Eart119 Homework #5 – Undamped forced oscillator

Write a short summary of your observations for each of the two problems below. Include all figures (with titles and labels) and the python scripts and functions you created.

Consider the initial value problem:

$$my'' + ky = F\cos(\omega t)$$

with the IC $y(0) = 0$, $y'(0) = 0$, for m=1, we can rewrite the equations as:

$$y'' + \omega_0^2 y = F\cos(\omega t)$$

where $\omega_0 = (k/m)^{1/2}$ – is the natural frequency of the oscillating system

I. Beats (25 points)

Let F = 0.5, $\omega_0=0.8$, $\omega=1$

a) For $\omega \neq \omega_0$ the exact solution has the general form (obtained using undetermined coefficients):

$$y = c_1\cos(\omega_0 t) + c_2\sin(\omega_0 t) + [F/(\omega_0^2 - \omega^2)]\cos(\omega t)$$

b) Solve for c_1 and c_2 using the initial conditions: $y(0) = 0$, $y'(0) = 0$

c) Write the general solution as product of sins (use: $\cos(\alpha) - \cos(\beta) = -2\sin((\alpha+\beta)/2)\sin((\alpha-\beta)/2)$)

d) Plot the analytical solutions for the given frequencies and amplitude of the forcing function. Observe that the solution exhibits two frequencies: the slow frequency, $(\omega-\omega_0)/2$ and fast frequency with $(\omega + \omega_0)/2$. Plot at least three periods of the slow frequency, together with the fast frequency.

e) Add the forcing function to the plot using a different color and transparency (e.g. alpha ~0.5)

f) Compare the analytical solutions to numerical solutions:
   a. Use forward Euler
   b. *BONUS*: Implement 4th order Runge-Kutta
   c. *BONUS*: Below which time steps, h, does the forward Euler approximation provide accurate solutions?

II. *Natural Frequency: $\omega_0 = \omega$* (15 points)

g) What happens if the frequency of the forcing function is in phase with the natural frequency of the oscillating system ($\omega \sim \omega_0$)?

h) Compare to the exact solution for $\boldsymbol{\omega = \omega_0}$:

$$y = c_1\cos(\omega_0 t) + c_2\sin(\omega_0) + \frac{F}{2\omega_0} t\sin(\omega_0 t)$$

i) Create a subplot (plt.subplot(212)) which shows the difference between numerical and exact solution.

j) In deriving the ODE, we used Hooke's law which assumes linearity between force and change in displacement. Is this still realistic when $\omega \sim \omega_0$? What do you think would happen in reality?