

Classical analog of electromagnetically induced transparency

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(Received 3 May 2001; accepted 27 August 2001)

We present a classical analog of electromagnetically induced transparency (EIT). In a system of just two coupled harmonic oscillators subject to a harmonic driving force, we reproduce the phenomenology observed in EIT. We also describe a simple experiment with two linearly coupled *RLC* circuits which can be incorporated into an undergraduate laboratory. © 2002 American

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[DOI: 10.1119/1.1412644]

I. INTRODUCTION

Imagine a medium that strongly absorbs a light beam of a certain frequency. Add a second light source, with a frequency that would also be absorbed by the medium, and the medium becomes transparent to the first light beam. This curious phenomenon is called electromagnetically induced transparency (EIT).¹ It usually takes place in vapors of three-level atoms. The light sources are lasers that drive two different transitions with one common level. Most authors attribute the effect to quantum interference in the medium, involving two indistinguishable paths leading to the common state. In addition to the phenomenon of electromagnetically induced transparency, the dispersive properties of the medium are significantly modified as has been recently demonstrated by the impressive reduction^{2–4} of the group velocity of a light pulse to only 17 m/s and the “freezing” of a light pulse in an atomic medium.^{5,6}

In this paper we develop a classical analog of electromagnetically induced transparency and discuss a simple experiment that can be carried out in an undergraduate physics laboratory. The stimulated resonance Raman effect has already been modeled classically in a system of three coupled pendula by Hemmer and Prentiss.⁷ Even though many aspects of electromagnetically induced transparency were already present in the stimulated Raman effect, as can be seen in Ref. 7, EIT had not been observed at that time,¹ and the dispersive features were not considered. Our model involves only two oscillators with linear coupling. The experiment is performed with *RLC* circuits. The interest of such an experiment and the purpose of this paper is to enable undergraduate students to develop their physical intuition for the coherent phenomena that occur in atomic systems.

II. THEORETICAL MODEL

We will focus our attention on the simulation of EIT in media composed of three-level atoms in the so-called Λ configuration interacting with two laser fields (see Fig. 1). The quantum states $|1\rangle$ and $|2\rangle$ represent the two ground states of the atom, and the state $|0\rangle$ is the excited atomic level.

The laser field that drives the atomic transition between the states $|1\rangle$ and $|0\rangle$ is the “pumping (or pump) laser,” and

the laser that drives the transition between the states $|2\rangle$ and $|0\rangle$ is the “probe laser.” A typical experiment consists of scanning the frequency of the probe laser and measuring its transmitted intensity. In the absence of the pump laser, one observes a standard absorption resonance profile. Under certain conditions, the addition of the pump laser prevents absorption in a narrow portion of the resonance profile, and the transmitted intensity as a function of the probe frequency has a narrow peak of induced transparency.

The effect depends strongly on the pump beam intensity. Typically the pump laser has to be intense so that the Rabi frequency Ω_1 associated with the transition from state $|1\rangle$ to $|0\rangle$ is larger than all damping rates present (associated with spontaneous emission from the excited state and other relaxation processes). One of the effects of the pump beam is to induce an ac-Stark splitting of the excited atomic state. The probe beam will therefore couple state $|2\rangle$ to two states instead of one. If the splitting (which varies linearly with the Rabi frequency Ω_1) is smaller than the excited state width, the two levels are indistinguishable, and one expects quantum interference in the probe absorption spectrum. As the Rabi frequency Ω_1 increases, the splitting becomes more pronounced and indistinguishability is lost. The absorption spectrum becomes a doublet called the Autler–Townes doublet.⁸ We will keep this picture in mind as we discuss a classical system with the same features.

