Assignment 4. Huanyu Li

question 4.

∵ $f: A \to A$ is a monotonic function.
on complete lattice $(A, \leq)$.
∴ for each $x, y \in A$. it holds.
if $x \leq y$ then $f(x) \leq f(y)$.

since f is a function from A to A.
∴ for each $x \in A$. $f(x) \in A$.
and $x \leq f(x)$.

By using standard induction.

when $\alpha = 0$ $f^0(x) = x \leq f^{0+1}(x)$.
so $f^\alpha(x) \leq f^{\alpha+1}(x)$ holds.

Assume when $\alpha = n-1$. $f^\alpha(x) \leq f^{\alpha+1}(x)$ holds.
so $f^{n-1}(x) \leq f^n(x)$.
We know that $f^{n-1}(x)$, $f^n(x) \in A$.
since f is a monotonic function on A
∴ $f(f^{n-1}(x)) \leq f(f^n(x))$.
∴ $f^n(x) \leq f^{n+1}(x)$

∴ $f^\alpha(x) \leq f^{\alpha+1}(x)$ holds for any ordinal α.