Math 105B Midterm Report

ID: 84698098
Name: Bao Hu

Purpose/Objective:

In this report, I will use the knowledge I learned from the lectures about numerical approximation to draw functions that approximate the shape of my left hand with all fingers.

Introduction:

I will implement the Clamped Cubic Spline Algorithm for finding the functions. Since the Clamped Cubic Spline Algorithm requires function values and the first derivative at endpoints, I will also implement the 3-point forward and backward differentiation for finding the slope at the endpoints for each function.

Procedure(Algorithm method):

Data Preparation:

To approximate the shape of my left hand, I collected the coordinates of some key points from the outline of each finger. Since the first two fingers from the left leaned to left a little bit and, similarly, the first two fingers from the right leaned to right a little bit, when drawing it with a tablet with my all my fingers naturally spread out, each individual finger might have the case that a single x value corresponds to 2 y values. So it's necessary to break down each finger to 2 curves. Therefore, 5*2 = 10 curves in total.

Since the Clamped Cubic Spline Algorithm requires slopes at the endpoints of each curve, I implemented the 3-point forward and backward differentiation algorithm for estimating the first derivatives.

3-Point Forward and Backward Differentiation:

To approximate the slope of the starting point of a curve I used the forward differentiation

$$f'(x_0) \approx \frac{-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)}{2h}$$

where x0 is the starting point.The distance between each point is h.

Similarly, the backward differentiation is as given

$$f'(x_0) \approx \frac{f(x_0 - 2h) - 4f(x_0 - h) + 3f(x_0)}{2h}$$

where x0 is the last point. The distance between each point is also h.

Clamped Cubic Spline Algorithm:

Clamped Cubic Spline is a numerical algorithm that interpolate a set of data points. It contains multiple cubic polynomial functions and these polynomials are continuous at the connected data points and also have the same first and second derivatives.

I chose this algorithm because approximating the graph of the hand may use many data points to capture the shape of all fingers. If I were to use other interpolation methods such as Lagrange polynomial interpolation or Hermite interpolation, it would give me a high-degree polynomial that might introduce the Oscillation Issue. This refers to the graph will deviate from the desired curve when the data points are unevenly distributed.

A good way to avoid this problem is to divide the entire interval to subintervals and construct low-degree polynomials for each subinterval. Clamped Cubic Spline Algorithm is a good choice in this case. It also provides precise control over the slope at the endpoints of the curve. In each subinterval, this method generates multiple cubic polynomials and connects them smoothly, resulting in a good shape of my hand.

Plotting the Results:

I plotted the resulting curves using different colors, 10 total, and adjusted the scale of the plot as the original hand drawing for better comparison.

Verification and Data Improvement:

I found some segments of curves not fitting the original hand drawing very well, so I add some more data points to improve it as best as I could.

Discussion and Results:

Resulting Functions:

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>> main
S1:
S1{1}(x) = (616*(x - 1)^2)/71 - (35*x)/2 + (2715*(x - 1)^3)/142 + 81/2
S1{2}(x) = (2861*(x - 6/5)^2)/142 - (4166*x)/355 - (30595*(x - 6/5)^3)/639 + 60496/1775
S1{3}(x) = (2285*(x - 3/2)^3)/71 - (1629*(x - 3/2)^2)/71 - (3571*x)/284 + 20369/568
S1{4}(x) = (3597*(x - 2)^2)/142 - (808*x)/71 - (1501*(x - 2)^3)/71 + 2255/71
S1{5}(x) = (311*(x - 5/2)^3)/71 - (453*(x - 5/2)^2)/71 - (541*x)/284 + 6681/568
S2:
S2{1}(x) = (25*x)/4 - (4750763536833979*(x - 1)^2)/562949953421312 + (1091588839595451*(x - 1)^3)/281474976710656 + 67/4
S2{2}(x) = (810113862884795*x)/1125899906842624 - (5903988072190505*(x - 3/2)^2)/2251799813685248 + (3564252546125437*(x - 3/2)^3)/1125899906842624 + 52738753846634191/2251799813685248
S2{3}(x) = (8500947782137927*x)/18014398509481984 + (2394384783092903*(x - 2)^2)/1125899906842624 - (103472712806997*(x - 2)^3)/35184372088832 + 1065380769422452381/45035996273704960
S2{4}(x) = (884697824217191*x)/2251799813685248 - (5144610863285905*(x - 5/2)^2)/2251799813685248 + (6295101455613325*(x - 5/2)^3)/4503599627370496 + 108166501563176445/4503599627370496
S2{5}(x) = (3039249963162083*(x - 3)^3)/2251799813685248 - (1693139086303645*(x - 3)^2)/9007199254740992 - (7596999972854869*x)/9007199254740992 + 1230847707180706043/45035996273704960
S2{6}(x) = (2067795086583607*(x - 7/2)^2)/1125899906842624 - (21548646209033*x)/1125899906842624 - (5177217533324885*(x - 7/2)^3)/562949953421312 + 55319935958751807/2251799813685248
S2{7}(x) = (1947334662152167*(x - 4)^3)/140737488355328 - (1682982189173881*(x - 4)^2)/140737488355328 - (2859789929806377*x)/562949953421312 + 31046710763315917/703687441776640
S2{8}(x) = (2476039608108739*(x - 9/2)^2)/281474976710656 - (468714337505675*x)/70368744177664 - (978124721748587*(x - 9/2)^3)/140737488355328 + 7033178804657635/140737488355328
S2{9}(x) = 18793147406952031/562949953421312 - (1833338228548087*(x - 5)^2)/1125899906842624 - (3933158878370781*(x - 5)^3)/9007199254740992 - (1732009649073683*x)/562949953421312
S2{10}(x) = (1121990684447741*(x - 11/2)^3)/140737488355328 - (5141611036485217*(x - 11/2)^2)/2251799813685248 - (5666091171542713*x)/1125899906842624 + 98355799905933811/2251799813685248
S3:
S3{1}(x) = (2727421488720965*(x - 51/10)^3)/1099511627776 - (7618428800170481*(x - 51/10)^2)/17592186044416 - (23*x)/2 + 1753/20
S3{2}(x) = (5473194345690219*(x - 26/5)^2)/17592186044416 - (6669337359340983*x)/281474976710656 - (313835302793553*(x - 26/5)^3)/274877906944 + 104997259157625419/703687441776640
S3{3}(x) = (1203864045049775*x)/281474976710656 - (2209773871783913*(x - 53/10)^2)/70368744177664 - (8036732218816923*(x - 53/10)^3)/8796093022208 + 8252799650289861/2814749767106560
S3{4}(x) = (1207488544182537*(x - 27/5)^3)/549755813888 - (5374482799236157*(x - 27/5)^2)/17592186044416 - (8279217982441693*x)/281474976710656 + 51688356527609411/281474976710656
S3{5}(x) = (6217407224916157*(x - 11/2)^2)/17592186044416 - (3465269450676849*x)/140737488355328 - (6913574284477239*(x - 11/2)^3)/4398046511104 + 44028938468369115/281474976710656
S3{6}(x) = (5204012692228717*(x - 28/5)^3)/4398046511104 - (2078881916456501*(x - 28/5)^2)/17592186044416 - (2471187262546183*x)/2251799813685248 + 75000681063507761/2814749767106560
S3{7}(x) = (2292488745015639*(x - 567/100)^2)/17592186044416 - (69658759832037*x)/281474976710656 - (3663585767900973*(x - 567/100)^3)/274877906944 + 609483344663843379/28147497671065600
S3{8}(x) = (8832989149052471*(x - 57/10)^3)/1099511627776 - (4702441319523535*(x - 57/10)^2)/4398046511104 - (7997951495709787*x)/281474976710656 + 512178230597589059/2814749767106560
S4:
S4{1}(x) = (11*x)/2 - (183148596203333*(x - 51/10)^2)/17592186044416 + (4096576069617013*(x - 51/10)^3)/562949953421312 + 19/20
S4{2}(x) = (5983635110848969*x)/9007199254740992 - (7558910359729891*(x - 11/2)^2)/4503599627370496 + (6328486955763349*(x - 11/2)^3)/1125899906842624 + 474611969065120861/18014398509481984
S4{3}(x) = (5291237251810961*x)/4503599627370496 + (7611821340509075*(x - 29/5)^2)/2251799813685248 - (1234817458561591*(x - 29/5)^3)/140737488355328 + 526597663430427027/22517998136852480