EIT-like phenomena with masses and springs

We will model the atom as a simple harmonic oscillator, consisting of particle 1 with mass m_1 attached to two springs with spring constants k_1 and K (see Fig. 2). The spring with constant k_1 is attached to a wall, while the other spring is attached to a second particle of mass m_2 and initially kept immobile at a fixed position. Particle 1 is also subject to a harmonic force $\mathcal{F}_s = Fe^{-i(\omega_s t + \phi_s)}$. If we analyze the power transferred from the harmonic source to particle 1 as a function of frequency ω_s , we will observe the standard resonance absorption profile discussed above [peaked at frequency $\omega_1^2 = (k_1 + K)/m$]. If we now allow particle 2 to move, subject only to the forces from the spring of constant K and a third spring of constant k_2 attached to a wall (see

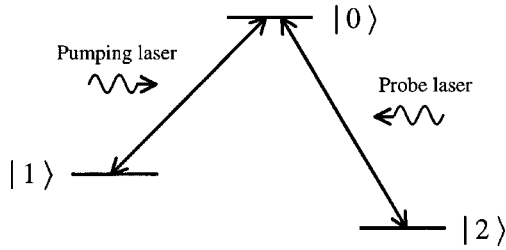


Fig. 1. The energy diagram of a three-level Λ -type atom interacting with two laser beams, coupling the two ground states to a common excited atomic state. Note that, if we connect each ground state to the excited state by a straight line, we draw the Greek letter Λ , hence the name.

Fig. 2), the absorption profile is modified. As we shall see, we can observe a profile whose features are similar to electromagnetically induced transparency evolving to an Autler–Townes-like doublet as a function of K . As a matter of fact, this doublet is the normal-mode splitting. For simplicity, we will consider the case $k_1 = k_2 = k$ and $m_1 = m_2 = m$.

The physical analogy between our model and the three-level atom coupled to two light fields is as follows. The atom is modeled as oscillator 1 (particle 1) with resonance frequency ω_1 . Because we have chosen $k_1 = k_2 = k$ and $m_1 = m_2 = m$, we have the analog of a degenerate Λ system (ground states $|1\rangle$ and $|2\rangle$ have the same energy). The pump field is simulated by the coupling of oscillator 1 to the second oscillator via the spring of constant K (recall that the quantized description of the field is in terms of harmonic oscillators). The probe field is then modeled by the harmonic force acting on particle 1.

To provide a quantitative description of the system, we write the equations of motion of particles 1 and 2 in terms of the displacements x_1 and x_2 from their respective equilibrium positions:

$$\ddot{x}_1(t) + \gamma_1 \dot{x}_1(t) + \omega^2 x_1(t) - \Omega_r^2 x_2(t) = \frac{F}{m} e^{-i\omega_s t}, \quad (1a)$$

$$\ddot{x}_2(t) + \gamma_2 \dot{x}_2(t) + \omega^2 x_2(t) - \Omega_r^2 x_1(t) = 0. \quad (1b)$$

We have set $\phi_s = 0$ for the probe force without loss of generality. We also let $\Omega_r^2 = K/m$, the frequency associated with the coherent coupling between the pumping oscillator and the oscillator modeling the atom; γ_1 is the friction constant associated with the energy dissipation acting on particle 1 (which simulates the spontaneous emission from the atomic excited state); and γ_2 is the energy dissipation rate of the pumping transition.

Because we are interested in the power absorbed by particle 1 from the probe force, we seek a solution for $x_1(t)$. Let us suppose that $x_1(t)$ has the form

$$x_1(t) = N e^{-i\omega_s t}, \quad (2)$$

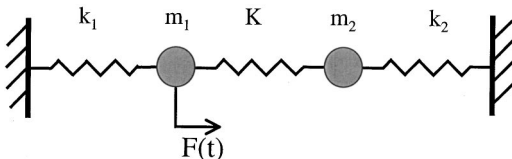


Fig. 2. The mechanical model used to simulate EIT.

