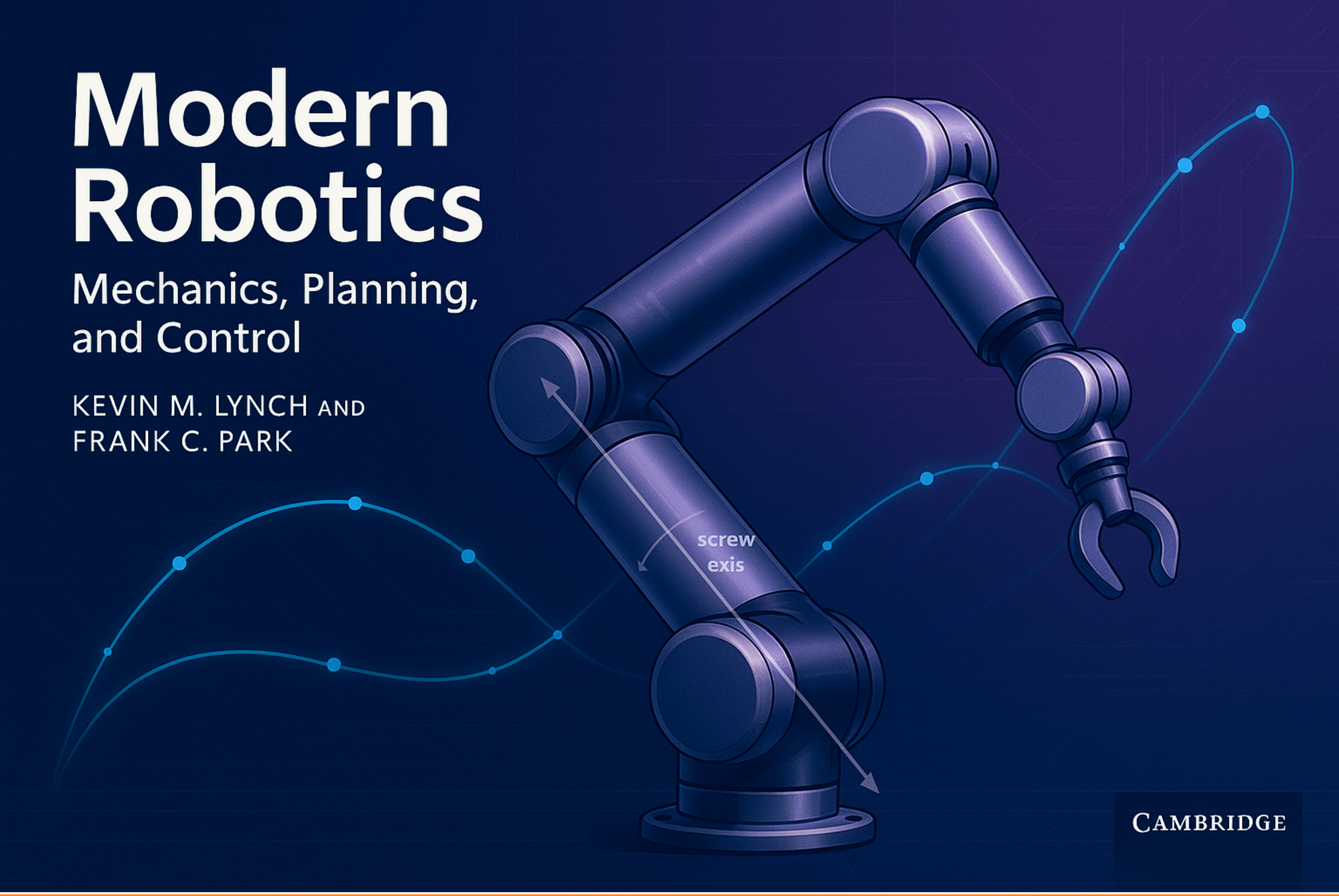

Modern Robotics Self-Assessment Collection

Modern Robotics: Mechanics, Planning, and Control

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Systematically master core concepts and methods of modern robotics through self-assessment problems

Contents

Chapter 1 Preview

Introduction

- *Core concerns of robotics: mechanics, planning, control*
- *Characteristics and applications of mechanisms: open chain vs closed chain*
- *Overview of actuator and sensor technologies*
- *Interdisciplinary nature of robotics*
- *Modern robotics development trends*
- *Learning objectives and challenges of this book*

Modern robotics is an interdisciplinary field that integrates mechanical engineering, electrical engineering, computer science, and mathematics. This textbook "Modern Robotics: Mechanics, Planning, and Control" by Kevin M. Lynch and Frank C. Park is a classic textbook in the field of robotics.

This self-assessment collection is strictly organized according to the textbook chapter sequence, covering the core content of the first 13 chapters:

1. **Chapter 1: Preview** - Overview of robotics and book framework
2. **Chapter 2: Configuration Space** - Mathematical description of robot configurations
3. **Chapter 3: Rigid-Body Motions** - Representation of rigid body motion in space
4. **Chapter 4: Forward Kinematics** - From joint variables to end-effector pose
5. **Chapter 5: Velocity Kinematics and Statics** - Jacobian matrix and force analysis
6. **Chapter 6: Inverse Kinematics** - From end-effector pose to joint variables
7. **Chapter 7: Kinematics of Closed Chains** - Parallel mechanisms and closed-loop constraints
8. **Chapter 8: Dynamics** - Relationship between motion and force
9. **Chapter 9: Trajectory Generation** - Path planning and time parameterization
10. **Chapter 10: Motion Planning** - Path search in configuration space
11. **Chapter 11: Robot Control** - Feedback control system design
12. **Chapter 12: Grasping and Manipulation** - Contact analysis and manipulation planning
13. **Chapter 13: Wheeled Mobile Robots** - Mobile platform kinematics and control

1.1 How to Use This Self-Assessment Collection

1. **Theoretical Learning**: First study the theoretical content of corresponding textbook chapters
2. **Concept Understanding**: Test understanding of basic concepts through self-assessment problems
3. **Mathematical Derivation**: Practice related mathematical derivations and calculations
4. **Practical Application**: Think about theoretical applications in actual robot systems
5. **Comprehensive Review**: Regularly review content from previous chapters

1.2 Learning Recommendations

- Emphasize mathematical foundations, especially linear algebra, differential geometry, and Lie group theory
- Combine theoretical learning with programming practice
- Focus on correspondence between physical intuition and mathematical description
- Actively think about advantages, disadvantages, and applicable scenarios of different methods
- Value integration and application of interdisciplinary knowledge

Chapter One Self-Assessment: Preview

1. **Detailed Core Concerns**

(a). In terms of "mechanics," what specific subtopics will this book discuss (e.g., kinematics, dynamics, statics)? What does each study?
(b). What different types of planning problems does "planning" usually involve? (e.g., geometry-based, time-based, dynamics-considering)
(c). What is the goal of "control"? How does it depend on mechanical models and planning results?

