsilon_3 = 21$
		3.39	$(T)\ r_{63} = 9.7$				
			$(T)\ n_o^3 r_{63} = 33$				
KD_2PO_4	$\bar{4}2m$	0.546	$(T)\ r_{63} = 26.8$		$n_o = 1.5079$		$(T)\ \epsilon_3 = 50$
(KD*P)			$(T)\ r_{41} = 8.8$		$n_e = 1.4683$		$(S)\ \epsilon_1 = \epsilon_2 = 58$
		0.633	$(T)\ r_{63} = 24.1$		$n_o = 1.502$		$(S)\ \epsilon_3 = 48$
					$n_e = 1.462$		
$(NH_4)H_2PO_4$	$\bar{4}2m$	0.546	$(T)\ r_{41} = 23.76$		$n_o = 1.5266$		$(T)\ \epsilon_1 = \epsilon_2 = 56$
(ADP)			$(T)\ r_{63} = 8.56$		$n_e = 1.4808$		$(T)\ \epsilon_3 = 15$
		0.633	$(T)\ r_{41} = 23.41$		$n_o = 1.5220$		$(S)\ \epsilon_1 = \epsilon_2 = 58$
			$(T)\ n_o^3 r_{63} = 27.6$		$n_e = 1.4773$		$(S)\ \epsilon_3 = 14$
$(NH_4)D_2PO_4$	$\bar{4}2m$	0.633	$(T)\ r_{41} = 40$		$n_o = 1.516$		
(AD*P)			$(T)\ r_{63} = 10$		$n_e = 1.475$		
$BaTiO_3$	4 mm	0.546	$(T)\ r_{51} = 1640$	$(S)\ r_{51} = 820$	$n_o = 2.437$		$(T)\ \epsilon_1 = \epsilon_2 = 3600$
($T_c = 395$ K)			$(T)\ r_c = 108$	$(S)\ r_c = 23$	$n_e = 2.365$		$(T)\ \epsilon_3 = 135$
$KTa_xNb_{1-x}O_3$ (KTN), $x = 0.35$ ($T_c = 40$–60°C)		0.633	$(T)\ r_{51} = 8000(T_c - 28)$		$n_o = 2.318$		
			$(T)\ r_c = 500(T_c - 28)$		$n_e = 2.277$		
			$(T)\ r_{51} = 3000(T_c - 16)$		$n_o = 2.318$		
			$(T)\ r_c = 700(T_c - 16)$		$n_e = 2.281$		
$Ba_{0.25}Sr_{0.75}Nb_2O_6$	4 mm	0.633	$(T)\ r_{13} = 67$	$(T)\ r_{51} = 42$	$n_o = 2.3117$		$\epsilon_3 = 3400$
($T_c = 395$ K)			$(T)\ r_{33} = 1340$	$(S)\ r_c = 1090$	$n_e = 2.2987$		(15 MHz)
α-HIO_3	222	0.633	$(T)\ r_{41} = 6.6$	$(S)\ r_{41} = 2.3$	$n_1 = 1.8365$		
			$(T)\ r_{52} = 7.0$	$(S)\ r_{52} = 2.6$	$n_2 = 1.984$		
			$(T)\ r_{63} = 6.0$	$(S)\ r_{63} = 4.3$	$n_3 = 1.960$		
$KNbO_3$	2 mm	0.633	$(T)\ r_{13} = 28$	$(T)\ r_{23} = 1.3$ $(T)\ r_{33} = 64$	$n_1 = 2.280$		
			$(T)\ r_{42} = 380$	$(S)\ r_{42} = 270$	$n_2 = 2.329$		
			$(T)\ r_{51} = 105$		$n_3 = 2.169$		
KIO_3	1	0.500	$r_{62} = 90$		$n_1 = 1.700$		
					$n_2 = 1.828$ (5893 Å)		
					$n_3 = 1.832$		

"(T)" = low frequency from dc through audio range: (S) = high frequency.

surface, we can determine their points of intersection (x_1, x_2, x_3) with the ellipsoid by requiring that at such points the radius vector be parallel to the normal, that is,

$$S_{ij}x_j = Sx_i \tag{9.1-15}$$

where S is a constant independent of i.

Writing out (9.1-15) in component form for $i = 1, 2, 3$ gives

$$\begin{aligned}(S_{11} - S)x_1 + S_{12}x_2 + S_{13}x_3 &= 0\\ S_{21}x_1 + (S_{22} - S)x_2 + S_{23}x_3 &= 0\\ S_{31}x_1 + S_{32}x_2 + (S_{33} - S)x_3 &= 0\end{aligned} \tag{9.1-16}$$

(9.1-16) constitutes a system of three homogeneous equations for the unknowns x_1, x_2, and x_3. The condition for a nontrivial solution is that the determinant of the coefficients vanishes, that is,

$$\det[S_{ij} - S\delta_{ij}] = 0 \tag{9.1-17}$$

This is a cubic equation in S. For real S_{ij}, which is the case with lossless crystals, the three roots S', S'', and S''' of (9.1-17) are real numbers. Having solved (9.1-17) we use the three roots, one at a time, in (9.1-16) to solve, to within a multiplicative constant, for the radius vector (x_1, x_2, x_3) to the point of intersection of the principal axis with the ellipsoid. The first vector, obtained by using S', is denoted by $\mathbf{X}'(x_1', x_2', x_3')$, the second by $\mathbf{X}''(x_1'', x_2'', x_3'')$, and the third, obtained from S''', is $\mathbf{X}'''(x_1''', x_2''', x_3''')$. Since the vectors satisfy (9.1-15), we have

$$S_{ij}x_j' = S'x_i' \tag{9.1-18}$$

with a similar relation applying to x_i'' and x_i'''.

It is an easy task to prove that the three principal axis vectors $\mathbf{X}'$, $\mathbf{X}''$, $\mathbf{X}'''$ are mutually orthogonal.

So far we have solved for the directions of the principal axes. Next we obtain their magnitudes. We multiply (9.1-18) by x_i'

$$S_{ij}x_i'x_j' = S'x_i'x_i' = S'|\mathbf{X}'|^2 \tag{9.1-19}$$

But the left side of (9.1-19) is, according to (9.1-13), equal to unity since the point (x_1', x_2', x_3') is on the ellipsoid (9.1-13). We can thus write

$$|\mathbf{X}'| = \frac{1}{\sqrt{S'}}$$

with similar results for $\mathbf{X}''$ and $\mathbf{X}'''$. The lengths of the principal axes of the index ellipsoid are thus $2(S')^{-1/2}$, $2(S'')^{-1/2}$, and $2(S''')^{-1/2}$. If we then express the equation of the index ellipsoid in terms of a Cartesian coordinate system whose axes are parallel to $\mathbf{X}'$, $\mathbf{X}''$, and $\mathbf{X}'''$, it becomes

$$S'x'^2 + S''y'^2 + S'''z'^2 = 1 \tag{9.1-20}$$

where the unit vectors $\mathbf{x}'$, $\mathbf{y}'$, and $\mathbf{z}'$ here are taken as parallel to $\mathbf{X}'$, $\mathbf{X}''$, and $\mathbf{X}'''$, respectively.

The bit of mathematics starting with (9.1-13) is referred to as the transformation of a quadratic form to a principal coordinate system. An equivalent description of this transformation is by the term matrix diagonalization. The original matrix being the ordered array of the coefficients S_{ij}

$$\mathbf{S} \equiv \begin{vmatrix} S_{11} & S_{12} & S_{13}\\ S_{21} & S_{22} & S_{23}\\ S_{31} & S_{32} & S_{33}\end{vmatrix} \tag{9.1-21}$$

The set of S', S'', and S''', which are the roots of (9.1-17), are the *eigenvalues* of the matrix $\mathbf{S}$, while the vectors $\mathbf{X}'$, $\mathbf{X}''$, and $\mathbf{X}'''$ are its eigenvectors. The term matrix diagonalization follows from the fact that if we express the quadric surface

$$S_{ij}x_ix_j = 1$$

whose coefficients form the matrix $\mathbf{S}$ of (9.1-21), in terms of a Cartesian coordinate system whose axes are $\mathbf{X}'$, $\mathbf{X}''$, and $\mathbf{X}'''$, it assumes the form (9.1-20) with the diagonal form of the matrix $\mathbf{S}$ as

$$\mathbf{S} = \begin{bmatrix} S' & 0 & 0\\ 0 & S'' & 0\\ 0 & 0 & S'''\end{bmatrix} \tag{9.1-22}$$

Example: Electrooptic Field in KH_2PO_4

To illustrate the method of matrix diagonalization, we use the example of KH_2PO_4(KDP) with a dc field along the crystal z axis, which was solved above in a somewhat less formal fashion.

The index ellipsoid is given by (9.1-7) as

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_zxy = 1 \tag{9.1-23}$$

The S_{ij} matrix is thus

$$S_{ij} = \begin{bmatrix} \dfrac{1}{n_o^2} & r_{63}E_z & 0\\ r_{63}E_z & \dfrac{1}{n_o^2} & 0\\ 0 & 0 & \dfrac{1}{n_e^2}\end{bmatrix} \tag{9.1-24}$$

The eigenvalues are given according to (9.1-17) as the roots of the equation

$$\det \begin{vmatrix} \dfrac{1}{n_0^2} - S & r_{63}E_z & 0 \\ r_{63}E_z & \dfrac{1}{n_0^2} - S & 0 \\ 0 & 0 & \dfrac{1}{n_e^2} - S \end{vmatrix} \tag{9.1-25}$$

which upon evaluation is

$$\left(\frac{1}{n_e^2} - S\right)\left[\left(\frac{1}{n_0^2} - S\right)^2 - (r_{63}E_z)^2\right] = 0$$

The roots are

$$S' = \frac{1}{n_e^2}$$

$$S'' = \frac{1}{n_0^2} + r_{63}E_z$$

$$S''' = \frac{1}{n_0^2} - r_{63}E_z \tag{9.1-26}$$

in agreement with (9.1-9). These roots are used, one at a time, in the equation

$$S_{ij}x_j = Sx_i \qquad i = 1, 2, 3 \tag{9.1-27}$$

to obtain the eigenvectors. Starting with S' we have

$$\left(\frac{1}{n_0^2} - \frac{1}{n_e^2}\right) x_1' + r_{63}E_z x_2' = 0$$

$$r_{63}E_z x_1' + \left(\frac{1}{n_0^2} - \frac{1}{n_e^2}\right) x_2' = 0$$

$$\left(\frac{1}{n_e^2} - \frac{1}{n_e^2}\right) x_3' = 0 \tag{9.1-28}$$

The first two equations above are satisfied by $x_1' = 0$ and $x_2' = 0$, while the third is satisfied by any value of x_3'. The eigenvector $\mathbf{X}'$ corresponding to $S'(=1/n_e^2)$ is thus parallel to the z axis. In a like fashion we substitute the value of S'' into (9.1-27) and find that the corresponding eigenvector $\mathbf{X}''$ is parallel to the direction $\mathbf{x} + \mathbf{y}$ while using S''' shows that $\mathbf{X}'''$ is parallel to $\mathbf{x} - \mathbf{y}$. Referring to the last two eigenvector directions as x' and y', we can rewrite the equation of the index ellipsoid in the x', y', z (principal) coordinate system as

$$\left(\frac{1}{n_0^2} - r_{63}E_z\right) x'^2 + \left(\frac{1}{n_0^2} + r_{63}E_z\right) y'^2 + \frac{z^2}{n_e^2} = 1 \tag{9.1-29}$$

where the quantities in parentheses are the eigenvalues given by (9.1-26). Equation (9.1-29) is the same as (9.1-9).

