

Computer Vision Homework 2 Report

涂宇清 522030910152

1. Programming Assignment

2. Written Assignment

2.1 Problem 1

a.

$$\begin{aligned} E(A, T) &= \sum_{i=1}^N \|Y_i - AX_i - T\|_2^2 \\ &= \sum_{i=1}^N (Y_i - AX_i - T)^T (Y_i - AX_i - T) \end{aligned}$$

To minimize $E(A, T)$, we need to take the derivative of $E(A, T)$ with respect to A and T and set them to zero.

$$\begin{aligned} \frac{\partial E(A, T)}{\partial A} &= \sum_{i=1}^N 2(AX_i + T - Y_i)X_i^T = O \\ \frac{\partial E(A, T)}{\partial T} &= \sum_{i=1}^N 2(AX_i + T - Y_i) = O \end{aligned}$$

It is given by the second equation that $T = \frac{1}{N} \sum_{i=1}^N (Y_i - AX_i) = \hat{Y} - A\hat{X}$. Then we can substitute T with $\hat{Y} - A\hat{X}$ in the first equation and get

$$\begin{aligned} \sum_{i=1}^N (AX_i + \hat{Y} - A\hat{X} - Y_i)X_i^T &= O \\ A \sum_{i=1}^N (X_i - \hat{X})X_i^T &= \sum_{i=1}^N (Y_i - \hat{Y})X_i^T \end{aligned}$$

And we know that

$$\begin{aligned}
\sum_{i=1}^N (X_i - \hat{X})(X_i - \hat{X})^T &= \sum_{i=1}^N (X_i - \hat{X})X_i^T - \sum_{i=1}^N (X_i - \hat{X})\hat{X}^T \\
&= \sum_{i=1}^N (X_i - \hat{X})X_i^T - \left(\sum_{i=1}^N X_i \hat{X}^T - N \hat{X} \hat{X}^T \right) \\
&= \sum_{i=1}^N (X_i - \hat{X})X_i^T
\end{aligned}$$

and

$$\sum_{i=1}^N (Y_i - \hat{Y})(X_i - \hat{X})^T = \sum_{i=1}^N (Y_i - \hat{Y})X_i^T$$

So we can get

$$\begin{aligned}
A \sum_{i=1}^N (X_i - \hat{X})X_i^T &= \sum_{i=1}^N (Y_i - \hat{Y})X_i^T \\
A \sum_{i=1}^N (X_i - \hat{X})(X_i - \hat{X})^T &= \sum_{i=1}^N (Y_i - \hat{Y})(X_i - \hat{X})^T \\
A(XX^T) &= YX^T \\
A &= (YX^T)(XX^T)^{-1}
\end{aligned}$$

To sum up, $\min(E(A, T))$ is given by $T^* = \hat{Y} - A^* \hat{X}$, $A^* = (YX^T)(XX^T)^{-1}$.

b. For every correspondence (x_i, y_i) , we have

$$\begin{aligned}
y_i &= Ax_i + T \\
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}
\end{aligned}$$

three equations totally. And A and T have 12 unknowns. So we need at least 4 correspondences to estimate the transformation.