SVML

Blatt 12

12.1

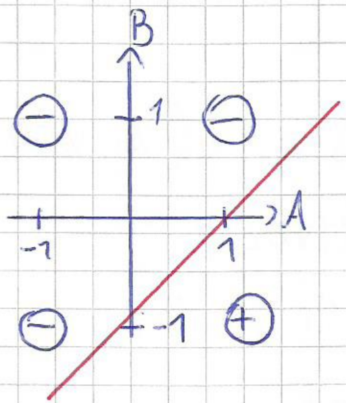
$\Rightarrow A \land \neg B$ (Perzeptron)

A	B	$\neg B$	$A \land \neg B$
1	1	-1	-1
1	-1	1	1
-1	1	-1	-1
-1	-1	1	-1

Lösungswege:

a) Probieren

b) Geometrie



$y = mx + b$

$B = mA + b$

$0 = A - B + b$

c) LGS

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} w_0 \\ w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$$

$$\vec{w} = (A^T A)^{-1} A^T \cdot b \quad \text{(oder Gauss-Alg)}$$

$$o_{A \land \neg B}(A, B) = \begin{cases} +1, & b + w_1 A + w_2 B > 0 \\ -1, & \text{sonst} \end{cases}$$

Bei „$\land$": $b = -1$

„$\lor$": $b = 1$

$\Rightarrow -1 + A - B$

$$o_{A \wedge \neg B}(A,B) = \begin{cases} +1, & \text{if } -0{,}1 + 0{,}5A - 0{,}5B > 0 \\ -1, & \text{sonst} \end{cases}$$

$x_1(A)$	$x_2(B)$	$\sum w_i x_i$	$o(x_1, x_2)$
1	1	-0,1	-1
1	-1	0,9	1
-1	1	-1,1	-1
-1	-1	-0,1	-1

2)

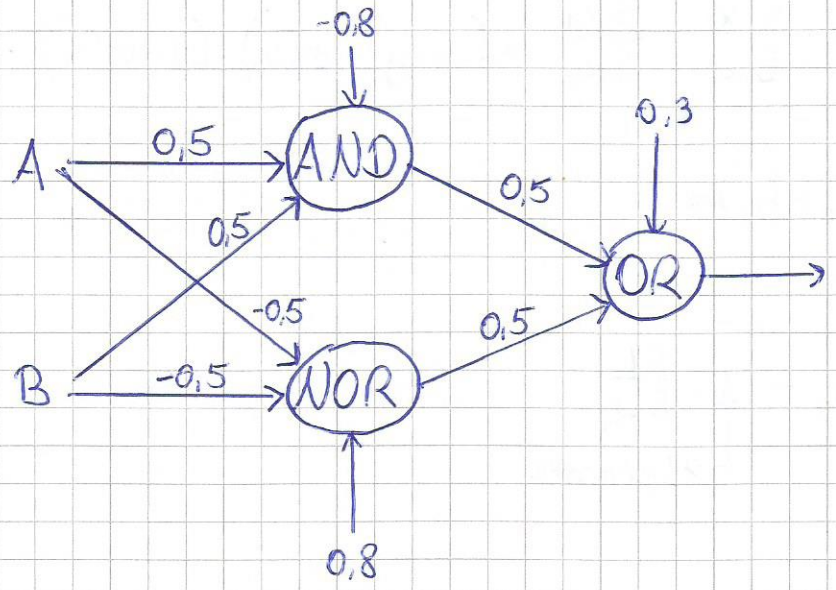
$$A \text{ XNOR } B = \neg(A \text{ XOR } B) = (A \wedge B) \vee (\neg A \wedge \neg B)$$

A	B	A XNOR B
1	1	1
1	-1	-1
-1	1	-1
-1	-1	1

$$o_{A \wedge B}(A,B) = \begin{cases} 1, & -0{,}8 + 0{,}5A + 0{,}5B > 0 \text{ (oder } -1 + A + B) \\ -1, & \text{sonst} \end{cases}$$

$$o_{\neg A \wedge \neg B}(A,B) = \begin{cases} +1, & -0{,}8 - 0{,}5A - 0{,}5B > 0 \\ -1, & \text{sonst} \end{cases}$$

$$o_{A \vee B}(A,B) = \begin{cases} 1, & 0{,}3 + 0{,}5A + 0{,}5B \\ -1 \end{cases}$$



12.2

1.) z.z.: $\frac{\delta}{\delta x}\sigma(x) = \sigma(x)\cdot(1-\sigma(x))$

$$\frac{\delta}{\delta x}\sigma(x) = \frac{\delta}{\delta x}\frac{1}{1+e^{-x}} = -\frac{1}{(1+e^{-x})^2}\cdot\frac{\delta}{\delta x}(1+e^{-x}) =$$

$$= -\frac{1}{(1+e^{-x})^2}\cdot(-e^{-x}) = \frac{e^{-x}}{(1+e^{-x})^2} = \underbrace{\frac{1}{1+e^{-x}}}_{\sigma(x)}\cdot\underbrace{\frac{e^{-x}}{1+e^{-x}}}_{1-\sigma(x)} =$$

$$\overset{*}{=} \sigma(x)\cdot(1-\sigma(x))$$

$$* \rightarrow 1-\frac{1}{1-e^{-x}} = \frac{1+e^{-x}-1}{1+e^{-x}} = \frac{e^{-x}}{1+e^{-x}}$$

2) $L(x_2, y) = \frac{1}{2}(x_2-y)^2$

a) $\frac{\delta}{\delta x_2}L = \frac{\delta}{\delta x_2}\frac{1}{2}(x_2-y)^2 = (x_2-y)\cdot\frac{\delta}{\delta x_2}(x_2-y) = x_2-y$

b) $\frac{\delta}{\delta x_1}L = \frac{\delta L}{\delta x_2}\cdot\frac{\delta x_2}{\delta x_1} = (x_2-y)\cdot\frac{\delta}{\delta x_1}(\sigma(x_1)-y) =$

$= (x_2-y)\cdot\sigma(x_1)\cdot(1-\sigma(x_1))$

c) $\frac{\delta L}{\delta \omega_1} = \frac{\delta L}{\delta x_2} \cdot \frac{\delta x_2}{\delta x_1} \cdot \frac{\delta x_1}{\delta \omega_1} = (x_2 - y) \cdot \sigma(x_1) \cdot (1 - \sigma(x_1)) \cdot$

$\cdot \frac{\delta}{\delta \omega_1}(\omega_1 x_0 + b_1) = (x_2 - y) \cdot (\sigma(x_1) \cdot (1 - \sigma(x_1)) \cdot x_0$

3.)

$\omega_1 \longleftarrow \omega_1 - \lambda \cdot \frac{\delta L}{\delta \omega_1}$

λ → Lernrate