Let's check the calculations...

$$\langle 0|\psi_A(x)\bar{\psi}_B(y)|0\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\frac{1}{\sqrt{2E_{p'}}}\langle 0|\left\{\sum_s a^s_{A\vec{p}}u^s_A(p)e^{-ip\cdot x} + \cancel{b^{s\dagger}_{A\vec{p}}v^s_A(p)e^{-ip\cdot x}}\right\}$$

$$\times\left\{\sum_s \cancel{b^s_{B\vec{p}'}\bar{v}^s_B(p')e^{-ip'\cdot y}} + a^{s\dagger}_{B\vec{p}'}\bar{u}^s_B(p')e^{ip'\cdot y}\right\}|0\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\frac{1}{2}\frac{1}{\sqrt{E_pE_{p'}}}\langle 0|\sum_s a^s_{A\vec{p}}u^s_A(p)\sum_s a^{s\dagger}_{B\vec{p}'}\bar{u}^s_B(p')e^{-i(p\cdot x-p'\cdot y)}|0\rangle$$

$$= \int \frac{d^3p\,d^3p'}{(2\pi)^6}\frac{1}{2}\frac{1}{\sqrt{E_pE_{p'}}}\sum_s\langle 0|u^s_A(p)\bar{u}^s_B(p')|0\rangle(2\pi)^3\delta^{(3)}(p-p')$$

$$\times e^{i(p\cdot x-p'\cdot y)}$$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\underbrace{\sum_s u^s_A(p)\bar{u}^s_B(p)}\,e^{-ip(x-y)}$$

$$(\not{p}+m)_{AB} = (\gamma^\mu p_\mu+m)_{AB} \quad \left(\text{spin sum relations}\right)$$

$$= (i\not{\partial}_x+m)_{AB}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}e^{-ip(x-y)}\,.\ \checkmark$$

Similarly, we can get the other relation too...

Oct 5, 2020

(1) Recall Dirac bispinor field...

$$\psi(x) = \sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left[a^r_{\vec{p}}\,u^r(\vec{p}) + b^{r\dagger}_{-\vec{p}}\,v^r(-\vec{p})\right]e^{i\vec{p}\cdot\vec{x}}$$

Use $\{a^r_{\vec{p}}, a^{s\dagger}_{\vec{p}'}\} = \{b^r_{\vec{p}}, b^{s\dagger}_{\vec{p}'}\} = (2\pi)^3\delta^{rs}\delta^{(3)}(\vec{p}-\vec{p}')$

and all other anti-comm $= 0$, derive the following:

$$\{\psi_a(\vec{x}), \psi_b^\dagger(\vec{y})\} = \delta_{ab}\,\delta^{(3)}(\vec{x}-\vec{y})$$

(2) The momentum operator is the Noether charge associated with spatial translation...

$$\vec{P} = -i\int d^3\vec{x}\;\psi^\dagger(x)\,\vec{\nabla}\psi(\vec{x})$$

Show that

$$\vec{P} = \sum_{r=1}^{2}\int\frac{d^3p}{(2\pi)^3}\,\vec{p}\left(a^{r\dagger}_{\vec{p}}a^r_{\vec{p}} + b^{r\dagger}_{\vec{p}}b^r_{\vec{p}}\right)$$

Oct 10, 2020

(1) Well...

$$\{\psi_a(x), \psi_b^\dagger(y)\}$$

$$= \psi_a(x)\,\psi_b^\dagger(y) + \psi_b^\dagger(y)\,\psi_a(x)$$

$$\cancel{\sum_{r=1}^{2}\int\frac{d^3p_a\,d^3p_b}{(2\pi)^6\,2\sqrt{E_{p_a}E_{p_b}}}\,e^{i(\vec{p}_a\cdot\vec{x} - \vec{p}_b\cdot\vec{y})}\left[a^r\right.}$$

To keep things clean...

$\psi_a(x)\,\psi_b^\dagger(y) + \psi_b^\dagger(x)\psi_a(x)$ which involves the factor...

$$= \sum_{r=1}^{2}\sum_{s=1}^{2}\left[\hat{a}^r_{p_a} u^r(p_a) + \hat{b}^{r\dagger}_{-p_a} v^r(-p_a)\right]\left[(\hat{a}^s_{p_b} u^s(p_b))^\dagger + (\hat{b}^{s\dagger}_{-p_b} v^s(-p_b))\right]$$

$$+ \sum_{r=1}^{2}\sum_{s=1}^{2}\left[(\hat{a}^s_{p_b} u^s(p_b))^\dagger + (\hat{b}^{s\dagger}_{-p_b} v^s(-p_b))^\dagger\right]\left[\hat{a}^r_{p_a} u^r(p_a) + \hat{b}^{r\dagger}_{-p_a} v^r(-p_a)\right]$$

$$= \sum_{r,s=1}^{2} \left\{ \{\hat{a}^r_{p_a}, \hat{a}^{s\dagger}_{p_b}\} u^r(p_a) u^{s\dagger}(p_b) + \{\hat{b}^r_{p_a}, \hat{b}^{\dagger s}_{p_b}\} v^r(p_a) v^s(p_b) \right\}$$

$$= \sum_{r,s=1}^{2} (2\pi)^3 \delta^{r,s} \delta^{(3)}(p_a - p_b) \left\{ u^r(p_a) u^{s\dagger}(p_b) + v^r(p_a) v^{s\dagger}(-p_b) \right\}$$

$$\Rightarrow \{\psi_a(x), \psi_b^\dagger(y)\}$$

$$= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{i\vec{p}(\vec{x}-\vec{y})} \sum_{r=1}^{2} \left\{ u^r(p) u^{r\dagger}(p) + v^r(-p) v^{r\dagger}(-p) \right\}$$

