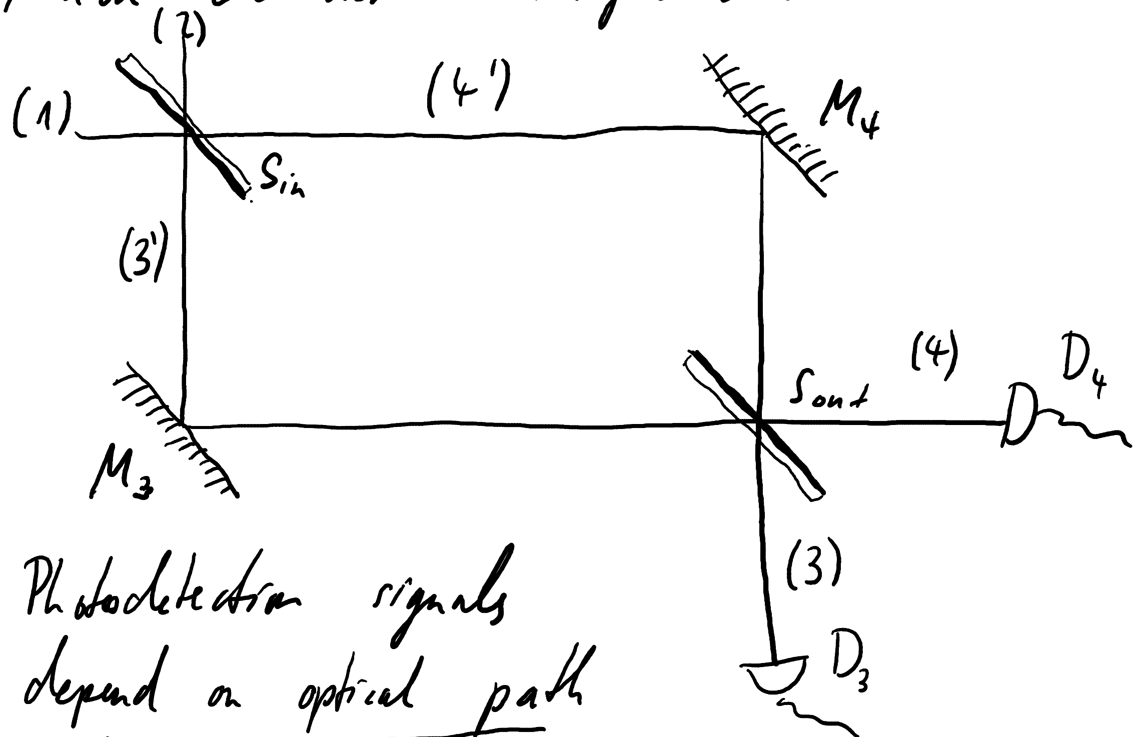
Mach-Zehnder Interferometer



Photodetection signals depend on optical path difference $\overline{S_{in} M_3 S_{out}} - \overline{S_{in} M_4 S_{out}}$

$$\hat{E}_3^{(+)} = \frac{1}{\sqrt{2}} \left(-\hat{E}_{3'}^{(+)} e^{ikL_{3'}} + \hat{E}_{4'}^{(+)} e^{ikL_{4'}} \right) \qquad k = \frac{\omega}{c}$$

$$\hat{E}_4^{(+)} = \frac{1}{\sqrt{2}} \left(\hat{E}_{3'}^{(+)} e^{ikL_{3'}} + \hat{E}_{4'}^{(+)} e^{ikL_{4'}} \right)$$

$$\vdots$$

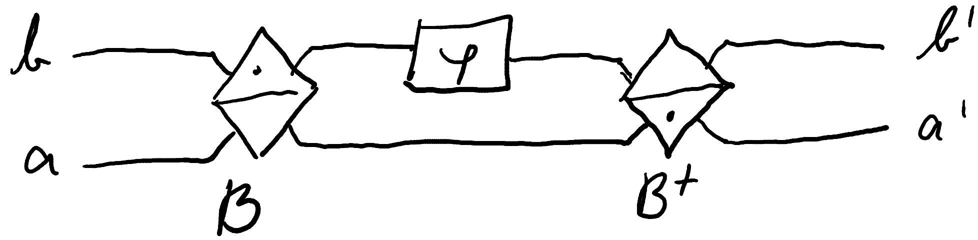
$$\hat{E}_3^{(+)} = e^{ik\bar{L}'} \left(-i \sin\left(\frac{k\delta L}{2}\right) \hat{E}_1^{+} - \cos\left(\frac{k\delta L}{2}\right) \hat{E}_2^{+} \right) e^{ikr_3}$$

$$\hat{E}_4^{(+)} = e^{ik\bar{L}'} \left(\cos\left(\frac{k\delta L}{2}\right) \hat{E}_1^{+} + i \sin\left(\frac{k\delta L}{2}\right) \hat{E}_2^{+} \right) e^{ikr_4}$$

with $\bar{L}' = (L_3' + L_4')/2$, and e^{ikr_3} and e^{ikr_4} stand for propagation from S_{out} to the detectors.

