CS 150 (Closed-book) Midterm Test II

May 24, Friday, 2019

Total: 50 points

QUESTION 1. [10 pts] Following the procedure given in class, convert the following regular expression to an ϵ-NFA:

$$R = 0^*(10^* + 11)0^*$$

Answer:

	ϵ	0	1
$\rightarrow q_0$	$\{q_1\}$	$\emptyset$	$\emptyset$
q_1	$\{q_2\}$	$\{q_1\}$	$\emptyset$
q_2	$\emptyset$	$\emptyset$	$\{q_3, q_4\}$
q_3	$\{q_6\}$	$\{q_3\}$	$\emptyset$
q_4	$\emptyset$	$\emptyset$	$\{q_5\}$
q_5	$\{q_6\}$	$\emptyset$	$\emptyset$
q_6	$\{q_7\}$	$\{q_6\}$	$\emptyset$
$^*q_7$	$\emptyset$	$\emptyset$	$\emptyset$

Note that the ϵ-NFA in the answer has been simplified a little bit. There could be a few more states with trivial ϵ-transitions, or fewer states with further simplification.

QUESTION 2. Convert the following DFA to a regular expression by eliminating its states in the order q_2, q_1:

	0	1
$\rightarrow *q_0$	q_2	q_1
q_1	q_2	q_0
q_2	q_1	q_2

Answer:

After eliminating state q_2, we have a new transition from q_1 to q_1 with label 01^*0 (2 pts), and the arc label from q_0 to q_1 becomes $1 + 01^*0$ (2 pts).

After eliminating state q_1, the arc label from q_0 to q_0 becomes (3 pts)

$$(1 + 01^*0)(01^*0)^*1$$

which leaves the final answer as (3 pts)

$$((1 + 01^*0)(01^*0)^*1)^*$$

Give partial credits for correct steps. Deduct at least 3 pts for eliminating states in a different order.

QUESTION 3. [10 pts] Prove that the following language is not regular using the Pumping Lemma:

$$L = \{0^i 1^j 2^{2i} \mid i, j \geq 1\}$$

Answer:

Let n be the constant in the Pumping Lemma. Consider $w = 0^n 12^{2n} \in L$. Let $w = xyz$ be any partition satisfying (i) $|y| > 0$ and $|xy| \leq n$. Clearly, $x = 0^s$ and $y = 0^t$ for some $t > 0$. Then $xyyz \notin L$, and thus a contradiction.

Give partial credits for correct/reasonable steps.

QUESTION 4. [10 pts] Convert the following DFA to the minimum-state equivalent DFA.

	0	1
$\rightarrow *q_0$	q_2	q_1
q_1	q_3	q_0
$*q_2$	q_0	q_4
q_3	q_1	q_3
q_4	q_3	q_2

Answer:

The three-state DFA for accepting binary numbers divisible by 3.

Give partial credits for correct steps.

QUESTION 5. [10 pts] Let $G = (\{S\}, \{(,)\}, P, S)$ be a CFG, where P contains the following rules:

$$S \to \epsilon | SS | S(S)$$

1. [3 pts] List five different strings in $L(G)$.
2. [2 pts] What is the language $L(G)$? (You may either give its name or characterize the strings in it.)
3. [5 pts] Show that the grammar G is ambiguous.

Answer:

1. For example, $\epsilon, (), ()(), (()), (())(), ()(())$ (3 pts).
2. Dyck language, or the language/set of balanced parentheses/brackets. (2 pts)
3. It is easy to give two parse trees for the string () with one starting with rule $S \to SS$ and the other starting with rule $S \to S(S)$ (5 pts). It is also okay to show two different leftmost derivations for some string in $L(G)$.