(a) $$T = \frac{1}{2} m \left(a^2 \dot{\varphi}^2 + \rho^2 \dot{\theta}^2\right)$$

Here ρ is a radius from the z axis,

$$\rho = b - a\cos\varphi$$

$$L = T = \frac{1}{2} m \left[a \dot{\varphi}^2 + (b - a\cos\varphi)^2 \dot{\theta}^2\right]$$

(b) $$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = m (b - a\cos\varphi)^2 \dot{\theta}$$

$$p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = m a^2 \dot{\varphi}$$

$$H = p_\theta \dot{\theta} + p_\varphi \dot{\varphi} - L = \frac{p_\theta^2}{2m (b - a\cos\varphi)^2} + \frac{p_\varphi^2}{2ma^2}$$

(c) θ is a cyclic coordinate. So p_θ is conserved.

H is conserved. $H = E$ (energy)

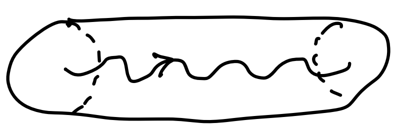
$$p_\varphi^2 = 2ma^2 \left(E - \frac{p_\theta^2}{2m(b - a\cos\varphi)^2}\right)$$

(d) $$2p_\varphi \frac{\partial p_\varphi}{\partial \varphi} = \frac{2a^2 p_\theta^2 \; a\sin\varphi}{(b - a\cos\varphi)^3}$$

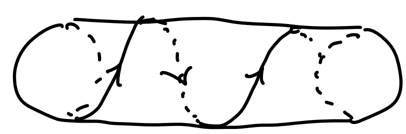
$$\left.\frac{\partial p_\varphi}{\partial \varphi}\right|_{\varphi = \pi + \varepsilon} = -\frac{a^3 p_\theta^2 \varepsilon}{(b+a)^3 p_\varphi} + \cdots \qquad \left(\sin(\pi + \varepsilon) = -\varepsilon + \cdots\right)$$

This looks like a restoring force. So we can get paths that oscillate around $\varphi = \pi$. Homerver, if p_φ is big enough, the mass will circle the torus.

(I)



(II)



What separates this two cases ?

The boundary is when $\varphi = 0$, $p_\varphi = 0$.

$$\Rightarrow \quad E^* = \frac{p_\theta^2}{2m(b-a)^2}.$$ When $E < E^*$, case (I) ; When $E > E^*$, (II).