Field values for number states:

$$|n_1 = 0, \ldots, n_{j-1} = 0, n_j, n_{j+1} = 0, \ldots \rangle \equiv |n_j\rangle$$

$$\langle n_j | \hat{\vec{E}}_\perp(\vec{r}) | n_j \rangle = \langle n_j | \hat{\vec{B}}(\vec{r}) | n_j \rangle = 0$$

$$c^2 \Delta \hat{\vec{B}}^2 = \Delta \hat{\vec{E}}_\perp^2 = \langle n_j | \hat{\vec{E}}_\perp(\vec{r})^2 | n_j \rangle = (2n_j + 1)\,\mathcal{E}_j^2$$

(so this is $(2n_j+1)$ times the $\Delta \hat{\vec{E}}_\perp^2$ in vacuum)

Coherent states (Quasi-classical states) Glauber

I want a state in which the E-field and B-field are as close to a classical state as possible.

$$\vec{E}_{cl} = \sum_j \mathcal{E}_j \vec{\varepsilon} \, (\underset{\substack{\uparrow \\ \text{a number}}}{\alpha_j} e^{-i\vec{k}_j \cdot \vec{r}} + \text{c.c.})$$

$$\vec{E}_{\text{quantum}} = \sum_j \mathcal{E}_j \vec{\varepsilon} \, (\hat{a}_j e^{-i\vec{k}_j \cdot \vec{r}} + \text{c.c.})$$

$\Rightarrow$ Define "coherent state $|\alpha_j\rangle$" as eigenstate of the annihilation operator with eigenvalue α_j:

$$\boxed{\hat{a}_j |\alpha_j\rangle = \alpha_j |\alpha_j\rangle} \quad \text{(for one mode, say } j)$$

Expand over number states:

$$|\alpha_j\rangle = \sum_{n_j=0}^{\infty} c_{n_j} |n_j\rangle$$

$$\hat{a}_j |\alpha_j\rangle = \sum_{n_j=1}^{\infty} c_{n_j} \sqrt{n_j}\, |n_j - 1\rangle = \sum_{n_j=0}^{\infty} c_{n_j+1} \sqrt{n_j+1}\, |n_j\rangle$$

$$\overset{!}{=} \alpha \sum_{n_j=0}^{\infty} c_{n_j} |n_j\rangle$$

$$\Rightarrow c_{n_j+1}\sqrt{n_j+1} = \alpha\, c_{n_j} \quad \text{or} \quad c_{n_j} = \frac{\alpha}{\sqrt{n_j}} c_{n_j - 1}$$

$$\Leftrightarrow c_{n_j} = \frac{\alpha_j^{n}}{\sqrt{n_j!}} c_{0_j}$$

$$\Rightarrow |\alpha_j\rangle = c_0 \sum_{n_j=0}^{\infty} \frac{\alpha_j^{n_j}}{\sqrt{n_j!}} |n_j\rangle .$$

Normalisation: $\langle \alpha_j | \alpha_j \rangle = 1 = |c_0|^2 \sum_{\substack{n_j=0\\ m_j=0}}^{\infty} \frac{\alpha_j^{n_j} \alpha_j^{* m_j}}{\sqrt{n_j! \, m_j!}} \underbrace{\langle m_j | n_j \rangle}_{\delta_{m_j n_j}}$

$$= |c_0|^2 \sum_{n_j=0}^{\infty} \frac{|\alpha_j|^{2n_j}}{n_j!} = |c_0|^2 e^{|\alpha_j|^2}$$

$\Rightarrow \quad c_0 = e^{-\frac{|\alpha_j|^2}{2}}$ up to a phase factor.

$$\Rightarrow \quad \boxed{|\alpha_j\rangle = e^{-\frac{|\alpha_j|^2}{2}} \sum_{n_j=0}^{\infty} \frac{\alpha_j^{n_j}}{\sqrt{n_j!}} |n_j\rangle}$$

Time evolution: Start at $t=0$ with $|\alpha_j\rangle$.

