Vector

n-tuple

$$V = [x_1, x_2, \dots, x_n] \quad 1 \times n$$

$$\begin{matrix} \| \\ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = [x_1, x_2, \dots, x_n]^T \\ n \times 1 \end{matrix}$$

<u>Field</u>

- Reals $\mathbb{R}$
- Complex $\mathbb{C}$

commutaivity

$w+z=z+w \quad z,w \in \mathbb{F}$

- Integer $\mathbb{Z}$

vector is an n-typle from a Field $F = \mathbb{R}$

$v \in \mathbb{R}^n$

$\|v\| = L$

$0 \in R^n$

$111001 \to (3,1)\,(3,0)$
$(1,1)$

Scalar Mult.

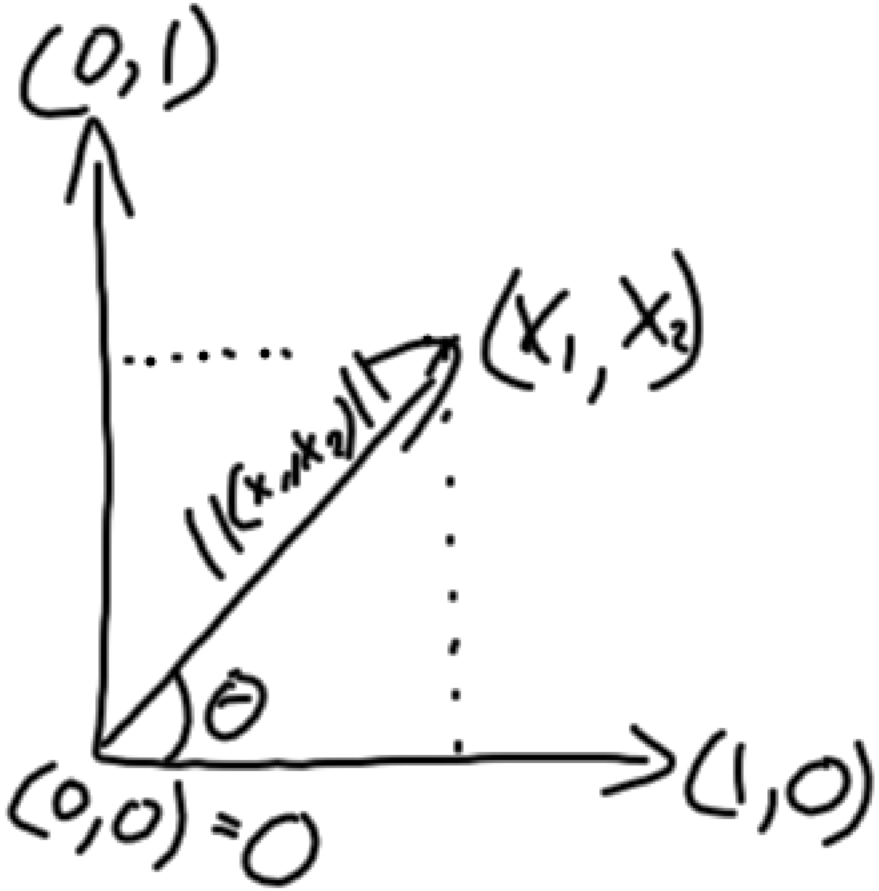
$a \in \mathbb{R}, v \in \mathbb{R}^n$

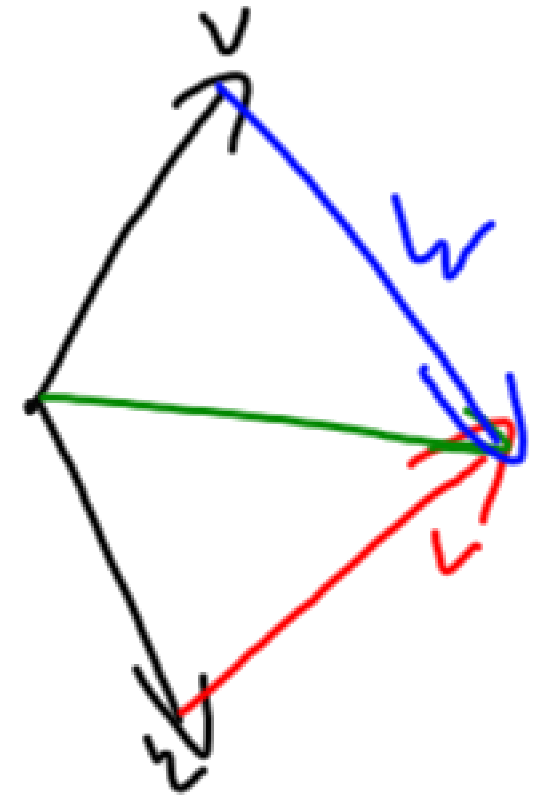
$$av = a[x_1, x_2, \dots, x_n]$$
$$= [ax_1, ax_2, \dots, ax_n] \in \mathbb{R}^n$$