Now, we want to convert $u^{s\dagger} \to \bar{u}$

$\to$ need γ^0. In particular, recall that $\gamma^0\gamma^0 = \mathbb{1}$

from page 63 $\Rightarrow$ and $\boxed{u^{s\dagger}(p)\gamma^0 = \bar{u}^s(p)}$, we have

$$\{\psi_a(x), \psi_b^\dagger(y)\} = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{i\vec{p}(\vec{x}-\vec{y})} \sum_{r=1}^{2} \left\{ u^r_p u^{r\dagger}_p \gamma^0 + v^r_{-p} v^{r\dagger}_{-p} \gamma^0 \right\}$$

$$= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{i\vec{p}(\vec{x}-\vec{y})} \sum_{r=1}^{2} \left\{ u^r_p \bar{u}^r_p + v^r_{-p} \bar{v}^r_{-p} \right\} \gamma^0 \quad \left(\text{spin sum}\right)$$

$$= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{i\vec{p}(\vec{x}-\vec{y})} \left(\underset{E\gamma^0 - \cancel{\vec{p}\cdot\vec{\gamma}}}{p_+\cdot\gamma} + \cancel{m\mathbb{I}} + \underset{E\gamma^0 + \cancel{\vec{p}\cdot\vec{\gamma}}}{p_-\cdot\gamma} - \cancel{m\mathbb{I}} \right) \gamma^0 \Big|_{\delta_{a,b}}$$

recall that only $\vec{p} \to -\vec{p}$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{i\vec{p}(\vec{x}-\vec{y})} \cancel{[2p\cdot\gamma]\gamma^0}$$

Rather

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{i\vec{p}(\vec{x}-\vec{y})} \left\{ E_p\cancel{\gamma^0} - \cancel{\vec{p}\cdot\vec{\gamma}} + E_p\cancel{\gamma^0} + \cancel{\vec{p}\cdot\vec{\gamma}} \right\} \cancel{\gamma^0}_{ab}$$

$$= \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}(\vec{x}-\vec{y})} = \delta_{ab}\,\delta^{(3)}(\vec{x}-\vec{y}).$$

So $\boxed{\{\psi_a(x),\ \psi_b^\dagger(y)\} = \delta_{ab}\,\delta^{(3)}(x-y)}$

~~(2) Let~~

$$\cancel{\vec{P} = -i\int d^3\vec{x}\ \psi^\dagger(\vec{x})\,\vec{\nabla}\psi(\vec{x}).}$$

~~Show that~~

~~$p = (p^0, \vec{p})$~~

$$\cancel{\vec{P} = \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3}\, \vec{p}\,\left(a_{\vec{p}}^{r\dagger} a_{\vec{p}}^r + b_{\vec{p}}^{r\dagger} b_{\vec{p}}^r\right).}$$

~~Well.~~

~~$ip\cdot x = \ldots + i\vec{p}\cdot\vec{x}$~~

$$\cancel{\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} e^{ip\cdot x} \sum_{r=1}^{2} \left\{ a_{\vec{p}}^r u_{\vec{p}}^r + b_{-\vec{p}}^{r\dagger} v_{-\vec{p}}^r \right\}.}$$

$$\cancel{\nabla\psi(x) = \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} (i\vec{p})}$$

(2) $$\Psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a^s_{\vec{p}} u^s_p e^{-ip\cdot x} + b^{s\dagger}_{\vec{p}} v^s_p e^{ip\cdot x} \right)$$

$$\rightarrow \quad \nabla \Psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (i\vec{p}) \left\{ a^s_{\vec{p}} u^s_p e^{-ip\cdot x} - b^{s\dagger}_{\vec{p}} v^s_p e^{ip\cdot x} \right\}$$

$$\Psi^\dagger(x) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{\sqrt{2E_q}} \sum_{r=1}^{2} \left\{ b^r_q v^{\dagger r}(q) e^{-iqx} + a^{r\dagger}_q u^{\dagger r}_q e^{iqx} \right\}$$

$$\rightarrow \quad \int d^3x \, (-i) \Psi^\dagger \nabla \Psi$$

$\delta^{(3)}(p-q)$

$$= \int d^3x \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \frac{1}{2} \frac{1}{\sqrt{E_p E_q}} e^{ix(p-q)} \vec{p} \times \sum_s \times \sum_r$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \frac{1}{E_p} \vec{p} \left\{ \sum_{s,r=1}^{2} \left(a^{r\dagger}_{\vec{p}} a^s_{\vec{p}} \underbrace{u^{\dagger r}_p u^s_p}_{2E_p \delta^{rs}} - b^r_{\vec{p}} b^{s\dagger}_{\vec{p}} v^{\dagger r}_p v^s_p \right) \right\}$$

other uv terms = 0

$$= \int \frac{d^3p}{(2\pi)^3} \frac{2E_p}{2E_p} \vec{p} \left(a^{r\dagger}_p a^r_p - b^r_p b^{r\dagger}_p \right)$$

$$= \int \frac{d^3p}{(2\pi)^3} \vec{p} \left(a^{r\dagger}_p a^r_p - b^r_p b^{r\dagger}_p \right)$$

Finally ... $\{ b^r_p, b^{r\dagger}_p \} = (2\pi)^3 \delta^{rr} (\vec{p}-\vec{p})$ ignore...

$$\Rightarrow \quad -b^r_p b^{r\dagger}_p = b^{r\dagger}_p b^r_p - (2\pi)^3 \delta^{rr} \delta(\vec{p}-\vec{p})$$

momentum op

$$\Rightarrow \quad \vec{P} = \int d^3x \, \Psi^\dagger(x) (-i\nabla) \Psi(x)$$

$$\boxed{\vec{P} = \int \frac{d^3p}{(2\pi)^3} \vec{p} \left(a^{r\dagger}_p a^r_p + b^{r\dagger}_p b^r_p \right)}$$

✓

More problems

(1) Let $\mathcal{U}$ be the following unitary op:

$$\mathcal{U} = \exp\left\{ \frac{-i\pi}{2} \sum_{r=1}^{2} \int \frac{d\vec{p}^{\,3}}{(2\pi)^3} \left(a_{\vec{p}}^{r\,\dagger} - b_{\vec{p}}^{r\,\dagger}\right)\left(a_{\vec{p}}^{r} - b_{\vec{p}}^{r}\right) \right\}$$

investigate the effect of $\mathcal{U}$ on $a_{\vec{p}}^{r}$ & $b_{\vec{p}}^{r}$. I.e. compute

$$\mathcal{U}^\dagger a_{\vec{p}}^{r}\, \mathcal{U} \quad \& \quad \mathcal{U}^\dagger b_{\vec{p}}^{r}\, \mathcal{U} .$$

What type of transform does $\mathcal{U}$ produce?