2. **Mechanism Characteristics and Applications**
   (a). What are the typical advantages and disadvantages of open-chain and closed-chain mechanisms in terms of accuracy, stiffness, workspace size, load capacity, etc.?
   (b). For the open-chain industrial manipulator in Figure 1.1(a) and the Stewart-Gough platform in Figure 1.1(b), think about how their emphasis on "mechanics, planning, control" differs in practical applications?
3. **Actuation and Perception Technology Considerations**
   (a). When selecting robot actuators (such as motors), besides output force/torque and speed range, what other important parameters need to be considered?
   (b). What types of errors or uncertainties typically exist in sensor information? How do these uncertainties affect robot system performance?
   (c). What advantages does the "RGB-D camera" mentioned in the book have compared to ordinary vision cameras? How does it help robots perceive the environment?
4. **Interdisciplinary Nature of Robotics**
   (a). Besides those mentioned in the book, what other disciplines does robotics have close connections with? Please provide examples.
   (b). Think about what roles artificial intelligence (AI) and computer vision (CV) play in modern robotics?
5. **Concept Distinction**
   (a). What are the similarities and differences between "robot mechanism" and "robot" in the context of this book?
   (b). What is the role of the "end-effector"? Please list at least three different types of end-effectors and their applications.
6. **Modern Robotics Development Trends**
   (a). What new requirements does Industry 4.0 and smart manufacturing place on robot technology?
   (b). What are the differences in technical challenges between service robots and industrial robots?
   (c). What are the key technologies for Human-Robot Collaboration?
   (d). How are machine learning and artificial intelligence changing traditional robot control methods?
7. **Learning Methods and Thinking Development**
   (a). Why is "systematic thinking" needed in robotics? How to cultivate this thinking in learning?
   (b). What is the importance of mathematical modeling in robotics?
   (c). How to balance theoretical learning and practical skill development?
   (d). Facing the interdisciplinary nature of robotics, how to develop effective learning strategies?
8. **Book Structure and Learning Planning**
   (a). Why does this book adopt the organizational structure of "mechanics $\rightarrow$ planning $\rightarrow$ control"? What is the logic of this structure?
   (b). What is the relationship between the first 8 chapters (basic theory) and later chapters (application extensions)?
   (c). Which chapters have strong interconnections? How should learning be arranged?
   (d). How to use the mathematical tools of this book to lay the foundation for subsequent professional courses and research work?

Chapter 2 Configuration Space

Introduction

- *Concept of degrees of freedom and Grübler's formula*
- *Geometric and topological properties of configuration space*
- *Explicit and implicit parametric representations*
- *Holonomic and nonholonomic constraints*
- *Relationships between task space, workspace, and configuration space*

Chapter Two Self-Assessment: Configuration Space

1. **Degrees of Freedom Calculation and Grübler's Formula**
   (a). Write the two forms of Grübler's formula and explain the meaning of each parameter. What are the values of m for planar and spatial mechanisms respectively?
   (b). For the planar mechanism shown in Figure 2.6 (with overlapping joints), please calculate its degrees of freedom using at least two different methods and verify the consistency of results.
   (c). For the parallelogram mechanism in Figure 2.7(a), the direct calculation result using Grübler's formula is 0, but it actually has 1 degree of freedom. Please explain why the formula fails.
   (d). When three links are connected to the same joint, how many joints are actually formed? How should this be handled when calculating degrees of freedom?
2. **Topological Properties of Configuration Space**
   (a). The C-space topological structure of a planar mobile robot is $\mathbb{R}^2 \times \mathbb{S}^1$. Please explain what $\mathbb{S}^1$ represents and why this structure?
   (b). For a point moving on a sphere surface, its C-space is $\mathbb{S}^2$. If this point can move in and out of the sphere interior, what is its C-space?
   (c). For a rigid body in n-dimensional space, derive the topological structure of its configuration space.
   (d). Explain why the topological structure of space is independent of coordinate representation choice.
3. **Constraint Analysis**
   (a). What are holonomic and nonholonomic constraints? Please provide examples to illustrate the difference between them.
   (b). Why are the constraint equations in the rolling coin problem nonholonomic? What impact does this have on its motion capabilities?
   (c). What types of joints can point constraints and surface constraints be equivalent to?
   (d). If a mechanism consists purely of revolute joints and all rotation axes intersect at one point, how should its degrees of freedom be analyzed?
4. **Practical Application Problems**
   (a). Three identical SRS open-chain manipulators grasp the same object. Find the degrees of freedom of this system. How does the degrees of freedom change if there are n such manipulators?
   (b). A moving platform is connected to a fixed base through n identical legs. For the moving platform to have 6 degrees of freedom, how many degrees of freedom should each leg have?
   (c). Analyze the mobile manipulator system in Figure 2.16, considering different situations such as base movement, manipulator fixedly grasping door handle, etc.
5. **Task Space and Workspace**
   (a). Distinguish between the concepts of task space, workspace, and configuration space.
   (b). What are the configuration space, task space, and workspace of a SCARA robot respectively?
   (c). Why can the task space of a painting robot be described as $\mathbb{R}^3 \times \mathbb{S}^2$ rather than $SE(3)$?

Chapter 3 Rigid-Body Motions

Introduction

- *Properties and operations of rotation matrices $SO(3)$*
- *Concept of angular velocity and spatial/body representations*
- *Exponential coordinates and Rodrigues' formula*
- *Homogeneous transformation matrices $SE(3)$*
- *Twists and wrenches*
- *Adjoint representation*

