Goldstein Ch10 #12

$$V(r) = \begin{cases} -V_0 & 0<r<r_0 \\ 0 & r>r_0 \end{cases} \qquad V_0>0$$

ⓐ use polar coord s in plane (r,θ) & (p_r, p_θ)

want oscillation or rotation

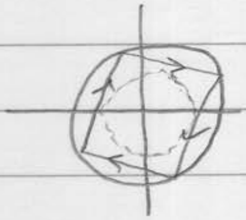
$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + V(r) = E \qquad (\text{conservative } H=T+V)$$

Case $E<0$ since $T>0$ need $V<0$ so $0<r<r_0$ to start

since E conserved always have $0 \le r(t) \le r_0$

θ cyclic so $p_\theta = \alpha_\theta$ = constant

Could be ⓘ rotation in (θ, p_θ)



θ increases

p_θ = const = periodic

here $r_1 < r < r_0$

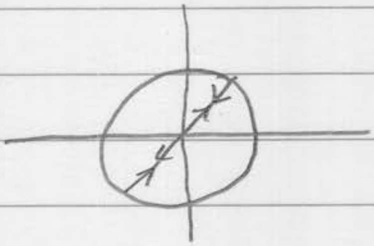
constrained away from zero

$$E = \frac{p_r^2}{2m} + \frac{\alpha_\theta^2}{2mr^2} - V_0 \quad , \quad p_r = \pm\sqrt{2m(V_0+E) - \frac{\alpha_\theta^2}{r^2}}$$

p_r also periodic

(r, p_r) is oscillation

(ii) special case, (θ, p_θ) is oscillation



θ periodic (either θ_0 or $\pi+\theta_0$)

$\alpha_\theta = p_\theta = 0$ so periodic

So for $-V_0 \le E < 0$ can always define action angle variables

initial conditions here: $0 < r(t=0) \le r_0$

$$-V_0 \le \frac{p_r^2}{2m} + \frac{\alpha_\theta^2}{2mr^2} - V_0 < 0 \quad \rightarrow \quad 0 < p_r^2 < 2mV_0 - \frac{\alpha_\theta^2}{r^2}$$

$$0 < p_r^2 < 2mV_0 - \frac{\alpha_\theta^2}{r_0^2}$$

$p_r^2 > 0$ so $\alpha_\theta^2 < 2mr_0^2 V_0$ required for negative E

Case $E \geq 0$ $\quad r$ is unbounded. Only possibility for action angle vars is r increasing, p_r periodic

If $\theta =$ constant, $p_\theta = \alpha_\theta = 0$, then $p_r =$ constant

hence periodic

Here (θ, p_θ) is dot in phase space (call it oscillation with 0 amplitude)

(r, p_r) is rotation

(Any other motion for $E > 0$ is unsuitable to action-angle vars)

Initial conditions here:

$$\underline{t=0} \quad p_r^2 = \begin{cases} (E+V_0)\,2m & r<r_0 \\ E\,(2m) & r>r_0 \end{cases} \qquad p_\theta = 0$$

then $\quad p_\theta = 0 \Rightarrow p_\theta(t) = 0$

p_r either constant if $t=0$ has $r>r_0$, or increases by one jump if $r<r_0$.

ⓑ $E<0$, $\quad r_1 < r < r_0$ case

$$J_\theta = \oint p_\theta \, d\theta = 2\pi \alpha_\theta$$

$$J_r = \oint p_r \, dr = \pm \oint dr \left(2m(V_0+E) - \alpha_\theta^2/r^2\right)^{1/2}$$

$$= \pm \left(2m(V_0+E)\right)^{1/2} \oint dr \, \frac{1}{r} \left(r^2 - r_1^2\right)^{1/2}$$

since $p_r = 0$ at $r = r_1$, $\quad r_1^2 = \dfrac{\alpha_\theta^2}{2m(V_0+E)}$

one period is $\quad \underset{p_r<0}{r_0 \to r_1} \underset{p_r>0}{\to r_0} \qquad$ doubles $r_1 \to r_0$ result

