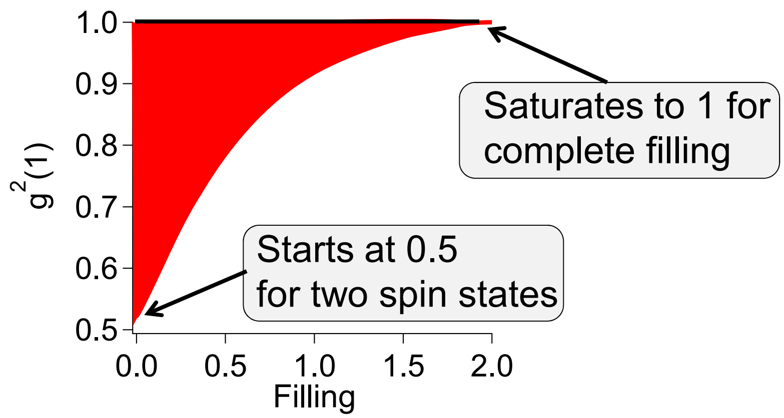
Real-space detection of Pauli hole

Density Correlations in a Lattice Gas

$$g_2(i) = \frac{\langle \hat{n}_i \hat{n}_0 \rangle}{\langle \hat{n}_i \rangle^2}$$

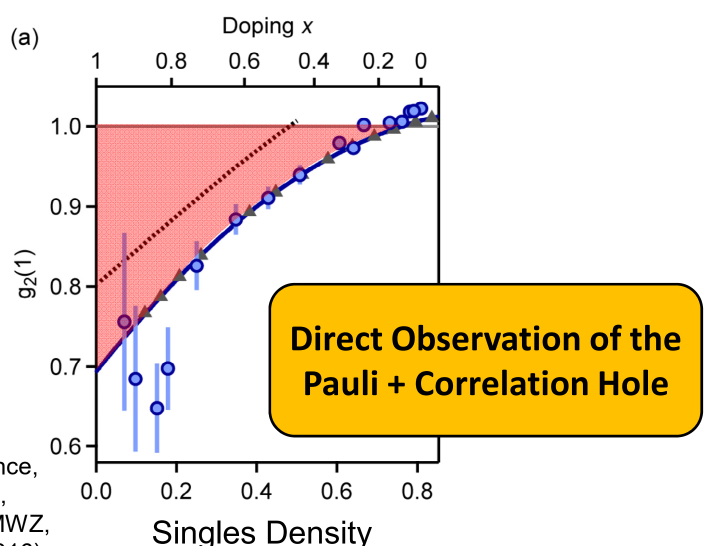
g_2 for fermions reduced below unity



Low filling: lattice spacing << interparticle distance
→ One site away one is deep inside the Pauli hole

Pair correlation function for Singles

$$g_2(r) = \langle \hat{m}_z^2(r)\hat{m}_z^2(0) \rangle / \langle \hat{m}_z^2 \rangle^2$$



Cheuk,Nichols, Lawrence, Okan, Zhang, Khatami, Trivedi, Paiva, Rigol, MWZ, Science 353, 1260, (2016)

Nonlinear Mach-Zehnder Interferometer

Linear optics: $\vec{P} = \varepsilon_0 \chi \vec{E}$

Non-linear $\quad \vec{P} = \varepsilon_0 \left(\chi^1 E + \underbrace{\chi^2 E^2}_{OPO} + \underbrace{\chi^3 E^3}_{Kerr} + \dots \right)$

Kerr effect: Intensity dependent phase shift

Cross-Phase Modulation Hamiltonian

$$H_{XPM} = -\chi \, a^+ a \, b^+ b$$

Use crystal of length L

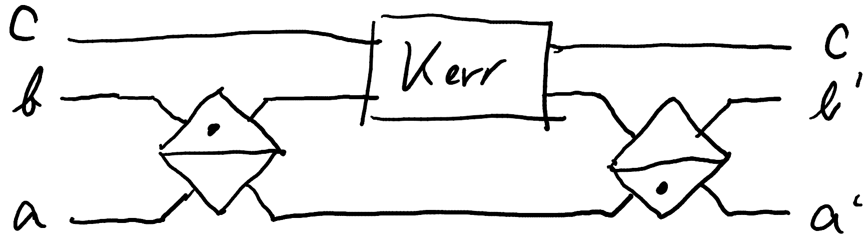
$$K = e^{-iHL} = e^{i\chi L a^+ a b^+ b}$$

Choose $\chi L = \pi$

$$K|00\rangle = |00\rangle$$
$$K|01\rangle = |01\rangle$$
$$K|10\rangle = |10\rangle$$
$$K|11\rangle = e^{i\chi L}|11\rangle = -|11\rangle$$

Kerr only gives a phase shift if I have photons in both rails.

