# MAT137 Lecture 25

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# Agenda

The substitution rule.

# The substitution rule

## Theorem

Let $u = g(x)$ be a differentiable function whose range is an interval $I$ and $f$ is continuous on $I$. Then

$$\int f(g(x))g'(x)\mathrm{d}x = \int f(u)\mathrm{d}u.$$

# The substitution rule

## The substitution rule

To apply the substitution rule we proceed as follows.

(i) Let
$$u = g(x),$$

then
$$\mathrm{d}u = g'(x)\mathrm{d}x,$$

(after this manipulation only the letter $u$ should appear, not the letter $x$).

(ii) Find an antiderivative (as an expression involving $u$).

(iii) Substitute $g(x)$ back for $u$.

## Example 1

Evaluate

$$\int \sqrt[3]{1 + x^4} \; x^{11} \mathrm{d}x.$$

*Hint.* Let $u = 1 + x^4$.

## Example 2

Evaluate

$$\int \frac{e^{2x}}{\sqrt{e^x + 1}} \mathrm{d}x.$$

*Hint.* Let $u = e^x + 1$.

# Example 3

Evaluate

$$\int \frac{\ln^2(\ln x)}{x \ln x} \, \mathrm{d}x.$$

*Hint.* Let $u = \ln x$.

# Example 4

Evaluate

$$\int \sec^5 x \tan^3 x \, \mathrm{d}x.$$

*Hint.* Let $u = \sec x$ or $u = \cos x$.

# Example 5

Evaluate
$$\int \frac{x}{x^4 + 2x^2 + 2}\mathrm{d}x.$$

*Hint.* Let $u = 1 + x^2$.

# Next Class: Thursday January 25

Watch videos 9.4, 9.5, 9.6, 9.7, 9.8, 9.9 in Playlist 9.