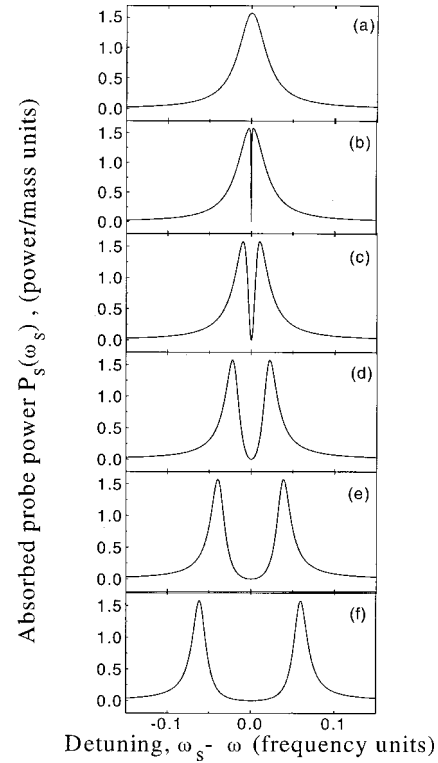


Fig. 3. The frequency dependence of the absorption of the probe energy by particle 1. The values of Ω_r are (a) 0.0, (b) 0.1, (c) 0.2, (d) 0.3, (e) 0.4, and (f) 0.5 in frequency units.

where N is a constant. After assuming a similar expression for $x_2(t)$ and substituting in Eq. (1a), we find

$$x_1(t) = \frac{(\omega^2 - \omega_s^2 - i\gamma_2\omega_s) F e^{-i\omega_s t}}{m[(\omega^2 - \omega_s^2 - i\gamma_1\omega_s)(\omega^2 - \omega_s^2 - i\gamma_2\omega_s) - \Omega_r^4]}. \quad (3)$$

Then if we compute the mechanical power $P(t)$ absorbed by particle 1 from the probe force \mathcal{F}_s ,

$$P(t) = F e^{-i\omega_s t} \dot{x}_1(t), \quad (4)$$

we find for the power absorbed during one period of oscillation of the probe force

$$P_s(\omega_s) = - \frac{2\pi i F^2 \omega_s (\omega^2 - \omega_s^2 - i\gamma_2\omega_s)}{m[(\omega^2 - \omega_s^2 - i\gamma_1\omega_s)(\omega^2 - \omega_s^2 - i\gamma_2\omega_s) - \Omega_r^4]}. \quad (5)$$

In Fig. 3 we show the real part of $P_s(\omega_s)$ for six values of the coupling frequency Ω_r expressed in frequency units. These curves were obtained using the values $\gamma_1 = 4.0 \times 10^{-2}$, $\gamma_2 = 1.0 \times 10^{-7}$, and $\omega_0 = \sqrt{k/m} = 2.0$, all expressed in the same frequency units. The amplitude F/m was equal to 0.1 force per mass units.

For $\Omega_r = 0$, we have a typical absorption profile, with a maximum probe power absorption for $\delta = 0$, where $\delta = \omega_s - \omega$ is the detuning between the probe and the oscillator frequencies. If we increase Ω_r to 0.1, we observe the appearance of a narrow dip in the absorption profile of $P_s(\omega_s)$. The zero absorption at the center frequency of the profile is a result of destructive interference between the normal modes of oscillation of the system.⁹ A further increase of the cou-

