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Assignment 3b (1).

X, Y are ordinals and $X < Y$. then X is a subset of Y (and also is a member of Y), Is every subset of Y an ordinals..

Yes.

Since Y is ordinal. the Y is a well order. then any subset of Y is a well order. . Every well order is order-isomorphic to an ordinal number.

Assignment 3b. (2).

1.7.
$A = (\mathbb{Z}, \prec)$ $0 \prec 1 \prec 2 \prec \cdots -3 \prec -2 \prec -1$

1.8. $(\mathbb{N}, \prec)$. $0 \prec 2 \prec 4 \prec \cdots 1 \prec 3 \prec 5 \prec \cdots$
$i \prec j$ iff i is even j is odd or
$i < j$. and $i\,j$ are both even
or both odd.

for 1.7.
construct such strange order
$B = (\mathbb{Z}, \prec)$ $0 \prec -1 \prec -2 \prec -3 \prec \cdots 3 \prec 2 \prec 1$
$i \prec j$ iff ① iff $i, j \leq 0$ and $i > j$
② iff $i, j > 0$ and $i > 0$
③ if $i < 0$, $j > 0$,
④ if $i = 0$ $j > 0$

$f(A \to B) = -x$.
$A \trianglelefteq B$

$f(B \to A) = -x$
$B \trianglelefteq A$

$\therefore A \cong B$.

Assignment 3b. (2) — 1.8

$$\boxed{f = x+2}$$

$B = (\mathbb{N}/\{0, 1\}, \prec)$. $2 \prec 4 \prec \cdots 3 \prec 5 \prec \cdots$

$i \prec j$ iff i is even j is odd

or $i < j$ and i, j are both even or both odd.

for each $a \prec b$ in $(\mathbb{N}, <)$.

$f(a+2) \prec f(b+2)$ in $B = (\mathbb{N}/\{0, 1\})$.

And for each $f(x) \prec f(y)$ in B $x, y \geq 2$.

$x-2 < y-2$ in $(\mathbb{N}, <)$.

Assignment 3b.

3.

The property does not hold for arbitrary posets.

~~Because poset is reflexive.~~

So ~~the poset itself is an order-isomorphism~~

Well-orders have this property.

because each well-order is order-isomorphic to an ordinal number. Then if two ~~order~~ well orders are order-isomorphic they are isomorphic to a same ordinal number.

But for arbitrary posets. it could exist many order-isomorphisms not an exactly one.

for example $(\mathbb{R}, \leq)$ and $(\mathbb{R}, \geq)$.

the order-isomorphisms could be

$$f(x) = -x$$

$$f(x) = -2x$$

or $(\mathbb{R}\setminus\{0\}, \leq)$ and $(\mathbb{R}\setminus\{0\}, \geq)$

$f(x) = \frac{1}{x}$, $f(x) = -x$