8.321 Recitation 8

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1 Modified Heisenberg Model

- Warmup: $H = A(\sigma_{z1} + \sigma_{z2}) + B\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
- Eigenvalues: $2A + B, B, -2A + B, -3B$. Eigenvectors: $|1,1\rangle, |1,0\rangle, |1,-1\rangle, |0,0\rangle$
- The model: $H = \sum_{i<j}^{N} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - g\sum_i^N \sigma_{zi}$ (N even)
- Eigenvalues: $2j(j+1) - 3N/2 - 2gm$. Eigenvectors $|j,m\rangle$. ($j = 0, 1, ..., N/2, m = -j \sim j$)
- Ground state ($g \geq 0$): $|j,j\rangle$ where $j = \max\{(g-1)/2, N/2\}$ rounded off to nearest integer
- Quantum paramagnetism