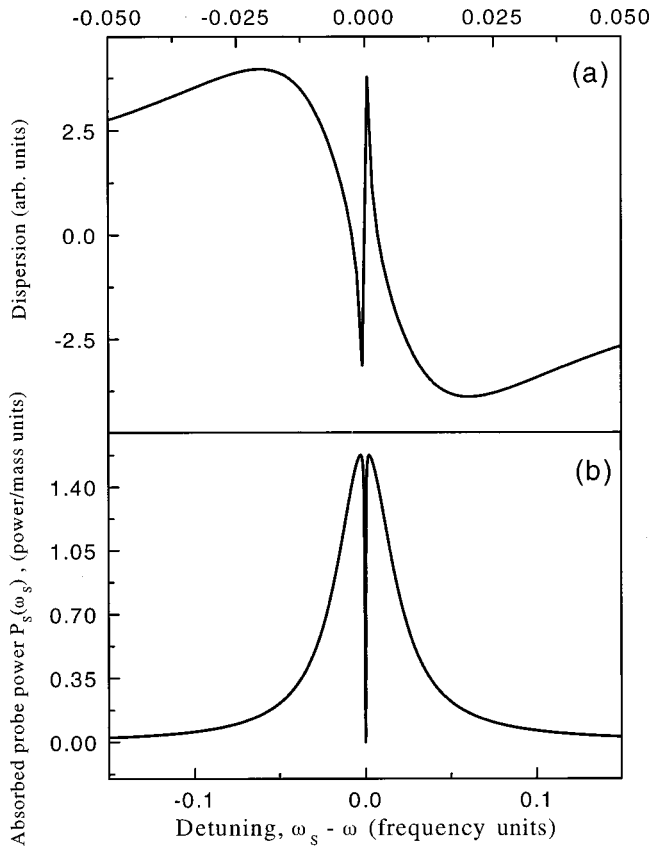


Fig. 4. The dispersive (a) and absorptive (b) responses of the probe power transferred to particle 1 for $\Omega_r=0.1$ in frequency units.

pling frequency leads to the appearance of two peaks in the probe power absorption profile, which are clearly separated for $\Omega_r=0.5$. In atomic systems this effect is called the Autler-Townes doublet.

It is important to mention that the dissipation rate γ_2 , associated with the energy loss of the pumping oscillator, must be much smaller than γ_1 in order to drive coherently particle 1's oscillation. In other words, there should be no significant increase of dissipation in the system by adding the pumping oscillator. For the case of EIT,¹⁰ the condition analogous to $\gamma_2 \ll \gamma_1$ is that the transfer rate of population from the atomic ground state $|1\rangle$ to $|2\rangle$ be negligible (see Fig. 1). In both situations, the violation of this condition obscures the observation of the induced transparency.

Another important result reproduced by this mechanical model is the dispersive behavior of the mass oscillator used to simulate the atom. The dispersive response is contained in the real part of the frequency dependence of the amplitude of oscillation $x_1(t)$ of particle 1, given by Eq. (3). In Fig. 4 we plot this quantity for $\Omega_r=0.1$. This value of Ω_r corresponds to the situation when the induced transparency is more pronounced and, as we can see from Fig. 4, the dispersion observed in the frequency interval where the induced transparency occurs is normal and has a very steep slope. This result coincides with that reported in an experimental realization of electromagnetically induced transparency.¹¹ It is also important to note that this very steep normal dispersion is responsible for the recently observed slow propagation of light,²⁻⁴ and the "freezing" of a light pulse in an atomic medium.^{5,6} It

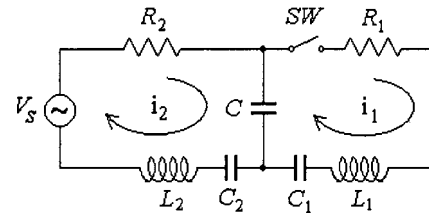


Fig. 5. The electrical circuit employed to investigate the induced transparency.

should therefore be possible to observe such propagation effects by considering absorption in a medium consisting of a series of mechanical "atom analogs."

Our theoretical model is not the only classical model that simulates EIT-like phenomena. As mentioned previously, in our model the pump field is replaced by a harmonic oscillator, simulating a quantum-mechanical description of the field. In most theoretical descriptions of EIT in atomic media, the pump and probe fields are classical. The mechanical analog to this description would then involve only one oscillator (one particle of mass m). In this latter model, it becomes apparent that EIT arises directly from destructive interference between the oscillatory forces driving the particle's movement. To keep the discussion simple, we have chosen not to present this description here. Furthermore, it is not related to the experimental results presented below.

