Quantitative Methods

March 12, 2024

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1 Preface

1.1 About the notes

- These notes aims to support my lecture at the HS Fresenius but are incomplete and no substitute for taking actively part in class. - A pdf version of these notes is available here
- I appreciate you reading it, and I appreciate any comments.
- This is work in progress so please check for updates regularly.
- Do not distribute without permission.

1.2 About the author

Figure 1.1: Prof. Dr. Stephan Huber[1]

I am a Professor of International Economics and Data Science at HS Fresenius, holding a Diploma in Economics from the University of Regensburg and a Doctoral Degree (summa cum laude) from the University of Trier. I completed postgraduate studies at the Interdisciplinary Graduate Center of Excellence at the Institute for Labor Law and Industrial Relations in the European Union (IAAEU) in Trier. Prior to my current position, I worked as a research assistant to Prof. Dr. Dr. h.c. Joachim Möller at the University of Regensburg, a post-doc at the Leibniz Institute for East and Southeast European Studies (IOS) in Regensburg, and a freelancer at Charles University in Prague.

Throughout my career, I have also worked as a lecturer at various institutions, including the TU Munich, the University of Regensburg, Saarland University, and the Universities of Applied Sciences in Frankfurt and Augsburg. Additionally, I have had the opportunity to teach abroad for the University of Cordoba in Spain, the University of Perugia in Italy, and the Petra Christian University in Surabaya, Indonesia. My published work can be found in international journals such as the Canadian Journal of Economics and the Stata Journal. For more information on my work, please visit my private homepage at hubchev.github.io.

[1] Source: https://sites.google.com/view/stephanhuber

1.3 Contact

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1.4 About this course

Workload of M-IBS 8 Quantitative & Qualitative Methods for Business

125 h = 56 h (in-class) + 21 h (guided private study hours) - 48 h (private self-study).

Workload of M-IBS 8.1 Quantitative Methods

62.5 h = 28 h (in-class) + 10.5 h (guided private study hours) - 24 h (private self-study).

Assessment

Students complete this module with a written exam of 120 minutes where 50% of the points stem from *M-IBS 8.1 Quantitative Methods* and 50% from *M-IBS 8.2 Qualitative Methods*. A passing grade in this module is achieved when the overall grade is greater than or equal to 4.0.

Learning outcomes:

After successful completion of the module, students are able to:

- assess and discuss coherent research paradigms, based on quantitative, qualitative, and mixed-methods research approaches,
- explain a broad set of quantitative and qualitative methods to collect, gather, illustrate, analyze, and interpret data,
- distinguish and discuss empirical strategies to identify causal mechanisms, causes, and effects.

How to prepare for the exam:

I am convinced that reading the lecture notes, preparing for class, taking actively part in class, and trying to solve the exercises without going straight to the solutions is the best method for students to

- maximize leisure time and minimize the time needed to prepare for the exam, respectively,
- getting long-term benefits out of the course,

- improve grades, and
- have more fun during lecture hours.

Literature:

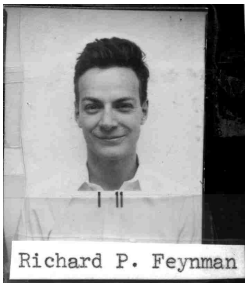
Cunningham [2021], Huntington-Klein [2022], Illowsky and Dean [2018], Békés and Kézdi [2021], Paldam [2021]

Content:

- Research design
  - Research design
  - How to measure socio-economical reality
  - How to identify causes of effects
  - How to identify effects of causes
  - The selection problem and ways to solve it (matching, natural experiments, laboratory experiments)
- Statistical toolbox
  - Types of data (cross-section, panel, time-series, georeferenced)
  - Types of variables (continuous, count, ordinal, categorical, qualitative)
  - Data sampling methods
  - Descriptive methods (data visualization, statistical moments, correlation)
  - Methods of statistical inference (distribution, statistical tests)
  - Mathematical and statistical software packages (R, Stata, SPSS, Excel, WolframAlpha, etc.)
- Methods
  - Data mining (graphical visualizations, cluster analysis, factor analysis)
  - Regression analysis (matching, instrument variables, difference in difference, fixed effects, regression discontinuity)
  - Other methods (time series analysis, spatial analysis, simulations, qualitative comparative analysis, etc.)

About how to learn (and prepare for the exam)

Figure 1.2: Richard P. Feynman's badge photo from Los Alamos National Laboratory[2]



[2]Source: Picture is taken from https://repository.aip.org/islandora/object/nbla%3A299600

Richard P. Feynman (see Figure 1.2):

> "I don't know what's the matter with people: they don't learn by understanding; they learn by some other way by rote, or something. Their knowledge is so fragile!"

Stephan Huber:

> "I agree with Feynman: The key to learning is understanding. However, I believe that there is no understanding without practice, that is, solving problems and exercises by yourself with a pencil and a blank sheet of paper without knowing the solution in advance."

- Study the lecture notes, i.e., try to understand the exercises and solve them yourself.
- Study the exercises, i.e., try to understand the logical rules and solve the problems yourself.
- Test yourself with past exams that you will find on ILIAS. The structure of the exam is more or less the same every semester.
- If you have the opportunity to form a group of students to study and prepare for the exam, make use of it. It is great to help each other, and it is very motivating to see that everyone has problems sometimes.
- If you have difficulties with some exercises and the solutions shown do not solve your problem, ask a classmate or contact me. I will do my best to help.

1.5 Personal note

Dear students,

If the title of this course "Quantitative & Qualitative Methods for Business" seems uninteresting to you, I assure you that it is actually quite exciting because it focuses on how we can use information to understand how the world and business works and how to interpret facts. The course will enhance your data literacy, help you think critically, and improve your personal decision-making skills.

One way we can do this is by understanding the differences between quantitative and qualitative data and how they can be used to inform our choices.

Quantitative data is information that can be measured, such as numbers and statistics, while qualitative data is information that cannot be measured and is often expressed in words or other non-numerical forms.

Both forms of information are crucial for making good decisions. Without sufficient information, it can be difficult to evaluate the options and potential outcomes of a decision, leading to poor or uninformed choices. In general, the more information a decision-maker has and the faster and better the information can be used, the better they will be to make a sound decision.

The methods we discuss in this course will help you systematically gather information and make sense of it.

Enjoy the course!

2 Doing research

2.1 What is research

Research often involves exploring unknown territory and seeking out new information through methods such as attending conferences, conducting interviews and experiments, and reading related research. This process can lead to the discovery of valuable techniques or insights that address important issues in society or science. Zora Neale Hurston [2010] (see Figure 2.1) paraphrased it beautifully:

> *"Research is formalized curiosity. It is poking and prying with a purpose."* [Hurston, 2010]

Figure 2.1: Zora Neale Hurston, 1891-1960[1]

Effective research is based on the principles of honesty, transparency and much more. A pithy yet profound quote from Scott Cunningham sums up this idea:

> *"True scientists do not collect evidence in order to prove what they want to be true or what others want to believe. That is a form of deception and manipulation called propaganda, and propaganda is not science. Rather, scientific methodologies are devices for forming a particular kind of belief. Scientific methodologies allow us to accept unexpected, and sometimes undesirable, answers."* [Cunningham, 2021, p. 10]

2.2 Everybody can do research

Before I go into how empirical research can and should be conducted, I would like to assert that each of us is a researcher in some sense and that you don't need a degree or a higher education to be a (good) researcher. Each of my four children (ages 2, 5, 6, and 8 (at the time of writing this)), for example, explores the world and learns something new every day. Even

[1]Source: Photography is taken from Library of Congress: Prints & Photographs Division, Carl van Vechten Collection, Reproduction Number LC-USZ62-54231, see: https://www.loc.gov/pictures/item/2004663047/

[2]Source: Image by macrovector on Freepik, see: https://www.freepik.com/free-vector/kindergarten-set-isolated-icons-with-toys-characters-kids-practicing-with-teacher-playing-games-vector-illustration_26760074.htm

Figure 2.2: Children as little researcher[2]

though none of my children is yet able to verify the novelty of their acquired knowledge and write it down in scientific form, I will claim that mine, like practically all children, are already little scientists. Why? Well, they explore unknown territory and search for information to discover new techniques that will make their lives pleasant, see Figure 2.2. Of course, they don't attend conferences or read journals to do this. They have never heard terms such as ontology, epistemology, axiology, or quantitative and qualitative methods. They are using methods that they have mastered for their age. They interview me, my wife and all other people around and they conduct experiments. For example, all my children liked to throw plates, cutlery, cups and alike from the table when they were about one year old. At first the throwing was just an accident, but they quickly found out that each throw was followed by a sound when the object touched the stone floor. My first son, in particular, took great delight in making these sounds. He threw everything within reach to the ground and giggled with joy at the clink he made when the object hit the ground. Perhaps he was also enjoying the attention he was getting from us parents through these actions. In any case, the behavior annoyed us. Wiping food scraps off the floor is not a nice thing to do. Unfortunately, at that time my son did not accept any argument to refrain from throwing. Neither a stern look nor a definite "no" helped to stop this behavior. Too great was the joy at the relationship he had figured out, which was, "I throw something off the table and it always clangs beautifully loud." So I started to do some research to figure out what I could do to stop him. The short answer I found can be summed up pretty well as "nothing". There is practically no good method to change the behavior without possibly negatively influencing his early childhood development. The reason is he did some research and we should not suppress that. Besides nature and material research he did social research: He found out that things fall to the ground (gravity), that things break and make different sounds (material research), and that other people notice him when he throws things (social research).

Once, when we were eating at a friend's house, my son (once again) threw everything off the table one after the other in unobserved moments. This time, however, it made no noise. The carpet under the table muffled everything. My son was irritated and at some point became really angry. Why? Well, his surely believed reality and his law "I throw something from the table and then it always clangs beautifully loud." was falsified. Soon he understood that his law only had to be adapted a little. It was then: "I throw something from the table and it clangs then beautifully loudly if a stone floor is under me." He repeated his experiments for a few more weeks, to check its validity. In the meantime he does other experiments trying to

contribute to his own knowledge.

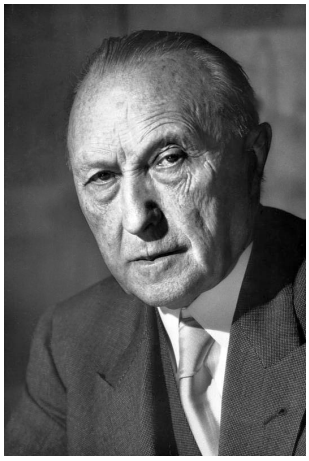
In general, the purpose of research is to find new knowledge or discover new ways to use existing knowledge in a creative way so as to generate new concepts, methodologies, inventions and understandings that -now or later- may be of some value for the human mankind. In simple terms, we aim to find something out. We aim to find a new law, a new relationship, a new insight. Or, we aim to challenge and revise existing insights on how the world works. You don't need a degree to do that. All you need is interest, open-mindedness, and a willingness to revise your ideas about how the world works. The latter is perhaps the most important skill you need to be a good researcher. Otherwise, one is a narrow-minded, and bigoted person who is too proud to follow up an insight with a change of mind.

I myself have a quick and happy tendency to change my views because it is a statement of a fresh understanding. Here are two more quotes from Mr. Keynes (see Figure 2.3) and Mr. Adenauer (see Figure 2.4), two historically slightly more significant people than me that are along the same lines and should convince you that changing your mind is not a sign of weakness, but of strength. Especially in science, the willingness to change one's mind is essential.

Figure 2.3: John Maynard Keynes (1883-1946)[3]

"When the facts change, I change my mind. What do you do, sir?"[4]

Figure 2.4: Konrad Adenauer (1876-1967)[5]



[3]Source: Photography is public domain and stems from https://de.wikipedia.org/wiki/John_Maynard_Keynes#/media/Datei:Ke

[4]This quote is often attributed to Keynes, but there is no clear evidence for it, see: https://quoteinvestigator.com/2011/07/22/keynes-change-mind/

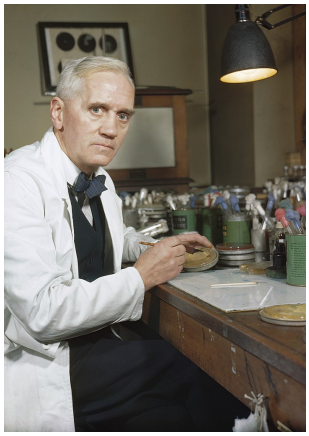
[5]Source: This photography from 1952 is public domain and stems from the Bundesarchiv, B 145 Bild-F078072-0004, Katherine Young, CC BY-SA 3.0 DE.

> *"What do I care about the rubbish I said yesterday? No one can stop me from getting smarter every day."* ("Was interessiert mich mein Geschwätz von gestern? … es kann mich doch niemand daran hindern, jeden Tag klüger zu werden.")[6]

2.3 It's difficult to do good research

Simply trying something and seeing what happens, like my children do, is a research method that relies on luck and chance. Before I go into more grown-up ways of doing research, I want to emphasize that the role of chance and serendipity in research is often downplayed and not acknowledged. The most well-known example of such research is the discovery of penicillin by Alexander Fleming (see Figure 2.5). In 1928, Fleming was studying the properties of staphylococcus bacteria when he noticed that a mold called Penicillium notatum had contaminated one of his bacterial cultures. He noticed that the mold seemed to be inhibiting the growth of the bacteria, and he began to investigate this further. Eventually, he was able to isolate and purify the active ingredient in the mold, which he named penicillin, and he discovered that it had powerful antibiotic properties. This discovery revolutionized the field of medicine and has saved countless lives.

Figure 2.5: Sir Alexander Fleming (1881-1955)[7]



Doing something on purpose and observing how things respond to the action can be considered a research strategy. Acting like a child or just waiting for something to happen by chance can also be considered a research strategy, and of course this can contribute greatly to knowledge. However, it are a naïve and poorly targeted strategies to conduct research. There are more grown-up research methods that are targeting more precisely the gaps in our knowledge and speed up innovation in the field where progress is desperately needed.

The processes of research and observation of phenomena should aim to maximize the probability of discovering new and intriguing findings. They should also ensure a high degree of confidence in the validity of our findings and reduce the likelihood that they will be disproved shortly afterwards. Transparency, scientific collaboration and open competition are crucial for efficient progress in science.

Take, for example, the scenario of a fatal disease. A naïve approach to finding a cure might be to try different things and observe who falls ill and who dies or is cured, hoping to stumble upon a cure through serendipitous observation. However, this method is unlikely to be effective or

[6] Freely quoted (and translated) from Weymar [1955, p. 521]
[7] Source: Photography is public domain and stems from https://en.wikipedia.org/wiki/File:Synthetic_Production_of_Penicillin_

practical. A more promising strategy would be to systematically study the disease and openly communicate research plans before they are implemented. This avoids unnecessary efforts and costs and accelerates the achievement of results.

For example, a laboratory should first seek to isolate the causative virus or bacterium in order to be able to grow and study it outside the danger to humans. Once this is done, we need a precise plan on how we can use all the available knowledge to cure the disease, protect people from infection, or help them survive the disease. In short, we need a strategic way to conduct research, i.e., a research strategy or design.

A *research strategy* is a general plan for conducting a study and a *research design* is a detailed plan for conducting the study. These words are frequently used interchangeably. A research strategy depends on many things including the question, the resources available, the current state of knowledge, the ambitions, whether quantitative or quantitative data are used, and what is considered to be the criteria of good research.

Before discussing some research strategies that can provide reasonable answers to certain types of questions, we should clarify how to ask a research question and what qualifies a research question.

2.4 Asking questions lika a good researcher

Unfortunately, there is no one research strategy that is appropriate for all questions and, what is worse, there is still controversy about what constitutes good research and how to properly ask a research question. In particular, this controversy takes place between researchers who use quantitative data and statistical methods and researchers who use qualitative data and methods.

Quantitative researchers are interested in both the causes of effects and the effects of causes. Experimental setups can allow to validate causes of effects and to measure the effects of causes. With observational data, however, it is often difficult to investigate the causes of effects. Thus, often quantitative research is more interested to quantify the effects of causes. Qualitative researchers also try to determine the causes of effects . However, their data analysis does rely less on statistical inference. A qualitative data set not necessarily requires (large) random samples or structured data (all the data that you can structure in a spreadsheet) in general, but allows to analyze selective and unstructured data (that is data in form of audio, video, text, images and alike). Qualitative research methods allow to classify these data into patterns or to interpret them in a meaningful way in order to arrive at results. Qualitative researchers are more concerned with the *why and how* of decision making and examine people's behavior, beliefs, perceptions of events, experiences, attitudes, interactions, and more in great depth.

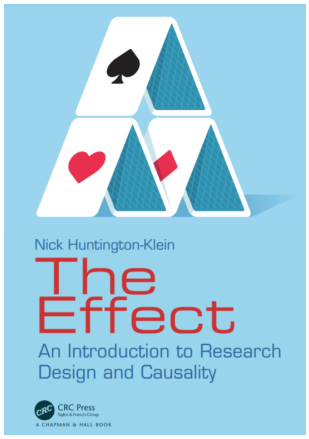
In empirical research, inductive and deductive are two different approaches to reasoning. *Inductive reasoning* is a process of collecting data from various sources, such as interviews, surveys or observations, and then use this data to identify patterns, themes, or relationships that can form the basis of a new hypothesis or theory. The goal of these exploratory studies, is to generate new ideas or insights about a topic, rather than testing a specific hypothesis. *Deductive reasoning* is a process in which the researcher starts with a general theory or hypothesis with the goal to test a specific hypothesis or theory. In most cases, a combination of both inductive and deductive reasoning may be used to formulate the research question and to design the empirical identification strategy.

In what follows, however, we focus on the criteria for *good research* that are more commonly used in evaluating the quality of quantitative research.

Exercise 2.1. The Effect ch.1+2

Read chapter 1 and 2 of Huntington-Klein [2022] and answer the questions below. The book (see Figure 2.6) is freely available at https://theeffectbook.net and here is the link to chapter 1: https://theeffectbook.net/ch-TheDesignofResearch.html

Figure 2.6: The Effect: An Introduction to Research Design and Causality[8]



1. What is the main focus of the book the author is writing about?

   a) Philosophy of science
   b) Qualitative research methods
   c) Empirical research and quantitative methods to identify and measure causal effects
   d) Statistics

2. What is the main challenge faced by quantitative empirical research, according to the author?

   a) Difficulty in obtaining accurate measurements
   b) Difficulty in interpreting measurements
   c) Difficulty in obtaining data that allows to answers the research question
   d) Difficulty in designing a research that gets a lot of attention

3. What is the author‘s main point about research questions?

   a) They should be well-defined, answerable, and understandable
   b) They should be simple and easy to answer
   c) They should be related to the world of traffic
   d) They should be related to the field of quantum mechanics

Please find solution here: Solution 2.1.

2.5 Features of good research

In order to make you a competent researcher who does not have to wait for a lucky chance but has a clear strategy, let’s discuss the criteria of a good research. Before I do that, however, I must make a disclaimer: there is a lack of consensus on what constitutes high-quality research in social sciences. In my experience, the practical benefits of such a tedious discussion are quite small. All I like to put forward is that I believe that all social science disciplines such as

[8]Source: Huntington-Klein [2022]

sociology, anthropology, psychology, economics, business administration, and education using quantitative methods agree that good research should be replicable, reproducible, transparent, reliable, and valid.

2.5.1 Reliability and validity

A research design is a plan to examine information in a systematic and controlled way so that the results of the research are *valid and reliable*.

Validity refers to the accuracy and truthfulness of research findings. In other words, if a study is valid, it should measure what it is intended to measure and produce results that are representative of the population being studied. Validity is important because it helps to ensure that the conclusions drawn from a study are supported by the data and are not based on flawed or biased methods.

Reliability refers to the consistency and stability of research findings. In other words, if a study is reliable, it should produce similar results if it is repeated using the same methods and conditions. Reliability is important because it helps to ensure that the results of a study are not simply due to chance or random error.

Both reliability and validity are important considerations in research, and researchers strive to maximize both in their studies. However, it is important to note that it is often difficult to achieve both at the same time, and trade-offs may need to be made between the two.

Note 1

A good research design should aim to minimize bias and maximize the reliability and validity of the research. It should also be appropriate for the research question being asked and the resources available to the researcher.

High reliability and low validity

An example of a study that has high reliability but low validity is a study that measures the weight of a group of people using a digital scale. If the scale is consistently accurate and produces the same weight measurements each time it is used, then the study has high reliability. However, if the scale is not calibrated correctly and produces inaccurate weight measurements, then the study has low validity.