$$J_r = 2\left(2m(V_0+E)\right)^{1/2} \underbrace{\int_{r_1}^{r_0} \frac{1}{r}\left(r^2 - r_1^2\right)^{1/2} dr}_{\sqrt{r_0^2 - r_1^2} - r_1 \cos^{-1}\left(\frac{r_1}{r_0}\right)}$$

$$= 2\left(2m(V_0+E)\right)^{1/2} \left[\left(r_0^2 - \frac{\alpha_\theta^2}{2m(V_0+E)}\right)^{1/2} - \frac{\alpha_\theta}{\sqrt{2m(V_0+E)}} \cos^{-1}\left(\frac{\alpha_\theta}{r_0\sqrt{2m(E+V_0)}}\right)\right]$$

to get the frequency use $\quad \dfrac{1}{\nu_r} = \dfrac{\partial J_r}{\partial E}$

Goldstein
Ch 12 #7

$$\Delta H = b\, p_x^2 p_y^2$$

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2} m \omega^2 (x^2+y^2)$$

one $\nu = \frac{\omega}{2\pi}$

$$x = \sqrt{\frac{J_x}{\pi m \omega}} \sin(2\pi(\nu t + \beta_x)) \quad , \quad y = \sqrt{\frac{J_y}{\pi m \omega}} \sin(2\pi(\nu t + \beta_y))$$

$$p_x = \sqrt{\frac{m J_x \omega}{\pi}} \cos(2\pi(\nu t + \beta_x)) \qquad p_y = \sqrt{\frac{m J_y \omega}{\pi}} \cos(2\pi(\nu t + \beta_y))$$

new vars $\{J_x, \beta_x\}, \{J_y, \beta_y\}$

$$H_0 = \frac{\omega}{2\pi}(J_x + J_y)$$

$$\dot{\beta}_x^{(1)} = \frac{\partial \Delta H}{\partial J_x}\bigg|_0 = \frac{\partial}{\partial J_x} b \left(\frac{m J_x \omega}{\pi}\right)\left(\frac{m J_y \omega}{\pi}\right) \cos^2(2\pi(\nu t+\beta_x)) \cos^2(2\pi(\nu t+\beta_y))$$

$$= b \frac{m^2\omega^2}{\pi^2} J_y^{(0)} \cos^2(2\pi(\nu t + \beta_x^{(0)})) \cos^2(2\pi(\nu t + \beta_y^{(0)}))$$

$x = y = 0$ at $t = 0$ so $\beta_x^{(0)} = \beta_y^{(0)} = 0$, $\overset{t=0}{p_x = p_y} \Rightarrow J_x^{(0)} = J_y^{(0)}$
$= \frac{\pi}{\omega} E$

over one period

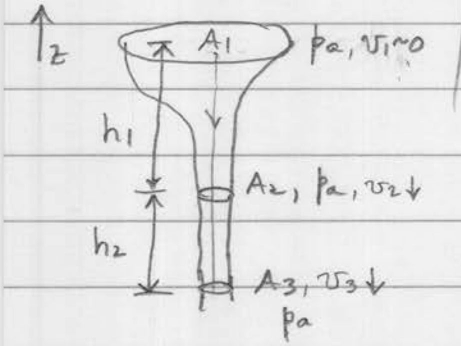
$$\overline{\dot{\beta}^{(1)}} = \frac{b\, m^2 \omega^2}{\pi^2} J_y^{(0)} \underbrace{\frac{1}{2\pi}\int_0^{2\pi} d\theta \cos^4(\theta)}_{= 3/8}$$

$$= \frac{3 b\, m^2 \omega^2 J_y^{(0)}}{8\pi^2}$$

$\nu_x^0 \rightarrow \nu_x + \overline{\dot{\beta}^{(1)}}$ is shift in ν_x

by symmetry its same for ν_y

$\boxed{\text{Smits } 4.36}$ ⓐ



continuity $\overset{0}{\cancel{\frac{\partial\rho}{\partial t}}} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$, $\overset{\text{Div.}}{\text{Thm}} \Rightarrow \rho\int_S ds\, \hat{n}\cdot\vec{v} = 0$

continuity $A_1 v_1 = A_2 v_2 = A_3 v_3$

Bernoulli (equal pressure)

$$\underset{0}{\cancel{\frac{v_1^2}{2}}} + g z_1 = \frac{v_2^2}{2} + g z_2 = \frac{v_3^2}{2} + g z_3$$

water at rest in funnel basin , $z_1 - z_2 = h_1$, $z_2 - z_3 = h_2$

ⓐ $\frac{A_3}{A_2} = \frac{v_2}{v_3}$, $v_2^2 = 2gh_1$, $v_3^2 - v_2^2 = 2gh_2$, $v_3^2 = 2g(h_1+h_2)$

