SVML

Blatt 10

10.2

1. Trainingsdaten $D = \{(x_1, t_1), (x_2, t_2), \ldots, (x_n, t_n)\}$

$$f(x, \omega) = \omega_0 x^0 + \omega_1 x^1 + \omega_2 x^2 (+ \ldots + \omega_n x^n)$$

$\hookrightarrow n = 2$

2. $$E(\omega) = \frac{1}{2} \sum_{n=0}^{3} \left[f(x, \omega) - t_n \right]^2 = \frac{1}{2} \sum_{n=0}^{3} \left[\sum_{m=0}^{2} \omega_m x^m - t_n \right]^2$$

3. $\Rightarrow$ $$\frac{\delta E(\omega)}{\delta \omega_i} = \frac{\delta}{\delta \omega_i} \frac{1}{2} \sum_{n=0}^{3} \left[\sum_{m=0}^{2} \omega_m x^m - t_n \right]^2 =$$

$$= \frac{1}{2} \cdot \sum_{n=0}^{3} \left[\frac{\delta}{\delta \omega_i} \left(\sum_{m=0}^{2} \omega_m x^m - t_n \right)^2 \right] =$$

$$= \frac{1}{2} \cdot \sum_{n=0}^{3} \left[2 \cdot \left(\sum_{m=0}^{2} \omega_m x^m - t_n \right) \cdot \frac{\delta}{\delta \omega_i} \left(\sum_{m=0}^{2} \omega_m x^m - t_n \right) \right] =$$

$$= \sum_{n=0}^{3} \left(\sum_{m=0}^{2} \omega_m x^m - t_n \right) \cdot \left(\sum_{m=0}^{2} \frac{\delta}{\delta \omega_i} \omega_m x^m - \underbrace{\frac{\delta}{\delta \omega_i} t_n}_{0} \right) \Bigg] =$$

$$= \sum_{n=0}^{3} \left[\left(\sum_{m=0}^{2} \omega_m x^m - t_n \right) \cdot x_n^i \right]$$

4. $$\frac{\delta E(\omega)}{\delta \omega_0} = x_0^0 \left(\omega_0 + x_0 \omega_1 + \omega_2 x_0^2 - t_0 \right) + x_1^0 \left(\omega_0 + \omega_1 x_1 + \omega_2 x_1^2 - t_1 \right) +$$
$$+ x_2^0 \left(\omega_0 + \omega_1 x_2 + \omega_2 x_2^2 - t_2 \right) + x_3^0 \left(\omega_0 + \omega_1 x_3 + \omega_2 x_3^2 - t_3 \right) =$$

$$= \omega_0 \left(x_0^0 + x_1^0 + x_2^0 + x_3^0 \right) + \omega_1 \left(x_0^1 + x_1^1 + x_2^1 + x_3^1 \right) +$$
$$+ \omega_2 \left(x_0^2 + x_1^2 + x_2^2 + x_3^2 \right) - \left(t_0 x_0^0 + t_1 x_1^0 + t_2 x_2^0 + t_3 x_3^0 \right) =$$
$$= \omega_0 (1 + 1 + 1 + 1) + \omega_1 (1 + 2 + 4 + 5) + \omega_2 (1 + 4 + 16 + 25) -$$
$$- (1 + 3 + 3 + 1) = 4\omega_0 + 12\omega_1 + 46\omega_2 - 8$$

$$\frac{\delta E(\omega)}{\delta \omega_1} = \omega_0(x_1+x_2+x_3+x_0) + \omega_1(x_0^2+x_1^2+x_2^2+x_3^2) +$$
$$+ \omega_2(x_1^3+x_2^3+x_3^3+x_0^3) - (t_0 x_0 + t_1 x_1 + t_2 x_2 + t_3 x_3) =$$

$$= 12\omega_0 + 46\omega_1 + 198\omega_2 - 24$$

$$\frac{\delta E(\omega)}{\delta \omega_2} = \omega_0(x_1^2+x_2^2+x_3^2+x_0^2) + \omega_1(x_0^3+x_1^3+x_2^3+x_3^3) +$$
$$+ \omega_2(x_0^4+x_1^4+x_2^4+x_3^4) - (t_0x_0^2+t_1x_1^2+t_2x_2^2+t_3x_3^2) =$$
$$= 46\omega_0 + 198\omega_1 + 898\omega_2 - 86$$

5. $\frac{\delta E(\omega)}{\delta \omega_i} = 0$ f.a. ω_i

$$\Rightarrow \frac{\delta E(\omega)}{\delta \omega_0} = 0 \Leftrightarrow 4\omega_0 + 12\omega_1 + 46\omega_2 = 8$$

$$\Rightarrow \frac{\delta E(\omega)}{\delta \omega_1} = 0 \Leftrightarrow 12\omega_0 + 46\omega_1 + 198\omega_2 = 24$$

$$\Rightarrow \frac{\delta E(\omega)}{\delta \omega_2} = 0 \Leftrightarrow 46\omega_0 + 198\omega_1 + 898\omega_2 = 86$$

