Contents

1 Contest

1.1 Template

template.h, 18 lines

```cpp
#include <bits/stdc++.h>
using namespace std;

#ifdef LOCAL
#include "cp/debug.h"
#else
#define debug(...)
#endif

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

int main() {
  cin.tie(nullptr)->sync_with_stdio(false);
  // freopen("input.txt", "r", stdin);
  // freopen("output.txt", "w", stdout);

  return 0;
}
```

1.2 Debug

Description: c++17 debug template, does not support: arrays (e.g. int arr[N], vector<int> dp[N]).

debug-cpp17.h, 135 lines

```cpp
#undef debug

template<typename T, typename G> ostream& operator<<(ostream& os, pair<T, G> p);

template<size_t N> ostream& operator<<(ostream& os, bitset<N> bs);

template<typename... Ts> ostream& operator<<(ostream& os, tuple<Ts...> tup);

template<typename T, typename = void> struct iterable_std_DA: false_type {};

template<typename T>
struct iterable_std_DA<T, void_t<decltype(declval<T>().begin(), declval<T>().end())>>
  : true_type {};

template<typename T, typename = void> struct non_iterable_std_DA: false_type {};

template<typename T>
struct non_iterable_std_DA<T, void_t<decltype(declval<T>().pop())>>: true_type {};

template<typename T, typename = void> struct stack_like: false_type {};

template<typename T>
struct stack_like<T, void_t<decltype(declval<T>().top())>>: true_type {};

template<typename T, typename = void> struct queue_like: false_type {};

template<typename T>
struct queue_like<T, void_t<decltype(declval<T>().front())>>: true_type {};

template<typename Container>
typename enable_if<iterable_std_DA<Container>::value &&
                   !is_same<Container, string>::value,
  ostream&>::type
operator<<(ostream& os, Container container);

template<typename Container>
typename enable_if<non_iterable_std_DA<Container>::value &&
                   !is_same<Container, string>::value,
  ostream&>::type
operator<<(ostream& os, Container container);

template<typename Container>
typename enable_if<iterable_std_DA<Container>::value &&
                   !is_same<Container, string>::value,
  ostream&>::type
operator<<(ostream& os, Container container) {
  os << "{";
  for (auto it = container.begin(); it != container.end(); ++it) {
    os << (it == container.begin() ? "" : ", ") << *it;
  }
  return os << "}";
}

template<typename Container>
typename enable_if<non_iterable_std_DA<Container>::value &&
                   !is_same<Container, string>::value,
  ostream&>::type
operator<<(ostream& os, Container container) {
  os << "{";
  if constexpr (stack_like<Container>::value) {
    bool first = true;
    while (!container.empty()) {
      if (!first) { os << ", "; }
      first = false;
      os << container.top(), container.pop();
    }
  } else if constexpr (queue_like<Container>::value) {
    bool first = true;
    while (!container.empty()) {
      if (!first) { os << ", "; }
      first = false;
      os << container.front(), container.pop();
    }
  } else {
    // maybe throw an error
  }
  return os << "}";
}

template<typename T, typename G> ostream& operator<<(ostream& os, pair<T, G> p) {
  return os << "(" << p.first << ", " << p.second << ")";
}

template<size_t N> ostream& operator<<(ostream& os, bitset<N> bs) {
  for (size_t i = 0; i < N; ++i) { os << (char) (bs[i] + '0'); }
  return os;
}

// https://en.cppreference.com/w/cpp/utility/integer_sequence
template<typename Tup, size_t... Is>
void print_tuple_impl(ostream& os, const Tup& tup, index_sequence<Is...>) {
  ((os << (Is == 0 ? "" : ", ") << get<Is>(tup)), ...);
}

template<typename... Ts> ostream& operator<<(ostream& os, tuple<Ts...> tup) {
  os << "(";
  print_tuple_impl(os, tup, index_sequence_for<Ts...>{});
  return os << ")";
}

// https://codeforces.com/blog/entry/125435
template<typename H, typename... T>
void debug(const char *names, H&& head, T&&...tail) {
  int i = 0;
  for (size_t bracket = 0; names[i] != '\0' && (names[i] != ',' || bracket != 0);
       i++) {
    if (names[i] == '(' || names[i] == '<' || names[i] == '{') {
      bracket++;
    } else if (names[i] == ')' || names[i] == '>' || names[i] == '}') {
      bracket--;
    }
  }
  cerr << "[", cerr.write(names, i) << " = " << head << "]";
  if constexpr (sizeof...(tail)) {
```

```
    while (names[i] != '\0' && names[i + 1] == ' ') { ++i; }
    cerr << " ";
    debug(names + i + 1, tail...);
  } else {
    cerr << '\n';
  }
}

using high_clock = chrono::high_resolution_clock;
auto start_time = high_clock::now();
int elapsed_time() {
  auto elapsed = high_clock::now() - start_time;
  start_time = high_clock::now();
  return chrono::duration_cast<chrono::milliseconds>(elapsed).count();
}

#define debug(...)                                       \
  {                                                      \
    cerr << __FUNCTION__ << ":" << __LINE__ << ": "; \
    debug(#__VA_ARGS__, __VA_ARGS__);                    \
  }
```

1.3 Java

template.java, 50 lines

```
import java.io.BufferedReader;
import java.util.StringTokenizer;
import java.io.IOException;
import java.io.InputStreamReader;
import java.io.PrintWriter;
import java.util.ArrayList;
import java.util.Arrays;
import java.util.Collections;
import java.util.Random;

public class Main {
    public static void main(String[] args) {
        FastScanner fs = new FastScanner();
        PrintWriter out = new PrintWriter(System.out);
        int n = fs.nextInt();
        out.println(n);
        out.close(); // don't forget this line.
    }
    static class FastScanner {
        BufferedReader br;
        StringTokenizer st;
        public FastScanner() {
            br = new BufferedReader(new InputStreamReader(System.in));
            st = null;
        }
        public String next() {
            while (st == null || st.hasMoreTokens() == false) {
                try {
                    st = new StringTokenizer(br.readLine());
                }
                catch (IOException e) {
                    throw new RuntimeException(e);
                }
            }
            return st.nextToken();
        }

        public int nextInt() {
            return Integer.parseInt(next());
        }

        public long nextLong() {
            return Long.parseLong(next());
        }

        public double nextDouble() {
            return Double.parseDouble(next());
        }
    }
}
```

1.4 sublime-build

c++17.sublime-build, 14 lines

```
// location: ~/.config/sublime-text/Packages/User/
// tip: sample file can be found at Tools > Developer > View Package File > 'C++
    Single File.sublime-build'
{
    "shell_cmd": "g++ -std=c++17 -DLOCAL -Wall -Wextra -Wfloat-equal -Wconversion
    -fmax-errors=3 \"${file}\" -o \"${file_path}/${file_base_name}.out\"",
    "file_regex": "^(..[^:]*):([0-9]+):?([0-9]+)?:? (.*)$",
    "working_dir": "${file_path}",
    "selector": "source.cpp, source.c++",
    "variants": [
        {
            "name": "build and run",
            "shell_cmd": "g++ -std=c++17 -DLOCAL -Wall -Wextra -Wfloat-equal
    -fmax-errors=3 \"${file}\" -o \"${file_path}/a.out\"; \"${file_path}/a.out\" <
    input > output 2> err"
        }
    ]
}
```

1.5 vscode

tasks.json, 25 lines

```
// location: ~/.vscode or ~/.config/Code/User/
{
    "version": "2.0.0",
    "tasks": [
        {
            "type": "shell",
            "label": "c++17 build",
            "command": "g++ -std=c++17 -DLOCAL -Wall -Wextra -Wfloat-equal
    -Wconversion -fmax-errors=3 \"${file}\" -o
    \"${fileDirname}/${fileBasenameNoExtension}.out\"",
            "group": {
                "kind": "build"
                // "isDefault": true
            },
        },
        {
            "type": "shell",
            "label": "c++17 build and run",
            "dependsOn": ["c++17 build"],
            "command": "\"${fileDirname}/${fileBasenameNoExtension}.out\" < input >
    output 2> err",
            "group": {
                "kind": "build"
                // "isDefault": true
            },
        }
    ]
}
```

2 Data structures

2.1 RMQ

Description: range minimum queries on a static array.
Time: $< O(N \log N), O(1) >$.

rmq.h, 20 lines

```
template<typename T> struct RMQ {
```

```
  static const int LOG = 21;
  int n;
  vector<vector<T>> table;
  RMQ(const vector<T>& arr) {
    n = (int) arr.size();
    table.assign(LOG, vector<T>(n));
    table[0] = arr;
    for (int j = 1; j < LOG; ++j) {
      for (int i = 0; i + (1 << j) - 1 < n; ++i) {
        table[j][i] = min(table[j - 1][i], table[j - 1][i + (1 << (j - 1))]);
      }
    }
  }
  T get(int l, int r) {
    assert(0 <= l && l <= r && r < n);
    int i = __lg(r - l + 1);
    return min(table[i][l], table[i][r - (1 << i) + 1]);
  }
};
```

2.2 Ordered set/map

ordered_set.h, 24 lines

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;

template<typename K, typename V, typename comp = less<K>>
using ordered_map = tree<K, V, comp, rb_tree_tag, tree_order_statistics_node_update>;

template<typename K, typename comp = less<K>>
using ordered_set = ordered_map<K, null_type, comp>;

const int INF = 0x3f3f3f3f;
void example() {
  vector<int> nums = {1, 2, 3, 5, 10};
  ordered_set<int> st(nums.begin(), nums.end());

  cout << *st.find_by_order(0) << '\n'; // 1
  assert(st.find_by_order(-INF) == st.end());
  assert(st.find_by_order(INF) == st.end());

  cout << st.order_of_key(2) << '\n'; // 1
  cout << st.order_of_key(4) << '\n'; // 3
  cout << st.order_of_key(9) << '\n'; // 4
  cout << st.order_of_key(-INF) << '\n'; // 0
  cout << st.order_of_key(INF) << '\n'; // 5
}
```

2.3 Dsu

dsu.h, 37 lines

```
struct Dsu {
  int n;
  vector<int> par, sz;
  Dsu(int _n): n(_n) {
    sz.resize(n, 1);
    par.resize(n);
    iota(par.begin(), par.end(), 0);
  }
  int find(int v) {
    // finding leader/parrent of set that contains the element v.
    // with {path compression optimization}.
    return (v == par[v] ? v : par[v] = find(par[v]));
  }
  bool same(int u, int v) { return find(u) == find(v); }
  bool unite(int u, int v) {
    u = find(u);
    v = find(v);
    if (u == v) { return false; }
    if (sz[u] < sz[v]) { swap(u, v); }
    par[v] = u;
    sz[u] += sz[v];
    return true;
  }
  vector<vector<int>> groups() {
    // returns the list of the "list of the vertices in a connected component".
    vector<int> leader(n);
    for (int i = 0; i < n; ++i) { leader[i] = find(i); }
    vector<int> id(n, -1);
    int count = 0;
    for (int i = 0; i < n; ++i) {
      if (id[leader[i]] == -1) { id[leader[i]] = count++; }
    }
    vector<vector<int>> result(count);
    for (int i = 0; i < n; ++i) { result[id[leader[i]]].push_back(i); }
    return result;
  }
};
```

2.4 MinQueue

Description: acts like normal std::queue except it supports get minimum value in current queue.

min_queue.h, 30 lines

```
template<typename T> struct MinQueue {
  vector<T> vals;
  int ptr = 0;
  vector<int> st;
  int ptr_idx = 0;
  void push(T val) {
    while ((int) st.size() > ptr_idx && vals[st.back()] >= val) { st.pop_back(); }
    st.push_back((int) vals.size());
    vals.push_back(val);
  }
  void pop() {
    assert(ptr < (int) vals.size());
    if (ptr_idx < (int) st.size() && st[ptr_idx] == ptr) { ptr_idx++; }
    ptr++;
  }
  T get() {
    assert(ptr_idx < (int) st.size());
    return vals[st[ptr_idx]];
  }
  int front() {
    assert(!empty());
    return vals[ptr];
  }
  int back() {
    assert(!empty());
    return vals.back();
  }
  bool empty() { return (ptr == (int) vals.size()); }
  int size() { return ((int) vals.size() - ptr); }
};
```

2.5 Segment tree

Description: A segment tree with range updates and sum queries that supports three types of operations:
- Increase each value in range [l, r] by x (i.e. a[i] += x).
- Set each value in range [l, r] to x (i.e. a[i] = x).
- Determine the sum of values in range [l, r].

segment_tree.h, 68 lines

