Bose–Fermi pair formation in the mixture of cold atomic gases

To cite this article: Toru Suzuki 2011 *J. Phys.: Conf. Ser.* **321** 012027

View the article online for updates and enhancements.

Bose-Fermi pair formation in the mixture of cold atomic gases

Toru Suzuki

Department of Physics, Tokyo Metropolitan University, Tokyo 192-0397, Japan

E-mail: `suzukitr@tmu.ac.jp`

Abstract. We study the role of the Bose-Fermi (BF) pair correlations in the cold BF-mixture of atomic gases. After summarizing the previous results on the formation of composite BF-pairs in the uniform BF mixture, we present an exactly solvable model where bosons and fermions are distributed over finite number of orbits and interact via attractive pairing-type interaction. We show that, as the interaction strength increases, BF pairs with different quantum numbers are gradually created in the ground state through a series of level crossings, until finally the system becomes a Fermi sea of BF-pairs.

1. Introduction

Recent development in the manipulation of cold fermionic atoms provided a means of realizing a crossover from Bardeen-Cooper-Schrieffer (BCS) superconducting phase to the Bose-Einstein Condensation (BEC) of bound fermion pairs. In these studies a full use of Feshbach resonance technique is made, whereby one adjusts magnetic field so as to bound or dissolve a number of fermion pairs. A similar situation may be expected with different types of particles as far as a similar technique is available. In fact a proposal to create composite particles in a Bose-Fermi(BF) mixture has been made[1, 2, 3, 4, 5], though no direct experimental signature has been reported up to now[6]. An obvious difference from the fermion-pair case is that because the BF-pairs are (composite) fermions one cannot expect a condensation of pairs in the latter. Instead, a new BF-pair can only be created with a different quantum number (e.g., the center-of-mass(CM) momentum), which requires an additional attraction to compensate the increase of energy (e.g., CM energy) of the pair. This suggests an extra complication in the theoretical prediction: In contrast to the fermion-pair case where Cooper-type instability signals a condensation, one has in principle to calculate the whole dispersion relation of BF-pairs in the presence of other particles and pairs. Main purpose of the present work is to make a solvable model study of the formation of BF-pairs in the BF-mixture. The model system is a schematic one which allows a calculation of exact eigenstates, so that one may exhibit a change from independent Bose-Fermi system to that of composite BF-pairs.

2. Boson-fermion Cooper problem

To set a physical basis we first summarize results of the previous studies on the formation of a single BF-pair in the uniform BF-mixture. In the absence of interactions the system at $T = 0$ consists of BEC and a Fermi sphere. Now one may switch on the attractive BF-interaction so that they can form a (quasi-)bound state. Since a BF pair obeys Fermi statistics, one would

expect that, for sufficiently strong binding of the pair, they may form a new Fermi sea as far as many-body correlations are not large. As the original noninteracting BEC-Fermi sea system and the new Fermi sea of BF pairs are quite different, the nature of the transition between the two extremes against, e.g., the strength of the interaction is not self-evident.

When the BF attractive interaction is not strong, one may start with a single boson and fermion, on top of the free BF-ground state, interacting via BF-effective interaction modified by the presence of the BF-medium. We start with the Hamiltonian

$$H = \sum_k \epsilon_k (b_k^\dagger b_k + f_k^\dagger f_k) + \sum V_{q'q} (bf)_{Pq'}^\dagger (bf)_{Pq} \tag{1}$$

where ϵ_k denotes kinetic energy of a particle with momentum k, and V is the pseudopotential matrix element between boson and fermion with total momentum P and relative momentum $q, (q')$, i.e., $(bf)_{Pq} \equiv b_{\frac{P}{2}+q}, f_{\frac{P}{2}-q}$. We may calculate dispersion relation of the BF-pair by studying the pole of the two-body propagator in the BF-medium:

$$G_{\mathrm{BF}} \sim \langle \theta(t) \{ (bf)_{Pq}(t), (bf)_{Pq'}^\dagger(t') \} \rangle. \tag{2}$$

Figure 1 shows the energy P_0 of the pole against CM-momentum P of the BF-pair for various interaction strength characterized by the scattering length a[4]. The latter is calculated with a suitable renormalization procedure for a short-range interaction.

