

Homework Assignment 2

Due Date: January 30, 2025 (Thursday), 1 p.m.

Recall that you have **6 free grace days** to be used on homeworks for the whole semester. If you're using one or more, please indicate how many you are using, and how many you have remaining.



Instruction: If you are asked to formulate a mathematical model, **clearly state the definition of decisions variables**, e.g., $x = \text{verbal description}$. Also, make sure that you write the final model in the following form.

$$\begin{aligned}
 &(\text{max or min}) \ c_1x_1 + c_2x_2 + \dots \\
 &\text{s.t. } a_{11}x_1 + a_{12}x_2 + \dots (\leq = \geq) b_1 \\
 &\quad \dots \\
 &\quad a_{m1}x_1 + a_{m2}x_2 + \dots (\leq = \geq) b_m \\
 &\quad x_i (\leq = \geq) 0 \text{ or } x_i \text{ unrestrained in sign, } \forall i = 1, 2, \dots
 \end{aligned}$$

Question 1 (10 Points)

A post office requires different numbers of full-time employees on different days of the week as given in the following table:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number of employees required	17	13	15	19	14	16	11

Union rules state that each full-time employee must work five consecutive days and receives two days off. For example, if one works from Monday to Friday, then he/she has Saturday and Sunday off, but if he/she works from Tuesday to Saturday, then Sunday and Monday next week are his/her days off. The post office wants to meet the schedule using only full-time employees. Formulate this problem as a linear program in order to minimize the number of employees to be hired.

Question 2 (10 Points)

Chad is a venture capitalist trying to decide how to divide up to \$12 million between investment A and investment B. Investment A has been returning 16% per year and investment B 20%. Naturally, Chad would like to maximize the annual return from his investments. Still, he wants to exercise some caution. No more that \$10 million should go into investment A and no more than \$5 million into investment B. Moreover, at least half as much should be invested in investment B as investment A, and at least half as much in investment A as investment B to maintain some balance.

- Formulate a linear program to decide Chad's optimal investment plan.
- Solve the problem graphically, i.e., graph the feasible region by hand and determine the optimal solution.

Question 3 (10 Points)

Campana is a food processing plant which manufactures corn and flour tortillas. They grind their own flour for the flour tortillas at a maximum rate of 2000 pounds per week. Each pack of flour tortillas requires 10 ounces of flour (1 pound = 16 ounce). They currently have a contract with Cobby Hobby which specifies that a delivery of 1200 pounds of corn is delivered every Monday. Each pack of corn tortillas requires $\frac{1}{4}$ pound of corn product. All the other ingredients are in plentiful supply. Finally, the labor force at Campana consists of 6 employees working full time (40 hours per week each).

Each pack of corn tortillas requires 2 minutes of labor, and each pack of flour tortillas requires 4 minutes of labor. Each pack of corn tortillas yields a profit of 50 cents, and each pack of flour tortillas yields a profit of \$0.75. Campana would like to know how many packs of corn tortillas and flour tortillas they should produce each week so as to achieve the highest possible profit.

- (a) Formulate a linear programming model for this problem.
- (b) Solve the problem graphically, i.e., graph the feasible region by hand and determine the optimal solution.

Question 4 (10 Points)

A company sells two models of its patented five-legged robots. The basic version uses a steel body, requires 30 minutes to assemble, and sells for a profit of \$180. The deluxe model takes 1.6 hours to assemble, has a titanium body, and sells for a profit of \$360. Over the next week the company has 350 legs, 50 steel bodies, 30 titanium bodies, and 60 hours of assembly available. The company wishes to determine a maximum profit production plan (assuming that everything produced can be sold).

- (a) Formulate a linear program to decide the company's optimal investment plan.
- (b) Solve the problem graphically, i.e., graph the feasible region by hand and determine the optimal solution.

Question 5 (10 Points)

A company produces and sells two different products. The demand for each product is unlimited, but the company is constrained by cash availability and machine capacity.

Each unit of the first and second product requires 1 and 1.5 machine hours, respectively. There are 10,000 machine hours available in the current production period. The production costs are \$3 and \$2 per unit of the first and second product, respectively. The sale prices of the first and second product are both \$6 per unit. The available cash is \$4,600, furthermore, 40% of the sales revenues from the first product and 30% of the sale revenues from the second product will be made available to finance operations during the current period.

- (a) Formulate a linear programming problem that maximizes net income.
- (b) Solve the problem graphically, i.e., graph the feasible region by hand and determine the optimal solution.
- (c) Suppose that the company could increase its available machine hours by 2,000 by spending \$400 for certain repairs. Should the investment be made?