9.2 ELECTROOPTIC RETARDATION

The index ellipsoid for KDP with **E** applied parallel to z is shown in Figure 9-2. If we consider propagation along the z direction, then according to the procedure described in Section 1.4 we need to determine the ellipse formed by the intersection of the plane $z = 0$ (in general, the plane that contains the origin and is normal to the propagation direction) and the ellipsoid. The equation of this ellipse is obtained from (9.1-9) by putting $z = 0$ and is

$$\left(\frac{1}{n_o^2} - r_{63}E_z\right) x'^2 + \left(\frac{1}{n_o^2} + r_{63}E_z\right) y'^2 = 1 \tag{9.2-1}$$

One quadrant of the ellipse is shown shaded in Figure 9-2, along with its minor and major axes, which in this case coincide with x' and y', respectively. It follows from Section 1.4 that the two allowed directions of polarization are x' and y' and that their indices of refraction are $n_{x'}$ and $n_{y'}$, which are given by (9.1-10) and (9.1-11).

We are now in a position to take up the concept of retardation. We consider an optical field that is incident normally on the $x'y'$ plane with its **E** vector along the x direction. We can resolve the optical field at $z = 0$ (input plane) into two mutually orthogonal components polarized along x' and y'. The x' component propagates as

$$e_{x'} = Ae^{i[\omega t - (\omega/c)n_{x'}z]}$$

which, using (9.1-10), becomes

$$e_{x'} = Ae^{i\{\omega t - (\omega/c)[n_o + (n_o^3/2)r_{63}E_z]z\}} \tag{9.2-2}$$

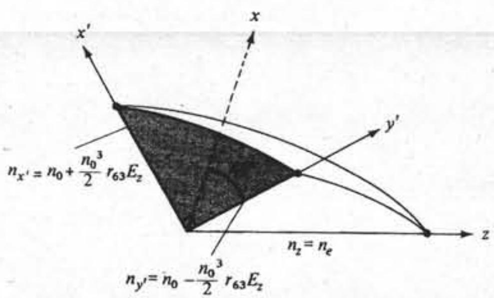


Figure 9-2 A section of the index ellipsoid of KDP, showing the principal dielectric axes x', y', and z due to an electric field applied along the z axis. The directions x' and y' are defined by Figure 9-1.

while the y' component is given by

$$e_{y'} = Ae^{i\{\omega t-(\omega/c)[n_o-(n_o^3/2)r_{63}E_z]z\}} \tag{9.2-3}$$

The phase difference at the output plane $z = l$ between the two components is called the *retardation*. It is given by the difference of the exponents in (9.2-2) and (9.2-3) and is equal to

$$\Gamma = \phi_{x'} - \phi_{y'} = \frac{\omega n_o^3 r_{63} V}{c} \tag{9.2-4}$$

where $V = E_z l$ and $\phi_{x'} = (\omega n_{x'}/c)l$.

Figure 9-3 shows $e_{x'}(z)$ and $e_{y'}(z)$ at some moment in time. Also shown are the curves traversed by the tip of the optical field vector at various points along the path. At $z = 0$, the retardation is $\Gamma = 0$ and the field is linearly polarized along x. At point e, $\Gamma = \pi/2$; thus, omitting a common phase factor, we have

$$e_{x'} = A \cos\left(\omega t - \frac{\pi}{2}\right) = A \sin \omega t$$

$$e_{y'} = A \cos \omega t \tag{9.2-5}$$

and the electric field vector is circularly polarized in the counterclockwise sense as shown in the figure. At point i, $\Gamma = \pi$ and thus

$$e_{x'} = A \cos(\omega t - \pi) = -A \cos \omega t$$

$$e_{y'} = A \cos \omega t$$

and the radiation is again linearly polarized, but this time along the y axis—that is, at 90° to its input direction of polarization.

The retardation as given by (9.2-4) can also be written as

$$\Gamma = \pi \frac{E_z l}{V_\pi} = \pi \frac{V}{V_\pi} \tag{9.2-6}$$

where V_π, the voltage yielding a retardation $\Gamma = \pi$,[4] is

$$V_\pi = \frac{\lambda}{2n_o^3 r_{63}} \tag{9.2-7}$$

where $\lambda = 2\pi c/\omega$ is the free space wavelength. Using, as an example, the value of r_{63} for ADP as given in Table 9-2, we obtain from (9.2-7)

$$(V_\pi)_{\text{ADP}} \approx 10{,}000 \text{ volts} \quad \text{at } \lambda = 0.5\ \mu\text{m}$$

[4] V_π is referred to as the "half-wave" voltage since, as can be seen in Figure 9-3c(i), it causes the two waves that are polarized along x' and y' to acquire a relative spatial displacement of $\Delta z = \lambda/2$, where λ is the optical wavelength.

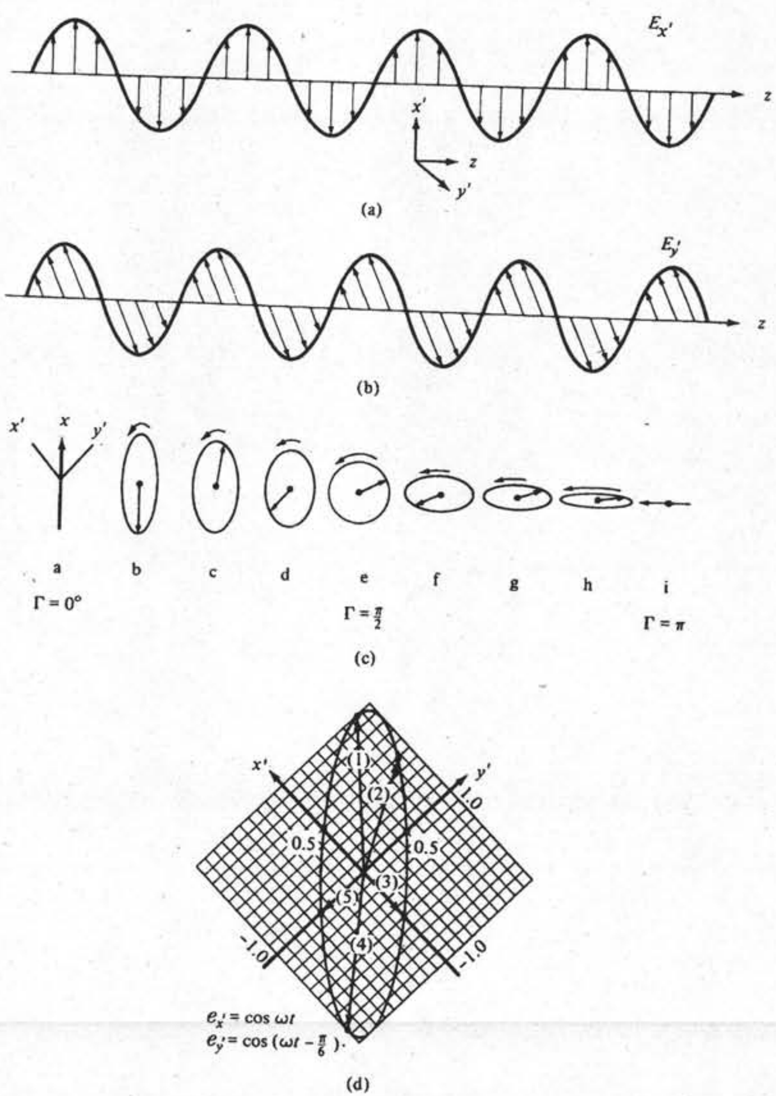


Figure 9-3 An optical field that is linearly polarized along x is incident on an electrooptic crystal having its electrically induced principal axes along x' and y'. (This is the case in KH_2PO_4 when an electric field is applied along its z axis.) (a) The component $e_{x'}$ at some time t as a function of the position z along the crystal. (b) $e_{y'}$ as a function of z at the same value of t as in (a). (c) The ellipses in the x'-y' plane traversed by the tip of the optical electric field at various points (a through i) along the crystal during one optical cycle. The arrow shows the instantaneous field vector at time t, while the curved arrow gives the sense in which the ellipse is traversed. (d) A plot of the polarization ellipse due to two orthogonal components $e_{x'} = \cos \omega t$ and $e_{y'} = \cos(\omega t - \pi/6)$. Also shown are the instantaneous field vectors at (1) $\omega t = 0°$, (2) $\omega t = 60°$, (3) $\omega t = 120°$, (4) $\omega t = 210°$, and (5) $\omega t = 270°$.

9.3 ELECTROOPTIC AMPLITUDE MODULATION

An examination of Figure 9-3 reveals that the electrically induced birefringence causes a wave launched at $z = 0$ with its polarization along x to acquire a y polarization, which grows with distance at the expense of the x component until at point i, at which $\Gamma = \pi$, the polarization becomes parallel to y. If point i corresponds to the output plane of the crystal and if one inserts at this point a polarizer at right angles to the input polarization—that is, one that allows only E_y to pass—then with the field on, the optical beam passes through unattenuated, whereas with the field off ($\Gamma = 0$), the output beam is blocked off completely by the crossed output polarizer. This control of the optical energy flow serves as the basis of the electrooptic amplitude modulation of light.

A typical arrangement of an electrooptic amplitude modulator is shown in Figure 9-4. It consists of an electrooptic crystal placed between two crossed polarizers, which, in turn, are at an angle of 45° with respect to the electrically induced birefringent axes x' and y'. To be specific, we show how this arrangement is achieved using a KDP crystal. Also included in the optical path is a naturally birefringent crystal that introduces a fixed retardation, so the total retardation Γ is the sum of the retardation due to this crystal and the electrically induced one. The incident field is parallel to x at the input face of the crystal, thus having equal-in-phase components along x' and y' that we take as

$$e_{x'} = A \cos \omega t$$
$$e_{y'} = A \cos \omega t$$

or, using the complex amplitude notation,

$$E_{x'}(0) = A$$
$$E_{y'}(0) = A$$

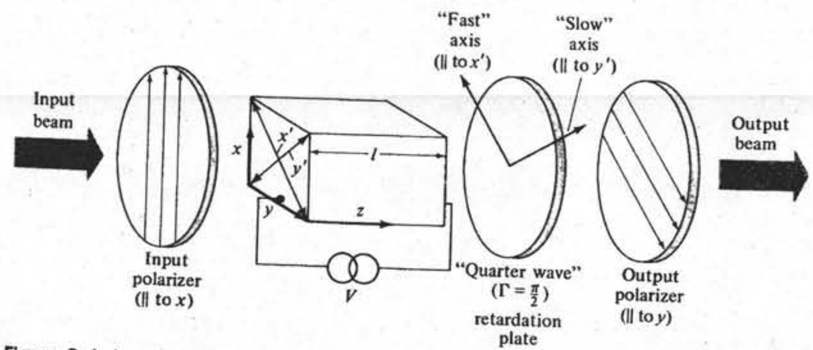


Figure 9-4 A typical electrooptic amplitude modulator. The total retardation Γ is the sum of the fixed retardation bias ($\Gamma_B = \pi/2$) introduced by the quarter-wave plate and that is attributable to the electrooptic crystal.

