Eart119 Homework 6

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May 2019

Part 1

In this problem the heat diffusion equation, equation 1, is solved numerically by using central second derivative for x and forward difference for t and Runge-Kutta integration method.

$$\frac{\delta u}{\delta t} = \alpha \frac{\delta^2 u}{\delta t^2} \tag{1}$$

The stability condition for the nummerical solution can be seen in equation 2.

$$\frac{\alpha \Delta t}{(\Delta x)^2} \tag{2}$$

I chose the initial condition so be $\bar{U}$ and therefore chose to phaseshift the Boundary conditions by 90° to match these. This made sense since we are interested in how it usually is not how it is if it has been day or night for longer than usual. I tracked the depth at which the deviations where smaller than 1 degree in either direction starting at the top. The result of different timesteps can be seen in figures 1, 2 and 3. The numerical stability was passed between timestep $0.002(24 \cdot 3600) = 172.8s$ and $0.005(24 \cdot 3600)$. This means that at timestep $0.001(24 \cdot 3600) = 86.4s$ we are well below the stability limit. From these simulations I expect to be safe at a depth of 2.85m.

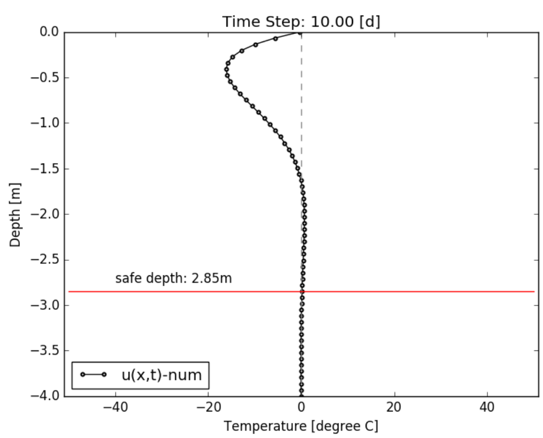

Figure 1: The graph shows the solution using timestepsize 86.4 seconds after 10 days. The stability factor is 0.19.

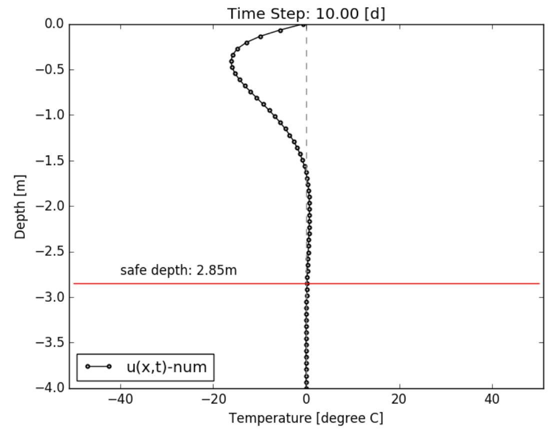

Figure 2: The graph shows the solution using timestepsize 172.8 seconds after 10 days. The stability factor is 0.38.

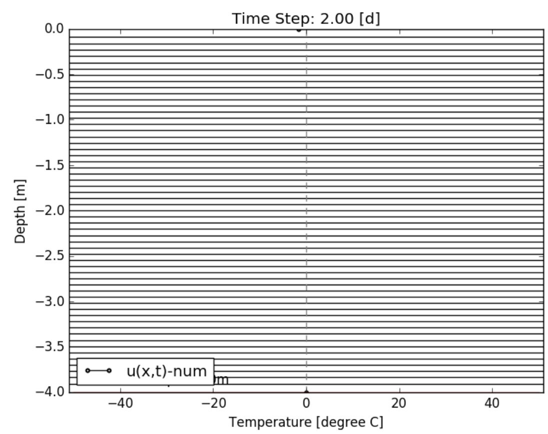

Figure 3: The graph shows the solution using timestepsize 172.8 seconds after 2 days. This solution is numerically unstable. The stability factor is 0.94.

Surprisingly the stability condition is never breached but as it approaches 1 the nummerical solution becomes unstable.

