GENERAL RELATIVITY & COSMOLOGY

A Quick Guide

Huan Bui

Colby College
Physics & Statistics
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1 Overview and Review

What is general relativity? It's a theory of gravity.

Replaces Newton's law of gravity, for heavy masses and high precision.

Keep in mind, GR is not compatible with Quantum Mechanics.

Question in Physics: how to reconcile GR and QM?

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