Daily Maths Questions

Ieng-Duan

Lobb Street
Brunswick
Australia

Question 1

Question 2

Question 3

Find the indefinite integral of the following

$$\int e^{e^{2016x}+6048x}dx \tag{0.0.1}$$

Solution
It's important to realise that $6048 = 3 \times 2016$. Let $u = 2016x$, then we have

$$\begin{aligned}\int e^{e^{2016x}+6048x}dx &= \int e^{e^u+3u}du\frac{1}{2016} \\ &= \frac{1}{2016}\int e^{e^u}(e^u)^3du.\end{aligned}$$

Let $w = e^u$, we have

$$\begin{aligned}\frac{1}{2016}\int e^{e^u}(e^u)^3du &= \frac{1}{2016}\int e^w w^3 dw\frac{du}{dw} \\ &= \frac{1}{2016}\int e^w w^3 dw\frac{1}{\frac{dw}{du}} \\ &= \frac{1}{2016}\int e^w w^3 dw\frac{1}{w} \\ &= \frac{1}{2016}\int e^w w^2 dw \\ &= \frac{1}{2016}(e^w w^2 - 2\int e^w w dw) \\ &= \frac{1}{2016}(e^w w^2 - 2[e^w w - \int e^w dw]) \\ &= \frac{1}{2016}(e^w w^2 - 2e^w w + e^w) \\ &= \frac{1}{2016}(e^{e^u}(e^u)^2 - 2e^{e^u}e^u + e^{e^u}) \\ &= \frac{1}{2016}(e^{e^{2016x}}(e^{2016x})^2 - 2e^{e^{2016x}}e^{2016x} + e^{e^{2016x}}) \\ &= \frac{1}{2016}(e^{e^{2016x}+4032x} - 2e^{e^{2016x}+2016x} + e^{e^{2016x}}).\end{aligned}$$

Question 4

Find the indefinite integral of the following

$$\int \frac{x^{-\frac{1}{2}}}{1+x^{\frac{1}{3}}}dx \tag{0.0.2}$$

Solution
The idea is to substitute x with another variable that will make the new variable have integer powers. A common multiple of 2 and 3 is 6, hence let $u = x^{\frac{1}{6}}$, then $x = u^6$, and we have

$$\begin{aligned}
\int \frac{x^{-\frac{1}{2}}}{1+x^{\frac{1}{3}}}dx &= \int \frac{u^{-3}}{1+u^2}du\frac{dx}{du} \\
&= \int \frac{u^{-3}}{1+u^2}6u^5du \\
&= 6\int \frac{u^2}{1+u^2}du \\
&= 6\int 1-\frac{1}{1+u^2}du \\
&= 6u - 6\tan^{-1}(u) + C \\
&= 6x^{\frac{1}{6}} - 6\tan^{-1}(x^{\frac{1}{6}}) + C.
\end{aligned}$$