At t: $|\Psi(t)\rangle = e^{-iHt/\hbar} |\alpha_j\rangle$

$$= e^{-|\alpha_j|^2/2} \sum_{n_j=0}^{\infty} \frac{\alpha_j^{n_j}}{\sqrt{n_j!}} e^{-i(n_j + \frac{1}{2})\omega_j t} |n_j\rangle$$

$$= e^{-i\frac{\omega_j t}{2}} e^{-|\alpha_j|^2/2} \sum_{n_j=0}^{\infty} \frac{\left(\alpha_j e^{-i\omega_j t}\right)^{n}}{\sqrt{n_j!}} |n_j\rangle$$

$$= e^{-i\frac{\omega_j t}{2}} \, |\alpha_j e^{-i\omega_j t}\rangle$$

Probability of finding the value $(n_j + \frac{1}{2})\hbar\omega_j$ for the energy (or the value n_j for the photon number) is time independent:

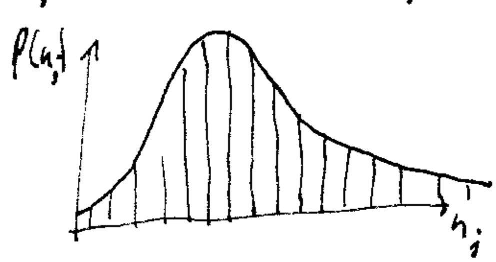
$$P(n_j) = |c_{n_j}|^2 = e^{-|\alpha_j|^2} \frac{|\alpha_j|^{2n_j}}{n_j!}$$

Mean photon number: $\bar{n}_j = \langle \hat{N}_j \rangle = \langle \alpha_j | \hat{a}_j^\dagger \hat{a}_j | \alpha_j \rangle = \alpha_j^* \alpha_j = |\alpha_j|^2$

(equivalently: $\langle \hat{N}_j \rangle = \sum_{n_j} n_j P(n_j) = \sum_{n_j} n_j e^{-|\alpha_j|^2} \frac{|\alpha_j|^{2n_j}}{n_j!} = |\alpha_j|^2 \sum_{n_j} e^{-|\alpha_j|^2} \frac{|\alpha_j|^{2n_j}}{n_j!} = |\alpha_j|^2$

So $\bar{n}_j = |\alpha_j|^2$

and $\boxed{P(n_j) = e^{-\bar{n}_j} \frac{\bar{n}_j^{\,n_j}}{n_j!}}$ Poissonian Distribution



$$\Delta n_j^2 = (\Delta \hat{N}_j)^2 = \langle \hat{N}_j^2 \rangle - \langle \hat{N}_j \rangle^2 = \langle \alpha_j | \hat{a}_j^+ \hat{a}_j \hat{a}_j^+ a_j | \alpha_j \rangle - |\alpha_j|^4$$

$$= \langle \alpha_j | \hat{a}_j^+ (\hat{a}_j^+ \hat{a}_j + 1) a_j | \alpha_j \rangle - |\alpha_j|^4$$

$$= |\alpha_j|^4 + \langle N_j \rangle - |\alpha_j|^4$$

$$= \langle N_j \rangle = \bar{n}_j$$

So $\boxed{\Delta n_j^2 = \bar{n}_j}$ (as it should be for Poissonian)

Electric field: Use $|\Psi(t)\rangle = e^{-i\frac{\omega_j t}{2}} | \alpha_j e^{-i\omega_j t} \rangle$

$$\langle \Psi(t) | \hat{\vec{E}}_\perp(\vec{r}) | \Psi(t) \rangle = i \sum_j \vec{\mathcal{E}}_j \left\{ \alpha_j e^{i(\vec{k}_j\cdot\vec{r} - \omega_j t)} - \alpha_j^* e^{-i(\vec{k}_j\cdot\vec{r} - \omega t)} \right\}$$

$$\langle \Psi(t) | \hat{\vec{E}}_\perp(\vec{r})^2 | \Psi(t) \rangle = \langle \Psi(t) | (\ldots a_j \ldots - \underbrace{a_j^+}_{\alpha_j^*} \ldots)(\ldots \underbrace{a_j}_{\alpha_j} \ldots - \ldots a_j^+ \ldots) | \Psi(t) \rangle$$

$$a_j a_j^+ = \underbrace{a_j^+}_{\alpha_j^*} \underbrace{a_j}_{\alpha_j} + 1$$

$$= \left(\langle \Psi(t) | \hat{E}_\perp(\vec{r}) | \Psi(t) \rangle \right)^2 + \mathcal{E}_j^2$$

$\Rightarrow \boxed{\Delta \hat{E}_\perp^2 = \mathcal{E}_j^2}$ just like in vacuum!