```
struct SegmentTree {
  int n;
  vector<long long> tree, lazy_add, lazy_set;
  SegmentTree(int _n): n(_n) {
    int p = 1;
    while (p < n) { p *= 2; }
    tree.resize(p * 2);
```

```
    lazy_add.resize(p * 2);
    lazy_set.resize(p * 2);
  }
  long long merge(const long long& left, const long long& right) {
    return left + right;
  }
  void build(int id, int l, int r, const vector<int>& arr) {
    if (l == r) {
      tree[id] += arr[l];
      return;
    }
    int mid = (l + r) >> 1;
    build(id * 2, l, mid, arr);
    build(id * 2 + 1, mid + 1, r, arr);
    tree[id] = merge(tree[id * 2], tree[id * 2 + 1]);
  }
  void push(int id, int l, int r) {
    if (lazy_set[id] == 0 && lazy_add[id] == 0) { return; }
    int mid = (l + r) >> 1;
    for (int child : {id * 2, id * 2 + 1}) {
      int range = (child == id * 2 ? mid - l + 1 : r - mid);
      if (lazy_set[id] != 0) {
        lazy_add[child] = 0;
        lazy_set[child] = lazy_set[id];
        tree[child] = range * lazy_set[id];
      }
      lazy_add[child] += lazy_add[id];
      tree[child] += range * lazy_add[id];
    }
    lazy_add[id] = lazy_set[id] = 0;
  }
  void update(
    int id, int l, int r, int u, int v, int amount, bool set_value = false) {
    if (r < u || l > v) { return; }
    if (u <= l && r <= v) {
      if (set_value) {
        tree[id] = 1LL * amount * (r - l + 1);
        lazy_set[id] = amount;
        lazy_add[id] = 0; // clear all previous updates.
      } else {
        tree[id] += 1LL * amount * (r - l + 1);
        lazy_add[id] += amount;
      }
      return;
    }
    push(id, l, r);
    int mid = (l + r) >> 1;
    update(id * 2, l, mid, u, v, amount, set_value);
    update(id * 2 + 1, mid + 1, r, u, v, amount, set_value);
    tree[id] = merge(tree[id * 2], tree[id * 2 + 1]);
  }
  long long get(int id, int l, int r, int u, int v) {
    if (r < u || l > v) { return 0; }
    if (u <= l && r <= v) { return tree[id]; }
    push(id, l, r);
    int mid = (l + r) >> 1;
    long long left = get(id * 2, l, mid, u, v);
    long long right = get(id * 2 + 1, mid + 1, r, u, v);
    return merge(left, right);
  }
};
```

2.6 Efficient segment tree

efficient_segment_tree.h, 26 lines

```
template<typename T> struct SegmentTree {
  int n;
  vector<T> tree;
  SegmentTree(int _n): n(_n), tree(2 * n) {}
  T merge(const T& left, const T& right) { return left + right; }
  template<typename G> void build(const vector<G>& initial) {
    assert((int) initial.size() == n);
    for (int i = 0; i < n; ++i) { tree[i + n] = initial[i]; }
    for (int i = n - 1; i > 0; --i) {
      tree[i] = merge(tree[i * 2], tree[i * 2 + 1]);
    }
  }
  void modify(int i, int v) {
    tree[i += n] = v;
    for (i /= 2; i > 0; i /= 2) { tree[i] = merge(tree[i * 2], tree[i * 2 + 1]); }
  }
  T get_sum(int l, int r) {
    // sum of elements from l to r - 1.
    T ret{};
    for (l += n, r += n; l < r; l /= 2, r /= 2) {
      if (l & 1) { ret = merge(ret, tree[l++]); }
      if (r & 1) { ret = merge(ret, tree[--r]); }
    }
    return ret;
  }
};
```

2.7 Persistent lazy segment tree

persistent_lazy_segment_tree.h, 65 lines

```
struct Vertex {
  int l, r;
  long long val, lazy;
  bool has_changed = false;
  Vertex() {}
  Vertex(int _l, int _r, long long _val, int _lazy = 0):
    l(_l), r(_r), val(_val), lazy(_lazy) {}
};
struct PerSegmentTree {
  vector<Vertex> tree;
  vector<int> root;
  int build(const vector<int>& arr, int l, int r) {
    if (l == r) {
      tree.emplace_back(-1, -1, arr[l]);
      return tree.size() - 1;
    }
    int mid = (l + r) / 2;
    int left = build(arr, l, mid);
    int right = build(arr, mid + 1, r);
    tree.emplace_back(left, right, tree[left].val + tree[right].val);
    return tree.size() - 1;
  }
  int add(int x, int l, int r, int u, int v, int amt) {
    if (l > v || r < u) { return x; }
    if (u <= l && r <= v) {
      tree.emplace_back(tree[x].l, tree[x].r, tree[x].val + 1LL * amt * (r - l + 1),
        tree[x].lazy + amt);
      tree.back().has_changed = true;
      return tree.size() - 1;
    }
    int mid = (l + r) >> 1;
    push(x, l, mid, r);
    int left = add(tree[x].l, l, mid, u, v, amt);
    int right = add(tree[x].r, mid + 1, r, u, v, amt);
    tree.emplace_back(left, right, tree[left].val + tree[right].val, 0);
    return tree.size() - 1;
  }
  long long get_sum(int x, int l, int r, int u, int v) {
    if (r < u || l > v) { return 0; }
    if (u <= l && r <= v) { return tree[x].val; }
    int mid = (l + r) / 2;
    push(x, l, mid, r);
```

```
    return get_sum(tree[x].l, l, mid, u, v) + get_sum(tree[x].r, mid + 1, r, u, v);
  }
  void push(int x, int l, int mid, int r) {
    if (!tree[x].has_changed) { return; }
    Vertex left = tree[tree[x].l];
    Vertex right = tree[tree[x].r];
    tree.emplace_back(left);
    tree[x].l = tree.size() - 1;
    tree.emplace_back(right);
    tree[x].r = tree.size() - 1;

    tree[tree[x].l].val += tree[x].lazy * (mid - l + 1);
    tree[tree[x].l].lazy += tree[x].lazy;

    tree[tree[x].r].val += tree[x].lazy * (r - mid);
    tree[tree[x].r].lazy += tree[x].lazy;

    tree[tree[x].l].has_changed = true;
    tree[tree[x].r].has_changed = true;
    tree[x].lazy = 0;
    tree[x].has_changed = false;
  }
};
```

2.8 Lichao tree

Description: A segment tree that allows inserting a new line and querying for minimum value over all lines at point x.
Usage: useful in convex hull trick.

lichao_tree.h, 41 lines

```
const long long INF_LL = (long long) 4e18;

struct Line {
  long long a, b;
  Line(long long _a = 0, long long _b = INF_LL): a(_a), b(_b) {}
  long long operator()(long long x) { return a * x + b; }
};

struct SegmentTree { // min query
  int n;
  vector<Line> tree;
  SegmentTree() {}
  SegmentTree(int _n): n(1) {
    while (n < _n) { n *= 2; }
    tree.resize(n * 2);
  }
  void insert(int x, int l, int r, Line line) {
    if (l == r) {
      if (line(l) < tree[x](l)) { tree[x] = line; }
      return;
    }
    int mid = (l + r) >> 1;
    bool b_left = line(l) < tree[x](l);
    bool b_mid = line(mid) < tree[x](mid);
    if (b_mid) { swap(tree[x], line); }
    if (b_left != b_mid) {
      insert(x * 2, l, mid, line);
    } else {
      insert(x * 2 + 1, mid + 1, r, line);
    }
  }
  long long query(int x, int l, int r, int at) {
    if (l == r) { return tree[x](at); }
    int mid = (l + r) >> 1;
    if (at <= mid) {
      return min(tree[x](at), query(x * 2, l, mid, at));
    } else {
      return min(tree[x](at), query(x * 2 + 1, mid + 1, r, at));
    }
  }
};
```

2.9 Old driver tree (Chtholly tree)

Description: An optimized brute-force approach to deal with problems that have operation of setting an interval to the same number.
Note: only works when inputs are random, otherwise it will TLE.

old_driver_tree.h, 58 lines

```
#include "../number-theory/modmul.h"
struct ODT {
  map<int, long long> tree;
  using It = map<int, long long>::iterator;

  It split(int x) {
    It it = tree.upper_bound(x);
    assert(it != tree.begin());
    --it;
    if (it->first == x) { return it; }
    return tree.emplace(x, it->second).first;
  }

  void add(int l, int r, int amt) {
    It it_l = split(l);
    It it_r = split(r + 1);
    while (it_l != it_r) {
      it_l->second += amt;
      ++it_l;
    }
  }

  void set(int l, int r, int v) {
    It it_l = split(l);
    It it_r = split(r + 1);
    while (it_l != it_r) { tree.erase(it_l++); }
    tree[l] = v;
  }

  long long kth_smallest(int l, int r, int k) {
    // return the k-th smallest value in range [l..r]
    vector<pair<long long, int>> values; // pair(value, count)
    It it_l = split(l);
    It it_r = split(r + 1);
    while (it_l != it_r) {
      It prev = it_l++;
      values.emplace_back(prev->second, it_l->first - prev->first);
    }
    sort(values.begin(), values.end());
    for (auto [value, cnt] : values) {
      if (k <= cnt) { return value; }
      k -= cnt;
    }
    return -1;
  }
  int sum_of_xth_power(int l, int r, int x, int mod) {
    It it_l = split(l);
    It it_r = split(r + 1);
    int res = 0;
    while (it_l != it_r) {
      It prev = it_l++;
      res =
        (res + 1LL * (it_l->first - prev->first) * modpow(prev->second, x, mod)) %
        mod;
    }
    return res;
  }
};
```

2.10 Disjoint sparse table

Description: range query on a static array.
Time: $O(1)$ per query.

disjoint_sparse_table.h, 37 lines

```
const int MOD = (int) 1e9 + 7;
struct DisjointSparseTable { // product queries.
  int n, h;
  vector<vector<int>> dst;
  vector<int> lg;
  DisjointSparseTable(int _n): n(_n) {
    h = 1; // in case n = 1: h = 0 !!.
    int p = 1;
    while (p < n) { p *= 2, h++; }
    lg.resize(p);
    lg[1] = 0;
    for (int i = 2; i < p; ++i) { lg[i] = 1 + lg[i / 2]; }
    dst.resize(h, vector<int>(n));
  }
  void build(const vector<int>& A) {
    for (int lv = 0; lv < h; ++lv) {
      int len = (1 << lv);
      for (int k = 0; k < n; k += len * 2) {
        int mid = min(k + len, n);
        dst[lv][mid - 1] = A[mid - 1] % MOD;
        for (int i = mid - 2; i >= k; --i) {
          dst[lv][i] = 1LL * A[i] * dst[lv][i + 1] % MOD;
        }
        if (mid == n) { break; }
        dst[lv][mid] = A[mid] % MOD;
        for (int i = mid + 1; i < min(mid + len, n); ++i) {
          dst[lv][i] = 1LL * A[i] * dst[lv][i - 1] % MOD;
        }
      }
    }
  }
  int get(int l, int r) {
    if (l == r) { return dst[0][l]; }
    int i = lg[l ^ r];
    return 1LL * dst[i][l] * dst[i][r] % MOD;
  }
};
```

2.11 Fenwick tree

Description: range update and range sum query.

fenwick_tree.h, 53 lines

