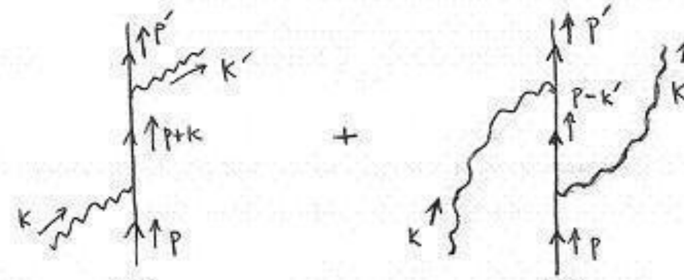
X-rays + gamma rays on electrons.

$e^- \gamma \to e^- \gamma$ (Compton scattering)

There two diagrams at lowest order



(I) (II)

Let $\varepsilon_\nu(k)$ + $\varepsilon_\nu(k')$ be the photon polarization vectors

(I) (II)

$$\bar{u}(p')(-ie\gamma^\mu)\varepsilon^*_\mu(k')\frac{i(\not p+\not k+m)}{(p+k)^2-m^2+i\varepsilon}(-ie\gamma^\nu)\varepsilon_\nu(k)u(p) + \bar{u}(p')(-ie\gamma^\nu)\varepsilon_\nu(k)\frac{i(\not p-\not k'+m)}{(p-k')^2-m^2+i\varepsilon}(-ie\gamma^\mu)\varepsilon^*_\mu(k')u(p)$$

$$= -ie^2\varepsilon^*_\mu(k')\varepsilon_\nu(k)\bar{u}(p')\left[\frac{\gamma^\mu(\not p+\not k+m)\gamma^\nu}{(p+k)^2-m^2+i\varepsilon} + \frac{\gamma^\nu(\not p-\not k'+m)\gamma^\mu}{(p-k')^2-m^2+i\varepsilon}\right]u(p)$$

We can simplify this a little. Note that $k^2 = k'^2 = m_\gamma^2 = 0$.

$p^2 = p'^2 = m^2$. So
$$(p+k)^2 = p^2 + 2p\cdot k + k^2 = m^2 + 2p\cdot k$$
$$(p-k')^2 = p^2 - 2p\cdot k' + k'^2 = -2p\cdot k' + m^2$$

Also $(\not p+m)\gamma^\nu u(p) = \gamma^\nu(-\not p+m)u(p) + \underset{\uparrow \atop p_\mu\{\gamma^\mu,\gamma^\nu\}}{2p^\nu}\, u(p)$

Since $\not p\, u(p) = m\cdot u(p)$, $(-\not p+m)u(p) = 0$. Therefore

$$(\not p+m)\gamma^\nu u(p) = 2p^\nu u(p)$$

So the amplitude is now:

$$i\mathcal{M} = -ie^2\, \varepsilon^*_\mu(k')\varepsilon_\nu(k)\, \bar{u}(p')\left[\frac{\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu}{2p\cdot k} + \frac{\gamma^\nu \not{k}' \gamma^\mu - 2\gamma^\nu p^\mu}{2p\cdot k'}\right]u(p)$$

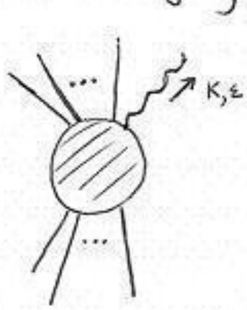
Photon polarizations + Ward identity

The interaction for QED has the general form

$$e\int d^4x\; j^\mu A_\mu$$ where j^μ is the conserved electric charge current

Let us consider a process with an outgoing photon with momentum k, polarization ε.

The amplitude can be written as



$$i\mathcal{M} = i\mathcal{M}^\mu(k)\,\varepsilon^*_\mu(k)$$

where

$$\mathcal{M}^\mu(k) \propto \int d^4x\; e^{ikx} \langle f|\, j^\mu(x)\, |i\rangle$$

final state, without photon under consideration

incoming state

Since $\partial_\mu j^\mu(x) = 0$,

$$k_\mu \mathcal{M}^\mu(k) \propto \int d^4x\; e^{ik\cdot x}\, k^\mu \langle f| j^\mu(x) |i\rangle = -i\int d^4x\; \partial_\mu\!\left(e^{ik\cdot x}\right) \langle f| j^\mu(x) |i\rangle$$

$$= i\int d^4x \, e^{ik\cdot x} \langle f | \partial_\mu j^\mu(x) | i \rangle = 0$$

This is a specific example of the Ward identity in QED.
When you replace the polarization vector for any external photon (can be incoming or outgoing) by the momentum k_μ, then the amplitude vanishes.

$$k_\mu \mathcal{M}^\mu(k) = 0.$$

More on this later.