$my'' + ky = F\cos(\omega t)$ IC: $y(0)=0$, $y'(0)=0$, $m=1$ $\rightarrow$ $y'' + \omega_0^2 y = F\cos(\omega t)$ where $\omega_0 = \left(\frac{k}{m}\right)^{1/2}$

natural frequency of oscillating system

a) $y = c_1\cos(\omega_0 t) + c_2\sin(\omega_0 t) + [F/(\omega_0^2-\omega^2)]\cos(\omega t)$

b) solve for c_1 and c_2 using IC:

when $y(0)=0$ and $y'(0)=0$

$$0 = c_1\cos(0) + c_2\sin(0) + \frac{F}{\omega_0^2-\omega^2}\cdot\cos(0)$$

$$0 = c_1 + \frac{F}{\omega_0^2-\omega^2}$$

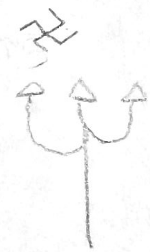
$$\boxed{c_1 = \frac{-F}{\omega_0^2-\omega^2}}$$ $\rightarrow$ plug c_1 back in to find c_2

$$y' = -\omega_0 c_1\sin(\omega_0 t) + c_2\omega_0\cos(\omega_0 t) + \frac{F}{\omega_0^2-\omega^2}\cdot\omega\cdot\sin(\omega t)$$

at IC:

$$0 = c_2\omega_0\cos(\omega_0 t)$$

$$\boxed{c_2 = 0}$$



c) write the general product solution as a product of sins

$$y = \frac{-F}{\omega_0^2-\omega^2}\cdot\cos(\omega_0 t) + \frac{F}{\omega_0^2-\omega^2}\cdot\cos(\omega t)$$

$$y = \frac{F}{\omega_0^2-\omega^2}\left[\cos(\omega t) - \cos(\omega_0 t)\right]$$ $\rightarrow$ using $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\times\sin\left(\frac{\alpha-\beta}{2}\right)$

$$\boxed{y = \frac{F}{\omega_0^2-\omega^2}\left[-2\sin\left(\frac{\omega t+\omega_0 t}{2}\right)\sin\left(\frac{\omega t-\omega_0 t}{2}\right)\right]}$$

n) $0 = C_1 + 0 + 0$

$y = C_2 \sin(\omega_0 t) + \left(\frac{F}{2\omega_0} t\right) \sin(\omega_0 t)$

$\dot{y} = C_2\omega_0 \cos(\omega_0 t) + \frac{F}{2\omega_0} \sin(\omega_0 t) + \frac{F}{2\omega_0} t \cdot \cos(\omega_0 t)$

$\dot{y}(0) = 0$

$0 = C_2\omega_0 + F L$

$\therefore \boxed{y = \frac{F}{2\omega_0} t \sin(\omega_0 t)}$

$\approx 0.3125 \cdot t \cdot \sin(\omega_0 \cdot t)$