The incident intensity is thus[5]

$$I_i \propto \mathbf{E} \cdot \mathbf{E}^* = |E_{x'}(0)|^2 + |E_{y'}(0)|^2 = 2A^2 \tag{9.3-1}$$

Upon emerging from the output face $z = l$, the x' and y' components have acquired, according to (9.2-4), a relative phase shift (retardation) of Γ radians, so we may take them as

$$\begin{aligned} E_{x'}(l) &= Ae^{-i\Gamma} \\ E_{y'}(l) &= A \end{aligned} \tag{9.3-2}$$

The total (complex) field emerging from the output polarizer is the sum of the y components of $E_{x'}(l)$ and $E_{y'}(l)$

$$(E_y)_o = \frac{-A}{\sqrt{2}}(e^{-i\Gamma} - 1) \tag{9.3-3}$$

which corresponds to an output intensity

$$\begin{aligned} I_o &\propto [(E_y)_o(E_y^*)_o] \\ &= \frac{A^2}{2}[(e^{-i\Gamma} - 1)(e^{i\Gamma} - 1)] = 2A^2 \sin^2 \frac{\Gamma}{2} \end{aligned}$$

where the proportionality constant is the same as in (9.3-1). The ratio of the output intensity to the input is thus

$$\frac{I_o}{I_i} = \sin^2 \frac{\Gamma}{2} = \sin^2 \left[\left(\frac{\pi}{2}\right) \frac{V}{V_\pi} \right] \tag{9.3-4}$$

The second equality in (9.3-4) was obtained from (9.2-6). The transmission factor (I_o/I_i) is plotted in Figure 9-5 against the applied voltage.

The process of amplitude modulation of an optical signal is also illustrated in Figure 9-5. The modulator is usually biased[6] with a fixed retardation $\Gamma = \pi/2$ to the 50 percent transmission point. A small sinusoidal modulation voltage would then cause a nearly sinusoidal modulation of the transmitted intensity as shown.

To treat the situation depicted by Figure 9-5 mathematically, we take

$$\Gamma = \frac{\pi}{2} + \Gamma_m \sin \omega_m t \tag{9.3-5}$$

where the retardation bias is taken as $\pi/2$, and Γ_m is related to the amplitude V_m of the modulation voltage $V_m \sin \omega_m t$ by (9.2-6); thus, $\Gamma_m = \pi(V_m/V_\pi)$.

---

[5]We recall here that the time average of the product of two harmonic fields Re $[Be^{i\omega t}]$ and Re $[Ce^{i\omega t}]$ is equal to $\frac{1}{2}$ Re $[BC^*]$.

[6]This bias can be achieved by applying a voltage $V = V_\pi/2$ or, more conveniently, by using a naturally birefringent crystal as in Figure 9-4 to introduce a phase difference (retardation) of $\pi/2$ between the x' and y' components.

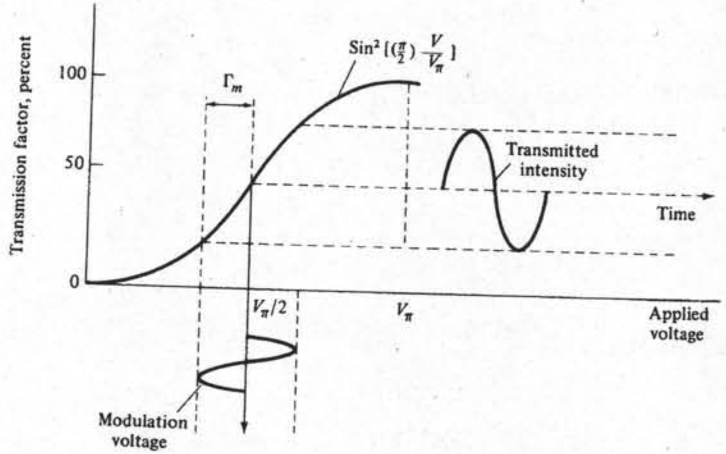


Figure 9-5 Transmission factor of a cross-polarized electrooptic modulator as a function of an applied voltage. The modulator is biased to the point $\Gamma = \pi/2$, which results in a 50 percent intensity transmission. A small applied sinusoidal voltage modulates the transmitted intensity about the bias point.

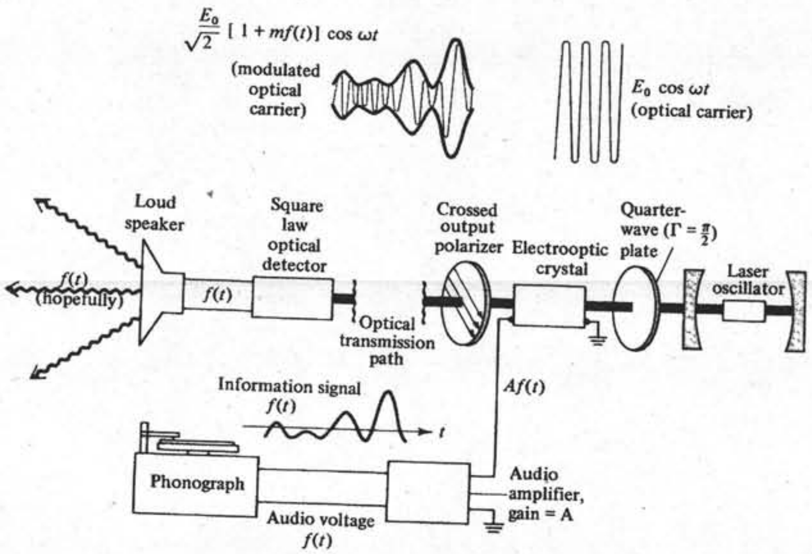


Figure 9-6 An optical communication link using an electrooptic modulator.

Using (9.3-4) we obtain

$$\frac{I_o}{I_i} = \sin^2\left(\frac{\pi}{4} + \frac{\Gamma_m}{2} \sin \omega_m t\right) \tag{9.3-6}$$

$$= \tfrac{1}{2}[1 + \sin (\Gamma_m \sin \omega_m t)] \tag{9.3-7}$$

which, for $\Gamma_m \ll 1$, becomes

$$\frac{I_o}{I_i} \simeq \frac{1}{2} (1 + \Gamma_m \sin \omega_m t) \tag{9.3-8}$$

so that the intensity modulation is a linear replica of the modulating voltage $V_m \sin \omega_m t$. If the condition $\Gamma_m \ll 1$ is not fulfilled, it follows from Figure 9-5 or from (9.3-7) that the intensity variation is distorted and will contain an appreciable amount of the higher (odd) harmonics. The dependence of the distortion of Γ_m is discussed further in Problem 9.3.

In Figure 9-6 we show how some information signal $f(t)$ (the electric output of a phonograph stylus in this case) can be impressed electrooptically as an amplitude modulation on a laser beam and subsequently be recovered by an optical detector. The details of the optical detection are considered in Chapter 11.

9.4 PHASE MODULATION OF LIGHT

In the preceding section we saw how the modulation of the state of polarization, from linear to elliptic, of an optical beam by means of the electrooptic effect can be converted, using polarizers, to intensity modulation. Here we consider the situation depicted by Figure 9-7, in which, instead of there being equal components along the induced birefringent axes (x' and y' in Figure 9-4), the incident beam is polarized parallel to one of them, x' say. In this case the application of the electric field does not change the state of polarization, but merely changes the output phase by

$$\Delta\phi_{x'} = -\frac{\omega l}{c} \Delta n_{x'}$$

where, from (9.1-10),

$$\Delta\phi_{x'} = -\frac{\omega n_o^3 r_{63}}{2c} E_z l \tag{9.4-1}$$

If the bias field is sinusoidal and is taken as

$$E_z = E_m \sin \omega_m t \tag{9.4-2}$$

then an incident optical field, which at the input ($z = 0$) face of the crystal is given by $e_{in} = A \exp(i\omega t)$, will emerge according to (9.2-2) as

$$e_{out} = A \exp\left\{i\left[\omega t - \frac{\omega}{c}\left(n_o + \frac{n_o^3}{2} r_{63} E_m \sin \omega_m t\right) l\right]\right\}$$

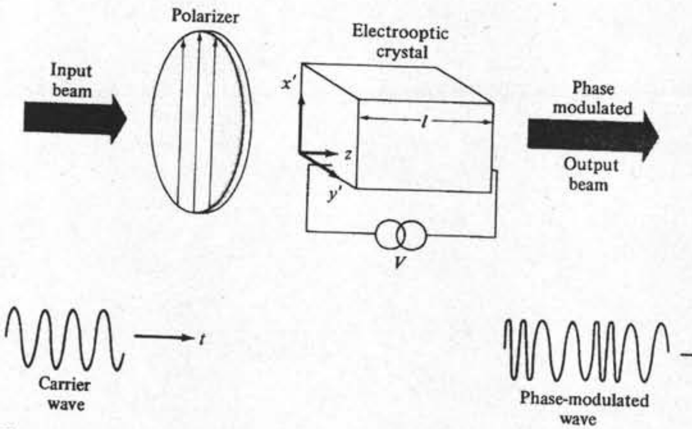


Figure 9-7 An electrooptic phase modulator. The crystal orientation and applied directions are appropriate to KDP. The optical polarization is parallel to an electrically induced principal dielectric axis (x').

where l is the length of the crystal. Dropping the constant phase factor, which is of no consequence here, we rewrite the last equation as

$$e_{\text{out}} = A \exp[i(\omega t + \delta \sin \omega_m t)] \tag{9.4-3}$$

where

$$\delta = \frac{-\omega n_o^3 r_{63} E_m l}{2c} = \frac{-\pi n_o^3 r_{63} E_m l}{\lambda} \tag{9.4-4}$$

is referred to as the phase modulation index. The optical field is thus phase-modulated with a modulation index δ. If we use the Bessel function identity

$$\exp(i\delta \sin \omega_m t) = \sum_{n=-\infty}^{\infty} J_n(\delta)\exp(in\omega_m t) \tag{9.4-5}$$

we can rewrite (9.4-3) as

$$e_{\text{out}} = A \sum_{n=-\infty}^{\infty} J_n(\delta)\, e^{i(\omega + n\omega_m)t} \tag{9.4-6}$$

which form gives the distribution of energy in the sidebands as a function of the modulation index δ. We note that, for $\delta = 0$, $J_0(0) = 1$ and $J_n(\delta) = 0$, $n \neq 0$. Another point of interest is that the phase modulation index δ as given by (9.4-4) is one half the retardation Γ as given by (9.2-4).

9.5 TRANSVERSE ELECTROOPTIC MODULATORS

In the examples of electrooptic retardation discussed in the two preceding sections, the electric field was applied along the direction of light propagation. This is the so-

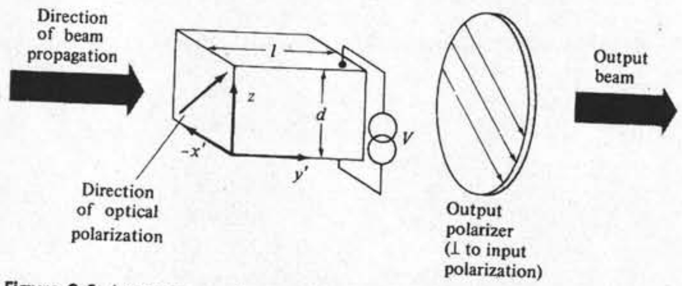


Figure 9-8 A transverse electrooptic amplitude modulator using a KH_2PO_4 (KDP) crystal in which the field is applied normal to the direction of propagation.

called longitudinal mode of modulation. A more desirable mode of operation is the transverse one, in which the field is applied normal to the direction of propagation. The reason is that in this case the field electrodes do not interfere with the optical beam, and the retardation, being proportional to the product of the field times the crystal length, can be increased by the use of longer crystals. In the longitudinal case the retardation, according to (9.2-4), is proportional to $E_z l = V$ and is independent of the crystal length l. Figures 9-1 and 9-2 suggest how transverse retardation can be obtained using a KDP crystal with the actual arrangement shown in Figure 9-8. The light propagates along y' and its polarization is in the $x'-z$ plane at 45° from the z axis. The retardation, with a field applied along z, is, from (9.1-10) and (9.1-12),

$$\begin{aligned}\Gamma &= \phi_{x'} - \phi_z \\ &= \frac{\omega l}{c}\left[(n_o - n_e) + \frac{n_o^3}{2} r_{63} \left(\frac{V}{d}\right)\right]\end{aligned} \tag{9.5-1}$$

where d is the crystal dimension along the direction of the applied field. We note that Γ contains a term that does not depend on the applied voltage. This point will be discussed in Problem 9-2. A detailed example of transverse electrooptic modulation using $\bar{4}3m$, cubic zinc-blende type crystals is given in Appendix C.

