Solutions to Homework 1

Question 1

1. We know by definition that $x \in A - B$ iff $x \in A$, but $x \notin B$. Also by definition, $\bar{B}$ is the set of elements not in B. Hence, $x \in A - B$ iff $x \in A$ and $x \in \bar{B}$. That is, $x \in A - B$ iff $x \in A \cap \bar{B}$.

2. We are given that $A \cap B = \emptyset$, so that if $x \in A$, then $x \notin B$. This is the same as saying that if $x \in A$, then $x \in \bar{B}$. Therefore $A \subseteq \bar{B}$, so that $A \cap \bar{B} = A$.

3. If $A = B$, we clearly have $A \times A = B \times B$. We prove the other direction by contradiction.

   Assume that $A \times A = B \times B$, but that $A \neq B$. If $A \neq B$, there is some element in one of A or B that is not in the other. Without loss of generality, let $a \in A$, but $a \notin B$. Now, $(a, a) \in A \times A$, but clearly, $(a, a) \notin B \times B$. This contradicts our assumption that $A \times A = B \times B$.

4. (a) $P(\emptyset) = \{\emptyset\}$
   (b) $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$
   (c) $P(P(P(\{\emptyset\}))) = P(\ P(\{\emptyset, \{\emptyset\}\})\ ) = P(\{\emptyset,\ \{\emptyset\},\ \{\{\emptyset\}\},\ \{\emptyset, \{\emptyset\}\}\}) =$
   $\{\emptyset,\ \{\emptyset\},\ \{\{\emptyset\}\},\ \{\{\{\emptyset\}\}\},\ \{\{\emptyset, \{\emptyset\}\}\},$
   $\{\emptyset, \{\emptyset\}\},\ \{\emptyset, \{\{\emptyset\}\}\},\ \{\emptyset, \{\emptyset, \{\emptyset\}\}\},\ \{\{\emptyset\}, \{\{\emptyset\}\}\},\ \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\},\ \{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\},$
   $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\},\ \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\},\ \{\emptyset, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\},\ \{\{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\},$
   $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}\}$

5. $\{\emptyset\} \times P(\emptyset) = \{\emptyset\} \times \{\emptyset\} = \{(\emptyset, \emptyset)\}$.

6. $\emptyset \times P(\emptyset) = \emptyset$.

Question 2

1. $\{1, 3, 5\} \oplus \{1, 2, 3\} = \{2, 5\}$

2. This follows from the definition of $A \oplus B$. If $x \in A$ or $x \in B$, $x \in A \cup B$. However, if $x \in A$ and $x \in B$, $x \in A \cap B$. Hence, $A \oplus B = A \cup B - A \cap B$.

3. Assume $x \in A \oplus B$. By definition of $A \oplus B$, one of the following conditions must hold:

   (a) $x \in A$ but $x \notin B$. In this case, $x \in A - B$, so $x \in (A - B) \cup (B - A)$.
   (b) $x \in B$, but $x \notin A$. In this case, $x \in B - A$, so $x \in (A - B) \cup (B - A)$.

   Conversely, assume $x \in (A - B) \cup (B - A)$. First note that no element of B is in $(A - B)$, and no element of A is in $(B - A)$. Hence, $(A - B)$ and $(B - A)$ are disjoint. Hence, $x \in A - B$, or $x \in B - A$, but not both. By definition of $A \oplus B$, $x \in A \oplus B$.

4. Use the immediately preceding result. $A \oplus B = (A - B) \cup (B - A) = (B - A) \cup (A - B) = B \oplus A$.

5. Let $x \in (A \oplus B) \oplus B$. By definition of $\oplus$, one of the following conditions must hold:

   (a) $x \in A \oplus B$, but $x \notin B$. Since $A \oplus B$ contains all $x \in A$ such that $x \notin B$, we conclude $x \in A$.
   (b) $x \notin A \oplus B$, but $x \in B$. Since $A \oplus B$ contains all $x \in B$ except those in $A \cap B$, we conclude that $x \in A \cap B$, that is, $x \in A$.

   This shows that $(A \oplus B) \oplus B \subseteq A$. We now show that $A \subseteq (A \oplus B) \oplus B$. Let $x \in A$. We have two possibilities

   (a) $x \in A, x \notin B$. In this case, $x \in (A \oplus B)$, and hence also in $(A \oplus B) \oplus B$.
   (b) $x \in A, x \in B$. Clearly, $x \in A \cap B$, so $x \notin A \oplus B$. But this means $x \in (A \oplus B) \oplus B$.

   Since $A \subseteq (A \oplus B) \oplus B$ and $(A \oplus B) \oplus B \subseteq A$, we must have $A = (A \oplus B) \oplus B$.

