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Assignment 4 (2).
question 2.

$(A, \leq)$ is a complete lattice.

so for each subset $B \subseteq A$.

there exist least upper bound of B.

for each $x \in B$. $x \leq \vee B$

$f: A \to A$ is a monotonic function

so for each $x, y \in A$.

if $x \leq y$. then $f(x) \leq f(y)$.

As a result ~~$f(x) \leq f(\vee B)$~~.

for each $x \in B$ we already know $x \leq \vee B$

$\therefore$ $f(x) \leq f(\vee B)$.

$\therefore$ $f(\vee B)$ is the least upper bound of $f(B)$

so. $f(\vee B) \geq \vee \{f(x) \mid x \in B\}$ holds for any subset $B \subseteq A$.