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Assignment 5-1.

So far I cannot find an example. of a complete. lattice $(A. \leq)$ and a monotone function $f: A \to A$ such that $f^{\omega}(\perp)$ is not a fixed point of f.

My idea is.

① every complete lattice $(A \leq)$ is a chain complete. partial order (ccpo).

② if a function $f: A \to A$ is monotonic on complete lattice $(A. \leq)$ it should be continuous.

Prove. of ②

for each $x, y \in A$. if $x \leq y$. then $f(x) \leq f(y)$

for any subset $B \subseteq A$. $\vee B$ exists and $\vee B$ is an element of B since the ~~realtition~~ relation on A is "$\leq$".

$\therefore \vee\{f(x) \mid x \in B\} = f(\vee B)$.

$\therefore f: A \to A$ is a continuous function

③ Based on definition of Kleene's fixed point theorem. $f^{\omega}(\perp)$ is the least fixed point of f.

Is there somthing wrong in ① or ② ?