Part 2

The analytical solution can be seen in equation 3, note that I switched to *sin* to match my boundary conditions.

$$U_{ana} = \bar{U} + \Delta\tilde{U}e^{-x\sqrt{\frac{\omega}{2\alpha}}}sin(-x\sqrt{\frac{\omega}{2\alpha}} + \omega t) \tag{3}$$

The numerical solution gets closer to the analytical solution as time progresses. This is due to my initial conditions where I assumed that the rock had an even temperature of $\bar{U}$ at $t = 0$. This becomes less and less significant the more time progresses so at the end of the 10 day simulation the numerical and the analytical solutions are very similar. This can be observed in figure 4 below, where the right image is the result at t=1 day and the image to the right is the result after t=10 days.

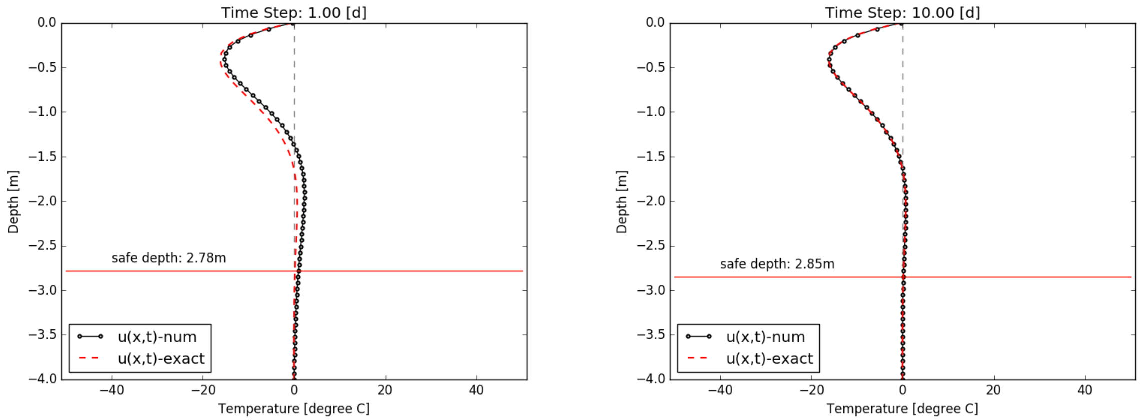

Figure 4: The graphs are the numerical solution using timestepsize 86.4 seconds and the analytical solution at the red dashed line.

Part 3

If we change the amplitude of the BC to 80 the solutions plotted in figure 5 are obtained. The solutions are plotted after 10 days using timestepsize 86.4 seconds. So I would be safe at a depth of 3.25m.

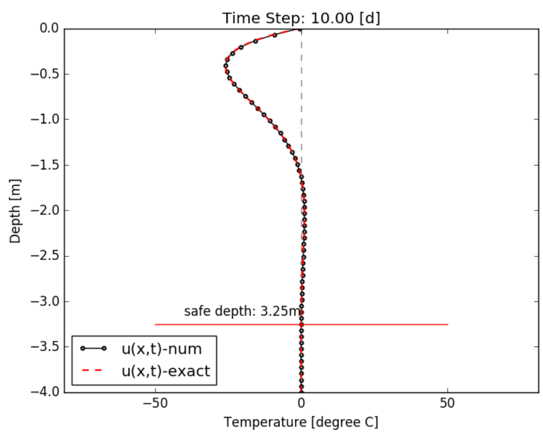

Figure 5: The figure shows the analytical and the analytical solution when the amplitude of the BC was changed to 80°. The plot is at t=10 days and the timestepsize used os 86.4 seconds.

Part 4

I can save each timestep by connecting them in a matrix. The result of that can be seen in figure 6. The safe depth is where there is no color change.

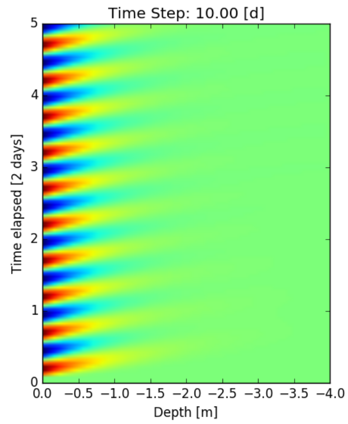

Figure 6: The temperature variation of the surface. The x-axis is depth and the y-axis is time. To get n even image the axis has units 2 days.