```
using tree_type = long long;
struct FenwickTree {
  int n;
  vector<tree_type> fenw_coeff, fenw;
  FenwickTree() {}
  FenwickTree(int _n): n(_n) {
    fenw_coeff.assign(n, 0); // fenwick tree with coefficient (n - i).
    fenw.assign(n, 0); // normal fenwick tree.
  }
  template<typename G> void build(const vector<G>& A) {
    assert((int) A.size() == n);
    vector<int> diff(n);
    diff[0] = A[0];
    for (int i = 1; i < n; ++i) { diff[i] = A[i] - A[i - 1]; }
    fenw_coeff[0] = (long long) diff[0] * n;
    fenw[0] = diff[0];
    for (int i = 1; i < n; ++i) {
      fenw_coeff[i] = fenw_coeff[i - 1] + (long long) diff[i] * (n - i);
      fenw[i] = fenw[i - 1] + diff[i];
    }
    for (int i = n - 1; i >= 0; --i) {
      int j = (i & (i + 1)) - 1;
      if (j >= 0) {
        fenw_coeff[i] -= fenw_coeff[j];
        fenw[i] -= fenw[j];
      }
    }
  }
  void add(vector<tree_type>& fenw, int i, tree_type val) {
    while (i < n) {
      fenw[i] += val;
      i |= (i + 1);
    }
  }
  tree_type __prefix_sum(vector<tree_type>& fenw, int i) {
    tree_type res{};
    while (i >= 0) {
      res += fenw[i];
      i = (i & (i + 1)) - 1;
    }
    return res;
  }
  tree_type prefix_sum(int i) {
    return __prefix_sum(fenw_coeff, i) - __prefix_sum(fenw, i) * (n - i - 1);
  }
  void range_add(int l, int r, tree_type val) {
    add(fenw_coeff, l, (n - l) * val);
    add(fenw_coeff, r + 1, (n - r - 1) * (-val));
    add(fenw, l, val);
    add(fenw, r + 1, -val);
  }
  tree_type range_sum(int l, int r) { return prefix_sum(r) - prefix_sum(l - 1); }
};
```

2.12 Fenwick tree 2D

Description: range update and range sum query on a 2D array.

fenwick_tree_2d.h, 40 lines

```
using tree_type = long long;
struct FenwickTree2D {
  int n, m;
  vector<vector<tree_type>> fenw[4];
  FenwickTree2D(int _n, int _m): n(_n), m(_m) {
    for (int i = 0; i < 4; i++) { fenw[i].resize(n, vector<tree_type>(m)); }
  }
  void add(int u, int v, tree_type val) {
    for (int i = u; i < n; i |= (i + 1)) {
      for (int j = v; j < m; j |= (j + 1)) {
        fenw[0][i][j] += val;
        fenw[1][i][j] += (u + 1) * val;
        fenw[2][i][j] += (v + 1) * val;
        fenw[3][i][j] += (u + 1) * (v + 1) * val;
      }
    }
  }
  void range_add(int r, int c, int rr, int cc, tree_type val) { // [r, rr] x [c, cc].
    add(r, c, val);
    add(r, cc + 1, -val);
    add(rr + 1, c, -val);
    add(rr + 1, cc + 1, val);
  }
  tree_type prefix_sum(int u, int v) {
    tree_type res{};
    for (int i = u; i >= 0; i = (i & (i + 1)) - 1) {
      for (int j = v; j >= 0; j = (j & (j + 1)) - 1) {
        res += (u + 2) * (v + 2) * fenw[0][i][j];
        res -= (v + 2) * fenw[1][i][j];
        res -= (u + 2) * fenw[2][i][j];
        res += fenw[3][i][j];
      }
```

```
    }
    return res;
  }
  tree_type range_sum(int r, int c, int rr, int cc) { // [r, rr] x [c, cc].
    return prefix_sum(rr, cc) - prefix_sum(r - 1, cc) - prefix_sum(rr, c - 1) +
           prefix_sum(r - 1, c - 1);
  }
};
```

2.13 Treap

treap.h, 94 lines

```
struct Node {
  int val, prior, cnt;
  bool rev;
  Node *left, *right;
  Node() {}
  Node(int _val):
    val(_val), prior(rng()), cnt(1), rev(false), left(nullptr), right(nullptr) {}
};

int get_cnt(Node *n) { return n ? n->cnt : 0; }
void pull(Node *& n) {
  if (!n) { return; }
  n->cnt = get_cnt(n->left) + get_cnt(n->right) + 1;
}
void push(Node *treap) {
  if (!treap || !treap->rev) { return; }
  treap->rev = false;
  swap(treap->left, treap->right);
  if (treap->left) { treap->left->rev ^= true; }
  if (treap->right) { treap->right->rev ^= true; }
}
struct Treap {
  Node *root;
  bool implicit_key;
  Treap(bool _implicit_key = true): root(nullptr), implicit_key(_implicit_key) {}
  bool go_right(Node *treap, int pos_or_val) {
    if (implicit_key) {
      int local_idx = get_cnt(treap->left);
      return local_idx <= pos_or_val;
    }
    return treap->val <= pos_or_val;
  }
  pair<Node *, Node *> split(Node *treap, int pos_or_val) {
    // normal treap -> Left: all nodes having val <= val
    // implicit treap -> Left: all nodes having index <= pos
    if (!treap) { return {}; }
    push(treap);
    if (go_right(treap, pos_or_val)) {
      if (implicit_key) { pos_or_val -= (get_cnt(treap->left) + 1); }
      auto pr = split(treap->right, pos_or_val);
      treap->right = pr.first;
      pull(treap);
      return {treap, pr.second};
    } else {
      auto pl = split(treap->left, pos_or_val);
      treap->left = pl.second;
      pull(treap);
      return {pl.first, treap};
    }
  }
  tuple<Node *, Node *, Node *> split(int u, int v) {
    auto [l, rem] = split(root, u - 1);
    auto [mid, r] = split(rem, v - (implicit_key ? u : 0));
    return {l, mid, r};
  }
  Node *merge(Node *l, Node *r) {
    push(l);
    push(r);
    if (!l || !r) { return (l ? l : r); }
    if (l->prior < r->prior) {
      l->right = merge(l->right, r);
      pull(l);
      return l;
    } else {
      r->left = merge(l, r->left);
      pull(r);
      return r;
    }
  }
  void insert(int pos, int val) {
    auto [l, r] = split(root, pos - 1);
    root = merge(merge(l, new Node(val)), r);
  }
  void insert(int val) { insert(val, val); }
  void erase(int u, int v) {
    auto [l, mid, r] = split(u, v);
    root = merge(l, r);
  }
  void reverse(int u, int v) {
    auto [l, mid, r] = split(u, v);
    mid->rev ^= true;
    root = merge(merge(l, mid), r);
  }
  int get_kth(Node *treap, int k) {
    if (!treap) { return (int) 1e9; }
    int sz = get_cnt(treap->left) + 1;
    if (sz == k) {
      return treap->val;
    } else if (sz < k) {
      return get_kth(treap->right, k - sz);
    }
    return get_kth(treap->left, k);
  }
};
```

2.14 Line container

Description: container that allow you can add lines in form $ax + b$ and query maximum value at x.

line_container.h, 40 lines

```
using num_t = int;
struct Line {
  num_t a, b; // ax + b
  mutable num_t x; // x-intersect with the next line in the hull
  bool operator<(const Line& other) const { return a < other.a; }
  bool operator<(num_t other_x) const { return x < other_x; }
};

struct LineContainer: multiset<Line, less<>> { // max-query
  // for doubles, use INF = 1 / 0.0
  static const num_t INF = numeric_limits<num_t>::max();

  num_t floor_div(num_t a, num_t b) { return a / b - ((a ^ b) < 0 && a % b != 0); }
  bool isect(iterator u, iterator v) {
    if (v == end()) {
      u->x = INF;
      return false;
    }
    if (u->a == v->a) {
      u->x = (u->b > v->b ? INF : -INF);
    } else {
      u->x = floor_div(v->b - u->b, u->a - v->a);
    }
    return u->x >= v->x;
  }
  void add(num_t a, num_t b) {
    auto z = insert({a, b, 0}), y = z++, x = y;
```

```
    while (isect(y, z)) { z = erase(z); }
    if (x != begin() && isect(--x, y)) {
      y = erase(y);
      isect(x, y);
    }
    while ((y = x) != begin() && (--x)->x >= y->x) { isect(x, erase(y)); }
  }
  num_t query(num_t x) {
    assert(!empty());
    auto it = *lower_bound(x);
    return it.a * x + it.b;
  }
};
```

3 Mathematics

3.1 Trigonometry

3.1.1 Sum - difference identities

$$\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$$
$$\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$$
$$\tan(u \pm v) = \frac{\tan(u) \pm \tan(v)}{1 \mp \tan(u)\tan(v)}$$

3.1.2 Sum to product identities

$$\cos(u) + \cos(v) = 2\cos(\frac{u+v}{2})\cos(\frac{u-v}{2}) \qquad \sin(u) + \sin(v) = 2\sin(\frac{u+v}{2})\cos(\frac{u-v}{2})$$
$$\cos(u) - \cos(v) = -2\sin(\frac{u+v}{2})\sin(\frac{u-v}{2}) \qquad \sin(u) - \sin(v) = 2\cos(\frac{u+v}{2})\sin(\frac{u-v}{2})$$

3.1.3 Product identities

$$\cos(u)\cos(v) = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$$
$$\sin(u)\sin(v) = -\frac{1}{2}[\cos(u+v) - \cos(u-v)]$$
$$\sin(u)\cos(v) = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

3.1.4 Double - triple angle identities

$$\sin(2u) = 2\sin(u)\cos(u) \qquad \sin(3u) = 3\sin(u) - 4\sin^3(u)$$
$$\cos(2u) = 2\cos^2(u) - 1 = 1 - 2\sin^2(u) \qquad \cos(3u) = 4\cos^3(u) - 3\cos(u)$$
$$\tan(2u) = \frac{2\tan(u)}{1 - \tan^2(u)} \qquad \tan(3u) = \frac{3\tan(u) - \tan^3(u)}{1 - 3\tan^2(u)}$$

3.2 Sums

$$\sum_{i=a}^{b} c^i = \frac{c^{b+1} - c^a}{c-1},\ c \neq 1 \qquad \sum_{i=1}^{n} i^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$$
$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2},\ c \neq 1 \qquad \sum_{i=1}^{n} i^7 = \frac{n^2(n+1)^2(3n^4+6n^3-n^2-4n+2)}{24}$$
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i = (a+b)^n$$
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=0}^{n} i\binom{n}{i} = n2^{n-1}$$
$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 \qquad \sum_{i=0}^{n} \frac{\binom{n}{i}}{i+1} = \frac{2^{n+1}-1}{n+1}$$
$$\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \qquad \sum_{k=0}^{m} \binom{n+k}{n} = \binom{n+m+1}{n+1}$$
$$\sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12} \qquad \sum_{i=k}^{n} \binom{i}{k} = \binom{n+1}{k+1}$$

3.3 Pythagorean triple

- A Pythagorean triple is a triple of positive integers a, b, and c such that $a^2 + b^2 = c^2$.
- If (a, b, c) is a Pythagorean triple, then so is (ka, kb, kc) for any positive integer k.
- A primitive Pythagorean triple is one in which a, b, and c are coprime.
- Generating Pythagorean triple
  - Euclid's formula: with arbitrary $0 < n < m$, then:
  $$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2$$
  form a Pythagorean triple.
  - To obtain primitive Pythagorean triple, this condition must hold: m and n are coprime, m and n have opposite parity.

4 String

4.1 Prefix function

Description: the prefix function of a string s is defined as an array pi of length n, where pi[i] is the length of the longest proper prefix of the sub-string s[0..i] which is also a suffix of this sub-string.
Time: $O(|S|)$.

prefix_function.h, 11 lines

```
vector<int> prefix_function(const string& s) {
  int n = (int) s.length();
  vector<int> pi(n);
  pi[0] = 0;
```

```
  for (int i = 1; i < n; ++i) {
    int j = pi[i - 1]; // try length pi[i - 1] + 1.
    while (j > 0 && s[j] != s[i]) { j = pi[j - 1]; }
    if (s[j] == s[i]) { pi[i] = j + 1; }
  }
  return pi;
}
```

4.2 Z function

Description: for a given string 's', z[i] = longest common prefix of 's' and suffix starting at 'i'. z[0] is generally not well defined (this implementation below assume z[0] = 0).
Time: $O(n)$.

z_function.h, 15 lines