We find that even before the formation of the bound state (i.e., $P_0 \leq -E_F$ at $P = 0$), a Cooper type BF-pair can be created in the ground state. This interpretation is consistent with the result of the binding energy calculation performed within a (renormalized) mean-field framework[4]. The latter shows a rapid decrease of energy below zero before the parameter $(k_F a)^{-1}$ signals a formation of a BF bound state.
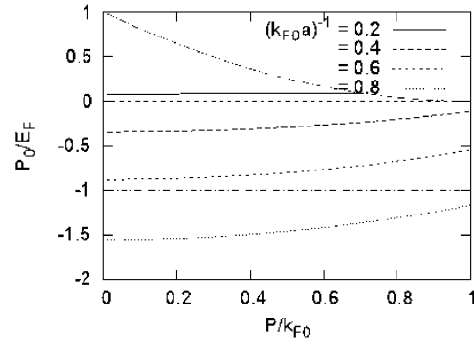

Figure 1. Pole energy P_0 against CM momentum

3. Solvable model study

We now consider a solvable boson-fermion pairing model which exhibits a formation of BF pairs, and study the behavior of the ground state as function of the BF-pairing strength. To construct such models we need two quantum numbers, i.e., the one which characterizes the intrinsic structure of the BF pair and the one which distinguishes the state of pairs. Let us call these quantum numbers as $k(= 1, 2, \ldots, D)$ and $p(= 1, 2, \ldots, M)$, respectively. Thus the model is a system of bosons and fermions distributed over the single-particle states which consist of M orbits, each orbit having degeneracy D, for both bosons $(b^\dagger, b)$ and fermions $(f^\dagger, f)$. With a simple Bose-Fermi pairing type interaction, the model Hamiltonian is given by

$$H = \sum_{p,k} (\epsilon_p^b b_{pk}^\dagger b_{pk} + \epsilon_p^f f_{pk}^\dagger f_{pk}) + \sum_p G_{p'p} C_{p'}^\dagger C_p,, \quad C_p^\dagger \equiv \sum_{k=1}^D b_{pk}^\dagger f_{pk}^\dagger. \tag{3}$$

where $\epsilon^{b(f)}$ are the boson (fermion) single-particle energies, while G_{qp} denotes an attractive BF interaction. The boson-fermion pair operators $C^\dagger, C$ are defined in each orbit p and satisfies $(C_p)^2 = 0$, though they are not pure fermions as their anticommutator is different from unity. We assume that the interaction is diagonal in p, i.e., $G_{pq} = -G\delta_{pq}$ so that the pairs may be formed in each orbit with the same internal structure. Note that a presence of nondiagonal interaction in the quantum number p would create a single very collective composite BF pair[7]. The model can be solved first by finding eigenstates in each orbit and then by combining these to obtain the whole system with given quantum numbers.

The problem is to find the ground state from many competing configurations with various number of BF pairs for each value of the interaction strength. For a weak attraction, favorable configurations consist of many seniority states, where 'seniority' means a number of unpaired particles. To obtain exact eigenvalues and eigenfunctions with given boson and fermion seniorities, we used a method of partial fermionization which is an extension of the partial bosonization. The latter has been developed to study quasiparticle excitations in the pairing problem of finite nuclei[8].

We first calculate the number of states. We consider a system with equal number N of bosons and fermions, i.e., $N_b = N_f = N$. Particles are distributed over the $L = DM$ single-particle states, so that the total number of states is given by $W_{\text{tot}} = (L + N - 1)!L/(L - N)!(N!)^2$. For the present pairing type interaction the eigenstates are characterized by boson and fermion seniority numbers ν_p^b, ν_p^f in each orbit p. For $N_b = N_f$ the total seniority number are given by $\nu_f = \nu_b(= \nu) = N - n_C$, where $\nu_{b,f} = \sum_p \nu_p^{b,f}$ and $n_C = \sum_p n_p^C$ is the sum of the number of BF pairs n_p^C in the orbit p. The total number of states are then given by

$$W_{\text{tot}} = \sum_{\nu=\nu_{\min}}^{N} W(\nu), \quad W(\nu) = \sum_{p=0}^{p_{\max}} \binom{M}{\lambda}\binom{M-\lambda}{p}\binom{L-M}{\nu-p}\binom{L+\nu-p-1}{\nu}, \qquad (4)$$

where $\nu_{\min} = \max(N - M, 0), p_{\max} = \min(M - \lambda, \nu)$. The quantity $\nu_{\min}$ gives the minimum number of unpaired particles, and for $N \le M$, where all particles can form a pair, $\nu_{\min} = 0$ and $p_{\max} = \nu$. $W(\nu)$ is the number of states with seniority ν. For instance, for $D = 4, M = N = 10$, where total number of states is about 10^{20}, there is only one N-pair state with $\nu = 0$, a free ground state, i.e., bosonic BEC and Fermi degenerate state, hides in the huge number of large seniority states with $W(\nu = 9) \sim 10^{18}$.