III. EIT-LIKE PHENOMENA IN COUPLED *RLC* CIRCUITS: A SIMPLE UNDERGRADUATE LABORATORY EXPERIMENT

An experiment to observe the predictions of Sec. II, although possible, would not be straightforward. Instead, we use the well-known analogy between mechanical oscillators and electric circuits to perform a simple experiment. The electrical analog to the system of Fig. 2 is the circuit shown in Fig. 5, where the circuit mesh consisting of the inductor L_1 and the capacitors C_1 and C simulates the pumping oscillator, and the resistor R_1 determines the losses associated with that oscillator. The atom is modeled using the resonant circuit formed by the inductor L_2 and the capacitors C_2 and C ; the resistor R_2 represents the spontaneous radiative decay from the excited level. The capacitor C , belonging to both circuit meshes, models the coupling between the atom and the pumping field, and determines the Rabi frequency associated with the pumping transition. In this case, the probe field is simulated by the frequency-tunable voltage source V_s .

The circuit mesh used to model the atom has only one resonance frequency representing the energy of the atomic excited level. That is, the probability of excitation of this circuit will be maximal when the applied harmonic force is on resonance. Because in this case we have two possible paths to accomplish this excitation, we are dealing with the analog of a three-level atom in the Λ configuration. Namely, the oscillator corresponding to the atom analog can be excited directly by the applied voltage V_s or by the coupling to the pumping oscillator.

Here again the induced transparency is investigated by analyzing the frequency dependence of the power $P_2(\omega_s)$ transferred from the voltage source V_s to the resonant circuit $R_2 L_2 C_e$,

Table I. Correspondences between the mechanical and electrical parameters.

Mechanical model	Electrical model
γ_2 (γ_2)	R_1/L_1 (R_2/L_2)
m_2 (m_2)	L_1 (L_2)
k_2 (k_2)	$1/C_1$ ($1/C_2$)
K	$1/C$
x_2 (x_2)	q_1 (q_2)
$(F/m)e^{-i\omega_s t}$	$\mathcal{V}_S(t)/L_2$

$$P_2(\omega_s) = \Re\{\mathcal{V}_S \mathcal{I}_2^*\}, \quad (6)$$

where \mathcal{V}_S and \mathcal{I}_2 are, respectively, the complex representations of V_S and $i_2(t)$, and the equivalent capacitor C_{e2} is the series combination of C and C_2 :

$$C_{e2} = \frac{CC_2}{C + C_2}. \quad (7)$$

If we set $L_1 = L_2 = L$ ($m_1 = m_2 = m$ in the mechanical model) and write the equations for the currents $i_1(t) = \dot{q}_1(t)$ and $i_2(t) = \dot{q}_2(t)$ shown in Fig. 5, we find the following system of coupled differential equations for the charges $q_1(t)$ and $q_2(t)$:

$$\ddot{q}_1(t) + \gamma_1 \dot{q}_1(t) + \omega_1^2 q_1(t) - \Omega_r^2 q_2(t) = 0, \quad (8a)$$

$$\ddot{q}_2(t) + \gamma_2 \dot{q}_2(t) + \omega_2^2 q_2(t) - \Omega_r^2 q_1(t) = \frac{V_S(t)}{L_2}. \quad (8b)$$

Here $\gamma_i = R_i/L_i$ ($i = 1, 2$), $\omega_i^2 = 1/(L_i C_{e1})$, and $\Omega_r^2 = 1/(L_2 C)$. (C_{e1} is the equivalent capacitor given by the series combination of C and C_1). These equations coincide with Eq. (1) using the correspondences shown in Table I and $\omega_1 = \omega_2$. Therefore, both models describe the same physics.