Well...

$$\mathcal{U}^\dagger a_{\vec{p}}^{r}\, \mathcal{U} = \exp\Bigg\{ \underbrace{+\frac{i\pi}{2}\left(\sum_{r=1}^{2} \int \frac{d\vec{p}^{\,3}}{(2\pi)^3} \left(a_{\vec{p}}^{r\,\dagger} - b_{\vec{p}}^{r\,\dagger}\right)\left(a_{\vec{p}}^{r} - b_{\vec{p}}^{r}\right)\right)^\dagger}_{X^\dagger} \Bigg\}$$

$$\cdot\, a_{\vec{p}}^{r} \,\cdot\, \exp\Bigg\{ \underbrace{\frac{i\pi}{2} \sum_{r=1}^{2} \int \frac{d\vec{p}^{\,3}}{(2\pi)^3} \left(a_{\vec{p}}^{r\,\dagger} - b_{\vec{p}}^{r\,\dagger}\right)\left(a_{\vec{p}}^{r} - b_{\vec{p}}^{r}\right)}_{X} \Bigg\}$$

$\Rightarrow$ no good way to do this except for power series,

Recall that $e^{x} = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$, $e^{-x} = \sum_{n=0}^{\infty} \frac{1}{n!} (-x)^n$

So

$$\mathcal{U}^\dagger a_{\vec{p}}^{r}\, \mathcal{U} = \left(\sum_{n=0}^{\infty} \frac{(X^\dagger)^n}{n!}\right) a_{\vec{p}}^{r} \left(\sum_{m=0}^{\infty} \frac{(X)^m}{m!}\right) = \sum_{n,m=0}^{\infty} \frac{1}{n!\,m!} (X^\dagger)^n a_{\vec{p}}^{r} X^m$$

but notice that $\mathcal{U}$ unitary iff X hermitian.

$$\rightarrow X^\dagger = X \rightarrow \mathcal{U}^\dagger a_{\vec{p}}^{r}\, \mathcal{U} = \sum_{n,m=0}^{\infty} \frac{1}{n!\,m!} X^n a_{\vec{p}}^{r} X^m .$$

Useful theorem:

$$e^X Y e^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots$$

& Identity relating comm & anti-comm:

$$[AB, C] = ABC - CAB$$

$$= ABC + ACB - ACB - CAB$$

$$= A\{B,C\} - \{A,C\}B.$$

We ~~get~~ will need to compute $\cancel{X}$, a_p^s

$$U^\dagger a_{\vec q}^{r} U = a_{\vec q}^{r} + [X, a_{\vec q}^{r}] + \frac{1}{2!}[X,[X,a_{\vec q}^{r}]] + \cdots$$

$\rightarrow$ need to compute

$$[X, a_{\vec q}^{r}] = X a_{\vec q}^{r} - a_{\vec q}^{r} X = \cdots = ?$$

$$= \left(+ \frac{i\pi}{2} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \left\{ (a_{\vec p}^{s\dagger} - b_{\vec p}^{s\dagger})(a_{\vec p}^{s} - b_{\vec p}^{s}) a_{\vec q}^{r} - \left\{ a_{\vec q}^{r} (a_{\vec p}^{s\dagger} - b_{\vec p}^{s\dagger})(a_{\vec p}^{s} - b_{\vec p}^{s}) \right\} \right\} \right)$$

$$= \frac{i\pi}{2} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \Big[(a_{\vec p}^{s\dagger} - b_{\vec p}^{s\dagger}) \underbrace{\{ a_{\vec p}^{s} - b_{\vec p}^{s}, a_{\vec q}^{r} \}}_{\to 0} - \{ a_{\vec p}^{s\dagger} - b_{\vec p}^{s\dagger}, a_{\vec q}^{r} \} (a_{\vec p}^{s} - b_{\vec p}^{s}) \Big]$$

$$= \frac{i\pi}{2} \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \left[-(2\pi)^3 \delta^{rs} \delta^{(3)}(\vec p - \vec q)(a_{\vec p}^{s} - b_{\vec p}^{s}) \right]$$

$$= -\frac{i\pi}{2} (a_{\vec q}^{r} - b_{\vec q}^{r})$$

Next term,

$$[X,[X,a_q^r]] = [X, \tfrac{-i\pi}{2}(a_q^r - b_q^r)]$$

$$= \tfrac{-i\pi}{2}[X, a_q^r] + \tfrac{i\pi}{2}[X, b_q^r] = \ldots$$

$$= 2\left(\tfrac{i\pi}{2}\right)^2 (a_q^r - b_q^r)$$

So... $e^X a_p^r e^{-X} = a_q^r - \frac{i\pi}{2}(a_q^r - b_q^r) + 2\left(\frac{i\pi}{2}\right)^2 (a_q^r - b_q^r)$

what's next? $+ \ldots ?$

each step $\rightarrow$ $+2\left(\frac{i\pi}{2}\right)$ & alt $(-)$ sign.