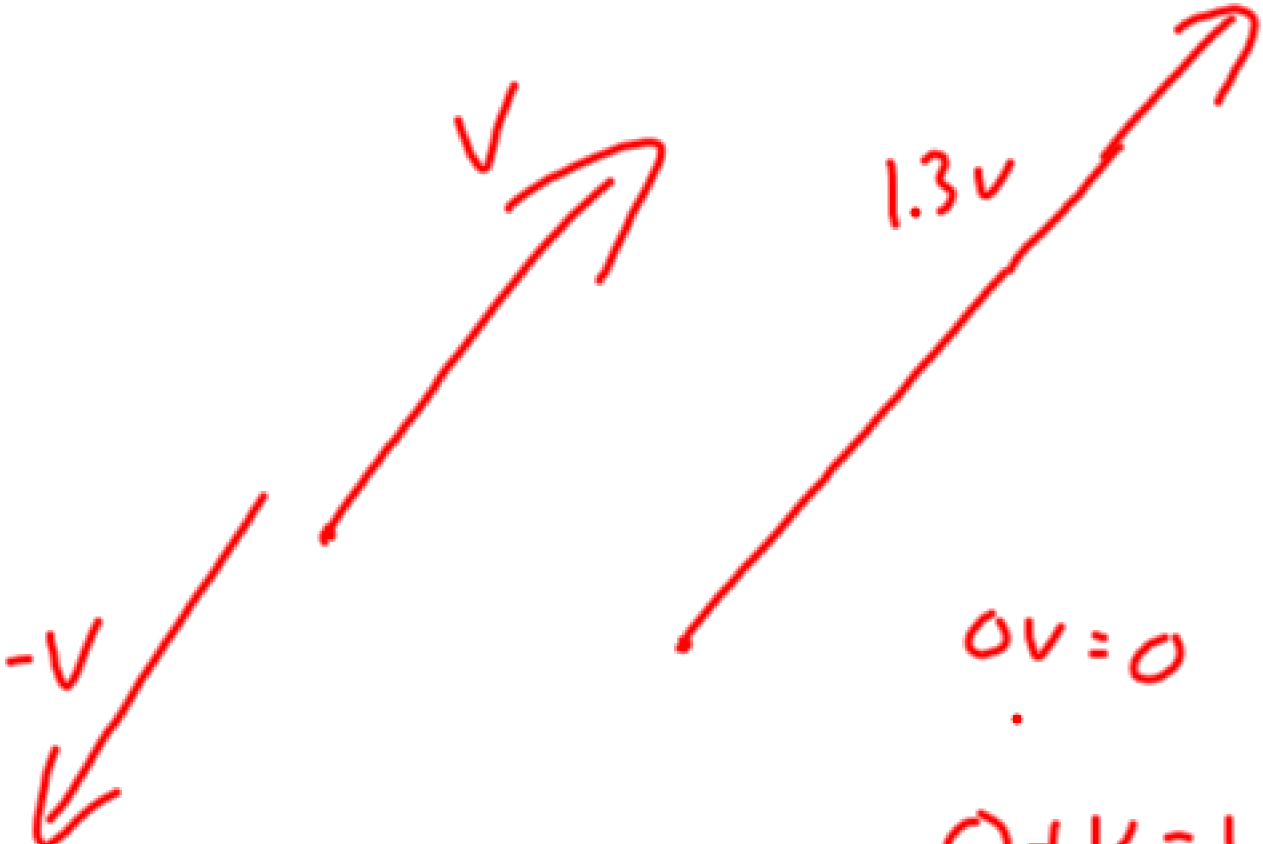
Addition

$v, w \in \mathbb{R}^n$

$$v + w = z = [x_1 + y_1, x_2 + y_2, \dots, x_n + y_n]$$
$$\in \mathbb{R}^n$$


(0,1)
(x1, x2)
||(x1,x2)||
θ
(0,0) = O
(1,0)