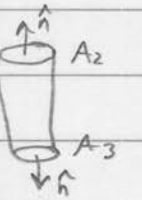
$v_2 = \sqrt{2gh_1}$ $\qquad v_3 = \sqrt{2g(h_1+h_2)}$

$$\frac{A_3}{A_2} = \sqrt{\frac{h_1}{h_1+h_2}}$$

ⓑ Mom. Conservation: $\overset{\text{Steady}\ 0}{\cancel{\frac{\partial}{\partial t}(\rho\vec{v})}} + \vec{\nabla}\cdot\hat{T} = \vec{f} = \rho\vec{g}$

$$\int_S \underbrace{ds\,\hat{n}\cdot\hat{T}}_{\text{vector}} = \int_{Vol} dV \underbrace{\vec{\nabla}\cdot\hat{T}}_{\text{vector}} = \int_{Vol} dV\,\rho\vec{g} = V\rho\vec{g}$$

$\hat{T}_{ki} = \delta_{ki}\, p + v_k v_i \rho$, take surface



so V = desired volume

fluid passes only through A2 & A3

so only these have non-zero velocities

$\hat{n}\cdot\hat{T}\big|_2 = \hat{z}\,p + \underbrace{\vec{v}_2}_{(-v_2\hat{z})}\underbrace{\hat{z}\cdot\vec{v}_2}_{(-v_2)}\rho$, $\quad \hat{n}\cdot\hat{T}\big|_3 = -\hat{z}\cdot\hat{T}\big|_3 = -\hat{z}\,p + \underbrace{\vec{v}_3}_{(-v_3\hat{z})}\underbrace{(-\hat{z}\cdot\vec{v}_3)}_{(+v_3)}\rho$

pressure term equal with opposite signs so cancel

v_2 = constant across A2 for ideal fluid , v_3 = constant on A3

So

$$V\cancel{\rho}(-g\hat{z}) = \int_{A_2} ds\,\hat{n}\cdot\hat{T} + \int_{A_3} ds\,\hat{n}\cdot\hat{T} = (v_2^2 A_2 - v_3^2 A_3)\,\hat{z}\,\cancel{\rho}$$

$$\text{Volume} = \frac{A_2 v_2^2}{(-g)}\left(1 - \frac{v_3^2}{v_2^2}\frac{A_3}{A_2}\right) = \frac{A_2(2\cancel{g}h_1)}{(-\cancel{g})}\left(1 - \frac{(h_1+h_2)}{h_1}\sqrt{\frac{h_1}{h_1+h_2}}\right)$$

$$= 2A_2 h_1\left(\sqrt{\frac{h_1+h_2}{h_1}} - 1\right)$$

7.12 Smits

$u = v_x = 2cxy$ $v_z = 0$

$v = v_y = c(a^2 + x^2 - y^2)$

ⓐ Incompressible?

$$\vec{\nabla}\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 2cy - 2cy = 0$$ Yes.

ⓑ $\vec{\nabla}\times\vec{v} = ?$

$$\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & 0 \end{vmatrix} = \hat{x}(0) - \hat{y}(0) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

$$= \hat{z}(c\,2x - 2cx) = 0$$ so irrotational

ⓒ $\vec{v} = \vec{\nabla}\phi$, $v_x = \frac{\partial \phi}{\partial x}$ so $\phi = cx^2y + g(y)$

$v_y = \frac{\partial \phi}{\partial y}$, $\phi = c(a^2 + x^2)y - \frac{cy^3}{3}$ ← this agrees with above, so works

← (fixed)

8.24 Smits

$F_D = f(V, L, B, \rho, \mu, g)$ $\mu =$ "υ"

