9/8/21

Today — Introduction

- classical physics
- quantum physics
- Example of QM: 2-state systems & Stern-Gerlach expt.

1 Fundamental Concepts

1.1 Introduction

Theoretical framework of classical physics

* state (at fixed t) is ~~dep~~ fixed by a point $\{x^i, p_i\}$ in phase space. (flat space, or more generally a symplectic mfld)

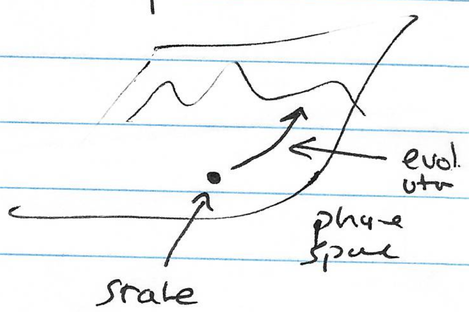
eg. (x, y, z, p_x, p_y, p_z)

* Poisson bracket $\{x^i, p_j\} = \delta_{ij}$ eg. $\{x, p_x\} = 1$, $\{x, p_y\} = 0$

$$\Rightarrow \{F, G\} = \sum_i \left(\frac{\partial F}{\partial x^i}\frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i}\frac{\partial G}{\partial x^i} \right)$$

* Observables: functions on phase space

$\Theta(x^i, p_j)$



eg. $\dfrac{p_x^2 + p_y^2 + p_z^2}{2m} = KE$

$U(x, y, z) = \frac{k}{2}(x^2 + y^2 + z^2)$

$H = U + KE$

* Hamiltonian $H(x, p)$ defines dynamics

$\dot{q} = \{q, H\}$ for q any fn on phase space

1st order ODE's.

- Describes all of mechanics
  E & M
  includes fluids, materials, ...

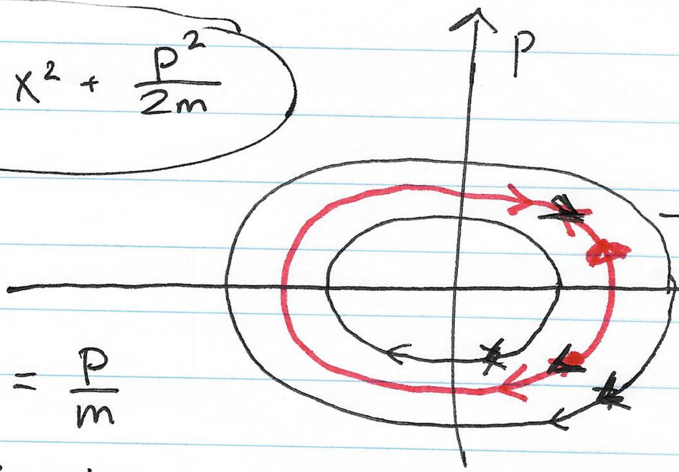
- Simple, intuitive framework

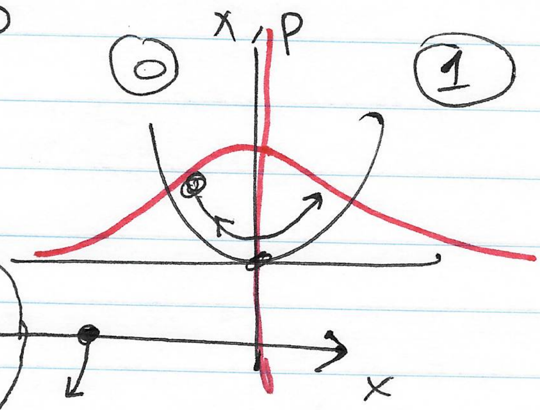
- Deterministic

- Time-reversible

Example: Classical 1D SHO

$$\boxed{H = \frac{k}{2}x^2 + \frac{p^2}{2m}}$$





$\dot{x} = \{x, H\} = \frac{p}{m}$

$\dot{p} = \{p, H\} = -kx$
$(= +m\ddot{x})$

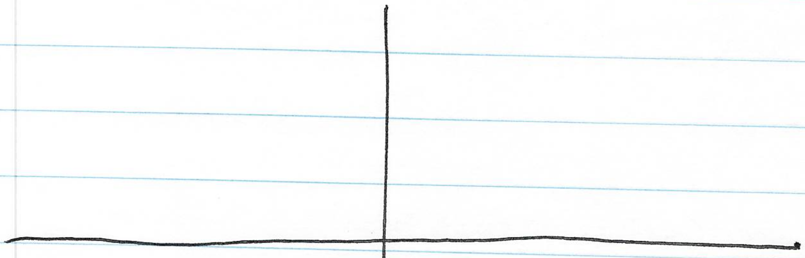
coordinates x, p

observable: x, p

H, $KE = \frac{p^2}{2m}$

$f(x,p) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$

"RHS observable"



in class. physics:

if you know the state (x_i, p_i)

$\Rightarrow$ every observable has a fixed value $\mathcal{O}(x,p)$

1.1 Intro (continued)

Theoretical framework of quantum physics

* State defined by vector $|v\rangle$ in (complex) vector space $\mathcal{H}$
  [really, a ray (magn. & overall phase unimportant); $\mathcal{H}$ often idealized as ∞-dim Hilbert space.]

* Observables are Hermitian operators (matrices) $\mathcal{O}^{+} = (\mathcal{O}^{T})^{*} = \mathcal{O}$

* Dynamics: $|v(t)\rangle = e^{-iHt/\hbar} |v(0)\rangle$
  [H hermitian, $\hbar$ constant]

* "Collapse postulate" (simple version; subtleties: degeneracy, cts spectrum

  If $|\psi\rangle = \sum \alpha_i |\lambda_i\rangle$, $A|\lambda_i\rangle = \lambda_i |\lambda_i\rangle$, $\lambda_i \neq \lambda_j$
  (assume $\sum |\alpha_i|^2 = 1$)
  then with prob. $|\alpha_i|^2$, measure $A = \lambda_i$, $|\psi\rangle = |\lambda_i\rangle$ after measure

This framework describes all quantum systems and all experiments to date that do not involve gravitational forces.
[Note: including QFT]

- atomic spectra (quantization of E)
- semiconductors, transistors (quantum tunneling)
- thermodynamics (QM → entropy S → temp T)
- Quantum Info systems (quantum communication, q. computing ...)