Another example of a research design that has high reliability but low validity is a study that uses a highly reliable measurement tool, such as a standardized test, to measure a concept that is not directly related to the research question being asked. For example, a study that uses a standardized math test to measure students' critical thinking skills may have high reliability because the test is consistently accurate and produces similar scores each time it is administered. However, the study may have low validity because the math test is not an appropriate tool for measuring critical thinking skills. As a result, the results of the study may not be representative of the students' true critical thinking abilities.

High validity and low reliability

An example of a study that has high validity but low reliability is a study that asks people to self-report their eating habits. While the study may produce accurate and representative results about people's eating habits, the self-reported data may vary from person to person and may not be consistent over time. As a result, the study has high validity but low reliability.

Another example of a study that has high validity but low reliability is a study that uses a highly valid measurement tool, such as a survey, to measure a concept that is directly related to the research question being asked. However, the study may have low reliability because the survey is not administered consistently or the responses are not accurately recorded. For example, a study that uses a survey to measure students' attitudes towards school may have high validity because the survey is relevant to the research question and accurately measures the students' attitudes. However, if the survey is not administered consistently or the responses are not accurately recorded, the study may have low reliability. As a result, the results of the study may not be representative of the students' true attitudes towards school.

Trade-offs between reliability and validity

In research design, trade-offs may need to be made between reliability and validity. For example, a study that uses a highly reliable measurement tool may not be valid if the tool is not appropriate for the research question being asked. Similarly, a study that uses a highly valid measurement tool may not be reliable if the tool is prone to producing inconsistent results. As a result, researchers must carefully consider both reliability and validity when designing a study and make trade-offs as necessary to maximize the overall quality of the research.

2.5.2 Generalizability

Coming back to my little son who threw everything within reach to the ground and giggled with joy at the clink he made when the object hit the ground. He identified a cause-and-effect relationship through an experiment in an controlled environment. His law "I throw something off the table and it always clangs" worked in our home. To our regret, it was replicateable and he really tried hard to falsify it. Moreover, his study was reasonable valid as his study design, conduct, and analysis could answer his research questions without bias (at least ignoring the other noises that his sibling and parents make coincidentally during his experiment). Scientist call this *internal validity*. However, he also found out that when he leaves our home, things are sometimes a bit different, for example, if there is a carpet under the table. Thus, his insights from our home findings can't be generalized to other contexts, at least not without further specifications. Scientist call this *external validity*.

> **Note 2**
>
> Internal validity examines whether the study design, conduct, and analysis answer the research questions without bias. External validity examines whether the study findings can be generalized to other contexts.

2.5.3 Replicability, reproducibility, transparency, and other criteria

It must be possible to repeat the research conducted for several reasons. For example, if you can repeat a study with slightly changed parameters, you are able to improve its external validity and show that the conclusions drawn are reliable. To be able to repeat a study, everything that is important for drawing a conclusion from the research has to be mentioned. This is what we call *transparency*. Moreover, everything in the study must have been done in such a way that we can check the results for truth. In the best case, it is possible to *reproduce* the results in the same way they were obtained in the study. Sometimes this is not possible because, for example, we can never really ask the same people again in a survey, and even if we found the same people, they would have gotten older and not be the same people as before. In such a case, it should at least be possible to *replicate* the research. This means that we can basically do the same thing in a setting that differs only in those things that we cannot avoid to be different. For example, by interviewing a group of people who match the people in the study to replicate them on all the important characteristics like age.

In an empirical quantitative research study, for example, the data and the code written to process the data and analyze it should be accessible to everyone.

In a qualitative study, all sources of information should be stated, and the circumstances leading to a conclusion should be fully explained. For example, all transcripts of interviews conducted should be made available. The researcher should provide rich and detailed descriptions of the data and the context in which it was collected. Research should be provided with rich, nuanced, and multi-layered accounts of social phenomena by describing and interpreting the meanings, beliefs, and practices of the people being studied. That is known as *thick description*. Researchers typically employ a variety of methods such as participant observation, in-depth interviews, and document analysis, and they often use multiple sources of data to triangulate their findings. The goal is to provide a holistic and broad understanding of the phenomenon being studied, rather than a narrow view from the researcher's perspective.

There are some other criteria of good research that are worth mentioning:

Credibility

The research should be trustworthy and believable, and the researcher should provide detailed descriptions of the methods used to ensure transparency.

Reflective Practice

The researcher should engage in reflexive practice throughout the research process, which means to be critically aware of oneself, one's own assumptions, and one's own role in the research process.

Triangulation

The researcher should use multiple methods, sources, and perspectives to increase the credibility of the findings (also see *thick description* above).

Transferability

The conclusions drawn from looking at mostly unstructured data in qualitative research can hardly be generalized in a strict sense, as they depend crucially on the context of the object of study. For example, generalizability is essentially impossible in a qualitative case study, since everything depends on the specific situation of an individual, a company, or a group of people considered in the specific setting. This means that in a case study or interview, we may be looking at only a few or even a single observation that cannot be considered *representative* of the larger population, as generalizability does. Transferability, on the other hand, gives the reader the ability to transfer the findings into other contexts. The ability to transfer contextual findings to other cases is a goal of qualitative research, and the author of a study should attempt to offer the information in a way that allows the reader to transfer the findings to the setting or situation with which he or she is familiar.

2.6 The role of resources, data and ethics

There are several types of research designs, including experimental designs, quasi-experimental designs, and observational designs. Each of these designs took advantage of various empirical methods and statistical procedures. We will discuss some of them later on. The choice of research design, of course, should depend on the research question being asked, the resources available, and the type of data that is being collected. The research design should also take into account any ethical considerations that may be relevant to the research. The research design should be chosen so that it is well suited to answer the research question. For example, if one is interested in the question "Why do some people get sick with a certain disease and others do not?" then an observational study design to determine possible *causes of effect* may be appropriate. These identified potential causes should then be verified followed by an experimental study. Relatively, a statistical analysis should be used which would allow the *effects of causes* to be evaluated. The aim should be to identify necessary and sufficient circumstances to develop a disease. Also circumstances should be described that favor a disease.

If the question is a "how" question, for example, "How do parents feel when their child throws everything off the table?" then interviews might be an appropriate study design. If available resources such as time, funding, and staff are limited, you might also consider conducting an (online) survey in which parents are asked standardized questions about their feelings. In any way, the chosen research design must be feasible given the resources available.

In answering a question, a researcher should know, state, and discuss all the assumptions and unexamined beliefs that led him to his conclusion. However, since resources for conducting and explaining research are limited, special attention should be paid to what are called *critical assumptions*. These are assumptions that must be true in reality, otherwise the research is meaningless. Therefore, researchers should make great efforts to identify and validate these assumptions.

The type of data that is being collected is another important factor to consider when choosing a research design. Different types of data, such as quantitative data, qualitative data, or a combination of both, may require different methods of collection and analysis. For example, quantitative data, such as numerical data, can be collected through methods such as surveys and analyzed using statistical techniques, whereas qualitative data, such as interview transcripts, may require more interpretive methods of analysis.

Finally, the researcher should also take into account any ethical considerations that may be relevant to the research. For example, if the study involves human subjects, the researcher must

ensure that the study is conducted in accordance with ethical principles such as informed consent and confidentiality. Additionally, the researcher should ensure that the potential benefits of the study outweigh any potential risks to the subjects.

Exercise 2.2. Features of research

1. Which of the following best defines reliability in research?

    a) The extent to which a measurement tool produces consistent results
    b) The extent to which a study's results accurately reflect the concept being measured
    c) The extent to which a study's results can be generalized to other populations
    d) The extent to which a study's results are statistically significant

2. Which of the following best defines validity in research?

    a) The extent to which a measurement tool produces consistent results
    b) The extent to which a study's results accurately reflect the concept being measured
    c) The extent to which a study's results can be generalized to other populations
    d) The extent to which a study's results are statistically significant

3. Which of the following is an example of a study with high reliability but low validity?

    a) A study that uses a highly reliable measurement tool to measure a concept that is directly related to the research question being asked
    b) A study that uses a highly valid measurement tool to measure a concept that is not directly related to the research question being asked
    c) A study that uses a highly reliable measurement tool to measure a concept that is not directly related to the research question being asked
    d) A study that uses a highly valid measurement tool to measure a concept that is directly related to the research question being asked

4. Which of the following is an example of a study with high validity but low reliability?

    a) A study that uses a highly reliable measurement tool to measure a concept that is directly related to the research question being asked
    b) A study that uses a highly valid measurement tool to measure a concept that is not directly related to the research question being asked
    c) A study that uses a highly reliable measurement tool to measure a concept that is not directly related to the research question being asked
    d) A study that uses a highly valid measurement tool to measure a concept that is directly related to the research question being asked

5. What does internal validity examine in a study?

    a) The ability to replicate the study
    b) The generalizability of the study's findings
    c) Whether the study design, conduct, and analysis answer the research questions without bias
    d) All of the above

6. What does external validity examine in a study?

    a) The ability to replicate the study
    b) The generalizability of the study's findings
    c) Whether the study design, conduct, and analysis answer the research questions without bias
    d) None of the above

7. What is transparency in research?

    a) The ability to replicate a study
    b) The generalizability of the study's findings
    c) The availability and accessibility of the data and materials used in a study for others to review
    d) The ethical considerations of the research

8. What are the different types of research design discussed in the text?

    a) Experimental designs, quasi-experimental designs, and observational designs
    b) Experimental designs and descriptive designs
    c) Quasi-experimental designs and observational designs
    d) None of the above

9. Why is replicability important in a study?

    a) To be able to repeat a study with slightly changed parameters and thus improve the external validity
    b) To be able to check the results of the study for truth.
    c) To be able to reproduce the results in the same way they were obtained in the study
    d) All of the above

Please find solution here: Solution 2.2.

2.7 Glossary

- **Generalizability:** The extent to which the results of a study can be applied to other populations or contexts.
- **Internal validity:** The degree to which a study's results can be attributed to the specific variables or factors being studied, and not to other extraneous factors.
- **External validity:** The degree to which a study's results can be generalized to other populations or contexts outside of the specific sample or setting of the study.
- **Quantitative data:** Data that can be measured and quantified.
- **Qualitative data:** Data that cannot be easily measured or quantified.
- **Quantitative research:** A research approach that uses statistical methods and experiments to determine the causes of effects, to quantify the effects of causes, or to describe data.
- **Qualitative research:** A research approach that uses unstructured data and methods to examine, for example, people's behavior, beliefs, and experiences in depth, rather than quantifying results.
- **Reflective Practice:** A form of self-evaluation used to analyze one's own thoughts and actions.
- **Reliability:** The consistency of a study's results to produce similar results when repeated.
- **Research design:** A detailed plan for conducting a study, frequently used interchangeably with research strategy.
- **Research method:** A procedure used to conduct a study or investigation to gain knowledge or understanding about a particular topic.
- **Research question:** A question or problem that a study aims to answer or solve.
- **Research strategy:** A general plan for conducting a study, frequently used interchangeably with research design.
- **Replicability:** The ability of a study to be repeated with new data.

- **Reproducibility:** The ability of a study to be repeated and produce the same results, often used interchangeably with replicability.
- **Serendipity:** The role of luck and unexpected events in research.
- **Thick Description:** A detailed narrative used to explain a situation and its context.
- **Credibility:** A quality criterion in qualitative research, which refers to confidence in the truth value of the data and interpretations of them.
- **Transparency:** The degree to which a study's methods and data are easily accessible and understandable to others, allowing for the study to be independently evaluated and replicated.
- **Triangulation:** A method used in qualitative research to verify the accuracy of data by combining multiple sources of information.
- **Validity:** The degree to which a study measures what it is intended to measure, and the extent to which the results of the study can be considered accurate and meaningful.
- **Structured data:** Data that can be easily organized and analyzed in a structured format, such as a spreadsheet.
- **Unstructured data:** Data that cannot be easily organized and analyzed in a structured format, such as text, images, and audio.

Solutions to excercises

Solution 2.1. Solution to exercise Exercise 2.1.

1. c), 2. c), 3. a)

Solution 2.2. Solution to exercise Exercise 2.2

1. a), 2. b), 3. c), 4. d), 5. c), 6. b), 7. c), 8. a), 9. d)

3 Identification

In empirical research, identification refers to the process of establishing a clear and logical relationship between a cause and an effect. This involves demonstrating that the cause is responsible for the observed effect, and that there are no other factors that could potentially explain the effect. The goal of identification is to provide strong evidence that a particular factor is indeed the cause of a particular outcome, rather than simply coincidentally happen. In order to identify a cause-and-effect relationship, researchers can use experimental or non-experimental, that is, observational data, or both. The next section discusses how to generate or collect this data.

3.1 Data acquisition

There are several ways to get data which allows you to (hopefully) identify a cause-and-effect relationship:

3.1.1 Interviews

An interview is normally a one-on-one verbal conversation. Interviews are conducted to learn about the participants' experiences, perceptions, opinions, or motivations. The relationship between the interviewer and interviewee must be taken into account and other circumstances (place, time, face to face, email, etc.) should be taken into account. There are three types of interviews structured, semi-structured, and unstructured. *Structured interviews* use a set list of questions and hence are like a verbal surveys. In *unstructured interviews* the interviewer doesn't use predetermined questions but only a list of topics to address. Semi-structured interviews are the middle ground. Semi-structured interviews require the interviewer to have a list of questions and topics pre-prepared, which can be asked in different ways with different interviewee/s. Semi-structured interviews increase the flexibility and the responsiveness of the interview while keeping the interview on track, increasing the reliability and credibility of the data. Semi-structured interviews are one of the most common interview techniques.

Structured interviews use a predetermined list of questions that must be asked in a specific order, improving the validity and trustworthiness of the data but lowering respondent response. Structured interviews resemble verbal questionnaires. In unstructured interviews, the interviewer has a planned list of subjects to cover but no predetermined interview questions. In exchange for less reliable data, this makes the interview more adaptable. Long-term field observation studies may employ unstructured interviews. The middle ground are interviews that are semi-structured. In semi-structured interviews, the interviewer must prepare a list of questions and themes that can be brought up in various ways with various interviewees.

Interviews allow you to address a cause-and-effect relationship fairly directly, and it can be a good idea to interview experts and ask some *why* and *how* questions to gather initial knowledge about a particular topic before further elaborating your research strategy. For example, I interviewed kindergarten teachers with many years of experience working with children, as well

as other parents, to get information on how to solve the problem of my children throwing plates around the dining room. However, findings based on interviews are not very valid or reliable because the personal perceptions of both the interviewer and the interviewee can have an impact on the conclusions drawn. For example, I received very different tips and explanations because of the personal experiences of the people I interviewed. Unfortunately, I could not really ask my son why he was misbehaving. His vocabulary was too limited at the time, and even if he could speak, he would probably refuse to tell me the truth.

3.1.2 Surveys

In contrast to an interview a survey can be sent out to many different people. Surveys can be used to identify a cause-and-effect relationship by asking questions about both the cause and the effect and examining the responses. For example, if a researcher wanted to determine whether there is a relationship between a person's level of education and their income, they could conduct a survey asking participants about their education level and their income. If the data shows that participants with higher levels of education tend to have higher incomes, it suggests that education may be a cause of higher income. However, it is important to note that surveys can only establish a correlation between variables, but it is difficult to claim that correlations that where found through the survey imply a causal relationship. To establish a causal relationship, a researcher would need to use other methods, such as an experiment, to control for other potential factors that might influence the relationship that the respondent does not see.

3.1.3 Case studies

Case studies involve in-depth examination of a single case or a small number of cases in order to understand a particular phenomenon. Case studies can be conducted using both quantitative and qualitative methods, depending on the research question and the data being analyzed. While it is reasonable to find causal effects in the particular case, it is problematic to generalize the causal relationship. To establish a general causal relationship, a researcher would need to use other methods, such as an experiment, to control for other potential factors that might influence the relationship that the respondent does not see.

3.1.4 Experiments

One way to clearly identify a cause-and-effect relationship is through experiments, which involve manipulating the cause (the independent variable) and measuring the effect (the dependent variable) under controlled conditions (we will later on define precisely what is meant here). Experiments can be conducted using both quantitative and qualitative methods. Here are some examples:

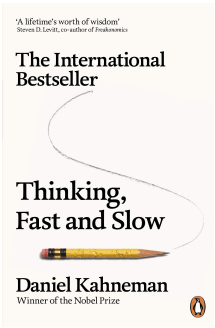
- A medical study in which a new drug is tested on a group of patients, while a control group receives a placebo.
- An educational study in which a group of students is taught a new method of learning, while a control group is taught using the traditional method.
- An agricultural study in which a group of crops is treated with a new fertilization method, while a control group is not treated.

- A study to determine the effect of a new training program on employee productivity might involve randomly assigning employees to either a control group that does not receive the training, or an experimental group that does receive the training. By comparing the productivity of the two groups, the researchers can determine if the new training program had a causal effect on employee productivity.
- A study to determine the effect of a new advertising campaign on sales might involve randomly assigning different groups of customers to be exposed to different versions of the campaign. By comparing the sales of the different groups, the researchers can determine if the advertising campaign had a causal effect on sales.
- In *experimental economics*, experimental methods are used to study economic questions. In a lab-like environment data are collected to investigate the size of certain effects, to test the validity of economic theories, to illuminate market mechanisms, or to examine the decision making of people. Economic experiments usually motivates and rewards subjects with money. The overall goal is to mimic real-world incentives and investigate things that cannot be captured or identified in the field.
- In *behavioral economics*, laboratory experiments are also used to study decisions of individuals or institutions and to test economic theory. However, it is done with a focus on cognitive, psychological, emotional, cultural, and social factors.

Figure 3.1: Daniel Kahneman and his best selling book[1]

(a) Daniel Kahneman

(a) Thinking, Fast and Slow



Source: https://commons.wikimedia.org/wiki/File:Daniel_Kahneman_(3283955327)_(cropped).jpg

In 2002 the Nobel Prize of Economics was awarded to Vernon L. Smith, I quote The Royal Swedish Academy of Sciences [2002], "for having established *laboratory experiments* as a tool in empirical economic analysis, especially in the study of alternative market mechanisms" and Daniel Kahneman "for having integrated insights from psychological research into economic science, especially concerning human judgment and decision-making under uncertainty".

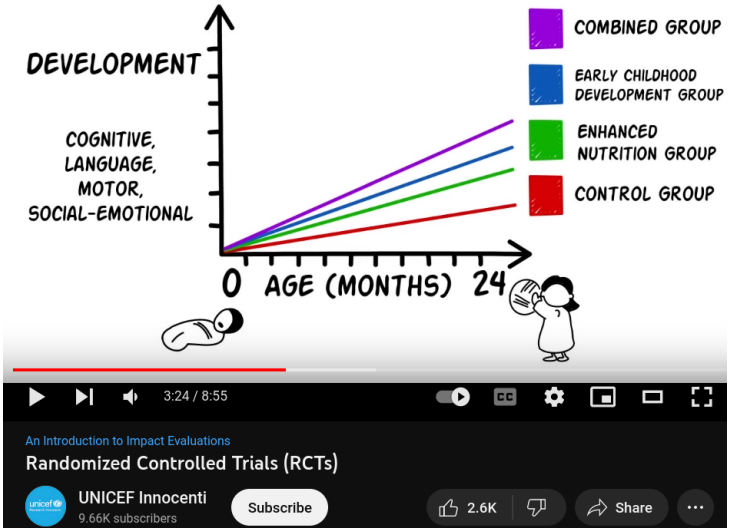
The strength of evidence from a controlled experiment is generally considered to be strong. However, the external validity, i.e., the generalizability, should be considered as well. External validity is sometimes low because effects that you can identify and measure in a lab are sometimes only of minor importance in the field.

There are different types of experiments:

Randomized controlled trials (RCTs) are a specific type of an experiment that involve randomly assigning participants to different treatment groups and comparing the outcomes of those groups. RCTs are often considered the gold standard of experimental research because they provide a high degree of control over extraneous variables and are less prone to bias.

For a better explanation and some great insights into what an RCT actually is, please watch the video produced by UNICEFInnocenti and published on the YouTube channel of UNICEF's dedicated research center, see https://youtu.be/Wy7qpJeozec and Figure 3.4.

Figure 3.4: Randomized Controlled Trials (RCTs)[2]



Quasi-experiments involve the manipulation of an independent variable, but do not involve random assignment of participants to treatment groups. Quasi-experiments are less controlled than RCTs, but can still provide valuable insights into cause-and-effect relationships.

