Classical Mechanics III (8.09 & 8.309) Fall 2020
Final Practice Problems

Massachusetts Institute of Technology
Physics Department
Not to be handed in
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Announcements for Exam Preparation

- The final exam will cover all materials in the course, Chapters 1-7.
- I suggest looking through your problem sets. You may well encounter problems that test similar concepts to the ones we studied there. Also you will encounter short answer problems, so also read through your lecture notes.
- I have posted the 2018 and 2019 final exams as practice problems. Solutions have also been posted.
- Problems related to chapters 1-4 of our lecture notes can be found in Goldstein or from Thornton and Marion.
- The problems on the next page are intended as practice for the material on: Hamilton-Jacobi and Action-Angle variables, Perturbation Theory, Fluids, and Chaos. Some of them are short. I will be posting solutions for all these problems.
- If you happen to want to look at even more problems, look at Smits for fluid problems and look at Strogatz for nonlinear dynamics/chaos problems. I have posted the end of book solutions from Strogatz.

Practice Problems

1. **Action-Angle Variables**

   Goldstein Ch.10 #12. Here r is the radial coordinate, and assume $V_0 > 0$.

   For part (b) only consider the case $-V_0 < E < 0$ where the angular motion is a rotation, find $J_r = J_r(E)$ and explain how you would use this result to find the frequency ν_r without actually carrying out the algebra. Note that

   $$\int_a^b \frac{1}{r}(r^2 - a^2)^{1/2} = \sqrt{b^2 - a^2} - a\cos^{-1}(a/b)$$

2. **Perturbation Theory**

   Goldstein Ch.12 #7. Here $\Delta H = bp_x^2 p_y^2$. At $t = 0$ take $x = y = 0$ and $p_x = p_y$. Note that

   $$\int_0^{2\pi} \frac{d\theta}{2\pi}\cos^4\theta = \frac{3}{8}$$

3. **Fluids** Smits problems (pages are posted):

   4.36 (ideal fluid flow),
   7.12 (parts a-c, characterizing a flow),
   8.24 (dimensional analysis, take $\mu = \nu$ kinematic viscosity)
   9.37 (pipe flow with viscosity, take $\mu = \nu$ kinematic viscosity)

4. **Nonlinear dynamics & Chaos** Strogatz problems (available in posted readings):

   3.4.4 (a bifurcation),
   5.2.5 (fixed point in 2-dim for linear system),
   6.3.9 (parts a-c only, more fixed points in 2-dim),
   6.5.3 (system with conserved $H(x, y)$),
   7.1.2 (stability of limit cycles)