Chapter Three Self-Assessment: Rigid-Body Motions

1. **Properties of Rotation Matrices** $SO(3)$
   (a). Prove Proposition 3.3: The inverse of a rotation matrix equals its transpose, and its inverse is also a rotation matrix.
   (b). Prove Proposition 3.4: The product of two rotation matrices is still a rotation matrix.
   (c). Does multiplication of rotation matrices satisfy the commutative law? Under what special circumstances does it satisfy the commutative law?
   (d). Explain the adjacent subscript cancellation rule and its application conditions.
   (e). How to represent coordinate frame orientation using rotation matrices? Explain the meaning of subscripts in R_{ab}.
2. **Angular Velocity**
   (a). How to express $\dot{R}(t)$ using body angular velocity ω_b and rotation matrix $R(t)$?
   (b). How to express $\dot{R}(t)$ using spatial angular velocity ω_s and rotation matrix $R(t)$?
   (c). What is the conversion relationship between body angular velocity ω_b and spatial angular velocity ω_s?
   (d). Prove Proposition 3.8: $R[\omega]R^T = [R\omega]$. What is the geometric meaning of this property?
   (e). If $R' = RR_{ab}$, which coordinate system's axis is the rotation about? What about $R'' = R_{ab}R$?
3. **Exponential Coordinates and Rotation**
   (a). Write Rodrigues' formula and explain its geometric meaning.
   (b). Given rotation matrix R, how to calculate its matrix logarithm $[\hat{\omega}]\theta$?
   (c). What is the geometric meaning of the magnitude $||\hat{\omega}\theta|| = |\theta|$ of exponential coordinates?
   (d). Why can the exponential coordinates of $SO(3)$ be represented as a solid ball of radius π with antipodal points on the sphere being equivalent?
   (e). Calculate the rotation matrix corresponding to $\hat{\omega}\theta = (1, 2, 0)$.
4. **Homogeneous Transformation Matrices** $SE(3)$
   (a). Prove the formula for the inverse of homogeneous transformation matrix: $T^{-1} = \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix}$.
   (b). What is the physical meaning of homogeneous transformation matrix multiplication T_1T_2?
   (c). Prove that homogeneous transformation T preserves distances and angles between vectors.
   (d). How to use homogeneous transformation matrices for coordinate transformation and rigid body displacement?
5. **Twists**
   (a). What are the physical meanings of each component in body twist $V_b = (\omega_b, v_b)$ and spatial twist $V_s = (\omega_s, v_s)$?
   (b). Derive the adjoint transformation relationship between body twist and spatial twist.
   (c). What is the normalized definition of screw axis S? Distinguish the representation of rotation axis and translation axis.
   (d). What is the geometric meaning of rigid body motion exponential coordinates $S\theta$? Its relationship with Chasles-Mozzi theorem?
   (e). Given homogeneous transformation matrix T, how to calculate its matrix logarithm $[S]\theta$?

6. **Wrenches and Adjoint Representation**
   (a). Definition of moment and force components in wrench $F = (m, f)$.
   (b). Derive the transformation relationship of wrenches between different coordinate systems, and prove based on power invariance.
   (c). Write the matrix form of adjoint representation $[Ad_T]$ and prove its basic properties.
   (d). What are the wrench representations of ”pure moment” and ”pure force”?

Chapter 4 Forward Kinematics

Introduction

- *Spatial and body forms of Product of Exponentials (PoE) formula*
- *Methods for determining screw axes*
- *Comparison between PoE and DH parameter methods*
- *URDF format and its applications*
- *Forward kinematics examples of typical robots*

Chapter Four Self-Assessment: Forward Kinematics

1. **PoE Formula Principles**
   (a). In the PoE spatial form $T(\theta) = e^{[S_1]\theta_1} \cdots e^{[S_n]\theta_n} M$, why are the exponential terms left-multiplied?
   (b). In the PoE body form $T(\theta) = Me^{[B_1]\theta_1} \cdots e^{[B_n]\theta_n}$, why are the exponential terms right-multiplied?
   (c). What is the conversion relationship between spatial screw axes S_i and body screw axes B_i?
   (d). Why are both the sets of spatial screw axes and body screw axes unique (given coordinate system choice)?
2. **Screw Axis Calculation**
   (a). For revolute joints, how to determine their spatial screw axis $S_i = (\omega_i, v_i)$?
   (b). What is the form of spatial screw axis for prismatic joints?
   (c). If the base coordinate system changes, how do the screw axes and zero configuration M change?
   (d). How to extract screw axis information from robot's URDF description?
3. **Comparison between PoE and DH Parameters**
   (a). List at least three advantages of PoE formula compared to DH parameter method.
   (b). What is the main advantage of DH parameter method? Under what circumstances is this advantage important?
   (c). How to convert DH parameter representation to PoE form?
   (d). Why is PoE formula more suitable for modern robot control algorithms?
4. **Example Calculations**
   (a). For RRRP SCARA robot, determine its M, S_i and B_i, and calculate end-effector pose at given joint angles using both PoE forms.
   (b). Analyze 3R spatial robot: If the end-effector coordinate origin position changes, how do M and S_i change?
   (c). For UR5 robot, when a certain joint angle changes, which screw axes will change?
   (d). Calculate the end-effector pose of RPH robot at $\theta = (\pi/2, 3, \pi)$.
5. **Special Mechanism Analysis**
   (a). Determination of screw axes for PRRRRR, RRRRPR, RRPPRR and other mechanisms.
   (b). When all joint axes intersect at one point, what special properties does forward kinematics have?
   (c). Analyze the kinematic characteristics of spherical wrist (three rotation axes intersecting).
   (d). How to handle mechanisms containing complex joints such as universal (U), helical (H) joints?

Chapter 5 Velocity Kinematics and Statics

Introduction

- *Definition and physical meaning of Jacobian matrix*
- *Spatial Jacobian and body Jacobian*
- *Kinematic singularity analysis*
- *Manipulability ellipsoid and force ellipsoid*
- *Static duality relationship*
- *Analytical Jacobian and geometric Jacobian*