Orthonormality: $$|\langle \beta | \alpha \rangle|^2 = \left| e^{-\frac{|\alpha|^2}{2}} e^{-\frac{|\beta|^2}{2}} \sum_{nm} \frac{\alpha^n \beta^{*m}}{\sqrt{n!}\sqrt{m!}} \underbrace{\langle m|n\rangle}_{\delta_{mn}} \right|^2$$

$$= e^{-|\alpha|^2} e^{-|\beta|^2} \left| e^{\alpha\beta^*} \right|^2$$

$$= e^{-|\alpha-\beta|^2}$$ (so they are not orthogonal)

Closure: $$\frac{1}{\pi} \int \underbrace{d(\mathrm{Re}\,\alpha)\, d(\mathrm{Im}\,\alpha)}_{d^2\alpha} |\alpha\rangle\langle\alpha| = \mathbb{1}$$

Decomposition: $|\psi\rangle = \int d^2\alpha \; c_\alpha \, |\alpha\rangle$ with $c_\alpha = \frac{1}{\pi}\langle\alpha|\psi\rangle$

This decomposition is <u>not</u> unique as

$\langle\alpha|\alpha'\rangle \neq 0$ but $|\langle\alpha|\alpha'\rangle|^2 = e^{-|\alpha-\alpha'|^2} \longrightarrow 0$ if α and α' are widely separated

$\Longrightarrow$ basis α is over complete

Quasi - Probability:

Expand the density operator in the $|\alpha\rangle$'s:

$$\rho = \int d^2\alpha \, P_\rho(\alpha) \, |\alpha\rangle\langle\alpha|$$

Since $\rho^\dagger = \rho$ and $\mathrm{Tr}\,\rho = 1$ We have

$$\int \frac{d^2\alpha}{\pi} \langle\alpha|\rho|\alpha\rangle = \int d^2\alpha \, P_\rho(\alpha) = 1.$$

$\boxed{P_\rho(\alpha) \approx \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle.}$ Define $Q_\rho(\alpha) \equiv \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle$

$$\langle\alpha|\rho|\alpha\rangle = \int d^2\alpha' \, Q_\rho(\alpha') \, |\langle\alpha|\alpha'\rangle|^2 .$$

$$= \int d^2\alpha' \, Q_\rho(\alpha') \, e^{-|\alpha-\alpha'|^2}$$

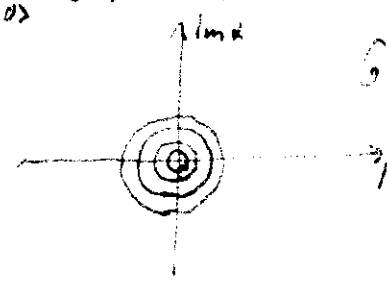
peaked around $\alpha = \alpha'$

$$\approx Q_\rho(\alpha) \underbrace{\int d^2\alpha' \, e^{-|\alpha-\alpha'|^2}}_{=\pi}$$

$$= \pi \, Q_\rho(\alpha).$$

Example 1: $\rho = |0\rangle\langle 0|$ vacuum

$$Q_{|0\rangle}(\alpha) = \frac{1}{\pi}|\langle\alpha|0\rangle|^2 = \frac{1}{\pi}e^{-|\alpha|^2}$$



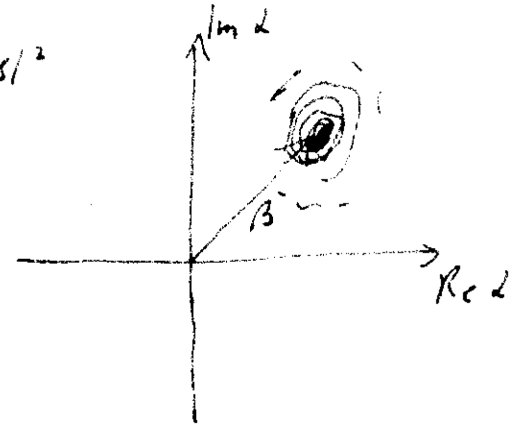
Gaussian centered at $\alpha = 0$ with width of order $\underline{1}$. (Reflecting $\Delta E_{1i}^2 = \mathcal{E}_i^2$)

