8.321 Recitation 16

Shing Yan Li

November 9, 2021

1 Free Particle

- (Normalizable) Wave packet: $\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i\left(kx - \hbar k^2 t/2m\right)} dk$

- Phase velocity $v_p = \omega/k$ vs group velocity $v_g = d\omega/dk$

- Gaussian wave packet: $\phi(k) = \left(\frac{2a}{\pi}\right)^{1/4} e^{-a(k-k_0)^2}$

- $\psi(x,t) = \frac{1}{(2\pi a)^{1/4}} \exp\left[\left(-\frac{x^2}{4a} + i\left(k_0 x - \frac{\hbar k_0^2}{2m}t\right)\right)/\left(1 + \frac{i\hbar t}{2ma}\right)\right]/\sqrt{1 + \frac{i\hbar t}{2ma}}$

- Wave packet spreading: $|\psi(x,t)|^2 = \frac{1}{\sqrt{2\pi a(t)}} \exp\left[-\frac{(x - \hbar k_0 t/m)^2}{2a(t)}\right]$ where $a(t) = a\left(1 + \left(\frac{\hbar t}{2ma}\right)^2\right)$, $\langle \Delta x \rangle \langle \Delta p \rangle = \frac{\hbar}{2}\sqrt{\frac{a(t)}{a}}$