Homework Assignment 1

Due Date: January 23, 2025 (Thursday), 1 p.m.

Recall that you have **6 free grace days** to be used on homeworks for the whole semester. If you're using one or more, please indicate how many you are using, and how many you have remaining.

Instruction: If you are asked to formulate a mathematical model, **clearly state the definition of decisions variables**, e.g., *x = verbal description*. Also, make sure that you write the final model in the following form.

$$(\text{max or min}) \ c_1x_1 + c_2x_2 + \dots$$
$$\text{s.t. } a_{11}x_1 + a_{12}x_2 + \dots \ (\leq = \geq) \ b_1$$
$$\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots \ (\leq = \geq) \ b_m$$
$$x_i \ (\leq \geq) \ 0 \text{ or } x_i \text{ unrestrained in sign}, \ \forall i = 1, 2, \dots$$

Question 1 (10 Points)

The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1, and each unit of product 2, up to 60 units, gives a profit of $2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out. Formulate a linear programming model for this problem.

Question 2 (10 Points)

The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is $5 per unit on special risk insurance and $2 per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work-Hours per Unit		Work-Hours Available
	Special Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

Formulate a linear programming model for this problem.

Question 3 (10 Points)

Weenies and Buns is a food processing plant which manufactures hot dogs and buns. They grind their own flour for the buns at a maximum rate of 200 pounds per week. Each bun requires 0.1 pound of flour. They currently have a contract with Pigland, Inc., which specifies that a delivery of 800 pounds of pork product is delivered every Monday. Each hot dog requires 1/4 pound of pork product. All the other ingredients in the hot dogs and buns are in plentiful supply. Finally, the labor force at Weenies and Buns consists of 5 employees working full time (40 hours per week each). Each hot dog requires 3 minutes of labor, and each bun requires 2 minutes of labor. Each hot dog yields a profit of $0.88, and each bun yields a profit of $0.33. Weenies and Buns would like to know how many hot dogs and how many buns they should produce each week so as to achieve the highest possible profit. Formulate a linear programming model for this problem.

Question 4 (10 Points)

Bob and Helen run a daycare for preschoolers. They are trying to decide what to feed the children for lunch. They would like to keep their costs down, but also need to meet the nutritional requirements of the children. They have already decided to go with chicken salad, which contains chicken breast, tomatoes, dried cherries, walnuts and avocado. The nutritional content of each ingredient and its cost are given in the table below.

Food Item	Total Fat (g)	Total Carbohydrate (mg)	Protein (g)	Vitamin C (mg)	Calories	Cost ($)
Chicken Breast (1 cup)	5	119	43	0	231	2.5
Tomatoes (1 cup)	0.4	7.8	1.8	27.4	36	0.23
Dried Cherries (1 cup)	0.5	19	1.6	20	77	1.5
Walnuts (1 cup)	52	11	12	1	523	1.6
Avocado (1 cup)	22	13	3	15	240	2.1

The nutritional requirements are as follows. Each child should receive between 350 and 700 calories. The lunch should contain between 30 to 40 grams of fat, and between 100 to 200 milligrams of total carbohydrate. Each child should consume at least 50 milligrams (mg) of vitamin C and 16 grams (g) of protein. Furthermore, for practical reasons (to call it chicken salad), each child will have at least one cup of chicken breast. Bob and Helen would like to select the food choices for the children which minimize cost while meeting the above requirements. Formulate a linear programming model for this problem.

Question 5 (10 Points)

Larry Edison is the director of the Computer Center for Buckly College. He now needs to schedule the staffing of the center. It is open from 8 A.M. until midnight. Larry has monitored the usage of the center at various times of the day, and determined that the following number of computer consultants are required:

Time of Day	Minimum Number of Consultants Required to Be on Duty
8 A.M.–noon	4
Noon–4 P.M.	8
4 P.M.–8 P.M.	10
8 P.M.–midnight	6

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for 8 consecutive hours in any of the following shifts: morning (8 A.M.–4 P.M.), afternoon (noon–8 P.M.), and evening (4 P.M.–midnight). Full-time consultants are paid $40 per hour. Part-time consultants can be hired to work any of the four shifts listed in the above table. Part-time consultants are paid $30 per hour. An additional requirement is that during every time period, there

must be at least 2 full-time consultants on duty for every part-time consultant on duty. Larry would like to determine how many full-time and how many part-time workers should work each shift to meet the above requirements at the minimum possible cost. Formulate a linear programming model for this problem.