Natural experiments involve the observation of naturally occurring events or situations that provide an opportunity to study cause-and-effect relationships. Natural experiments are often used when it is not possible or ethical to manipulate variables experimentally.

In a **laboratory experiment**, researchers manipulate an independent variable and measure the effect on a dependent variable in a controlled laboratory setting. This allows for greater control over extraneous variables, but the results may not generalize to real-world situations.

In a **field experiment**, researchers manipulate an independent variable and measure the effect on a dependent variable in a natural setting, rather than in a laboratory. This allows researchers to study real-world phenomena, but it can be more difficult to control for extraneous variables.

3.1.5 Observational data

Figure 3.5: Observational data[3]

Observational data are data that had been observed before the research question was asked or being collected independently from the study. To understand how observational data can be

[2]Source: https://youtu.be/Wy7qpJeozec
[3]Source: https://pixabay.com/images/id-5029286/

used to constitute a causal relationship is a bit tricky because there is only one world and only one reality at a time. In other words, we usually miss a counterfactual which we can use for a comparison. Take, for example, the past COVID-19 pandemic, where you chose to be vaccinated or not. Regardless of what you chose, we will never find out what would have happened to you if you had chosen differently. Maybe you would have died, maybe you would have gotten more or less sick, or maybe you wouldn't have gotten sick at all. We don't know, and it's impossible to find out because it's impossible to observe the counterfactual outcomes. This makes it difficult to establish causality from observational data. However, ingenious minds have found reasonable procedures and methods to extract some level of knowledge from observational data that allows us to infer causal relationships from observational data where we cannot directly observe the counterfactual outcome. We will come back to these methods later on.

In the upcoming sections, however, we will discuss experimental research designs including *randomized controlled trials* (RCTs) which are considered to be the "gold standard for measuring the effect of an action" [Taddy, 2019, p. 128]. RCTs can be used, for example, to study the effectiveness of drugs by observing people randomly assigned to three groups, one taking the pill (or treatment), a second receiving a placebo, and a third taking nothing. If the first group responds in any way differently than the other groups, the drug has an effect. Before explaining an RCT in more detail, we need to be clear about the fundamental problem of causal inference. This will be discussed in the following.

Exercise 3.1. Methods used in economic research

Read Paldam [2021] which is freely available here and answer the following questions:

a) List the eight types of research methods described in the paper and provide the description found in the paper.
b) Read the following statements and discuss whether they are true or not, and if the latter, correct them:

   i) The annual production of research papers in economics in the year 2017 has reached about 100 papers in top journals, and about 1,400 papers in the group of good journals. The production has grown with 3.3% per year, and thus it has doubled the last twenty years.
   ii) The upward trend in publication must be due to the large increase in the importance of publications for the careers of researchers, which has greatly increased the production of papers. There has also been a large increase in the number of researches, but as citations are increasingly skewed toward the top journals it has not increased demand for papers correspondingly.
   iii) Four trends are significant: The fall in theoretical papers and the rise in classical papers. There is also a rise in the share of statistical method and event studies. It is surprising that there is no trend in the number of experimental studies.
   iv) Book reviews have dropped to less than 1/3. Perhaps, it also indicates that economists read fewer books than they used to. Journals have increasingly come to use smaller fonts and larger pages, allowing more words per page. The journals from North-Holland Elsevier have managed to cram almost two old pages into one new one. This makes it easier to publish papers, while they become harder to read.
   v) About 50% of papers in the sample considered in Paldam [2021] belong to the economic theory class, about 6% are experimental studies, and about 43% are empirical studies based on data inference.
   vi) The papers in economic theory have increased from 33.6% to 59.5% – this is the largest change for any of the eight subgroups. It is highly significant in the trend test.

c) Explain what is meant with *theory fatigue* and discuss the reasons that lead to that fatigue.
d) According to Paldam [2021]: What factors contribute to the immediate relevance of research papers for policymakers?

See Solution 3.1.

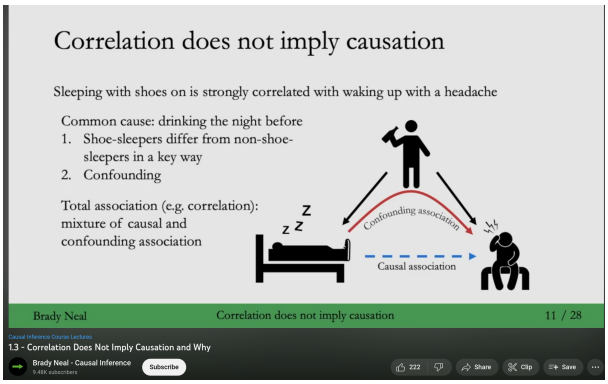
3.2 Correlation does not imply causation

Correlation refers to a statistical relationship between two variables, where one variable tends to increase or decrease as the other variable also increases or decreases. However, just because two variables are correlated does not necessarily mean that one variable causes the other. This is known as the *correlation does not imply causation* principle.

For example, it may be observed that the number of storks in a particular area is correlated with the birth rate of babies in that area. However, this does not mean that the presence of storks causes an increase in the birth rate. It is possible that both the number of storks and the number of babies born are influenced by other factors, such as the overall population density or economic conditions in the area.

Therefore, it is important to carefully consider all possible explanations (confounders) for a correlation and to use empirical evidence to determine the true cause-and-effect relationship between variables.

Figure 3.6: Correlation does not imply causation[4]



Tip 1

Watch the video of Brady Neal's lecture *Correlation Does Not Imply Causation and Why*. Alternatively, you can read chapter 1.3 of his lecture notes [Neal, 2020] which you find here.

3.3 The fundamental problem of causal inference

> Cunningham [2021, ch. 1.3]: *"It is my firm belief, which I will emphasize over and over in this book, that without prior knowledge, estimated causal effects are rarely, if ever, believable. Prior knowledge is required in order to justify any claim of a causal*

[4]Source: https://youtu.be/DFPm_a-_uJM
[5]Source: Cunningham [2021]

Figure 3.7: Causal Inference: The Mixtape[5]

finding. And economic theory also highlights why causal inference is necessarily a thorny task."

As Cunningham [2021] explains in his book (see Figure 3.7), it is very hard to claim causality. In the following section, I will paraphrase briefly two aspects why it is so difficult to claim to have found a causal effect. One reason for that is, that it is rather difficult to find or generate the right data and to use them properly so that the result is not biased. First, I will discuss Simpson's Paradox as an example how easy it is to interpret the data falsely. It will provide an idea on how difficult it is to analyze observational data meaningful and that we need to have a theory when looking on data. Above that, we should try to challenge the assumptions on which the theory is build on. After that I will briefly discuss the fundamental problem of causal inference as a problem of missing counterfactual data.

Exercise 3.2. Causal inference ch.1

Please read chapter 1 (Introduction) of Cunningham [2021] and answer the following questions. The book is freely available on https://mixtape.scunning.com/ and the link to chapter 1 is https://mixtape.scunning.com/01-introduction.

1. What are some common misconceptions about causality that the author addresses in chapter 1?
2. What is the role of randomization in causal inference, as described in the book?

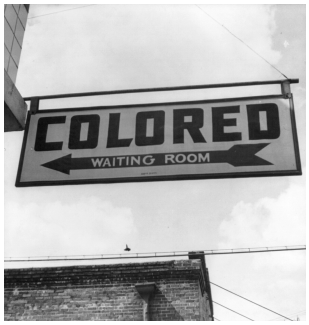
See Solution 3.2.

3.3.1 Simpsons Paradox

Discrimination is bad. Whenever we see it, we should try to find ways to overcome it. De jure segregation mandated the separation of races by law is clearly discriminatory. Other forms of discrimination, however, are often more difficult to spot and as long we don't have good evidence for discrimination, we should not judge prematurely. That means we should be sure that we see an act of making unjustified distinctions between individuals based on some categories to which they belong or perceived to belong. For example, if men and women are treated differently

[6]Source: The photography is public domain and stems from the Library of Congress Prints and Photographs Division Washington, see: http://hdl.loc.gov/loc.pnp/pp.print.

Figure 3.8: Discrimination[6]



without an acceptable reason, we consider it discriminative. For example, UC Berkeley was accused of discrimination in 1973 because it admitted only 35% of female applicants but 44% of male applicants overall. The difference was statistical significant. However, it turned out that the selection of students was not discriminative against women but agains men accordingly to Bickel et al. [1975]. Who conclude there was just a "tendency of women to apply to graduate departments that are more difficult for applicants of either sex to enter" [Bickel et al., 1975, p. 403]. Figure @ref(fig:berkley) taken from Bickel et al. [1975, page 403] visualizes this fact.

Here you can read the summary of their remarkable study:

> *"Examination of aggregate data on graduate admissions to the University of California, Berkeley, for fall 1973 shows a clear but misleading pattern of bias against female applicants. Examination of the disaggregated data reveals few decision-making units that show statistically significant departures from expected frequencies of female admissions, and about as many units appear to favor women as to favor men. If the data are properly pooled, taking into account the autonomy of departmental decision making, thus correcting for the tendency of women to apply to graduate departments that are more difficult for applicants of either sex to enter, there is a small but statistically significant bias in favor of women. The graduate departments that are easier to enter tend to be those that require more mathematics in the undergraduate preparatory curriculum. The bias in the aggregated data stems not from any pattern of discrimination on the part of admissions committees, which seem quite fair on the whole, but apparently from prior screening at earlier levels of the educational system. Women are shunted by their socialization and education toward fields of graduate study that are generally more crowded, less productive of completed degrees, and less well funded, and that frequently offer poorer professional employment prospects."*

Exercise 3.3. Graduate admissions

Read the first three pages of Bickel et al. [1975], i.e., pages 398-400, and answer the following questions. The article can be found here.

a) Describe the two assumptions that must be true in order to prove that UC Berkeley discriminates against women or men overall.
b) Table 1, shows that 277 fewer women and 277 more men were admitted than we would have expected under the two assumptions. Show how this number was calculated.
c) Explain the analogy with fish that illustrates the danger of pooling data.

See Solution 3.3.

[7]Source: Bickel et al. [1975, p. 403]

Figure 3.9: Proportion of applicants that are women plotted against proportion of applicants admitted[7]

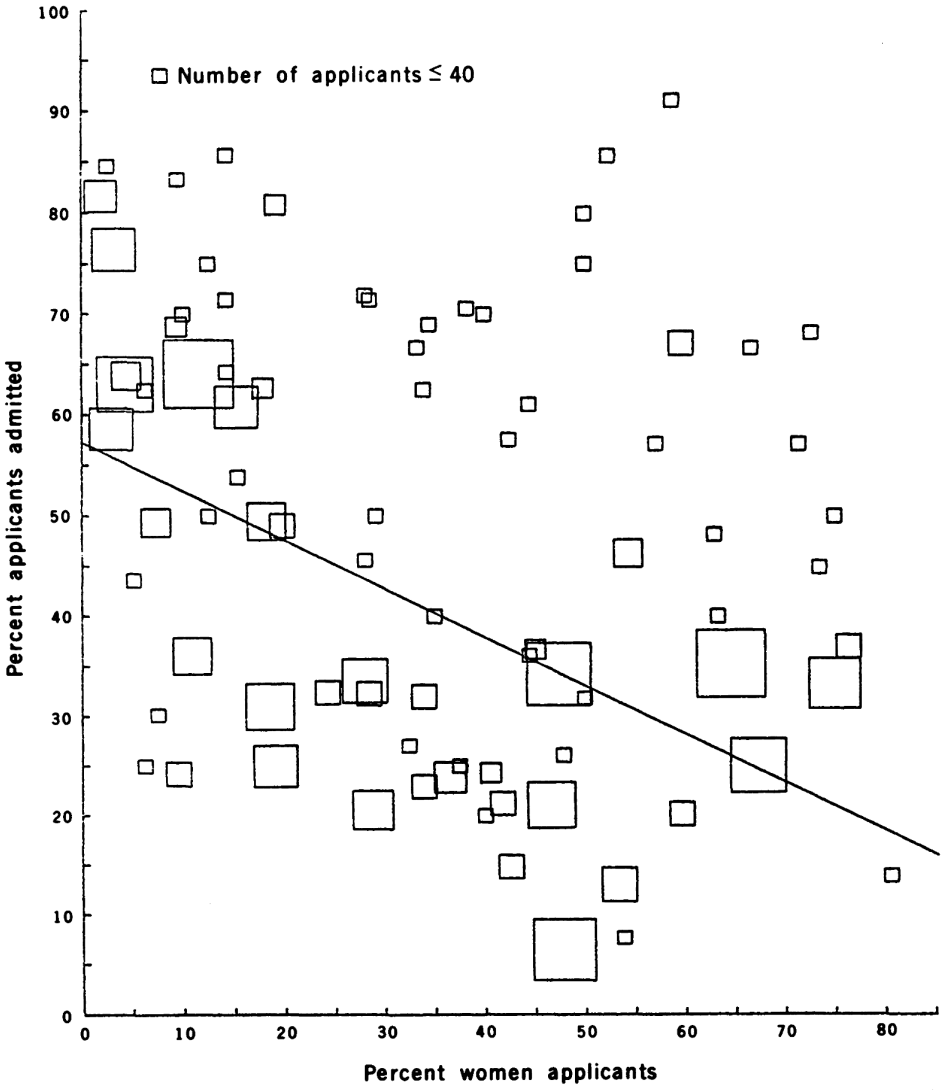


Fig. 1. Proportion of applicants that are women plotted against proportion of applicants admitted, in 85 departments. Size of box indicates relative number of applicants to the department.

Exercise 3.4. Simpson's Paradox

1. What is Simpson's Paradox?
    a) A phenomenon in which the direction of a relationship between two variables changes when a third variable is introduced
    b) A phenomenon in which the strength of a relationship between two variables changes when a third variable is introduced
    c) The phenomenon where correlation appears to be present in different groups of data, but disappears or reverses when the groups are combined
2. What is a potential cause of Simpson's Paradox?
    a) Differences in the variance of the two variables
    b) Differences in the correlation of the two variables
    c) Confounding variables
    d) Differences in the sample size of the two variables

See Solution 3.4.

3.3.2 Rubin causal model

> Keele [2015, page 314]: *"An identification analysis identifies the assumptions needed for statistical estimates to be given a causal interpretation."*

If we are interested in the causal effect of a certain treatment on an outcome, we need to compare the outcome of the individuals who received the treatment to the outcome of the individuals who did not receive the treatment. However, if the counterfactual outcome is missing for some individuals, we cannot make this comparison and therefore cannot estimate the causal effect. Unfortunately, the counterfactual is usually non-existing. For example, if we want to measure the effect of a vaccine we never can have a person who is vaccinated and not vaccinated at the same time. Formally, we have either $Y_i(1)$ or $Y_i(1)$, where Y_i denotes the effect/output of individual i in case of being vaccinated (1) and not vaccinated (0).

Thus, the so-called individual treatment effect (ITE) does not exist for person i:

$$ITE_i = Y_i(1) - Y_i(0)$$

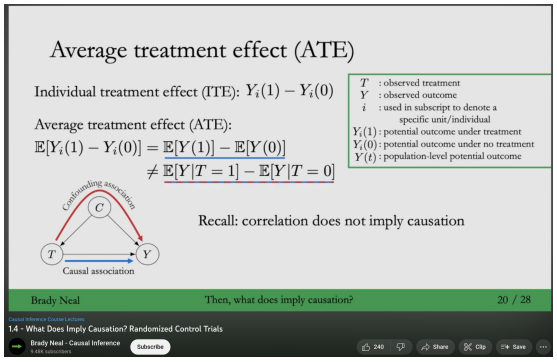
The Rubin Causal Model, also known as the potential outcomes framework, is a statistical framework for analyzing causality in the context of missing data. Table 3.1 is taken from Neal [2020] and shows some example data to illustrate that the fundamental problem of causal inference is actually a missing data problem. The Model goes back to Donald B. Rubin (born 1943) a statistician and is now a widely used method for causal inference. The basic premise of the Rubin Causal Model is that for each individual in a study, there are two potential outcomes: the outcome that would occur if the individual were exposed to a certain treatment or intervention (the "treatment group"), and the outcome that would occur if the individual were not exposed to that treatment (the "control group"). The key idea is that these potential outcomes can be used to infer causality by comparing the outcomes between the treatment and control groups even if we do not have a full set of data.

Table 3.1: Example data to illustrate that the fundamental problem of causal inference

i	T	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	0	?	0	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
6	1	1	1	?	?

Tip 2

Figure 3.10: Average treatment effect (ATE)



Watch the video of Brady Neal's lecture What Does Imply Causation? Randomized Control Trials (see Figure 3.10). Alternatively, you can read Neal [2020, ch. 2] of his lecture notes, see here.

Under certain assumptions, the Rubin Causal Model allows for the estimation of the Average Treatment Effect (ATE), which is the difference in the expected outcomes between the treatment and control groups, given by the formula:

$$ATE \triangleq \mathbb{E}[Y(1) - Y(0)]$$

Several methods exist for estimating the ATE within the Rubin Causal Model, and this course will explore some of them. When applied correctly, this model can yield valuable insights into causal relationships and enhance decision-making processes. However, it's important to recognize that the Rubin Causal Model is subject to certain limitations and assumptions. These assumptions must be satisfied to ensure the validity of the model's inferences. Section Section 3.4 addresses some of these critical assumptions.

To get the average treatment effect (ATE) we can take the average of the individual treatment effects (ITE):

$$ATE \triangleq \mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[\underbrace{Y_i(1) - Y_i(0)}_{ITE}] \tag{3.1}$$

3.4 Its difficult to overcome the fundamental problem

In the following we will discuss conditions that need to hold in order to empirically draw causal conclusions from the ATE without bias. This is important because equation Equation 3.1 does very often not hold when using observational data.

3.4.1 Ignorability

Referring to table Table 3.1, Brady Neal [2020] wrote:

> *"what makes it valid to calculate the ATE by taking the average of the Y(0) column, ignoring the question marks, and subtracting that from the average of the Y(1) column, ignoring the question marks?" This ignoring of the question marks (missing data) is known as ignorability. Assuming ignorability is like ignoring how people ended up selecting the treatment they selected and just assuming they were randomly assigned their treatment"* [Neal, 2020, p. 9]

Randomized controlled trials (RCTs) are characterized by randomly assigning individuals to different treatment groups and comparing the outcomes of those groups. Thus, they are build on the assumption of *ignorability*

$$(Y(1), Y(0)) \perp T$$

which allows to write the ATE as follows:

$$\begin{aligned} \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] &= \mathbb{E}[Y(1) \mid T = 1] - \mathbb{E}[Y(0) \mid T = 0] && (3.2) \\ &= \mathbb{E}[Y \mid T = 1] - \mathbb{E}[Y \mid T = 0].(\#eq : feq2) && (3.3) \end{aligned}$$

Another perspective on this assumption is the concept of exchangeability. Exchangeability refers to the idea that the treatment groups can be interchanged such that if they were switched, the new treatment group would have the same outcomes as the old treatment group, and the new control group would have the same outcomes as the old control group.

3.4.2 Unconfoundedness

While randomized controlled trials (RCTs) assume the concept of ignoreability, most observational data present challenges in drawing causal conclusions due to the presence of confounding factors that affect both (1) the likelihood of individuals being part of the treatment group and (2) the observed outcome. For instance, regional factors can affect both the number of storks and the number of babies born in a region. These factors are typically referred to as confounders, which we discussed in Section 3.2 as having the potential to create the illusion of a causal impact where none exists. However, empirical methods are available *to control for* these confounders and prevent the violation of the ignoreability assumption.

Exercise 3.5. Treatment effects

Read sections 2.1 and 2.3 of Neal [2020].

1. What is the individual treatment effect (ITE)?
2. What is the average treatment effect (ATE)?
3. How is the ATE calculated?
4. Can the ATE be used to determine the effect of a treatment on an individual level?

5. What are some potential sources of bias when estimating the ATE?

See Solution 3.5.

3.5 Statistical control requires causal justification

> **Tip 3**
>
> Read Wysocki et al. [2022] which is freely available here.

Scientific research revolves around challenging our own views and findings. A good researcher does not merely present their results; instead, they engage in discussions about potential limitations and pitfalls to draw valid conclusions. Engaging in polemics goes against the essence of good research. We should not conceal potential weaknesses in our scientific strategy or empirical approach; rather, we should emphasize their existence. Even if this disappoints individuals seeking easy answers, it is crucial to acknowledge these limitations. The Catalogue of Bias is an excellent resource that provides insight into various potential pitfalls and challenges encountered during research, which may sometimes be difficult to completely rule out.