$$\rightarrow \quad U^\dagger a_p^r U = a_p^r - \frac{i\pi}{2}(a_p^r - b_p^r) + \frac{1}{2!}2\left(\frac{i\pi}{2}\right)^2 (a_p^r - b_p^r),$$

$$- \frac{1}{3!}4\left(\frac{i\pi}{2}\right)^3 (a_p^r - b_p^r) + \frac{1}{4!}\left(\frac{i\pi}{2}\right)^4 6(\ldots) + \ldots$$

$$= a_p^r \left\{1 - \frac{i\pi}{2} + \frac{1}{2!}2\left(\frac{i\pi}{2}\right)^2 - \frac{1}{3!}4\left(\frac{i\pi}{2}\right)^3 + \frac{6}{4}\left(\frac{i\pi}{2}\right)^4 + \ldots\right\}$$

check this!

$$+ b_p^r \left\{\frac{i\pi}{2} - \frac{1}{2!}2\left(\frac{i\pi}{2}\right)^2 + \frac{1}{3!}4\left(\frac{i\pi}{2}\right)^3 - \frac{6}{4}\left(\frac{i\pi}{2}\right)^4 + \ldots\right\}$$

$$= a_p^r \left\{1 - \tfrac{1}{2}\cdot 2\right\} + b_p^r \cdot \left\{\tfrac{1}{2}\cdot 2\right\}$$

$$= b_p^r \quad \rightarrow \quad \boxed{U^\dagger a_p^r U = b_p^r}$$

$\rightarrow$ <u>U corresponds to charge conjugation!</u>

2) using Dirac annihilation & creation ops, construct P for which

$$P^\dagger a^r_{\vec p} P = a^r_{-\vec p} \quad \& \quad P^\dagger b^r_{\vec p} P = b^r_{-\vec p}.$$

Last time, we find that

$$[X, a^r_{\vec p}] = -\left(\frac{i\pi}{2}\right)\left(a^r_{\vec p} - b^r_{\vec p}\right) \longrightarrow \text{target}$$

$\rightarrow$ we want X for which

$$\begin{cases} [X, a^r_{\vec p}] = -\left(\frac{i\pi}{2}\right)\left(a^r_{\vec p} - a^r_{-\vec p}\right). \\ [X, b^r_{\vec p}] = -\left(\frac{i\pi}{2}\right)\left(b^r_{\vec p} + b^r_{-\vec p}\right). \end{cases}$$

$\rightarrow$ Let

$$P = \exp\left\{ -\frac{i\pi}{2}\sum_{r=1}^{2}\int \frac{d^3p}{(2\pi)^3}\left\{ a^{r\dagger}_{\vec p}\left(a^r_{\vec p} - a^r_{-\vec p}\right) + b^{r\dagger}_{\vec p}\left(b^r_{\vec p} + b^r_{-\vec p}\right)\right\}\right\}$$

$\uparrow$

check this, like last time

$\rightarrow$ should work! :)

Obv a^r_p only int. w/ 1st term $a^\dagger(\,)a = 0$

$aa^\dagger(\,) = \delta(\sim)$ ✓

Same with b^r_p ✓.

Interacting Fields & Feynman Diagrams

To get better description of the real world, need to include interactions in the theory.

To preserve causality, new terms may involve only products of fields at the same spacetime point!

↳ $\phi^4(x)$ ✓, but not $\phi(x)\,\phi(y)$.

$$\rightarrow \quad \mathcal{H}_{int} = \int d^3x \; \mathcal{H}_{int}[\phi(x)] = -\int d^3x \; \mathcal{L}_{int}[\phi(x)]$$

→ insist that $\mathcal{H}_{int}$ is a fn only of the fields, not of their derivatives.

→ Common ex in particle physics & CMT:

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4}$$

<u>note</u>

$\pi(x)$ is still $\partial_0\phi_0(x)$ since there are not any interaction terms involving $\partial_0\phi$

λ: dimensionless "coupling constant".

→ In general, adding interactions produces divergences

However, no matter what the true physics looks like at high momenta or short distances, the low momentum / long distance physics is well-approximated by an "effective" FT.

with "renormalizable" interactions.

→ those interactions where coupling constant are has dimensions $\boxed{d > 0}$

$[\text{Mass}]^d$ where $d > 0$.

Ex $\quad -\frac{1}{2} m^2 \phi^2 \quad \& \quad \frac{\lambda}{4!} \phi^4$ same dim

→ $\lambda \sim [\text{mass}]^0$ → Renormalizable.

But $\quad -\frac{\lambda_6}{6!} \phi^6$ → not renormalizable.

since $\lambda \sim [\text{mass}]^{-2}$.

$\boxed{\text{Perturbation Expansion}}$

Let $H = H_0 + H_{int} \sim\!\!\to = \int d^3x \, \frac{\lambda}{4!} \phi^4(x)$

H_0 ↑ KG, free

→ we will generate power series in λ.

At any t_0, we can write

$$\phi(t_0, \vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left\{ a_{\vec{p}} \, e^{ip\cdot x} + a^\dagger_{\vec{p}} \, e^{-ip\cdot x} \right\}$$

$$\equiv \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left\{ \underset{\substack{\parallel\!\mid \\ a_{\vec{p}}}}{\tilde{a}_{\vec{p}}} \, e^{i\vec{p}\cdot\vec{x}} + \tilde{a}^\dagger_{\vec{p}} \, e^{-i\vec{p}\cdot\vec{x}} \right\}$$

where we've let $a_{\vec{p}}$ absorb $e^{\pm iEt_0}$

The Heisenberg field is then given by:

$$\rightarrow \boxed{\phi(t,\vec{x}) = e^{iH(t-t_0)}\,\phi(t_0,x)\,e^{-iH(t-t_0)}}$$

If there's no interaction then we have

$$\rightarrow \phi_{free}(t,\vec{x}) = e^{iH_0(t-t_0)}\,\phi(t_0,x)\,e^{-iH_0(t-t_0)}$$

$$= \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left\{a_p e^{-ip\cdot x} + a_p^\dagger e^{+ip\cdot x}\right\}\Bigg|_{\substack{x^0 = t-t_0\\ p^0 = E_{\vec{p}}}}$$

Define this to be $\phi_I(t,\vec{x})$, the interaction picture field

the interaction picture field $\equiv$ Heisenberg field when $\lambda = 0$.