ⓐ $[F_D] = \frac{kg\,m}{s^2}$, $[V] = \frac{m}{s}$, $[L] = m$, $[B] = m$, $[\rho] = \frac{kg}{m^3}$, $[\mu] = \frac{m^2}{s}$

$[g] = m/s^2$

↑ also could take $\mu = \eta = \upsilon\rho$ as valid interpretation

dimless: L/B, v^2/gL, $R = \frac{VL}{\mu}$, $[\rho v^2 L^2] = [F_D]$

ⓐ $$F_D = \rho v^2 L^2\, h\left(\frac{L}{B}, \frac{VL}{\mu}, \frac{v^2}{gL}\right)$$

ⓑ need same $\left(L/B \;\&\; \frac{VL}{\mu} \;\&\; v^2/gL\right)$ as the ship

ⓒ $L' = \frac{1}{25}L = 4m$, $B' = \frac{1}{25}B$ → $v'^2 = v^2\, L'/L = 100/25 = 4$

$v = 10 m/s$, $v^2/L = v'^2/L'$ $v' = 2\ m/s$

$\frac{v'L'}{\mu'} = \frac{vL}{\mu}$, $\frac{\mu'}{\mu} = \frac{v'L'}{vL} = \frac{2}{10}\frac{1}{25} = \frac{1}{125}$

with same ρ

$$\frac{F_D'}{F_D} = \frac{v'^2L'^2}{v^2L^2} = \frac{1}{(125)^2}$$

$\boxed{\text{9.37 Smits}}$, steady flow $\qquad w >> D$

Naivier Stokes $\qquad (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{v}$

$$\vec{v} = v_x(y)\,\hat{x}$$

$$v_x(y) = u = U_0\left[1-\left(\frac{2y}{D}\right)^2\right]$$

$$(\vec{v}\cdot\vec{\nabla})\vec{v} \propto \frac{\partial}{\partial x} v_x(y) = 0$$

ⓐ $$\frac{1}{\rho}\frac{\partial}{\partial x}p = \nu\frac{\partial^2}{\partial y^2}v_x(y) = \nu U_0\left(\frac{-4}{D^2}\right)\overset{\frac{\partial^2}{\partial y^2}y^2}{\overset{\downarrow}{2}} = -\frac{8\nu U_0}{D^2}$$

shear stress $\qquad \sigma'_{ik} = \underbrace{\eta}_{\nu\rho}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\underbrace{\vec{\nabla}\cdot\vec{v}}_{0}\right)$

at wall: $\quad \sigma'_{xy} = \sigma'_{yx} = \nu\rho\frac{\partial v_x}{\partial y}\Big|_{y=\pm D/2} = \nu\rho\left(\frac{-4}{D^2}\right)\left(\overset{\overset{y}{\downarrow}}{\pm\frac{D}{2}}\right)U_0$

$\hat{x}$ friction force on wall from shear

$$\tau_w = (-\hat{y})\cdot\sigma'_{yx}\Big|_{y=D/2} + (+\hat{y})\cdot\sigma'_{yx}\Big|_{y=-D/2} = \nu\rho\left(\frac{-4}{D^2}\right)\left(-\frac{D}{2}-\frac{D}{2}\right)U_0$$

$$\tau_w = \nu\rho\frac{4U_0}{D^2}D = \nu\rho\frac{8U_0}{D^2}\frac{D}{2}$$

so $\quad \frac{\partial}{\partial x}p = -\frac{8\rho\nu U_0}{D^2} = -\frac{2}{D}\tau_w \quad$ ✓

ⓑ $\quad \frac{\partial p}{\partial x} = -\frac{8U_0\nu}{D^2} \qquad$ found in ⓐ $\qquad \nu = \mu$

ⓒ $\quad \vec{\nabla}\times\vec{v}\Big|_z = \frac{\partial v_x}{\partial y} = \nu\rho\left(\frac{-4}{D^2}\right)y \qquad$ linearly with y

Strogatz 3.4.4

$$\dot{x} = x + \frac{rx}{1+x^2} = x\left(1+\frac{r}{1+x^2}\right)$$

$$\dot{x} = x\left(\frac{1+r+x^2}{1+x^2}\right)$$ numerator has subcritical pitchfork form

fixed pts: $x=0$ (always), $1+r<0$ then $x = \pm\sqrt{-(1+r)}$
so $r<-1$

check stability:

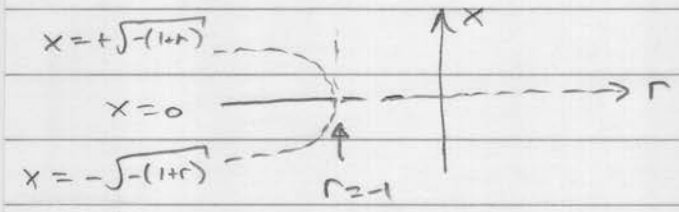
near $x \simeq 0$ $\dot{x} = x(1+r)$ stable $1+r<0$ ie $r<-1$
unstable $1+r>0$ $r>-1$

$x = +\sqrt{-(1+r)} + \eta$ for $r<-1$

$$\dot{x} = \frac{x\left(x+\sqrt{-(1+r)}\right)\left(x-\sqrt{-(1+r)}\right)}{1+x^2}$$

so $$\dot{\eta} = \frac{\sqrt{-(1+r)}}{-r}\, 2\sqrt{-(1+r)}\,(\eta) = \underbrace{\frac{2(1+r)}{-r}}_{>0}\eta$$ unstable

$x = -\sqrt{-(1+r)} + \eta$, similar, unstable



Strogatz 5.2.5

$\dot{x} = 3x-4y$
$\dot{y} = x-y$, $$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \underbrace{\begin{pmatrix}3 & -4\\ 1 & -1\end{pmatrix}}_{M}\begin{pmatrix}x\\ y\end{pmatrix}$$

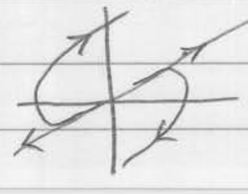
$\tau = \mathrm{tr}\, M = 2$

$\Delta = \det M = -3+4 = +1$, $\tau^2 - 4\Delta = 0$, $\tau > 0$

$\vec{x} = 0$ is unstable degenerate node

$\lambda_\pm = \tau/2 = +1$ $(M-\lambda\mathbb{1}) \Rightarrow \begin{pmatrix}2 & -4\\ 1 & -2\end{pmatrix}\begin{pmatrix}a_1\\ a_2\end{pmatrix} = 0$ $a_1 = 2a_2$

$\begin{pmatrix}2\\ 1\end{pmatrix}$ only eigenvector



$\boxed{\text{Strogatz } 6.3.9}$ $\qquad \dot{x} = y^3 - 4x$

$\qquad\qquad\qquad\qquad \dot{y} = y^3 - y - 3x$

ⓐ $\dot{x} = 0$ fixed $\qquad y^3 = 4x$

$\dot{y} = 0$ pts $\qquad 4x - 3x - y = 0$ so $x = y$, $x^3 = 4x$

- $x = y = 0$ or $x = y = +2$ or $x = y = -2$

<u>$x=y=0$</u> $\quad \dot{x} = -4x \qquad \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -4 & 0 \\ -1 & -3 \end{pmatrix} \qquad \tau = -7$, $\tau^2 - 4\Delta = 1$

$\dot{y} = -y - 3x \qquad\qquad \Delta = 12$

stable node

<u>$x=y=2$</u> $\quad x = 2 + \eta \qquad \dot{\eta} = (2+\xi)^3 - 4(2+\eta) \simeq \cancel{8} + 12\xi - \cancel{8} - 4\eta$

$y = 2 + \xi \qquad \dot{\xi} = (2+\xi)^3 - (2+\xi) - 3(2+\eta) \simeq \cancel{8} + 12\xi - \cancel{2} - \xi - \cancel{6} - 3\eta$

$= 11\xi - 3\eta$

$M = \begin{pmatrix} -4 & +12 \\ -3 & +11 \end{pmatrix} \qquad \tau = 7 \qquad \tau^2 - 4\Delta = 49 + 32 = 81$

$\Delta = -44 + 36 = -8$, saddle node

$\lambda = -1$ has $\begin{pmatrix} -3 & 12 \\ -3 & 12 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0 \quad a_1 = 4a_2 \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $\lambda_\pm = \frac{7}{2} \pm \frac{1}{2} 9 = -1$ or $\boxed{+8}$ has $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