Solutions to the exercises

Solution 3.1. Methods used in economic research (Exercise 3.1)

a) List the eight types of research methods described in the paper and provide the description found in the paper {.unlisted .unnumbered}

1. **Economic theory:** Papers are where the main content is the development of a theoretical model. The ideal theory paper presents a (simple) new model that recasts the way we look at something important.
2. **Statistical technique, incl. forecasting** Papers reporting new estimators and tests are published in a handful of specialized journals in econometrics and mathematical statistics. Some papers compare estimators on actual data sets. If the demonstration of a methodological improvement is the main feature of the paper, it belongs to this subgroup, but if the economic interpretation is the main point of the paper, it belongs to the classical empirical studies or newer techniques group.
3. **Surveys, incl. meta-studies** When the literature in a certain field becomes substantial, it normally presents a motley picture with an amazing variation, especially when different schools exist in the field. They are of two types, where the second type is still rare:
    1. **Assessed surveys** where the author reads the papers and assesses what the most reliable results are. Such assessments require judgment that is often quite difficult to distinguish from priors, even for the author of the survey.
    2. **Meta-studies** which are quantitative surveys of estimates of parameters claimed to be the same. These types of studies have two levels: The basic level collects and codes the estimates and studies their distribution. This is a rather objective exercise where results seem to replicate rather well. The second level analyzes the variation between the results. This is less objective.

4. **Experiments in laboratories** Most of these experiments take place in a laboratory, where the subjects communicate with a computer, giving a controlled, but artificial, environment. A number of subjects are told a (more or less abstract) story and paid to react in either of a number of possible ways. A great deal of ingenuity has gone into the construction of such experiments and in the methods used to analyze the results. Lab experiments do allow studies of behavior that are hard to analyze in any other way, and they frequently show sides of human behavior that are difficult to rationalize by economic theory. However, everything is artificial – even the payment while participants usually receive real money for participation and their performance. In some cases, the stories told are so elaborate and abstract that framing must be a substantial risk. In addition, experiments cost money, which limits the number of subjects. It is also worth pointing to the difference between expressive and real behavior. It is typically much cheaper for the subject to 'express' nice behavior in a lab than to be nice in the real world.

5. **Event studies (field experiments and natural experiments)** Event studies are studies of real world experiments. They are of two types:

    1. **Field experiments** analyze cases where some people get a certain treatment and others do not. The 'gold standard' for such experiments is double blind random sampling, where everything (but the result!) is announced in advance. Experiments with humans require permission from the relevant authorities, and the experiment takes time too. In the process, things may happen that compromise the strict rules of the standard. Controlled experiments are expensive, as they require a team of researchers.
    2. **Natural experiments** take advantage of a discontinuity in the environment, i.e., the period before and after an (unpredicted) change of a law, an earth-quake, etc. Methods have been developed to find the effect of the discontinuity. Often, such studies look like classical empirical studies with many controls that may that may or may not belong. Thus, the problems discussed under the classic empirical studies also apply here.

6. **Descriptive, deductions from data** In a descriptive study, researcher use an existing sample and hence, they have no control over the data generating process as it is usually the case with experiments. Descriptive studies are deductive. The researcher describes the data aiming at finding structures that tell a story, which can be interpreted. The findings may call for a formal test. If one clean test follows from the description, the paper can still be classified as a descriptive study. If more elaborate regression analysis is used, however, it can also be classified as a classical empirical study. Descriptive studies often contain a great deal of theory. Some descriptive studies present a new data set developed by the author to analyze a debated issue. In these cases, it is often possible to make a clean test, so to the extent that biases sneak in, they are hidden in the details of the assessments made when the data are compiled.

7. **Classical empirical studies** Typically have three steps: It starts by a theory, which is developed into an operational model. Then it presents the data set, and finally it runs regressions. The significance levels of the t-ratios on the coefficient estimated assume that the regression is the first meeting of the estimation model and the data. In practice, we all know that this is rarely the case. The classical method is often just a presentation technique. The great virtue of the method is that it can be applied to real problems outside academia. The relevance comes with a price: The method is quite flexible as many choices have to be made, and they often give different results. Preferences and interests, may affect these choices.

8. **Newer techniques** Partly as a reaction to the problems of classical empirical methods, the last 3–4 decades have seen a whole set of newer empirical techniques. They include different types of vector autoregression (VAR)[8], Bayesian techniques, causality and co-integration tests, Kalman filters, hazard functions, etc. The main reason for the lack of success for the new empirics is that it is quite bulky to report a careful set of co-integration tests or VARs, for example, and they often show results that are far from useful in the sense that they are unclear and difficult to interpret.

b) Read the following statements and discuss whether they are true or not, and if the latter, correct them:

Statements i) and vi) are false, all others are correct.

i) *The numbers are wrong:* The annual production of research papers in economics in the year 2017 has now reached about **1,000** papers in top journals, and about **14,000** papers in the group of good journals. The production has grown with 3.3% per year, and thus it has doubled the last twenty years.

ii) *Statement is correct:* The upward trend in publication must be due to the large increase in the importance of publications for the careers of researchers, which has greatly increased the production of papers. There has also been a large increase in the number of researches, but as citations are increasingly skewed toward the top journals it has not increased demand for papers correspondingly.

iii) *Statement is correct:* Four trends are significant: The fall in theoretical papers and the rise in classical papers. There is also a rise in the share of statistical method and event studies. It is surprising that there is no trend in the number of experimental studies.

iv) *Statement is correct:* Book reviews have dropped to less than 1/3. Perhaps, it also indicates that economists read fewer books than they used to. Journals have increasingly come to use smaller fonts and larger pages, allowing more words per page. The journals from North-Holland Elsevier have managed to cram almost two old pages into one new one. This makes it easier to publish papers, while they become harder to read.

v) *Statement is correct:* About 50% of papers in the sample considered in Paldam [2021] belong to the economic theory class, about 6% are experimental studies, and about 43% are empirical studies based on data inference.

vi) *Economic theory is not on the rise:* The papers in economic theory have **dropped from 59.5% to 33.6%** – this is the largest change for any of the eight subgroups. It is highly significant in the trend test.

vii) "Theory fatigue" is a term used to describe the decreasing attractiveness of theoretical research among journals, researchers and political decision-makers. This trend goes hand in hand with the increasing importance of empirical research. Policy makers are finding it increasingly difficult to engage with variations of existing theoretical models, and researchers often struggle to systematically summarize the findings of theoretical work, making it difficult to draw definitive conclusions on specific topics. In addition, theoretical work can be unconvincing to a wider audience that must rely on the reasonableness of complex and sometimes unrealistic assumptions. The credibility of theoretical research often depends on how realistic the initial assumptions are and how plausible the conclusions are. If neither aspect is grounded in reality, there is a danger that the research becomes

[8] A VAR is a statistical model used to capture the relationship between multiple quantities as they change over time.

an abstract exercise that provides new insights into the real world, but which are difficult to communicate to the layperson.

viii) A research paper that policymakers find appealing typically offers estimates of a crucial effect that decision-makers outside of academia are keen to understand. Papers that target policymakers should put an emphasis on distilling the core findings into a short executive summary tailored for decision-makers, facilitating their understanding and application of the research insights.

Solution 3.2. Causal inference ch.1 (Exercise 3.2)

1. Some common misconceptions about causality that the author addresses in chapter 1 include confusion between correlation and causality, and the belief that observational studies cannot (hardly) establish causality without prior knowledge. He says that human beings "engaging in optimal behavior are the main reason correlations almost never reveal causal relationships, because rarely are human beings acting randomly" which is crucial for identifying causal effects.
2. The role of randomization in causal inference, as described in the book, is that it helps to control for confounding variables and allows for the estimation of causal effects.

Solution 3.3. Graduate admissions (Exercise 3.3)

a) Assumption 1 is that in any given discipline male and female applicants do not differ in respect of their intelligence, skill, qualifications, promise, or other attribute deemed legitimately pertinent to their acceptance as students. It is precisely this assumption that makes the study of "sex bias" meaningful, for if we did not hold it any differences in acceptance of applicants by sex could be attributed to differences in their qualifications, promise as scholars, and so on. (…) Assumption 2 is that the sex ratios of applicants to the various fields of graduate study are not importantly associated with any other factors in admission. [Bickel et al., 1975, page 398]

b) Expectations were taken based on the overall acceptance rate of about 0.416 and multiplied by the total observed numbers of applicants admitted and rejected. For example: $(3738 + 4704) \cdot 0.41666 \approx 3460$ and $(3738 + 4704) \cdot (1 - 0.41666) \approx 4981$. Taking the difference of these two measures gives the number to be explained.

c) The analogy is explained on page 400:

> "Picture a fishnet with two different mesh sizes. A school of fish, all of identical size (assumption 1), swim toward the net and seek to pass. The female fish all try to get through the small mesh, while the male fish all try to get through the large mesh. On the other side of the net all the fish are male. Assumption 2 said that the sex of the fish had no relation to the size of the mesh they tried to get through. It is false."

The UC Berkley case is just one of many examples to illustrate that uniformity of group assignment of individuals is a necessary condition to ensure that pooling of data does not lead to misleading conclusions when using statistics. The phenomenon of obtaining different results depending on whether one considers the data pooled or unpooled is often referred to as the *Simpson Paradox*.

Solution 3.4. Simpson's Paradox (Exercise 3.4)

1. a), 2. c) and d)

Solution 3.5. Solution to exercise Exercise 3.5

1. The individual treatment effect (ITE) is a measure of the effect of a treatment or intervention on an individual level. It represents the difference in the outcome for an individual who receives the treatment versus the outcome for that same individual if they had not received the treatment.
2. The average treatment effect (ATE) is a measure of the difference in the expected outcomes between a treatment group and a control group. It represents the overall effect of a treatment on the population as a whole.
3. The ATE is calculated by taking the difference between the average outcome for the treatment group and the average outcome for the control group.
4. No, the ATE is a population-level measure and cannot be used to determine the effect of a treatment on an individual level. To determine the effect of a treatment on an individual level, you would need to use techniques such as propensity score matching or instrumental variables.
5. Some potential sources of bias when estimating the ATE include selection bias, measurement bias, and unobserved confounding variables. To mitigate these biases, researchers may use randomization or other advanced statistical techniques such as propensity score matching or instrumental variables to control for these potential sources of bias.

4 Statistics

Learning objectives:

In this chapter, we will explore several key concepts essential for conducting empirical analysis through statistical inference. Specifically, we will cover:

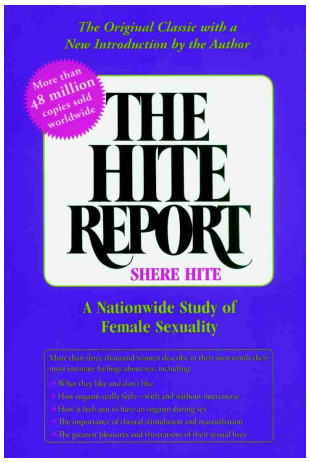
- The defining characteristics of a sample suitable for empirical analysis and statistical inference.
- The distinction between a sample and a population, and why understanding this difference is critical.
- How to identify biased samples and understand the implications of bias in empirical research.
- The differences between random sampling and random assignment, including their respective roles in research.
- Why a simple random sample is considered the gold standard in sampling methods due to its advantageous properties.
- Various methods for data collection and drawing samples, highlighting the strengths and weaknesses of each approach.

4.1 Sampling

4.1.1 The Hite Report

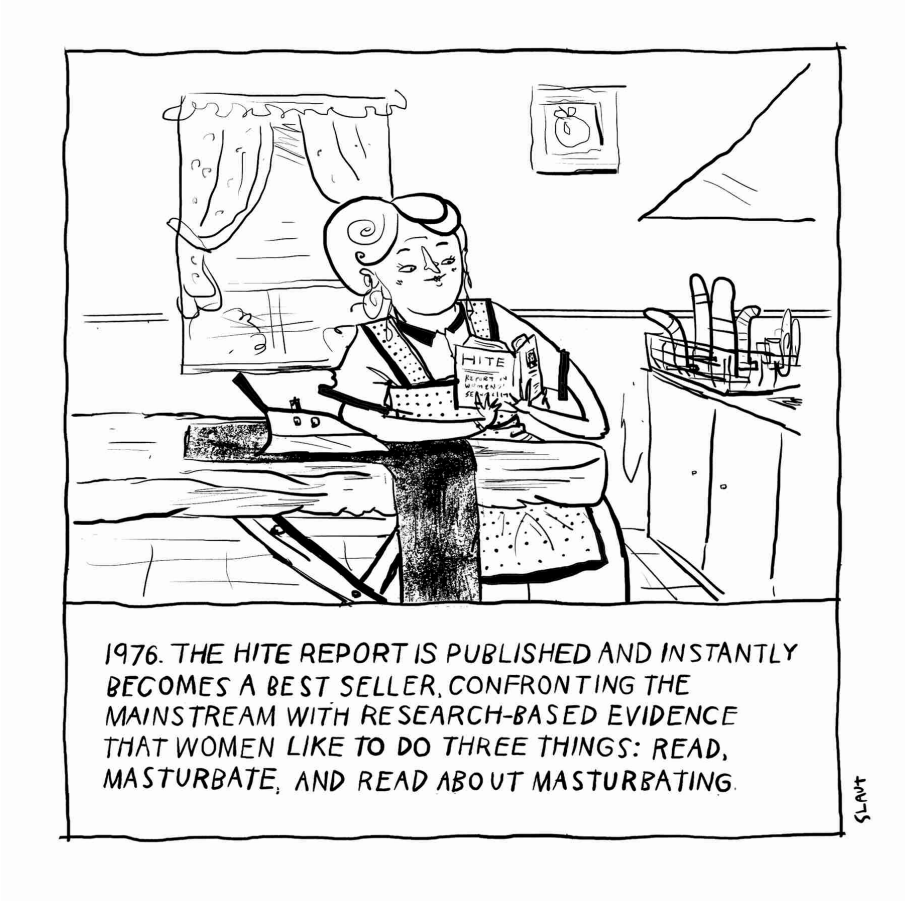
In 1976, when the *The Hite Report* [see Hite, 1976] was published it instantly became a best seller. Hite used an individualistic research method. Thousands of responses from anonymous questionnaires were used as a framework to develop a discourse on human responses to gender and sexuality. The following comic concludes the main results.

Figure 4.1: The Hite [1976] Report



[1]Picture is taken from https://www.theparisreview.org/blog/2017/07/21/great-moments-literacy-hite-report.

Figure 4.2: Comic on the Hite Report[1]



The picture of womens' sexuality in Hite [1976] was probably a bit biased as the sample can hardly be considered to be a **random and unbiased** one:

- Less than 5% of all questionnaires which were sent out were filled out and returned (response bias).
- The questions were only sent out to women's organizations (an opportunity sample).

Thus, the results were based on a sample of women who were highly motivated to answer survey's questions, for whatever reason.

4.1.2 Sample design

In statistics and quantitative research methodology, a sample is a group of individuals or objects that are collected or selected from a statistical population using a defined procedure. The elements of a sample are called sample points, sampling units, or observations.

Usually, the population is very large, and therefore, conducting a census or complete enumeration of all individuals in the population is either impractical or impossible. Therefore, a sample is taken to represent a manageable subset of the population. Data is collected from the sample, and statistics are calculated to make inferences or extrapolations from the sample to the population.

In statistics, we often rely on a **sample**, that is, a small subset of a larger set of data, to draw inferences about the larger set. The larger set is known as the **population** from which the sample is drawn.

Researchers adopt a variety of sampling strategies. The most straightforward is **simple random sampling**. Such sampling requires every member of the population to have an equal chance of being selected into the sample. In addition, the selection of one member must be independent of the selection of every other member. That is, picking one member from the population must not increase or decrease the probability of picking any other member (relative to the others). In this sense, we can say that simple random sampling chooses a sample by pure chance. To check your understanding of simple random sampling, consider the following example. What is the population? What is the sample? Was the sample picked by simple random sampling? Is it biased?

4.1.2.1 Random sampling

Random sampling is a sampling procedure by which each member of a population has an equal chance of being included in the sample. Random sampling ensures a representative sample. There are several types of random sampling. In simple random sampling, not only each item in the population but each sample has an equal probability of being picked. In systematic sampling, items are selected from the population at uniform intervals of time, order, or space (as in picking every one-hundredth name from a telephone directory). Systematic sampling can be biased easily, such as, for example, when the amount of household garbage is measured on Mondays (which includes the weekend garbage). In stratified and cluster sampling, the population is divided into strata (such as age groups) and clusters (such as blocks of a city) and then a proportionate number of elements is picked at random from each stratum and cluster. Stratified sampling is used when the variations within each stratum are small in relation to the variations between strata. Cluster sampling is used when the opposite is the case. In what follows, we assume simple random sampling. Sampling can be from a finite population (as in

picking cards from a deck without replacement) or from an infinite population (as in picking parts produced by a continuous process or cards from a deck with replacement).

In statistics, a **simple random sample** is a subset of individuals (a sample) chosen from a larger set (a population). Each individual is chosen randomly and entirely by chance, such that each individual has the same probability of being chosen at any stage during the sampling process, and each subset of k individuals has the same probability of being chosen for the sample as any other subset of k individuals.

The simple random sample has two important properties:

1. **UNBIASED:** Each unit has the same chance of being chosen.
2. **INDEPENDENCE:** Selection of one unit has no influence on the selection of other units.

Exercise 4.1. Random sampling

- What is meant by random sampling (simple random sample)?
- What is its importance?
- Why is having a large sample always better than having a small(er) one?

See Solution 4.1

4.1.3 Other sampling methods

Systematic sampling

Systematic sampling (a.k.a. interval sampling) relies on arranging the study population according to some ordering scheme and then selecting elements at regular intervals through that ordered list. Systematic sampling involves a random start and then proceeds with the selection of every k^{th} element from then onwards.

Accidental sampling / opportunity sampling / convenience sampling

These sampling methods describe a type of nonprobability sampling which involves the sample being drawn from that part of the population which is close to hand. That is, a population is selected because it is readily available and convenient.

Stratified sampling

Since simple random sampling often does not ensure a representative sample, a sampling method called stratified random sampling is sometimes used to make the sample more representative of the population. This method can be used if the population has a number of distinct groups. In stratified sampling, you first identify members of your sample who belong to each group. Then you randomly sample from each of those subgroups in such a way that the sizes of the subgroups in the sample are proportional to their sizes in the population. Let‘s take an example: Suppose you were interested in views of capital punishment at an urban university. You have the time and resources to interview 200 students. The student body is diverse with respect to age; many older people work during the day and enroll in night courses (average age is 39), while younger students generally enroll in day classes (average age of 19). It is possible that night students have different views about capital punishment than day students. If 70% of the students were day students, it makes sense to ensure that 70% of the sample consisted of day students. Thus, your sample of 200 students would consist of 140 day students and 60 night students. The proportion of day students in the sample and in the population (the entire university) would be

the same. Inferences to the entire population of students at the university would therefore be more secure.

Cluster sampling

Sometimes it is more cost-effective to select respondents in groups (clusters) of similar respondents. Sampling is often clustered by geography, or by time periods.

4.1.3.1 Random assignment

In experimental research, populations are often hypothetical. For example, in an experiment comparing the effectiveness of a new anti-depressant drug with a placebo, there is no actual population of individuals taking the drug. In this case, a specified population of people with some degree of depression is defined and a random sample is taken from this population. The sample is then randomly divided into two groups; one group is assigned to the treatment condition (drug) and the other group is assigned to the control condition (placebo). This random division of the sample into two groups is called random assignment. **Random assignment is critical for the validity of an experiment**. For example, consider the bias that could be introduced if the first 20 subjects to show up at the experiment were assigned to the experimental group and the second 20 subjects were assigned to the control group. It is possible that subjects who show up late tend to be more depressed than those who show up early, thus making the experimental group less depressed than the control group even before the treatment was administered. In experimental research of this kind, failure to assign subjects randomly to groups is generally more serious than having a non-random sample. Failure to randomize (the former error) invalidates the experimental findings. A non-random sample (the latter error) simply restricts the generalizability of the results.

4.1.4 Sample size

The sample size is an important feature of any empirical study in which the goal is to make inferences about a population from a sample. In practice, the sample size used in a study is usually determined based on the cost, time, or convenience of collecting the data, and the need for it to offer sufficient statistical power.