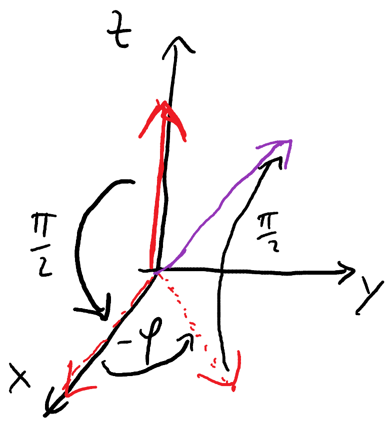
We can describe this using our beamsplitters + phase shifters!

Dual-rail photon representation of a qubit allows to discuss interferometers as gates



$B^{+}B = \mathbb{1}$

$$|OUT\rangle = B^{+} \; P \; B \; |IN\rangle$$
$$= R_y\left(\tfrac{\pi}{2}\right) R_z(-\varphi) \; R_y\left(-\tfrac{\pi}{2}\right)|IN\rangle$$
$$= R_x(-\varphi)\,|IN\rangle \qquad \varphi = k\,\delta L$$



$\varphi = 0$ balanced

$\varphi = \pi$ swap $\Rightarrow$ inverts qubit.

Let's take $|\Psi_{in}\rangle = |\varphi_1\rangle \otimes |0_2\rangle$

Photodetection signal $w_3(r_3,t) = s\,\left\| \hat{E}_3^{(+)}(r_3)\,|\Psi_{in}\rangle \right\|^2$

(s – sensitivity of detector) $\qquad = s\,\sin^2\!\left(\frac{k\,\delta L}{2}\right) \left\| \hat{E}_1^{(+)} |\varphi_1(t)\rangle \right\|^2$

Only depends on path difference δL, not on $\bar{L}'$ or where the detector stands. However, it depends on the state of the field.

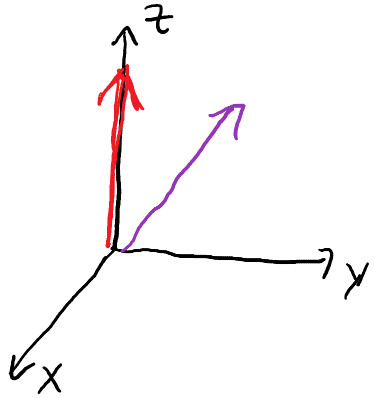
Coherent state input: $|\varphi_1(1)\rangle = |\alpha e^{-i\omega t}\rangle$

$$\Rightarrow \quad W_3 = W_1 \sin^2\left(\frac{k\delta L}{2}\right)$$

with $W_1 = s\,\mathcal{E}^2|\alpha|^2$ the photodetection probability that would be detected at the incoming port.

$\Rightarrow$ same result as for classical optics

Now let's take $|\varphi_1\rangle = |1\rangle$, i.e. we start with qubit $\begin{bmatrix}1\\0\end{bmatrix}$. This is aligned along z on Bloch sphere



Detection on D_3 measures

$$W_3 = W_1 \sin^2\left(\frac{k\delta L}{2}\right)$$

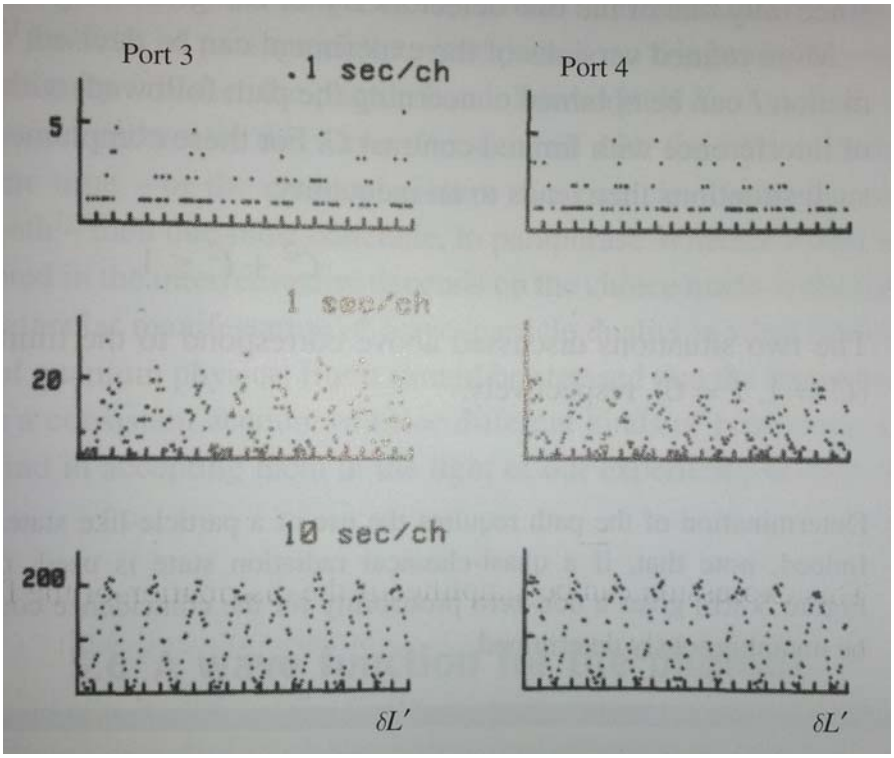
with $W_1 = s\,\mathcal{E}^2$

$$W_4 = W_1 \cos^2\left(\frac{k\delta L}{2}\right)$$

(D_3 and D_4 measure S_z)