```
vector<int> z_function(const string& s) {
  int n = (int) s.size();
  vector<int> z(n);
  z[0] = 0;
  // [l, r)
  for (int i = 1, l = 0, r = 0; i < n; ++i) {
    if (i < r) { z[i] = min(r - i, z[i - l]); }
    while (i + z[i] < n && s[z[i]] == s[i + z[i]]) { ++z[i]; }
    if (i + z[i] > r) {
      l = i;
      r = i + z[i];
    }
  }
  return z;
}
```

4.3 Counting occurrences of each prefix

Description: count the number of occurrences of each prefix in the given string.
Time: $O(n)$.

counting_occur_of_prefix.h, 12 lines

```
#include "prefix_function.h"
vector<int> count_occurrences(const string& s) {
  vector<int> pi = prefix_function(s);
  int n = (int) s.size();
  vector<int> ans(n + 1);
  for (int i = 0; i < n; ++i) { ans[pi[i]]++; }
  for (int i = n - 1; i > 0; --i) { ans[pi[i - 1]] += ans[i]; }
  for (int i = 0; i <= n; ++i) { ans[i]++; }
  return ans;
  // Input: ABACABA
  // Output: 4 2 2 1 1 1 1
}
```

4.4 Knuth–Morris–Pratt algorithm

Description: searching for a sub-string in a string.
Time: $O(N + M)$.

KMP.h, 11 lines

```
#include "prefix_function.h"
vector<int> KMP(const string& text, const string& pattern) {
  int n = (int) pattern.length();
  string combined = pattern + '$' + text;
  vector<int> pi = prefix_function(combined);
  vector<int> indices;
  for (int i = 0; i < (int) combined.length(); ++i) {
    if (pi[i] == n) { indices.push_back(i - 2 * n); }
  }
  return indices;
}
```

4.5 Suffix array

Description: suffix array is a sorted array of all the suffixes of a given string.

Usage:
- sa[i] = starting index of the i-th smallest suffix.
- rank[i] = rank of the suffix starting at 'i'.
- lcp[i] = longest common prefix between 'sa[i - 1]' and 'sa[i]'
- for arbitrary 'u v', let i = rank[u] - 1, j = rank[v] - 1 (assume i < j), then longest_common_prefix(u, v) = min(lcp[i + 1], lcp[i + 2], ..., lcp[j])

Time: $O(N \log N)$.

suffix_array.h, 47 lines

```
struct SuffixArray {
  string s;
  int n, lim;
  vector<int> sa, lcp, rank;
  SuffixArray(const string& _s, int _lim = 256):
    s(_s), n(s.length() + 1), lim(_lim), sa(n), lcp(n), rank(n) {
    s += '$';
    build();
    kasai();
    sa.erase(sa.begin());
    lcp.erase(lcp.begin());
    rank.pop_back();
    s.pop_back();
  }
  void build() {
    vector<int> nrank(n), norder(n), cnt(max(n, lim));
    for (int i = 0; i < n; ++i) {
      sa[i] = i;
      rank[i] = s[i];
    }
    for (int k = 0, rank_cnt = 0; rank_cnt < n - 1;
         k = max(1, k * 2), lim = rank_cnt + 1) {
      for (int i = 0; i < n; ++i) {
        norder[i] = (sa[i] - k + n) % n;
        cnt[rank[i]]++;
      }
      for (int i = 1; i < lim; ++i) { cnt[i] += cnt[i - 1]; }
      for (int i = n - 1; i >= 0; --i) { sa[--cnt[rank[norder[i]]]] = norder[i]; }
      rank[sa[0]] = rank_cnt = 0;
      for (int i = 1; i < n; ++i) {
        int u = sa[i], v = sa[i - 1];
        int nu = (u + k) % n, nv = (v + k) % n;
        if (rank[u] != rank[v] || rank[nu] != rank[nv]) { ++rank_cnt; }
        nrank[sa[i]] = rank_cnt;
      }
      for (int i = 0; i < rank_cnt + 1; ++i) { cnt[i] = 0; }
      rank.swap(nrank);
    }
  }
  void kasai() {
    for (int i = 0, k = 0; i < n - 1; ++i, k = max(0, k - 1)) {
      int j = sa[rank[i] - 1];
      while (s[i + k] == s[j + k]) { k++; }
      lcp[rank[i]] = k;
    }
  }
};
```

4.6 Suffix array slow

Description: an easier and shorter implementation of suffix array but run a bit slower.
Time: $O(N \log^2 N)$.

suffix_array_slow.h, 41 lines

```
struct SuffixArraySlow {
  string s;
  int n;
  vector<int> sa, lcp, rank;
  SuffixArraySlow(const string& _s):
    s(_s), n((int) s.size() + 1), sa(n), lcp(n), rank(n) {
```

```cpp
    s += '$';
    build();
    kasai();
    sa.erase(sa.begin());
    lcp.erase(lcp.begin());
    rank.pop_back();
    s.pop_back();
  }
  bool comp(int i, int j, int k) {
    return make_pair(rank[i], rank[(i + k) % n]) <
           make_pair(rank[j], rank[(j + k) % n]);
  }
  void build() {
    vector<int> nrank(n);
    for (int i = 0; i < n; ++i) {
      sa[i] = i;
      rank[i] = s[i];
    }
    for (int k = 0; k < n; k = max(1, k * 2)) {
      stable_sort(sa.begin(), sa.end(), [&](int i, int j) { return comp(i, j, k); });
      for (int i = 0, cnt = 0; i < n; ++i) {
        if (i > 0 && comp(sa[i - 1], sa[i], k)) { ++cnt; }
        nrank[sa[i]] = cnt;
      }
      rank.swap(nrank);
    }
  }
  void kasai() {
    for (int i = 0, k = 0; i < n - 1; ++i, k = max(0, k - 1)) {
      int j = sa[rank[i] - 1];
      while (s[i + k] == s[j + k]) { ++k; }
      lcp[rank[i]] = k;
    }
  }
};
```

4.7 Manacher's algorithm

Description: for each position, computes d[0][i] = half length of longest palindrome centered on i (rounded up), d[1][i] = half length of longest palindrome centered on i and i - 1.
Time: $O(N)$.

manacher.h, 23 lines

```cpp
array<vector<int>, 2> manacher(const string& s) {
  int n = (int) s.size();
  array<vector<int>, 2> d;
  for (int z = 0; z < 2; ++z) {
    d[z].resize(n);
    int l = 0, r = 0;
    for (int i = 0; i < n; ++i) {
      int mirror = l + r - i + z;
      d[z][i] = (i < r ? min(d[z][mirror], r - i) : 0);
      int L = i - d[z][i] - z, R = i + d[z][i];
      while (L >= 0 && R < n && s[L] == s[R]) {
        d[z][i]++;
        L--;
        R++;
      }
      if (R > r) {
        l = L;
        r = R;
      }
    }
  }
  return d;
}
```

4.8 Trie

Description: a rooted tree in which each edge is labeled with a character.
Usage:
- Check if a string exists in the set of strings.

Time: $O(N)$ for each operation where N is the length of the string.

trie.h, 36 lines

```cpp
struct Trie {
  const static int ALPHABET = 26;
  const static char minChar = 'a';
  struct Vertex {
    int next[ALPHABET];
    bool leaf;
    Vertex() {
      leaf = false;
      fill(next, next + ALPHABET, -1);
    }
  };
  vector<Vertex> trie;
  Trie() { trie.emplace_back(); }

  void insert(const string& s) {
    int i = 0;
    for (const char& ch : s) {
      int j = ch - minChar;
      if (trie[i].next[j] == -1) {
        trie[i].next[j] = trie.size();
        trie.emplace_back();
      }
      i = trie[i].next[j];
    }
    trie[i].leaf = true;
  }
  bool find(const string& s) {
    int i = 0;
    for (const char& ch : s) {
      int j = ch - minChar;
      if (trie[i].next[j] == -1) { return false; }
      i = trie[i].next[j];
    }
    return (trie[i].leaf ? true : false);
  }
};
```

4.9 Hashing

hash61.h, 56 lines

```cpp
struct Hash61 {
  static const uint64_t MOD = (1LL << 61) - 1;
  static uint64_t BASE;
  static vector<uint64_t> pw;
  uint64_t addmod(uint64_t a, uint64_t b) const {
    a += b;
    if (a >= MOD) { a -= MOD; }
    return a;
  }
  uint64_t submod(uint64_t a, uint64_t b) const {
    a += MOD - b;
    if (a >= MOD) { a -= MOD; }
    return a;
  }
  uint64_t mulmod(uint64_t a, uint64_t b) const {
    uint64_t low1 = (uint32_t) a, high1 = (a >> 32);
    uint64_t low2 = (uint32_t) b, high2 = (b >> 32);

    uint64_t low = low1 * low2;
    uint64_t mid = low1 * high2 + low2 * high1;
    uint64_t high = high1 * high2;

    uint64_t ret =
      (low & MOD) + (low >> 61) + (high << 3) + (mid >> 29) + (mid << 35 >> 3) + 1;
    // ret %= MOD:
```

```
    ret = (ret >> 61) + (ret & MOD);
    ret = (ret >> 61) + (ret & MOD);
    return ret - 1;
  }
  void ensure_pw(int m) {
    int sz = (int) pw.size();
    if (sz >= m) { return; }
    pw.resize(m);
    for (int i = sz; i < m; ++i) { pw[i] = mulmod(pw[i - 1], BASE); }
  }

  vector<uint64_t> pref;
  int n;
  template<typename T> Hash61(const T& s) { // strings or arrays.
    n = (int) s.size();
    ensure_pw(n);
    pref.resize(n + 1);
    pref[0] = 0;
    for (int i = 0; i < n; ++i) {
      pref[i + 1] = addmod(mulmod(pref[i], BASE), s[i]);
    }
  }
  inline uint64_t operator()(const int from, const int to) const {
    assert(0 <= from && from <= to && to < n);
    // pref[to + 1] - pref[from] * pw[to - from + 1]
    return submod(pref[to + 1], mulmod(pref[from], pw[to - from + 1]));
  }
};
mt19937 rnd((unsigned int) chrono::steady_clock::now().time_since_epoch().count());
uint64_t Hash61::BASE = (MOD >> 2) + rnd() % (MOD >> 1);
vector<uint64_t> Hash61::pw = vector<uint64_t>(1, 1);
```

4.10 Minimum rotation

Description: finds the lexicographically smallest rotation of a string.
Usage: rotate(v.begin(), v.begin() + minRotation(v), v.end())
Time: $O(N)$.

min_rotation.h, 19 lines

```
#pragma once

int minRotation(string s) {
  int a = 0, n = (int) s.size();
  s += s;
  for (int b = 0; b < n; ++b) {
    for (int k = 0; k < n; ++k) {
      if (a + k == b || s[a + k] < s[b + k]) {
        b += max(0, k - 1);
        break;
      }
      if (s[a + k] > s[b + k]) {
        a = b;
        break;
      }
    }
  }
  return a;
}
```

5 Numerical

5.1 Fast Fourier transform

Description: a fast algorithm for multiplying two polynomials.
Time: $O(N \log N)$.

fast_fourier_transform.h, 47 lines