Recall that the definition of a random sample is a sample in which every member of the population has an equal chance of being selected. This means that the **sampling procedure** rather than the **results** of the procedure define what it means for a sample to be random. Random samples, especially if the sample size is small, are not necessarily representative of the entire population.

Larger sample sizes generally lead to increased precision when estimating unknown parameters. For example, if we wish to know the proportion of a certain species of fish that is infected with a pathogen, we would generally have a more precise estimate of this proportion if we sampled and examined 200 rather than 100 fish. Several fundamental facts of mathematical statistics describe this phenomenon, including the *law of large numbers* and the *central limit theorem*.

Note 3: The quality of data matters

A helpful slogan to keep in mind while scrutinizing statistical results is *garbage in, garbage out*. Regardless of how scientifically sound and visually appealing a statistic may appear, the formula used to derive it is oblivious to the quality of the data that underpins it. It is your responsibility to conduct a thorough examination. For example, if the data on which the statisitc is based emanates from a biased sample (one that favors certain individuals over others), a flawed design, unreliable data-collection protocols, or misleading questions, the margin of error becomes *bad*. If the bias is sufficiently severe, the outcomes become worthless.

4.1.5 Sample errors

Read the following examples[2]:

Example 1: You have been hired by the National Election Commission to examine how the American people feel about the fairness of the voting procedures in the U.S. Who will you ask?

It is not practical to ask every single American how he or she feels about the fairness of the voting procedures. Instead, we query a relatively small number of Americans, and draw inferences about the entire country from their responses. The Americans actually queried constitute our sample of the larger population of all Americans. The mathematical procedures whereby we convert information about the sample into intelligent guesses about the population fall under the rubric of inferential statistics. A sample is typically a small subset of the population. In the case of voting attitudes, we would sample a few thousand Americans drawn from the hundreds of millions that make up the country. In choosing a sample, it is therefore crucial that it not over-represent one kind of citizen at the expense of others. For example, something would be wrong with our sample if it happened to be made up entirely of Florida residents. If the sample held only Floridians, it could not be used to infer the attitudes of other Americans. The same problem would arise if the sample were comprised only of Republicans. ___Inferential statistics are based on the assumption that sampling is random}. We trust a random sample to represent different segments of society in close to the appropriate proportions (provided the sample is large enough; see below).

Example 2: We are interested in examining how many math classes have been taken on average by current graduating seniors at American colleges and universities during their four years in school. Whereas our population in the last example included all US citizens, now it involves just the graduating seniors throughout the country. This is still a large set since there are thousands of colleges and universities, each enrolling many students. It would be prohibitively costly to examine the transcript of every college senior. We therefore take a sample of college seniors and then make inferences to the ntire population based on what we find. To make the sample, we might first choose some public and private colleges and universities across the United States. Then we might sample 50 students from each of these institutions. Suppose that the average number of math classes taken by the people in our sample were 3.2. Then we might speculate that 3.2 approximates the number we would find if we had the resources to examine every senior in the entire population. But we must **be careful about the possibility that our sample is non-representative of the population**. Perhaps we chose an overabundance of math majors, or chose too many technical institutions that have heavy math requirements. Such bad sampling makes our sample unrepresentative of the population of all seniors. To

[2]The examples are taken from Lane [2023] and can be accessed here.

solidify your understanding of sampling bias, consider the following example. Try to identify the population and the sample, and then reflect on whether the sample is likely to yield the information desired.

Example 3: A substitute teacher wants to know how students in the class did on their last test. The teacher asks the 10 students sitting in the front row to state their latest test score. He concludes from their report that the class did extremely well. What is the sample? What is the population? Can you identify any problems with choosing the sample in the way that the teacher did?

In Example 3, the population consists of all students in the class. The sample is made up of just the 10 students sitting in the front row. **The sample is not likely to be representative of the population**. Those who sit in the front row tend to be more interested in the class and tend to perform higher on tests. Hence, the sample may perform at a higher level than the population.

Example 4: A coach is interested in how many cartwheels the average college freshmen at his university can do. Eight volunteers from the freshman class step forward. After observing their performance, the coach concludes that college freshmen can do an average of 16 cartwheels in a row without stopping.

In Example 4, the population is the class of all freshmen at the coach's university. The sample is composed of the 8 volunteers. The sample is poorly chosen because **volunteers are more likely to be able to do cartwheels** than the average freshman; people who can't do cartwheels probably did not volunteer! In the example, we are also not told of the gender of the volunteers. Were they all women, for example? That might affect the outcome, contributing to the non-representative nature of the sample.

Example 5: Sometimes it is not feasible to build a sample using simple random sampling. To see the problem, consider the fact that both Dallas and Houston are competing to be hosts of the 2012 Olympics. Imagine that you are hired to assess whether most Texans prefer Houston to Dallas as the host, or the reverse. Given the impracticality of obtaining the opinion of every single Texan, you must construct a sample of the Texas population. But now notice how difficult it would be to proceed by simple random sampling. For example, how will you contact those individuals who don't vote and don't have a phone? Even among people you find in the telephone book, how can you identify those who have just relocated to California (and had no reason to inform you of their move)? What do you do about the fact that since the beginning of the study, an additional 4,212 people took up residence in the state of Texas? As you can see, it is sometimes very difficult to develop a truly random procedure.

4.2 Descriptive statistics

Learning objectives:

- Calculate and interpret the arithmetic mean, median, mode, range, variance, and standard deviation of a dataset.
- Understand the concepts of skewness and kurtosis to describe the shape of data distribution.
- Distinguish between standard deviation and standard error, and compute the standard error of the mean.
- Learn the calculation and application of the coefficient of variation as a relative measure of variability.

- Grasp the fundamentals of covariance and the Pearson correlation coefficient to measure the relationship between two variables.
- Explore the Spearman rank correlation coefficient for non-parametric data analysis.
- Apply these statistical measures to real-world datasets through exercises and supplementary video materials.

4.2.1 Univariate data

4.2.1.1 Arithmetic mean

The arithmetic mean ($\bar{x}$) is calculated as the sum of all the values in a dataset divided by the total number of values:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

where $\bar{x}$ represents the arithmetic mean, x_i represents each individual value in the dataset, and n represents the total number of values in the dataset.

4.2.1.2 Median

The median is the middle value of a dataset when it is sorted in ascending or descending order. If the dataset has an odd number of values, the median is the middle value. If the dataset has an even number of values, the median is the average of the two middle values.

4.2.1.3 Mode

The mode is the value or values that appear most frequently in a dataset.

4.2.1.4 Range

The range is the difference between the maximum and minimum values in a dataset.

$$\text{Range} = \max(x_i) - \min(x_i)$$

where Range represents the range value, and x_i represents each individual value in the dataset.

4.2.1.5 Variance

The variance represents the average of the squared deviations of a random variable from its mean. It quantifies the extent to which a set of numbers deviates from their average value. Variance is commonly denoted as $Var(X)$, σ^2, or s^2. The calculation of variance is as follows:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

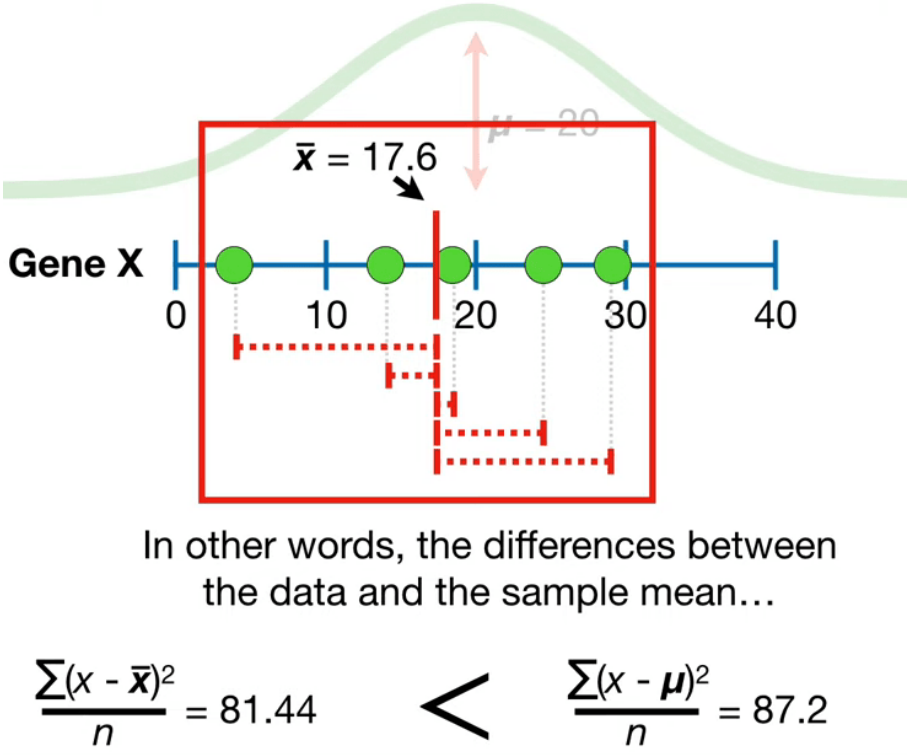
However, it is better to use

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2.$$

The use of $n-1$ instead of n in the formula for the sample variance is known as *Bessel's correction*, which corrects the bias in the estimation of the population variance, and some, but not all of the bias in the estimation of the population standard deviation. Consequently this way to calculate the variance and hence the standard deviation is called the *sample standard deviation* or the *unbiased estimation of standard deviation*. In other words, when working with a sample instead of the full population the limited number of observations tend to be closer to the *sample mean* than to the *population mean*, see Figure 4.3. Bessels Correction takes that into account.

For a detailed explanation, you can watch the video by StatQuest with Josh Starmer: Why Dividing By N Underestimates the Variance

Figure 4.3: Bias when using the sample mean[3]



[3]Picture is taken from the video https://youtu.be/sHRBg6BhKjI

4.2.1.6 Standard deviation

As the variance is hard to interpret, the standard deviation is a more often used measure of dispersion. A low standard deviation indicates that the values tend to be close to the mean. It is often abbreviated with sd, SD, or most often with the Greek letter sigma, σ. The underlying idea is to measure the average deviation from the mean. It is calculated as follows:

$$sd(x) = \sqrt{\sigma^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2} = \sigma$$

4.2.1.7 Standard error

The standard deviation (SD) measures the amount of variability, or dispersion, for a subject set of data from the mean, while the standard error of the mean (SEM) measures how far the sample mean of the data is likely to be from the true population mean. The SEM is always smaller than the SD. It matters because it helps you estimate how well your sample data represents the whole population.

The standard error of the mean (SEM) can be expressed as:

$$sd(\bar{x}) = \sigma_{\bar{x}} \ = s = \frac{\sigma}{\sqrt{n}}$$

where σ is the standard deviation of the population and n is the size (number of observations) of the sample.

Also see the video by StatQuest with Josh Starmer: Standard Deviation vs Standard Error, Clearly Explained!!!

4.2.1.7.0.1 * Why divide by the square root of n?

Let X_i be an independent draw from a distribution with mean $\bar{x}$ and variance σ^2. What is the variance of $\bar{x}$?

By definition:

$$\mathrm{Var}(x) = E\left[(x_i - E[x_i])^2\right] = \sigma^2$$

so

$$\begin{aligned}
\mathrm{Var}(\bar{x}) &= E\left[\left(\frac{\sum x_i}{n} - E\left[\frac{\sum x_i}{n}\right]\right)^2\right] \\
&= E\left[\left(\frac{\sum x_i}{n} - \frac{1}{n}E\left[\sum x_i\right]\right)^2\right] \\
&= \frac{1}{n^2}E\left[\left(\sum x_i - E\left[\sum x_i\right]\right)^2\right] \\
&= \frac{1}{n^2}E\left[\left(\sum x_i - \sum \bar{x}\right)^2\right] \\
&= \frac{1}{n^2}E\left[(x_1 + x_2 + \cdots + x_n - \underbrace{\bar{x} - \bar{x} - \cdots - \bar{x}}_{n \text{ terms}})^2\right] \\
&= \frac{1}{n^2}E\left[\sum (x_i - \bar{x})^2\right] \\
&= \frac{1}{n^2}\sum E\left(x_i - \bar{x}\right)^2 \\
&= \frac{1}{n^2}\underbrace{\sum \sigma^2}_{n\cdot\sigma^2} \\
&= \frac{1}{n}\sigma^2
\end{aligned}$$

and hence

$$sd(\bar{x}) = \sqrt{\mathrm{Var}(\bar{x})} = s = \frac{\sigma}{\sqrt{n}}$$

4.2.1.8 Coefficient of variation

The coefficient of variation (CoV) is a relative measure of variability and is calculated as the ratio of the standard deviation to the mean, expressed as a percentage:

$$CoV = \frac{\sigma}{\bar{x}}$$

where CoV represents the coefficient of variation, σ represents the standard deviation, and $\bar{x}$ represents the arithmetic mean.

4.2.1.9 Skewness

Skewness is a measure of the asymmetry of a distribution. There are different formulas to calculate skewness, but one common method is using the third standardized moment (γ_1):

$$\gamma_1 = \frac{\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\sigma}\right)^3}{n}$$

where γ_1 represents the skewness, x_i represents each individual value in the dataset, $\bar{x}$ represents the arithmetic mean, σ represents the standard deviation, and n represents the total number of values in the dataset.

4.2.1.10 Kurtosis

Kurtosis measures the peakedness or flatness of a probability distribution. There are different formulations for kurtosis, and one of the common ones is the fourth standardized moment. The formula for kurtosis is given by:

$$\text{Kurtosis} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2}$$

where Kurtosis represents the kurtosis value, x_i represents each individual value in the dataset, $\bar{x}$ represents the mean of the dataset, and n represents the total number of values in the dataset.

4.2.2 Bivariate data

4.2.2.1 Covariance

Covariance $Cov(X, Y)$ (or σ_{XY}) is a measure of the joint variability of two variables (x and y) and their observations i, respectively. The covariance is positive when larger values of one variable tend to correspond with larger values of the other variable, or when smaller values of one variable tend to correspond with smaller values of the other variable. On the other hand, a negative covariance suggests an inverse relationship, where larger values of one variable tend to correspond with smaller values of the other variable.

It's important to note that the magnitude of the covariance is influenced by the units of measurement, making it challenging to interpret directly. Additionally, the spread of the variables also affects the covariance. The formula for calculating covariance is as follows:

$$\text{Cov}(X, Y) = \sigma_{XY} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

where $cov(X, Y)$ represents the covariance, σ_{XY} is an alternative notation, x_i and y_i are the individual observations of variables X and Y, $\bar{x}$ and $\bar{y}$ are the means of variables X and Y, and n is the total number of observations.

> **Tip 4**
>
> To gain a better understanding of the concept and calculation of covariance, I highly recommend watching Josh Starmer's informative and visually engaging video titled Covariance and Correlation Part 1: Covariance.

4.2.2.2 The correlation coefficient (Bravais-Pearson)

The Pearson correlation coefficient measures the linear relationship between two variables. It is calculated as the covariance of the variables divided by the product of their standard deviations.

$$\rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

where ρ represents the Pearson correlation coefficient, $\text{Cov}(X, Y)$ denotes the covariance between variables X and Y, σ_X denotes the standard deviation of variable X, and σ_Y denotes the standard deviation of variable Y. It has a value between +1 and -1.

By dividing the covariance of X and Y by the multiplication of the standard deviations of X and Y, the correlation coefficient is normalized by having a minimum of -1 and a maximum of 1. Thus, it can fix the problem of the variance that the scale (unit of measurement) determines the size of the variance.

> **Tip 5**
>
> I highly recommend watching the video Pearson's Correlation, Clearly Explained!!! StatQuest with Josh Starmer. It provides a clear and engaging explanation of the meaning of correlation. The video features informative animations that help visualize the concept.

In interpreting correlations, it is important to remember that they…

1. … only reflect the strength and direction of linear relationships,
2. … do not provide information about the slope of the relationship, and
3. … fail to explain important aspects of nonlinear relationships.

Figure 4.4: Correlations are blind on some eye

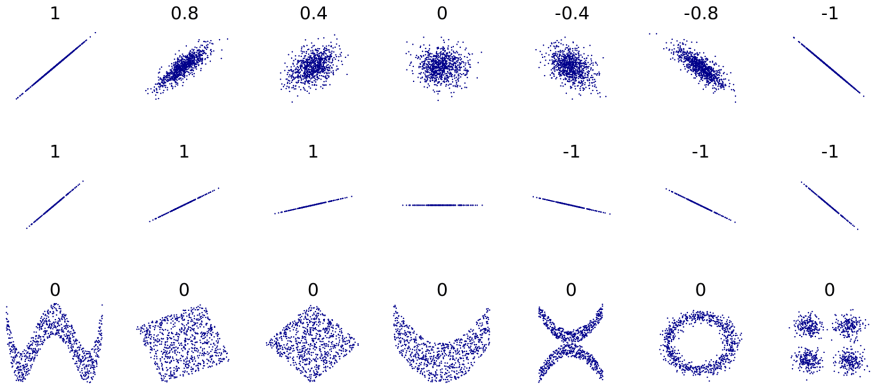


Figure 4.4 shows that correlation coefficients are limited in to explaining the relationship of two variables. For example, when the slope of a relationship is zero, the correlation coefficient becomes undefined due to the variance of Y being zero. Furthermore, Pearson's correlation coefficient is sensitive to outliers, and all correlation coefficients are prone to sample selection biases. It is crucial to be careful when attempting to correlate two variables, particularly when one represents a part and the other represents the total. It is also worth noting that small correlation values do not necessarily indicate a lack of association between variables. For example, Pearson's correlation coefficient can underestimates the association between variables exhibiting a quadratic relationship. Therefore, it is always advisable to examine scatterplots in conjunction with correlation analysis.

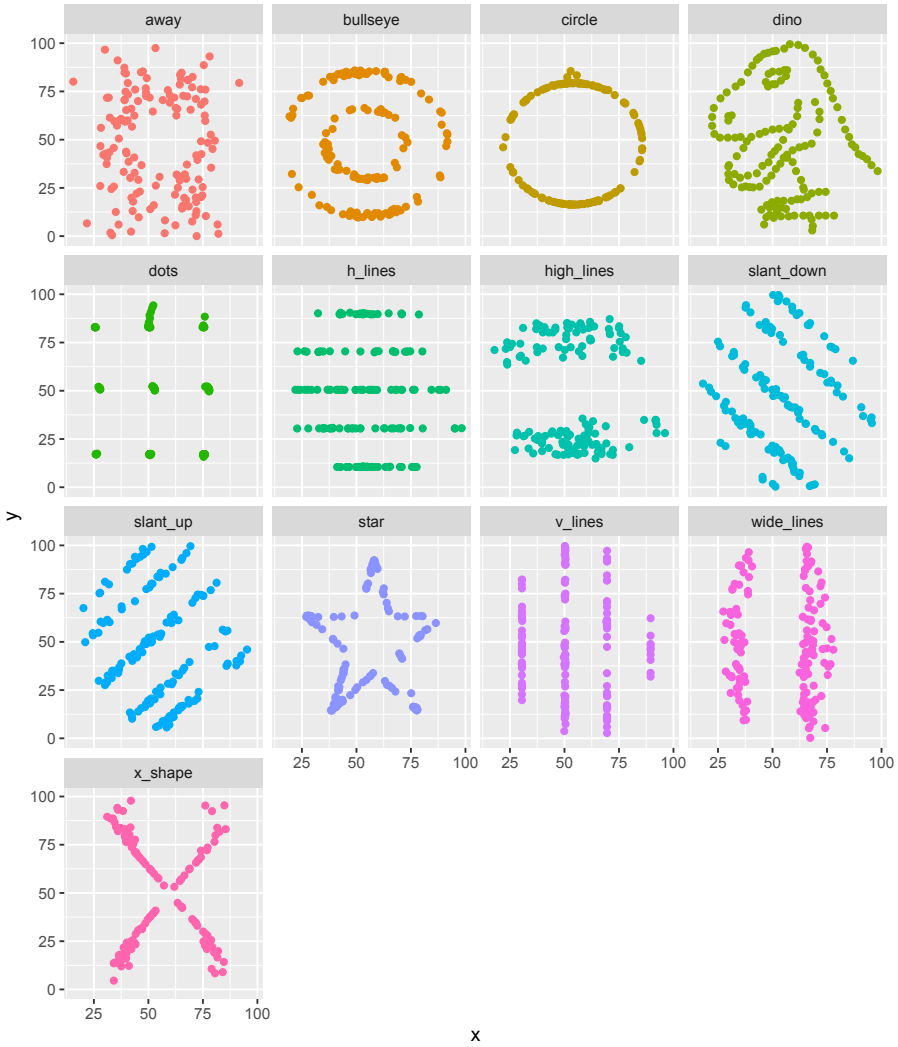
In Figure 4.5 you see various graphs that all have the same correlation coefficient and share other statistical properties like is investigated in Exercise 4.2.