Example 2: Coherent state $Q_\beta(\alpha)$

$$\rho = |\beta\rangle\langle\beta|$$

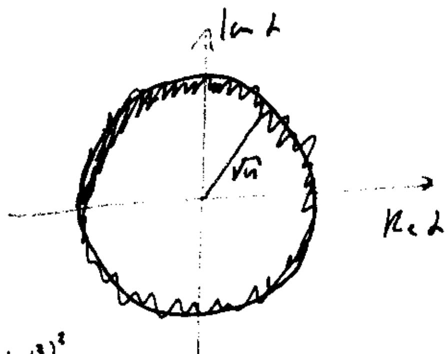
$$Q_\beta(\alpha) = \frac{1}{\pi}|\langle\alpha|\beta\rangle|^2 = \frac{1}{\pi}e^{-|\alpha-\beta|^2}$$

$\Rightarrow$ Gaussian centered at β



Example 3: Number state $|n\rangle$:

$Q_n(\alpha)$



a ring

radius $\sqrt{n}$

width 1 ($\frac{1}{c^2}$ radius)

$$= \frac{1}{\pi}|\langle n|\alpha\rangle|^2$$

$$= \frac{1}{\pi}e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}$$

$$\approx \frac{1}{\pi}\frac{1}{\sqrt{2\pi|\alpha|^2}}e^{-\frac{(n-|\alpha|^2)^2}{2|\alpha|^2}}$$

$$= \frac{1}{\pi}\frac{1}{\sqrt{2\pi n}}e^{-\frac{(|\alpha|^2-n)^2}{2n}} \approx \frac{1}{\pi}\frac{1}{\sqrt{2\pi n}}e^{-\frac{[(|\alpha|-\sqrt{n})(|\alpha|+\sqrt{n})]^2}{2n}} \approx \frac{1}{\pi}\frac{1}{\sqrt{2\pi n}}e^{-2(|\alpha|-\sqrt{n}}$$

Example 4: Thermal state in equilibrium

$$\rho = \frac{e^{-\beta H}}{Z} = \sum_n \frac{e^{-n\hbar\omega/k_BT}}{Z}|n\rangle\langle n|$$

$$Z = \sum_n e^{-n\hbar\omega/k_BT} = \frac{1}{1-e^{-\hbar\omega/k_BT}}$$

$$\rho_{th} = \sum_n P_n |n\rangle\langle n|$$

with $P_n = e^{-n\hbar\omega/k_BT}(1-e^{-\hbar\omega/k_BT})$

This describes:
- resistive filaments
- sun light
- gas discharges
- "chaotic light"

mean excitation

$$\langle n \rangle = \bar{n} = Tr(\rho\, a^+ a) = \sum_n n\, p_n = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

$$p_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}}$$ $\Rightarrow$ Planck's law using density of modes

most probable n: $n = 0$!

Variance: $\langle \Delta n^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2$

$\langle n^2 \rangle = \sum n^2 P_n$.

$\sum n(n-1) P_n = 2\bar{n}^2$

$\langle n^2 \rangle - \bar{n} = 2\bar{n}^2$

$\langle n^2 \rangle = 2\bar{n}^2 + \bar{n}$

$\Rightarrow \Delta n^2 = \bar{n}^2 + \bar{n}$

for poissonian it's $(\Delta n^2)_p = \bar{n}$

$P_n = x^n (1-x)$ with $x = e^{-\hbar\omega/kT}$

Trick: $n(n-1)\, p_n = (1-x)\, x^2 \frac{d^2}{dx^2} x^n$

$$\sum n(n-1)\, p_n = x^2 (1-x) \frac{d^2}{dx^2} \sum x^n$$

$$= x^2 (1-x) \frac{d^2}{dx^2} \frac{1}{1-x}$$

$$= x^2 (1-x) \frac{2}{(1-x)^3}$$

$$= \frac{2x^2}{(1-x)^2} = 2 \frac{1}{\left(\frac{1}{x} - 1\right)^2} = 2\bar{n}^2$$