v
w



$0v = 0$

$0 + v = v$

$L(v) = w$ <u>L</u>i<u>n</u>e<u>a</u>r <u>Map</u>

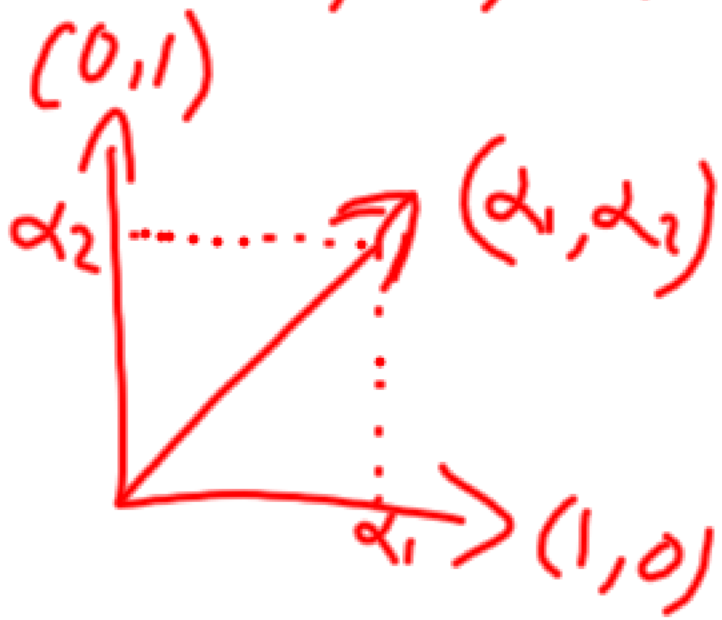
$\alpha L(v) = L(\alpha v)$

$L(v) + L(w) = L(v + w)$

$$v_1, v_2, \dots, v_n \in V$$

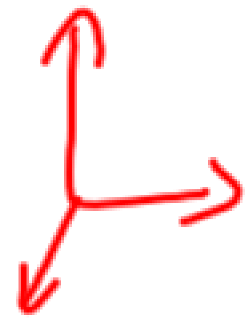
$$V' = \alpha_1 V_1 + \alpha_2 V_2 + \dots + \alpha_n v_n$$

$$\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$$



$$(\alpha_1, \alpha_2) = \alpha_1 (1,0) + \alpha_2 (0,1)$$
$$= (\alpha_1, \alpha_2)$$

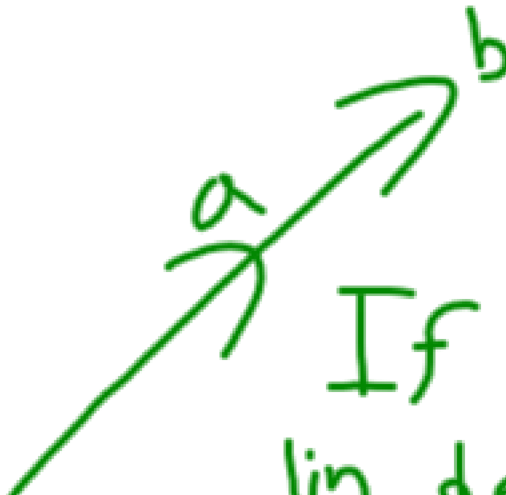
bases a set of linearly independent vectors, have to have exactly n

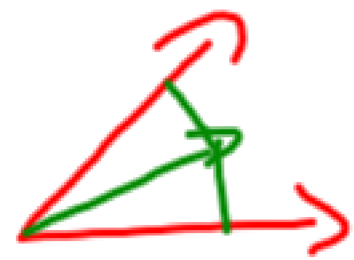


~~(a, 0)~~

$\frac{a}{b}(b, 0)$

$(0, c)$





If $v_1, v_2, \dots, v_K$

lin. dependent

$v_2 = \alpha_1 v_1 + \alpha_3 v_3 + \cdots +$

linear independence

$v_1, v_2, \cdots, v_K$

$v_2 \neq \alpha_1 v_1 + \alpha_3 v_3 + \cdots +$

$K = n$

Lin. indepence Def 2

$\alpha_1 v_1 + \alpha v_2 + \cdots + \alpha_n v_n = 0$

$\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$

Vector Space

existance bases → We can find a set of vector $v_1, \ldots, v_n$ that are linearly independent and span the space