Chapter Five Self-Assessment: Velocity Kinematics and Statics

1. **Jacobian Matrix Fundamentals**
   (a). Explain the statement "the i-th column of Jacobian matrix is the end-effector twist when the i-th joint moves with unit velocity."
   (b). How is the i-th column of spatial Jacobian $J_s(\theta)$ obtained from S_i through adjoint transformation?
   (c). How to calculate the i-th column of body Jacobian $J_b(\theta)$?
   (d). Why is spatial Jacobian related to "space-fixed" axes and body Jacobian related to "body-fixed" axes?
2. **Jacobian Transformation Relationships**
   (a). Derive the transformation relationship $J_s = [Ad_{T_{sb}}]J_b$.
   (b). Prove that the rank of Jacobian matrix is independent of choosing spatial or body Jacobian.
   (c). What is the intrinsic connection between this transformation relationship and single twist transformation?
   (d). Under what circumstances is $J_s = J_b$?
3. **Kinematic Singularities**
   (a). List and explain the five common types of singularities mentioned in the book.
   (b). Are all singularities "harmful"? Give examples of positive effects of singularities.
   (c). How to determine singular configurations through rank or determinant of Jacobian matrix?
   (d). Does kinematic singularity depend on the choice of end-effector coordinate system? Why?
   (e). Difference between boundary singularities and internal singularities and their effects on robot performance.
4. **Manipulability Analysis**
   (a). Derive the manipulability ellipsoid equation $V^T(JJ^T)^{-1}V = 1$.
   (b). Why is body Jacobian preferred when analyzing linear velocity manipulability?
   (c). What is the physical meaning of manipulability ellipsoid volume?
   (d). Relationship between force ellipsoid and manipulability ellipsoid principal axis lengths.
   (e). Explain the phenomenon "directions easy to generate velocity are hard to generate force."
   (f). Compare advantages and disadvantages of different manipulability measures (μ_1, μ_2, μ_3).
5. **Static Analysis**
   (a). Derive $\tau = J^T(\theta)F$ based on virtual work principle.
   (b). Uniqueness problem of joint torque solution for redundant robots ($n > 6$).
   (c). Range of external wrenches that underactuated robots ($n < 6$) can resist.
   (d). Role of Jacobian transpose in force control.
   (e). Robot payload capacity analysis methods.
6. **Analytical Jacobian**
   (a). Difference between analytical Jacobian J_a and geometric Jacobian J.
   (b). Under what circumstances is analytical Jacobian needed?
   (c). Role of matrix $A(r)$ in exponential coordinate analytical Jacobian.
   (d). How is the duality between velocity kinematics and statics reflected?

Chapter 6 Inverse Kinematics

Introduction

- *Complexity and challenges of inverse kinematics problem*
- *Conditions and methods for analytical solutions*
- *Numerical iterative algorithms*
- *Inverse kinematics of redundant robots*
- *Multiple solution handling and solution selection*

Chapter Six Self-Assessment: Inverse Kinematics

1. **Problem Nature and Challenges**
   (a). Why is inverse kinematics of 6DOF spatial robot much more complex than 3R planar arm?
   (b). How does multiple solution nature of inverse kinematics affect control and planning? How to utilize this property?
   (c). Besides target being outside workspace, what other factors cause inverse kinematics to have no solution?
   (d). How to analyze existence and uniqueness of inverse kinematics solutions?
2. **Analytical Inverse Kinematics**
   (a). What are the geometric features that make PUMA-type and Stanford-type robots analytically solvable?
   (b). Why can PUMA arm be decoupled into "inverse position" and "inverse orientation" subproblems?
   (c). How does shoulder offset d_1 affect θ_1 solution? What do "lefty" and "righty" solutions mean?
   (d). How are the three joint angles of spherical wrist solved from given orientation? Relationship with Euler angles?
   (e). Content and application conditions of Pieper's criterion.
3. **Numerical Inverse Kinematics**
   (a). Basic idea and iterative formula of Newton-Raphson method.
   (b). How to define pose "error"? Why can't we directly use $T_{sd} - T_{sb}$?
   (c). Which is more commonly used in numerical IK: body Jacobian or spatial Jacobian? Why?
   (d). Properties of pseudoinverse $J^{\dagger}$: minimum norm solution for redundant case, least squares solution for singular case.
   (e). Factors affecting convergence and initial value selection strategies.
   (f). Damped least squares and other numerical methods.
4. **Redundant Robots**
   (a). Properties and dimensions of solution space for redundant robot inverse kinematics.
   (b). How to utilize redundant degrees of freedom to optimize secondary objectives? Give examples.
   (c). Concept of Jacobian null space and its relationship with self-motion.
   (d). Mathematical expression and implementation of gradient projection method.
   (e). Applications of redundancy in obstacle avoidance and joint limit avoidance.
5. **Practical Application Considerations**
   (a). How to use previous time step solution as initial guess in continuous path tracking?
   (b). How is inverse kinematics simplified when controlling only position (arbitrary orientation)?
   (c). Effects of joint limits on inverse kinematics solution and handling methods.
   (d). Computational efficiency requirements of IK algorithms for real-time control.
   (e). Optimal solution selection strategies in multiple solution cases.

Chapter 7 Kinematics of Closed Chains

Introduction

- *Characteristics and complexity of closed-chain mechanisms*
- *Planar parallel mechanism analysis (3-RPR, etc.)*
- *Spatial parallel mechanisms (Stewart-Gough platform)*
- *Differential kinematics and constraint Jacobian*
- *Three types of singularities*

Chapter Seven Self-Assessment: Kinematics of Closed Chains

1. **Closed-Chain Mechanism Fundamentals**
   (a). Explain the complexity of closed-chain analysis from perspectives of degrees of freedom calculation, constraint equations, and forward/inverse kinematics difficulty.
   (b). Why is "inverse kinematics usually simpler than forward kinematics" for parallel mechanisms? Illustrate with concrete examples.
   (c). Definition, advantages and disadvantages of redundantly actuated parallel mechanisms.
   (d). Methods for establishing closed-loop constraint equations and solution strategies.
2. **Planar Parallel Mechanisms**
   (a). Derivation process of closed-loop constraint equations for 3-RPR mechanism.
   (b). Why does 3-RPR forward kinematics lead to sixth-order polynomial? Meaning of multiple solutions?
   (c). Analyze singular configurations of 3-RPR mechanism and their type determination.
   (d). Kinematic characteristics of other planar parallel mechanisms (3-RRR, 3-PRR, etc.).
3. **Spatial Parallel Mechanisms**
   (a). Direct solution method for Stewart-Gough platform inverse kinematics.
   (b). Why is Stewart platform forward kinematics extremely complex? Origin of 40 solutions?
   (c). Effects on degrees of freedom and performance when replacing SPS legs with UPS and other types.
   (d). Kinematic advantages of Delta robots, SCARA-type parallel mechanisms, etc.
4. **Differential Kinematics**
   (a). Besides kinematic Jacobian, what other type of Jacobian needs to be considered in closed-chain mechanisms?
   (b). Why is static analysis an effective method for deriving parallel mechanism Jacobian?
   (c). Meaning of each term in constraint equation differential form $H_a\dot{q}_a + H_p\dot{q}_p = 0$.
   (d). Physical meaning and consequences when H_p is singular.
5. **Closed-Chain Singularities**
   (a). Mathematical definition and physical meaning of actuator singularities. Difference between non-degenerate and degenerate singularities.
   (b). Characteristics of configuration space singularities and their relationship with actuator singularities.
   (c). Similarity between end-effector singularities and open-chain singularities.
   (d). Do various types of singularities depend on coordinate system choice and actuated joint selection?
   (e). Effects of singularities on parallel mechanism performance and control.
   (f). Why do parallel mechanisms usually have high stiffness and high precision?