<u>$x=y=-2$</u> Note: $x \to -x$ & $y \to -y$ is symmetry of eqtns

so saddle node here too

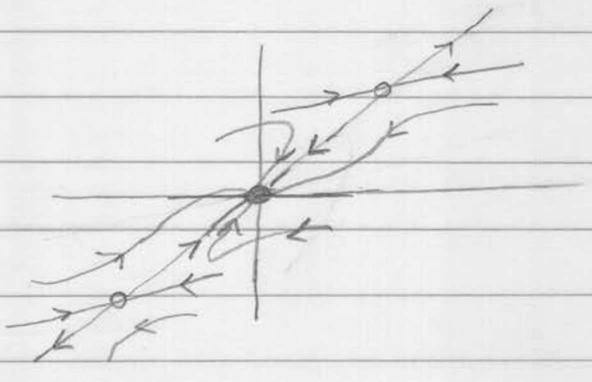
ⓑ $x = y \Rightarrow \quad \dot{x} = x^3 - 4x = x(x^2 - 4)$

$\dot{y} = x^3 - 4x$ too $\qquad$ so $\dot{x} = \dot{y}$ & $x = y$ as time increases

ⓒ let $w = x - y$, $\dot{w} = \dot{x} - \dot{y} = (y^3 - 4x) - (y^3 - y - 3x) = -(x - y) = -w$

$\therefore \quad w = Ae^{-t}$ decreases

ⓓ



$\boxed{\text{6.5.4 Strogatz}}$ $\ddot{x} = ax - x^2$

find conserved quantity & sketch

$\dot{x} = \omega \qquad \equiv f_x$

$\dot{\omega} = ax - x^2 \qquad \equiv f_\omega$

$$H(x,\omega) = \int^{\omega} d\omega' f_x - \int^{x} dx' f_\omega = \frac{\omega^2}{2} - \left(\frac{ax^2}{2} - \frac{x^3}{3}\right)$$

$$= \frac{\omega^2}{2} + \frac{x^2}{3}\left(x - 3\frac{a}{2}\right) = \frac{\omega^2}{2} + \frac{x^3}{3} - \frac{ax^2}{2}$$

fixed pts $\omega = 0$ & $x = 0$ or $\omega = 0$, $x = a$ $\qquad \frac{\partial^2 H}{\partial x^2} = 2x - a$

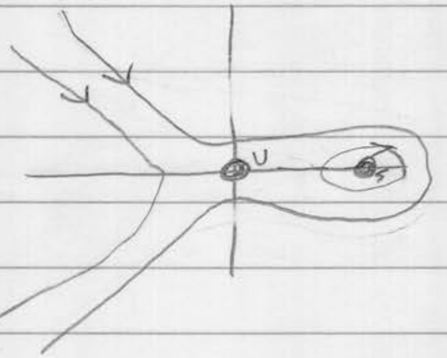
$\frac{\partial^2 H}{\partial x^2} = -a$ <0 for $a>0$ max/unstable