Can use this for a controlled swap



$|c\rangle = |0\rangle \qquad a', b' \longrightarrow a, b$

$\quad\;\; = |1\rangle \qquad a', b' \longrightarrow b, a \quad$ swap

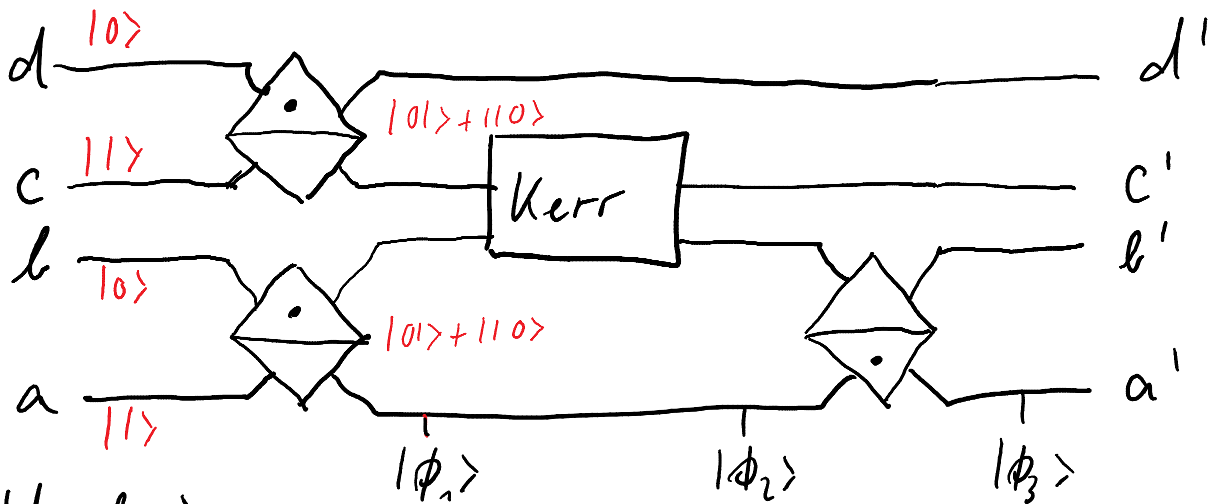
$$|OUT\rangle = \underbrace{B_{ab}^{+} \underbrace{K_{bc}}_{e^{i\chi L b^{+}b c^{+}c}} B_{ab}}_{e^{i\chi L c^{+}c \left(\frac{b^{+}-a^{+}}{\sqrt{2}}\right)\left(\frac{b-a}{\sqrt{2}}\right)}} |IN\rangle$$

$$= \dots \; e^{i\xi c^{+}c \, (a^{+}b - b^{+}a)/2} \; \dots \; |IN\rangle$$

phase shifts ($\rightarrow$ see Wiki)

$\Rightarrow$ It's a beam splitter with rotation angle $\xi c^{+}c$.

Creation of entangled states



$|dcba\rangle$

$$|\phi_1\rangle = (|01\rangle + |10\rangle)(|01\rangle + |10\rangle)$$

$$= |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle$$

$$\downarrow$$

$$|\phi_2\rangle = |0101\rangle - |0110\rangle + |1001\rangle + |1010\rangle$$

$$|\phi_3\rangle = |01\cancel{01}\rangle - |0110\rangle - |0110\rangle - |0\cancel{101}\rangle$$

$$+ |1001\rangle - |1\cancel{010}\rangle + |1\cancel{010}\rangle + |1001\rangle$$

$$= \frac{|1001\rangle - |0110\rangle}{\sqrt{2}}$$

Entangled State!

Entanglement

Captures the most "quantum" nature of quantum states of light.

EPR: QM not complete as intuition violated ?

John Bell: Correlations produced by entangled states are distinctly beyond classical mechanics

Useful resource:
- precision greater than with classical states
- exponential speed-up in quantum algorithms
- enables teleportation

Definition:

A bi-partite state $|\psi_{AB}\rangle$ of a composite system $A+B$ is entangled if and only if there do <u>not</u> exist states $|\psi\rangle_A$ and $|\psi\rangle_B$ such that

$$|\psi\rangle_{AB} = |\psi\rangle_A \otimes |\psi\rangle_B$$

$\uparrow$ tensor product.

Examples:
- $|00\rangle = |0\rangle \otimes |0\rangle$ — No
- $\dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$ — Yes
- $\dfrac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2}$ — No — $\dfrac{(|0\rangle + |1\rangle)}{\sqrt{2}} \otimes \dfrac{(|0\rangle + |1\rangle)}{\sqrt{2}}$
- $\dfrac{|00\rangle + |01\rangle + |10\rangle}{\sqrt{3}}$ — Yes

Most basic states:

- Singlet $|\Psi_{EPR}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$

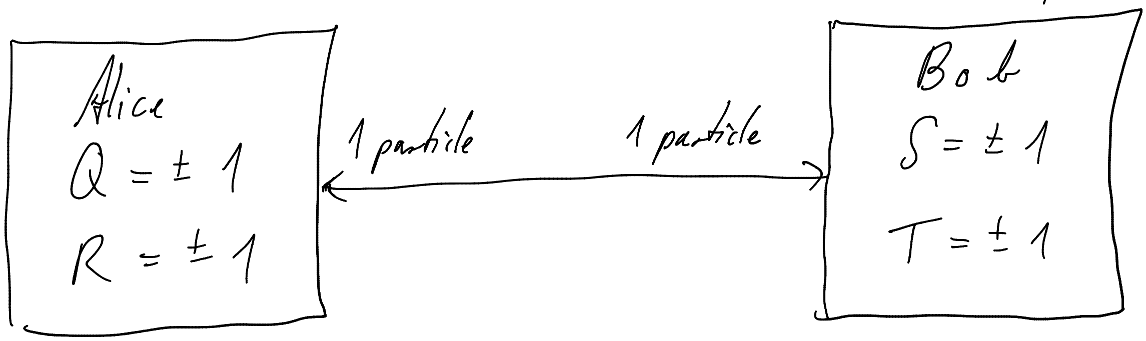
Careful: This is not just a single photon.
Beamsplitter produces such states, but $|0\rangle$ is the vacuum, no photons present.

Important
- to have this realized in two distinct systems
- two ports need to be separately manipulated

Ex:
- The two e^- of He are in a singlet, but under typical circumstances we cannot use this as a resource, not separable
- The polarizations of two photons in two modes
$$|\Psi_{EPR}\rangle = \frac{|HV\rangle + |VH\rangle}{\sqrt{2}}$$
- Two photons, each in two modes
$$|0_L\rangle = |01\rangle \quad ; \quad |1_L\rangle = |10\rangle$$
$$|\Psi_{EPR}\rangle = \frac{|0_L 1_L\rangle + |1_L 0_L\rangle}{\sqrt{2}} \qquad \text{good}$$

EPR and Bell Inequality

Give Alice and Bob each one photon from state $|\psi\rangle$ (quantum or classical); random bases Q, R (Alice)
S, T (Bob)



CHSH (Clauser, Horne, Shimony, Holt)

$$QS + RS + RT - QT = \underbrace{(Q+R)}_{\substack{0 \\ \text{or } \pm 2}} \underset{\pm 1}{S} + \underbrace{(R-Q)}_{\substack{\pm 2 \\ 0}} \underset{\pm 1}{T} = \pm 2$$

Def.: $p(q,r,s,t)$: probability that, before measurement, system is in state where $Q=q$, $R=r$, $S=s$, $T=t$.

$$\langle QS + RS + RT - QT \rangle = \sum_{qrst} p(q,r,s,t)(qs + rs + rt - qt)$$

$$\leq \sum_{qrst} p(q,r,s,t) \cdot 2 = 2.$$

$$\langle QS \rangle + \langle RS \rangle + \langle RT \rangle - \langle QT \rangle \leq 2$$

CHSH Inequality.

Quantum violation of CHSH inequality:

Use $|\Psi_{EPR}\rangle$

Alice measures:

$$Q = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$R = |0\rangle\langle 1| + |1\rangle\langle 0|$$

Bob measures:

$$S = \frac{-|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|}{\sqrt{2}}$$

$$T = \frac{|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| - |1\rangle\langle 1|}{\sqrt{2}}$$

Physically, if EPR represented by photon polarization, so that

$$|\Psi_{EPR}\rangle = \frac{|HV\rangle + |VH\rangle}{\sqrt{2}}$$

Q - linear polarization
R - circular polarization

S - first, rotate polarization by 45°, then measure linear pol.
T " then " circular pol.

Def. $\langle \dots \rangle = \langle \Psi_{EPR} | \dots | \Psi_{EPR} \rangle$

$$\langle QS \rangle = \langle RS \rangle = \langle RT \rangle = -\langle QT \rangle = \frac{1}{\sqrt{2}}$$

$$\langle QS \rangle + \langle RS \rangle + \langle RT \rangle - \langle QT \rangle = \boxed{2 \cdot \sqrt{2}}$$

Observed experimentally!

e.g.

$$QS = \frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right) \otimes \left(-|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$= \frac{1}{\sqrt{2}}\Big(-|00\rangle\langle 00| - |00\rangle\langle 01| - |01\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |10\rangle\langle 11| + |11\rangle\langle 10| - |11\rangle\langle 11| \Big)$$

$$\left\langle \frac{10+01}{\sqrt{2}} \right| QS \left| \frac{10+01}{\sqrt{2}} \right\rangle = \frac{1}{2}\Big(\langle 10|QS|10\rangle + \langle 10|QS|01\rangle + \langle 01|QS|10\rangle + \langle 01|QS|01\rangle \Big)$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(1 + 1\right) = \frac{1}{\sqrt{2}} \quad . \quad \text{etc.}$$

Implications for the classical world

One of the assumptions of the classical bound wrong.

- Assumption that Q, R, S, T have definite values which exist before the measurement
  → "Realism"
- Assumption that Alice's measurement does not influence the result of Bob's measurement
  → "Locality"

→ Assumption of <u>local</u> <u>reality</u> must be wrong!
(maybe each individually is wrong?).

Bell Inequality Violation with Two Remote Atomic Qubits

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