9.6 HIGH-FREQUENCY MODULATION CONSIDERATIONS

In the examples considered in the three preceding sections, we derived expressions for the retardation caused by electric fields of low frequencies. In many practical situations the modulation signal is often at very high frequencies and, in order to utilize the wide frequency spectrum available with lasers, may occupy a large bandwidth. In this section we consider some of the basic factors limiting the highest usable modulation frequencies in a number of typical experimental situations.

Consider first the situation described by Figure 9-9. The electrooptic crystal is placed between two electrodes with a modulation field containing frequencies near

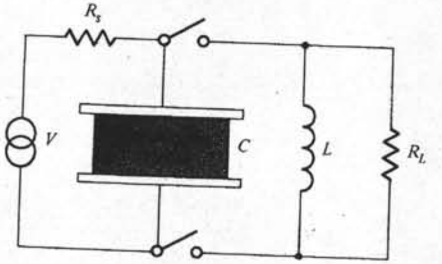


Figure 9-9 Equivalent circuit of an electrooptic modulation crystal in a parallel-plate configuration.

$\omega_0/2\pi$ applied to it. R_s is the internal resistance of the modulation source and C represents the parallel-plate capacitance due to the electrooptic crystal. If $R_s > (\omega_0 C)^{-1}$, most of the modulation voltage drop is across R_s and is thus wasted, since it does not contribute to the retardation. This can be remedied by resonating the crystal capacitance with an inductance L, where $\omega_0^2 = (LC)^{-1}$, as shown in Figure 9-9. In addition, a shunting resistance R_L is used so that at $\omega = \omega_0$ the impedance of the parallel RLC circuit is R_L, which is chosen to be larger than R_s so most of the modulation voltage appears across the crystal. The resonant circuit has a finite bandwidth—that is, its impedance is high only over a frequency interval $\Delta\omega/2\pi \simeq 1/2\pi R_L C$ (centered on ω_0). Therefore, the maximum modulation bandwidth (the frequency spectrum occupied by the modulation signal) must be less than

$$\frac{\Delta\omega}{2\pi} \simeq \frac{1}{2\pi R_L C} \tag{9.6-1}$$

if the modulation field is to be a faithful replica of the modulation signal.

In practice, the size of the modulation bandwidth $\Delta\omega/2\pi$ is dictated by the specific application. In addition, one requires a certain peak retardation Γ_m. Using (9.2-4) to relate Γ_m to the peak modulation voltage $V_m = (E_z)_m l$, we can show, with the aid of (9.6-1), that the power $V_m^2/2R_L$ needed in KDP-type crystals to obtain a peak retardation Γ_m is related to the modulation bandwidth $\Delta\nu = \Delta\omega/2\pi$ as

$$P = \frac{\Gamma_m^2 \lambda^2 A \epsilon \Delta\nu}{4\pi l n_0^6 r_{63}^2} \tag{9.6-2}$$

where $n_0 l$ is the length of the optical path in the crystal, A is the cross-sectional area of the crystal normal to l, and ϵ is the dielectric constant at the modulation frequency ω_0.

Transit-Time Limitations to High-Frequency Electrooptic Modulation

According to (9.2-4) the electrooptic retardation due to a field E can be written as

$$\Gamma = aEl \tag{9.6-3}$$

where $a = \omega n_0^3 r_{63}/c$ and l is the length of the optical path in the crystal. If the field E changes appreciably during the transit time $\tau_d = nl/c$ of light through the crystal, we must replace (9.6-3) by

$$\Gamma(t) = a \int_0^l e(z)\, dz = a \frac{c}{n} \int_{t-\tau_d}^{t} e(t')\, dt' \tag{9.6-4}$$

where c is the velocity of light and $e(t')$ is the instantaneous electric field. In the second integral we replace integration over z by integration over time, recognizing that the portion of the wave that reaches the output face $z = l$ at time t entered the crystal at time $t - \tau_d$. We also assumed that at any given moment the field $e(t)$ has the same value throughout the crystal.'

Taking $e(t')$ as a sinusoid

$$e(t') = E_m e^{i\omega_m t'}$$

we obtain from (9.6-4)

$$\begin{aligned}\Gamma(t) &= a \frac{c}{n} E_m \int_{t-\tau_d}^{t} e^{i\omega_m t'}\, dt' \\ &= \Gamma_0 \left[\frac{1 - e^{-i\omega_m \tau_d}}{i\omega_m \tau_d}\right] e^{i\omega_m t}\end{aligned} \tag{9.6-5}$$

where $\Gamma_0 = a(c/n)\tau_d E_m = alE_m$ is the peak retardation, which obtains when $\omega_m \tau_d \ll 1$. The factor

$$r = \frac{1 - e^{-i\omega_m \tau_d}}{i\omega_m \tau_d} \tag{9.6-6}$$

gives the decrease in peak retardation resulting from the finite transit time. For $r \simeq 1$ (that is, no reduction), the condition $\omega_m \tau_d \ll 1$ must be satisfied, so the transit time must be small compared to the shortest modulation period. The factor r is plotted in Figure 11-17.

If, somewhat arbitrarily, we take the highest useful modulation frequency as that for which $\omega_m \tau_d = \pi/2$ (at this point, according to Figure 11-17, $|r| = 0.9$) and we use the relation $\tau_d = nl/c$, we obtain

$$(\nu_m)_{\max} = \frac{c}{4nl} \tag{9.6-7}$$

which, using a KDP crystal ($n \simeq 1.5$) and a length $l = 1$ cm, yields $(\nu_m)_{\max} = 5 \times 10^9$ Hz.

Traveling-Wave Modulators

One method that can, in principle, overcome the transit-time limitation, involves applying the modulation signal in the form of a traveling wave [3], as shown in Figure 9-10. If the optical and modulation field phase velocities are equal to each other, then a portion of an optical wavefront will exercise the same instantaneous

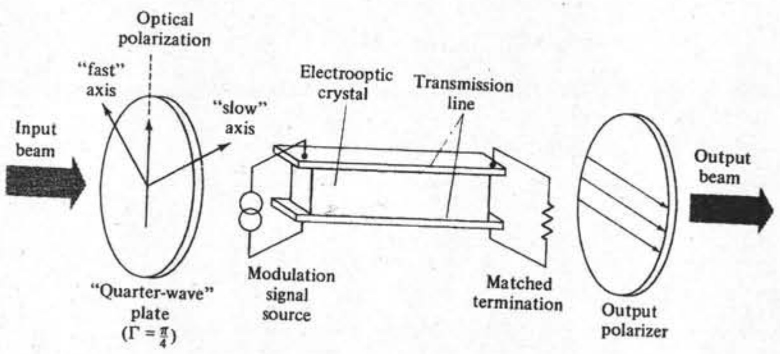


Figure 9-10 A traveling-wave electrooptic modulator.

electric field, which corresponds to the field it encounters at the entrance face, as it propagates through the crystal and the transit-time problem discussed above is eliminated. This form of modulation can be used only in the transverse geometry that was discussed in the preceding section, since the RF field in most propagating structures is predominantly transverse.

Consider an element of the optical wavefront that *enters* the crystal at $z = 0$ at time t. The position z of this element at some later time t' is

$$z(t') = \frac{c}{n}(t' - t) \tag{9.6-8}$$

where c/n is the optical phase velocity. The retardation exercised by this element is given similarly to (9.6-4) by

$$\Gamma(t) = \frac{ac}{n}\int_t^{t+\tau_d} e[t', z(t')]\, dt' \tag{9.6-9}$$

where $e[t', z(t')]$ is the instantaneous modulation field as seen by an observer traveling with the phase front. Taking the traveling modulation field as

$$e(t', z) = E_m e^{i[\omega_m t' - k_m z]}$$

we obtain, using (9.6-8),

$$e[t', z(t')] = E_m e^{i[\omega_m t' - k_m (c/n)(t'-t)]} \tag{9.6-10}$$

Recalling that $k_m = \omega_m / c_m$, where c_m is the phase velocity of the modulation field, we substitute (9.6-10) in (9.6-9) and, carrying out the simple integration, obtain

$$\Gamma(t) = \Gamma_0 e^{i\omega_m t}\left[\frac{e^{i\omega_m \tau_d(1-c/nc_m)} - 1}{i\omega_m \tau_d(1 - c/nc_m)}\right] \tag{9.6-11}$$

where $\Gamma_0 = alE_m = a(c/n)\tau_d E_m$ is the retardation that would result from a dc field equal to E_m.

The reduction factor

$$r = \frac{e^{i\omega_m \tau_d(1-c/nc_m)} - 1}{i\omega_m \tau_d(1 - c/nc_m)} \tag{9.6-12}$$

is of the same form as that of the lumped-constant modulator (9.6-6) except that τ_d is replaced by $\tau_d(1 - c/nc_m)$. If the two phase velocities are made equal so that $c/n = c_m$, then $r = 1$ and maximum retardation is obtained *regardless* of the crystal length.

The maximum useful modulation frequency is taken, as in the treatment leading to (9.6-7), as that for which $\omega_m \tau_d(1 - c/nc_m) = \pi/2$, yielding

$$(\nu_m)_{\max} = \frac{c}{4ln(1 - c/nc_m)} \tag{9.6-13}$$

which, upon comparison with (9.6-7), shows an increase in the frequency limit or useful crystal length of $(1 - c/nc_m)^{-1}$. The problem of designing traveling wave electrooptic modulators is considered in References [4–6].

For a more detailed treatment of electrooptic modulation including the traveling-wave and high-frequency cases, the student should consult Reference [11] as well as the treatment of Section 9.9.

9.7 ELECTROOPTIC BEAM DEFLECTION

The electrooptic effect is also used to deflect light beams [7]. The operation of such a beam deflector is shown in Figure 9-11. Imagine an optical wavefront incident on a crystal in which the optical path length depends on the transverse position x. This could be achieved by having the velocity of propagation—that is, the index of refraction n—depend on x, as in Figure 9-11. Taking the index variation to be a linear function of x, the upper ray A "sees" an index $n + \Delta n$ and hence traverses the crystal in a time

$$T_A = \frac{l}{c}(n + \Delta n)$$

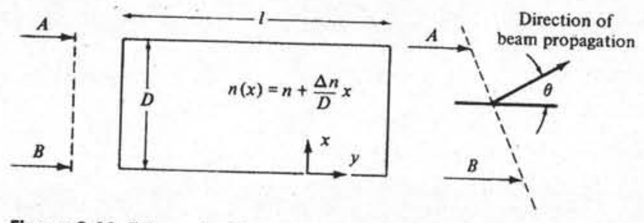


Figure 9-11 Schematic diagram of a beam deflector. The index of refraction varies linearly in the x direction as $n(x) = n_o + ax$. Ray B "gains" on ray A in passing through the crystal axis, thus causing a tilting of the wavefront by θ.