S4{4}(x) = (1659748820835427*x)/1125899906842624 - (2121213130841105*(x - 6)^2)/1125899906842624 + (2971757146865335*(x - 6)^3)/4503599627370496 + 12190727116843735/562949953421312
S4{5}(x) = (6886702675705381*(x - 7)^2)/72057594037927936 - (1415438322791125*x)/4503599627370496 - (4827232769145733*(x - 7)^3)/36028797018963968 + 148393756801180627/4503599627370496
S4{6}(x) = 2971838843600786277/90071992547409920 - (1898748907932955*(x - 15/2)^2)/18014398509481984 - (3729403461686037*(x - 15/2)^3)/1125899906842624 - (2875144955333903*x)/9007199254740992
S4{7}(x) = (3255812789388025*(x - 8)^3)/562949953421312 - (2856388499637433*(x - 8)^2)/562949953421312 - (818779380606769*x)/281474976710656 + 1874310543271729/35184372088832
S4{8}(x) = (8109322737778421*(x - 17/2)^2)/2251799813685248 - (8208350675239807*x)/2251799813685248 - (6414197849996735*(x - 17/2)^3)/140737488355328 + 265642751045450607/4503599627370496
S4{9}(x) = (5518015546763643*(x - 44/5)^3)/35184372088832 - (1316486348471481*(x - 44/5)^2)/35184372088832 - (1940755734034923*x)/140737488355328 + 25922281445932313/175921860444160
S4{10}(x) = (7977291918346771*(x - 9)^2)/140737488355328 - (5593945716571023*x)/562949953421312 - (5013583868999653*(x - 9)^3)/17592186044416 + 63293360377829383/562949953421312
S4{11}(x) = (1735881300696377*(x - 46/5)^3)/2199023255552 - (8043955326425741*(x - 46/5)^2)/70368744177664 - (755152544010923*x)/35184372088832 + 19215688046914909/87960930222080
S4{12}(x) = (2155126290064929*(x - 93/10)^2)/17592186044416 - (5810600418553791*x)/281474976710656 - (7791183103772687*(x - 93/10)^3)/35184372088832 + 592458709616973923/2814749767106560
S4{13}(x) = (2010688232959913*x)/1125899906842624 - (728914564267477*(x - 19/2)^2)/70368744177664 - (5654392658208147*(x - 19/2)^3)/70368744177664 + 1203420313253493/2251799813685248
S5:
S5{1}(x) = (5523946417946319*(x - 97/10)^2)/35184372088832 - 11*x + (6421147906212081*(x - 97/10)^3)/8796093022208 + 247/2
S5{2}(x) = (423*x)/10 + 376*(x - 49/5)^2 - 1990*(x - 49/5)^3 - 19827/50
S5{3}(x) = (289*x)/5 - 221*(x - 99/10)^2 - 570*(x - 99/10)^3 - 27411/50
S6:
S6{1}(x) = (149*x)/4 - (3026219839428985*(x - 10)^2)/35184372088832 + (1150892807094239*(x - 10)^3)/17592186044416 - 691/2
S6{2}(x) = (343591408683695*x)/1125899906842624 + (3411668654829855*(x - 21/2)^2)/281474976710656 - (1819871761790601*(x - 21/2)^3)/35184372088832 + 327588571998379577/11258999068426240
S6{3}(x) = (2651698882457209*x)/2251799813685248 - (7648188587740575*(x - 53/5)^2)/2251799813685248 + (6065790662877133*(x - 53/5)^3)/2251799813685248 + 224251529046778099/11258999068426240
S6{4}(x) = (1158129611844703*(x - 11)^3)/2251799813685248 - (2953918338304071*(x - 11)^2)/18014398509481984 - (2221089878216909*x)/9007199254740992 + 317165964439468239/9007199254740992
S6{5}(x) = (2735909250958091*(x - 23/2)^2)/4503599627370496 - (55915052958709*x)/2251799813685248 - (8627377780142957*(x - 23/2)^3)/4503599627370496 + 736013370724271887/22517998136852480
S6{6}(x) = (1669723464217363*(x - 12)^3)/281474976710656 - (5102578709628173*(x - 12)^2)/2251799813685248 - (7692908380133091*x)/9007199254740992 + 479059295612163917/11258999068426240
S6{7}(x) = (6919430232736869*(x - 123/10)^2)/2251799813685248 - (2756343276201325*x)/4503599627370496 - (7054731944162407*(x - 123/10)^3)/281474976710656 + 356036420746264339/90071992547409920
S6{8}(x) = (6275311547178947*(x - 25/2)^3)/140737488355328 - (3367910387405321*(x - 25/2)^2)/281474976710656 - (5382942211401787*x)/2251799813685248 + 1388940117177132239/22517998136852480
S6{9}(x) = (4162463469209389*(x - 127/10)^2)/281474976710656 - (4111657280515281*x)/2251799813685248 - (5168560520947281*(x - 127/10)^3)/70368744177664 + 1224742016495238063/22517998136852480
S6{10}(x) = (4346777179917209*(x - 13)^3)/70368744177664 - (3611088601550217*(x - 13)^2)/70368744177664 - (7197048882323727*x)/562949953421312 + 110450134072847811/562949953421312
S6{11}(x) = (2909077168325597*(x - 27/2)^2)/70368744177664 - (156329603362847*x)/8796093022208 - (4445963825186739*(x - 27/2)^3)/140737488355328 + 4546354732618565/17592186044416

