Problem 1. A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $\mathbf{B}(t) = B_0 \cos(\omega t)\,\hat{\mathbf{z}}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time. (30 pts)

Problem 2. A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in Fig. 1 Find the magnetic field in the gap, at a distance $s < a$ from the axis. (40 pts)

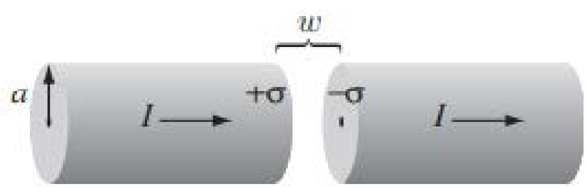


FIGURE 1

Problem 3. The preceding problem was an artificial model for the charging capacitor, designed to avoid complications associated with the current spreading out over the surface of the plates. For a more realistic model, imagine *thin* wires that connect to the centers of the plates (Fig. 2(a)). Again, the current I is constant, the radius of the capacitor is a, and the separation of the plates is $w \ll a$. Assume that the current flows out over the plates in such a way that the surface charge is uniform, at any given time, and is zero at $t = 0$. (50 pts)

(a) Find the electric field between the plates, as a function of t.

(b) Find the displacement current through a circle of radius s in the plane midway between the plates. Using this circle as your "Amperian loop," and the flat surface that spans it, find the magnetic field at a distance s from the axis.

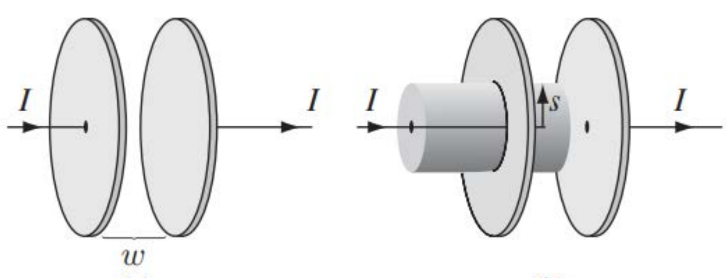
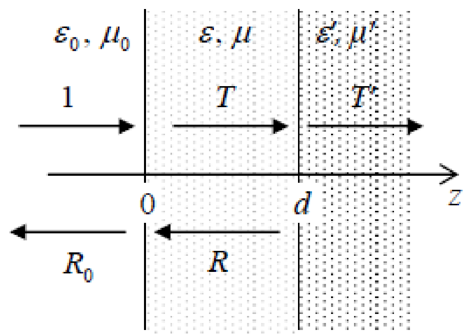


FIGURE 2

(c) Repeat part (b), but this time use the cylindrical surface in Fig. 2(b), which is open at the right end and extends to the left through the plate and terminates outside the capacitor. Notice that the displacement current through this surface is zero, and there are two contributions to I_{enc}.

Problem 4 A plane wave of frequency ω is normally incident, from free space, on a plane surface of a material with real electric permittivity ε' and magnetic permeability μ'. To minimize the wave's reflection from the surface, you may cover it with a layer, of thickness d, of another transparent material – see the figure on the right. Calculate the optimal values of ε, μ, and d. (60 pts)



Problem 5 A monochromatic, plane wave is incident from inside a medium with $\varepsilon\mu > \varepsilon_0\mu_0$ onto its plane surface, at an angle of incidence θ larger than the critical angle $\theta_c = \sin^{-1}(\varepsilon_0\mu_0/\varepsilon\mu)^{1/2}$. Calculate the depth δ of the evanescent wave penetration into the free space, and analyze its dependence on θ. Does the result depend on the wave's polarization? (30 pts)