The lower portion of the wavefront (that is, ray B) "sees" an index n and has a transit time

$$T_B = \frac{l}{c} n$$

The difference in transit times results in a lag of ray A with respect to B of

$$\Delta y = \frac{c}{n}(T_A - T_B) = l\frac{\Delta n}{n}$$

which corresponds to a deflection of the beam-propagation axis, as measured inside the crystal, at the output face of

$$\theta' = -\frac{\Delta y}{D} = -\frac{l\Delta n}{Dn} = -\frac{l}{n}\frac{dn}{dx} \tag{9.7-1}$$

where we replaced $\Delta n/D$ by dn/dx. The external deflection angle θ, measured with respect to the horizontal axis, is related to θ' by Snell's law

$$\frac{\sin\theta}{\sin\theta'} = n$$

which, using (9.7-1) and assuming $\sin\theta \simeq \theta \ll 1$ yields

$$\theta = \theta' n = -l\frac{\Delta n}{D} = -l\frac{dn}{dx} \tag{9.7-2}$$

A simple realization of such a deflector using a KH_2PO_4(KDP) crystal is shown in Figure 9-12. It consists of two KDP prisms with edges along the x', y', and z

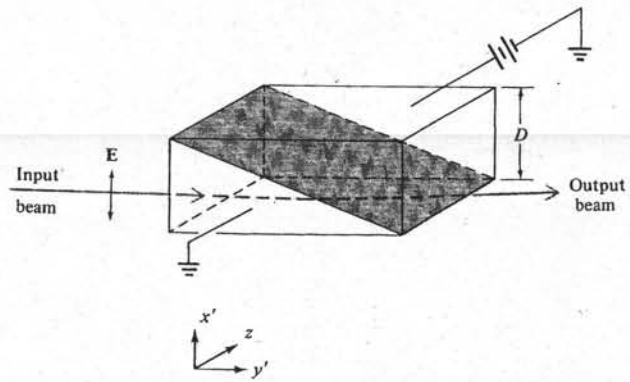


Figure 9-12 Double-prism KDP beam deflector. Upper and lower prisms have their z axes reversed with respect to each other. The deflection field is applied parallel to z.

directions.[7] The two prisms have their z axes opposite to one another, but are otherwise similarly oriented. The electric field is applied parallel to the z direction, and the light propagates in the y' direction with its polarization along x'. For this case the index of refraction "seen" by ray A, which propagates entirely in the upper prism, is given by (9.1-10) as

$$n_A = n_o + \frac{n_o^3}{2} r_{63} E_z$$

while in the lower prism the sign of the electric field with respect to the z axis is reversed so that

$$n_B = n_o - \frac{n_o^3}{2} r_{63} E_z$$

Using (9.7-2) with $\Delta n = n_A - n_B$, the deflection angle is given by

$$\theta = \frac{l}{D} n_o^3 r_{63} E_z \tag{9.7-3}$$

According to (2.5-18), every optical beam has a finite, far-field divergence angle that we call θ_{beam}. It is clear that a fundamental figure of merit for the deflector is not the angle of deflection θ that can be changed by a lens, but the factor N by which θ exceeds θ_{beam}. If one were, as an example, to focus the output beam, then N would correspond to the number of resolvable spots that can be displayed in the focal plane using fields with a magnitude up to E_z.

To get an expression for N we assume that the crystal is placed astride the "waist" of a Gaussian (fundamental) beam with a spot size ω_0. According to (2.5-18) the far-field diffraction angle in air is

$$\theta_{\text{beam}} = \frac{\lambda}{\pi\omega_0}$$

Such a beam can be passed through a crystal with height $D = 2\omega_0$ so that, using (9.7-3), the number of resolvable spots is

$$N = \left|\frac{\theta}{\theta_{\text{beam}}}\right| = \frac{\pi l n_o^3 r_{63}}{2\lambda} E_z \tag{9.7-4}$$

It follows directly from (9.7-4), the details being left as a problem, that an electric field that induces a birefringent retardation (in a distance l) $\Delta\Gamma = \pi$ will yield $N \simeq 1$. Therefore, fundamentally, the electrooptic extinction of a beam, which according to (9.3-4) requires $\Gamma = \pi$, is equivalent to a deflection by one spot diameter.

[7] These are the principal axes of the index ellipsoid when an electric field is applied along the z direction as described in Section 9.1 and in the example of Section 9.5.

The deflection of an optical beam by diffraction from a sound wave is discussed in Chapter 12. Electrooptic modulation in thin dielectric waveguides [8–10] is discussed in Chapter 13.

9.8 ELECTROOPTIC MODULATION—COUPLED WAVE ANALYSIS

The analysis of electrooptic modulation in Section 9.3 was based on an approach that requires one to determine the propagating electromagnetic eigenmodes of the crystal in the presence of the (low-frequency) electric field. In Section 9.2, we found as a special case the two eigenmodes and their indices in the presence of a dc field $\hat{E}_m = \hat{e}_z E_z$ for propagation along the z axis of KH_2PO_4.

The formalism of this section uses a different point of view, that of the coupled modes approach, in which the modulation field is viewed as a perturbation that causes exchange of power, coupling, between the eigenmodes of the crystal in the absence of an applied modulating field. The two approaches are, of course, formally equivalent as will be shown in Section 13.9. The author, however, prefers the coupled-mode approach since it dispenses with the need to diagonalize, as in (9.1-25), the permeability tensor. In addition, it lends itself more easily to an accurate analysis in situations in which the modulation field is a high-frequency field and the transit time of the optical field is not small compared to the modulation period.

We start with Equation (1.2-9), which using (1.2-3) can be written as

$$\mathbf{e} \cdot \mathbf{i} = -\nabla \cdot (\mathbf{e} \times \mathbf{h}) - \mathbf{h} \cdot \frac{\partial \mathbf{b}}{\partial t} - \mathbf{e} \cdot \frac{\partial \mathbf{d}}{\partial t}$$

Using the Gauss theorem (1.2-10a) to integrate the last equation over an arbitrary volume V

$$-\int_V \nabla \cdot (\mathbf{e} \times \mathbf{h})\, dV = -\int_s (\mathbf{e} \times \mathbf{h}) \cdot \mathbf{n}\, da$$
$$= \int_V (\mathbf{e} \cdot \mathbf{i})\, dV + \int_V \left(\mathbf{h} \cdot \frac{\partial \mathbf{b}}{\partial t} + \mathbf{e} \cdot \frac{\partial \mathbf{d}}{\partial t} \right) dV$$

where S is the surface bounding V. If we further assume that the material medium is linear (i.e., ϵ and μ are independent of the field strengths), we can rewrite the last result as

$$-\int_s (\mathbf{e} \times \mathbf{h}) \cdot \mathbf{n}\, da = \int_V (\mathbf{e} \cdot \mathbf{i})\, dV + \frac{d}{dt} \int_V \frac{1}{2} (\mathbf{e} \cdot \mathbf{d} + \mathbf{h} \cdot \mathbf{b}) dV \quad (9.8\text{-}1)$$

The usual interpretation of (9.8-1) is that the left side represents the electromagnetic power flowing into V, the term $(\mathbf{e} \cdot \mathbf{i})$ represents the losses due to conduction (ohmic losses), while the last term represents the temporal rate of change of the electromagnetic energy stored in V. It follows that the energy density (J/m^3) due to the electric field is

$$\omega_e = \frac{1}{2} \mathbf{E} \cdot \mathbf{D} \quad (9.8\text{-}2)$$

where to conform with the notation of this section we changed $\mathbf{e} \to \mathbf{E}$, $\mathbf{h} \to \mathbf{H}$. In a linear (but not necessarily isotropic) medium we can write

$$D_i = \epsilon_{ij} E_j, \qquad E_i = \eta_{ij} D_j, \qquad \overline{\overline{\eta}} \equiv \overline{\overline{\epsilon}}^{-1} \quad (9.8\text{-}3)$$

so that

$$2\omega_e = \eta_{ij} D_j D_i \quad (9.8\text{-}4)$$

Let

$$\frac{D_i}{\sqrt{2\omega_e \epsilon_0}} \longrightarrow x_i$$

so that the last equation becomes

$$\epsilon_0 \eta_{ij} x_i x_j = 1$$

which, after adopting the Voigt notation

$$11 \to 1,\ 22 \to 2,\ 33 \to 3,\ 23 \to 4,\ 13 \to 5,\ 12 \to 6 \quad (9.8\text{-}5)$$

becomes

$$\epsilon_0 \left[\eta_1 x^2 + \eta_2 y^2 + \eta_3 z^2 + 2\eta_4 yz + 2\eta_5 xz + 2\eta_6 xy \right] = 1 \quad (9.8\text{-}6)$$

This equation is identical to that of the optical indicatrix (9.1-2), provided we associate

$$\left(\frac{1}{n^2} \right)_i \longrightarrow \epsilon_0 \eta_i \quad (9.8\text{-}7)$$

So that the definition (9.1-3) of the linear electrooptic effect can be written as $\epsilon_0 \Delta \eta_i = r_{ik} E_k^{(0)}$ or, restoring the full subscript labeling,

$$\epsilon_0 \Delta \eta_{ij} = r_{ijk} E_k^{(0)} \quad (9.8\text{-}8)$$

where $E_k^{(0)}$ is a dc or low-frequency field and summation over repeated indices is assumed.

Returning for a moment to (9.8-3), we will assume that $\overline{\overline{\epsilon}}$ and $\overline{\overline{\eta}}$ are expressed initially in the principal dielectric coordinate system where, with no applied field, $E_k^{(0)} = 0$, ϵ_{ij}, η_{ij} are zero when $i \neq j$, i.e., $\overline{\overline{\epsilon}}$ and $\overline{\overline{\eta}}$ are diagonal. We use the rule for matrix inversion to obtain

$$\eta_{ii} = (\epsilon_{ii})^{-1}$$
$$\underset{i \neq j}{\eta_{ij}} = \frac{-\epsilon_{ji}}{\epsilon_{ii}\epsilon_{jj}} = \frac{-\epsilon_{ij}}{\epsilon_{ii}\epsilon_{jj}} \quad (9.8\text{-}9)$$

and after combining (9.8-8) and (9.8-9)

$$\Delta \epsilon_{ji} = -\frac{\epsilon_{ii}\epsilon_{jj}}{\epsilon_0} r_{ijk} E_k^{(0)}$$
$$(r_{ijk} = r_{jik}) \quad (9.8\text{-}10)$$

which is our principal result. It expresses the linear electrooptic effect as the first-order perturbation in the elements of the dielectric tensor.

Now consider the effect of applying a low-frequency field $E_k^{(0)}$ to a crystal in the presence of an optical field $E_j^{(\omega)}(\mathbf{r}, t)$ using

$$D_i^{(\omega)} = \epsilon_{ij}E_j^{(\omega)} = \epsilon_0 E_i^{(\omega)} + P_i^{(\omega)}$$

or

$$P_i^{(\omega)} = (\epsilon_{ij} - \epsilon_0\delta_{ij})E_j^{(\omega)} \tag{9.8-11}$$

so that a perturbation $\Delta\epsilon_{ij}$ causes a perturbation in the medium optical polarization of

$$\begin{aligned}\Delta P_i^{(\omega)} &= (\Delta\epsilon_{ij})E_j^{(\omega)} \\ &= -\frac{\epsilon_{ii}\epsilon_{jj}}{\epsilon_0} r_{ijk}E_j^{(\omega)}E_k^{(0)}\end{aligned} \tag{9.8-12}$$

The effect of the dc (low-frequency) field $E_k^{(0)}$ on the optical field propagating through the crystal is thus represented by an effective optical polarization $\Delta P_i^{(\omega)}$ of (9.8-12). The latter can now be used in Maxwell's equations to investigate the effect of the dc field on optical propagation. This will be done next.

