Protocol Idea & Questions

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1 Protocol Idea

After our meeting, I spent some more time thinking about the idea of using measurements and local unitaries to "shrink" and turn the ansatz (of large system size) to the ground state (a smaller system). Now, I want to run it through you again to see if it makes sense do this.

Suppose we want to prepare a k-qubit TFIM critical ground state. What I'm thinking is perhaps we can start with a ring-like cluster state of $k \times 2^n$ qubits in for some n. The goal is to achieve the k-qubit critical state after $\sim n$ measurement layers, where $n \sim \log(k)$.

Here's the protocol I'm thinking of:

- To start, we measure every other qubit on the ring in the X-basis, leaving the unmeasured $k \times 2^{(n-1)}$ qubits in the highly entangled GHZ state (up to some bit flips). The effective system size is halved.
- Apply a set of non-uniform local unitaries $\{\mathcal{U}_i\}_{i=1}^{p}$ across the unmeasured qubits. The set of these unitaries will be parameterized by $p > 2$ parameters (where 2 is what a single layer of QAOA has).
- Compute the cost function to find a new parameters for the next set of unitaries for the next layer of measurements (which shrinks the effective system size down further).(*)
- Repeat until convergence.

(*): There's one detail regarding computing the cost function, though, because now we have not only the full system size ($L_{\text{full}} = k \times 2^n$ qubits) but also an effective system size which approaches k as we go through the layers of measurements ($L_{\text{eff}} \downarrow k$).

Unlike in QAOA where $L_{\text{full}} = L_{\text{eff}}$ (since the number of effective qubits doesn't change), here $L_{\text{eff}} \downarrow k$ following each layer of measurements. As a result, we don't have a trial $|\psi_{\text{trial}}\rangle$in the same Hilbert space as $|\psi_{\text{target}}\rangle$ to compute the cost function

$$\langle\psi_{\text{target}}|\,\mathcal{H}\,|\psi_{\text{trial}}\rangle$$

as in QAOA. To circumvent this issue, I think we can embed $|\psi_{\text{target}}\rangle$ into a larger Hilbert space by adjoining it with ancilla qubits $\bigotimes|+\rangle$ so that $|\psi_{\text{target}}\rangle\,|\bigotimes+\rangle$ and $|\psi_{\text{trial}}\rangle$ are in the same Hilbert space. With this, we can compute the cost function:

$$\langle\psi_{\text{target}}|\left\langle\bigotimes+\right|(\mathcal{H}\otimes\mathcal{I})\,|\psi_{\text{trial}}\rangle\,.$$

I believe that by minimizing this new quantity at each layer, we will still be on the right track to minimizing the original cost function $\langle\psi_{\text{target}}|\,\mathcal{H}\,|\psi_{\text{trial}}\rangle$. The intuition here being that the ancilla qubits and the identity operator don't affect the optimization.

If this protocol works and if n scales as $\log(k)$ then we have a protocol whose depth scales as $\log(k)$. Not only that, the number of qubits required also doesn't scale too fast ($\sim k^\alpha$ where $1 < \alpha < 2$).

2 Questions

I want to give this idea a test, but I want to run it through you before perhaps trying to implement it.

- Am I explaining the protocol clearly enough? Does the protocol make sense?
- Is there a limitation or potential problem here that you can foresee?
- Do you see a potential issue with having too many qubits in the beginning (the full system size in this protocol scales as $\sim k^\alpha$ where $1 < \alpha < 2$)? If that's the case, we can also choose not to measure half the remaining qubits at each layer.