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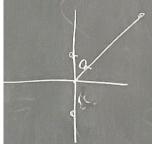
(a) $H = \dfrac{p^2}{2m} - Gm\left(\dfrac{m_1}{\sqrt{r^2+(z-a)^2}} + \dfrac{m_2}{\sqrt{r^2+(z+a)^2}}\right)$

$p^2 = p_r^2 + p_z^2 + \dfrac{p_\phi^2}{r^2}$



(b) $H\left(r, z, \phi, \dfrac{\partial W}{\partial r}, \dfrac{\partial W}{\partial z}, \boxed{\dfrac{\partial W}{\partial \phi}}\right) = \alpha_1 = E$

ϕ is separable from r and z. r and z are not separable.

$\Rightarrow p_\phi$ is constant

(c) Denoting $s_1 = \sinh v$, $s_2 = \sin u$, $c_1 = \cosh v$, $c_2 = \cos u$

$$\sqrt{r^2+(z\pm a)^2} = a\sqrt{(s_1 s_2)^2 + (c_1 c_2 \pm 1)^2} = a\sqrt{(c_1^2-1)(1-c_2^2)+(c_1c_2\pm1)^2} = a(c_1 \pm c_2)$$

(Noticing that $c_1 \geq 1 \geq c_2$)

$$V = -\frac{Gm}{a}\left(\frac{m_1}{c_1-c_2} + \frac{m_2}{c_1+c_2}\right) = -\frac{Gm\,(m_1+m_2)c_1+(m_1-m_2)c_2}{a\,(c_1^2-c_2^2)}$$

$$\begin{cases} \dot r = a(c_1 s_2 \dot v + s_1 c_2 \dot u) \\ \dot z = a(s_1 c_2 \dot v - c_1 s_2 \dot u) \end{cases} \Rightarrow T = \frac{m}{2}\left(\dot r^2 + \dot z^2 + r^2 \dot\phi^2\right)$$

$$= \frac{ma^2}{2}(c_1^2 s_2^2 + s_1^2 c_2^2)(\dot v^2 + \dot u^2) + \frac{m}{2} r^2 \dot\phi^2$$

$$\begin{cases} p_v = ma^2(c_1^2 s_2^2 + s_1^2 c_2^2)\,\dot v \\ p_u = \cdots\cdots\cdots\,\dot u \end{cases} \Longrightarrow = \frac{p_v^2 + p_u^2}{2ma^2\underbrace{(c_1^2 s_2^2 + s_1^2 c_2^2)}} + \frac{p_\phi^2}{2ma^2 s_1^2 s_2^2}$$

$$= c_1^2(1-c_2^2) + (c_1^2-1)c_2^2$$

$$= c_1^2 - c_2^2$$

$$H = \frac{1}{c_1^2-c_2^2}\left(\frac{p_u^2+p_v^2}{2ma^2} + \frac{p_\phi^2}{2ma^2}\frac{c_1^2-c_2^2}{s_1^2 s_2^2} - \frac{Gm}{a}\left[(m_1+m_2)c_1 + (m_1-m_2)c_2\right]\right)$$

$$\frac{1}{s_1^2} + \frac{1}{s_2^2}$$

(d) $H\left(v, u, \dfrac{\partial W}{\partial v}, \dfrac{\partial W}{\partial u}\right) = E$

$$\Rightarrow \left[\frac{\left(\frac{\partial W}{\partial v}\right)^2}{2ma^2} + \frac{p_\phi^2}{2ma^2 s_1^2} - \frac{Gm(m_1+m_2)c_1}{a} - c_1 E^2\right]$$

$$+\left[\frac{\left(\frac{\partial W}{\partial u}\right)^2}{2ma^2} + \frac{p_\phi^2}{2ma^2 s_2^2} - \frac{Gm(m_1-m_2)c_2}{a} - c_2 E^2\right] = 0$$

3. $\dot u = -\{u, H\} + \dfrac{\partial u}{\partial t} = 0$

$\Rightarrow$ Constant of motion