>0 for $a<0$ min/stable

$\frac{\partial^2 H}{\partial x^2} = a$, >0 $a>0$ unstable

<0 $a<0$ stable

$a>0$



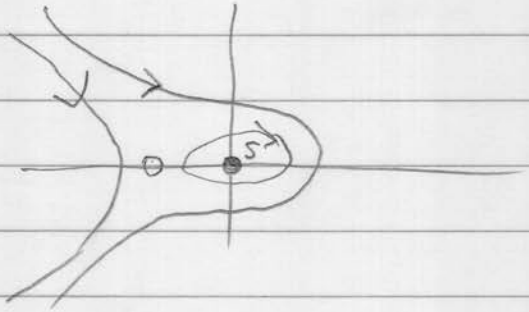
Hamiltonian / Conservative E system

so stable center about ●s

●u = unstable

●s = stable

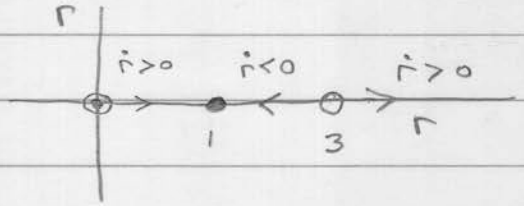
$a<0$

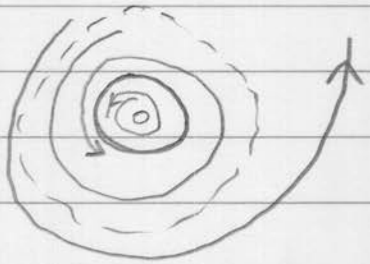


7.1.2 Stragotz $\dot{r} = r(1-r^2)(9-r^2)$ $\dot{\theta} = 1$

$r \geq 0$

$r = 1$ & $r = 3$ are limit cycles, $r=0$ fixed pt





Strogatz 6.5.3.

$$\ddot{x} = \cancel{ax - x^2}\, a - e^x$$

Find conserved quantity

$$\cancel{x}\, \ddot{x} = \cancel{ax - x^2}\, a - e^x$$

$$\cancel{x}\, \ddot{x} = -\frac{d}{dx}\left[-ax + e^x\right]$$

Let $V(x) = -ax + e^x$

Hence we have

$$\ddot{x} = -\frac{dV}{dx}$$

$$\dot{x}\,\ddot{x} = -\dot{x}\frac{dV}{dx}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\dot{x}^2}{2}\right) = -\frac{d}{dt}V(x)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\dot{x}^2}{2} + V(x)\right) = 0$$

Hence $E = \frac{\dot{x}^2}{2} + V(x) = \frac{\dot{x}^2}{2} - ax + e^x$

is conserved.

Let $y = \dot{x} \Rightarrow \dot{y} = a - e^x$

Jacobian matrix for this system is $A = \begin{pmatrix} \frac{\partial y}{\partial x} & \frac{\partial y}{\partial y} \\ \frac{\partial(a-e^x)}{\partial x} & \frac{\partial(a-e}{\partial y} \end{pmatrix}$

$\dot{x} = y$

$\dot{y} = a - e^x$

$$A = \begin{pmatrix} 0 & 1 \\ -e^x & 0 \end{pmatrix}$$

Hence if ~~there~~ there are fixld points then they are centers

as $\Delta = \det A = e^{x} > 0$

Recall that for a conservative system we only have centers and saddles as fixed points.

$\Delta > 0 \Rightarrow$ centers [only if there are fixed points in the first place!]

Now let's find the fixed points for this system

$\dot{x} = y$

$\dot{y} = a - e^{x}$

$\dot{x} = \dot{y} = 0 \Rightarrow \quad y = 0, \; x = \ln a$

$\downarrow$

Only when $a > 0$

For $a = 0$ or $a < 0$

$\longrightarrow$ No fixed points

Lets consider the case of $a > 0$ first.