4.2.2.3 Rank correlation coefficient (Spearman)

Spearman's rank correlation coefficient is a measure of the strength and direction of the monotonic relationship between two variables. It can be calculated for a sample of size n by converting the n raw scores X_i, Y_i to ranks $\mathrm{R}(X_i), \mathrm{R}(Y_i)$, then using the following formula:

[4]This graph was produced employing the `datasauRus` R package.

Figure 4.5: These diagrams all have the same statistical properties[4]



$$r_s = \rho_{\mathrm{R}(X),\mathrm{R}(Y)} = \frac{\mathrm{cov}(\mathrm{R}(X), \mathrm{R}(Y))}{\sigma_{\mathrm{R}(X)} \sigma_{\mathrm{R}(Y)}},$$

where ρ denotes the usual Pearson correlation coefficient, but applied to the rank variables, $\mathrm{cov}(\mathrm{R}(X), \mathrm{R}(Y))$ is the covariance of the rank variables, $\sigma_{\mathrm{R}(X)}$ and $\sigma_{\mathrm{R}(Y)}$ are the standard deviations of the rank variables.

If all n ranks are distinct integers, you can use the handy formula:

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

where ρ denotes the correlation coefficient, $\sum d_i^2$ is the sum of squared differences between the ranks of corresponding pairs of variables, and n represents the number of pairs of observations.

The coefficient ranges from -1 to 1. A value of 1 indicates a perfect increasing monotonic relationship, while a value of -1 indicates a perfect decreasing monotonic relationship. A value of 0 suggests no monotonic relationship between the variables.

Spearman's rank correlation coefficient is a non-parametric measure and is often used when the relationship between variables is not linear or when the data is in the form of ranks or ordinal categories.

Exercise 4.2. DatasauRus

The following exercise shows how to create Figure 4.5 using the programming language R.

Figure 4.6: The logo of the DatasauRus package[5]

a) Load the packages `datasauRus` and `tidyverse`. If necessary, install these packages.

b) The package `datasauRus` comes with a dataset in two different formats: `datasaurus_dozen` and `datasaurus_dozen_wide`. Store them as `ds` and `ds_wide`.

c) Open and read the R vignette of the `datasauRus` package. Also open the R documentation of the dataset `datasaurus_dozen`.

d) Explore the dataset: What are the dimensions of this dataset? Look at the descriptive statistics.

e) How many unique values does the variable `dataset` of the tibble `ds` have? Hint: The function unique() return the unique values of a variable and the function length() returns the length of a vector, such as the unique elements.

f) Compute the mean values of the `x` and `y` variables for each entry in `dataset`. Hint: Use the group_by() function to group the data by the appropriate column and then the summarise() function to calculate the mean.

[5]Source: https://github.com/jumpingrivers/datasauRus.

g) Compute the standard deviation, the correlation, and the median in the same way. Round the numbers.

h) What can you conclude?

i) Plot all datasets of `ds`. Hide the legend. Hint: Use the `facet_wrap()` and the `theme()` function.

j) Create a loop that generates separate scatter plots for each unique datatset of the tibble `ds`. Export each graph as a png file.

k) Watch the video Animating the Datasaurus Dozen Dataset in R from The Data Digest on YouTube.

Please find the solutions here.

Exercise 4.3. Summary statistics

Calculate for the following datasets: the mode, the median, the 20% quantile, the range, the interquartile range, the variance, the arithmetic mean, the sample standard deviation, the coefficient of variation.

a) For ten participants in a scientific conference the age has been noted: [25, 21, 18, 37, 56, 89, 46, 23, 21, 34.]
b) A random sample of 128 visitors of the Cupcake festival yielded the following frequencies regarding the cupcake consumption during their visit:

Table 4.1: Random sample of 128 visitors

Cupcages consumed	1	2	3	4	5	6
Abs. freq.	2	30	37	28	23	8

See Solution 4.2.

Exercise 4.4. Summary statistics in spreadsheet software

Given is the following datset: [0, 0, 40, 50, 50, 60, 70, 90, 100, 100.] Compute the following summary statistics of the data set using a spreadsheet software like *Excel* or *LibreOffice Calc*: mean, median, mode, quartiles (Q1, Q2, Q3), range, interquartile range, variance, standard deviation, mean absolute deviation, coefficient of variation and skewness.

See Solution 4.4.

Exercise 4.5. Guess the summary statistics

Given are the following variables:

Table 4.2: Some variables with observations

a	b	c	d	e
97	70	1	1	970
98	80	50	2	980
99	90	50	3	990
100	100	50	4	1000

a	b	c	d	e
101	110	50	5	1010
102	120	50	6	1020
103	130	99	7	1030

Rank the variables without calculating concrete numbers accordingly to the values of the following descriptive statistics: mode, median, mean, range, variance, standard deviation, coefficient of variation.

See Solution 4.3.

Solutions to the exercises

Solution 4.1. Solution to exercise Exercise 4.1

Random sampling is a sampling procedure by which each member of a population has an equal chance of being included in the sample. Random sampling ensures a representative sample. There are several types of random sampling. In simple random sampling, not only each item in the population but each sample has an equal probability of being picked. In systematic sampling, items are selected from the population at uniform intervals of time, order, or space (as in picking every one-hundredth name from a telephone directory). Systematic sampling can be biased easily, such as, for example, when the amount of household garbage is measured on Mondays (which includes the weekend garbage). In stratified and cluster sampling, the population is divided into strata (such as age groups) and clusters (such as blocks of a city) and then a proportionate number of elements is picked at random from each stratum and cluster. Stratified sampling is used when the variations within each stratum are small in relation to the variations between strata. Cluster sampling is used when the opposite is the case. In what follows, we assume simple random sampling. Sampling can be from a finite population (as in picking cards from a deck without replacement) or from an infinite population (as in picking parts produced by a continuous process or cards from a deck with replacement). The larger the sample gets, the closer we get to the population and hence, we reduce the bias of having a non-randomly selected sample.

Solution 4.2. Solution to exercise Exercise 4.3

Metric	1	2
Mode	21	3
Median	29.5	3
P20	21	2
Range	71	5
IQR	25	1.5
Arithmetic mean	37	3.5
σ^2	485.33	1.5591
σ	22.030	1.248621
COV	0.5954	2.4027

Solution 4.3. Solution to exercise Exercise 4.5

varlabel	a	b	c	d	e
Variance	2 (4.6666)	4 (466.66)	5 (800.33)	1 (4.6666)	3 (466.66)
COV	1 (.02160)	3 (.21602)	5 (.56580)	4 (.54006)	2 (.02160)
Mean	3 (100)	4 (100)	2 (50)	1 (4)	5 (1000)
Median	3 (100)	4 (100)	2 (50)	1 (4)	5 (1000)
Range	1 (6)	4 (60)	5 (98)	2 (6)	3 (60)
SD	1 (2.1602)	4 (21.602)	5 (28.290)	2 (2.1602)	3 (21.602)

Solution 4.4. Solution to exercise Exercise 4.4

Mittelwert	56
Standardfehler	11,4697670227235
Modus	0
Median	55
Erstes Quartil	42,5
Drittes Quartil	85
Varianz	1315,55555555556
Standardabweichung	36,2705880232945
Kurtosis	-0,731538352420953
Schräge	-0,417051115341008
Bereich	100
Minimum	0
Maximum	100
Summe	560
Anzahl	10

5 Regression analysis

5.1 Simple linear regression

The linear regression analysis is a widely used technique for predictive modeling. Its purpose is to establish a mathematical equation that relates a continuous response variable, denoted as y, to one or more independent variables, represented by x. The objective is to create a regression model that enables the prediction of the value of y based on known values of x.

To ensure meaningful predictions, it is important to have an adequate number of observations, denoted as i, available for the variables of interest.

The linear regression model can be expressed as:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,$$

where the index i denotes the individual observations, ranging from $i = 1$ to n. The variable y_i represents the dependent variable, also known as the regressand. The variable x_i represents the independent variable, also referred to as the regressor. β_0 denotes the intercept of the population regression line, a.k.a. the constant. β_1 denotes the slope of the population regression line. Lastly, ϵ_i refers to the error term or the residual, which accounts for the deviation between the predicted and observed values of y_i.

By fitting a linear regression model, one aims to estimate the values of β_0 and β_1 in order to obtain an equation that best captures the relationship between y and x.

While the correlation coefficient and the slope in simple linear regression are similar in many ways, it's important to note that they are not identical. The correlation coefficient measures the strength and direction of the linear relationship between variables in a broader sense, while the slope in simple linear regression specifically quantifies the change in the dependent variable associated with a unit change in the independent variable.

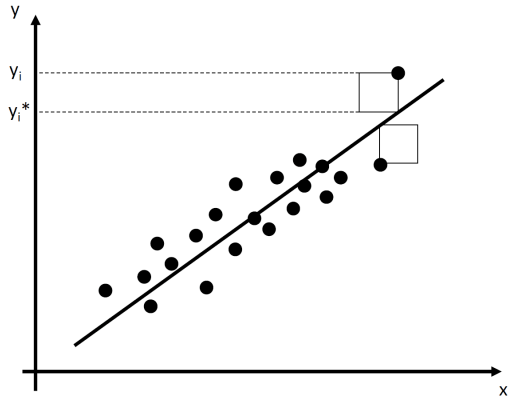
5.1.1 Estimating the coefficients of the linear regression model

In practice, the intercept and slope of the regression are unknown. Therefore, we must employ data to estimate the unknown parameters, β_0 and β_1. The method we use is called the ordinary least squared (OLS) method. The idea is to minimize the sum of the squared differences of all y_i and y_i^* as sketched in figure Figure 5.1.

Thus, we minimize the squared residuals by choosing the estimated coefficients $\hat{\beta}_0$ and $\hat{\beta}_1$

$$\begin{aligned} \min_{\hat{\beta}_0, \hat{\beta}_1} \sum_{i=1} \epsilon_i^2 &= \sum_{i=1} \left[y_i - \underbrace{(\hat{\beta}_0 + \hat{\beta}_1 x_i)}_{\text{predicted values} \equiv y_i^*} \right]^2 \\ \Leftrightarrow &= \sum_{i=1} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 \end{aligned}$$

Figure 5.1: The fitted line and the residuals



Minimizing the function requires to calculate the first order conditions with respect to alpha and beta and set them zero:

$$\frac{\partial \sum_{i=1} \epsilon_i^2}{\partial \beta_0} = 2 \sum_{i=1} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$
$$\frac{\partial \sum_{i=1} \epsilon_i^2}{\partial \beta_1} = 2 \sum_{i=1} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) x_i = 0$$

This is just a linear system of two equations with two unknowns β_0 and β_1, which we can mathematically solve for β_0:

$$\sum_{i=1} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$
$$\Leftrightarrow \hat{\beta}_0 = \frac{1}{n} \sum_{i=1} (y_i - \hat{\beta}_1 x_i)$$
$$\Leftrightarrow \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

and for β_1:

$$
\begin{aligned}
&\sum_{i=1}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)x_i = 0 \\
\Leftrightarrow &\sum_{i=1} y_i x_i - \underbrace{\hat{\beta}_0}_{\bar{y}-\hat{\beta}_1\bar{x}} x_i - \hat{\beta}_1 x_i^2 = 0 \\
\Leftrightarrow &\sum_{i=1} y_i x_i - (\bar{y} - \hat{\beta}_1\bar{x})x_i - \hat{\beta}_1 x_i^2 = 0 \\
\Leftrightarrow &\sum_{i=1} y_i x_i - \bar{y}x_i - \hat{\beta}_1\bar{x}x_i - \hat{\beta}_1 x_i^2 = 0 \\
\Leftrightarrow &\sum_{i=1}(y_i - \bar{y} - \hat{\beta}_1\bar{x} - \hat{\beta}_1 x_i)x_i = 0 \\
\Leftrightarrow &\sum_{i=1}(y_i - \bar{y})x_i - \hat{\beta}_1(\bar{x} - x_i)x_i = 0 \\
\Leftrightarrow &\sum_{i=1}(y_i - \bar{y})x_i = \hat{\beta}_1 \sum_{i=1}(\bar{x} - x_i)x_i \\
\Leftrightarrow &\hat{\beta}_1 = \frac{\sum_{i=1}(y_i - \bar{y})x_i}{\sum_{i=1}(\bar{x} - x_i)x_i} \\
\Leftrightarrow &\hat{\beta}_1 = \frac{\sum_{i=1}(y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}(\bar{x} - x_i)^2} \\
\Leftrightarrow &\hat{\beta}_1 = \frac{\sigma_{x,y}}{\sigma_x^2}
\end{aligned}
$$

The estimated regression coefficient $\hat{\beta}_1$ equals the covariance between y and x divided by the variance of x.

The formulas presented above may not be very intuitive at first glance. The online version of the book Hanck et al. [2020] offers a nice interactive application in the box The OLS Estimator, Predicted Values, and Residuals that helps to understand the mechanics of OLS. You can add observations by clicking into the coordinate system where the data are represented by points. Once two or more observations are available, the application computes a regression line using OLS and some statistics which are displayed in the right panel. The results are updated as you add further observations to the left panel. A double-click resets the application, that means, all data are removed.

5.1.2 The least squares assumptions

OLS performs well under a quite broad variety of different circumstances. However, there are some assumptions which need to be satisfied in order to ensure that the estimates are normally distributed in large samples.

The Least Squares Assumptions should fulfill the following assumptions:

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \ i = 1, \dots, n$$

- The error term ϵ_i has conditional mean zero given $X_i : E(u_i|X_i) = 0$.
- $(X_i, Y_i), i = 1, \dots, n$ are independent and identically distributed (i.i.d.) draws from their joint distribution.
- Large outliers are unlikely: X_i and Y_i have nonzero finite fourth moments. That means, assumption 3 requires that X and Y have a finite kurtosis.

5.1.3 Measures of fit

After fitting a linear regression model, a natural question is how well the model describes the data. Visually, this amounts to assessing whether the observations are tightly clustered around the regression line. Both the coefficient of determination and the standard error of the regression measure how well the OLS Regression line fits the data.

R^2 is the fraction of the sample variance of Y_i that is explained by X_i. Mathematically, the R^2 can be written as the ratio of the explained sum of squares to the total sum of squares. The explained sum of squares (ESS) is the sum of squared deviations of the predicted values $\hat{Y}_i$, from the average of the Y_i. The total sum of squares (TSS) is the sum of squared deviations of the Y_i from their average. Thus we have

$$ESS = \sum_{i=1}^{n} \left(\hat{Y}_i - \overline{Y}\right)^2, \tag{5.1}$$

$$TSS = \sum_{i=1}^{n} \left(Y_i - \overline{Y}\right)^2, \tag{5.2}$$

$$R^2 = \frac{ESS}{TSS}. \tag{5.3}$$

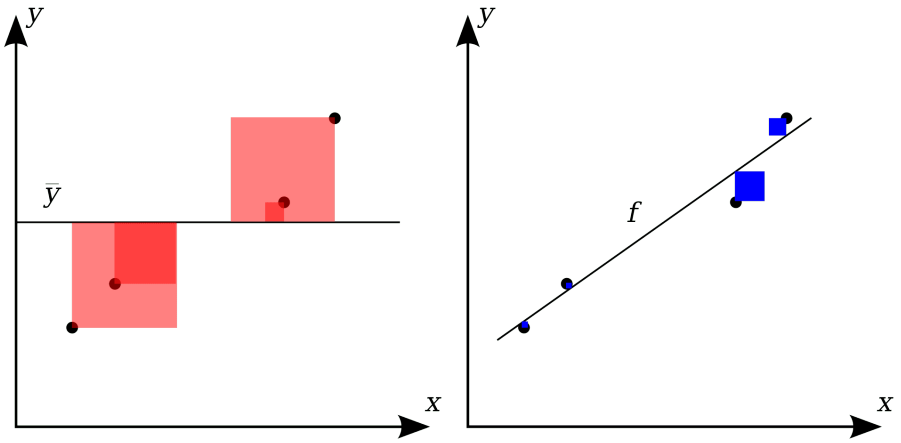
Since $TSS = ESS + SSR$ we can also write

$$R^2 = 1 - \frac{SSR}{TSS}$$

with

$$SSR = \sum_{i=1}^{n} \epsilon^2.$$

Figure 5.2: Total sum of squares and sum of squared residuals



R^2 lies between 0 and 1. It is easy to see that a perfect fit, i.e., no errors made when fitting the regression line, implies $R2 = 1$ since then we have $SSR = 0$. On the contrary, if our estimated regression line does not explain any variation in the Y_i, we have $ESS = 0$ and consequently $R^2 = 0$. Figure 5.2 represents the relationship of TTS and SSR.

5.2 Multiple linear regression

Having understood the simple linear regression model, it is important to broaden our scope beyond the relationship between just two variables: the dependent variable and a single regressor. Our goal is to causally interpret the measured association of two variables, which requires certain conditions as explained in Chapter 3.

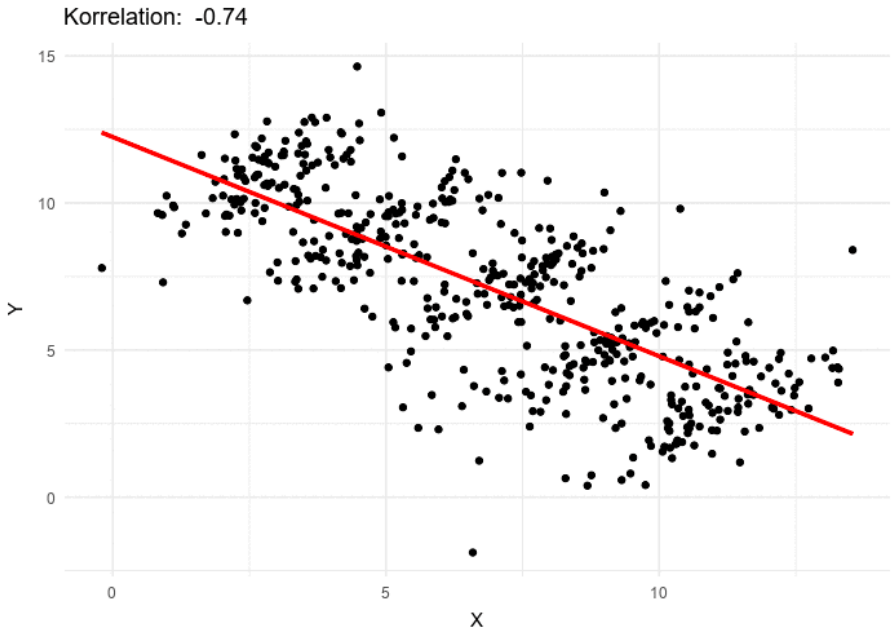
5.2.1 Simpson's paradox

To illustrate this concept, let's revisit the phenomenon known as Simpson's paradox. Simpson's paradox occurs when the overall association between two categorical variables differs from the association observed when we consider the influence of one or more other variables, known as controlling variables. This paradox highlights three key points:

1. It challenges the assumption that statistical relationships are fixed and unchanging, showing that the relationship between two variables can vary depending on the set of variables being controlled.
2. Simpson's paradox is part of a larger class of association paradoxes, indicating that similar situations can arise in various contexts.
3. It serves as a reminder of the potential pitfalls of making causal inferences in nonexperimental studies, emphasizing the importance of considering confounding variables.

Thus, it is important to consider confounding variables to ensure valid and reliable causal interpretations in research, particularly in nonexperimental settings.