Existance of $0 \in V$

$0 + v = v$

$0 \cdot v = 0$

- Closure under scalar mult. addition

$v, w \in V \Leftarrow\!\Rightarrow v + w \in V$

$\alpha v \in V \Leftarrow\!\Rightarrow v \in V$

$$\begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{m,1} & \cdots & a_{m,n} \end{bmatrix}$$

$T \in \mathcal{L}(V, W)$

$T : V \to W$

$T : V \to V$

$(v_1, \ldots, v_n)$ for V

$(w_1, \ldots, w_m)$ for W

$\alpha T(x) = T(\alpha x)$

$T(v) = w$

$T(v_k) = a_{1,k} w_1 + \cdots + a_{m,k} w_m$

$$\begin{array}{cc} & \begin{array}{ccccc} v_1 & \cdots & v_k & \cdots & v_n \end{array} \\ \begin{array}{c} w_1 \\ \vdots \\ w_m \end{array} & \begin{bmatrix} & & a_{1k} & & \\ & & \vdots & & \\ & & a_{mk} & & \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{array}{cc} T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) & \\ T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) & \end{array} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\overset{2\times 2}{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}} \overset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}}$$

$$= \underset{2\times 1}{\begin{bmatrix} \cos\theta x - \sin\theta y \\ x\sin\theta + y\cos\theta \end{bmatrix}}$$

Transforms and Camera Models

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

CS 4475

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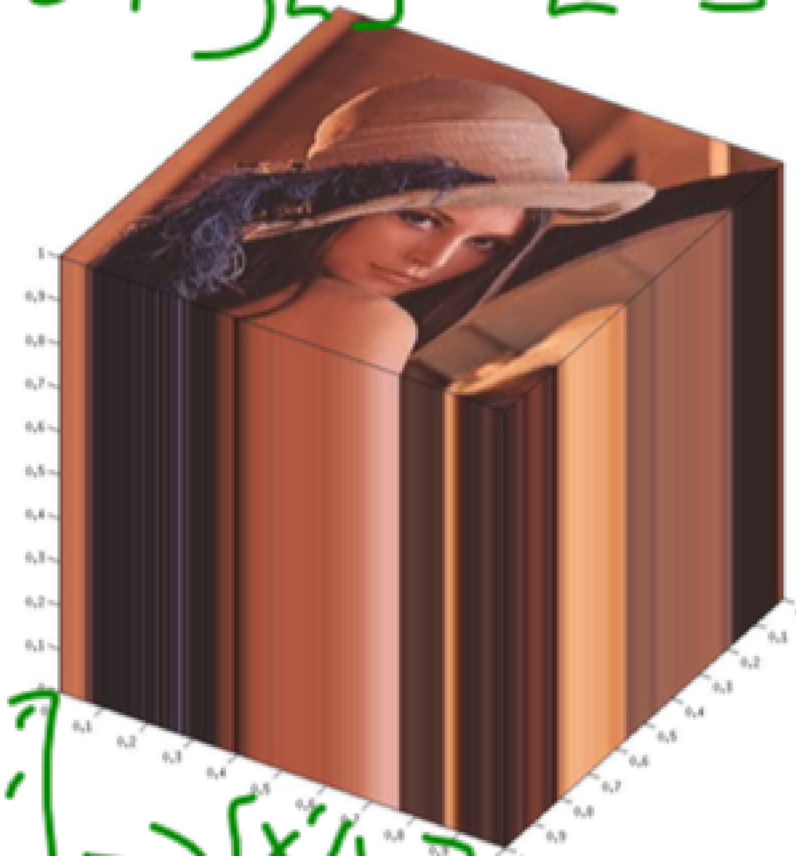
Homogeneous Coordinates

$$\begin{bmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



$$\begin{bmatrix} a & b & t_x \\ c & d & t_y \\ e & f & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} \rightarrow \begin{bmatrix} x'/w \\ y'/w \end{bmatrix}$$