Q3)

1. Entity types: BANK, ACCOUNT, CUSTOMER, LOAN
2. Weak entity type: BANK-BRANCH. Partial key: BranchNo. Identifying relationship: BRANCHES.
3. The partial key BranchNo in BANK-BRANCH specifies that the same BranchNo value may occur under different BANKs. The identifying relationship BRANCHES specifies that BranchNo values are uniquely assigned for those BANK-BRANCH entities that are related to the same BANK entity. Hence, the combination of BANK Code and BranchNo together constitute a full identifier for a BANK-BRANCH.
4. Relationships and their constraints

a) Branches
   - Relationship between Bank and Bank_Branch entity sets.
   - A bank has one or more bank_branches (i.e atleast one relationship ≥ 1).
   - A bank_branch should be assigned to exactly one bank (i.e exactly one = 1)
   - Constraints: key and participation

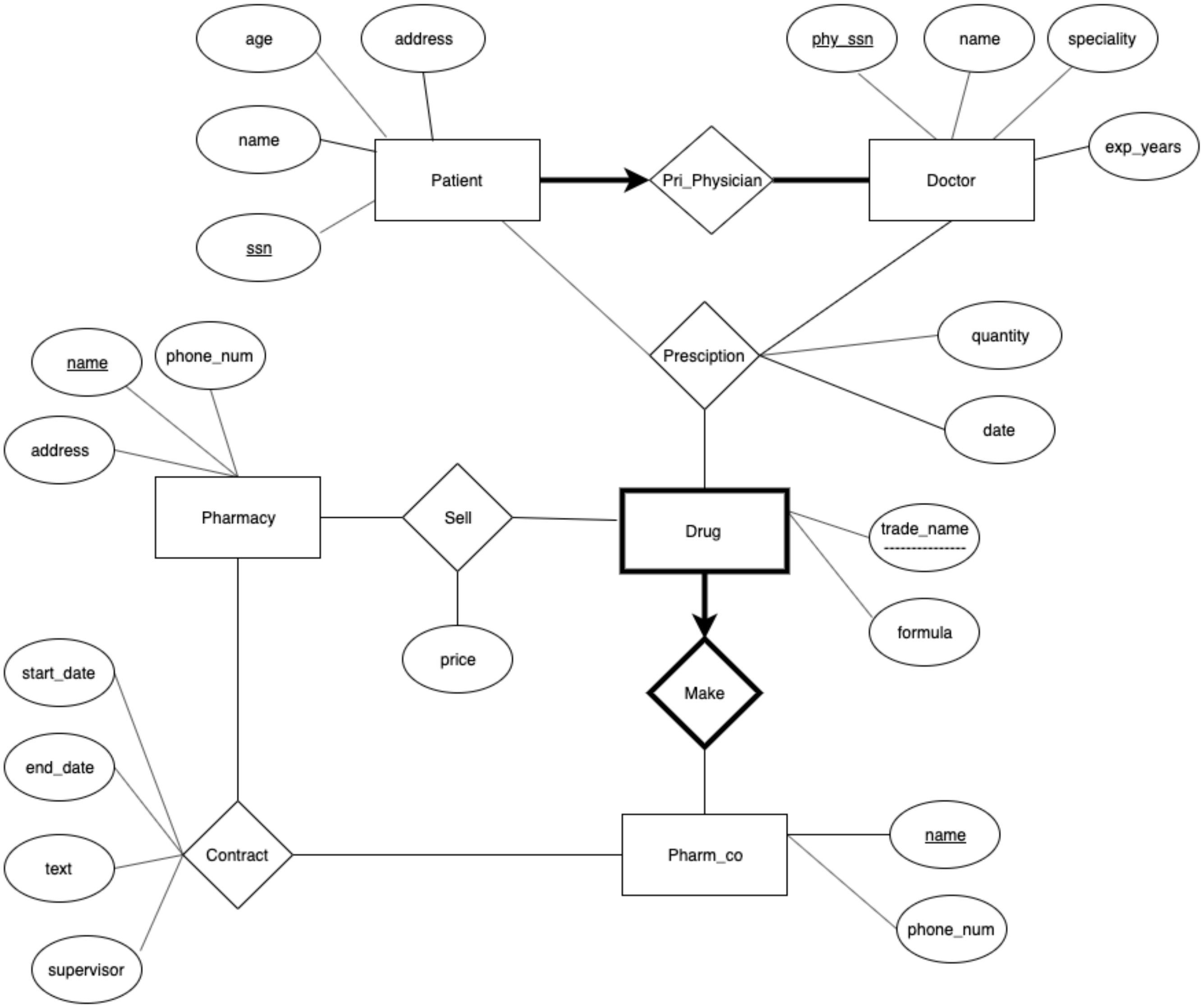
b) Accts
   - Relationship between Bank_Branch and Account entity sets.
   - An account is associated with one or more bank_branches (i.e atleast one relationship ≥ 1)
   - A bank_branch can have 0 or more Accounts (i.e ≥ 0)
   - Constraints: participation

c) Loans
   - Relationship between Bank_Branch and Loan entity sets.
   - A Loan is associated with one or more bank_branches(i.e atleast one relationship ≥ 1)
   - A bank_branch can have 0 or more Loans (i.e ≥ 0)
   - Constraints: participation

d) A_C
   - Relationship between Account and Customer entity sets.
   - An account is associated with one or more customer(i.e atleast one relationship ≥ 1)
   - A customer can have 0 or more Accounts (i.e ≥ 0)
   - Constraints: participation

e) L_C
   - Relationship between Loan and Customer entity sets.
   - A Loan is associated with one or more bank_branches(i.e atleast one relationship ≥ 1)
   - A bank_branch can have 0 or more Loans (i.e ≥ 0)
   - Constraints: participation

Q4)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