Assignment · 2. 12.

Huanyu Li

(1) $\Rightarrow$ (2).

Since $(A, \leq)$ is a complete lattice.

Based on the definition of complete lattice, every subset of a complete lattice has both glb and lub.

so $\wedge B$ exists for every $B \subseteq A$,

(2) $\Rightarrow$ (1).

since $\wedge B$ exists for every $B \subseteq A$.

Letting $D \subseteq A$ as the upper bound of B.

Then $B = \{x \in A \mid x \leq D\}$

since $D \subseteq A$ $\wedge D$ exists. $D = \{d \in A \mid d \geq m\}$

$\wedge D$ is indeed $\vee B$

so Both $\vee B$ and $\wedge B$ exist for. $B \subseteq A$.

$\therefore (A, \leq)$ is a complete lattice.