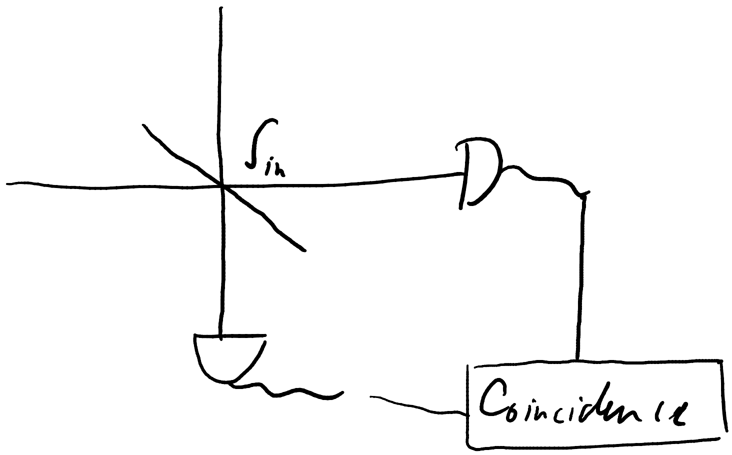
$\Rightarrow$ Even for a one-photon state, which one can regard as quasi-particle like, interference fringes will be observed.

"A photon can interfere with itself"



P. Grangier, A. Aspect
from Grangier, Roger, Aspect, EPL 1(4) (1986), p. 173

If instead we place detectors after the first beamsplitter:



Measure coincidence, i.e. probability for joint detection: It's 0, as we measure $a^2|1\rangle = 0$.

$\Rightarrow$ Particle-like behaviour. One photon only clicks once.

Tempting to conclude the photon goes on one side or the other of S_{in}.

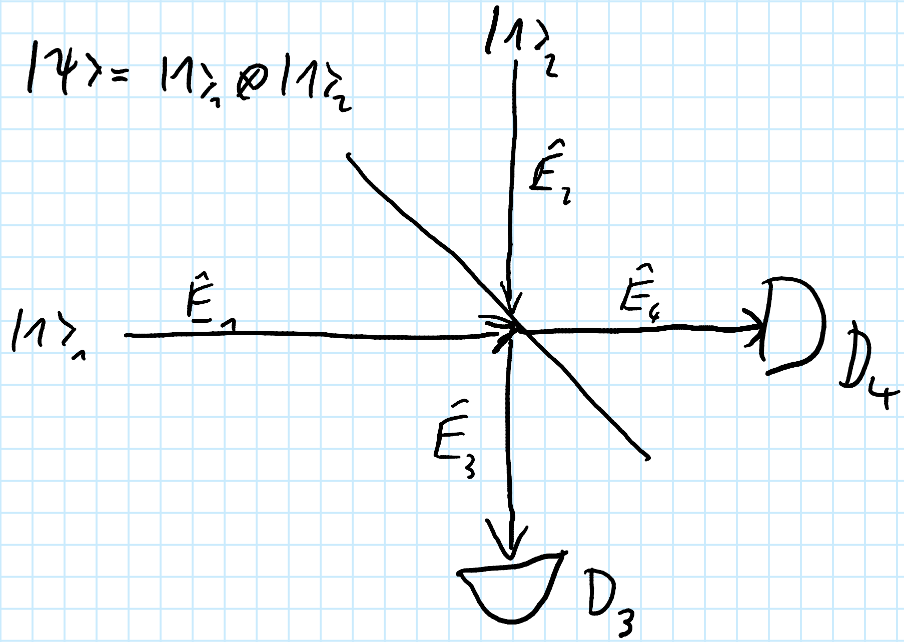
But in the interferometer setup, we see interference, naturally interpreted as a wave that divides on S_{in} and propagates along both arms.

$\Rightarrow$ Wave-particle duality

Bohr's complementarity: Contradictory classical behaviour only occurs in incompatible exp. setups.

Two Photons, one at each BS input:

$|\psi\rangle = |1\rangle_1 \otimes |1\rangle_2$



Detector D_3: $\overline{i_3} \propto \| \hat{E}_3^{(+)} |\psi\rangle \|^2$

$$\hat{E}_3^{(+)} |\psi\rangle \propto \frac{a-b}{\sqrt{2}} |1\rangle \otimes |1\rangle$$
$$= \frac{|1\rangle \otimes |0\rangle - |0\rangle \otimes |1\rangle}{\sqrt{2}}$$
$$\| \hat{E}_3^{(+)} |\psi\rangle \|^2 = \mathcal{E}^2 \left(\frac{1}{2} + \frac{1}{2} \right)$$

same at D_4 $\Rightarrow$ Measures sum of individual probabilities. No interference. Looks like classical particles distributed randomly by the beamsplitter