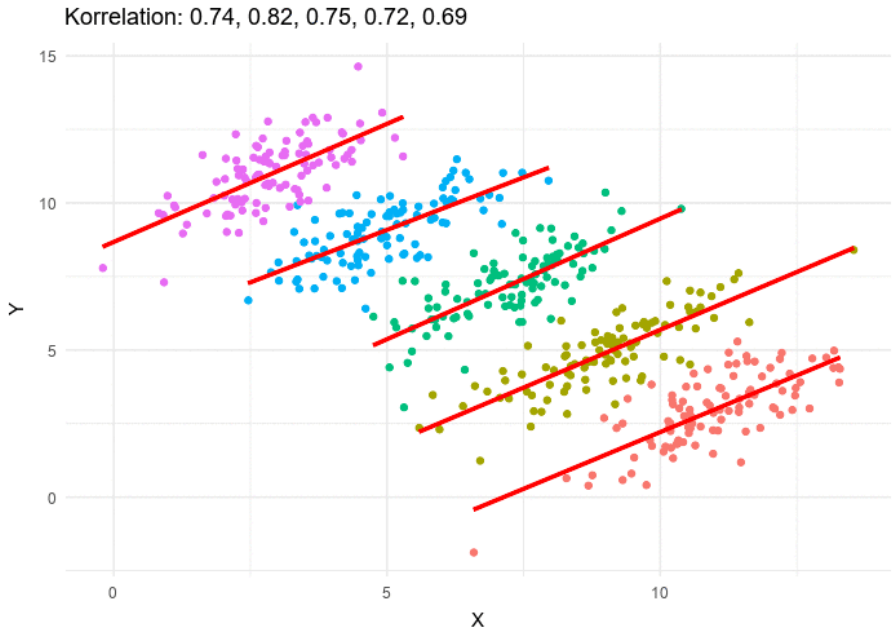
Figure 5.3: Simpsons paradox and the power of controlling variables (1)



The multiple regression model can be expressed as:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i, \ i = 1, \ldots, n.$$

Figure 5.4: Simpsons paradox and the power of controlling variables (2)



To estimate the coefficients of the multiple regression model, we seek to minimize the sum of squared mistakes by choosing estimated coefficients $\beta_0, \beta_1, \dots, \beta_k$ such that:

$$\sum_{i=1}^{n}(Y_i - b_0 - b_1 X_{1i} - b_2 X_{2i} - \cdots - b_k X_{ki})^2$$

This demands matrix notation which goes beyond the scope of this introduction.

5.2.2 Gauss-Markov and the best linear unbiased estimator

The Gauss-Markov assumptions, also known as the classical linear regression assumptions, are a set of assumptions that underlie the ordinary least squares (OLS) method for estimating the parameters in a linear regression model. These assumptions ensure that the OLS estimators are unbiased, efficient, and have desirable statistical properties.

The Gauss-Markov assumptions are as follows:

1. **Linearity**: The relationship between the dependent variable and the independent variables is linear in the population model. This means that the true relationship between the variables can be represented by a linear equation.
2. **Independence**: The errors (residuals) in the regression model are independent of each other. This assumption ensures that the errors for one observation do not depend on or influence the errors for other observations.
3. **Strict exogeneity**: The errors have a mean of zero conditional on all the independent variables. In other words, the expected value of the errors is not systematically related to any of the independent variables.

4. **No perfect multicollinearity**: The independent variables are not perfectly correlated with each other. Perfect multicollinearity occurs when one independent variable is a perfect linear combination of other independent variables, leading to problems in estimating the regression coefficients.
5. **Homoscedasticity**: The errors have constant variance (homoscedasticity) across all levels of the independent variables. This assumption implies that the spread or dispersion of the errors is the same for all values of the independent variables.
6. **No endogeneity**: The errors are not correlated with any of the independent variables. Endogeneity occurs when there is a correlation between the errors and one or more of the independent variables, leading to biased and inefficient estimators.
7. **No autocorrelation**: The errors are not correlated with each other, meaning that there is no systematic pattern or relationship between the errors for different observations.

These assumptions collectively ensure that the OLS estimators are unbiased, efficient, and have minimum variance among all linear unbiased estimators. Violations of these assumptions can lead to biased and inefficient estimators, invalid hypothesis tests, and unreliable predictions. Therefore, it is important to check these assumptions when using the OLS method and consider alternative estimation techniques if the assumptions are violated.

5.2.3 Confounding and control variables

> **Tip 7: Statistical control requires causal justification**
>
> Read Section 3.5 (again).

A confounding variable is a factor that was not accounted for or controlled in a study but has the potential to influence the results. In other words, the true effects of the treatment or intervention can be obscured or muddled by the presence of this variable.

For instance, let's consider a scenario where two groups of individuals are observed: one group took vitamin C daily, while the other group did not. Over the course of a year, the number of colds experienced by each group is recorded. It might be observed that the group taking vitamin C had fewer colds compared to the group that did not. However, it would be incorrect to conclude that vitamin C directly reduces the occurrence of colds. Since this study is observational and not a true experiment, numerous confounding variables are at play. One potential confounding variable could be the individuals' level of health consciousness. Those who take vitamins regularly might also engage in other health-conscious behaviors, such as frequent handwashing, which could independently contribute to a lower risk of catching colds.

To address confounding variables, researchers employ control measures. The idea is to create conditions where confounding variables are minimized or eliminated. In the aforementioned example, researchers could pair individuals who have similar levels of health consciousness and randomly assign one person from each pair to take vitamin C daily (while the other person receives a placebo). Any differences observed in the number of colds between the groups would be more likely attributable to the vitamin C, compared to the original observational study. Well-designed experiments are crucial as they actively control for potential confounding variables.

Consider another scenario where a researcher claims that eating seaweed prolongs life. However, upon reading interviews with the study subjects, it becomes apparent that they were all over 100 years old, followed a very healthy diet, slept an average of 8 hours per day, drank ample water, and engaged in regular exercise. In this case, it is not possible to determine whether longevity

was specifically caused by seaweed consumption due to the presence of numerous confounding variables. The healthy diet, sufficient sleep, hydration, and exercise could all independently contribute to longer life. These variables act as confounding factors.

A common error in research studies is to fail to control for confounding variables, leaving the results open to scrutiny. The best way to head off confounding variables is to do a well-designed experiment in a controlled setting. Observational studies are great for surveys and polls, but not for showing cause-and-effect relationships, because they don't control for confounding variables.

Control variables are usually variables that you are not particularly interested in, but that are related to the dependent variable. You want to remove their effects from the equation. A control variable enters a regression in the same way as an independent variable – the method is the same.

Tip 7

Nick Huntington-Klein offers Causal Inference Animated Plots on his homepage. Read this page and consider the animated graphs.

5.2.4 Omitted variable bias and ceteris paribus

From the Gauss-Markov theorem we know that if the OLS assumptions are fulfilled, the OLS estimator is (in the sense of smallest variance) the best linear conditionally unbiased estimator (BLUE). However, OLS estimates can suffer from omitted variable bias when any regressor, X, is correlated with any omitted variable that matters for variable Y.

For omitted variable bias to occur, two conditions must be fulfilled:

1. X is correlated with the omitted variable.
2. The omitted variable is a determinant of the dependent variable Y.

In regression analysis, "ceteris paribus" is a Latin phrase that translates to "all other things being equal" or "holding everything else constant." It is a concept used to examine the relationship between two variables while assuming that all other factors or variables remain unchanged.

When we say *ceteris paribus* in the context of regression analysis, we are isolating the effect of a specific independent variable on the dependent variable while assuming that the values of the other independent variables remain constant. By holding other variables constant, we can focus on understanding the direct relationship between the variables of interest.

For example, consider a regression analysis that examines the relationship between income (dependent variable) and education level (independent variable) while controlling for age, gender, and work experience. By stating *ceteris paribus*, we are assuming that age, gender, and work experience remain constant, and we are solely interested in understanding the impact of education level on income.

Exercise 5.1. Look at the Output

In Figure 5.5 you see an excerpt of a regression output taken from a statistical program named *Stata*. Some t-values and p-values are missing.

Figure 5.5: Regression output

price	Coef.	Std. Err.	t	P>|t|
weight	3.464706	.630749	5.49	0.000
mpg	21.8536	74.22114		
foreign	3673.06	683.9783	5.37	
_cons	-5853.696	3376.987		0.087

a) Calculate the t-value of the coefficient `mpg`. Is the coefficient at a level of $\alpha = 0.05$ statistically significant?

b) Is the coefficient foreign at a level of $\alpha = 0.05$ statistically significant?

c) Is the constant at a level of $\alpha = 0.05$ statistically significant?

Exercise 5.2. Look at Stata Output

In Figure 5.6 you find two regression outputs from Stata. Try to interpret the p-values and the confidence intervals. How are the t-values calculated. Can you use the *magic number* 1.96 to say if a corresponding estimated coefficient is statistically significant, or not? Which estimated model is *better*?

Exercise 5.3. Explain the weight

In the following exercise you need to use the programming language R.

a) Write down your name, your matriculation number, and the date.

b) Set your working directory.

c) Clear your global environment.

d) Load the following package: `tidyverse`

```
library("tidyverse")
```

The following table stems from a survey carried out at the Campus of the German Sport University of Cologne at Opening Day (first day of the new semester) between 8:00am and 8:20am. The survey consists of 6 individuals with the following information:

Table 5.1: Dataset collected in Cologne

id	sex	age	weight	calories	sport
1	f	21	48	1700	60
2	f	19	55	1800	120
3	f	23	50	2300	180
4	m	18	71	2000	60
5	m	20	77	2800	240
6	m	61	85	2500	30

Figure 5.6: Stata regression output

(a) Output A

```
. reg weight height sex_n
```

Source	SS	df	MS	Number of obs	=	23
				F(2, 20)	=	7.02
Model	1122.42818	2	561.214089	Prob > F	=	0.0049
Residual	1599.05008	20	79.9525041	R-squared	=	0.4124
				Adj R-squared	=	0.3537
Total	2721.47826	22	123.703557	Root MSE	=	8.9416

weight	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
height	.5923091	.2671132	2.22	0.038	.0351207	1.149498
sex_n	-5.78938	4.477272	-1.29	0.211	-15.12881	3.550045
_cons	-23.74037	50.40112	-0.47	0.643	-128.8753	81.39453

(a) Output B

```
. reg weight height sex_n I_sex_height
```

Source	SS	df	MS	Number of obs	=	23
				F(3, 19)	=	6.02
Model	1326.55663	3	442.185544	Prob > F	=	0.0046
Residual	1394.92163	19	73.4169278	R-squared	=	0.4874
				Adj R-squared	=	0.4065
Total	2721.47826	22	123.703557	Root MSE	=	8.5684

weight	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
height	-.6916747	.8114546	-0.85	0.405	-2.390069	1.006719
sex_n	-153.9612	88.96469	-1.73	0.100	-340.1665	32.244
I_sex_height	.8536423	.5119438	1.67	0.112	-.2178683	1.925153
_cons	201.104	143.2315	1.40	0.176	-98.6829	500.8909

Data Description:

- **id:** Variable with an anonymized identifier for each participant.
- **sex:** Gender, i.e., the participants replied to be either male (m) or female (f).
- **age:** The age in years of the participants at the time of the survey.
- **weight:** Number of kg the participants pretended to weight.
- **calories:** Estimate of the participants on their average daily consumption of calories.
- **sport:** Estimate of the participants on their average daily time that they spend on doing sports (measured in minutes).

Which type of data do we have here? (Panel data, repeated cross-sectional data, cross-sectional data, time Series data)

Store each of the five variables in a vector and put all five variables into a dataframe with the title `df`. If you fail here, read in the data using this line of code:

```
df <- read_csv("https://raw.githubusercontent.com/hubchev/courses/main/dta/df-calories.csv")
```

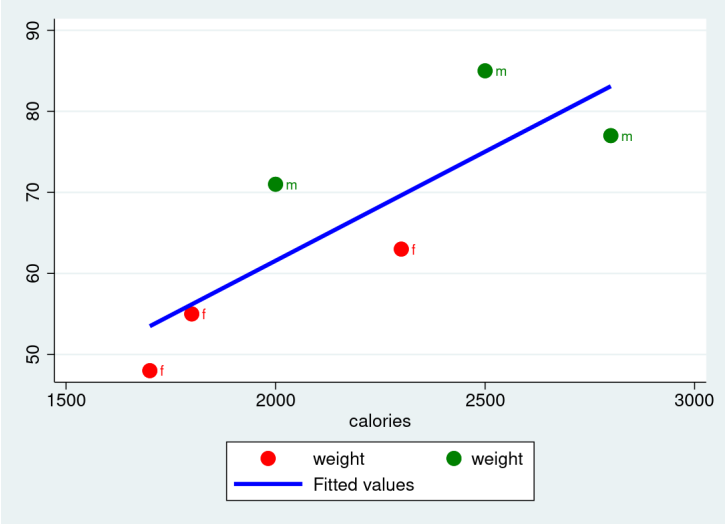
g) Show for all numerical variables the summary statistics including the mean, median, minimum, and the maximum.

h) Show for all numerical variables the summary statistics including the mean and the standard deviation, **separated by male and female**. Use therefore the pipe operator.

i) Suppose you want to analyze the general impact of average calories consumption per day on the weight. Discuss if the sample design is appropriate to draw conclusions on the population. What may cause some bias in the data? Discuss possibilities to improve the sampling and the survey, respectively.

j) The following plot visualizes the two variables weight and calories. Discuss what can be improved in the graphical visualization.

Figure 5.9: Weight vs. Calories



k) Create a scatterplot matrix to visualize relationships between all numerical variables in the dataset.

l) Calculate the Pearson Correlation Coefficient for the following pairs of variables:

- `calories` and `sport`
- `weight` and `calories`

This will help in understanding the strength and direction of the linear relationship between these variables.

m) Generate a scatterplot with `weight` on the y-axis and `calories` on the x-axis. Include a linear fit to the data and label the points with the `sex` variable. This visualization can provide insights into the relationship between calorie consumption and weight, differentiated by gender.

n) Estimate the following regression specification using the Ordinary Least Squares (OLS) method:

$$weight_i = \beta_0 + \beta_1 calories_i + \epsilon_i$$

```
# OLS Regression
reg_base <- lm(weight ~ calories, data = df)
summary(reg_base)
```

o) Interpret the results. In particular, interpret how many kg the estimated weight increases—on average and ceteris paribus—if calories increase by 100 calories. Additionally, discuss the statistical properties of the estimated coefficient $\hat{\beta}_1$ and the meaning of the *Adjusted R-squared*.

p) OLS estimates can suffer from omitted variable bias. State the two conditions that need to be fulfilled for omitted bias to occur.

q) Discuss potential confounding variables that may cause omitted variable bias. Given the dataset above how can some of the confounding variables be *controlled for*?

Solutions are provided here.

Exercise 5.4. Causal inference and animated plots

Nick Huntington-Klein [2022] has created wonderful animated graphs that give great and quick insights into how causal inference works. Please read his online chapter on *Causal Inference Animated Plots* and discuss

a) what he means when he speaks of *closing the back-door path* and *controlling for*,
b) what methods exist to *close the back-door path*, and
c) why it is sometimes necessary to omit variables from an estimated regression model.

6 Hands on experiments

6.1 Natural experiments

In social science, a *natural experiment* is a research design that exploits naturally occurring circumstances or events to study the effects of an intervention or treatment. In these experiments, the treatment is not manipulated by the researcher, but is instead determined by exogenous, or external, factors. *Exogenous variations* refer to changes in the independent variable that are not caused by the researcher's actions but instead occur naturally or through some external factor. These variations are often unpredictable and occur without the intervention of the researcher, making them an ideal source of variation to study the causal effects of a treatment or intervention. One example of a natural experiment is the partition of Germany after World War II, which created two economies that were initially similar but experienced vastly different economic and institutional environments. Another example is the introduction of a new policy or technology in one state or country but not in another, allowing for a comparison of outcomes before and after the treatment. A natural experiment might involve comparing the outcomes of two groups of people who were exposed to different levels of air pollution due to a policy change or a natural disaster. In this case, the variation in air pollution levels is exogenous, since it is not controlled by the researcher but rather determined by external factors.

By leveraging these exogenous variations, researchers can better estimate the causal effects of an intervention or treatment, and provide evidence for policy-makers to make more informed decisions. In the following, we will get known to some studies that are based on natural experiments.

Exercise 6.1. Natural experiments in research

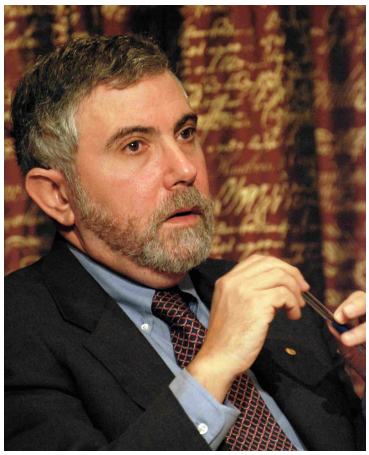
a) Think of other natural experiments that can be scientifically exploited.
b) Download Sieweke and Santoni [2020], see here.
c) Read Sieweke and Santoni [2020, section 3.1] and study Sieweke and Santoni [2020, table V.].

6.2 Empirical evidence: Bombing

In their article "Bones, Bombs, and Break Points: The Geography of Economic Activity," Davis and Weinstein [2002] explore another natural experiment that has shaped the economic geography of the world: the natural endowment of different regions with physical and institutional factors that affect their productivity and attractiveness to economic activity. Using a spatial econometric model, they test the relative importance of three such factors: climate, natural resources, and political borders. They find that political borders, such as the ones that emerged from colonialism or ethnic conflict, have the strongest effect on economic activity, even after controlling for other factors. This has important implications for policy, as it suggests that changing the institutional environment of a region can have a significant impact on its economic performance.

Overall, Davis and Weinstein's article demonstrates the power of natural experiments to shed light on important economic questions and inform policy debates. By examining the historical and geographical factors that have shaped economic activity around the world, they offer valuable insights into the mechanisms that drive economic growth and inequality.

Figure 6.1: Paul Krugman *1953[1]



Before you read this article, let me explain that the theory which this article elaborates and tests goes back to the 2008 nobel-prize winner Paul Krugman (*1953) who founded the so-called *New Economic Geography* (NEG). Here is an excerpt of how the Royal Swedish Academy of Sciences summarizes Krugman's contribution to the field (RSA2008Prize, 3):

> Economic geography deals not only with what goods are produced where, but also with the distribution of capital and labor over countries and regions. The approach Krugman used in his foreign trade theory – the assumption of economies of scale in production and a preference for diversity in consumption – was also found to be appropriate for analyzing geographical issues. This allowed Krugman to integrate two disparate fields in a cohesive model.
>
> The embryo of the theory which would come to be called the "new economic geography'' had already appeared in Krugman's 1979 article. In the final pages, he asks what would happen if foreign trade became impossible, for instance due to excessively high transport costs or other obstacles. His line of reasoning is as follows. If two countries are exactly alike, then welfare will be the same in both countries. But if the countries are alike in all respects except that one of them has a slightly larger population than the other, then the real wages of labor will be somewhat higher in the country with more inhabitants. The reason is that firms in the more highly populated country can make better use of economies of scale, which implies lower prices to consumers and/or greater diversity in the supply of goods. This, in turn, enhances the welfare of consumers. As a result, labor, i.e., consumers, will tend to move to the country with more inhabitants, thereby increasing its population. Real wages and the supply of goods will then continue to increase even more in that country, thereby giving rise to further migration, and so on.
>
> Twelve years would pass, however, before Krugman reconsidered these ideas. In an article published in 1991, he developed these concepts into a comprehensive theory of location of labor and firms. Here, he assumes that although trade is possible, it

[1] Source: https://commons.wikimedia.org/wiki/File:Paul_Krugman-press_conference_Dec_07th,_2008-8.jpg.

is obstructed due to transport costs. Otherwise, labor is free to move to the country or region which can offer the highest welfare, in terms of real wages and diversity of goods. Firms' location decisions imply a trade-off between utilizing economies of scale and saving on transport costs.

Concentration or Decentralization?

The above considerations evolved into the so-called core-periphery model, which shows that the relation between economies of scale and transport costs can result in either concentration or decentralization of communities. Under certain conditions, the forces which contribute to concentration will dominate. Regional imbalances arise and most of the population will be concentrated in a high-technology core, whereas a small minority will inhabit the periphery and live off agriculture. Such a mechanism could underlie the explosive urbanization witnessed today throughout the world, with rapidly growing megacities surrounded by increasingly depopulated rural areas. This is not necessarily the only possibility, however. Under different conditions, the forces which give rise to decentralization will dominate. This promotes somewhat more balanced development. Krugman's model can be used to account for the mechanisms at work in both directions. For example, his model indicates that declining transport costs easily generate concentration and urbanization – which seems particularly noteworthy since transport costs have exhibited a declining trend throughout the twentieth century.

