MATH 302 HW2

Hubert King

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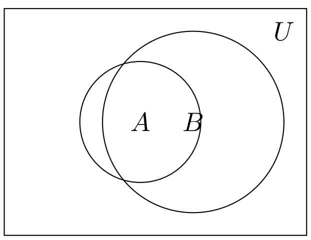
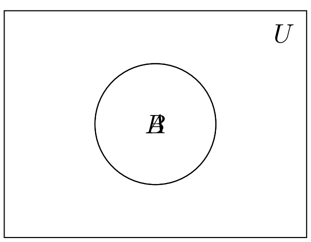
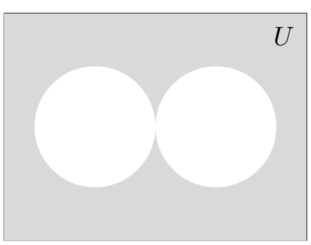
Problem 1



Problem 2



Problem 3



Problems 4-10

4. If $A \subseteq B$ **then** $A \cup B = B$

Proof 1 *First, we show that* $B \subseteq A \cup B$. *Let* $x \in B$. *By definition of union,* $x \in A \cup B$.
Next, we show that $A \cup B \subseteq B$. *Let* $x \in A \cup B$. *Then either:*

- $x \in A$*: Since* $A \subseteq B$, *we have* $x \in B$.
- $x \in B$*: Then* $x \in B$ *directly.*

Thus, $A \cup B = B$.

5. If $A \subseteq B$ **then** $B^c \subseteq A^c$

Proof 2 *Let* $x \in B^c$. *This means* $x \notin B$. *Since* $A \subseteq B$, *any element in* A *must be in* B. *Thus,* $x \notin A$. *Therefore,* $x \in A^c$, *proving that* $B^c \subseteq A^c$.

6. If $A \subseteq B$ **then** $B \setminus (B \setminus A) = A$

Proof 3 *First, we show* $A \subseteq B \setminus (B \setminus A)$. *Let* $x \in A$. *Since* $A \subseteq B$, *we have* $x \in B$. *Moreover,* $x \notin B \setminus A$ *since* $x \in A$. *Thus,* $x \in B \setminus (B \setminus A)$.

Next, we show $B \setminus (B \setminus A) \subseteq A$. *Let* $x \in B \setminus (B \setminus A)$. *Then* $x \in B$ *and* $x \notin B \setminus A$, *implying* $x \in A$. *Thus,* $B \setminus (B \setminus A) = A$.

7. $A \subseteq B \cap C$ **if and only if** $A \subseteq B$ **and** $A \subseteq C$

Proof 4 $(\Rightarrow)$ *Assume* $A \subseteq B \cap C$. *Let* $x \in A$. *Then* $x \in B \cap C$, *meaning* $x \in B$ *and* $x \in C$. *Thus,* $A \subseteq B$ *and* $A \subseteq C$.
$(\Leftarrow)$ *Assume* $A \subseteq B$ *and* $A \subseteq C$. *Let* $x \in A$. *Since* $A \subseteq B$, *we have* $x \in B$, *and since* $A \subseteq C$, *we have* $x \in C$. *Thus,* $x \in B \cap C$, *so* $A \subseteq B \cap C$.

8. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof 5 $(\subseteq)$ *Let* $x \in A \cap (B \cup C)$. *Then* $x \in A$ *and* $x \in B \cup C$. *Since* $x \in B \cup C$, *we have* $x \in B$ *or* $x \in C$. *Thus,* $x \in (A \cap B)$ *or* $x \in (A \cap C)$. *Therefore,* $x \in (A \cap B) \cup (A \cap C)$.

$(\supseteq)$ *Let* $x \in (A \cap B) \cup (A \cap C)$. *Then* $x \in A \cap B$ *or* $x \in A \cap C$. *In either case,* $x \in A$ *and* $x \in B \cup C$. *Thus,* $x \in A \cap (B \cup C)$.

9. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof 6 $(\subseteq)$ *Let* $x \in A \cup (B \cap C)$. *Then* $x \in A$ *or* $x \in B \cap C$. *If* $x \in A$, *then* $x \in A \cup B$ *and* $x \in A \cup C$. *If* $x \in B \cap C$, *then* $x \in B$ *and* $x \in C$, *implying* $x \in A \cup B$ *and* $x \in A \cup C$. *Thus,* $x \in (A \cup B) \cap (A \cup C)$.

$(\supseteq)$ *Let* $x \in (A \cup B) \cap (A \cup C)$. *Then* $x \in A \cup B$ *and* $x \in A \cup C$. *If* $x \in A$, *then clearly* $x \in A \cup (B \cap C)$. *Otherwise,* $x \in B$ *and* $x \in C$, *implying* $x \in B \cap C$. *Thus,* $x \in A \cup (B \cap C)$.

10. $A = (A \cap B) \cup (A \setminus B)$

Proof 7 ($\subseteq$) *Let* $x \in A$. *Then either* $x \in B$ *or* $x \notin B$. *If* $x \in B$, *then* $x \in A \cap B$. *If* $x \notin B$, *then* $x \in A \setminus B$. *Thus,* $x \in (A \cap B) \cup (A \setminus B)$.

($\supseteq$) *Let* $x \in (A \cap B) \cup (A \setminus B)$. *If* $x \in A \cap B$, *then* $x \in A$. *If* $x \in A \setminus B$, *then* $x \in A$. *Thus,* $x \in A$.