Now, look at Heisenberg field...

$$\phi(t,\vec{x}) = e^{iH(t-t_0)}\,\phi(t_0,\vec{x})\,e^{-iH(t-t_0)}$$

$$= \underbrace{e^{iH(t-t_0)}\,e^{-iH_0(t-t_0)}}\,\phi_I(t,\vec{x})\,e^{+iH_0(t-t_0)}\,e^{-iH(t-t_0)}$$

$$= \mathcal{U}^\dagger(t,t_0)\,\phi_I(t,\vec{x})\,\mathcal{U}(t,t_0)$$

time evolution operator

→ Evolve the operator on $\phi_I(t,\vec{x})$

OR

Evolve the states by $\mathcal{U}(t,t_0) \rightsquigarrow \mathcal{U}|\phi\rangle$ ...

→ now, we want to express $\mathcal{U}(t,t_0)$ entirely in ϕ_I

To do this, note that $\mathcal{U}(t,t_0)$ solves SE:

$$i\frac{\partial}{\partial t}\mathcal{U}(t,t_0) = e^{iH_0(t-t_0)}(H-H_0)e^{-iH(t-t_0)}$$

$$= e^{iH_0(t-t_0)} H_{int}\, e^{-iH(t-t_0)}$$

$$= \underbrace{e^{iH_0(t-t_0)} H_{int}\, e^{-iH_0(t-t_0)}}\, e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$$

$$= H_I(t)\,\mathcal{U}(t,t_0)$$

where

$$H_I(t) = e^{iH_0(t-t_0)} \mathcal{H}_{int}\, e^{-iH_0(t-t_0)}$$

$$= \int d^3x \frac{\lambda}{4!}\phi_I^4 \quad \boxed{?}$$

$$= \int d^3x\, e^{iH_0(t-t_0)} \frac{\lambda}{4!}\phi^4 e^{-iH_0(t-t_0)}$$

$$= \int d^3x \frac{\lambda}{4!}\phi_I^4 \quad \checkmark$$

→ this is the Hamiltonian in the interaction picture.

So since $\mathcal{U}$ solves the SE:

$$i\partial_t \mathcal{U}(t,t_0) = H_I(t)\,\mathcal{U}(t,t_0),$$

$\mathcal{U}$ must look like

$$\boxed{\mathcal{U}(t,t_0) \sim \exp\{-iH_I t\}}$$

More carefully, we actually have that

$$\boxed{\mathcal{U}(t,t_0) = T\exp\left\{-i\int_{t_0}^{t} dt'\, H_I(t')\right\}}$$ Dyson's formula

time-ordering symbol.

Why T? Why ordering? $\Rightarrow$ B/c $H(t_1) \not\leftrightarrow H(t_2)$ when $t_1 \neq t_2$.

"T" puts the <u>latest</u> operators on the left.

hence $i\partial_t \mathcal{U}(t,t_0) = \underline{H_I(t)}\,\mathcal{U}(t,t_0)$.

As a pwr series in λ:

$$\mathcal{U}(t,t_0) = 1 + (-i)\int_{t_0}^{t} dt'\, H_I(t') + \frac{(-i)^2}{2!}\int_{t_0}^{t}\int_{t_0}^{t} dt_1 dt_2\, T\{H_1(t_1)H_2(t_2)\}$$
$$+ \frac{(-i)^3}{3!}\int_{t_0}^{t}\int_{t_0}^{t}\int_{t_0}^{t} dt_1\, dt_2\, dt_3\, T\{H_1(t_1)\,H_2(t_2)\,H_3(t_3)\} + \ldots$$

Note "the time-ordering of the exponential is just ~~the time-ordering~~ the Taylor series of the terms time-ordered..."

$\rightarrow$ Now, we want to generalize $\mathcal{U}(t,t_0)$ to $\mathcal{U}(t,t')$

reference time

This generalization is natural...

$$\boxed{\mathcal{U}(t,t') \equiv T\left\{\exp\left[-i\int_{t'}^{t} dt''\, \mathcal{H}_I(t'')\right]\right\} \quad (t \geq t')}$$

Then we see that b/c both t, t' are variables... we find:

$$\boxed{\begin{aligned} i\partial_t\, \mathcal{U}(t,t') &= \mathcal{H}_I(t)\,\mathcal{U}(t,t') \\ i\partial_{t'}\, \mathcal{U}(t,t') &= -\,\mathcal{U}(t,t')\,\mathcal{H}_I(t'). \end{aligned}}$$

and thus...

$$\boxed{\mathcal{U}(t,t') = e^{iH_0(t-t_0^*)}\, e^{-iH(t-t')}\, e^{-iH_0(t-t_0^*)}}$$

So $\mathcal{U}$ is unitary.

Further, for $t_1 \geq t_2 \geq t_3$,

$$\begin{aligned} \mathcal{U}(t_1,t_2)\,\mathcal{U}(t_2,t_3) &= \mathcal{U}(t_1,t_3) \\ \mathcal{U}(t_1,t_3)\left[\mathcal{U}(t_2,t_3)\right]^\dagger &= \mathcal{U}(t_1,t_2) \end{aligned}$$

Now, let $|0\rangle$ ~~be~~ be gnd state of $\mathcal{H}_0$

$|\Omega\rangle$ be gnd state of $\mathcal{H}$

$|n\rangle$ be ~~gnd~~ label all $|E_n\rangle$ of $\mathcal{H}$.

Then, $(E_0 = \langle\Omega|H|\Omega\rangle)$

$$e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_nT}|n\rangle\langle n|0\rangle$$

Assume $H_0|0\rangle = 0$ & consider $T\to\infty$ limit.