Using the method of partial fermionization we can write down the exact eigenvalues for the Hamiltonian. They are characterized by the number of BF pairs n_p^C and boson and fermion seniorities $\nu_p^{b,f}$ as

$$E = \sum_p (\epsilon_p^b + \epsilon_p^f - GQ_p)n_p^C + \sum_p \epsilon_p^b n_p^b + \sum_p \epsilon_p^f n_p^f, \qquad Q_p = D + n_p^b - n_p^f, \qquad (5)$$

where $n_p^{b,f} = n_p^C + \nu_p^{b,f}$ are the number of bosons/fermions in the orbit p. The eigenstates have large degeneracies, which in general depend on the choice of ϵ_k.

In the numerical example below, we take an equal spacing single-particle orbits

$$\epsilon_p^b = \epsilon_p^f \equiv (p-1)\epsilon \qquad (p = 1, 2, \cdots, M) \qquad (6)$$

and consider the case of equal number $N_b = N_f = N$ of bosons and fermions distributed over $M = N$ orbits. We study the ground state for a fixed value of the pairing strength G by comparing low-lying configurations of particles for various seniority numbers n_C of BF pairs, and thus for the number $\nu = \nu_b = \nu_f$ of unpaired particles.

Figure 2 shows the energies of low-lying 100 states for $D = 10, M = N = 20$ against G.

It shows that the character of the ground state changes from high seniority state at small G to low one at large G through a series of level crossings. To understand the result we shall simplify the expression (5) for the ground state, by assuming that for a given n_C (and hence ν), each of the BF pairs, unpaired bosons and fermions independently takes the lowest energy configuration by neglecting blocking effect. Thus we consider that BF pairs occupy the lowest n_C orbits, all unpaired bosons occupy the $p = 1$ state, while unpaired fermions are distributed over the lowest $p_F = \nu/D$ orbits, where we assume p_F to be an integer.
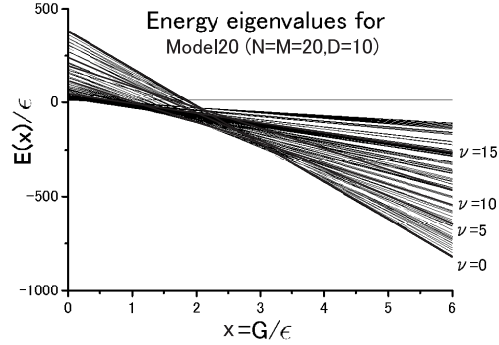

Figure 2.
Energies of selected eigenstates as functions of the interaction strength $x = G/\epsilon$.

The lowest energy E_{n_C} for a given number n_C takes the form

$$E_{n_C} = \epsilon_1(N - n_C) + 2\sum_{p=1}^{n_C} \epsilon_p + D\sum_{p=1}^{p_F} \epsilon_p - GD \max(p_F, n_C). \qquad (7)$$

The value of $G = G_{\max}$ for which all the particles form BF pairs (i.e., the completion of the composite Fermi sea) is obtained by equating E_N and E_{N-1}. This gives the value

$$G_{\max} = \frac{2}{D}(\epsilon_N - \epsilon_1), \quad \text{i.e.,} \quad x_{\max} = \frac{G_{\max}}{\epsilon} \sim \frac{2N}{D}. \qquad (8)$$

This condition shows that the interaction energy $\sim GD$ just compensates the single particle energy of the last orbit $p = N$ of the BF pair. For the above parameters we obtain $x_{\max} \simeq 4$ in agreement with Fig.2. For intermediate values $0 \leq G \leq G_{\max}$ the ground state experiences a sequence of level crossings from no pair ($n_C = 0, \nu = N$) to the fully paired ($n_C = N, \nu = 0$) states. The crossing is studied by comparing E_{n_C} and E_{n_C-1} for a general value of n_C. The envelope of these crossings can be calculated from eq.(7), which gives $E/\epsilon \simeq -(Dx)^2/4$ up to $x = x_{\max}$ which reproduces well the calculated results for the ground state behavior.

Acknowledgments

References
[1] Storozhenko A, Schuck P, Suzuki T, Yabu H and Dukelsky J. 2005 *Phys.Rev.* A **71** 063617.
[2] Yabu H, Takayama T, Suzuki T and Schuck P 2004 *Nucl.Phys.* **A738** 273.
[3] Kagan M Yu, Brodsky I V, Efremov D V and Klaptsov A V 2004 *Phys. Rev.* A**70** 023607.
[4] Watanabe T, Suzuki T and Schuck P 2008 *Phys. Rev.* **A78** 033601.
[5] Watanabe T 2010 Ph.D. thesis, Tokyo Metropolitan University.
[6] An instability of a BF-mixture was reported in Modguno G et. al. 2002 *Science* **297** 2240. This phenomenon has been interpreted as a collapse of the system due to attractive interaction.
[7] Dukelsky J and Pittel S 1990 *Phys.Rev.* C**42** 2030 and private communication.
[8] Suzuki T and Matsuyanagi K 1976 *Prog.Theor.Phys.* **56** 115.