Joint detections: Measure $\overline{i_3 i_4}$ (say via $\overline{(i_3 - i_4)^2}$)

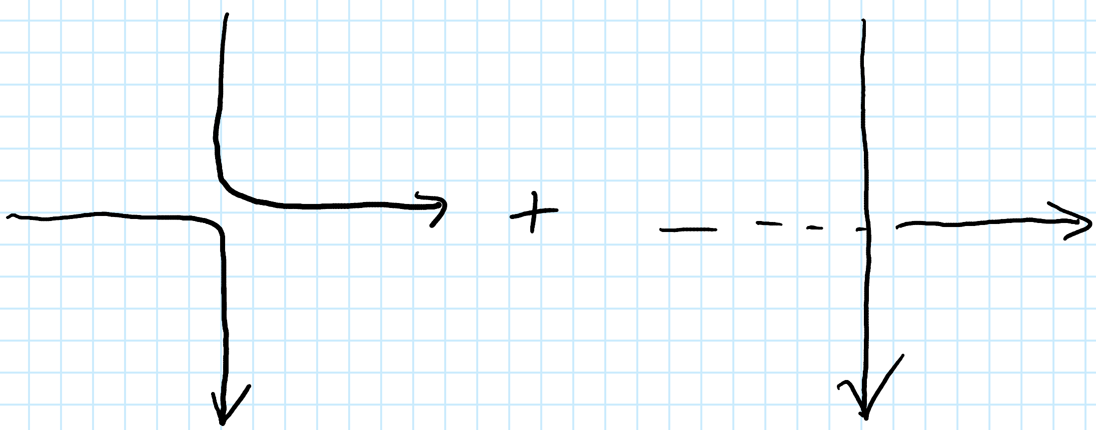
$$\hat{E}_3^{(+)} \hat{E}_4^{(+)} |\psi\rangle \propto \frac{a-b}{\sqrt{2}} \frac{a+b}{\sqrt{2}} |1\rangle \otimes |1\rangle$$
$$= \frac{1}{2} \left(a^2 - b^2 \right) |1\rangle \otimes |1\rangle = \underline{\underline{0}}$$

⇒ When photons perfectly overlap on the beamsplitter, they never leave by different ports, they leave <u>together</u> either by port 3 or by port 4.

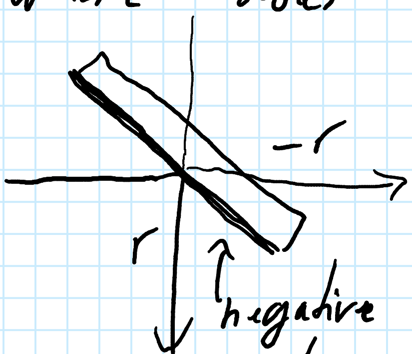
⇒ quantum coalescence

or Hong-Ou-Mandel interference

Two processes lead to the same result (one photon on each detector), so we need to add their amplitudes



Where does the negative sign come from?



negative reflection coefficient at coated glass surface.

Same sign that we have between $\hat{E}_3^{(+)} \propto \frac{a-b}{\sqrt{2}}$

and $\hat{E}_4^{(+)} \propto \frac{a+b}{\sqrt{2}}$.

Hong - Ou - Mandel interference:

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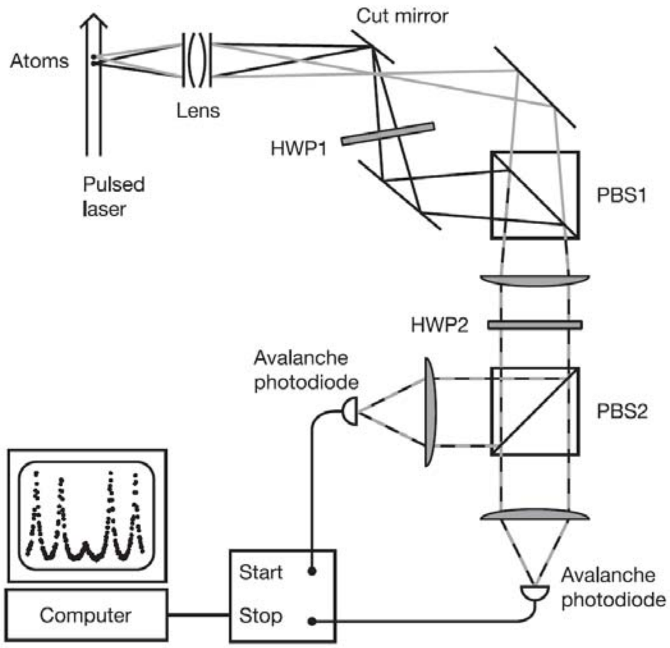
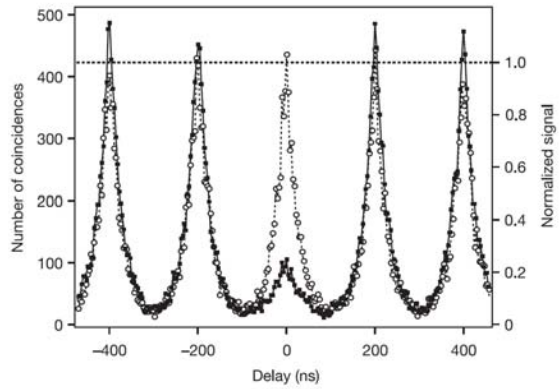
LETTERS