$\updownarrow$

$T\to\infty(1-i\varepsilon)$

Then e^{-iE_nT} dies slowest for $n=0$, and so...

$$\to\ e^{-iHT}|0\rangle \to e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle$$

assume $\langle\Omega|0\rangle\neq 0$

So

$$\boxed{|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0T}\langle\Omega|0\rangle\right)^{-1} e^{-iHT}|0\rangle}$$

Now, since T large, we can shift it by a small constant...

$$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left\{e^{-iE_0(T+t_0)}\langle\Omega|0\rangle\right\}^{-1} e^{-iH(T+t_0)}|0\rangle$$

$$= \lim_{T\to\infty(1-i\varepsilon)} \left\{e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right\}^{-1} \underbrace{e^{-iH(t_0-(-T))}}_{\mathcal{U}(t_0,-T)}|0\rangle$$

$$= \lim_{T\to\infty(1-i\varepsilon)} \left\{e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right\}^{-1} e^{-iH(t_0-(-T))}\underbrace{e^{-iH_0(-T-t_0)}|0\rangle}_{=|0\rangle \text{ since } H_0|0\rangle=0}$$

$$\to\ |\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\mathcal{U}(t_0,-T)|0\rangle}{e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle}$$

Similarly,

$$\boxed{\langle\Omega| = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|\,\mathcal{U}(T,t_0)}{e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle}}$$

So, putting these together gives a correlation function...

For $x^0 > y^0 > t_0$, we have

$$\to \langle\Omega|\phi(x)\phi(y)|\Omega\rangle$$

$$= \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|\,\mathcal{U}(T,t_0)}{e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle} \overbrace{\left(\mathcal{U}(x^0,t_0)\right)^\dagger \phi_I(x)\,\mathcal{U}(x^0,t_0)}^{\phi(x)}$$

$$\times \left(\mathcal{U}(y^0,t_0)\right)^\dagger \phi_I(y)\,\mathcal{U}(y^0,t_0) \times$$

$$\frac{\mathcal{U}(t_0,-T)}{e^{-iE_0(T+t_0)}\langle\Omega|0\rangle}$$

$$= \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|\,\mathcal{U}(T,x^0)\phi_I(x)\,\mathcal{U}(x^0,y^0)\phi_I(y)\,\mathcal{U}(y^0,-T)|0\rangle}{|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}}$$

awkward...

so divide the whole thing by $1 = \langle\Omega|\Omega\rangle$

$$1 = \langle\Omega|\Omega\rangle = \frac{\langle 0|\,\overbrace{\mathcal{U}(T,t_0)\,\mathcal{U}(t_0,-T)}^{\mathcal{U}(T,-T)}|0\rangle}{|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}}$$

to get (for $x^0 > y^0$)

$$\boxed{\langle\Omega|\phi(x)\phi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|\,\mathcal{U}(T,x^0)\phi_I(x)\,\mathcal{U}(x^0,y^0)\phi_I(y)\,\mathcal{U}(y^0,-T)|0\rangle}{\langle 0|\,\mathcal{U}(T,-T)|0\rangle}}$$

So, we have that, by replacing U's with Dyson's formula (w/ time-ordering) ...

$$\boxed{\langle \Omega | T\{\phi(x)\phi(y)\} | \Omega \rangle = \lim_{T \to \infty(1-i\epsilon)} \frac{\langle 0 | T\{\phi_I(x)\phi_I(y) \exp\{-i\int_{-T}^{T} dt\, H_I(t)\}\} | 0 \rangle}{\langle 0 | T \exp\{-i\int_{-T}^{T} dt\, H_I(t)\} | 0 \rangle}}$$

So, looks like the term

$$\exp\left\{ -i \int_{-T}^{T} dt\, H_I(t) \right\}$$ ~~can~~ ~~important~~ & can be found, so

Wick's theorem

→ So, we have reduced the problem of calc. correlation functions to evaluating

$$\boxed{\langle 0 | T\{ \phi_I(x_1)\phi_I(x_2) \cdots \phi_I(x_n) \} | 0 \rangle}$$

→ This is the vacuum exp-value of time-ordered products of finite number of field operators.

$n = 2 \Rightarrow$ get Feynman operator.

$n > 2 \rightarrow$ can use brute force, but there are also ways to simplify calculations.

Now, we study $\boxed{\langle 0 | T\{ \phi_I(x)\phi_I(y) \} | 0 \rangle}$

Recall that

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$$\phi_I(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left\{ a_{\vec p}\, e^{-ip\cdot x} + a^\dagger_{\vec p}\, e^{+ip\cdot x} \right\}$$

Call $\phi_I^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a_{\vec p}\, e^{-ip\cdot x}$

and $\phi_I^-(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} a^\dagger_{\vec p}\, e^{+ip\cdot x}$

which is ~~useful~~ b/c

$\phi_I^+(x)|0\rangle = 0$, $\langle 0|\phi_I^-(x) = 0$.

only has annihilation ops — only has creation ops.

→ For $x^0 > y^0$,

$$T\{\phi_I(x)\phi_I(y)\} \overset{x^0>y^0}{=} \phi_I^+(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^-(y) + \phi_I^+(x)\phi_I^-(y) + \phi_I^-(x)\phi_I^+(y)$$

$$= \phi_I^+(x)\phi_I^+(y) + \phi_I^-(y)\phi_I^+(x) + \phi_I^-(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^-(y) + [\phi_I^+(x), \phi_I^-(y)]$$

every of these terms has the form $a^\dagger_{\vec p} a^\dagger_{\vec q} a_{\vec k} a_{\vec l}$

i.e. creation ops always on the left.

→ "Normal order" → has vanishing vacuum exp value.

What can we say about the commutator?