Once the current $i_2(t)$ (or \mathcal{I}_2) is known, the power $P_2(\omega_s)$ as a function of the frequency ω_s of the applied voltage, when the switch SW in Fig. 5 is closed, is given by

$$P_2(\omega_s) = \frac{p_1(\omega_s)}{p_1(\omega_s)^2 + p_2(\omega_s)^2} |A_S|^2, \quad (9)$$

where

$$p_1(\omega_s) = R_2 + \frac{R_1/(\omega_s C)^2}{R_1^2 + (\omega_s L_1 - 1/(\omega_s C_{e1}))^2}, \quad (10)$$

$$p_2(\omega_s) = (\omega_s L_2 - 1/(\omega_s C_{e2})) - \frac{(1/(\omega_s C)^2)(\omega_s L_1 - 1/(\omega_s C_{e1}))}{R_1^2 + (\omega_s L_1 - 1/(\omega_s C_{e1}))^2}, \quad (11)$$

and A_S is the amplitude of the applied voltage. On the other hand, when the switch is open, we have

$$P_2(\omega_s) = \frac{R_2}{R_2^2 + (\omega_s L_2 - 1/(\omega_s C_{e2}))^2} |A_S|^2. \quad (12)$$

There are many ways of measuring the power $P_2(\omega_s)$. We have chosen to measure the current flowing through the inductor L_2 , which has the same frequency dependence as $P_2(\omega_s)$. We actually measure the voltage drop across the inductor L_2 and integrate it to find a voltage proportional to

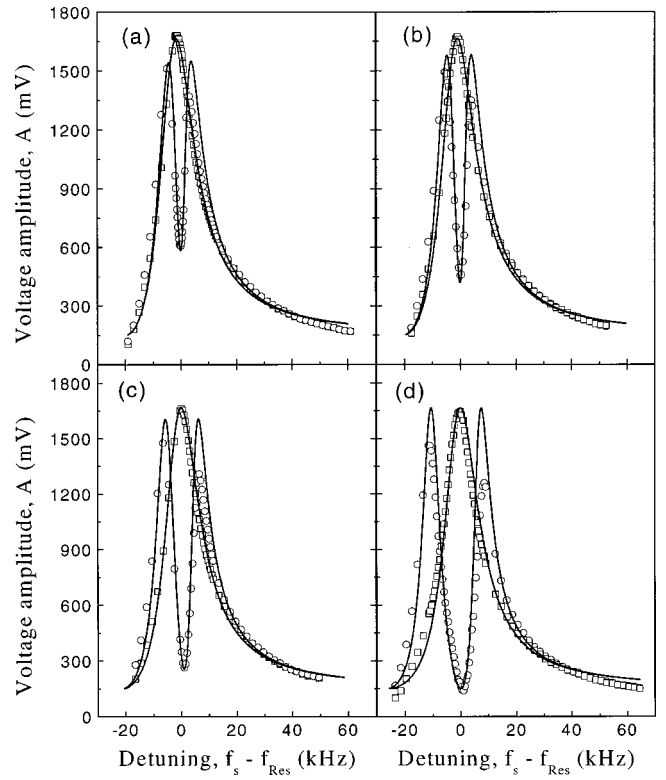


Fig. 6. The power transferred to the circuit $R_2 L_2 C_{e2}$ as a function of the frequency ω_s for different values of the coupling capacitor C . The values of C are (a) $C = 0.196 \mu\text{F}$, with resonance frequency $f_{\text{res}} = \omega_2/2\pi = 20.0 \text{ kHz}$, (b) $C = 0.150 \mu\text{F}$, $f_{\text{res}} = 19.5 \text{ kHz}$, (c) $C = 0.096 \mu\text{F}$, $f_{\text{res}} = 21.5 \text{ kHz}$, (d) $C = 0.050 \mu\text{F}$, $f_{\text{res}} = 26.5 \text{ kHz}$. The open squares correspond to measurements made with the switch SW open (see Fig. 5), which is equivalent to turning the pump off. The open circles correspond to measurements made with the switch closed. The solid curves are a comparison to the theory presented in Sec. III. The evolution from EIT to the Autler–Townes regime is clearly observed.

the current flowing through the inductor. This voltage is an oscillatory signal at the frequency ω_s . We are interested in the amplitude of this signal, which we read from an oscilloscope.