```
const double PI = acos(-1);
using Comp = complex<double>;
int reverse_bit(int n, int lg) {
  int res = 0;
  for (int i = 0; i < lg; ++i) {
    if (n & (1 << i)) { res |= (1 << (lg - i - 1)); }
  }
  return res;
}
void fft(vector<Comp>& a, bool invert = false) {
  int n = (int) a.size();
  int lg = 0;
  while (1 << (lg) < n) { ++lg; }
  for (int i = 0; i < n; ++i) {
    int rev_i = reverse_bit(i, lg);
    if (i < rev_i) { swap(a[i], a[rev_i]); }
  }
  for (int len = 2; len <= n; len *= 2) {
    double angle = 2 * PI / len * (invert ? -1 : 1);
    Comp w_base(cos(angle), sin(angle));
    for (int i = 0; i < n; i += len) {
      Comp w(1);
      for (int j = i; j < i + len / 2; ++j) {
        Comp u = a[j], v = a[j + len / 2];
        a[j] = u + w * v;
        a[j + len / 2] = u - w * v;
        w *= w_base;
      }
    }
  }
  if (invert) {
    for (int i = 0; i < n; ++i) { a[i] /= n; }
  }
}
vector<int> mult(vector<int>& a, vector<int>& b) {
  vector<Comp> A(a.begin(), a.end()), B(b.begin(), b.end());
  int n = (int) a.size(), m = (int) b.size(), p = 1;
  while (p < n + m) { p *= 2; }
  A.resize(p), B.resize(p);
  fft(A, false);
  fft(B, false);
  for (int i = 0; i < p; ++i) { A[i] *= B[i]; }
  fft(A, true);
  vector<int> res(n + m - 1);
  for (int i = 0; i < n + m - 1; ++i) { res[i] = (int) round(A[i].real()); }
  return res;
}
```

6 Number Theory

6.1 Euler's totient function

- Euler's totient function, also known as **phi-function** $\phi(n)$ counts the number of integers between 1 and n inclusive, that are **coprime to** n.
- Properties:
  - Divisor sum property: $\sum_{d|n} \phi(d) = n$.
  - $\phi(n)$ is a **prime number** when $n = 3, 4, 6$.
  - If p is a prime number, then $\phi(p) = p - 1$.
  - If p is a prime number and $k \geq 1$, then $\phi(p^k) = p^k - p^{k-1}$.
  - If a and b are **coprime**, then $\phi(ab) = \phi(a) \cdot \phi(b)$.

- In general, for **not coprime** a and b, with $d = gcd(a,b)$ this equation holds: $\phi(ab) = \phi(a) \cdot \phi(b) \cdot \frac{d}{\phi(d)}$.
- With $n = p_1^{k_1} \cdot p_2^{k_2} \cdots p_m^{k_m}$:

$$\phi(n) = \phi(p_1^{k_1}) \cdot \phi(p_2^{k_2}) \cdots \phi(p_m^{k_m})$$
$$= n \cdot \left(1 - \frac{1}{p_1}\right) \cdot \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_m}\right)$$

- Application in Euler's theorem:
  - If $\gcd(a, M) = 1$, then:

$$a^{\phi(M)} \equiv 1 \pmod{M} \Rightarrow a^n \equiv a^{n \bmod \phi(M)} \pmod{M}$$

  - In general, for arbitrary a, M and $n \geq \log_2 M$:

$$a^n \equiv a^{\phi(M) + [n \bmod \phi(M)]} \pmod{M}$$

Time: $O(N \log N)$.

phi_euler_totient_function.h, 10 lines

```
const int MAXN = (int) 2e5;
int etf[MAXN + 1];
void sieve(int n) {
  for (int i = 0; i <= n; ++i) { etf[i] = i; }
  for (int i = 2; i <= n; ++i) {
    if (etf[i] == i) {
      for (int j = i; j <= n; j += i) { etf[j] -= etf[j] / i; }
    }
  }
}
```

6.2 Mobius function

- For a positive integer $n = p_1^{k_1} \cdot p_2^{k_2} \cdots p_m^{k_m}$:

$$\mu(n) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } \exists k_i > 1 \\ (-1)^m & \text{otherwise} \end{cases}$$

- Properties:
  - $\sum_{d|n} \mu(d) = [n = 1]$.
  - If a and b are **coprime**, then $\mu(ab) = \mu(a) \cdot \mu(b)$.
  - Mobius inversion: let f and g be arithmetic functions:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right) g(d)$$

.

Time: $O(N \log N)$.

mobius_function.h, 8 lines

```
const int MAXN = (int) 2e5;
int mu[MAXN + 1];
void sieve(int n) {
  mu[1] = 1;
  for (int i = 1; i <= n; ++i) {
    for (int j = 2 * i; j <= n; j += i) { mu[j] -= mu[i]; }
  }
}
```

6.3 Primes

Approximating the number of primes up to n:

n	$\pi(n)$	$\frac{n}{\ln n - 1}$
100 ($1e^2$)	25	28
500 ($5e^2$)	95	96
1000 ($1e^3$)	168	169
5000 ($5e^3$)	669	665
10000 ($1e^4$)	1229	1218
50000 ($5e^4$)	5133	5092
100000 ($1e^5$)	9592	9512
500000 ($5e^5$)	41538	41246
1000000 ($1e^6$)	78498	78030
5000000 ($5e^6$)	348513	346622

($\pi(n)$ = the number of primes less than or equal to n, $\frac{n}{\ln n - 1}$ is used to approximate $\pi(n)$).

6.4 Wilson's theorem

A positive integer n is a prime if and only if:

$$(n-1)! \equiv n - 1 \pmod{n}$$

6.5 Zeckendorf's theorem

The Zeckendorf's theorem states that every positive integer n can be represented uniquely as a sum of one or more distinct non-consecutive Fibonacci numbers. For example:

$$64 = 55 + 8 + 1$$
$$85 = 55 + 21 + 8 + 1$$

6.6 Bitwise operation

- $a + b = (a \oplus b) + 2(a \,\&\, b)$
- $a \mid b = (a \oplus b) + (a \,\&\, b)$
- $a \,\&\, (b \oplus c) = (a \,\&\, b) \oplus (a \,\&\, c)$
- $a \mid (b \,\&\, c) = (a \mid b) \,\&\, (a \mid c)$
- $a \,\&\, (b \mid c) = (a \,\&\, b) \mid (a \,\&\, c)$
- $a \mid (a \,\&\, b) = a$
- $a \,\&\, (a \mid b) = a$
- $n = 2^k \Leftrightarrow !(n \,\&\, (n-1)) = 1$
- $-a = \sim a + 1$
- $4i \oplus (4i+1) \oplus (4i+2) \oplus (4i+3) = 0$

- Iterating over all subsets of a set and iterating over all submasks of a mask:

mask.h, 18 lines

```
int n;
void mask_example() {
  for (int mask = 0; mask < (1 << n); ++mask) {
    for (int i = 0; i < n; ++i) {
```

```
      if (mask & (1 << i)) {
        // do something...
      }
    }
    // Time complexity: O(n * 2^n).
  }
  for (int mask = 0; mask < (1 << n); ++mask) {
    for (int submask = mask;; submask = (submask - 1) & mask) {
      // do something...
      if (submask == 0) { break; }
    }
    // Time complexity: O(3^n).
  }
}
```

6.7 Modmul

Description: calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \le a, b \le c \le 7.2 \cdot 10^{18}$.
Note: this runs roughly 2x faster than the naive (__int128_t) a * b % M.
Time: $O(1)$ for modmul, $O(\log b)$ for modpow.

modmul.h, 15 lines

```
#pragma once

uint64_t modmul(uint64_t a, uint64_t b, uint64_t mod) {
  int64_t ret = a * b - mod * uint64_t(1.L / mod * a * b); // overflow is fine!
  return ret + mod * (ret < 0) - mod * (ret >= (int64_t) mod);
}
uint64_t modpow(uint64_t a, uint64_t b, uint64_t mod) {
  uint64_t ans = 1;
  while (b > 0) {
    if (b & 1) { ans = modmul(ans, a, mod); }
    a = modmul(a, a, mod);
    b /= 2;
  }
  return ans;
}
```

6.8 Miller–Rabin

Description: Miller–Rabin primality test, this algorithm works for number up to $7e^{18}$.

miller_rabin.h, 26 lines

```
using num_t = long long;
int small_primes[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 73, 113, 193, 311,
  313, 407521, 299210837};
bool miller_rabin(num_t a, num_t d, int s, num_t mod) {
  num_t x = modpow(a, d, mod);
  if (x == mod - 1 || x == 1) { return true; }
  for (int i = 0; i < s - 1; ++i) {
    x = modmul(x, x, mod);
    if (x == mod - 1) { return true; }
  }
  return false;
}
bool is_prime(num_t n) {
  if (n < 4) { return n > 1; }
  num_t d = n - 1;
  int s = 0;
  while (d % 2 == 0) {
    d >>= 1;
    s++;
  }
  for (int a : small_primes) {
    if (n == a) { return true; }
    if (n % a == 0 || !miller_rabin(a, d, s, n)) { return false; }
  }
  return true;
}
```

6.9 Pollard's rho algorithm

Description: Pollard's rho is an efficient algorithm for integer factorization. The algorithm can run smoothly with n upto $1e^{18}$, but be careful with overflow for larger n (e.g. $1e^{19}$).

pollard_rho.h, 52 lines

```
#include "miller_rabin.h"
#include "modmul.h"

uint64_t f(uint64_t x, int c, uint64_t mod) { // f(x) = (x^2 + c) % mod.
  x = modmul(x, x, mod) + c;
  if (x >= mod) { x -= mod; }
  return x;
}
uint64_t pollard_rho(uint64_t n, int c) {
  // algorithm to find a random divisor of `n`.
  // using random function: f(x) = (x^2 + c) % n.
  uint64_t x = 2, y = x, d;
  long long p = 1;
  int dist = 0;
  while (true) {
    y = f(y, c, n);
    dist++;
    d = __gcd(max(x, y) - min(x, y), n);
    if (d > 1) { break; }
    if (dist == p) {
      dist = 0;
      p *= 2;
      x = y;
    }
  }
  return d;
}
void factorize(uint64_t n, vector<uint64_t>& factors) {
  if (n < 2) { return; }
  if (is_prime(n)) {
    factors.emplace_back(n);
    return;
  }
  uint64_t d = n;
  for (int c = 2; d == n; c++) { d = pollard_rho(n, c); }
  factorize(d, factors);
  factorize(n / d, factors);
}
vector<uint64_t> gen_divisors(vector<pair<uint64_t, int>>& factors) {
  vector<uint64_t> divisors = {1};
  for (auto& x : factors) {
    int sz = (int) divisors.size();
    for (int i = 0; i < sz; ++i) {
      uint64_t cur = divisors[i];
      for (int j = 0; j < x.second; ++j) {
        cur *= x.first;
        divisors.push_back(cur);
      }
    }
  }
  return divisors; // this array is NOT sorted yet.
}
```

6.10 Segment divisor sieve

Description: computes the number of divisors for each number in range [L, R].

segment_divisor_sieve.h, 14 lines

```
const int MAXN = (int) 1e6; // R - L + 1 <= N.
int divisor_count[MAXN + 3];
void segment_divisor_sieve(long long L, long long R) {
  for (long long i = 1; i <= (long long) sqrt(R); ++i) {
    long long start1 = ((L + i - 1) / i) * i;
    long long start2 = i * i;
    long long j = max(start1, start2);
```

```
    if (j == start2) {
      divisor_count[j - L] += 1;
      j += i;
    }
    for (; j <= R; j += i) { divisor_count[j - L] += 2; }
  }
}
```

6.11 Linear sieve

Description: finding primes and computing value for multiplicative function in $O(N)$.
Time: $O(N)$ (but the factor may be large).

linear_sieve.h, 44 lines

```
const int N = (int) 2e6 + 3;
bool is_prime[N + 1];
int spf[N + 1]; // smallest prime factor
int lpf[N + 1]; // largest prime factor
int cntp[N + 1]; // number of prime factor
int phi[N + 1]; // euler's totient function
int mu[N + 1]; // mobius function
int func[N + 1]; // a multiplicative function, f(p^k) = k
int k[N + 1]; // k[i] = the power of the smallest prime factor of i
int pw[N + 1]; // pw[i] = p^k[i] where p is the smallest prime factor of i
vector<int> primes;

void linear_sieve(int n = N) {
  spf[0] = spf[1] = lpf[0] = lpf[1] = -1;
  phi[1] = mu[1] = func[1] = 1;
  for (int x = 2; x <= n; ++x) {
    if (spf[x] == 0) {
      primes.push_back(x);
      is_prime[x] = true;
      spf[x] = lpf[x] = x;
      cntp[x] = 1;
      phi[x] = x - 1, mu[x] = -1, func[x] = 1;
      k[x] = 1, pw[x] = x;
    }
    for (int p : primes) {
      if (p > spf[x] || x * p > n) { break; }
      spf[x * p] = p, lpf[x * p] = lpf[x];
      cntp[x * p] = cntp[x] + 1;
      if (p == spf[x]) {
        phi[x * p] = phi[x] * p;
        mu[x * p] = 0;
        func[x * p] = func[x / pw[x]] * (k[x] + 1);
        k[x * p] = k[x] + 1;
        pw[x * p] = pw[x] * p;
      } else {
        phi[x * p] = phi[x] * phi[p];
        mu[x * p] = mu[x] * mu[p]; // or -mu[x]
        func[x * p] = func[x] * func[p];
        k[x * p] = 1;
        pw[x * p] = p;
      }
    }
  }
}
```

6.12 Bitset sieve

Description: sieve of eratosthenes for large n (up to 10^9).
Time: time and space tested on codeforces:
- For $n = 10^8$: 200 ms, 6 MB.
- For $n = 10^9$: 4000 ms, 60 MB.

bitset_sieve.h, 19 lines