The Wave Equation

Starting with Maxwell's equations as in Section 8.2, we have

$$\nabla^2\mathbf{E}(\mathbf{r}, t) - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r}, t) - \mu_0\frac{\partial^2}{\partial t^2}\mathbf{P} = 0 \tag{9.8-13}$$

The polarization vector $\mathbf{P}$ is taken as the sum

$$\mathbf{P} = \epsilon_0\bar{\bar{\chi}}\mathbf{E} + \Delta\mathbf{P}$$

where $\bar{\bar{\chi}}$ is the second-rank linear susceptibility tensor and $\Delta\mathbf{P}$ the perturbation in $\mathbf{P}$ as given by (9.8-12). Using the last expression in (9.8-13) leads to

$$\begin{aligned}\nabla^2\mathbf{E} - \mu_0\frac{\partial^2}{\partial t^2}(\bar{\bar{1}} + \bar{\bar{\chi}})\mathbf{E} - \mu_0\frac{\partial^2}{\partial t^2}(\Delta\mathbf{P}) \\ = \nabla^2\mathbf{E} - \mu_0\frac{\partial^2}{\partial t^2}\bar{\bar{\epsilon}}\mathbf{E} - \mu_0\frac{\partial^2}{\partial t^2}(\Delta\mathbf{P})\end{aligned} \tag{9.8-14}$$

defining the dielectric tensor

$$\bar{\bar{\epsilon}} = \epsilon_0(\bar{\bar{1}} + \bar{\bar{\chi}})$$

the wave equation (9.8-13) becomes

$$\nabla^2\mathbf{E} - \mu_0\frac{\partial^2}{\partial t^2}\bar{\bar{\epsilon}}E - \mu_0\frac{\partial^2}{\partial t^2}(\Delta\mathbf{P}) = 0 \tag{9.8-15}$$

Most of the scenarios of electrooptic modulation may be described as an exchange of power between two optical eigenfields.[8] These can be two mutually orthogonal transverse polarizations of a field propagating along, say, the $\hat{\zeta}$ direction. As a result we take the total field as

$$\begin{aligned}\mathrm{E}(r, t) &= \tfrac{1}{2}\hat{e}_1A_1(\mathbf{r}, t)e^{i(\omega t - k_1\zeta)} \\ &\quad + \tfrac{1}{2}\hat{e}_2A_2(\mathbf{r}, t)e^{i(\omega t - k_2\zeta)} + \text{c.c.}\end{aligned} \tag{9.8-16}$$

where A_1 and A_2 are the "slowly" varying complex amplitudes of the fields that are polarized along the $\hat{e}_1$ and $\hat{e}_2$ directions normal to $\hat{\zeta}$ and

$$k_{1,2} = (\omega/c)n_{1,2}$$

Substitution of the last equation in (9.8-15) gives

$$\begin{aligned}&\hat{e}_1e^{i(\omega t - k_1\zeta)}\left[\left(\frac{\partial^2}{\partial\zeta^2} - 2ik_1\frac{\partial}{\partial\zeta} - k_1^2\right) - \mu_0\epsilon_{11}\left(\frac{\partial^2}{\partial t^2} + 2i\omega\frac{\partial}{\partial t} - \omega^2\right)\right]\frac{A_1(\zeta, t)}{2} \\ &\quad + \hat{e}_2e^{i(\omega t - k_2\zeta)}\left[\left(\frac{\partial}{\partial\zeta^2} - 2ik_2\frac{\partial}{\partial\zeta} - k_2^2\right) - \mu_0\epsilon_{22}\right. \\ &\quad \left.\times\left(\frac{\partial^2}{\partial t^2} + 2i\omega\frac{\partial}{\partial t} - \omega^2\right)\right]\frac{A_2(\zeta, t)}{2} + \text{c.c.} \\ &\quad = i\mu_0\frac{\partial^2}{\partial t^2}\Delta P(\mathbf{r}, t)\end{aligned} \tag{9.8-17}$$

where we assume that $\hat{e}_1$ and $\hat{e}_2$ are unit vectors along the principal dielectric axes (this is the case in nearly all experimental situations) and used

$$\epsilon_{11} = \epsilon_0(1 + \chi_{11}) \qquad \epsilon_{22} = \epsilon_0(1 + \chi_{22})$$

Recognizing that $k_i^2 \equiv \omega_i^2\mu_0\epsilon_{11}$, $n_i = \sqrt{\epsilon_{ii}/\epsilon_0}$ $(i = 1, 2)$, and making the slowly varying envelope approximation $\partial^2/\partial\zeta^2 \ll k\partial/\partial\zeta$ and $\partial^2/\partial t^2 \ll \omega\partial/\partial t$, we obtain

$$\left(\frac{\partial}{\partial\zeta} + \frac{n_1}{c}\frac{\partial}{\partial t}\right)A_1(\zeta, t) = \frac{i\mu_0}{k_1}e^{-i(\omega t - k_1\zeta)}\frac{\partial^2}{\partial t^2}[\Delta P(\mathbf{r}, t)]_1 \tag{9.8-18a}$$

The subscript 1 on the right side signifies that we need only consider that part of $\Delta\mathbf{P}$ that contains the factor $\exp[i(\omega t - k_1\zeta)]$. The rest average out to zero.

$$\left(\frac{\partial}{\partial\zeta} + \frac{n_2}{c}\frac{\partial}{\partial t}\right)A_2(\zeta, t) = \frac{i\mu_0}{k_2}e^{-i(\omega t - k_2\zeta)}\frac{\partial^2}{\partial t^2}[\Delta P(\mathbf{r}, t)]_2 \tag{9.8-18b}$$

These are our basic working equations. They can be used to analyze most of the situations arising in electrooptic or acoustooptic modulation.

---

[8] We use the term *eigen (self) field* in the sense of the eigen modes of quantum mechanics. It is used here to describe a propagating monochromatic field, say, along z, that except for a propagation delay does not depend on z.

We will next show how Equations (9.8-18) are used in the important cases of electrooptic phase and amplitude modulation with a traveling modulation field.

9.9 PHASE MODULATION

In this case we have a traveling modulation field with some polarization, say, k, so that in (9.8-12) we take

$$E_k^{(0)} = E_{mk} \sin (\omega_m t - k_m \zeta) \tag{9.9-1}$$

For pure phase modulation of, say, wave 1, it is necessary that the perturbation polarization $(\Delta P)_1$ involve only A_1 and not A_2. This polarization is then given by (9.8-12) as

$$\Delta P_1(\zeta, t) = -\frac{(\epsilon_{11})^2}{2\epsilon_0} r_{11k} A_1(\zeta, t) e^{i(\omega t - k_1 \zeta)} E_{mk} \sin (\omega_m t - k_m \zeta) + \text{c.c.} \tag{9.9-2}$$

So that Equation (9.8-18a) becomes

$$\left(\frac{\partial}{\partial \zeta} + \frac{n_1}{c}\frac{\partial}{\partial t}\right) A_1(\zeta, t) = i\beta \sin (\omega_m t - k_m \zeta) A_1(\zeta, t) \tag{9.9-3}$$

$$\begin{aligned} \beta &\equiv + \frac{k_1 n_1^2}{2} r_{11k} E_{mk} \\ &= + \frac{\omega}{2c} n_1^3 r_{11k} E_{mk} \end{aligned} \tag{9.9-4}$$

In deriving (9.9-3) we took advantage of the fact that $\omega \ggg \omega_m$ (typically $\omega \sim 10^{15}$, $\omega_m < 10^{11}$), and replaced $\partial^2/\partial t^2$ on the right side of (9.8-17) by $-\omega^2$.

Equation (9.9-3) is a first-order linear partial differential equation and can be integrated by a change of variables

$$\begin{aligned} u &= \zeta + \frac{c}{n} t \\ v &= \zeta - \frac{c}{n} t \end{aligned} \tag{9.9-5}$$

$$(n \equiv n_1)$$

Using

$$\begin{aligned} \frac{\partial}{\partial \zeta} &= \frac{\partial u}{\partial \zeta}\frac{\partial}{\partial u} + \frac{\partial v}{\partial \zeta}\frac{\partial}{\partial v} = \frac{\partial}{\partial u} + \frac{\partial}{\partial v} \\ \frac{\partial}{\partial t} &= \frac{\partial u}{\partial t}\frac{\partial}{\partial u} + \frac{\partial v}{\partial t}\frac{\partial}{\partial v} = \frac{c}{n}\left(\frac{\partial}{\partial u} - \frac{\partial}{\partial v}\right) \end{aligned} \tag{9.9-6}$$

Equation (9.9-3) becomes $(A_1 \equiv A)$.

$$2\frac{\partial}{\partial u} A = i\beta \sin \left[\frac{n\omega_m}{2c}(u - v) - \frac{k_m}{2}(u + v)\right] A \tag{9.9-7}$$

By treating u and v as independent variables, an integration of (9.9-7) yields

$$A(\zeta, t) = C\left(\zeta - \frac{c}{n} t\right) \exp \left[-i \frac{\beta c}{\omega_m (n - n_m)} \cos (\omega_m t - k_m \zeta)\right] \tag{9.9-8}$$

where C is an arbitrary function and $n_m = c k_m / \omega_m$ is the index of refraction at ω_m. The boundary condition at the input ($\zeta = 0$) face of the crystal is

$$A(0, t) = A_0, \tag{9.9-9}$$

where A_0 is an arbitrary constant. This condition requires that the function C be of the form

$$C\left(\zeta - \frac{c}{n} t\right) = A_0 \exp \left[i \frac{\beta c}{\omega_m (n - n_m)} \cos \left(\omega_m t - \frac{n}{c}\omega_m \zeta\right)\right] \tag{9.9-10}$$

The mode amplitude $A(\zeta, t)$ is then, according to Equations (9.9-8) and (9.9-10), given by

$$\begin{aligned} A(\zeta, t) = A_0 \exp \\ \times \left\{ i \frac{\beta c}{\omega_m (n - n_m)} \left[\cos \left(\omega_m t - \frac{\omega_m}{c} n\zeta\right) - \cos \left(\omega_m t - \frac{\omega_m}{c} n_m \zeta\right)\right]\right\} \end{aligned} \tag{9.9-11}$$

By using the trigonometric identity

$$\cos \alpha - \cos \beta = -2 \sin \tfrac{1}{2}(\alpha + \beta) \sin \tfrac{1}{2}(\alpha - \beta)$$

the amplitude at the output face ($\zeta = L$) of the crystal can be written as

$$A(L, t) = A_0 \exp [i\delta \sin (\omega_m t - \phi)] \tag{9.9-12}$$

where

$$\delta = \beta L \frac{\sin \dfrac{\omega_m}{2c}(n_m - n)L}{\dfrac{\omega_m}{2c}(n_m - n)L} \tag{9.9-13}$$

$$\phi = \frac{\omega_m}{2c}(n + n_m)L \tag{9.9-14}$$

If there is no further perturbation beyond $\zeta = L$, then the emerging beam can be written (using 9.8-16) as

$$E_1(L, t) = A_0 \exp \{i[\omega t + \delta \sin (\omega_m t - \phi) - kL]\} \tag{9.9-15}$$

$$k = \frac{\omega n}{c}$$

This is our main result. The output consists of a phase-modulated wave with a modulation index δ. The value of δ given by Equation (9.9-13) for this case is no longer proportional to the length of the crystal, L, and is reduced from its maximum value βL by a factor

$$\eta \equiv \frac{\sin \Delta L}{\Delta L}$$

where

$$\Delta \equiv \frac{\omega_m}{2c}(n_m - n) = \frac{\omega_m}{2}\left(\frac{1}{v_m} - \frac{1}{v_0}\right) \tag{9.9-16}$$

in which $v_0 = c/n_1$ and $v_m = c/n_m$ are the phase velocities of the light and modulating wave, respectively. Physically, this reduction factor is due to the mismatch of the phase velocities of the waves. In the event that the light wave and the modulating wave are traveling with the same phase velocities, the light wave will experience a constant modulating field as it propagates through the electrooptic crystal. The reduction factor η in this case is unity; that is, there is no reduction in the modulation index δ. The modulation index in this case is linearly proportional to the length of the crystal. When the phase velocities are not equal, δ becomes a periodic function of the length of the crystal. A plot of δ versus L is shown in Figure 9-13. The maximum δ occurs at the condition when

$$\frac{\omega_m}{2c}|n_m - n|L = \frac{\pi}{2} \tag{9.9-17}$$

with the maximum modulation index, δ_{max}, given by

$$\delta_{max} = \frac{\omega}{\omega_m}\frac{n^3}{|n - n_m|} r_{11k}E_{mk} \tag{9.9-18}$$