↑ just a number, there's no creation / annihilation op. in it!

$$[\phi_I^+(x), \phi_I^-(y)] = \langle 0 | [\phi_I^+(x), \phi_I^-(y)] | 0 \rangle$$

$$= \langle 0 | \phi_I^+(x)\phi_I^-(y) | 0 \rangle = \langle 0 | \phi_I(x)\phi_I(y) | 0 \rangle .$$

With these, we can write

$$T\{\phi_I(x)\phi_I(y)\} = \phi_I^+(x)\phi_I^+(y) + \phi_I^-(x)\phi_I^+(y) + \phi_I^-(y)\phi_I^+(x) + \phi_I^-(x)\phi_I^-(y) + \langle 0 | \phi_I(x)\phi_I(y) | 0 \rangle$$

Now, <u>define</u> the "normal ordering symbol" N

s.t. N takes the string a, $a^\dagger$'s and rearranges them so that $a^\dagger$'s are on the left

Ex.

$$\begin{cases} N(a_{\vec p}^\dagger a_{\vec p}) = a_{\vec p}^\dagger a_{\vec p} \\ N(a_{\vec p} a_{\vec p}^\dagger) = a_{\vec p}^\dagger a_{\vec p} . \\ N(a_{\vec p} a_{\vec q} a_{\vec p}^\dagger) = a_{\vec p}^\dagger a_{\vec p} a_{\vec q} \end{cases}$$

ordering for $a_{\vec p}$, $a_{\vec q}$ doesn't matter since they commute.

→ <u>Note</u> N is not a well-defined mathematical operation

e.g. $N([a_{\vec p}, a_{\vec q}^\dagger]) \neq N((2\pi)^3 \delta^{(3)}(\vec p - \vec q))$

→ it is only a lexicographic convention.

Now, let us consider general x^0, y^0, then

$$T\{\phi_I(x)\phi_I(y)\} = N\{\phi_I(x)\,\phi_I(y)\} + \begin{cases} [\phi_I^+(x), \phi_I^-(y)] & \text{for } x^0 > y^0 \\ [\phi_I^+(y), \phi_I^-(x)] & \text{for } x^0 < y^0 \end{cases}$$

→ Let us define the <u>contraction of $\phi_I(x)$, $\phi_I(y)$</u> as

$$\overbrace{\phi_I(x)\phi_I(y)} = \begin{cases} [\phi_I^+(x), \phi_I^-(y)] & x^0 \geq y^0 \\ [\phi_I^+(y), \phi_I^-(x)] & x^0 < y^0 \end{cases}$$

then notice that, from our previous observation,

$$\overbrace{\phi_I(x)\phi_I(y)} = \begin{cases} \langle 0|\, \phi_I(x)\phi_I(y)\,|0\rangle & x^0 \geq y^0 \\ \langle 0|\, \phi_I(y)\phi_I(x)\,|0\rangle & y^0 > x^0. \end{cases}$$

So,

Feynman propagator

$$\overbrace{\phi_I(x)\phi_I(y)} = \langle 0|\, T\{\phi_I(x)\phi_I(y)\}\,|0\rangle$$

$$= D_F(x-y) \rightsquigarrow \text{Feynman propagator.}$$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

With these, we have that

$\longrightarrow$ leaving out the I's subscripts

$$T\{\phi(x)\phi(y)\} = N\{\phi(x)\phi(y) + \overline{\phi(x)\phi(y)}\}$$

$$\overline{\phi(x)\phi(y)} = \overline{\phi(y)\phi(x)}$$

$\rightarrow$ $T\{\phi(x)\phi(y)\} = N\{\phi(x)\phi(y)\} +$ "contraction".

In fact, the generalization of this is called

<u>Wick's Theorem</u>

$$\boxed{\begin{aligned} &T\{\phi(x_1)\phi(x_2)\cdots\phi(x_m)\} \\ &= N\{\phi(x_1)\phi(x_2)\cdots\phi(x_m) + \text{all possible contractions}\}\end{aligned}}$$

<u>Ex</u> $T\{\phi_1\phi_2\phi_3\phi_4\}$ $\quad(\phi_n = \phi(x_n))$

$$\begin{aligned} = N\{&\phi_1\phi_2\phi_3\phi_4 + \\ &+ \wick{\c1\phi_1\c1\phi_2\phi_3\phi_4} + \wick{\phi_1\c1\phi_2\c1\phi_3\phi_4} + \wick{\phi_1\phi_2\c1\phi_3\c1\phi_4} \\ &\wick{\c1\phi_1\phi_2\phi_3\c1\phi_4} + \wick{\c1\phi_1\phi_2\c1\phi_3\phi_4} + \wick{\phi_1\c1\phi_2\phi_3\c1\phi_4} + \wick{\c1\phi_1\c1\phi_2\c2\phi_3\c2\phi_4} \\ &+ \wick{\c1\phi_1\c2\phi_2\c1\phi_3\c[below]2\phi_4} + \wick{\c[below]1\phi_1\c2\phi_2\c2\phi_3\c[below]1\phi_4}\}\end{aligned}$$

what does $N\{\overline{\phi_1\phi_2}\phi_3\phi_4\}$ mean?

$$N\{\overline{\phi_1\phi_2}\phi_3\phi_4\} = \overline{\phi_1\phi_2}\,N\{\phi_3\phi_4\}$$

$$= D_F(x_1 - x_2)\,N\{\phi_3\phi_4\}.$$

Proof → prove by induction. $n=2$ is good (Feynman)

→ assume this holds for $n-1$.

Let $W(\phi_1 \ldots \phi_n) = N\{\phi_1 \phi_2 \ldots \phi_n + \text{all possible contr.}\}$

To prove $W(\phi_1 \ldots \phi_n) = T\{\phi_1 \phi_2 \ldots \phi_n\}$.

W/l/o/g: let $x_1^0 \geq x_2^0 \geq \ldots \geq x_n^0$.

Then $T\{\phi_1 \ldots \phi_n\} = \phi_1 T\{\phi_2 \ldots \phi_n\}$
$$= \phi_1 W(\phi_2 \ldots \phi_n)$$ since $n-1$ true.