```
const int N = (int) 1e8;
bitset<N / 2 + 1> isPrime;
void sieve(int n = N) {
  isPrime.flip();
  isPrime[0] = false;
  for (int i = 3; i <= (int) sqrt(n); i += 2) {
    if (isPrime[i >> 1]) {
      for (int j = i * i; j <= n; j += 2 * i) { isPrime[j >> 1] = false; }
    }
  }
}
void example(int n) {
  sieve(n);
  int primeCnt = (n >= 2);
  for (int i = 3; i <= n; i += 2) {
    if (isPrime[i >> 1]) { primeCnt++; }
  }
  cout << primeCnt << '\n';
}
```

6.13 Block sieve

Description: a very fast sieve of eratosthenes for large n (up to 10^9).
Time: time and space tested on codeforces:
- For $n = 10^8$: 160 ms, 60 MB.
- For $n = 10^9$: 1600 ms, 505 MB.

block_sieve.h, 25 lines

```
const int N = (int) 1e8;
bitset<N + 1> is_prime;
vector<int> fast_sieve() {
  const int S = (int) sqrt(N), R = N / 2;
  vector<int> primes = {2};
  vector<bool> sieve(S + 1, true);
  vector<array<int, 2>> cp;
  for (int i = 3; i <= S; i += 2) {
    if (sieve[i]) {
      cp.push_back({i, i * i / 2});
      for (int j = i * i; j <= S; j += 2 * i) { sieve[j] = false; }
    }
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto& [p, idx] : cp) {
      for (; idx < S + L; idx += p) { block[idx - L] = true; }
    }
    for (int i = 0; i < min(S, R - L); ++i) {
      if (!block[i]) { primes.push_back((L + i) * 2 + 1); }
    }
  }
  for (int p : primes) { is_prime[p] = true; }
  return primes;
}
```

7 Combinatorics

7.1 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{n!(n+1)!}$$

$$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i},\ C_0 = 1,\ C_n = \frac{4n-2}{n+1} C_{n-1}$$

- The first 12 Catalan numbers ($n = 0, 1, 2, \ldots, 11$):

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786$$

- Applications of Catalan numbers:
  - difference binary search trees with n vertices from 1 to n.
  - rooted binary trees with $n+1$ leaves (vertices are not numbered).
  - correct bracket sequence of length $2*n$.
  - permutation $[n]$ with no 3-term increasing subsequence (i.e. doesn't exist $i<j<k$ for which $a[i]<a[j]<a[k]$).
  - ways a convex polygon of $n+2$ sides can split into triangles by connecting vertices.

7.2 Fibonacci numbers

$$F_n = \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{otherwise} \end{cases}$$

- The first 20 Fibonacci numbers ($n = 0, 1, 2, \ldots, 19$):

$$F_n = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181$$

- Binet's formula:

$$F_n = \frac{\varphi^n - \psi^n}{\varphi - \psi} = \frac{\varphi^n - \psi^n}{\sqrt{5}}$$

where $\varphi = \dfrac{1+\sqrt{5}}{2}, \psi = \dfrac{1-\sqrt{5}}{2}$

- Properties:

$$F_{2n+1} = F_n^2 + F_{n+1}^2$$
$$F_{2n} = F_{n-1} \cdot F_n + F_n \cdot F_{n+1}$$
$$F_{n+1} \cdot F_{n-1} - F_n^2 = (-1)^n$$
$$n \mid m \Leftrightarrow F_n \mid F_m$$
$$\gcd(F_n, F_m) = F_{\gcd(n,m)}$$

7.3 Stirling numbers of the first kind

Number of permutations of n elements which contain exactly k permutation cycles.

$$S(0,0) = 1$$
$$S(n,k) = S(n-1,k-1) + (n-1)S(n-1,k)$$
$$\sum_{k=0}^{n} S(n,k)x^k = x(x+1)(x+2)\ldots(x+n-1)$$

7.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k non-empty groups.

$$S(n,1) = S(n,n) = 1$$
$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$
$$S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$$

7.5 Derangements

Permutation of the elements of a set, such that no element appears in its original position (no fixied point). Recursive formulas:

$$D(n) = (n-1)[D(n-1) + D(n-2)] = nD(n-1) + (-1)^n$$

8 Geometry

8.1 Fundamentals

8.1.1 Point

point.h, 64 lines

```
#pragma once

const double PI = acos(-1);
const double EPS = 1e-9;
typedef double ftype;
struct Point {
  ftype x, y;
  Point(ftype _x = 0, ftype _y = 0): x(_x), y(_y) {}
  Point& operator+=(const Point& other) {
    x += other.x;
    y += other.y;
    return *this;
  }
  Point& operator-=(const Point& other) {
    x -= other.x;
    y -= other.y;
    return *this;
  }
  Point& operator*=(ftype t) {
    x *= t;
    y *= t;
    return *this;
  }
  Point& operator/=(ftype t) {
    x /= t;
    y /= t;
    return *this;
  }
  Point operator+(const Point& other) const { return Point(*this) += other; }
  Point operator-(const Point& other) const { return Point(*this) -= other; }
  Point operator*(ftype t) const { return Point(*this) *= t; }
  Point operator/(ftype t) const { return Point(*this) /= t; }
  Point rotate(double angle) const {
    return Point(x * cos(angle) - y * sin(angle), x * sin(angle) + y * cos(angle));
  }
  friend istream& operator>>(istream& in, Point& t);
  friend ostream& operator<<(ostream& out, const Point& t);
  bool operator<(const Point& other) const {
    if (fabs(x - other.x) < EPS) { return y < other.y; }
    return x < other.x;
  }
};

istream& operator>>(istream& in, Point& t) {
  in >> t.x >> t.y;
  return in;
}
ostream& operator<<(ostream& out, const Point& t) {
  out << t.x << ' ' << t.y;
```

```
  return out;
}

ftype dot(Point a, Point b) { return a.x * b.x + a.y * b.y; }
ftype norm(Point a) { return dot(a, a); }
ftype abs(Point a) { return sqrt(norm(a)); }
ftype angle(Point a, Point b) { return acos(dot(a, b) / (abs(a) * abs(b))); }
ftype proj(Point a, Point b) { return dot(a, b) / abs(b); }
ftype cross(Point a, Point b) { return a.x * b.y - a.y * b.x; }
bool ccw(Point a, Point b, Point c) { return cross(b - a, c - a) > EPS; }
bool collinear(Point a, Point b, Point c) { return fabs(cross(b - a, c - a)) < EPS; }
Point intersect(Point a1, Point d1, Point a2, Point d2) {
  double t = cross(a2 - a1, d2) / cross(d1, d2);
  return a1 + d1 * t;
}
```

8.1.2 Line

line.h, 80 lines

```
#include "point.h"

struct Line {
  double a, b, c;
  Line(double _a = 0, double _b = 0, double _c = 0): a(_a), b(_b), c(_c) {}
  friend ostream& operator<<(ostream& out, const Line& l);
};
ostream& operator<<(ostream& out, const Line& l) {
  out << l.a << ' ' << l.b << ' ' << l.c;
  return out;
}
void PointsToLine(const Point& p1, const Point& p2, Line& l) {
  if (fabs(p1.x - p2.x) < EPS) {
    l = {1.0, 0.0, -p1.x};
  } else {
    l.a = -(double) (p1.y - p2.y) / (p1.x - p2.x);
    l.b = 1.0;
    l.c = -l.a * p1.x - l.b * p1.y;
  }
}
void PointsSlopeToLine(const Point& p, double m, Line& l) {
  l.a = -m;
  l.b = 1;
  l.c = -l.a * p.x - l.b * p.y;
}
bool areParallel(const Line& l1, const Line& l2) {
  return fabs(l1.a - l2.a) < EPS && fabs(l1.b - l2.b) < EPS;
}
bool areSame(const Line& l1, const Line& l2) {
  return areParallel(l1, l2) && fabs(l1.c - l2.c) < EPS;
}
bool areIntersect(Line l1, Line l2, Point& p) {
  if (areParallel(l1, l2)) { return false; }
  p.x = -(l1.c * l2.b - l1.b * l2.c) / (l1.a * l2.b - l1.b * l2.a);
  if (fabs(l1.b) > EPS) {
    p.y = -(l1.c + l1.a * p.x);
  } else {
    p.y = -(l2.c + l2.a * p.x);
  }
  return 1;
}
double distToLine(Point p, Point a, Point b, Point& c) {
  double t = dot(p - a, b - a) / norm(b - a);
  c = a + (b - a) * t;
  return abs(c - p);
}
double distToSegment(Point p, Point a, Point b, Point& c) {
  double t = dot(p - a, b - a) / norm(b - a);
  if (t > 1.0) {
    c = Point(b.x, b.y);
  } else if (t < 0.0) {
    c = Point(a.x, a.y);
  } else {
    c = a + (b - a) * t;
  }
  return abs(c - p);
}
bool intersectTwoSegment(Point a, Point b, Point c, Point d) {
  ftype ABxAC = cross(b - a, c - a);
  ftype ABxAD = cross(b - a, d - a);
  ftype CDxCA = cross(d - c, a - c);
  ftype CDxCB = cross(d - c, b - c);
  if (ABxAC == 0 || ABxAD == 0 || CDxCA == 0 || CDxCB == 0) {
    if (ABxAC == 0 && dot(a - c, b - c) <= 0) { return true; }
    if (ABxAD == 0 && dot(a - d, b - d) <= 0) { return true; }
    if (CDxCA == 0 && dot(c - a, d - a) <= 0) { return true; }
    if (CDxCB == 0 && dot(c - b, d - b) <= 0) { return true; }
    return false;
  }
  return (ABxAC * ABxAD < 0 && CDxCA * CDxCB < 0);
}
void perpendicular(Line l1, Point p, Line& l2) {
  if (fabs(l1.a) < EPS) {
    l2 = {1.0, 0.0, -p.x};
  } else {
    l2.a = -l1.b / l1.a;
    l2.b = 1.0;
    l2.c = -l2.a * p.x - l2.b * p.y;
  }
}
```

8.1.3 Circle

circle.h, 16 lines

```
#include "point.h"

int insideCircle(const Point& p, const Point& center, ftype r) {
  ftype d = norm(p - center);
  ftype rSq = r * r;
  return fabs(d - rSq) < EPS ? 0 : (d - rSq >= EPS ? 1 : -1);
}
bool circle2PointsR(const Point& p1, const Point& p2, ftype r, Point& c) {
  double h = r * r - norm(p1 - p2) / 4.0;
  if (fabs(h) < 0) { return false; }
  h = sqrt(h);
  Point perp = (p2 - p1).rotate(PI / 2.0);
  Point m = (p1 + p2) / 2.0;
  c = m + perp * (h / abs(perp));
  return true;
}
```

8.1.4 Triangle

triangle.h, 33 lines

```
#include "line.h"
#include "point.h"

double areaTriangle(double ab, double bc, double ca) {
  double p = (ab + bc + ca) / 2;
  return sqrt(p) * sqrt(p - ab) * sqrt(p - bc) * sqrt(p - ca);
}
double rInCircle(double ab, double bc, double ca) {
  double p = (ab + bc + ca) / 2;
  return areaTriangle(ab, bc, ca) / p;
}
double rInCircle(Point a, Point b, Point c) {
  return rInCircle(abs(a - b), abs(b - c), abs(c - a));
}
bool inCircle(Point p1, Point p2, Point p3, Point& ctr, double& r) {
  r = rInCircle(p1, p2, p3);
  if (fabs(r) < EPS) { return false; }
```