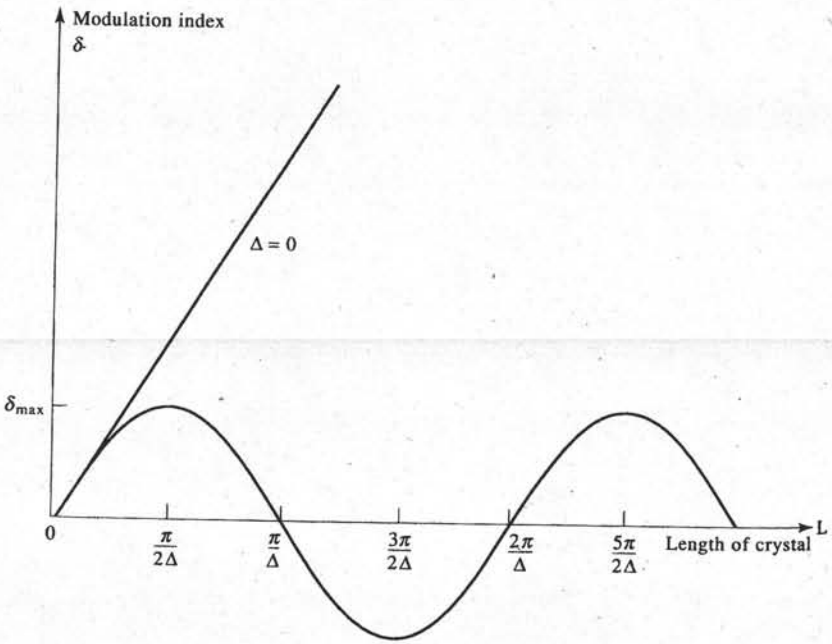


Figure 9-13 The modulation index δ versus the length L of the crystal.

Example: $LiNbO_3$ Phase Modulator

Referring to Figure 9-14, we consider a rectangular $LiNbO_3$ rod with its input and output planes perpendicular to the y axis. The z direction is a principal dielectric axis of the crystal. An RF field with an **E** vector parallel to the z axis is applied to the crystal. Both the RF field and the optical beam are propagating in the y direction. An input polarizer in front of the input plane ensures that the light is polarized along the z direction of the crystal. The modulation index δ is given, according to Equation (9.9-13), by

$$\delta = \frac{\omega}{2c} n_e^3 r_{33} E_z L \frac{\sin \frac{\omega_m}{2c}(n_m - n_e)L}{\frac{\omega_m}{2c}(n_m - n_e)L} \tag{9.9-19}$$

where n_e is the extraordinary index of refraction of the crystal, L is the length of the crystal, and r_{33} is the relevant electrooptic coefficient. Let $\omega_m/2\pi = 6$ GHz, $n_m = 1.84$, and $n_e = 2.2$; then the maximum modulation occurs at $L = 6.8$ cm according to Equation (9.9-17). The symmetry group of $LiNbO_3$ is 3 m. From Table 9.2 it follows that the relevant electrooptic coefficient for the structure shown in Figure 9-14 is r_{33}.

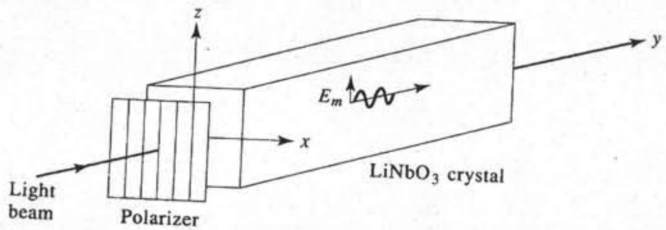


Figure 9-14 A $LiNbO_3$ rod used as the electrooptic crystal for phase modulation. The modulating RF field is polarized along the z axis and is propagating along the y axis.

Amplitude Modulation (advanced topic)

We now consider the case when the orthogonally polarized normal modes A_1 and A_2 are coupled (i.e., exchange power) by the applied modulation electric field. This occurs when the perturbation polarization (ΔP_1) in (9.8-18) is proportional to A_2 while, at the same time, ΔP_2 is proportional to A_1. In this case, electromagnetic energy is exchanged between the coupled modes as the wave propagates in the crystal. The magnitudes of the mode amplitudes are therefore functions of space and time. The mode amplitudes satisfy the coupled-mode equation (9.8-18). We will now consider a case of pure amplitude modulation. Again we assume that a traveling modulation wave $E_{mk} \sin(\omega_m t - k_m\zeta)$ yielding the coupled-mode equations (9.8-18). In our new language of mode coupling, amplitude modulation takes place when the presence of a modulation field

$$\mathbf{E}_{mod} = \hat{\mathbf{e}}_k E_{mk} \sin(\omega_m t - k_m z) \tag{9.9-20}$$

causes power exchange between the modes 1 and 2. An inspection of the coupled-mode equations (9.8-18) shows that this coupling occurs when the presence of $\mathbf{E}_{mod}$ creates a perturbation $\Delta\epsilon_{ij}(i \neq j)$, since this, according to (9.8-13), will couple mode $i(1)$ to $j(2)$ and vice versa. Using (9.8-13), the condition for pure amplitude modulation is that the set of conditions

$$\begin{aligned} r_{11k}E_k^{(0)} &= r_{22k}E_k^{(0)} = 0 \\ r_{12k}E_k^{(0)} &= r_{21k}E_k^{(0)} \neq 0 \end{aligned} \tag{9.9-21}$$

be satisfied for some Cartesian direction k. If we take the modulation field as in (9.9-1), the coupled-mode equations (9.8-18) become

$$\begin{aligned} \left(\frac{\partial}{\partial\zeta} + \frac{n_1}{c}\frac{\partial}{\partial t}\right) A_1 &= i\kappa \sin(\omega_m t - k_m\zeta) A_2 e^{i(k_1-k_2)\zeta} \\ \left(\frac{\partial}{\partial\zeta} + \frac{n_2}{c}\frac{\partial}{\partial t}\right) A_2 &= i\kappa \sin(\omega_m t - k_m\zeta) A_1 e^{-i(k_1-k_2)\zeta} \end{aligned} \tag{9.9-22}$$

where

$$\kappa = \frac{n_1^2 n_2^2}{n_1 + n_2}\frac{\omega}{c} r_{12k} E_{mk} \tag{9.9-23}$$

and summation over k is assumed. In the case when $n_1 = n_2$, the coupled equations become

$$\begin{aligned} \left(\frac{\partial}{\partial\zeta} + \frac{n}{c}\frac{\partial}{\partial t}\right) A_1 &= i\kappa \sin(\omega_m t - k_m\zeta) A_2 \\ \left(\frac{\partial}{\partial\zeta} + \frac{n}{c}\frac{\partial}{\partial t}\right) A_2 &= i\kappa \sin(\omega_m t - k_m\zeta) A_1 \end{aligned} \tag{9.9-24}$$

with

$$\kappa = \frac{\omega n^3}{2c} r_{12k} E_{mk} \tag{9.9-25}$$

The solutions of the coupled mode-equations (9.9-22) for the general case $(n_1 \neq n_2)$ are very complicated and will not be discussed here. For the case when $n_1 = n_2 = n$, the general solution of the coupled-mode equations is given by

$$\begin{aligned} A_1(\zeta, t) &= C_1\left(\zeta - \frac{c}{n}t\right) \cos\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos(\omega_m t - k_m\zeta)\right] \\ &\quad + C_2\left(\zeta - \frac{c}{n}t\right) \sin\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos(\omega_m t - k_m\zeta)\right] \\ A_2(\zeta, t) &= iC_2\left(\zeta - \frac{c}{n}t\right) \cos\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos(\omega_m t - k_m\zeta)\right] \\ &\quad - iC_1\left(\zeta - \frac{c}{n}t\right) \sin\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos(\omega_m t - k_m\zeta)\right] \end{aligned} \tag{9.9-26}$$

where C_1 and C_2 are arbitrary functions. Let the boundary condition at the input $(\zeta = 0)$ face of the crystal be

$$\begin{aligned} A_1(0, t) &= A_0 \\ A_2(0, t) &= 0 \end{aligned} \tag{9.9-27}$$

These conditions correspond to the case where the input polarizer is parallel to $\hat{\mathbf{e}}_1$ (one of the unperturbed principal axes).

Let $\zeta = 0$ in Equation (9.9-26), then the boundary condition (9.9-27) becomes

$$\begin{aligned} &C_1\left(-\frac{c}{n}t\right) \cos\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\omega_m t\right] + C_2\left(-\frac{c}{n}t\right) \sin\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\omega_m t\right] = A_0 \\ &C_1\left(-\frac{c}{n}t\right) \sin\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\omega_m t\right] - C_2\left(-\frac{c}{n}t\right) \cos\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\omega_m t\right] = 0 \end{aligned} \tag{9.9-28}$$

This gives at $\zeta = 0$

$$\begin{aligned} C_1\left(-\frac{c}{n}t\right) &= A_0 \cos\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\omega_m t\right] \\ C_2\left(-\frac{c}{n}t\right) &= A_0 \sin\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\omega_m t\right] \end{aligned} \tag{9.9-29}$$

Equations (9.9-29) give the functions C_1 and C_2 at $\zeta = 0$. Since C_1 and C_2 are, in general, functions of $\zeta - (c/n)t$, they are given by

$$C_1\left(\zeta - \frac{c}{n}t\right) = A_0 \cos\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\left(\omega_m t - \frac{\omega_m}{c} n\zeta\right)\right]$$

$$C_2\left(\zeta - \frac{c}{n}t\right) = A_0 \sin\left[\frac{\kappa c}{\omega_m(n - n_m)} \cos\left(\omega_m t - \frac{\omega_m}{c} n\zeta\right)\right] \quad (9.9\text{-}30)$$

Substituting Equations (9.9-30) into Equations (9.9-26), the mode amplitudes become

$$A_1(\zeta, t) = A_0 \cos\left\{\frac{\kappa c}{\omega_m(n - n_m)}\left[\cos(\omega_m t - k_m\zeta) - \cos\left(\omega_m t - \frac{\omega_m}{c} n\zeta\right)\right]\right\}$$

$$A_2(\zeta, t) = iA_0 \sin\left\{\frac{\kappa c}{\omega_m(n - n_m)}\left[\cos\left(\omega_m t - \frac{\omega_m}{c} n\zeta\right) - \cos(\omega_m t - k_m\zeta)\right]\right\} \quad (9.9\text{-}31)$$