Chapter 8 Dynamics

Introduction

- *Basic concepts and equation forms of dynamics*
- *Lagrangian method*
- *Newton-Euler recursive algorithm*
- *Dynamic parameter identification*
- *Computational efficiency optimization*
- *Applications of dynamics in control*

Chapter Eight Self-Assessment: Dynamics

1. **Dynamics Fundamentals**
   (a). What is the core problem studied by robot dynamics?
   (b). Definition and difference between forward dynamics and inverse dynamics.
   (c). Physical meaning of each term in standard dynamics equation form $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$.
   (d). Why is dynamics the theoretical foundation of robot control?
2. **Lagrangian Method**
   (a). Basic form and derivation approach of Lagrangian dynamics equation.
   (b). How to calculate kinetic and potential energy of robot systems?
   (c). Important properties of mass matrix $M(q)$ (symmetry, positive definiteness, etc.).
   (d). Characteristics and calculation methods of Coriolis and centripetal force term $C(q, \dot{q})$.
   (e). Role of Christoffel symbols in dynamics.
3. **Newton-Euler Method**
   (a). Basic idea and advantages of Newton-Euler recursive algorithm.
   (b). What does forward recursion calculate? What does backward recursion calculate?
   (c). How to handle base motion of robots (moving base, floating base)?
   (d). Computational complexity comparison between recursive algorithm and Lagrangian method.
   (e). Applications of adjoint representation and screws in the algorithm.
4. **Dynamic Parameters**
   (a). What dynamic parameters are included? Methods for representing inertial parameters.
   (b). Concept and importance of minimal parameter set.
   (c). Design principles for parameter identification experiments and requirements for excitation trajectories.
   (d). Main factors affecting identification accuracy.
   (e). Parameter validation methods based on physical consistency.
5. **Computational Optimization**
   (a). Comparison of advantages and disadvantages between symbolic computation and numerical computation in dynamics.
   (b). Main methods for improving computational efficiency of dynamics.
   (c). Performance requirements of real-time control for dynamics computation.
   (d). Under what circumstances can certain dynamic terms be ignored?
   (e). Applications of parallel computing in dynamics solution.
6. **Control Applications**
   (a). Specific role of dynamic models in trajectory tracking control.
   (b). Principle, advantages, and implementation methods of feedforward control.
   (c). How to handle dynamic parameter uncertainties in adaptive control?
   (d). Dynamic foundation of force/position hybrid control.
   (e). Principle and limitations of computed torque control method.

Chapter 9 Trajectory Generation

Introduction

- *Difference between path and trajectory*
- *Time parameterization methods*
- *Polynomial time scaling*
- *Trapezoidal velocity profiles*
- *S-curve velocity profiles*
- *Trajectory optimization under constraints*

Chapter Nine Self-Assessment: Trajectory Generation

1. **Basic Concepts**
   (a). What is the difference between path and trajectory? Why is it necessary to distinguish these two concepts in robot control?
   (b). What is time scaling? Why is it necessary to combine geometric paths with time parameters?
   (c). What constraints need to be considered in trajectory generation? How do these constraints affect trajectory shape?
   (d). Explain the physical meaning of each term in $\dot{X} = \frac{dX}{ds}\frac{ds}{dt}$.
2. **Polynomial Time Scaling**
   (a). For point-to-point motion, if we require zero velocity and acceleration at both start and end, what is the minimum polynomial order needed?
   (b). Derive the cubic polynomial time scaling $s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ satisfying boundary conditions: $s(0) = 0, s(T) = 1, \dot{s}(0) = 0, \dot{s}(T) = 0$.
   (c). What advantages does fifth-order polynomial time scaling have compared to cubic polynomial? When is it necessary to use higher-order polynomials?
   (d). If we also require zero acceleration and jerk at start and end, what polynomial order is needed?
3. **Trapezoidal Velocity Profiles**
   (a). Draw the velocity, acceleration, and jerk curves of trapezoidal velocity profiles.
   (b). Prove that for given maximum velocity v and maximum acceleration a, trapezoidal time scaling can minimize motion time.
   (c). Derive the necessary condition for trapezoidal profile to reach maximum velocity: $\frac{v^2}{a} \leq 1$.
   (d). Given velocity v and total time T, prove the necessary condition for three-phase trapezoidal motion: $vT \leq 2$.
4. **Practical Applications**
   (a). Consider linear path $\theta(s) = \theta_{start} + s(\theta_{end} - \theta_{start})$ from $(0, 0)$ to $(\pi, \pi/3)$. Given joint velocity limits $|\dot{\theta}_i| \leq 2$ rad/s and acceleration limits $|\ddot{\theta}_i| \leq 0.5$ rad/s², find the fastest motion time using cubic time scaling.
   (b). For an elliptical path, starting from $(0, 0)$, clockwise through $(2, 1), (4, 0), (2, -1)$ back to $(0, 0)$, write the parametric expression.
   (c). Cylindrical helical path: $x = \cos(2\pi s), y = \sin(2\pi s), z = 2s$, time scaling $s(t) = \frac{1}{4}t + \frac{1}{8}t^2$, find $\dot{X}$ and $\ddot{X}$.
5. **Advanced Trajectory Planning**
   (a). What advantages do S-curve velocity profiles have compared to trapezoidal profiles? In what application scenarios are S-curve profiles more suitable?
   (b). How to handle trajectory planning through multiple path points? What are the continuity conditions at via points?
   (c). In environments with obstacles, how to combine path planning with trajectory generation?
   (d). What is the basic idea of real-time trajectory modification? How to achieve fast response while ensuring smoothness?

Chapter 10 Motion Planning

Introduction

- ❑ *Motion planning in configuration space*
- ❑ *Graph search algorithms: A*, Dijkstra*
- ❑ *Sampling-based methods: RRT, PRM*
- ❑ *Potential field methods*
- ❑ *Motion planning under constraints*