Quasi-probability: $Q_{th}(\alpha)$

$$\rho_{th} = \sum_n p_n |n\rangle\langle n|$$

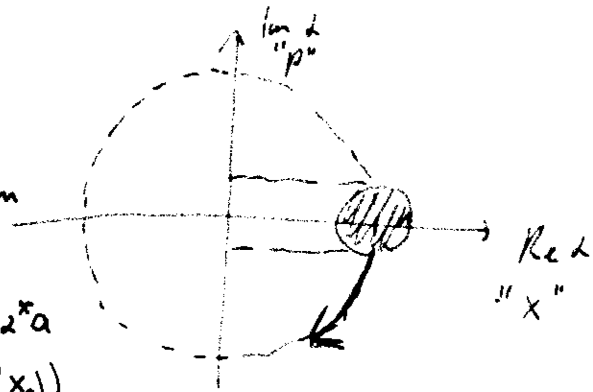
$$Q_{th}(\alpha) = \frac{1}{\pi} \langle \alpha | \rho_{th} | \alpha \rangle = \frac{1}{\pi} \sum_n p_n |\langle \alpha | n \rangle|^2$$

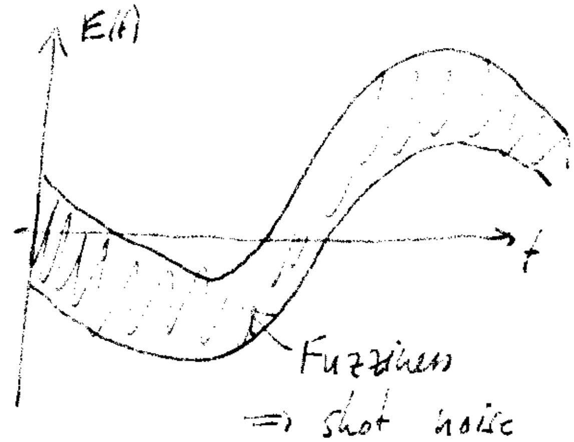
$$= \frac{1}{\pi} \frac{1}{\bar{n}+1} e^{-\frac{|\alpha|^2}{\bar{n}+1}}$$

gaussian centered at origin, width $\sqrt{\bar{n}+1}$ $\left(\frac{1}{e}\right)$

Coherent state time dependence $e^{-iHt/\hbar}|\alpha\rangle = e^{-i\frac{\omega t}{2}}|\alpha e^{-i\omega t}\rangle$

$Q_\rho(\alpha(t))$





Displaced vacuum
$|\alpha\rangle = \hat{D}(\alpha)|0\rangle$
with $\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$
(like $\exp(-i\frac{\hat{p}x_0}{\hbar})$)

E-field $\propto \langle\alpha|a - a^\dagger|\alpha\rangle = \alpha - \alpha^* \propto \mathrm{Im}\,\alpha$

Other distributions:

$$Q_\rho(\alpha) = \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle$$

$P_\rho(\alpha)$ defined if $\rho = \int d^2\alpha\, P_\rho(\alpha)\,|\alpha\rangle\langle\alpha|$
Glauber representation.

$$Q_\rho(\alpha) = \frac{1}{\pi}\int d^2\alpha'\, P_\rho(\alpha')\,|\langle\alpha|\alpha'\rangle|^2$$
$$= \frac{1}{\pi}\int d^2\alpha'\, P_\rho(\alpha')\, e^{-|\alpha-\alpha'|^2}$$

if $P_\rho(\alpha')$ varies smoothly compared to $e^{-|\alpha-\alpha'|^2}$,

$$Q_\rho(\alpha) \approx P_\rho(\alpha)\,\underbrace{\frac{1}{\pi}\int d^2\alpha'\, e^{-|\alpha-\alpha'|^2}}_{1}.$$

But careful, $P_\rho(\alpha)$ can become negative!

Wigner distribution: W adapted to calculate mean values of symmetrized products $b+b^\dagger$ and $i(b-b^\dagger)$
⇒ want this for E-field!
P adapted to normal order $(a^\dagger)^m a^n$
Q " " anti-normal order $a^m (a^\dagger)^n$.