Numerous research papers inspired by Krugman's New Economic Geography (NEG) focus on the origins and implications of the so-called first and second nature effects. These effects are used to explain the uneven distribution of economic activity both across countries and within regions of a country. Two primary explanations have been investigated: (1) *Fundamentals*, which refer to differences in the fundamental productivity of locations, and (2) *Agglomeration forces*, which are related to the proximity of economic agents that boost productivity and make a location more attractive.

These two mechanisms are obviously not exclusive and can both operate simultaneously. The key empirical question is to what extent observed patterns of economic activity are explained by these two mechanisms. Understanding whether fundamentals or agglomeration forces are responsible for the pattern of economic activity has significant implications for the persistence of spatial equilibria and policy making.

For instance, suppose only agglomeration forces are at play. In that case, the location of economic activity is relatively arbitrary, and a particular location is attractive mainly because other workers are choosing to locate there. This phenomenon is similar to selecting a nightclub: club A is crowded and everybody wants to be there only because it was the club had attracted the first person that night.

In contrast, if only fundamentals are in operation, the distribution of activity is determined by the distribution of these underlying factors. When agglomeration forces dominate fundamentals, the spatial distribution of activity becomes a matter of political interest. For example, regional policies can aim to move the distribution of economic activity between different equilibria. Using a temporary subsidy, regions can try to attract a 'critical mass' of economic activity. Once established, this critical mass will make the location more attractive, even when the subsidy has ended.

This approach is similar to nightclub policies, such as offering free entry or other discounts to the first people who are searching for a club. These incentives are an attempt to attract a

critical mass of party-goers, making the nightclub more attractive and increasing the likelihood that other party-goers will choose the same club later.

Exercise 6.2. The impact of nuclear bombs on agglomerations

Background: The bombing of Japan during the Second World War devastated 66 cities that were targeted, leading to the destruction of approximately half of their housing stock and the deaths of around 300,000 Japanese. Although Hiroshima and Nagasaki are more well-known due to the nuclear bombs that were dropped on them, the majority of cities in the sample suffered little to no destruction, including several large cities. Despite the scale of the bombing, it was clearly a temporary shock that did not change the fundamental attractiveness of locations. However, the nuclear radiation in Hiroshima and Nagasaki could be considered an exception.

Data: The data set used in the paper consists of information on 303 Japanese cities with a population exceeding 30,000 in 1925. Population figures are recorded every five years, with the exception of the 1945 census, which took place in 1947. One way to gauge the intensity of the shocks experienced by the cities is by looking at the number of dead or missing residents.

a) Read Davis and Weinstein [2002] which can be downloaded here and explain the implications of Figure 6.2:

The results of Davis and Weinstein (2002) show a striking persistence of city size even after terrible wartime destruction. This has important implications for attempts to use regional policy to shift economic activity between spatial equilibria. The following quote sums it up well: > "An important practical question, then, is whether such spatial catastrophes are theoretical curiosa or a central tendency in the data. Our results provide an unambiguous answer: Even nuclear bombs have little effect on relative city sizes over the course of a couple of decades. The theoretical possibility of spatial catastrophes due to temporary shocks is not a central tendency borne out in the data." Davis and Weinstein [2002, p. 1284]

The use of bombing as a temporary shock, however, can be seen as problematic: Although bombing causes casualties and destroys housing and social structures, legal property rights and operating licenses are not destroyed. Therefore, rebuilding cities may prove less burdensome than building new settlements from scratch. Moreover, even after a nuclear explosion, functioning infrastructure may still be in place, which could make rebuilding at the old site less costly.

b) Read Bleakley and Lin [2012] and discuss how this is confronting the insights from Davis and Weinstein [2002]. It is freely available here

6.3 Field experiments: Would you work more if wages are high?

Unlike laboratory experiments, which are conducted in a controlled environment, field experiments are conducted in real-world settings, such as schools, workplaces, and communities. They allow for testing causality by controlling the independent variable while observing the dependent variable in a naturalistic setting. They are a valuable tool for testing the effectiveness of policies and interventions in real-world situations. The level of external validity is usually much higher than alternative methods, meaning that the results can be generalized to other similar contexts beyond the specific setting where the experiment was conducted. In addition, field experiments can help identify unintended consequences of policies or interventions that might not be observable in laboratory experiments or observational studies.

Figure 6.2: Two figures from Davis and Weinstein [2002]

(a) Figure 1 of Davis and Weinstein [2002]

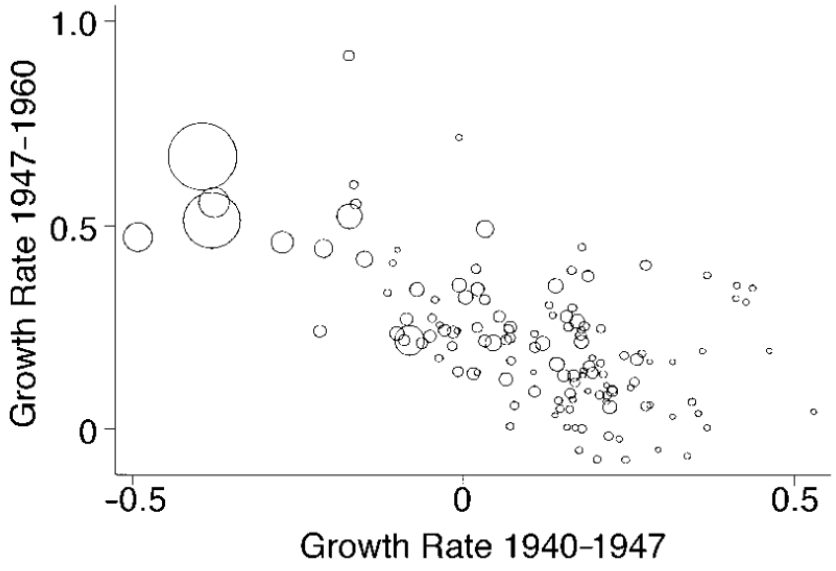


FIGURE 1. EFFECTS OF BOMBING ON CITIES WITH MORE THAN 30,000 INHABITANTS

(a) Figure 2 of Davis and Weinstein [2002]

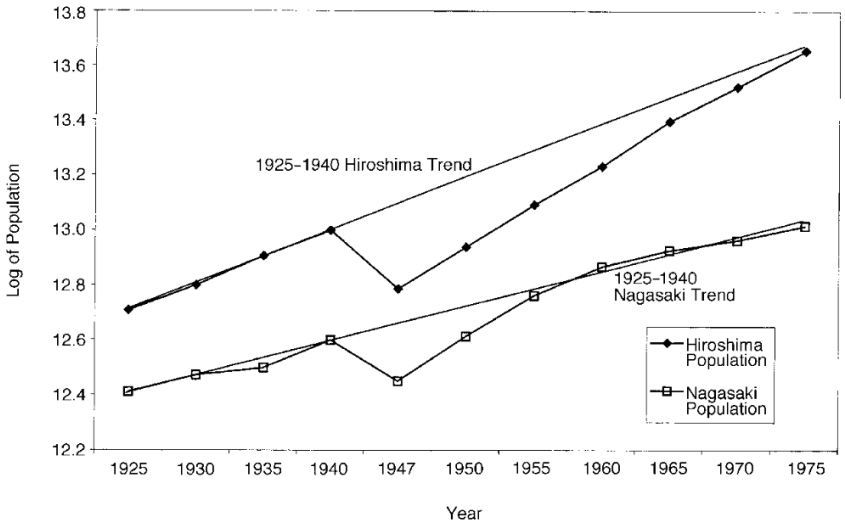


FIGURE 2. POPULATION GROWTH

Another advantage of field experiments is that they can be used to test theories in contexts where observational studies may be limited. For example, a theory may predict that a certain policy or intervention will have a specific effect, but it may be difficult to test this theory through observational studies due to confounding variables or selection bias.

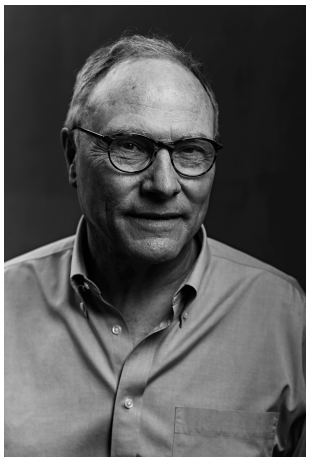
However, field experiments can be costly and time-consuming to conduct. Please give Harrison and List [2004] article a quick read, as it explains the nature and advantages of field experiments well.

Exercise 6.3. Field experiments in research

a) Read Fehr and Goette [2007]. Summarize the results.
b) Explain the identification strategy and the experimental design.
c) Read Bandiera et al. [2011]. Can you think of field studies that organizations could conduct to improve their business?

7 Hands on observational data: Difference in difference

Figure 7.1: David Card (*1956)[1]



David Card is one of the most influential labor economist of the 20th century and Nobel laureate of 2021. He is well-known for his research on the effects of the minimum wage on employment, which challenged the traditional view that increasing the minimum wage leads to a decrease in employment. In his article *Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania* [Card and Krueger, 1994] he and Alan Krueger (1960-2019) used a *natural experiment* to examine the effect of an increase in the minimum wage on employment. In particular, they identified a treatment group (restaurants in New Jersey) and a control group (restaurants in eastern Pennsylvania) to measure the effect of increasing the minimum wage that was increased in New Jersey but not in Pennsylvania. This increase did not lead to a decrease in employment, which contradicted the widely held view that increasing the minimum wage would lead to job loss. The empirical method that they used is called *difference in difference* and we discuss it in the following section.

The difference in difference (DiD) method allows to estimate the causal effect of a treatment or intervention. In particular, it is popular to study the impact of policy changes and other interventions on a specific outcome of interest.

The basic idea behind the DiD method is to compare the change in an outcome variable between a treatment group and a control group over time. The treatment group is the group that is exposed to the intervention or treatment, while the control group is a group that is not exposed to the intervention. The difference in the change in the outcome variable between the two groups is then used to estimate the causal effect of the intervention.

To use the DiD method, researchers typically collect data on the outcome variable of interest for both the treatment and control groups before and after the intervention. This data is then used to calculate the difference in the change in the outcome variable between the two groups.

[1] Source: https://davidcard.berkeley.edu/.

For example, if a study aims to examine the effect of a new policy on the employment rate, it should collect data on the employment rate for a group of individuals living in a region where the policy was implemented, and for a group living in a similar region where the policy was not implemented. The study can then compare the change in the employment rate for the two groups, before and after the implementation of the policy. The difference in the change in the employment rate between the two groups can be used to estimate of the causal effect of the policy on employment.

It is important to note that DiD assumes that there are no other factors that could be affecting the outcome variable of interest and that the treatment and control groups are similar in all ways except for the intervention. To control for these assumptions researchers can use statistical techniques such as matching to ensure that treatment and control groups are similar before the intervention.

DiD is useful when we only have observational data and in situations where it is not possible or ethical to randomly assign individuals to a treatment or control group, for example, in the case of policy changes.

Tip 8: Differences-in-Differences and Rubin causal model

Figure 7.2: Differences-in-Differences

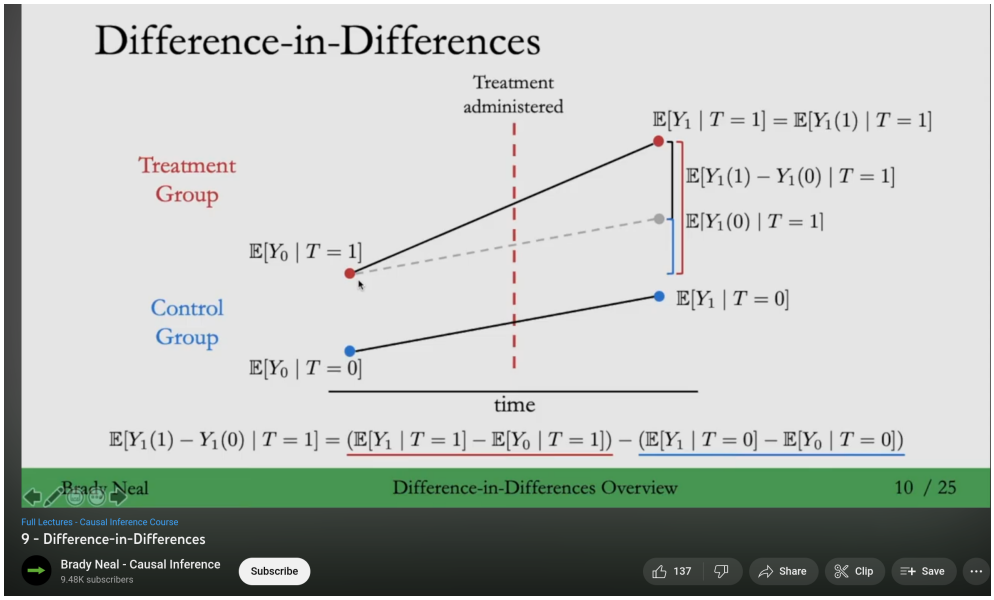
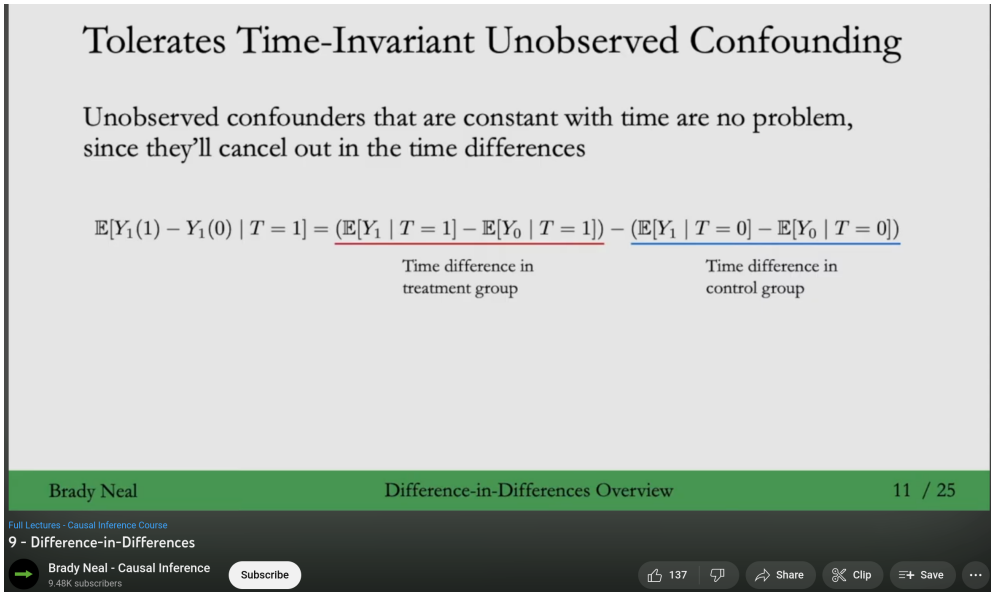


Figure 7.3: Tolerate time-invariant unobserved confounding



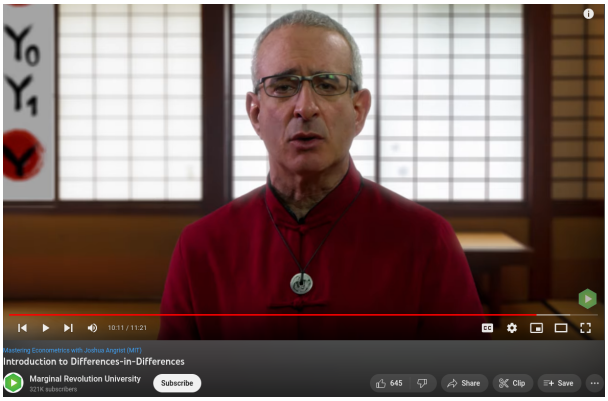
Figures 7.2 and 7.3 stem from a video of Brady Neal's lecture on *Difference-in-Differences*. Please watch this video.

Exercise 7.1. Mastering DiD with Joshua Angrist

Watch the video *Introduction to Differences-in-Differences* and answer the following questions. The video is part of a course called *Mastering Econometrics with Joshua Angrist (MIT)* produced

[2]The picture stems from the video https://youtu.be/eiffOVbYvNc

Figure 7.4: Josh Angrist (*1960): Nobel Prize winner of economics in 2021[2]



by Marginal Revolution University. In it, Josh Angrist introduces differences within differences using one of the worst economic events in history: the Great Depression.

1. In the video, the treatment being examined is:

    a) Bank failure.
    b) "Easy" money.
    c) "Tight" money.
    d) Differences-in-Differences.
    e) None of the above.

2. If the treatment were effective, which outcome would we expect to observe?

    a) Fewer bank failures.
    b) Increased bank failures.
    c) Continued parallel trends.
    d) No differences in any variables unrelated to bank failure.
    e) None of the above.

3. Practically, how is DD (Differences-in-Differences) typically implemented?

    a) Non-parametric statistical techniques.
    b) Randomized trials.
    c) Regression analysis.
    d) Instrumental variables.
    e) None of the above.

See Solution 7.1

Solutions to the exercises

Solution 7.1. Solution to exercise Exercise 7.1

Answers: 1. b), 2. a), 3. c)

8 Publish

A clear and sound empirical methodology forms the foundations of any empirical research work. However, more than a rigorous methodology is needed to ensure the wide dissemination of a study. Two aspects are crucial here: accessibility and dissemination.

The first aspect, the public accessibility of the paper, is relatively easy to achieve. Nowadays, anyone can upload their work online to ensure its availability. The second aspect, dissemination, is more complex. There are various strategies to increase the popularity of a paper, but the traditional and arguably most effective method is publication in a high-impact journal with a large readership. The successful placement of a paper in such a journal is almost an art form. The paper must fulfill several criteria as well as possible: a sound methodology, a clear contribution, thematic relevance and a compelling text that appeals to the journal's readership are just some of the key benchmarks.

How can you learn to publish a paper successfully? It is crucial to study papers that have gained recognition and make an effort to recognize the reasons for their success. This exercise not only sharpens your writing skills, but also improves your perceptiveness as a researcher, allowing you to select important research topics and complete projects more efficiently.

Prestigious journals such as the American Economic Review and the Journal of Economic Literature, published by the American Economic Association (AEA), set the gold standard in academic publishing. This section looks at the AEA's publishing standards. The path to publication with the AEA, particularly for empirical research, has changed considerably and is now governed by strict criteria. It is imperative that authors familiarize themselves with the key considerations and guidelines for submitting empirical research to an AEA journal. Although standards may vary across disciplines, the AEA's exacting standards have influenced a wide range of social science journals. Engagement with these standards is invaluable as it provides guidance in designing and planning research that meets these rigorous requirements.

While each AEA journal has its specific focus and requirements, all aim for integrity, clarity, and replicability of empirical research. Authors should start by carefully reviewing the author guidelines for their targeted journal, paying close attention to any specific mandates regarding empirical work. For Randomized Controlled Trials (RCTs), for example, a registration is required for all applicable submissions prior to submitting, see RCT Registry Policy.

Tip 9: The AEA's registry for randomized controlled trials

Visit www.socialscienceregistry.org and inform yourself about some registered RCTs.

The AEA has placed a significant emphasis on the transparency and replicability of empirical research. Authors are required to ensure that their data and methodologies are openly available and clearly described, allowing other researchers to replicate their results. This commitment to transparency extends to the publication of data sets, code, and detailed methodological appendices, which must accompany the submitted manuscript.

Authors should:

- **Justify their choice of methodology**: The paper should clearly explain why the chosen method is appropriate for the research question at hand.
- **Detail the analytical process**: From data collection to analysis, each step should be meticulously detailed, allowing for the study's replication.
- **Address potential limitations**: No empirical study is without its limitations. Authors should openly discuss these, including any biases, measurement errors, or external factors that may impact the results.

While statistical significance is a key metric for empirical analysis, the AEA also places a strong emphasis on the economic relevance and implications of the findings. Authors should not only present statistically significant results but also explain their economic significance.

The American Economic Association (AEA) employs a rigorous double-blind peer review process that ensures that submissions are peer-reviewed without revealing the identity of the authors or reviewers. This procedure serves to ensure the integrity and impartiality of the evaluation. Authors should be prepared to receive feedback and constructive criticism, which often requires a revision of the manuscript. A willingness to engage constructively with this feedback is crucial to refining the work and improving its chances of publication.

Tip 10: AEA Policies

Read the *AEA Data and Code Policies and Guidance*.
Read the *Submission Guidelines of the American Economic Review*.

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