Using next the trigonometric identity

$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$

and the relation $k_m = (\omega_m/c)n_m$, the mode amplitudes at the output plane ($\zeta = L$) of the crystal become

$$A_1(L, t) = A_0 \cos[\delta \sin(\omega_m t - \phi)]$$

$$A_2(L, t) = iA_0 \sin[\delta \sin(\omega_m t - \phi)] \quad (9.9\text{-}32)$$

where

$$\delta = \kappa L \frac{\sin \frac{\omega_m}{2c}(n - n_m)L}{\frac{\omega_m}{2c}(n - n_m)L} \quad (9.9\text{-}33)$$

and ϕ is given by Equation (9.9-14). We notice that δ in Equation (9.9-33) is identical to the phase modulation index (9.9-13) in its dependence on the length of the crystal L. Therefore, all the discussion of the phase-velocity matching for phase modulation can also be applied to amplitude modulation. In particular, maximum modulation occurs when condition (9.9-17) is satisfied and the maximum modulation depth is given by

$$\delta_{\max} = \frac{\omega}{\omega_m} \frac{n^3}{|n - n_m|} r_{12k} E_{mk} \quad (9.9\text{-}34)$$

It is interesting to compare the final result (9.9-32) with our previous formalism that led to (9.3-2) and (9.3-3). We associate the direction x of Figure 9-4 with the direction "1" of this section and y with "2." Using $\Gamma(t) = \Gamma_m \sin \omega_m t$ after reverting to real-time notation

$$E_x(L) = \frac{1}{\sqrt{2}}[(E_{x'}(L) + E_{y'}(L)] = \frac{A}{\sqrt{2}} \mathrm{Re}[e^{i(\Gamma_m/2)\sin\omega_m t} + e^{-i(\Gamma_m/2)\sin\omega_m t}]$$
$$= \sqrt{2} A \cos\left(\frac{\Gamma_m}{2}\sin\omega_m t\right) \quad (9.9\text{-}35)$$

$$E_y(L) = \frac{1}{\sqrt{2}}[E_{x'}(L) - E_{y'}(L)] = i\sqrt{2} A \sin\left(\frac{\Gamma_m}{2}\sin\omega_m t\right) \quad (9.9\text{-}36)$$

If we use the definition (9.2-4), we have $\Gamma_m/2 = \kappa L$. It follows that Equations (9.9-35, 9.9-36) reduce to the form of (9.9-32) for the phase-matched case $n = n_m$, where $\delta = \kappa L = \Gamma_m/2$. Equations (9.9-35 and 9.9-36), however, account accurately for the important case when $n \neq n_m$, i.e., the phase velocity of the modulation field is different from that of the optical wave. The "exact" analysis also gives us the correct form of the phase delay ϕ.

Problems

9.1 Derive the equations of the ellipses traced during one period by the optical field vector as shown in Figure 9-3(c) for $\Gamma = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}$.

9.2 Discuss the consequence of the field-independent retardation $(\omega l/c_c)(n_o - n_e)$ in Equation (9.5-1) on an amplitude modulator such as that shown in Figure 9-4.

9.3 Use the Bessel-function expansion of $\sin[a \sin x]$ to express (9.3-7) in terms of the harmonics of the modulation frequency ω_m. Plot the ratio of the third harmonic ($3\omega_m$) of the output intensity to the fundamental as a function of Γ_m. What is the maximum allowed Γ_m if this ratio is not to exceed 10^{-2}? (*Answer:* $\Gamma_m < 0.5$.)

9.4 Show that, if a phase-modulated optical wave is incident on a square-law detector, the output contains no alternating currents.

9.5 Using References [4] and [5], design a partially loaded KDP traveling wave phase modulator that operates at $\nu_m = 10^9$ Hz and yields a peak phase excursion of $\delta = \pi/3$. What is the modulation power?

9.6 Derive the expression [similar to Equation (9.6-2)] for the modulation power of a transverse $\bar{4}3m$ crystal electrooptic modulator of the type described in the Appendix B.

9.7 Derive an expression for the modulation power requirement [corresponding to Equation (9.6-2)] for a GaAs transverse modulator.

9.8 Show that if a ray propagates at an angle $\theta(\ll 1)$ to the z axis in the arrangement of Figure 9-4, it exercises a birefringent contribution to the retardation.

$$\Delta\Gamma_{\text{birefringent}} = \frac{\omega l}{2c} n_0 \left(\frac{n_0^2}{n_e^2} - 1\right) \theta^2$$

which corresponds to a change in index

$$n_0 - n_e(\theta) = \frac{n_0 \theta^2}{2} \left(\frac{n_0^2}{n_e^2} - 1\right)$$

9.9 Derive an approximate expression for the maximum allowable beam-spreading angle in Problem 9.8 for which $\Delta\Gamma_{\text{birefringent}}$ does not interfere with the operation of the modulator. *Answer:*

$$\theta < \left[\frac{\lambda}{4ln_0\{n_0^2/n_e^2 - 1\}}\right]^{1/2}$$

9.10 Consider the index ellipsoid S defined by

$$S_{ij}x_i x_j = 1$$

Show that the vector **N** defined by

$$N_i = S_{ij}x_j$$

is perpendicular to S at the point (x_1, x_2, x_3) on S.

9.11 Consider the case of a KH_2PO_4(KDP) crystal with an applied field along the x axis. Show that in the new principal dielectric axes coordinate system (x', y', z'), x' coincides with x while y' and z' are in the y–z plane, but rotated from their original positions by θ, where

$$\tan 2\theta = \frac{2r_{41}E_x}{1/n_0^2 - 1/n_e^2}$$

Show that in the x, y', z' system the equation for the index ellipsoid is

$$\frac{x^2}{n_0^2} + \left(\frac{1}{n_0^2} + r_{41}E_x \tan\theta\right) y'^2 + \left(\frac{1}{n_e^2} - r_{41}E_x \tan\theta\right) z'^2 = 1$$

9.12 An optical beam with amplitude E_0 and frequency $\omega/2\pi$ is split, equally, in two. One of the beams is left as is, while the other is phase modulated according to

$$\Delta\phi = a + \delta \cos\omega_m t \ (\omega_m \ll \omega)$$

The two beams are then recombined coherently (the whole procedure can be accomplished by a Michelson–Morley, or a Mach–Zehnder, interferometer with a phase modulator placed in one arm).

a. Express the recombined field in the form

$$E_{\text{rec}} = f(t)e^{i(\alpha + \beta \cos\omega_m t)}$$

b. Show that for $a = \pi/2$, $\delta \ll 1$

$$E_{\text{rec}} \approx E_0\left(1 + \frac{\delta}{2}\cos\omega_m t\right)e^{i\left(\frac{\pi}{4} + \frac{\delta}{2}\cos\omega_m t\right)}$$

c. Obtain the (approximate) optical spectrum of the output beam.

d. Derive the intensity modulation characteristics

$$\frac{|E_{\text{rec}}|^2}{|E_0|^2}$$

for the general case.

e. Using the results from **d**, determine how we can obtain a nearly linear modulation response in which the detected photocurrent, $I_{\text{det}} \propto |E_{\text{rec}}|^2$, is proportional to the modulation signal $\delta \cos\omega_m t$.

9.13 In Section 9.1 show that the three principal vectors X', X'', and X''' are perpendicular to each other.

9.14 Let x, y, z be the principal dielectric axes of a crystal with dielectric tensor elements ϵ_{xx}, ϵ_{yy}, ϵ_{zz}. Consider a new coordinate system ξ, η, z where ξ and η are rotated at an angle θ about the z axis (the z axis is the same in both systems). Show that the $\bar{\bar{\epsilon}}$ tensor in the new system is

	ξ	η	z
ξ	$\epsilon_{xx} + \delta \sin^2\theta$	$\delta/2 \sin(2\theta)$	0
η	$\delta/2 \sin(2\theta)$	$\epsilon_{xx} + \delta\cos^2\theta$	0
z	0	0	ϵ_{zz}

where $\delta \equiv \epsilon_{yy} - \epsilon_{xx}$.

9.15 Consider a crystal with principal dielectric axes x, y, z and corresponding ϵ_{xx}, ϵ_{yy}, ϵ_{zz}. Let the application of an electric field (or strain) cause an off-diagonal element ϵ_{zy} to appear.

a. Show the new principal dielectric axes are rotated about the x axis by an angle

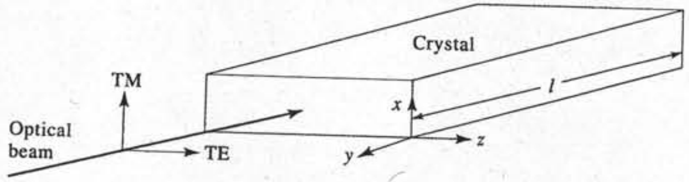
$$\theta \simeq \frac{\epsilon_{zy}}{\epsilon_{yy} - \epsilon_{zz}} \ (\epsilon_{zy} \ll \epsilon_{yy}, \epsilon_{zz})$$

b. Show that in KDP the application of a dc field $\mathbf{E} = \hat{\mathbf{e}}_x E_x$ causes a rotation β of the z and y principal axes about the x axis where

$$\beta = -\frac{n_e^2 n_0^2 r_{41} E_x}{n_0^2 - n_e^2}$$

9.16

a. Design an electrooptic waveguide modulator in a $LiTaO_3$ crystal as shown.



Show that the phase retardation $\Gamma \equiv \theta_{TE} - \theta_{TM}$ is given by

$$\Gamma = \frac{\omega l}{c}\,[n_0^3 r_{13} - n_e^3 r_{33}]\, E_z$$

b. Describe how you will use the waveguide as: (1) an amplitude modulator, (2) a phase modulator. Calculate the requisite modulation voltage assuming a width in the z direction of 5 μm and $\lambda = 0.6328\ \mu$m.

9.17

a. Design a polarization switch (mode coupler) TE↔TM using $LiNbO_3$, the crystal geometry of problem 9.15, and a dc field parallel to the x axis. Show how you can overcome the velocity mismatch problem ($n_0 \neq n_e$) by using a spatially periodic dc field

$$E_x = E_0 \cos \frac{2\pi}{\Lambda} y$$

with a proper choice of the period Λ.

b. What is the value of Λ at $1 = 1.15\ \mu$m? (See Table 9-2 for dispersion data.)

References

1. See, for example, J. F. Nye, *Physical Properties of Crystals.* New York: Oxford, 1957, p. 123.
2. See, for example, A. Yariv, *Quantum Electronics.* New York: Wiley, 1967, Chap. 14.
3. Peters, L. C., "Gigacycle bandwidth coherent light traveling-wave phase modulators," *Proc. IEEE* 51:147, 1963.
4. Rigrod, W. W., and I. P. Kaminow, "Wide-band microwave light modulation," *Proc. IEEE* 51:137, 1963.
5. Kaminow, I. P., and J. Lin, "Propagation characteristics of partially loaded two-conductor transmission lines for broadband light modulators," *Proc. IEEE* 51:132, 1963.
6. White, R. M., and C. E. Enderby, "Electro-optical modulators employing intermittent interaction," *Proc. IEEE* 51:214, 1963.

7. Fowler, V. J., and J. Schlafer, "A survey of laser beam deflection techniques," *Proc. IEEE* 54:1437, 1966.
8. Hall, D., A. Yariv, and E. Garmire, "Observation of propagation cutoff and its control in thin optical waveguides," *Appl. Phys. Lett.* 17:127, 1970.
9. Hall, D., A. Yariv, and E. Garmire, "Optical guiding and electrooptic modulation in GaAs epitaxial layers," *Opt. Commun.* 1:403, 1970.
10. Hammer, J. M., and W. Phillips, "Low-loss single mode optical waveguide and efficient high-speed modulators of $LiNb_xTa_{1-x}O_3$ on $LiTaO_3$," *Appl. Phys. Lett.* 24:545, 1974.
11. Yariv, A., and P. Yeh, *Optical Waves in Crystals.* New York: Wiley-Interscience, 1983, Chaps. 7 and 8.