Fluctuations and Noise

Heisenberg uncertainty principle

$$\Delta P \Delta Q \geq \frac{\hbar}{2}$$

recall $P = \frac{i}{\sqrt{2}}(a^+ - a)\cdot\sqrt{\hbar m \omega} = i\underbrace{\sqrt{\frac{\hbar\varepsilon_0}{2}\left(\frac{2\pi}{L}\right)^3\omega}}_{P_0}\,(a^+ - a)$

$(P = -\varepsilon_0 \mathcal{E}_\perp)$ $\quad Q = \sqrt{\frac{\hbar}{2m\omega}}\,(a^+ + a) = \underbrace{\sqrt{\frac{\hbar}{2\varepsilon_0\left(\frac{2\pi}{L}\right)^3\omega}}}_{Q_0}\,(a^+ + a)$

$(Q = A_\perp)$

$$\Rightarrow \langle\alpha|P|\alpha\rangle = iP_0(\alpha^* - \alpha)$$

$$\begin{aligned}
\langle\alpha|P^2|\alpha\rangle &= -P_0^2\langle\alpha|(a^+-a)(a^+-a)|\alpha\rangle \\
&= -P_0^2\langle\alpha|(\alpha^*-a)(a^+-\alpha)|\alpha\rangle \\
&= -P_0^2\langle\alpha|-\alpha^*\alpha + \alpha^2 + \alpha^{*2} - aa^+|\alpha\rangle \\
&= -P_0^2\langle\alpha|-\alpha^*\alpha + \alpha^2 + \alpha^{*2} - a^+a - 1|\alpha\rangle \\
&= -P_0^2\langle\alpha|-\alpha^*\alpha + \alpha^2 + \alpha^{*2} - \alpha^*\alpha - 1|\alpha\rangle \\
&= -P_0^2\langle\alpha|(\alpha^*-\alpha)(\alpha^*-\alpha)|\alpha\rangle + P_0^2 \\
&= (\langle\alpha|P|\alpha\rangle)^2 + P_0^2
\end{aligned}$$

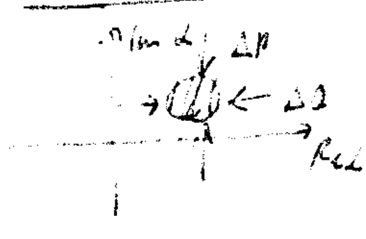
$$\langle\alpha|Q|\alpha\rangle = Q_0(\alpha^* + \alpha)$$

$$\langle\alpha|Q^2|\alpha\rangle = (\langle\alpha|Q|\alpha\rangle)^2 + Q_0^2$$

$$\Rightarrow \Delta P^2 = P_0^2 \; ; \quad \Delta Q^2 = Q_0^2$$

$$\Delta P \Delta Q = P_0 Q_0 = \frac{\hbar}{2}$$

$\Rightarrow$ Coherent states are minimum uncertainty states



area $\Delta P \cdot \Delta Q = \frac{\hbar}{2}$

$$\Delta(\mathrm{Re}\,\alpha)\,\Delta(\mathrm{Im}\,\alpha) = \frac{1}{4}$$

Another useful measure

$g^{(2)}(\tau)$ second order temporal coherence function

classical: $g^{(2)}_{cl}(\tau) = \frac{\langle I(t)\, I(t+\tau)\rangle}{\langle I\rangle^2}$

Homework #1: $g^{(2)}_{cl}(\tau) \geq 1$

In quantum mechanics: Use operators
Instead of $I(t)$ use $\hat{E}^2(t)$?

Note: Often, we are interested in processes where photons are absorbed by an atom initially in the ground state. The resonant part of that interaction involves exciting the atom and destroying a photon, i.e. it involves

$$\hat{\vec{E}}^{(+)}(\vec{r}=\vec{0}, t) = i \sum_j \sqrt{\frac{\hbar\omega_j}{2\varepsilon_0 V}}\, a_j\, e^{-i\omega_j t}$$

and not

$$\hat{\vec{E}}^{(-)}(\vec{r}=0, t) = -i \sum_j \sqrt{\frac{\hbar\omega_j}{2\varepsilon_0 V}}\, a_j^+\, e^{+i\omega_j t}$$

So typically, correlation functions are defined in normal order, with the a's to the right of the a^+'s.

If one is interested in a spontaneous or stimulated emission, we define the corresponding correlators involving $\hat{\vec{E}}^{(-)}$ on the right side, and $\vec{E}^{(+)}$ on the left.

