8.(3)09 Section 2

September 17, 2021

1 Particle on a cylinder

Consider a particle free to move on the surface of a cylinder with radius R and axis along the $\hat{z}$ direction that is subject to a central force $\vec{F} = -k\vec{r}$.

(a)

Write a Lagrangian for this system and ensure all constraints are imposed.

(b)

Find the equations of motion and any conserved quantities of the system.

(c)

Describe the motion of this system in words and/or pictures.

2 Point transformation

Consider the following Lagrangian:

$$L = \frac{1}{2}m\left(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2\right) - \frac{k}{2}\left(ax^2 + 2bxy + cy^2\right), \quad a, b, c \in \mathbb{R}, \qquad m > 0, k > 0, b^2 \neq ac$$

(a)

Write L in terms of a two-dimensional vector $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$.

(b)

Find a point transformation for $\vec{r}$ to new generalized coordinates $\vec{q}$ whose components have uncoupled equations of motion. Determine the Lagrangian for these new coordinates. *Hint: consider the properties of real symmetric matrices.*

(c)

Consider the two special cases $a = c = 0$, $b \neq 0$ and $b = 0$, $a + c = 0$. Determine the physical systems described by the Lagrangian in these cases.

(d)

Describe why we need $b^2 \neq ac$.