CS 150 (Closed-book) Midterm Test I

May 3, Friday, 2019

Total: 50 points

QUESTION 1. [10 pts] Design a DFA to accept the following language:

$$L = \{x \mid x \in \{0,1\}^*, \text{ the number of } 0's \text{ in } x \text{ is divisible by 4 and the number of } 1's \text{ in } x \text{ is divisible by 2}\}$$

Answer:

	0	1
$* \rightarrow q_{0,0}$	$q_{1,0}$	$q_{0,1}$
$q_{0,1}$	$q_{1,1}$	$q_{0,0}$
$q_{1,0}$	$q_{2,0}$	$q_{1,1}$
$q_{1,1}$	$q_{2,1}$	$q_{1,0}$
$q_{2,0}$	$q_{3,0}$	$q_{2,1}$
$q_{2,1}$	$q_{3,1}$	$q_{2,0}$
$q_{3,0}$	$q_{0,0}$	$q_{3,1}$
$q_{3,1}$	$q_{0,1}$	$q_{3,0}$

Give partial credits for DFAs that handles the number of 0's or the number of 1's correctly.

QUESTION 2. [10 pts] Design an ϵ-NFA to accept the following language:

$$L = \{x \mid x \in \{0,1\}^*,\ x \text{ begins or ends with } 101\}$$

Answer:

	ϵ	0	1
$\rightarrow q_0$	$\{q_1, q_2\}$	$\emptyset$	$\emptyset$
q_1	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_5\}$	$\emptyset$
q_5	$\emptyset$	$\emptyset$	$\{q_7\}$
$*q_7$	$\emptyset$	$\{q_7\}$	$\{q_7\}$
q_2	$\emptyset$	$\{q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\emptyset$	$\{q_4\}$
q_4	$\emptyset$	$\{q_6\}$	$\emptyset$
q_6	$\emptyset$	$\emptyset$	$\{q_8\}$
$*q_8$	$\emptyset$	$\emptyset$	$\emptyset$

Note that the answer is not unique.

QUESTION 3. [10 pts] Convert the following NFA to a DFA:

	0	1
$\rightarrow q_0$	$\{q_1\}$	$\{q_0, q_1\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$
$*q_2$	$\{q_1\}$	$\emptyset$

Answer:

	0	1
$\rightarrow \{q_0\}$	$\{q_1\}$	$\{q_0, q_1\}$
$\{q_1\}$	$\{q_1, q_2\}$	$\{q_2\}$
$*\{q_2\}$	$\{q_1\}$	$\emptyset$
$\{q_0, q_1\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$*\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$*\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\emptyset$	$\emptyset$	$\emptyset$

It's okay to includue inaccessible states (*i.e.*, $\{q_0, q_2\}$). Give partial credits for correct steps.

QUESTION 4. [10 pts] Give a regular expression for the following language:

$$L = \{x \mid x \in \{0,1\}^*,\ x \text{ has at most two } 0'\text{s between consecutive } 1'\text{s}\}$$

For example, the language constains strings $000, 01000, 00100101101000$, but not strings like 010001 or 10001011.

Answer:

$$0^* + 0^*1((\epsilon + 0 + 00)1)^*0^*$$

The regex is not unique. Give partial credits for regex's containing some correct components.

QUESTION 5. [10 pts] Prove or disprove the following identities. Note that, you can disprove an identity by means of a counterexample.

1. $(0^*1^*)^* = (0^*1)^* + (01^*)^*$
2. $1(01)^* = (10)^*1$

Answer:

1. False (2 pts). *E.g.*, 10 (2 pts).
2. True (2 pts). Consider any string $w \in L(1(01)^*)$. Clearly, $w = 1(01)^n$ for some integer $n \geq 0$. But, such a string can also be rewritten as $w = (10)^n 1$. Hence, $w \in L((10)^*1)$ and $L(1(01)^*) \subseteq L((10)^*1)$. Similarly, we can prove that $L((10)^*1) \subseteq L(1(01)^*)$. (4 pts)