EE 020

Homework 8

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Problem 1 (Chapter 7.1 Problem 24)

24. $\begin{bmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{bmatrix}$ $\quad \det(\lambda I - A) = 0$

$$|\lambda I - A| = \begin{bmatrix} \lambda-3 & -2 & -3 \\ 3 & \lambda+4 & -9 \\ 1 & 2 & \lambda-5 \end{bmatrix}$$

$$= (\lambda-3)\begin{vmatrix} \lambda+4 & -9 \\ 2 & \lambda-5 \end{vmatrix} - (-2)\begin{vmatrix} 3 & -9 \\ 1 & \lambda-5 \end{vmatrix} + 3\begin{vmatrix} 3 & \lambda+4 \\ 1 & 2 \end{vmatrix}$$

$$= (\lambda-3)[(\lambda+4)(\lambda-5)+18] + 2[3(\lambda-5)+9] + 3(6-(\lambda+4))$$

$$= (\lambda-3)[\lambda^2-\lambda-20+18] + 2[3\lambda-15+9] + 3(6-\lambda-4)$$

$$|\lambda I - A| = (\lambda-3)(\lambda^2-\lambda-2) + 2(3\lambda-6) + 3(2-\lambda)$$

$$= (\lambda^3-\lambda^2-2\lambda-3\lambda^2+3\lambda+6) + 6\lambda-12+6-3\lambda$$

$$= \lambda^3 - 4\lambda^2 + 4\lambda$$

$$= \lambda(\lambda-2)^2 \quad = 0$$

(b) $\lambda(\lambda-2)^2 = 0$ eigenvalues are 0,2,2

$\lambda_1 = 0 \quad \lambda_2 = 2 \quad \lambda_3 = 2. \quad (\lambda I - A)x = 0 \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$\lambda = 0 \cdot \quad \begin{bmatrix} -3 & -2 & 3 \\ 3 & 4 & -9 \\ 1 & 2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1+x_3 \\ x_2-3x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

From the above system $\begin{cases} x_1+x_3=0 \\ x_2-3x_3=0 \end{cases}$ let $x_3 = s$ then $x_1 = -s \quad x_2 = 3s$

$\lambda_1 = 0: \quad x_1 = \begin{bmatrix} -s \\ 3s \\ s \end{bmatrix} = s\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

$\lambda = 2 \quad \begin{bmatrix} -1 & -2 & 3 \\ 3 & 6 & -9 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} x_1 = -2s+3t \\ x_2 = s \\ x_3 = t \end{matrix}$

$\lambda_2 = 2 \quad x_{2,3} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

Problem 4 (chapter 7.2. problem 12)

12. $A = \begin{bmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{bmatrix}$

$\boxed{\lambda I - A} = \begin{bmatrix} \lambda-3 & -2 & 3 \\ 3 & \lambda+4 & -9 \\ 1 & 2 & \lambda-5 \end{bmatrix}$

$= (\lambda-3)(\lambda^2-\lambda-20+18)+2(3\lambda-15+9)+3(6-\lambda-4)$

$= (\lambda-3)(\lambda^2-\lambda-2)+2(3\lambda-6)+3(2-\lambda)$

$= \lambda^3-4\lambda^2+4\lambda$

$= \lambda(\lambda^2-4\lambda+4)$

$= \lambda(\lambda-2)^2$

$\lambda = 0, 2, 2$ eigenvalue.

$\lambda = 0$ $\begin{bmatrix} 0-3 & -2 & 3 \\ 3 & 0+4 & -9 \\ 1 & 2 & 0-5 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $-3x_1-2x_2+3x_3=0$

$2x_2-6x_3=0$

let $x_3 = s$ then $x_2 = 3s$ $x_1 = -s$ $x = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

$\lambda = 2$ $\begin{bmatrix} 2-3 & -2 & 3 \\ 3 & 2+4 & -9 \\ 1 & 2 & 2-5 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $-x_1-2x_2+3x_3=0$

$x_3 = s$ $x_2 = t$.

$x_1 = -2t+3s$

$x_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ $x_3 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ thus: total eigenvectors for $\lambda = 0, 2, 2$ are

$P = \begin{bmatrix} -1 & -2 & 3 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{3}{2} \\ \frac{-3}{2} & -2 & \frac{9}{2} \\ \frac{-1}{2} & -1 & \frac{5}{2} \end{bmatrix}$

$\det(P) = -1(1-0)+2(3-0)+3(0-1) = 2.$

$P^{-1}AP = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{3}{2} \\ \frac{-3}{2} & -2 & \frac{9}{2} \\ \frac{-1}{2} & -1 & \frac{5}{2} \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & -3 \\ -3 & -4 & 9 \\ -1 & -2 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 & 3 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Problem 6 Mathlab Questions

study * Matlab functions poly() and roots(), Lets find eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ -4 & 4 & 3 \end{bmatrix}$$

See next page for outputs

Problem 11:

Study Matlab function eig(). Find eigenvalues and eigenvectors of the matrix.

$$\begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ -4 & 4 & 3 \end{bmatrix}$$

See next page for outputs.