Chapter Ten Self-Assessment: Motion Planning

1. **Motion Planning Fundamentals**
   (a). Why is motion planning usually performed in configuration space rather than workspace?
   (b). Explain the concept of C-obstacle. How to compute C-obstacles from workspace obstacles?
   (c). What are the differences between completeness, probabilistic completeness, and resolution completeness in motion planning?
   (d). How does the curse of dimensionality affect the performance of motion planning algorithms?
2. **Graph Search Algorithms**
   (a). Compare the advantages and disadvantages of Dijkstra algorithm and A* algorithm. What conditions must the heuristic function in A* algorithm satisfy?
   (b). Explain the meaning of each term in A* algorithm's $f(n) = g(n) + h(n)$. Why must the heuristic function satisfy the consistency condition?
   (c). In discretized configuration space, how to choose appropriate resolution? What problems arise if resolution is too high or too low?
   (d). What are the strategies for path replanning in dynamic environments? What is the basic idea of D* algorithm?
3. **Sampling-Based Methods**
   (a). What are the basic steps of RRT algorithm? Why is RRT said to have probabilistic completeness?
   (b). Compare RRT and RRT* algorithms. How does RRT* ensure asymptotic optimality?
   (c). What do the construction and query phases of PRM (Probabilistic Roadmap) method do respectively?
   (d). How to improve basic RRT algorithm to handle dynamic constraints and differential constraints?
4. **Potential Field Methods**
   (a). What is the basic idea of artificial potential field method? How to design attractive and repulsive potential fields?
   (b). How to solve the local minimum problem in potential field methods?
   (c). What advantages do navigation functions have compared to traditional potential fields?
   (d). How to combine potential field methods with other planning algorithms?
5. **Special Applications**
   (a). What special considerations are there for motion planning of nonholonomic constraint systems (such as cars)?
   (b). What additional challenges does motion planning for multi-robot systems face?
   (c). What are the strategies for motion planning in dynamic environments?
   (d). What methods exist for motion planning considering uncertainties?

Chapter 11 Robot Control

Introduction

- *Control system fundamentals*
- *Linear time-invariant system analysis*
- *PID controller design*
- *Robot dynamics control*
- *Adaptive control and robust control*

Chapter Eleven Self-Assessment: Robot Control

1. **Control Theory Fundamentals**
   (a). What is the transfer function of a second-order underdamped system? How do natural frequency ω_n and damping ratio ζ affect system response?
   (b). Derive the 2% settling time for a second-order system: $t = \frac{4}{\zeta\omega_n}$. What is the 5% settling time?
   (c). What is the relationship between overshoot and damping ratio? How to reduce overshoot by adjusting parameters?
   (d). How to determine system stability? What are the application conditions of Routh-Hurwitz criterion?
2. **PID Control**
   (a). What are the roles of each component in PID controller: what do proportional, integral, and derivative terms do respectively?
   (b). What are the basic steps of Ziegler-Nichols tuning method?
   (c). What is integral windup phenomenon? How to solve it through anti-windup techniques?
   (d). The derivative term is sensitive to noise. How to improve the implementation of derivative term in practice?
3. **Robot Dynamics Control**
   (a). What is the basic idea of computed torque control? How to achieve system linearization?
   (b). Derive the mathematical model of independent joint control. Under what circumstances can coupling terms be ignored?
   (c). What is the role of feedforward control in robot control? How to design feedforward compensators?
   (d). What is the relationship between inverse dynamics control and computed torque control? What should be noted in implementation?
4. **Advanced Control Methods**
   (a). What is the basic idea of adaptive control? How to design parameter adaptation laws?
   (b). What are the advantages and disadvantages of sliding mode control? How to reduce chattering in sliding mode control?
   (c). What are the differences between robust control and adaptive control? What is the basic idea of H_∞ control?
   (d). What are the application prospects of machine learning in robot control?
5. **Practical Considerations**
   (a). How do sensor noise and delays affect control system performance? How to design filters?
   (b). How to handle actuator saturation and dead-zone nonlinearities?
   (c). How to design controllers to handle model uncertainties and external disturbances?
   (d). What are the hardware and software requirements for real-time control systems?

Chapter 12 Grasping and Manipulation

Introduction

- ❏ *Contact models and friction cones*
- ❏ *Force closure and form closure*
- ❏ *Grasp planning and grasp quality evaluation*
- ❏ *Manipulation planning*
- ❏ *Assembly stability analysis*

Chapter Twelve Self-Assessment: Grasping and Manipulation

1. **Contact Models**
   (a). What are the differences in constraint characteristics between point contact, line contact, and surface contact?
   (b). What is the mathematical expression of Coulomb friction model? What is the geometric meaning of friction cone?
   (c). In planar cases, how to determine the instantaneous center of rotation (CoR) from wrenches?
   (d). Duality relationship of wrenches: why are force and motion said to be dual?
2. **Grasp Analysis**
   (a). What is the difference between form closure and force closure? Which condition is stronger?
   (b). In planar cases, what is the necessary condition for n contact points to achieve form closure?
   (c). Derive the general conditions for form closure in three-dimensional space. Why are more than 7 contact points needed?
   (d). Application of Screw theory in grasp analysis: how to describe contact constraints using screws?
3. **Grasp Quality**
   (a). What are the evaluation metrics for grasp quality? What are their respective physical meanings?
   (b). What is grasp robustness? How to quantify grasp resistance to disturbances?
   (c). What methods exist for determining optimal grasp positions?
   (d). How to consider object weight and inertial properties in grasp planning?
4. **Manipulation Planning**
   (a). What is quasi-static manipulation? What are the applicable conditions of quasi-static assumption?
   (b). How to model kinematic constraints of rolling and sliding?
   (c). How to plan contact state transitions during manipulation?
   (d). What are the special considerations for bimanual coordinated manipulation?
5. **Assembly and Stability**
   (a). How to analyze assembly stability? What is the relationship between center of gravity and support polygon?
   (b). What is the basic idea of first-order analysis method?
   (c). How to use graphical methods to analyze stability of planar assemblies?
   (d). How to calculate contact force distribution in multi-body systems?

Chapter 13 Wheeled Mobile Robots

Introduction

- *Kinematic models of wheeled robots*
- *Handling nonholonomic constraints*
- *Odometry and localization*
- *Trajectory tracking control*
- *Mobile manipulation robots*

Chapter Thirteen Self-Assessment: Wheeled Mobile Robots

1. **Kinematic Modeling**
   (a). Derive the kinematic equations for differential drive robots. How to express no-slip rolling constraints?
   (b). What are the geometric constraints of Ackermann steering mechanism? How to avoid wheel skidding?
   (c). What are the working principles of omniwheels and mecanum wheels? How do they achieve omnidirectional motion?
   (d). Compare the motion capabilities of different wheeled configurations: differential drive, Ackermann, omnidirectional, etc.
2. **Nonholonomic Constraints**
   (a). Why are the constraints of wheeled robots nonholonomic? What impact does this have on control?
   (b). How to analyze reachability of nonholonomic systems? What is the role of Lie brackets?
   (c). What is the concept and standard form of chained systems?
   (d). How to convert nonholonomic systems to chained form for control design?
3. **Localization and Odometry**
   (a). What are the error sources in wheel odometry? How to reduce cumulative errors?
   (b). What are the roles of gyroscopes and accelerometers in mobile robot localization?
   (c). What are the basic principles of visual odometry? What advantages does it have compared to wheel odometry?
   (d). What are the basic methods for multi-sensor fusion localization?
4. **Trajectory Tracking Control**
   (a). Design trajectory tracking controller for differential drive robots. How to handle nonholonomic constraints?
   (b). Applications of backstepping control in mobile robot control?
   (c). How to design controllers to ensure convergence of tracking errors?
   (d). What is the difference between path following and trajectory tracking? What is the principle of pure pursuit algorithm?
5. **Mobile Manipulation Robots**
   (a). What are the coordination control strategies for mobile base and manipulator?
   (b). How to allocate motion between mobile base and manipulator? How to utilize redundancy?
   (c). Stability considerations in mobile manipulation: how to prevent tipping?
   (d). What challenges does mobile manipulation planning face in dynamic environments?