So $T\{\phi_1 \ldots \phi_n\} = \underbrace{\phi_1^- W(\phi_2 \ldots \phi_n) + W(\phi_2 \ldots \phi_n)\phi_1^+}_{X}$
$$+ \underbrace{[\phi_1^+, W]}_{Y}$$

Let $X \equiv \phi_1^- W + W\phi_1^+$; $Y \equiv [\phi_1^+, W]$.

$X + Y$ are normal ordered: X contains all contractions in $W(\phi_1 \ldots \phi_n)$ which does not contract ϕ_1 w/ anything.

Y contains all contractions in $W(\phi_1 \ldots \phi_n)$ which contracts ϕ_1 with something.

So $T(\phi_1 \phi_2 \ldots \phi_n) = W(\phi_1 \ldots \phi_n)$.

(we won't worry too much abt this proof.

→ the main idea is the theorem itself).

$\square$

In any case, we have another way to explicitly write out the result of Wick's Thm:

$$T\{\phi_1\phi_2\cdots\phi_n\} = N\left\{\exp\left[\frac{1}{2}\sum_{\substack{i,j=1\\ i\neq j}}^{n} \phi_i\phi_j \frac{\partial}{\partial\phi_i}\frac{\partial}{\partial\phi_j}\right]\phi_1\cdots\phi_n\right\}$$

we'll see why later on.

$\boxed{\text{Feynman Diagrams}}$

Wick's Thm allows ~~us~~ to write

$$\langle 0|T\{\phi_1\cdots\phi_n\}|0\rangle$$

in terms of sums and products of Feynman propagators.

→ Now, we will develop ~~a~~ diagrammatic expressions.

Recall that

$$T\{\phi_1\phi_2\phi_3\phi_4\} = N\{\phi_1\phi_2\phi_3\phi_4 + \text{all possible contractions}\}$$

But the only contribution to

$\langle 0|T\{\phi_1\phi_2\phi_3\phi_4\}|0\rangle$ is when <u>all</u> the ϕ's are contracted.

This b/c whenever things are in normal order, the exp value vanishes. → $N(\phi_1\phi_2\phi_3\phi_4) = \phi_1\phi_2 N(\phi_3\phi_4)$

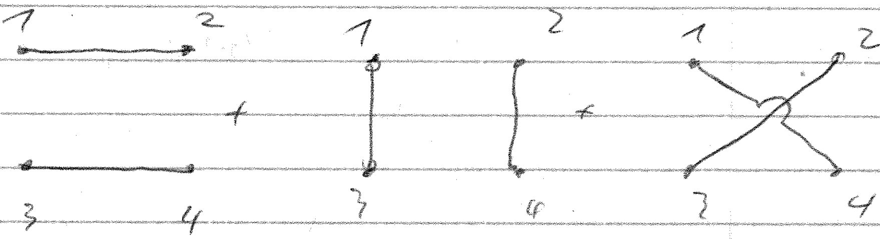
→ to "escape" from normal order, ϕ's have to be contracted

This means that

$$T\{\phi_1\phi_2\phi_3\phi_4\} = \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\phi_4$$

→ Can write this as Feynman diagrams...

$T\{\phi_1\phi_2\phi_3\phi_4\} =$



↳ Interpretation

Particles are created at 2 spacetime points, each propagates to one of the other points, then get annihilated.

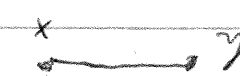
→ total amplitude of the process is the sum of the diagrams.

Well... what about something like...

$$\langle 0|\, T\left\{\phi(x)\,\phi(y)\exp\left\{-i\int_{-\infty}^{\infty} dt\, H_I(t)\right\}\right\}|0\rangle \,?$$

- Well... as a pwr series in λ, the lowest order term is

$$\langle 0|\, T\{\phi(x)\phi(y)\}|0\rangle = D_F(x-y) \,.$$



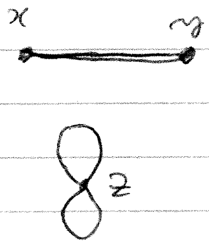
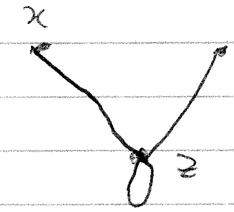
- 1st order $\langle 0|\, T\{\phi(x)\phi(y)\}(-i)\left(\int_{-\infty}^{\infty} dt\, H_I(t)\right)_{dt} |0\rangle$

$$= \langle 0|\, T\{\phi(x)\phi(y)\,(-i)\int d^4z\, \tfrac{\lambda}{4!}\phi^4(z)\}|0\rangle$$

$$= \cdots \frac{-i\lambda}{4!}\int d^4z\, \langle 0| T\{ \phi(x)\phi(y) \overbrace{\phi(z)\phi(z)\phi(z)\phi(z)}^{\phi^4(z)} \} |0\rangle$$

$$= \frac{-i\lambda}{4!}\int d^4z \left\{ \phi(x)\phi(y)\cdot\{\phi(z)\phi(z)\phi(z)\phi(z) + \phi_z\phi_z\phi_z\phi_z + \phi_z\phi_z\phi_z\phi_z \} + \phi(x)\phi(y)\phi(z)\phi(z)\phi(z)\phi(z) \right\}$$

$\hookrightarrow$ 12 terms that are identical

=  +  $\longrightarrow$ # propagators

γ γ

$\int d^4z\, D_F(x-z) D_F(z-y)\, D_F(z-z)$

$\uparrow$

12 of these.

$\int d^4z\, D_F(x-y)\, D_F(z-z)\, D_F(z-z)$

$\hookrightarrow$ each contraction D_F is a line.

each spacetime point is a dot.

$\rightarrow$ but need to distinguish "external" and "internal" points.

$\downarrow$ x, y $\downarrow$ z

Each internal point is associated w/ a factor of $-i\lambda\int d^4z$, with combinatorial factor...

How do we count these combinatorial factors?

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