Euler-Lagrange Equations in Partial Differential Equations

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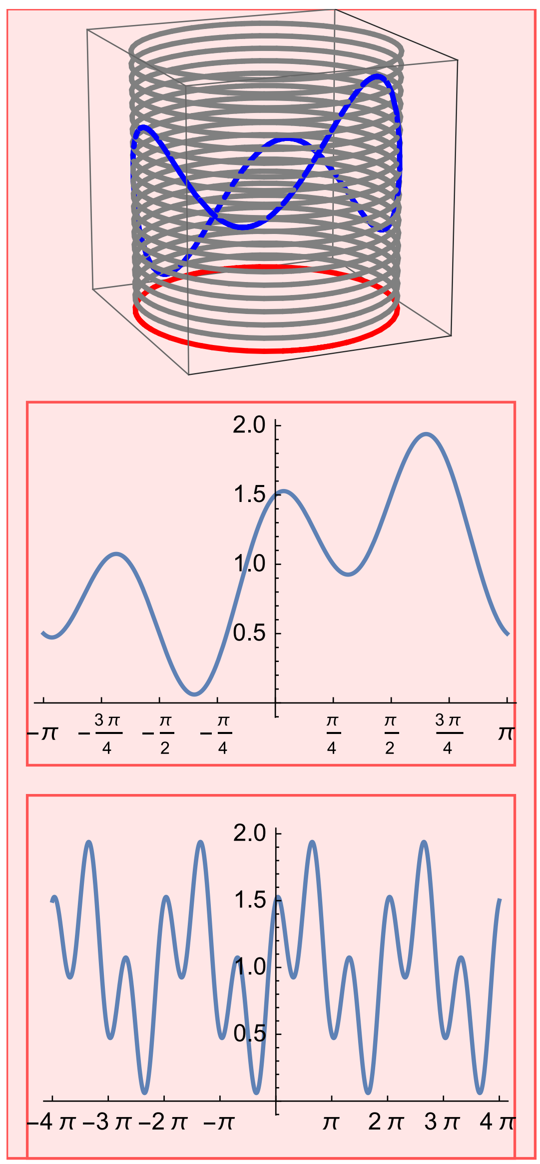
May 18, 2019



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Contents

1 Introduction

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$$e^{i\pi} + 1 = 0.$$

The equation

$$e^{it} = \cos(t) + i\sin(t) \tag{1}$$

which holds for all $t \in \mathbb{R}$ shall be referred to again. In fact, the equation (1) is called Euler's identity.

2 Functional Analysis

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2.1 Semigroups and their infinitesimal generators

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Definition 1. *Suppose that X is a Banach space and $D(A) \subseteq X$ is a linear subspace of X. By a linear operator A on X with domain $D(A)$, we mean a function $A : D(A) \to X$ that is $\mathbb{C}$-linear. We shall say that A is densely defined if $D(A)$ is a dense subset of X with respect to the norm topology.*

Definition 2 (Closed operator)**.** *A linear operator C on X with domain $D(C)$ is said to be closed if for all $\{x_n\}_n \subseteq D(C)$ such that*

$$x_n \to x \qquad \text{and} \qquad Cx_n \to y$$

as $n \to \infty$, we have

$$x \in D(C) \qquad \text{and} \qquad Cx = y.$$

Here is a remark:

Remark 1. *It's silly to make a remark about a remark.*

Now I can talk about the remark, Remark 1 while refering to

Proposition 1 (Basic semigroup facts)**.** *Let $\{T_t\}_{t\geqslant 0}$ be a semigroup on X.*

1. *There are constants $C \geqslant 1$ and $\gamma \geqslant 0$ such that*

$$\|T_t\|_{op} \leqslant Ce^{t\gamma}$$

for all $t \geqslant 0$.

2. *For each $x \in X$, the map $t \to T_t x$ from $[0, \infty)$ into X is continuous.*

Proof. 1. First we establish the existence of C. Suppose that for some sequence of non-negative real numbers $t_n \to 0$ we have $\|T_{t_n}\|_{op} \to \infty$. Then by the uniform boundedness principle, there is $x \in X$ for which

$$\lim_{n\to\infty} \|T_{t_n} x\| = \infty.$$

This cannot be true in view of Property *iii* of Definition 1. Consequently, there must be $C \geqslant 1$ and $\delta > 0$ for which

$$\|T_t\|_{op} \leqslant C \tag{2}$$

for all $t \in [0, \delta]$. Using the semigroup property, it follows that for any $t \geqslant 0$ and natural number n,

$$T_t = T_{nt/n} = (T_{t/n})^n$$

and therefore

$$\|T_t\|_{op} \leqslant \|T_{t/n}\|_{op}^n. \tag{3}$$

So for any $t \in [0, \infty)$ choose a natural number n for which $(n - 1)\delta \leqslant t < n\delta$. Combining (2) and (3) we have

$$\|T_t\|_{op} \leqslant \|T_{t/n}\|_{op}^n \leqslant C^n = CC^{n-1} \leqslant CC^{t/\delta} = Ce^{\gamma t}$$

where $\gamma = (\log(C))/\delta \geqslant 0$. This proves the first part of the proposition.

For the second, observe that for any $x \in X$, $t \in [0, \infty)$ and $h > 0$,

$$\|T_{t+h}x - T_t x\| = \|T_t(T_h x - x)\| \leqslant Ce^{\gamma t}\|T_h x - x\|$$

where we have used the semigroup property. By an appeal to Property *iii*. of Definition 2, the proof is complete. You can also cite references [1] and [2] here. The references [3] and [4]might also be useful. □

References

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