Appendix A Self-Assessment Answers

A.1 Chapter One Answers: Preview

A.1.1 Detailed Core Concerns

1. **Mechanics subtopics:**
   - **Kinematics**: Studies motion without considering forces, including relationships between position, velocity, and acceleration
   - **Statics**: Studies balance of forces and moments without motion
   - **Dynamics**: Studies relationships between force and motion, including mass and inertia factors
2. **Different types of planning:**
   - **Geometric path planning**: Finding collision-free paths in configuration space
   - **Trajectory planning**: Adding time parameters to paths, considering velocity and acceleration constraints
   - **Dynamic planning**: Motion planning considering actuator limits and dynamic constraints
3. **Control goals and dependencies:** Control aims to enable robots to accurately execute planned motions. It depends on:
   - Mechanical models providing mathematical system description
   - Planning results providing desired motion trajectories
   - Sensor feedback providing actual state information

A.1.2 Mechanism Characteristics and Applications

1. **Open-chain vs closed-chain comparison:**

Property	Open-chain	Closed-chain
Accuracy	Error accumulation	High precision
Stiffness	Relatively low	High stiffness
Workspace	Large	Relatively small
Load capacity	Medium	High

2. **Application emphasis differences:**
   - **Open-chain industrial manipulator**: Emphasizes flexibility and large workspace, kinematics and trajectory planning are more important
   - **Stewart-Gough platform**: Emphasizes high precision and high stiffness, mechanical analysis and precision control are more important

A.2 Chapter Two Answers: Configuration Space

A.2.1 Degrees of Freedom Calculation and Grübler's Formula

1. **Two forms of Grübler's formula:**

$$\text{DoF} = m(N-1) - \sum_{i=1}^{J} c_i \tag{A.1}$$

$$\text{DoF} = m(N-1-J) + \sum_{i=1}^{J} f_i \tag{A.2}$$

   Where: $m = 3$ for planar mechanisms, $m = 6$ for spatial mechanisms; N: number of links; J: number of joints.
2. **Overlapping joint handling:** When three links connect to the same location, it forms 2 joints, not 1 joint.

A.3 Chapter Three Answers: Rigid-Body Motions

A.3.1 Properties of Rotation Matrices $SO(3)$

1. **Proof of Proposition 3.3:** For rotation matrix R, we have $R^T R = I$ and $\det(R) = 1$.

$$R^T = R^{-1} \quad \text{(orthogonality)} \tag{A.3}$$

$$(R^{-1})^T(R^{-1}) = (RR^T)^{-1} = I^{-1} = I \tag{A.4}$$

$$\det(R^{-1}) = \det(R^T) = \det(R) = 1 \tag{A.5}$$

2. **Proof of Proposition 3.4:** Let $R_1, R_2 \in SO(3)$:

$$(R_1R_2)^T(R_1R_2) = R_2^T R_1^T R_1 R_2 = R_2^T R_2 = I \tag{A.6}$$

$$\det(R_1R_2) = \det(R_1)\det(R_2) = 1 \times 1 = 1 \tag{A.7}$$

3. **Commutativity of rotation matrix multiplication:** Generally does not satisfy commutativity. Special cases where it does:
   - One of them is the identity matrix
   - Both rotations are about the same axis
4. **Adjacent subscript cancellation rule:** Applies to rotation matrix multiplication and rotation matrix-vector multiplication, e.g., $R_{ab}R_{bc} = R_{ac}$
5. **Meaning of R_{ab}:** Represents the orientation of coordinate frame $\{b\}$ relative to coordinate frame $\{a\}$, or the rotation transformation from $\{b\}$ to $\{a\}$.

A.4 Chapter Four Answers: Forward Kinematics

A.4.1 PoE Formula Principles

1. **Left multiplication in spatial form:** In $T(\theta) = e^{[S_1]\theta_1} \cdots e^{[S_n]\theta_n} M$, each $e^{[S_i]\theta_i}$ represents motion about space-fixed axis S_i at current pose, hence left multiplication.
2. **Right multiplication in body form:** In $T(\theta) = Me^{[B_1]\theta_1} \cdots e^{[B_n]\theta_n}$, each $e^{[B_i]\theta_i}$ represents motion about body-fixed axis B_i, hence right multiplication.
3. **Screw axis conversion:** $B_i = [Ad_{M^{-1}}]S_i$, where M is the home configuration transformation matrix.
4. **Uniqueness:** Given coordinate system choice and joint axis definitions, both spatial and body screw axis sets are unique.

A.5 Chapter Five Answers: Velocity Kinematics and Statics

A.5.1 Jacobian Matrix Fundamentals

1. **Physical meaning of Jacobian columns:** The i-th column J_i represents the instantaneous twist (linear and angular velocity) of the end-effector when $\dot{\theta}_i = 1$ and all other joint velocities are zero.
2. **Spatial Jacobian calculation:**

$$J_s = [J_{s1}\ J_{s2}\ \cdots\ J_{sn}] \tag{A.8}$$

$$J_{si} = [Ad_{T_{0i-1}}]S_i \tag{A.9}$$

3. **Body Jacobian calculation:**

$$J_b = [J_{b1}\ J_{b2}\ \cdots\ J_{bn}] \tag{A.10}$$

$$J_{bi} = [Ad_{T_{in}^{-1}}]B_i \tag{A.11}$$

A.6 Chapter Six Answers: Inverse Kinematics

A.6.1 Problem Nature and Challenges

1. **Complexity differences:** 6DOF spatial robot inverse kinematics is much more complex than 3R planar arm due to:
   - Increased complexity of nonlinear equation systems
   - Dramatically increased number of multiple solutions
   - More difficult handling of singularities
2. **Multiple solution effects and utilization:**
   - **Effects**: Increases control complexity, requires solution selection strategies
   - **Utilization**: Obstacle avoidance, joint limit avoidance, energy optimization
3. **Other causes of no solution:**
   - Joint limit constraints
   - Numerical problems near singularities
   - Self-collision constraints

