(172) Claim $n!\, Q(k,n) \le \binom{k-1}{n-1}$

distributing k i'd things to n i'd recepients, each gets at least 1.

distributing k things to n identical recepients s.t. each gets at least one, no repeats, ~~permute rece.~~ then permute recepients.

Obvious since partitioning k into n distinct parts and then permute is "included" in distributing k into n parts ~~[illegible]~~ s.t. each part gets at least one.

(I really don't know how else to explain this...)

(175) ~~174: $P(k,n) = Q(k+n, n)$~~

~~$\to Q(k,n) = Q((k-n)+n, n) = P(k-n, n) = Q$~~

(175) $Q(k,n) = ?$ $\to$ So $\boxed{Q(k,n) = Q(k-n,n) + Q(k-n,n-1)}$

- If all parts have size ≥ 2, then remove largest column, and left w/ n parts $\to$ get $Q(k-n,n)$
- If there is a part of size 1, then remove largest column gives $n-1$ parts $\to$ get $Q(k-n,n-1)$

Supp 5 $P(n,2) = ?$

If n is even, then can split $\frac{n}{2} + \frac{n}{2}$

or have $\frac{n}{2} - 1$ options

$\Rightarrow$ get $\frac{n}{2}$ choices

If n is odd, then have $\left\lfloor \frac{n}{2} \right\rfloor$ choices

So $P(n,2) = \left\lfloor \frac{n}{2} \right\rfloor$ $n \geq 0$.

Supp 6

If $k = 3$ then $P(k, k-2) = P(3,1) = 1$

If $k \geq 3$ then $P(k, k-2) = \sum_i P(k-(k-2), i)$

$= \sum_i P(2, i)$

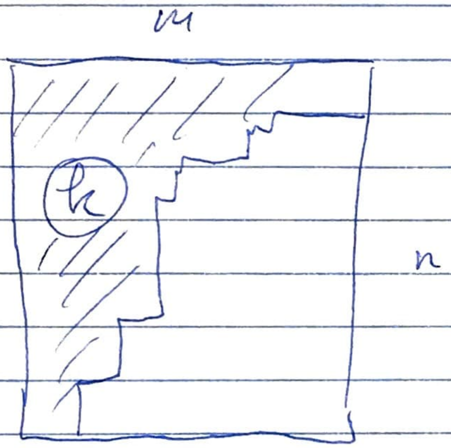
$= 1 + 1 = 2$.

Can also consider $k = 2$ but this is trivial

7 Claim: $p_m(k, n)$ (parts of size at most m)

$= p$ ($mn - k$ ... into no more than n parts of size at most $m-1$).

Pf



For each partition of k into n parts s.t. max size is m, we have a (unique) partition of $(mn-k)$ into no more than n parts of size at most $m-1$

(by Young's diagram)

So, # partitions of k into n parts s.t. max size is m

is **equal** to the # partitions of $(mn-k)$ into no more than n parts of size at most $m-1$.