Here: We ask for the probability of absorbing two photons (and consider 1 mode only here)

$$\sum_f |\langle \Psi_f | a a | \Psi_i\rangle|^2 = \langle \Psi_i | a^+ a^+ a a | \Psi_i\rangle$$

$$\Rightarrow g^{(2)}(\tau) = \frac{\langle a^+(0) a^+(\tau) a a\rangle}{\langle a^+ a\rangle^2} \overset{HW}{=} \frac{\langle n^2\rangle - \langle n\rangle}{\langle n\rangle^2}$$ independent of τ, single mode!

$$g^{(2)} = \frac{\langle a^+ a^+ a a \rangle}{\langle a^+ a \rangle^2} = \frac{\langle n^2 \rangle - \langle n \rangle}{\langle n \rangle^2}$$

$g^{(2)}(0) < 1$ is now possible.

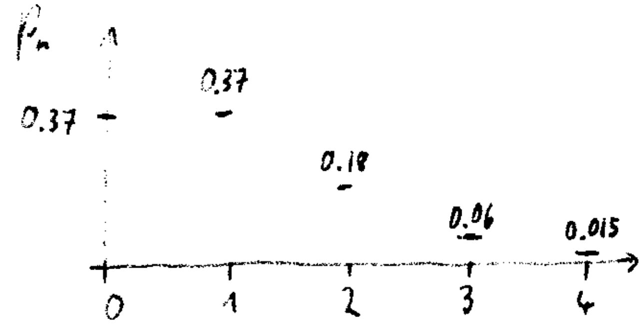
Fano factor $\quad F = \underbrace{\frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}}_{= 1 \text{ for poissonian distribution}} - 1$

	$\langle n^2 \rangle$	F	$g^{(2)}(0)$
thermal	$2\bar{n}^2 + \bar{n}$	$\bar{n}$ super poissonian	2
coherent	$\bar{n}^2 + \bar{n}$	0 poissonian	1
$\vert n \rangle$	$\bar{n}^2$	-1 sub-poissonian	$1 - \frac{1}{n} < 1$ [0 for $n = 1$]

The single photon

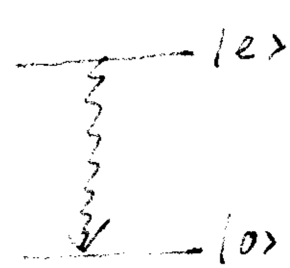
$|\alpha\rangle$ with $\alpha = 1$ is NOT a single photon

$$p_n = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} = \frac{1}{e} \frac{1}{n!}$$

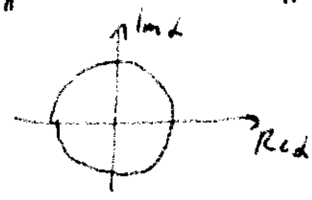


$|n\rangle$ is an eigenstate of $a^+ a$ with $n = 1$.

single atoms emit single photons:



$$Q_{|1\rangle}(\alpha) = \frac{1}{\pi} |\langle \alpha | 1 \rangle|^2 = \frac{|\alpha|^2}{\pi} e^{-|\alpha|^2}$$

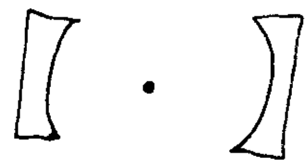


Ring

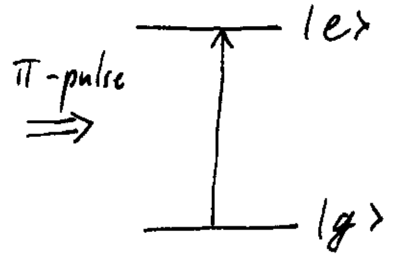
No phase $\quad \langle E \rangle = 0$

How to create single photons?