A.7 Chapter Seven Answers: Kinematics of Closed Chains

A.7.1 Closed-Chain Mechanism Fundamentals

1. **Complexity analysis:**
   - **Degrees of freedom calculation**: Need to consider closed-loop constraints
   - **Constraint equations**: Nonlinear constraint equation systems
   - **Forward kinematics**: Usually more difficult than inverse kinematics
2. **Why inverse kinematics is simpler:** For parallel mechanisms, given end-effector pose, each chain's inverse kinematics can be solved independently.
3. **Redundant actuation:**
   - **Definition**: Number of actuators exceeds mechanism degrees of freedom
   - **Advantages**: Improve stiffness, eliminate singularities, increase load capacity
   - **Disadvantages**: Complex control, increased cost, internal force issues

A.7.2 Planar Parallel Mechanisms

1. **3-RPR constraint equations:** For three RPR chains, each provides one constraint equation:
$$||p + R \cdot r_i - b_i||^2 = l_i^2, \quad i = 1, 2, 3 \tag{A.12}$$
   where p is platform position, R is platform attitude, r_i are connection points on platform, b_i are connection points on base.
2. **Generation of sixth-order polynomial:** Three quadratic constraint equations solved simultaneously, after elimination yield sixth-order polynomial in platform angle, theoretically up to 6 solutions.
3. **Singular configuration analysis:** 3-RPR singularities usually belong to configuration space singularities, occurring when three chains approach collinearity.

A.8 Chapter Eight Answers: Dynamics

A.8.1 Dynamics Fundamentals

1. **Core problem:** Studies relationships between force and motion, including how to calculate motion from forces and vice versa.
2. **Standard equation form:** $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$

- $M(q)$: Mass matrix (inertial effects)
- $C(q,\dot{q})$: Coriolis and centrifugal force matrix
- $G(q)$: Gravity vector
- τ: Joint torques

3. **Control theory foundation:** Dynamics model provides the mathematical description of the system needed for controller design.

A.8.2 Lagrange Method

1. **Basic form:**

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \tag{A.13}$$

$$L = T - V \quad \text{(Lagrangian function)} \tag{A.14}$$

2. **Kinetic and potential energy calculation:**

$$T = \frac{1}{2}\sum_{i=1}^{n} \dot{q}^T M_i(q)\dot{q} \tag{A.15}$$

$$V = \sum_{i=1}^{n} m_i g^T p_i(q) \tag{A.16}$$

3. **Mass matrix properties:**
   - Symmetry: $M(q) = M^T(q)$
   - Positive definiteness: $M(q) > 0$
   - Boundedness: $\lambda_{\min} I \leq M(q) \leq \lambda_{\max} I$
4. **Christoffel symbols:** $C_{ijk} = \frac{1}{2}\left(\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i}\right)$

A.9 Chapter Nine Answers: Trajectory Generation

A.9.1 Basic Concepts

1. **Path vs trajectory:**
   - **Path**: Geometric trajectory without time information, e.g., $\theta(s), s \in [0,1]$
   - **Trajectory**: Motion description with time information, e.g., $\theta(t), t \in [0,T]$
   - Need to distinguish in control: path determines motion direction, trajectory determines motion speed
2. **Role of time scaling:** Combines geometric paths with time parameters to generate executable motion trajectories satisfying velocity and acceleration constraints.
3. **Trajectory constraints:**
   - Joint velocity and acceleration limits
   - Actuator torque limits
   - Path geometric constraints
   - Smoothness requirements
4. **Velocity relationship:** $\dot{X} = \frac{dX}{ds}\frac{ds}{dt}$, where $\frac{dX}{ds}$ is path tangent vector, $\frac{ds}{dt}$ is time scaling velocity.

A.10 Chapter Ten Answers: Motion Planning

A.10.1 Motion Planning Fundamentals

1. **Configuration space advantages:**
   - Robot simplified to a point
   - Obstacles mapped to C-obstacles

- Convenient for applying graph search algorithms

2. **C-obstacles:** Mapping of workspace obstacles in configuration space, considering robot shape and pose.
3. **Completeness types:**
   - **Completeness**: If solution exists, it will be found
   - **Probabilistic completeness**: Probability of finding solution approaches 1 as time increases
   - **Resolution completeness**: Complete at given resolution
4. **Curse of dimensionality:** Search space grows exponentially with dimension, algorithm complexity increases dramatically.

A.11 Chapter Eleven Answers: Robot Control

A.11.1 Control Theory Fundamentals

1. **Second-order system transfer function:** $G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
   ω_n affects response speed, ζ affects overshoot and oscillation.
2. **Settling time:** 2% settling time: $t = \frac{4}{\zeta\omega_n}$; 5% settling time: $t = \frac{3}{\zeta\omega_n}$

A.12 Chapter Twelve Answers: Grasping and Manipulation

A.12.1 Contact Models

1. **Contact types:**
   - **Point contact**: 3 constraints (planar), 6 constraints (spatial)
   - **Line contact**: 4 constraints (planar), 10 constraints (spatial)
   - **Surface contact**: 5 constraints (planar), 12 constraints (spatial)
2. **Coulomb friction:** $|f_t| \leq \mu f_n$, where f_t is tangential force, f_n is normal force, μ is friction coefficient. Friction cone represents range of possible contact forces.
3. **Instantaneous center of rotation:** For planar twist $V = (\omega_z, v_x, v_y)$:
$$\text{CoR} = \begin{bmatrix} -v_y/\omega_z \\ v_x/\omega_z \end{bmatrix} \tag{A.17}$$
4. **Duality relationship:** Force and motion are dual in the Screw theory framework, satisfying $F^T V = \text{power}$.

A.13 Chapter Thirteen Answers: Wheeled Mobile Robots

A.13.1 Kinematic Modeling

1. **Differential drive kinematics:**
$$\dot{x} = \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L)\cos\theta \tag{A.18}$$
$$\dot{y} = \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L)\sin\theta \tag{A.19}$$
$$\dot{\theta} = \frac{r}{2L}(\dot{\phi}_R - \dot{\phi}_L) \tag{A.20}$$
   No-slip rolling constraint: $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$

A.14 Numerical Methods

- Numerical integration: Runge-Kutta methods and multistep methods
- Nonlinear equation solving: Newton method and quasi-Newton methods

- Optimization methods: gradient methods, Newton methods, and constrained optimization
- Interpolation and fitting: polynomial interpolation and spline functions
- Numerical linear algebra: LU decomposition, QR decomposition, and SVD