S7:
S7{1}(x) = (708130513574281*(x - 14)^2)/1125899906842624 - (70368744177655*x)/562949953421312 + (1684406788466277*(x - 14)^3)/1125899906842624 + 4996180836614081/281474976710656
S7{2}(x) = (6328743904192083*x)/1125899906842624 + (720168859871639*(x - 15)^2)/140737488355328 - (4908039805245789*(x - 15)^3)/562949953421312 - 74664960239714013/1125899906842624
S7{3}(x) = (2364017537648255*x)/562949953421312 - (8962768536764257*(x - 31/2)^2)/1125899906842624 + (2128474410228057*(x - 31/2)^3)/140737488355328 - 49640645623400801/1125899906842624
S7{4}(x) = (2134028249975149*x)/281474976710656 + (4144731096493107*(x - 16)^2)/281474976710656 - (2783288062230285*(x - 16)^3)/140737488355328 - 1729407970953581/17592186044416
S7{5}(x) = (525956813280707*x)/70368744177664 - (8410266180395497*(x - 33/2)^2)/562949953421312 + (91321778927969*(x - 33/2)^3)/8796093022208 - 13415925164314147/140737488355328
S7{6}(x) = (2893339430282747*x)/9007199254740992 + (5705993547032445*(x - 17)^2)/9007199254740992 - (5472266415269785*(x - 17)^3)/36028797018963968 + 1073620839245521833/45035996273704960
S7{7}(x) = (1893320756113027*x)/2251799813685248 + (7307787282612551*(x - 35/2)^2)/18014398509481984 - (8777881765249909*(x - 35/2)^3)/9007199254740992 + 335201612531053683/22517998136852480
S7{8}(x) = (2321882670910475*x)/4503599627370496 - (2378232251642147*(x - 18)^2)/2251799813685248 + (8934725621858599*(x - 18)^3)/72057594037927936 + 46657050372363165/2251799813685248
S7{9}(x) = (2474656027199833*(x - 94/5)^3)/1125899906842624 - (3416255660005503*(x - 94/5)^2)/4503599627370496 - (527036682465171*x)/562949953421312 + 66710495605750781/1407374883553280
S7{10}(x) = (5045837610548149*(x - 19)^2)/9007199254740992 - (8789921661335307*x)/9007199254740992 - (8953194910759261*(x - 19)^3)/562949953421312 + 2168108047528520981/45035996273704960
S7{11}(x) = (5483517847486647*(x - 96/5)^3)/35184372088832 - (5056552095796279*(x - 96/5)^2)/562949953421312 - (5990430211443429*x)/2251799813685248 + 28280311656358063/351843720888320
S7{12}(x) = (1970729516376601*x)/9007199254740992 + (664510424129375*(x - 193/10)^2)/17592186044416 - (3638681473368919*(x - 193/10)^3)/17592186044416 + 2240744186468944679/90071992547409920
S7{13}(x) = (1292783921112489*(x - 39/2)^3)/4398046511104 - (6074793839567875*(x - 39/2)^2)/70368744177664 - (2671816417053521*x)/281474976710656 + 120526388914305367/562949953421312
S8:
S8{1}(x) = (35*x)/4 + (1845660527084537*(x - 91/5)^2)/35184372088832 - (7029279379870867*(x - 91/5)^3)/17592186044416 - 573/4
S8{2}(x) = (4083817574430859*x)/562949953421312 - (2371907100838043*(x - 183/10)^2)/35184372088832 + (3946849303151027*(x - 183/10)^3)/8796093022208 - 651637124039224157/5629499534213120
S8{3}(x) = (4071665513798931*x)/562949953421312 + (147769503933943*(x - 92/5)^2)/2199023255552 - (6953329000921319*(x - 92/5)^3)/17592186044416 - 81333776586284213/703687441776640
S8{4}(x) = (620283534291533*x)/70368744177664 - (3615370675219525*(x - 37/2)^2)/70368744177664 + (6788745965898821*(x - 37/2)^3)/70368744177664 - 20346847234213153/140737488355328
S8{5}(x) = (1367809044604791*x)/1125899906842624 + (2952377598409181*(x - 75/4)^2)/140737488355328 - (5537929228105313*(x - 75/4)^3)/140737488355328 - 17017285425319901/4503599627370496
S8{6}(x) = (4870425596083545*x)/1125899906842624 - (4804277290679215*(x - 19)^2)/562949953421312 - (6698245635264843*(x - 19)^3)/2251799813685248 - 70020088188734875/1125899906842624
S8{7}(x) = (156277256356075*x)/281474976710656 - (2904507067984469*(x - 96/5)^2)/281474976710656 + (547597239091717*(x - 96/5)^3)/137438953472 + 86553553141619/8796093022208
S8{8}(x) = (2069230035501659*x)/70368744177664 + (5166167649406043*(x - 77/4)^2)/8796093022208 - (4127960400492337*(x - 77/4)^3)/549755813888 - 153419738222703967/281474976710656
S8{9}(x) = (8957288682215181*x)/281474976710656 - (4740937311775707*(x - 193/10)^2)/8796093022208 + (5294266147569039*(x - 193/10)^3)/2199023255552 - 1664017471024079053/2814749767106560
S8{10}(x) = (6448728261228017*(x - 97/5)^2)/35184372088832 - (1054728106484455*x)/281474976710656 - (1887034876745349*(x - 97/5)^3)/2199023255552 + 134959723627428231/1407374883553280