```
  Line l1, l2;
  double ratio = abs(p2 - p1) / abs(p3 - p1);
  Point p = p2 + (p3 - p2) * (ratio / (1 + ratio));
  PointsToLine(p1, p, l1);
  ratio = abs(p1 - p2) / abs(p2 - p3);
  p = p1 + (p3 - p1) * (ratio / (1 + ratio));
  PointsToLine(p2, p, l2);
  areIntersect(l1, l2, ctr);
  return true;
}
double rCircumCircle(double ab, double bc, double ca) {
  return ab * bc * ca / (4.0 * areaTriangle(ab, bc, ca));
}
double rCircumCircle(Point a, Point b, Point c) {
  return rCircumCircle(abs(b - a), abs(c - b), abs(a - c));
}
```

8.1.5 Convex hull

convex_hull.h, 17 lines

```
#include "point.h"

vector<Point> CH_Andrew(vector<Point>& Pts) { // overall O(n log n)
  int n = Pts.size(), k = 0;
  vector<Point> H(2 * n);
  sort(Pts.begin(), Pts.end());
  for (int i = 0; i < n; ++i) {
    while ((k >= 2) && !ccw(H[k - 2], H[k - 1], Pts[i])) { --k; }
    H[k++] = Pts[i];
  }
  for (int i = n - 2, t = k + 1; i >= 0; --i) {
    while ((k >= t) && !ccw(H[k - 2], H[k - 1], Pts[i])) { --k; }
    H[k++] = Pts[i];
  }
  H.resize(k);
  return H;
}
```

8.1.6 Polygon

polygon.h, 45 lines

```
#include "point.h"

double perimeter(const vector<Point>& P) {
  double ans = 0.0;
  for (int i = 0; i < (int) P.size() - 1; ++i) { ans += abs(P[i] - P[i + 1]); }
  return ans;
}
double area(const vector<Point>& P) {
  double ans = 0.0;
  for (int i = 0; i < (int) P.size() - 1; ++i) {
    ans += (P[i].x * P[i + 1].y - P[i + 1].x * P[i].y);
  }
  return fabs(ans) / 2.0;
}
bool isConvex(const vector<Point>& P) {
  int n = (int) P.size();
  if (n <= 3) { return false; }
  bool firstTurn = ccw(P[0], P[1], P[2]);
  for (int i = 1; i < n - 1; ++i) {
    if (ccw(P[i], P[i + 1], P[(i + 2) == n ? 1 : i + 2]) != firstTurn) {
      return false;
    }
  }
  return true;
}
int insidePolygon(Point pt, const vector<Point>& P) {
  int n = (int) P.size();
  if (n <= 3) { return -1; }
  bool on_polygon = false;
  for (int i = 0; i < n - 1; ++i) {
    if (fabs(abs(P[i] - pt) + abs(pt - P[i + 1]) - abs(P[i] - P[i + 1])) < EPS) {
      on_polygon = true;
    }
  }
  if (on_polygon) { return 0; }
  double sum = 0.0;
  for (int i = 0; i < n - 1; ++i) {
    if (ccw(pt, P[i], P[i + 1])) {
      sum += angle(P[i] - pt, P[i + 1] - pt);
    } else {
      sum -= angle(P[i] - pt, P[i + 1] - pt);
    }
  }
  return fabs(sum) > PI ? 1 : -1;
}
```

9 Linear algebra

9.1 Gauss elimination

Time: $O(\min(n, m) \cdot nm)$ or $O(n^3)$ in case $n = m$.

gauss_elimination.h, 37 lines

```
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or a big number
int gauss(vector<vector<double>> a, vector<double>& ans) {
  int n = (int) a.size();
  int m = (int) a[0].size() - 1;
  vector<int> where(m, -1);
  for (int col = 0, row = 0; col < m && row < n; ++col) {
    int sel = row;
    for (int i = row; i < n; ++i) {
      if (abs(a[i][col]) > abs(a[sel][col])) { sel = i; }
    }
    if (abs(a[sel][col]) < EPS) { continue; }
    for (int i = col; i <= m; ++i) { swap(a[sel][i], a[row][i]); }
    where[col] = row;

    for (int i = 0; i < n; ++i) {
      if (i != row) {
        double c = a[i][col] / a[row][col];
        for (int j = col; j <= m; ++j) { a[i][j] -= a[row][j] * c; }
      }
    }
    ++row;
  }
  ans.assign(m, 0);
  for (int i = 0; i < m; ++i) {
    if (where[i] != -1) { ans[i] = a[where[i]][m] / a[where[i]][i]; }
  }
  for (int i = 0; i < n; ++i) {
    double sum = 0;
    for (int j = 0; j < m; ++j) { sum += ans[j] * a[i][j]; }
    if (abs(sum - a[i][m]) > EPS) { return 0; }
  }
  for (int i = 0; i < m; ++i) {
    if (where[i] == -1) { return INF; }
  }
  return 1;
}
```

9.2 Gauss determinant

Description: computing determinant of a square matrix A by applying Gauss elimination to produces a row echolon matrix B, then the determinant of A is equal to product of the elements of the diagonal of B.

Time: $O(N^3)$.

gauss_determinant.h, 28 lines

```
const double EPS = 1e-9;
double determinant(vector<vector<double>> A) {
  int n = (int) A.size();
  double det = 1;
  for (int i = 0; i < n; ++i) {
    // find non-zero cell
    int k = i;
    for (int j = i + 1; j < n; ++j) {
      if (abs(A[j][i]) > abs(A[k][i])) { k = j; }
    }
    if (abs(A[k][i]) < EPS) {
      det = 0;
      break;
    }
    if (i != k) {
      swap(A[i], A[k]);
      det = -det;
    }
    det *= A[i][i];
    for (int j = i + 1; j < n; ++j) { A[i][j] /= A[i][i]; }
    for (int j = 0; j < n; ++j) {
      if (j != i && abs(A[j][i]) > EPS) {
        for (int k = i + 1; k < n; ++k) { A[j][k] -= A[i][k] * A[j][i]; }
      }
    }
  }
  return det;
}
```

9.3 Bareiss determinant

Description: Bareiss algorithm for computing determinant of a square matrix A with integer entries using only integer arithmetic.

Usage:

- Kirchhoff's theorem: finding the number of spanning trees.

Time: $O(N^3)$.

bareiss_determinant.h, 28 lines

```
long long determinant(vector<vector<long long>> A) {
  int n = (int) A.size();
  long long prev = 1;
  int sign = 1;
  for (int i = 0; i < n - 1; ++i) {
    // find non-zero cell
    if (A[i][i] == 0) {
      int k = -1;
      for (int j = i + 1; j < n; ++j) {
        if (A[j][i] != 0) {
          k = i;
          break;
        }
      }
      if (k == -1) { return 0; }
      swap(A[i], A[k]);
      sign = -sign;
    }
    for (int j = i + 1; j < n; ++j) {
      for (int k = i + 1; k < n; ++k) {
        assert((A[j][k] * A[i][i] - A[j][i] * A[i][k]) % prev == 0);
        A[j][k] = (A[j][k] * A[i][i] - A[j][i] * A[i][k]) / prev;
      }
    }
    prev = A[i][i];
  }
  return sign * A[n - 1][n - 1];
}
```

10 Graph

10.1 Bellman-Ford algorithm

Description: single source shortest path in a weighted (negative or positive) directed graph.

Time: $O(VE)$.

bellman_ford.h, 34 lines

```
const int64_t INF = (int64_t) 2e18;
struct Edge {
  int u, v; // u -> v
  int64_t w;
  Edge() {}
  Edge(int _u, int _v, int64_t _w): u(_u), v(_v), w(_w) {}
};
int n;
vector<Edge> edges;
vector<int64_t> bellmanFord(int s) {
  // dist[stating] = 0.
  // dist[u] = +INF, if u is unreachable.
  // dist[u] = -INF, if there is a negative cycle on the path from s to u.
  // -INF < dist[u] < +INF, otherwise.
  vector<int64_t> dist(n, INF);
  dist[s] = 0;
  for (int i = 0; i < n - 1; ++i) {
    bool any = false;
    for (auto [u, v, w] : edges) {
      if (dist[u] != INF && dist[v] > w + dist[u]) {
        dist[v] = w + dist[u];
        any = true;
      }
    }
    if (!any) { break; }
  }
  // handle negative cycles
  for (int i = 0; i < n - 1; ++i) {
    for (auto [u, v, w] : edges) {
      if (dist[u] != INF && dist[v] > w + dist[u]) { dist[v] = -INF; }
    }
  }
  return dist;
}
```

10.2 Articulation point and Bridge

Description: finding articulation points and bridges in a simple undirected graph.

Time: $O(V + E)$.

articulation_point_and_bridge.h, 39 lines

```
const int N = (int) 1e5;
vector<int> g[N];
int num[N], low[N], dfs_timer;
bool joint[N];
vector<pair<int, int>> bridges;
void dfs(int u, int prev = -1) {
  low[u] = num[u] = ++dfs_timer;
  int child = 0;
  for (int v : g[u]) {
    if (v == prev) { continue; }
    if (num[v]) {
      low[u] = min(low[u], num[v]);
    } else {
      dfs(v, u);
      low[u] = min(low[u], low[v]);
      child++;
      if (low[v] >= num[v]) { bridges.emplace_back(u, v); }
      if (prev != -1 && low[v] >= num[u]) { joint[u] = true; }
    }
  }
```

```
  if (prev == -1 && child > 1) { joint[u] = true; }
}

int solve() {
  int n, m;
  cin >> n >> m;
  for (int i = 0; i < m; ++i) {
    int u, v;
    cin >> u >> v;
    u--;
    v--;
    g[u].push_back(v);
    g[v].push_back(u);
  }
  for (int i = 0; i < n; ++i) {
    if (!num[i]) { dfs(i); }
  }
  return 0;
}
```

10.3 Topo sort

Description: a topological sort of a directed acyclic graph is a linear ordering of its vertices such that for every directed edge from vertex u to vertex v, u comes before v in the ordering.
Note: if there are cycles, the returned list will have size smaller than n.
Time: $O(V + E)$.

topo_sort.h, 21 lines

```
vector<int> topo_sort(const vector<vector<int>>& g) {
  int n = (int) g.size();
  vector<int> indeg(n);
  for (int u = 0; u < n; ++u) {
    for (int v : g[u]) { indeg[v]++; }
  }
  queue<int> q; // Note: use min-heap to get the smallest lexicographical order.
  for (int u = 0; u < n; ++u) {
    if (indeg[u] == 0) { q.emplace(u); }
  }
  vector<int> topo;
  while (!q.empty()) {
    int u = q.front();
    q.pop();
    topo.emplace_back(u);
    for (int v : g[u]) {
      if (--indeg[v] == 0) { q.emplace(v); }
    }
  }
  return topo;
}
```

10.4 Strongly connected components

10.4.1 Tarjan's Algorithm

Description: Tarjan's algorithm finds strongly connected components (SCC) in a directed graph. If two vertices u and v belong to the same component, then scc_id[u] == scc_id[v].
Time: $O(V + E)$.

tarjan.h, 27 lines

```
const int N = (int) 5e5;
vector<int> g[N], st;
int low[N], num[N], dfs_timer, scc_id[N], scc;
bool used[N];
void Tarjan(int u) {
  low[u] = num[u] = ++dfs_timer;
  st.push_back(u);
  for (int v : g[u]) {
    if (used[v]) { continue; }
    if (num[v] == 0) {
      Tarjan(v);
      low[u] = min(low[u], low[v]);
    } else {
      low[u] = min(low[u], num[v]);
    }
  }
  if (low[u] == num[u]) {
    int v;
    do {
      v = st.back();
      st.pop_back();
      used[v] = true;
      scc_id[v] = scc;
    } while (v != u);
    scc++;
  }
}
```

10.4.2 Kosaraju's algorithm

Description: Kosaraju's algorithm finds strongly connected components (SCC) in a directed graph in a straightforward way. Two vertices u and v belong to the same component iff scc_id[u] == scc_id[v]. This algorithm generates connected components numbered in topological order in corresponding condensation graph.
Time: $O(V + E)$.

kosaraju.h, 34 lines

