What the SIGMA: "Strategic (Caffeine) Intake for Gaining Maximum Alertness".

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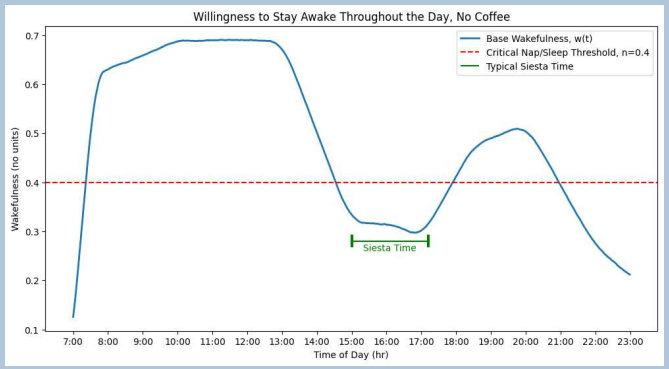
The Problem

The consumption of coffee is a ubiquitous ritual for enhancing alertness and cognitive function. However, the timing of coffee intake can significantly influence its efficacy due to the body's varying energy levels throughout the day. This entails a need to investigate the optimal timing of coffee consumption, to maximize wakefulness and avoid unnecessary napping temptation.

Introducing SIGMA

Employing a quantitative approach, we model the pharmacokinetics of caffeine absorption and its subsequent impact on alertness levels, using a novel combination of linear ingress and exponential decay functions to simulate caffeine's effects from ingestion to metabolization. The SIGMA project proposes actionable guidelines that can be customized for individuals seeking to optimize their mental performance through mathematically-informed coffee consumption.



An Optimization Perspective

Let $w(t)$ be a continuous function representing wakefulness over time from t_{wake} to t_{sleep}. Define $n = 0.4$ as the critical napping threshold, and let $c_{t_i}(t)$ denote the wakefulness enhancement due to coffee consumed at time t_i, defined elsewhere. Then, The optimal coffee consumption time t_O is then given by the following continuous optimization problem:

$$t_O = \underset{t_i \in [t_{\text{wake}}, t_{\text{sleep}}]}{\arg\max} \left(\min_{t \in [t_{\text{wake}}, t_{\text{sleep}}]} \{ w(t) + c_{t_i}(t) - n \} \right)$$

where $c_{t_i}(t)$ has no effect before t_i, impacting wakefulness only from t_i onward.

The intuition behind this formalization is to find the time of consumption that leads to the highest minimum wakefulness throughout the day. In other words, of all possible coffee ingestion times, t_O is the time that will help an individual feel the least tired when at their tiredest.

Wakefulness from Caffeine

To define $c_{t_i}(t)$, we incorporate both a linear ingress phase and an exponential decay phase, corresponding to the dynamics of coffee consumption, absorption, and its diminishing effects over time.
Define the coffee effect function $c_{t_i}(t)$ as follows, where t_i is the time of coffee consumption, A is the maximum effect of coffee, $\Delta t_{\text{ingress}}$ is the duration of the linear ingress phase, and τ is the half-life of the exponential decay:

$$c_{t_i}(t) = \begin{cases} 0 & \text{for } t < t_i \\ \frac{A}{\Delta t_{\text{ingress}}}(t - t_i) & \text{for } t_i \le t < t_i + \Delta t_{\text{ingress}} \\ A \cdot e^{-\lambda(t - t_i - \Delta t_{\text{ingress}})} & \text{for } t \ge t_i + \Delta t_{\text{ingress}} \end{cases}$$

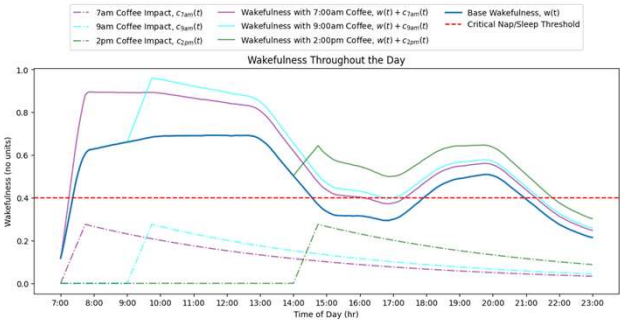
where $\lambda = \frac{\ln(2)}{\tau}$ is the decay constant derived from the half-life τ. Here, τ is taken to be 5 hours, as per the existing literature.*

Numerical Results

Given that an individual's experience of wakefulness is unique, a generic model was created to approximate a single example of energy levels throughout the day (see Figure 1 in the bottom left). Using this model, various caffeine ingestion times were tested and used to calculate t_O.
For our experiments, we set the following hyperparameters:

$$\tau = 5,\ t_{wake} = 07{:}30,\ t_{sleep} = 21{:}00,$$
$$\Delta t_{ingress} = 0.5\ hrs, n = 0.4, A = 0.275.$$

Some wakefulness trajectories are plotted below for visualization. As can be seen, the coffee ingestion at 2:00pm keeps the wakefulness levels above the critical napping threshold, making it a strong candidate ingestion time. Code for reproducing results is available at https://github.com/hubarruby/optimalcaffeine



What Was Learned

With the hyperparameters set as defined above, the **optimal coffee ingestion time was numerically estimated to be t_O = 1:54pm.** The worst time to consume coffee was found to be 4:48pm.

* https://go.drugbank.com/drugs/DB00201#pharmacology The half life of coffee is reported to be somewhere between 3 and 7 hours. As such, a mean value of 5 was used.