S8{11}(x) = (8068092739112687*x)/1125899906842624 - (5218078294299575*(x - 39/2)^2)/70368744177664 + (6376665530988099*(x - 39/2)^3)/8796093022208 - 1305313906298429453/11258999068426240
S8{12}(x) = (7928318918174403*x)/562949953421312 + (5042959490036041*(x - 98/5)^2)/35184372088832 - (5900628750712781*(x - 98/5)^3)/4398046511104 - 354006942343490387/1407374883553280
S9:
S9{1}(x) = (266953852556451*(x - 91/5)^3)/17179869184 - (6935446521605721*(x - 91/5)^2)/4398046511104 + 16
S9{2}(x) = (3315581416562143*(x - 73/4)^2)/4398046511104 - (361986510504363*x)/8796093022208 - (7274674579800467*(x - 73/4)^3)/1099511627776 + 26917596476062147/35184372088832
S9{3}(x) = (3217558733381375*(x - 183/10)^3)/1099511627776 - (1049223331318199*(x - 183/10)^2)/4398046511104 - (2165611231679447*x)/140737488355328 + 414602728883531441/1407374883553280
S9{4}(x) = (7050495269685229*(x - 367/20)^2)/35184372088832 - (4868539015703047*x)/281474976710656 - (1702525065311265*(x - 367/20)^3)/2199023255552 + 1854307813173575689/5629499534213120
S9{5}(x) = (5923810724482945*(x - 369/20)^3)/70368744177664 - (280406260952167*(x - 369/20)^2)/8796093022208 - (250885670003799*x)/562949953421312 + 222055301518303591/11258999068426240
S9{6}(x) = (3379717907200215*(x - 93/5)^2)/562949953421312 - (2435828089064601*x)/562949953421312 - (2021462233554797*(x - 93/5)^3)/562949953421312 + 257494259721180053/2814749767106560
S9{7}(x) = (3815852907737869*(x - 19)^2)/2251799813685248 - (5618845083285875*x)/4503599627370496 - (3825702792986791*(x - 19)^3)/72057594037927936 + 151794052856136585/4503599627370496
S9{8}(x) = (410120869594803*x)/562949953421312 + (7201314251264723*(x - 98/5)^2)/4503599627370496 + (386960379626993*(x - 98/5)^3)/9007199254740992 - 6233280007145539/1407374883553280
S9{9}(x) = (2283722212220165*x)/1125899906842624 + (3716745239520459*(x - 20)^2)/2251799813685248 - (5887429816372061*(x - 20)^3)/2251799813685248 - 42456356516550013/1407374883553280
S9{10}(x) = (3868617301681743*x)/2251799813685248 - (5114399485037633*(x - 41/2)^2)/2251799813685248 + (7654328787792645*(x - 41/2)^3)/562949953421312 - 106821913654190759/4503599627370496
S9{11}(x) = (3526082075003705*x)/1125899906842624 + (8924293439170151*(x - 83/4)^2)/1125899906842624 - (2507111614851493*(x - 83/4)^3)/140737488355328 - 238621616696861563/4503599627370496
S9{12}(x) = (8455122744623081*x)/2251799813685248 - (764797031242351*(x - 21)^2)/140737488355328 + (2537549958381569*(x - 21)^3)/562949953421312 - 148284180059176477/2251799813685248
S9{13}(x) = (7216583787200871*x)/4503599627370496 + (5864069101531335*(x - 429/20)^2)/9007199254740992 - (4236957562092903*(x - 429/20)^3)/9007199254740992 - 1834906549045434779/90071992547409920
S9{14}(x) = (4259651149621725*x)/2251799813685248 - (4507643503687857*(x - 22)^2)/36028797018963968 - (3751275875089153*(x - 22)^3)/9007199254740992 - 29967664043199615/1125899906842624
S9{15}(x) = (2908733400571059*x)/2251799813685248 - (7879207451082455*(x - 113/5)^2)/9007199254740992 + (1637339477780411*(x - 113/5)^3)/4503599627370496 - 148542889169709827/11258999068426240
S9{16}(x) = (1725853385021869*x)/2251799813685248 - (7899185408818965*(x - 23)^2)/18014398509481984 + (1225249358583307*(x - 23)^3)/4503599627370496 - 138255545555324599/11258999068426240
S9{17}(x) = (6355724282768353*x)/9007199254740992 + (3401903447090359*(x - 24)^2)/9007199254740992 - (5304712906937849*(x - 24)^3)/4503599627370496 + 73125568019549/1125899906842624
S9{18}(x) = (56267449045373*x)/281474976710656 - (6256117636861595*(x - 49/2)^2)/4503599627370496 + (8911118534819317*(x - 49/2)^3)/4503599627370496 + 34909645954827103/2814749767106560
S9{19}(x) = (2655000897957723*x)/9007199254740992 + (3555280082683691*(x - 25)^2)/2251799813685248 - (2724179292321011*(x - 25)^3)/562949953421312 + 447247623290380433/45035996273704960
S9{20}(x) = (6907235702045873*(x - 128/5)^3)/140737488355328 - (8029405411013817*(x - 128/5)^2)/1125899906842624 - (854796027389619*x)/281474976710656 + 1041713004111539/10995116277760