Gaußsches Eliminationsverfahren:

$$\left(\begin{array}{ccc|c} 4 & 12 & 46 & 8 \\ 12 & 46 & 198 & 24 \\ 46 & 198 & 898 & 46 \end{array}\right) \quad \text{II} - 3\cdot\text{I}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 4 & 12 & 46 & 8 \\ 0 & 10 & 60 & 0 \\ 46 & 198 & 898 & 46 \end{array}\right) \quad \text{III} - 11\,\text{I}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 4 & 12 & 46 & 8 \\ 0 & 10 & 60 & 0 \\ 2 & 66 & 392 & -2 \end{array}\right) \quad \text{III} - \tfrac{1}{2}\text{I}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 4 & 12 & 46 & 8 \\ 0 & 10 & 60 & 0 \\ 0 & 60 & 369 & -6 \end{array}\right) \quad \text{III} - 6\cdot\text{II}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 4 & 12 & 46 & 8 \\ 0 & 10 & 60 & 0 \\ 0 & 0 & 9 & -6 \end{array}\right)$$

$$\Rightarrow 9\omega_2 = -6 \Rightarrow \omega_2 = -\frac{2}{3}$$
$$\Rightarrow 10\omega_1 - \frac{60\cdot 2}{3} = 0$$
$$\Rightarrow \omega_1 = 4$$

$$\Rightarrow 4\omega_0 + 4\cdot 12 - \frac{2}{3}\cdot 46 = 8$$
$$\Rightarrow \omega_0 = -\frac{7}{3}$$

$\Rightarrow y = -\frac{7}{3} + 4x - \frac{2}{3}x^2$

$$f(x_0) = -\frac{7}{3} + 4 - \frac{2}{3} = 1 = f_0$$
$$f(x_1) = -\frac{7}{3} + 4 \cdot 2 - \frac{2}{3} \cdot 2^2 = 3 = f_1$$
$$f(x_2) = -\frac{7}{3} + 4 \cdot 4 - \frac{2}{3} \cdot 4^2 = 3 = f_2$$
$$f(x_3) = -\frac{7}{3} + 4 \cdot 5 - \frac{2}{3} \cdot 5^2 = 1 = f_3$$

10.1

$$\begin{pmatrix} 0{,}2 & 0{,}1 & 0{,}2 & 0{,}9 & 0{,}1 \\ 0{,}1 & i & 0{,}7 & j & 0{,}6 \\ 0{,}1 & 0{,}1 & 0{,}2 & 0{,}5 & 0{,}8 \end{pmatrix}$$

$g_{max} = 240$

$g_i = 120, \quad g_j = 180$

$c_s = 0{,}5, \quad c_d = -0{,}5$

Initialisierung:

- $p^{(0)}(i) = \frac{g_i}{g_{max}} = \frac{120}{240} = 0{,}5$
- $p^{(0)}(j) = \frac{g_j}{g_{max}} = \frac{180}{240} = 0{,}75$

Betrag eines Nachbarpixels i_k zum Wert $p(i)$:

$$\begin{aligned} q(i) &= c_s \cdot p(i_k) + c_d \cdot (1 - p(i_k)) = \\ &= 0{,}5 \cdot p(i_k) + 0{,}5 \cdot (1 - p(i_k)) = \\ &= 0{,}5 \cdot p(i_k) + 0{,}5\, p(i_k) - 0{,}5 = \\ &= p(i_k) - 0{,}5 \end{aligned}$$

Änderung von $p(i)$ aufgrund der Beträge der Nachbarpixel:

$$\Delta p(i) = \frac{1}{8} \cdot \sum_{k=1}^{8} q(i_k) = \frac{1}{8} \cdot \sum_{k=1}^{8} (p(i_k) - 0{,}5)$$

Normalisierung der Wsl. $p^{(r)}(i)$ nach Iteration r:

$$p^{(r)}(i) = \frac{p^{(r-1)}(i) \cdot (1+\Delta p(i))}{p^{(r-1)}(i) \cdot (1+\Delta p(i)) + (1-p^{(r-1)}(i)) \cdot (1-\Delta p(i))}$$

Iteration 1:

$$\Delta p(i) = \frac{1}{8} \cdot (-0{,}3 - 0{,}4 - 0{,}3 - 0{,}4 - 0{,}2 - 0{,}4 - 0{,}4 - 0{,}3) =$$
$$= \frac{1}{8} \cdot (-2{,}3) = -0{,}2875$$

$$\Delta p(j) = \frac{1}{8} \cdot (-0{,}3 + 0{,}4 - 0{,}4 + 0{,}2 + 0{,}1 - 0{,}3 + 0 + 0{,}3) =$$
$$= 0$$

$$\Rightarrow p^{(1)}(i) = \frac{p^{(0)}(i) \cdot (1+\Delta p(i))}{p^{(0)}(i) \cdot (1+\Delta p(i)) + (1-p^{(0)}(i)) \cdot (1-\Delta p(i))} = \frac{0{,}5 \cdot 0{,}7125}{0{,}5 \cdot 0{,}7125 + 0{,}5 \cdot 1{,}2875} =$$

$$= 0{,}35625$$

$$\Rightarrow p^{(1)}(j) = \frac{0{,}75 \cdot 1}{0{,}75 \cdot 1 + 0{,}25 \cdot 1} = 0{,}75$$

Iteration 2:

$\Delta p(i)$ und $\Delta p(j)$ wie in Iteration 1

$$\Rightarrow p^{(2)}(i) = \frac{0{,}35625 \cdot 0{,}7125}{0{,}35625 \cdot 0{,}7125 + 0{,}64375 \cdot 1{,}2875} = 0{,}2344 \Rightarrow \text{Vordergrund}$$

$$\Rightarrow p^{(2)}(j) = 0{,}75 \Rightarrow \text{Hintergrund}$$