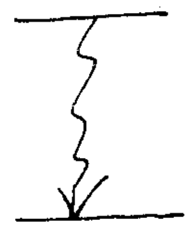
1 atom or ion



Cavity $\rightarrow$ enhance probability to emit into cavity mode

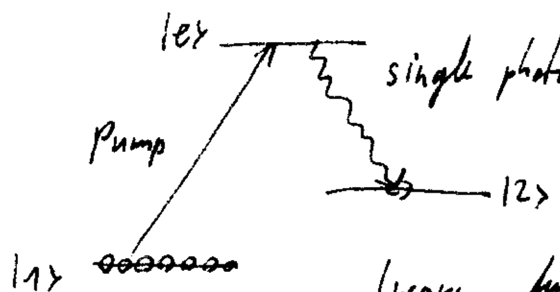


within Γ^{-1}



- Heralded Photon

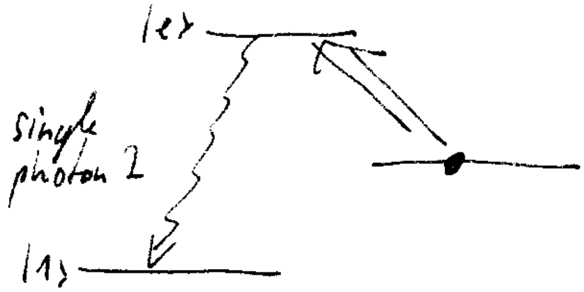
Many Atoms



$\Rightarrow$ 1 atom in state $|2\rangle$

(wave function $|\psi\rangle = \dfrac{|1000\ldots\rangle + |010\ldots\rangle + |0010\ldots\rangle}{\sqrt{N}}$)

$\Rightarrow$ gives you super-radiant enhancement of emission into one mode (n times stronger coupling to cavity)

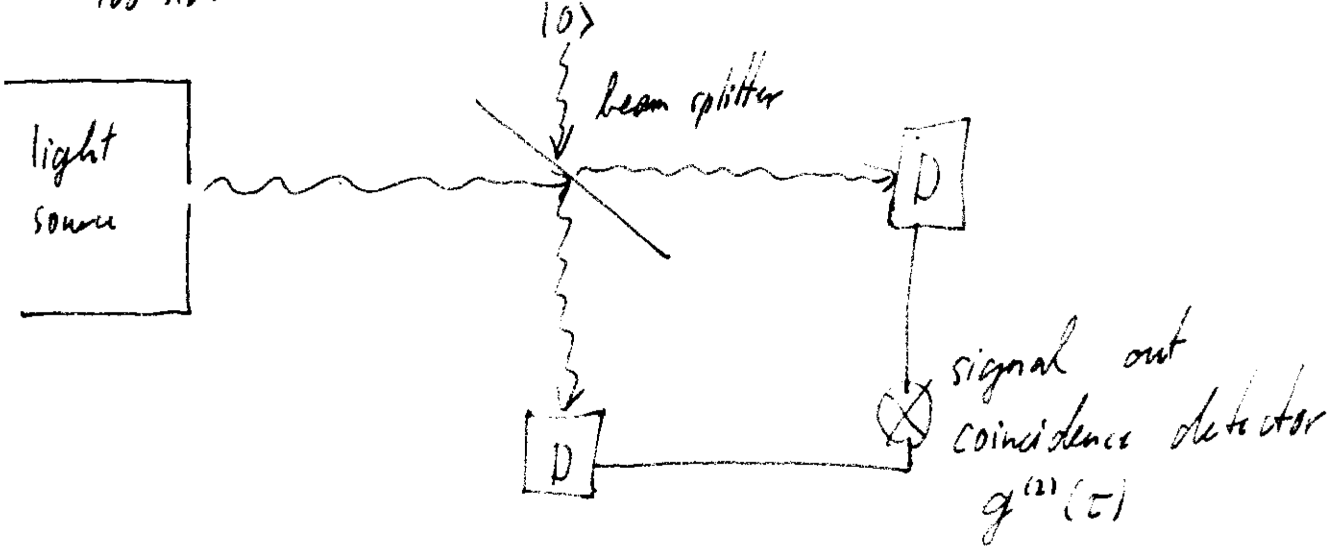


heralded by photon 1

see Vuletic group

Hanbury Brown - Twiss experiment 1956

Landmark experiment in the 1950's
first experiment to look at g_2
to look at successive photons cannot use single photomultiplier
→ too slow ⇒ use beam splitter and use two photodetectors.



classically:
intensity splits equally. $\cdot\, g^{(2)} = 1$ coherent state
$\cdot\, g^{(2)} = 2$ thermal state

quantum mechanically:
$|n=1\rangle$: photon can only go to only one detector
$$g^{(2)}(0) = 0$$

see homework # 1.

Note: $g^{(2)} = 2$ is of q.m. origin, but comes out classically. Why? Because all we need is the superposition principle, deriving from the boson nature of photons, but already obeyed by classical fields.