MATH 302 HW1

Hubert King

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Problem 1

We show this through a truth table.

A	B	C	A $\wedge B$	(A $\wedge B) \vee C$	$\neg((A \wedge B) \vee C)$	$(\neg A \vee \neg B) \wedge \neg C$
0	0	0	0	0	1	1
0	0	1	0	1	0	0
0	1	0	0	0	1	1
0	1	1	0	1	0	0
1	0	0	0	0	1	1
1	0	1	0	1	0	0
1	1	0	1	1	0	0
1	1	1	1	1	0	0

Problem 2

Let P be the statement that the defendant is guilty. Let Q be the statement that there is an accomplice. When the defense says "that's not true" he is saying the third column is true, and that can only be the case if the defendant is guilty and there is no accomplice.

P	Q	P $\Rightarrow$ Q	$\neg(A \Rightarrow B)$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

Problem 3

Let A be the statement that a person is diurnal. Let us assume that if a person is not diurnal, they must be nocturnal. Let the B be the statement that a person is awake. Let C be the statement that a person's beliefs are true. Let D be the statement that a person believes they are awake.

A	B	C	D
1	1	1	1
1	0	0	1
0	1	0	0
0	0	1	0

The table shows that diurnal people always think they are awake, and nocturnal people always think they are asleep. When A is true, D is true, and vice versa.

Problem 4

Let A be the statement that a person is diurnal. Let B be the statement that their belief is true. Let C be the statement that they are awake.

A	B	C
1	1	1
0	0	1

If the person is diurnal, the person believes (correctly) that they are indeed diurnal, and thus must be awake. If the person is nocturnal, the person believes (incorrectly) that they are diurnal, and thus must also be awake. We don't know whether their belief is true, but we do know the person is awake.

Problem 5

Let A be the event that KulpA is diurnal. Let B be the event that KulpB is diurnal.

Let C be the event that KulpA can at some point believe that both KulpA and KulpB are nocturnal, and D the event that KulpB currently believes that both KulpA and KulpB are diurnal. Let E be the event that KulpA is awake, and we know from the problem that if KulpA is not awake then KulpB is awake.

A	B	C	D	E
1	0	1	1	0
0	1	1	1	0
1	1	1	1	0
0	0	1	1	0

For C and D to be true, E must always be false (i.e. KulpB is awake.

Problem 6

$$\neg(\forall x \exists y B(x,y))$$
$$\Rightarrow \exists x \neg(\exists y B(x,y))$$
$$\Rightarrow \exists x \forall y \neg B(x,y)$$
$$\Rightarrow \neg S = \exists x \forall y \neg B(x,y)$$

Problem 7

Player 1 winning strategy:

$$\forall y \exists x : W(x,y)$$

The negation is as follows:

$$\neg[\forall y \exists x : W(x,y)] = \exists y \forall x : \neg W(x,y)$$

Player 2 winning strategy:

$$\forall x \exists y : \neg W(x,y)$$

The negation is as follows:

$$\neg[\forall x \exists y : \neg W(x,y)] = \exists x \forall y : \neg\neg W(x,y) = \exists x \forall y : W(x,y)$$

Problem 8

Let $M(w,x,y,z)$ be the statement that player 2 wins.

Player 1 winning strategy:

$$\exists w \forall x \exists y \forall z W(w,x,y,z)$$

The negation:

$$\neg[\exists w \forall x \exists y \forall z W(w,x,y,z)] = \forall w \exists x \forall y \exists z \neg W(w,x,y,z) = \forall w \exists x \forall y \exists z M(w,x,y,z)$$

Player 2 winning strategy:

$$\forall w \exists x \forall y \exists z M(w,x,y,z)$$

The negation:

$$\neg[\forall w \exists x \forall y \exists z M(w,x,y,z)] = \exists w \forall x \exists \forall z \neg M(w,x,y,z) = \exists w \forall x \exists \forall z W(w,x,y,z)$$