PH312: Physics of Fluids (Prof. McCoy) – Reflection

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Feb 26 2021

1. Explain why particle paths, streamlines, and streaklines are identical for *steady flow*.

(a) In steady flow, the velocity field is time-independent, i.e., $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}(\mathbf{r})$, and thus streamlines (which are just velocity field lines) are also time-independent. Let a particle p be moving instantaneously along some streamline S. Because S is unchanging and that the tangent of S at p is always along the velocity of p, p must move along S.

(b) Let p be moving along S. Suppose p moves onto some streamline $S_1 \neq S$. From (a), we know that S_1 and S must cross, but this is impossible because p will then have velocities in two direction. Two streamlines cannot intersect except at a position of zero velocity.

(c) Let particles $\{p\}$ make up some streakline. By definition, each of $\{p\}$ has passed through one point belonging to some streamline S. From (b), we know that none of $\{p\}$ leaves S, and so all of $\{p\}$ must follow S.

2. To transform the flow field in Figure 6.3(a) where flow is viewed from the bank to Figure 6.3(b) where flow is viewed from the ship, we add $-\mathbf{U}$ (which points Left-Right) to each velocity vector $\mathbf{u}'$ in (a). For example, the point p in (a) with $\mathbf{u}'$ pointing Up-Left becomes $\mathbf{u}$ = Up-Left + Right $\approx$ Up-Right. The velocity may change direction in some cases depending on how $\mathbf{U}$ to compares to $\mathbf{u}'$ in direction and amplitude. For example, the regions in (a) where the flow is Right-to-Left are where it was previously moving downstream in (b) **more slowly** than $\mathbf{U}$.

3.

(a) $a = b_i c_{ij} d_j$ ✓

(b) $a = b_i c_i + d_j$ ✗

The LHS has no free index, but the RHS does. So, this is not allowed according to the convention (p.28, K&C: the free index must appear on both sides). If we want to say that the component d_j is equal to the scalar $a - b_i c_i$ for all j then we have to be explicit to avoid confusion.

(c) $a_i = \delta_{ij} b_i + c_i$ ✗

The contraction $\delta_{ij} b_i$ leaves b_j with free index j, which is incompatible with the free index i on a_i and c_i.

(d) $a_k = b_k c + d_i e_{ik} = c b_k + d_i e_{ik}$ ✓

(e) $a_i = b_i + c_{ij} d_{ij} e_i$ ✗

The index i appears 3 times in the second term on the RHS. This can cause confusion.

4. Let A be a second-order tensor and R be a rotation matrix. Because $R^\top R = \mathbb{I}$, i.e., $(R^\top R)_{mn} = \delta_{mn}$, we have

$$A'_{ii} = R_{im} R_{in} A_{mn} = {R^\top}_{mi} R_{in} A_{mn} = (R^\top R)_{mn} A_{mn} = \delta_{mn} A_{mn} = A_{mm} = A_{ii}.$$

So, A_{ii}, or the trace of the matrix of A, is invariant under coordinate rotations.

5. Let R be a rotation matrix. Because $R^\top R = \mathbb{I} = R R^\top$, i.e., $(RR^\top)_{ij} = \delta_{ij}$, we have

$$\delta'_{ij} = R_{im} R_{jn} \delta_{mn} = R_{im} R_{jm} = R_{im} {R^\top}_{mj} = (RR^\top)_{ij} = \delta_{ij}.$$

So, δ_{ij} is an isotropic tensor.