MAT137 Lecture 11

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Agenda

- Differentiable implies continuous
- Proofs of differentiation rules

Differentiability

Consider the function f given by

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Show that f is differentiable at 0.

Continuity vs Differentiability

Give an example of a function that is continuous at 0 but not differentiable at 0.

Continuity vs Differentiability

Let

$$f(x) = \begin{cases} x, & x \text{ is rational,} \\ 0, & x \text{ is irrational.} \end{cases}$$

(a) Show that f is continuous at 0.

(b) Show that f is NOT differentiable at 0.

Continuity vs Differentiability

Let

$$f(x) = \begin{cases} x \sin(1/x), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

(a) Show that f is continuous at 0.

(b) Show that f is NOT differentiable at 0.

Quotient Rule

Suppose that f and g are differentiable at a and $g(a) \neq 0$. Show that

$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - g'(a)f(a)}{[g(a)]^2}.$$

Next Class: Thursday Oct 19

Watch videos 8, 9, 10, 11, 12 in Playlist 3.