S10:
S10{1}(x) = x + (7807027590469151*(x - 20)^2)/18014398509481984 - (3903513795234563*(x - 20)^3)/9007199254740992 - 17
S10{2}(x) = (637960682438305*x)/1125899906842624 - (7807027590469115*(x - 21)^2)/9007199254740992 + (2927635346425917*(x - 21)^3)/2251799813685248 - 8893574703833909/1125899906842624
S10{3}(x) = (1538828402229951*x)/562949953421312 + (853893642707559*(x - 22)^2)/281474976710656 - (660540225809883*(x - 22)^3)/4503599627370496 - 15519737540976181/281474976710656
S10{4}(x) = (1237884699711805*x)/281474976710656 + (204803820213135*(x - 557/25)^2)/70368744177664 - (5730592941247693*(x - 557/25)^3)/562949953421312 - 129456106246575397/1407374883553280
S10{5}(x) = (4729193503009371*x)/1125899906842624 - (8575043118073511*(x - 45/2)^2)/2251799813685248 + (3445140601967355*(x - 45/2)^3)/1125899906842624 - 197051108939624959/2251799813685248
S10{6}(x) = (6173737060237003*x)/2251799813685248 - (4747580069064513*(x - 114/5)^2)/4503599627370496 - (3473975955870717*(x - 114/5)^3)/4503599627370496 - 306867016159804211/5629499534213120
S10{7}(x) = (5015782489071863*x)/2251799813685248 - (853995705323367*(x - 23)^2)/562949953421312 + (6059776308196083*(x - 23)^3)/4503599627370496 - 96222698832328241/2251799813685248
S10{8}(x) = (5367595230870559*x)/2251799813685248 + (1927874374270919*(x - 119/5)^2)/1125899906842624 - (125038593366617*(x - 119/5)^3)/35184372088832 - 526153841789334121/11258999068426240
S10{9}(x) = (1152752160370393*x)/562949953421312 - (2873607611007157*(x - 121/5)^2)/1125899906842624 + (1295850383412977*(x - 121/5)^3)/140737488355328 - 108520763966645393/2814749767106560
