Signalverarbeitung und maschinelles Lernen

Blatt 2

(1) 1.1

z.z.: $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$$|z_1 z_2| = |(x_1 + iy_1) \cdot (x_2 + iy_2)| = |x_1x_2 - y_1y_2 + (x_1y_2 + x_2y_1) \cdot i| =$$
$$= \sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + x_2y_1)^2} = \sqrt{x_1^2x_2^2 + y_1^2y_2^2 - 2x_1x_2y_1y_2 + x_1^2y_2^2 + x_2^2y_1^2 + 2x_1x_2y_1y_2} = \sqrt{x_1^2 \cdot (x_2^2 + y_2^2) + y_1^2 \cdot (x_2^2 + y_2^2)} =$$
$$= \sqrt{(x_1^2 + y_1^2) \cdot (x_2^2 + y_2^2)} = \sqrt{(x_1^2 + y_1^2)} \cdot \sqrt{(x_2^2 + y_2^2)} = |z_1| \cdot |z_2|$$

$\square$

1.2

z.z.: $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$$\left|\frac{z_1}{z_2}\right| = \frac{|r_1 \cdot e^{i\varphi_1}|}{|r_2 \cdot e^{i\varphi_2}|} = \left|\frac{r_1}{r_2} \cdot e^{i\varphi_1} \cdot e^{-i\varphi_2}\right| = \left|\frac{r_1}{r_2} \cdot e^{i(\varphi_1 - \varphi_2)}\right| =$$
$$= \frac{r_1}{r_2} \ast = \frac{|r_1 \cdot e^{i\varphi_1}|}{|r_2 \cdot e^{i\varphi_2}|}$$

1.3

→ Polardarstellung, für Identitäten mit Beträgen

(2) 2.1 $\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$, $\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$

z.z.: $r \cdot e^{i\varphi} = r(\cos(\varphi) + i \cdot \sin(\varphi))$

$$r(\cos(\varphi) + i \cdot \sin(\varphi)) = \frac{r}{2} \cdot \left(\frac{e^{i\varphi} + e^{-i\varphi}}{1} + e^{i\varphi} - e^{-i\varphi}\right) =$$
$$= \frac{r}{2} \cdot (2 \cdot e^{i\varphi}) = r \cdot e^{i\varphi}$$

$\square$

2.2

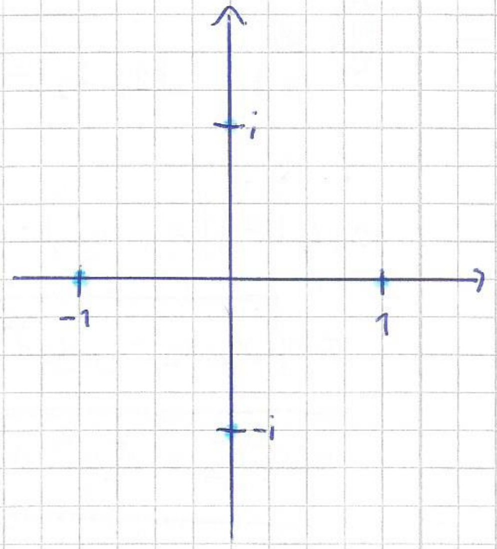
$$z_1 \cdot z_2 = r_1 \cdot e^{i\varphi_1} \cdot r_2 \cdot e^{i\varphi_2} = (r_1 \cdot r_2) \cdot e^{i(\varphi_1+\varphi_2)}$$

2.3

a) $i^n,\ n \in \mathbb{Z}$

n	...	-2	-1	0	1	2	3	4
i^n	...	-1	$-i$	1	i	-1	$-i$	1

$\Rightarrow$



Polar: $i^n = e^{i \cdot \frac{\pi}{2} \cdot n} \Rightarrow r = 1,\ \varphi = n\frac{\pi}{2}$

Kart: $\cos\left(\frac{\pi}{2}n\right) + i \cdot \sin\left(\frac{\pi}{2} \cdot n\right)$

b) $$\frac{2}{1-i\sqrt{3}} = \frac{2(1+i\sqrt{3})}{(1-i\sqrt{3})(1+i\sqrt{3})} = \frac{2 \cdot (1+i\sqrt{3})}{1+3} =$$

$$= \frac{1}{2} + i \cdot \frac{\sqrt{3}}{2} \Rightarrow \text{Kart.}$$

Polar: $r = |z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$

$\cos(\varphi) = \frac{x}{r} \Rightarrow \varphi = \arccos\left(\frac{x}{r}\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\Rightarrow z = 1 \cdot e^{i \cdot \frac{\pi}{3}}$

c) $$z^n = \underbrace{\left(e^{i\frac{\pi}{3}}\right)^n = e^{i \cdot \frac{\pi}{3} \cdot n}}_{\text{Polar}} = \underbrace{\cos\left(\frac{\pi}{3}n\right) + i \cdot \sin\left(\frac{\pi}{3} \cdot n\right)}_{\text{Kart.}}$$

d) $$\cancel{\frac{x+iy}{x-iy} = \frac{(x+iy)^2}{x^2+y^2}} \quad \frac{x+iy}{y-ix} = \frac{(x+iy)(y+ix)}{y^2+x^2} =$$

$$= \frac{xy - yx + i(x^2+y^2)}{x^2+y^2} = i \Rightarrow e^{i \cdot \frac{\pi}{2}}$$