Quantum interference between two single photons emitted by independently trapped atoms

J. Beugnon[1], M. P. A. Jones[1], J. Dingjan[1], B. Darquié[1], G. Messin[1], A. Browaeys[1] & P. Grangier[1]





LETTERS

Quantum interference of photon pairs from two remote trapped atomic ions

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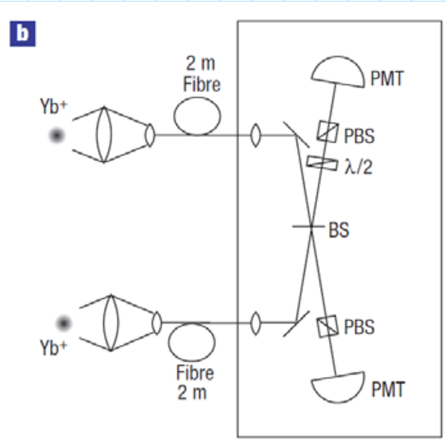
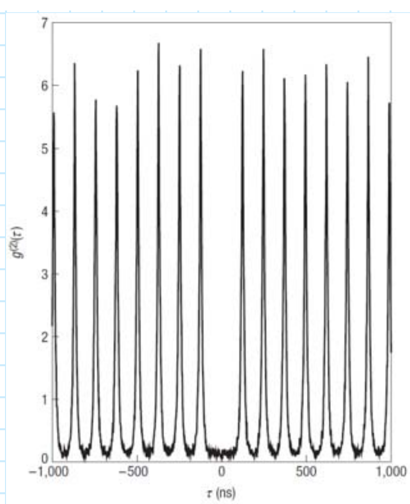
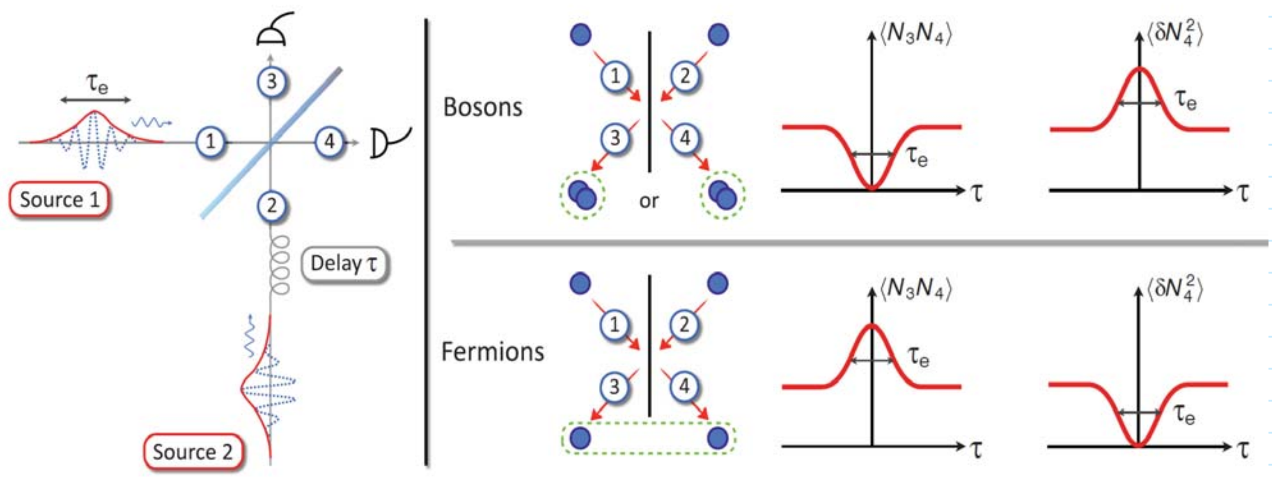