```
const int N = (int) 1e5;
vector<int> g[N], rev_g[N], vers;
int scc_id[N];
bool vis[N];
int n, m;

void dfs1(int u) {
  vis[u] = true;
  for (int v : g[u]) {
    if (!vis[v]) { dfs1(v); }
  }
  vers.push_back(u);
}
void dfs2(int u, int id) {
  scc_id[u] = id;
  vis[u] = true;
  for (int v : rev_g[u]) {
    if (!vis[v]) { dfs2(v, id); }
  }
}
void Kosaraju() {
  for (int i = 0; i < n; ++i) {
    if (!vis[i]) { dfs1(i); }
  }
  memset(vis, 0, sizeof vis);
  int scc_cnt = 0;
  // iterating in reverse order
  for (int i = n - 1; i >= 0; --i) {
    int u = vers[i];
    if (!vis[u]) { dfs2(u, ++scc_cnt); }
  }
  cout << scc_cnt << '\n';
  for (int i = 0; i < n; ++i) { cout << scc_id[i] << " \n"[i == n - 1]; }
}
```

10.5 K-th smallest shortest path

Description: finding the k-th smallest shortest path from vertex s to vertex t, each vertex can be visited more than once.

k_smallest_shortest_path.h, 20 lines

```
using adj_list = vector<vector<pair<int, int>>>;
vector<long long> k_smallest(const adj_list& g, int k, int s, int t) {
  int n = (int) g.size();
  vector<long long> ans;
```

```
  vector<int> cnt(n);
  using pli = pair<long long, int>;
  priority_queue<pli, vector<pli>, greater<pli>> pq;
  pq.emplace(0, s);
  while (!pq.empty() && cnt[t] < k) {
    int u = pq.top().second;
    long long d = pq.top().first;
    pq.pop();
    if (cnt[u] == k) { continue; }
    cnt[u]++;
    if (u == t) { ans.push_back(d); }
    for (auto [v, cost] : g[u]) { pq.emplace(d + cost, v); }
  }
  assert(k == (int) ans.size());
  return ans;
}
```

10.6 Eulerian path

10.6.1 Directed graph

Description: Hierholzer's algorithm. An Eulerian path in a directed graph is a path that visits all edges exactly once. An Eulerian cycle is a Eulerian path that is a cycle.
Time: $O(E)$.

eulerian_path_directed.h, 16 lines

```
vector<int> find_path_directed(const vector<vector<int>>& g, int s) {
  int n = (int) g.size();
  vector<int> stack, cur_edge(n), vertices;
  stack.push_back(s);
  while (!stack.empty()) {
    int u = stack.back();
    stack.pop_back();
    while (cur_edge[u] < (int) g[u].size()) {
      stack.push_back(u);
      u = g[u][cur_edge[u]++];
    }
    vertices.push_back(u);
  }
  reverse(vertices.begin(), vertices.end());
  return vertices;
}
```

10.6.2 Undirected graph

Description: Hierholzer's algorithm. An Eulerian path in a undirected graph is a path that visits all edges exactly once. An Eulerian cycle is a Eulerian path that is a cycle.
Time: $O(E)$.

eulerian_path_undirected.h, 21 lines

```
struct Edge {
  int to;
  list<Edge>::iterator reverse_edge;
  Edge(int _to): to(_to) {}
};
vector<int> vertices;
void find_path(vector<list<Edge>>& g, int u) {
  while (!g[u].empty()) {
    int v = g[u].front().to;
    g[v].erase(g[u].front().reverse_edge);
    g[u].pop_front();
    find_path(g, v);
  }
  vertices.emplace_back(u); // reversion list.
}
void add_edge(vector<list<Edge>>& g, int u, int v) {
  g[u].emplace_front(v);
  g[v].emplace_front(u);
  g[u].front().reverse_edge = g[v].begin();
  g[v].front().reverse_edge = g[u].begin();
}
```

10.7 HLD

HLD.h, 68 lines

```
const int INF = 0x3f3f3f3f;
template<class SegmentTree>
struct HLD { // vertex update and max query on path u -> v
  int n;
  vector<vector<int>> g;
  SegmentTree seg_tree;
  vector<int> par, top, depth, sz, id;
  int timer = 0, root = 0;
  bool VAL_IN_EDGE = false;
  HLD() {}
  HLD(int _n): n(_n), g(n), seg_tree(n), par(n), top(n), depth(n), sz(n), id(n) {}
  void build() {
    dfs_sz(root);
    dfs_hld(root);
  }
  void add_edge(int u, int v) {
    g[u].push_back(v);
    g[v].push_back(u);
  }
  void dfs_sz(int u) {
    sz[u] = 1;
    for (int& v : g[u]) { // MUST BE ref for the swap below
      par[v] = u;
      depth[v] = depth[u] + 1;
      g[v].erase(find(g[v].begin(), g[v].end(), u));
      dfs_sz(v);
      sz[u] += sz[v];
      if (sz[v] > sz[g[u][0]]) { swap(v, g[u][0]); }
    }
  }
  void dfs_hld(int u) {
    id[u] = timer++;
    for (int v : g[u]) {
      top[v] = (v == g[u][0] ? top[u] : v);
      dfs_hld(v);
    }
  }
  int lca(int u, int v) {
    while (top[u] != top[v]) {
      if (depth[top[u]] > depth[top[v]]) { swap(u, v); }
      v = par[top[v]];
    }
    // now u, v is in the same heavy-chain
    return (depth[u] < depth[v] ? u : v);
  }
  void set_vertex(int v, int x) { seg_tree.set(id[v], x); }
  void set_edge(int u, int v, int x) {
    if (u != par[v]) { swap(u, v); }
    seg_tree.set(id[v], x);
  }
  void set_subtree(int v, int x) {
    // modify segment_tree so that it supports range update
    seg_tree.set_range(id[v] + VAL_IN_EDGE, id[v] + sz[v] - 1, x);
  }
  int query_path(int u, int v) {
    int res = -INF;
    while (top[u] != top[v]) {
      if (depth[top[u]] > depth[top[v]]) { swap(u, v); }
      int cur = seg_tree.query(id[top[v]], id[v]);
      res = max(res, cur);
      v = par[top[v]];
    }
    if (depth[u] > depth[v]) { swap(u, v); }
    int cur = seg_tree.query(id[u] + VAL_IN_EDGE, id[v]);
```

```
    res = max(res, cur);
    return res;
  }
};
```

10.8 DSU on tree

dsu_on_tree.h, 28 lines

```
const int nmax = (int) 2e5 + 1;
vector<int> adj[nmax];
int sz[nmax]; // sz[u] is the size of the subtree rooted at u
bool big[nmax];

void add(int u, int p, int del) {
  // do something...
  for (int v : adj[u]) {
    if (big[v] == false) { add(v, u, del); }
  }
}

void dsuOnTree(int u, int p, int keep) {
  int bigC = -1;
  for (int v : adj[u]) {
    if (v != p && (bigC == -1 || sz[bigC] < sz[v])) { bigC = v; }
  }
  for (int v : adj[u]) {
    if (v != p && v != bigC) { dsuOnTree(v, u, 0); }
  }
  if (bigC != -1) {
    big[bigC] = true;
    dsuOnTree(bigC, u, 1);
  }
  add(u, p, 1);
  if (bigC != -1) { big[bigC] = false; }
  if (keep == 0) { add(u, p, -1); }
}
```

10.9 2-SAT

Description: finds a way to assign values to boolean variables a, b, c,.. of a 2-SAT problem (each clause has at most two variables) so that the following formula becomes true: $(a \mid b) \& (\sim a \mid c) \& (b \mid \sim c) \ldots$
Usage:
- TwoSat twosat(number of boolean variables);
- twosat.either(a, ~b); // a is true or b is false
- twosat.solve(); // return true iff the above formula is satisfiable

Time: $O(V + E)$ where V is the number of boolean variables and E is the number of clauses.

two_sat.h, 46 lines

```
struct TwoSat {
  int n;
  vector<vector<int>> g, tg; // g and transpose of g
  vector<int> comp, order;
  vector<bool> vis, vals;
  TwoSat(int _n): n(_n), g(2 * n), tg(2 * n), comp(2 * n), vis(2 * n), vals(n) {}
  void either(int u, int v) {
    u = max(2 * u, -2 * u - 1);
    v = max(2 * v, -2 * v - 1);
    g[u ^ 1].push_back(v);
    g[v ^ 1].push_back(u);
    tg[v].push_back(u ^ 1);
    tg[u].push_back(v ^ 1);
  }
  void set(int u) { either(u, u); }
  void dfs1(int u) {
    vis[u] = true;
    for (int v : g[u]) {
      if (!vis[v]) { dfs1(v); }
    }
    order.push_back(u);
  }
  void dfs2(int u, int scc_id) {
    comp[u] = scc_id;
    for (int v : tg[u]) {
      if (comp[v] == -1) { dfs2(v, scc_id); }
    }
  }
  bool solve() {
    for (int i = 0; i < 2 * n; ++i) {
      if (!vis[i]) { dfs1(i); }
    }
    fill(comp.begin(), comp.end(), -1);
    for (int i = 2 * n - 1, scc_id = 0; i >= 0; --i) {
      int u = order[i];
      if (comp[u] == -1) { dfs2(u, scc_id++); }
    }
    for (int i = 0; i < n; ++i) {
      int u = i * 2, nu = i * 2 + 1;
      if (comp[u] == comp[nu]) { return false; }
      vals[i] = comp[u] > comp[nu];
    }
    return true;
  }
  vector<bool> get_vals() { return vals; }
};
```

11 Misc.

11.1 Ternary search

Description: given an unimodal function $f(x)$, find the maximum/minimum of $f(x)$. Unimodal means the function strictly increases/decreases first, reaches a maximum/minimum (at a single point or over an interval), and then strictly decreases/increases.

ternary_search.h, 27 lines

```
const double eps = 1e-9;
template<typename T> inline T func(T x) { return x * x; }

// these two functions below find min, for find max: change '<' below to '>'.
double ternary_search(double l, double r) { // min
  while (r - l > eps) {
    double mid1 = l + (r - l) / 3;
    double mid2 = r - (r - l) / 3;
    if (func(mid1) < func(mid2)) {
      r = mid2;
    } else {
      l = mid1;
    }
  }
  return l;
}
int ternary_search(int l, int r) { // min
  while (l < r) {
    int mid = l + (r - l) / 2;
    if (func(mid) < func(mid + 1)) {
      r = mid;
    } else {
      l = mid + 1;
    }
  }
  return l;
}
```

11.2 Matrix

matrix.h, 37 lines

```
using matrix_type = int;
const int MOD = (int) 1e9 + 7;
struct Matrix {
```

```
  static const matrix_type INF = numeric_limits<matrix_type>::max();
  int N, M;
  vector<vector<matrix_type>> mat;

  Matrix(int _N, int _M, matrix_type v = 0): N(_N), M(_M) {
    mat.assign(N, vector<matrix_type>(M, v));
  }
  static Matrix identity(int n) { // return identity matrix.
    Matrix I(n, n);
    for (int i = 0; i < n; ++i) { I[i][i] = 1; }
    return I;
  }

  vector<matrix_type>& operator[](int r) { return mat[r]; }
  const vector<matrix_type>& operator[](int r) const { return mat[r]; }

  Matrix& operator*=(const Matrix& other) {
    assert(M == other.N); // [N x M] [other.N x other.M]
    Matrix res(N, other.M);
    for (int r = 0; r < N; ++r) {
      for (int c = 0; c < other.M; ++c) {
        long long square_mod = (long long) MOD * MOD;
        long long sum = 0;
        for (int g = 0; g < M; ++g) {
          sum += (long long) mat[r][g] * other[g][c];
          if (sum >= square_mod) { sum -= square_mod; }
        }
        res[r][c] = sum % MOD;
      }
    }
    mat.swap(res.mat);
    return *this;
  }
};
```