In Fig. 6 we show the measured amplitudes corresponding to four values of the coupling capacitor C . For each value of C a measurement was made with the switch open (open square) and with the switch closed (open circle). In Table II we give the specifications of the electronic components used in the experiment.

As we can see from Fig. 6, in the open switch configuration (no pumping), we have a maximum coupling of electrical power from the voltage source V_S to the resonant circuit $R_2 L_2 C_{e2}$ at the resonant frequency (zero detuning). When

Table II. Specifications of the electronic components used in the experiment.

Electronic component	Value
R_1	0 Ω
R_2	51.7 Ω
L_1	1000 μH
L_2	1000 μH
C_1	0.1 μF
C_2	0.1 μF

the switch is closed, that is, when the pumping circuit (pumping force) is acting on the resonant circuit $R_2L_2C_{e2}$, the absorption of the electrical power of the voltage source at zero detuning is depressed. This fact corresponds to the transparent condition and is more pronounced when the value of the coupling capacitor is reduced, corresponding to an increment of the Rabi frequency of the pumping field in electromagnetically induced transparency in atomic systems.

Here, as in the experiments with atoms and light, the observed transparency can be interpreted as a destructive interference. In this case, the interference is between the power given by the voltage source to the resonant circuit $R_2L_2C_{e2}$, and the power transferred to the same circuit from the other oscillator representing the pumping force. As explained in Sec. II, it can also be viewed as an interference between the two possible excitation paths corresponding to the normal modes of the coupled oscillators. For the minimum value of C used in our experiment [see Fig. 6(d)], we observe two absorption peaks, which are the classical analog of the Autler–Townes effect and correspond to the splitting of these normal modes. We should also point out that the smallest coupling value we used [Fig. 6(a)] does not lead to an infinitely narrow transparency peak as would be expected from the value $R_1=0$. This behavior is probably due to internal residual series resistances of the components we used.

The solid lines shown in Fig. 6 represent the theoretical results obtained using Eq. (9) or (12). These curves are not expected to fit the experimental data exactly because our measurements are affected by the frequency response of the integrator used to derive the voltage proportional to the current in the inductor L_2 . It is not our purpose to analyze in detail the deviations of our system from the ideal model proposed.

It is also possible to measure the dispersive response of our atom analog. One has to analyze the relative phase between the oscillating current flowing through inductor L_2 and the phase of the applied voltage. This measurement is best performed with a lock-in amplifier. Because this equipment is not always available in undergraduate laboratories, we prefer to describe a simple procedure that allows us to observe the qualitative features that are predicted theoretically (see Fig. 4). We observe the oscillatory signal on the oscilloscope corresponding to the current flowing through the inductor, with the trigger signal from the voltage source $V_S(t)$. The phase of the sinusoidal signal is observed to “jump” as we scan ω_s across the absorption resonance and transparency region. If we scan by increasing the value of ω_s , we observe three abrupt phase variations, with the intermediate one being opposite to the other two. This behavior is exactly what we would expect from Fig. 4.

IV. CONCLUSION

We have shown that EIT can be modeled by a totally classical system. Our results extend the results of Hemmer and Prentiss⁷ to EIT-like phenomena, with the use of only two coupled oscillators instead of three. We also treat the dispersive response of the classical oscillator used to model the three-level atom. We performed an experiment with coupled RLC circuits and observed EIT-like signals and the classical analog of the Autler–Townes doublet as a function of the coupling between the two RLC circuits. The experiment can be performed in undergraduate physics laboratories and should help form students’ physical intuition on coherent phenomena that take place in atomic vapors.

ACKNOWLEDGMENTS

This work was supported by the Brazilian agencies FAPESP, CAPES, and CNPq.

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⁹As discussed, the induced transparency results from the existence of two possible paths for the absorption of the probe energy to excite the oscillation of particle 1. We can see these paths in Eq. (5) by rewriting it as a superposition of the normal modes of oscillation of the coupled oscillators.

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