$a > 0 \rightarrow$ ~~(0~~ $(\ln a, 0)$

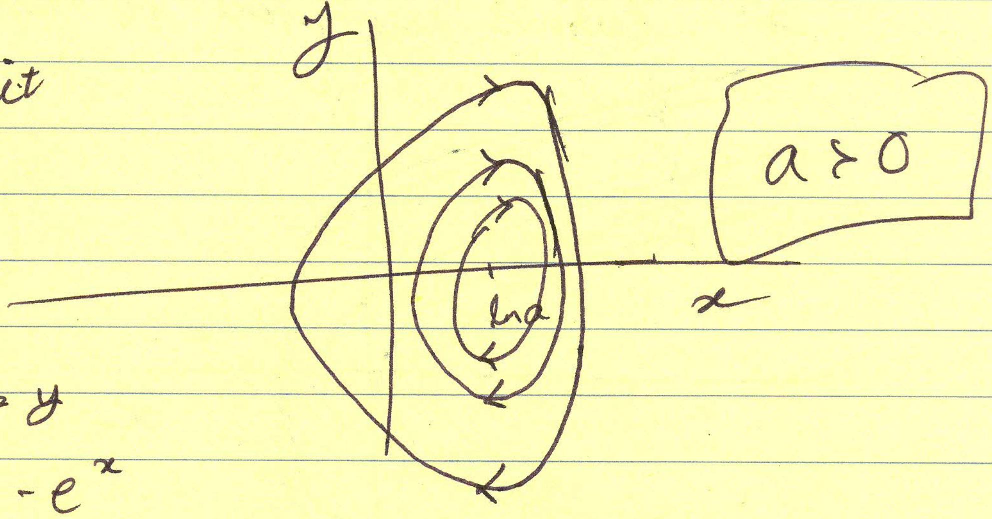
$\downarrow$

$\boxed{\text{Center}}$

$$E = \frac{y^2}{2} - ax + e^x$$ is constant along the closed contours

$$\left.\frac{d^2E}{dx^2}\right|_{x=\ln a} = a > 0$$ → E has a minimum at ~~x~~ $(0, \ln a)$

Phase portrait

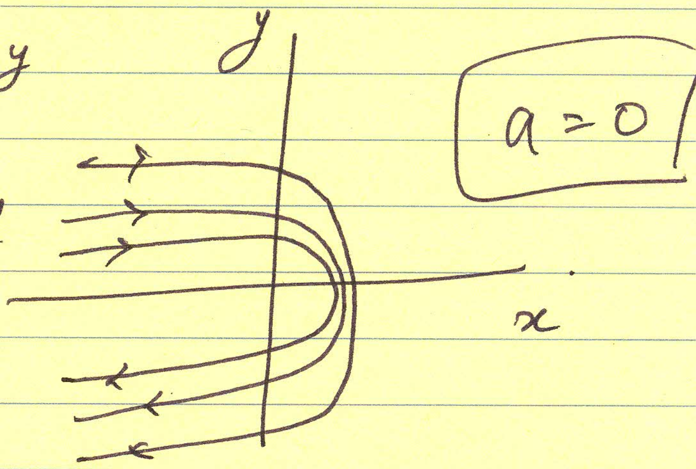


Note that $\dot{x} = y$
$\dot{y} = a - e^x$

Note For large x, $-e^x$ dominates in $\dot{y}$ and we get very steep descent. The curve turns right ~~or~~ left only when y is large enou

For a = 0 ~~$y \overset{?}{=} x$~~ $\dot{x} = y$
$\dot{y} = -e^x$

We have no fixed points



Note For $x \to -\infty$
$\dot{y} \to 0 \Rightarrow$ ~~the~~ curves get horizontal for $x \to -\infty$

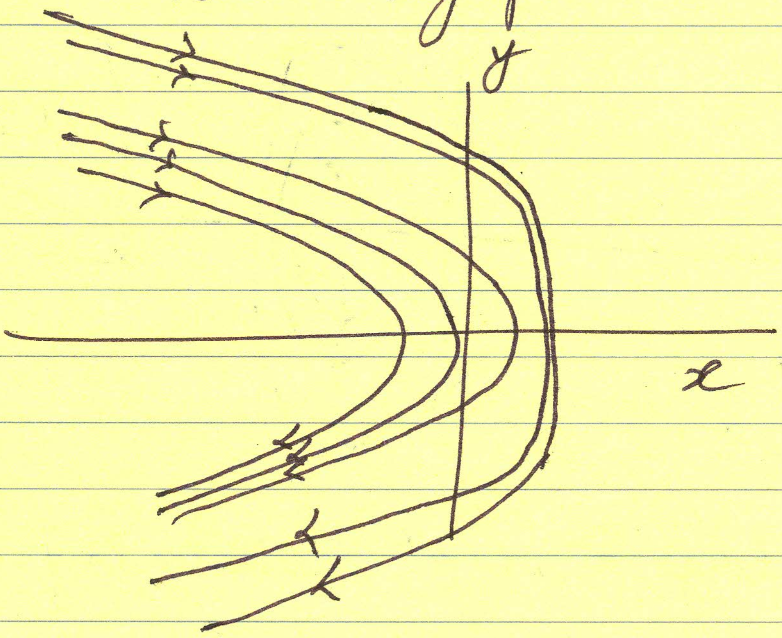
$a < 0$

$$\dot{x} = y$$

$$\dot{y} = -|a| - e^{x}$$

as $x \to -\infty$ $\quad \dot{y} \to -|a|$

$\Rightarrow$ curves asymptote to lines $y = -|a|x + $Con



$\boxed{a < 0}$

~~For Contour plots for conservative system~~