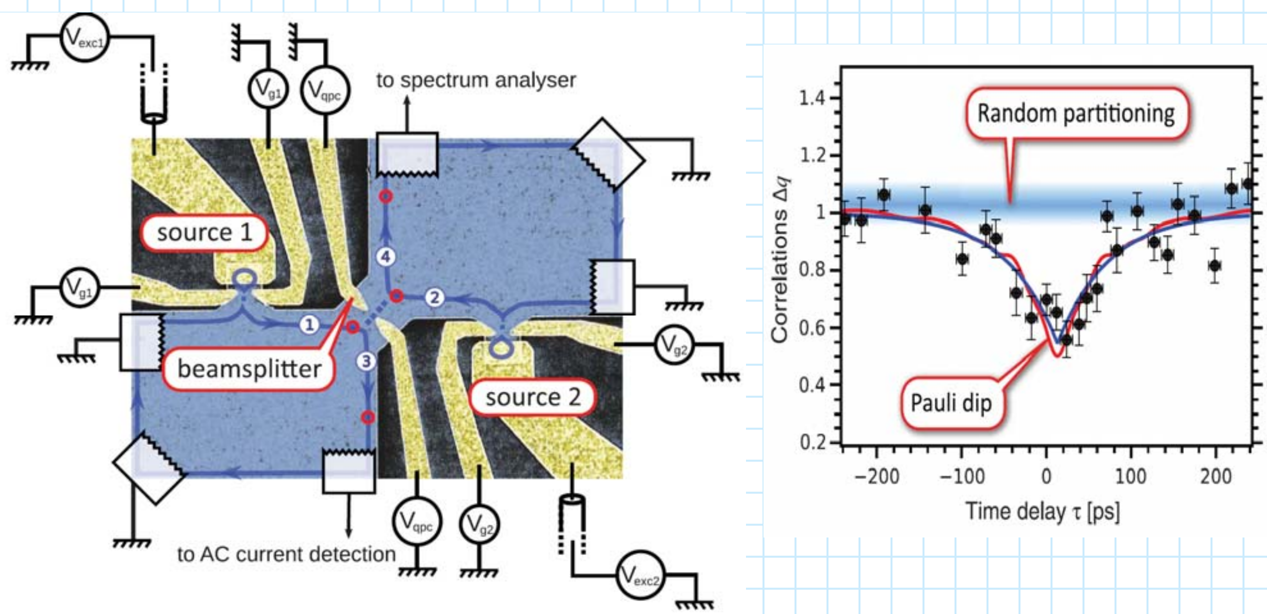


Hong - Ou - Mandel interference:

Coherence and Indistinguishability of Single Electrons Emitted by Independent Sources

E. Bocquillon,[1] V. Freulon,[1] J.-M Berroir,[1] P. Degiovanni,[2] B. Plaçais,[1] A. Cavanna,[3] Y. Jin,[3] G. Fève[1]*



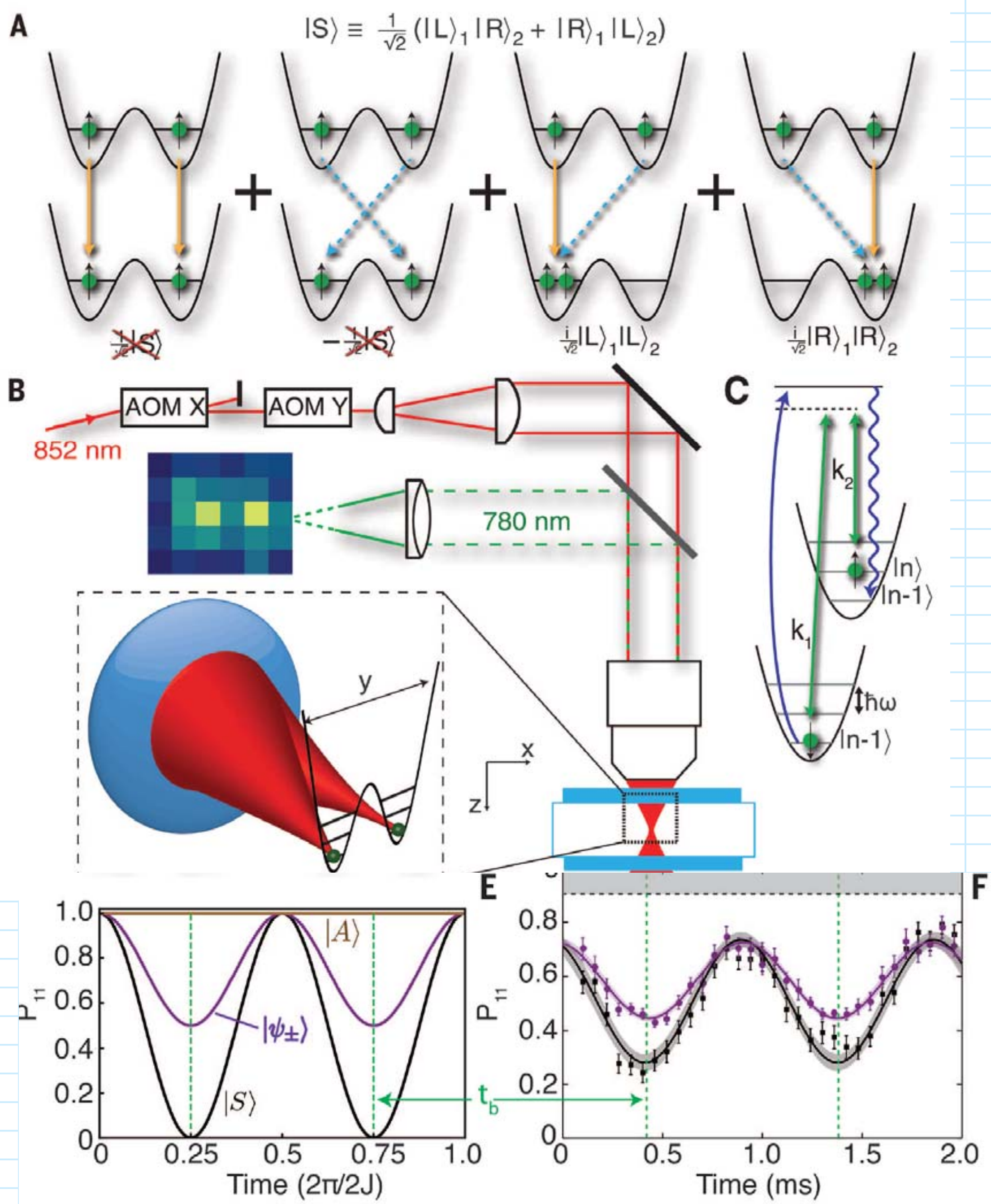


Hong - Ou - Mandel interference:

QUANTUM MECHANICS

Two-particle quantum interference in tunnel-coupled optical tweezers

A. M. Kaufman,[1,2] B. J. Lester,[1,2] C. M. Reynolds,[1,2] M. L. Wall,[1,2] M. Foss-Feig,[3]
K. R. A. Hazzard,[1,2] A. M. Rey,[1,2] C. A. Regal[1,2]*



Hong-Ou-Mandel interference:

Strongly Correlated Quantum Walks in Optical Lattices

Philipp M. Preiss,[1] Ruichao Ma,[1] M. Eric Tai,[1] Alexander Lukin,[1] Matthew Rispoli,[1] Philip Zupancic,[1] Yoav Lahini,[1,2] Rajibul Islam,[1] and Markus Greiner[1,*]

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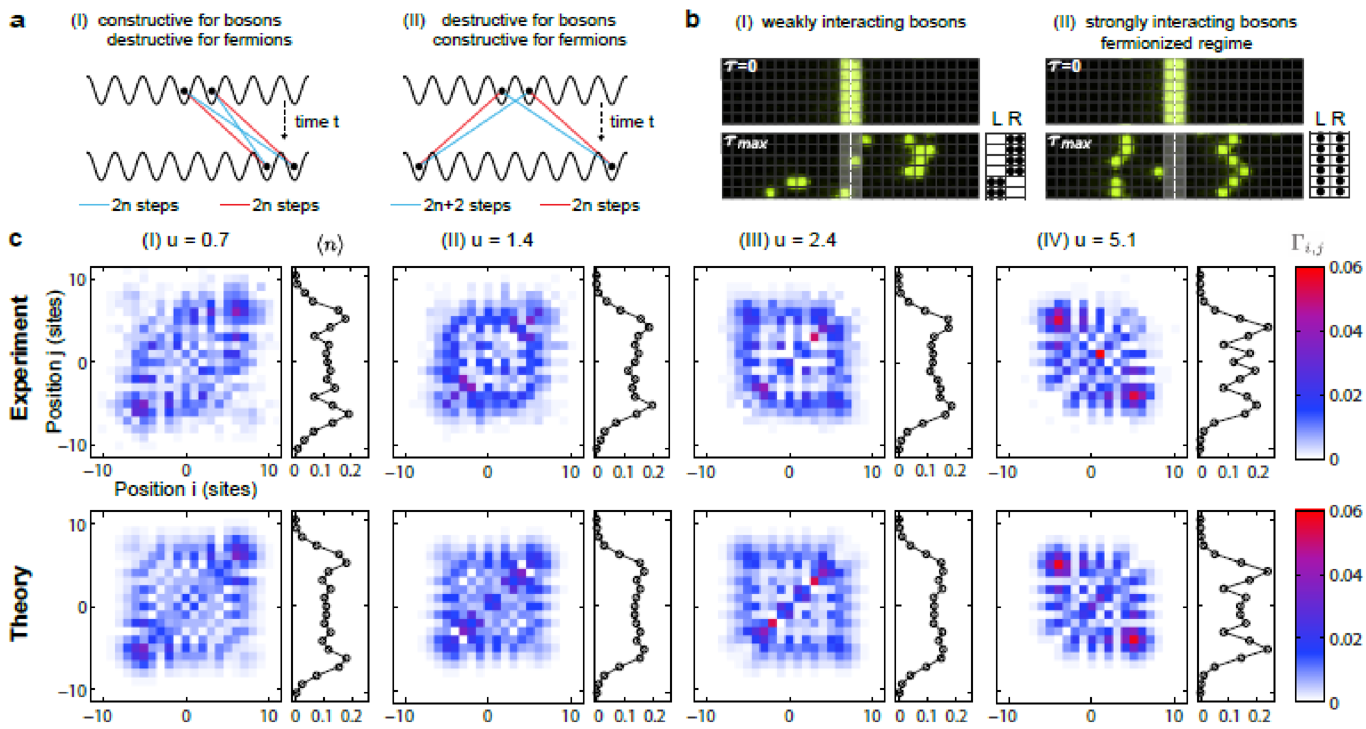


Nonlinear Mach-Zehnder Interferometer

Linear optics: $\vec{P} = \varepsilon_0 \chi \vec{E}$

Non-linear $\quad \vec{P} = \varepsilon_0 \left(\chi^1 E + \underbrace{\chi^2 E^2}_{OPO} + \underbrace{\chi^3 E^3}_{Kerr} + \dots \right)$