S10{10}(x) = (1245670926060233*x)/281474976710656 + (4783278034878743*(x - 123/5)^2)/562949953421312 - (989449518733821*(x - 123/5)^3)/35184372088832 - 136329025302769299/1407374883553280
S10{11}(x) = (2504909990103033*x)/562949953421312 - (2357718672482953*(x - 124/5)^2)/281474976710656 + (6154461379442031*(x - 124/5)^3)/562949953421312 - 68504272950097703/703687441776640
S10{12}(x) = (2931736811143675*x)/1125899906842624 + (6986988413792747*(x - 503/20)^2)/2251799813685248 - (4732452271236471*(x - 503/20)^3)/562949953421312 - 231882328417866761/4503599627370496
S10{13}(x) = (2366755992547457*x)/2251799813685248 - (7864391244071217*(x - 511/20)^2)/1125899906842624 + (4702343583401321*(x - 511/20)^3)/35184372088832 - 533872368086176127/45035996273704960
S10{14}(x) = (4023655214726551*x)/281474976710656 + (5151287847327115*(x - 103/4)^2)/70368744177664 - (6997384428391891*(x - 103/4)^3)/2199023255552 - 396422088607352769/1125899906842624
```

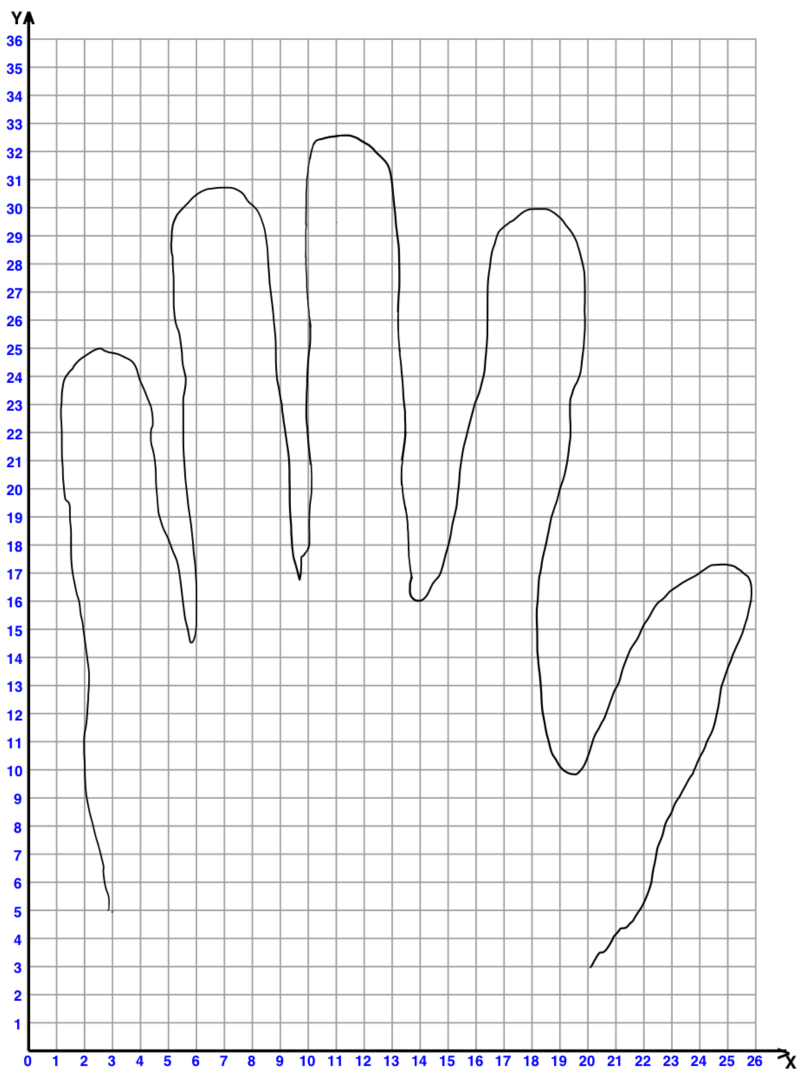
As we can see, there are many polynomial functions. Each of them is a cubic polynomial. The whole graph is represented by the notation Si(j), where i is the number of the subintervals, i = 1, 2, …, 10 in this case, and j varies by different numbers of data points within each subinterval.

j = ni - 1, where ni is the number of data points in i-th subinterval. The algorithm ensures the smoothness at each connected point.

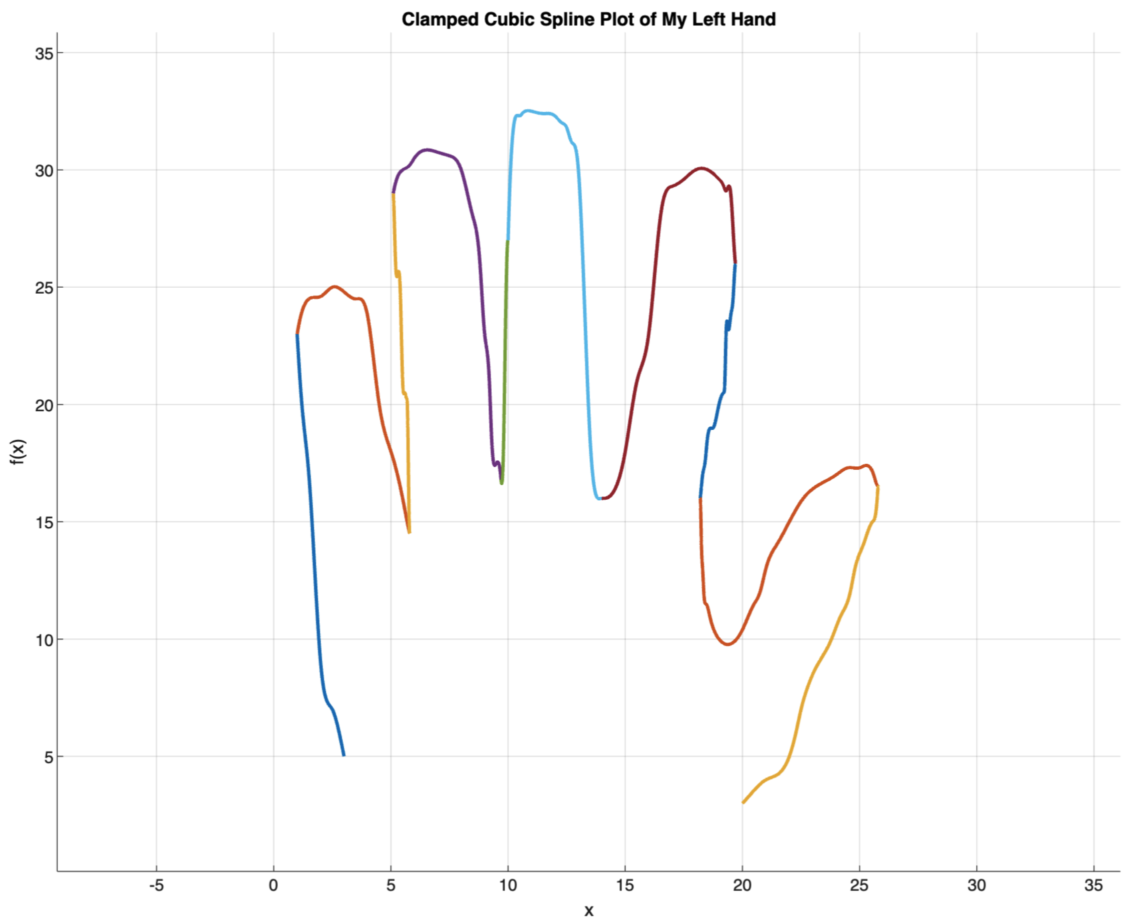
Plot the Functions and Compare:

The resulting polynomial functions are difficult to read, so let's instead look at the resulting plotting graph from these functions and compare it with the original hand drawing.

Let's first look at the original hand drawing:



And here is the Clamped Cubic Spline Plot of my left hand:



As we can see, there are 10 different curves with different colors, and the functions plot is pretty similar to the original hand drawing which means the algorithm implementation and data collection was successful.

There are some minor differences between these two. One of the most noticeable differences is the middle finger. The original hand drawing's middle finger has a more complex shape since it bends slightly and changes direction many times. In contrast, the spline plot's middle finger has a more straight and smooth shape.

This difference is due to the challenges of collecting data points for the middle finger. Especially, for some x values within the middle finger interval, there are multiple corresponding y values, and to properly capture these details, I have to divide the middle finger to more segmentations.

Due to the complexity of precisely dividing the data and collecting the data points, I chose to divide the middle finger to only 2 segmentations, which is the same as I did for all the other fingers, but those are less complicated. And finally I yielded this reasonable approximation of my left hand's shape.

Conclusion:

In this report, I successfully implemented the Clamped Cubic Spline Algorithm and 3-point Forward and Backward Differentiation to approximate the shape of my left hand. I carefully collected the data points and was able to generate a smooth spline plot, and it's similar to my left hand's shape.