CS 150

HW5

Xia Hua

86118335

1. Convert the grammer

$$S \to AaAb$$
$$A \to aS \mid bA \mid a$$

to a PDA that accepts the same language by empty stack,

First create the CFG:

$G = \{\{S, A\}, \{a, b\}, A', S, q\}$

let $A' = \{ S \to AaAb, A \to aS \mid bA \mid a \}$

So $P_a = (\delta, \{a,b\}, \{S, A, a, b\}, q, S\}$

we can find those 4 values as:

$$\begin{cases} \delta(q, \varepsilon, S) = \{(q, Aa, Ab)\} \\ \delta(q, \varepsilon, A) = \{(q, aS), (q, bA), (q, a)\} \\ \delta(q, a, a) = \{(q, \varepsilon)\} \\ \delta(q, b, b) = \{(q, \varepsilon)\} \end{cases}$$

2. ① $\delta(q,0,z_0)=\{(q,xz_0)\}$

② $\delta(q,0,x)=\{(q,xx)\}$

③ $\delta(q,1,x)=\{(q,x)\}$

④ $\delta(q,\varepsilon x)=\{(p,\varepsilon)\}$

⑤ $\delta(p,e,x)=\{(p,\varepsilon)\}$

⑥ $\delta(p,1,x)=\{(p,xx)\}$

⑦ $\delta(p,1,z_0)=\{(p,\varepsilon)\}$

Assume the new CFG is a
According to the relations on the left, we can create a new a is $(V,\{0,1\},R,S)$,
V can be represent as
$\{(pxp)(pz_0p),(zxq),S$
$(qxp)(qz_0q)(qz_0p)\}$

Define (qz_0q) is B
(qxq) is D
(pxp) is F

A: $(qz_0q)\to 0(qxq)(qz_0q)$

$A\to 0EA$

$(qz_0p)\to 0(qxq)(qz_0p)$

$B\to 0CB$

$(qz_0p)\to 0(qxp)(pz_0p)$

$B\to 0DF$

$(qxq)\to 0(qxq)(qz_0p)$

$C\to 0CC$

$(qxp)\to 0(qxq)(qxp)$

$D\to 0CD$

$(qxp)\to 0(qxp)(pxp)$

$D\to 0DF$

$(qxq)\to 1(qxq)$

$C\to 1C$

$(qxp)\to 1(qxq)$

$D\to 1C$

$(qxp)\to\varepsilon$

$D\to\varepsilon$

$(pxp)\to\varepsilon$

$E\to\varepsilon$

$(pxp)\to 1(pxp)(pxp)$

$E\to 1EE$

$(pz_0p)\to 1\varepsilon$

$F\to\varepsilon$

Therefore combine left rules
CFG will be:

$S\to A\mid B$

$A\to 0CA$

$B\to 0CB\mid 0DF$

$C\to 0CC\mid 1C$

$D\to 0CD\mid 0DF\mid 1C\mid\varepsilon$

$E\to 1EE\mid\varepsilon$

$F\to\varepsilon$

3.(a)

$C \to S \Rightarrow C \to S$

$A \to C \Rightarrow A \to C$

$B \to S \mid A \Rightarrow B \to S \mid A$

$S \to 0A0 \mid 1B1 \mid BB \Rightarrow$

$S \in \{ 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB \mid B \}$

for the ε-production:

$S \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB \mid B \mid$

$A \to C$

$B \to S \mid A$

$C \to S$

(b) By the question we know that the possible route for traverse is

$A \to C \mid C \to S \mid B \to S \mid$

$A \to C \to S \to B.$

Summarize to:

$S \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB$

also the A; B, C, are the same to S:

$A \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB$

$B \to \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$C \to \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

(c)

$S \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid BB$

(d) CNF noticed we have a new rule. that $A \to 0$, $B \to 1$.

So change the previous rules that is. $C \to SA$, $D \to SB$

the new CNF is:

$S \to AA \mid AC \mid BB \mid BD \mid SS$

including above new rules.

4. (b) $\{a^n b^n c^i \mid i \le n\}$

let $w = a^k b^k c^k$ $\quad k \in L$.

$w = uvxwxy$ $\quad |vwx| \le k$

$w' = uv^2wy^2x$ $\quad |vx| > 0$

① for vwx contain c but not b, $w' = uv^2x^2y$, the b's inside w' is less than c's inside the w', so we can conduce that $i > n$, so we can conclude that $w' \notin L$. ☆

② vwx does not have c, but must have at least one 1.

$w' = w|v^2wy^2x$, we will get more a or b which is $w' \notin L$ ☆

So for conclusion is not content-free.

(c) $w = 0^k \in L$ and $w = uvwxy$.

① $|vwx| \le k$

② $|vx| > 0$

$w' = uwx$

$\Downarrow$

$w' = 0^{k-a}$ (for every $a > 0$, k is real numbq)

But we can easily found out $k-a$ is not possibe to be a unique prime number, therefore. $w' \notin L$.

So the total language is not context-free.

5. (a) $L_1 \rightarrow G_1$: $S \rightarrow AC \mid \varepsilon$
$A \rightarrow aAbb \mid \varepsilon$
$C \rightarrow cC \mid \varepsilon$

$L_2 \rightarrow G_2$: $S \rightarrow AB \mid \varepsilon$
$A \rightarrow Aa \mid \varepsilon$
$B \rightarrow bBcc \mid \varepsilon$

(b) $L_1 \cap L_2$ is not CFG.

Inside the L_2 language, the number of b's is equal the number of 2a's, the number of C's is equal the number of 2b's.

Inside the L_1 language, the number of b's is equal the number of a's, the number of C's is equal the number of 2b's.

We can conclude the property that $L_1 \cap L_2$:

$C = 2b = 4a$. ∈ all represent the numbers in language not actual value.

$L = \{ a^n b^{2n} c^{4n} \mid$ for n is natural numbers $\}$

Let's prove the CFG is not context free,

$w = uvwxy$.

$w' = uwy$

① $|vwx| \leq n$.
⇓
Only contains C the total numbers of $(b+c) \leq 6n-1$
So $w' \notin L$

② $|vx| > 0$
⇓
must contains at lease 1a ; the total numbers $(a+b) = 3n-1$.
So $w' \notin L$.

Therefore the language is not context-free.
the $L_1 \cap L_2$ is not CFG.

6.(b) baaab

S,C				
S,A,C	S,C			
/	S,A,C	B		
A,S	B	B	S,C	
B	A,C	A,C	A,C	B

w = b a a a b ⇒ S, C

(c) aabab:

S,C				
S,A	B			
B	B	S,C		
B	S,C	A,S	S,C	
A,C	A,C	B	A,C	B

w = a a b a b ⇒ S, C.