Kerr effect: Intensity dependent phase shift

Cross-Phase Modulation Hamiltonian

$$H_{XPM} = -\chi \, a^+ a \, b^+ b$$

Use crystal of length L

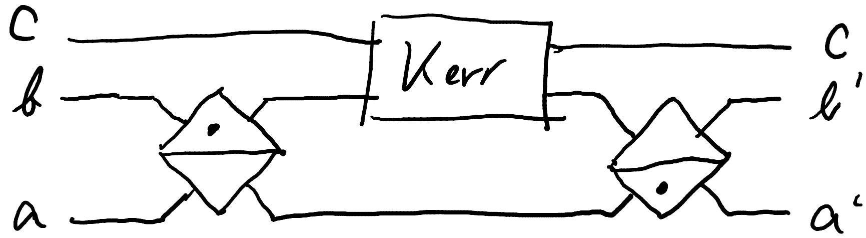
$$K = e^{-iHL} = e^{i\chi L a^+ a b^+ b}$$

Choose $\chi L = \pi$

$$K|00\rangle = |00\rangle$$
$$K|01\rangle = |01\rangle$$
$$K|10\rangle = |10\rangle$$
$$K|11\rangle = e^{i\chi L}|11\rangle$$
$$= -|11\rangle$$

Kerr only gives a phase shift if I have photons in both rails.

Can use this for a controlled swap



$|c\rangle = |0\rangle \qquad a', b' \longrightarrow a, b$

$\quad\;\; = |1\rangle \qquad a', b' \longrightarrow b, a \qquad$ swap

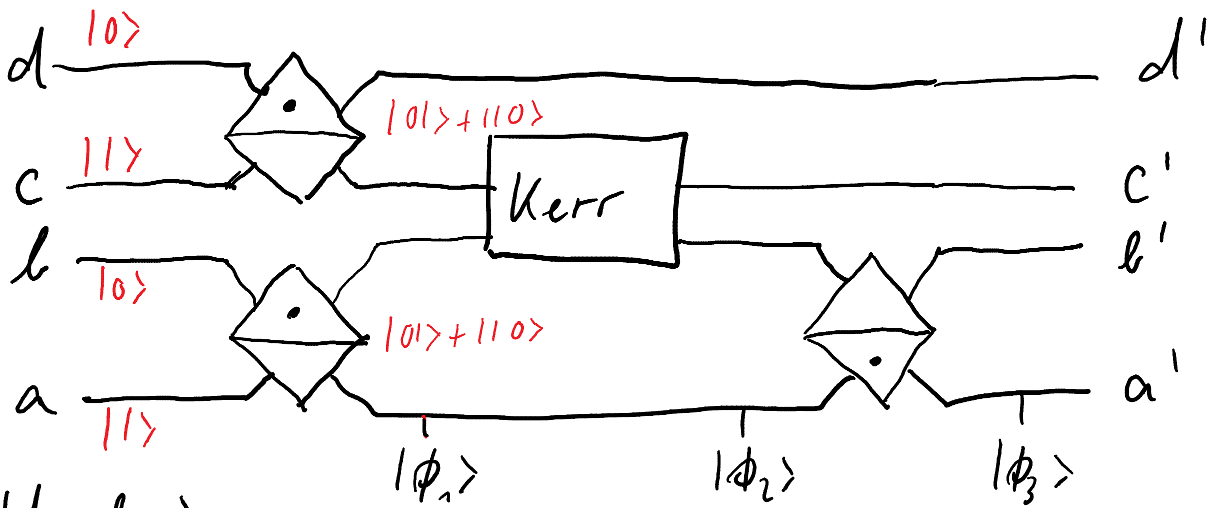
$$|OUT\rangle = \underbrace{B^{+}_{ab} \underbrace{K_{bc}}_{e^{i\chi L b^{+}b c^{+}c}} B_{ab}}_{e^{i\chi L c^{+}c \left(\frac{b^{+}-a^{+}}{\sqrt{2}}\right)\left(\frac{b-a}{\sqrt{2}}\right)}} |IN\rangle$$

$$= \ldots \; e^{i\xi c^{+}c \, (a^{+}b - b^{+}a)/2} \; \ldots \; |IN\rangle$$

phase shifts ($\rightarrow$ see Wiki)

$\Rightarrow$ It's a beam splitter with rotation angle $\xi c^{+}c$.

Creation of entangled states



$|dc\,ba\rangle$

$$|\phi_1\rangle = (|01\rangle + |10\rangle)(|01\rangle + |10\rangle)$$

$$= |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle$$

$$\downarrow$$

$$|\phi_2\rangle = |0101\rangle - |0110\rangle + |1001\rangle + |1010\rangle$$

$$|\phi_3\rangle = \cancel{|0101\rangle} - |0110\rangle - |0110\rangle - \cancel{|0101\rangle}$$
$$+ |1001\rangle - \cancel{|1010\rangle} + \cancel{|1010\rangle} + |1001\rangle$$
$$= \frac{|1001\rangle - |0110\rangle}{\sqrt{2}}$$

Entangled State!