Math 302 , Problem Set 1 Due January 23,2025 6 P.M.

On problems 1-5 you may use truth tables to ensure you have given a complete explanation of your answer. In problems 7 and 8 be careful to give explanations of what you are doing. Try writing everything in complete sentences, etc.

1. Show using truth-tables that

$$\neg((A \wedge B) \vee C),$$

is the same as

$$(\neg A \vee \neg B) \wedge \neg C.$$

2. A prosecutor and a defense attorney discuss a case.

Prosecutor: If the defendant is guilty, then there is an accomplice.

Defense Attorney (responding to Prosecutor): That's not true.

Explain that the Defense attorney's statement is dumb, by proving from it the guilt of the defendant.

3. Assume that people are always either awake or asleep but not both, and that they always have a belief regarding whether they are asleep or awake. We say that a person is *diurnal* if everything they believe while they are awake is true and everything they believe while they are asleep is false. Contrariwise, we say that a person is *nocturnal* if everything they believe while they are awake is false and everything they believe while they are asleep is true. Prove that a diurnal person always believes they are awake, while a nocturnal person always believes they are asleep.

4. With everything as in problem 2, a person who is either nocturnal or diurnal believes they are diurnal. Can you determine if the belief is true? Can you determine if the person is awake or asleep?

5. Two people named KulpA and KulpB are each either nocturnal or diurnal. At some point, KulpA believed that both KulpA and KulpB are nocturnal, while KulpB believes that both KulpA and KulpB are diurnal. Assume that exactly one of KulpA and KulpB is asleep and exactly one is awake. Which one is asleep?

6. Consider the statement

$$S = \forall x \exists y B(x, y),$$

where $B(x, y)$ is a statement taking two variables. Derive the negation $\neg S$ step by step by writing

$$S = \forall x(\exists y B(x, y)),$$

and moving the $\neg$ through one quantifier at a time.

7. Two players, “Player one” and “Player two” play a game in which ”Player one” makes a move x and “Player two” knowing this move chooses a countermove y. Let $W(x, y)$ be the statement that “Player one” wins if the moves x, y are made. Suppose that if “Player one” does not win then “Player two” wins. Write formally the statement that “Player two” has a winning strategy using quantifiers, connectives and the statement $W(x, y)$. Also write formally the statement that “Player One” has a winning strategy. Show that these statements are negations of one another.

8. The two players in problem 6 play a more complicated game. First “Player one” makes a move w, then “Player two” responds with a move x then “Player one” responds with a move y and finally “Player two” responds with a move z. Let $W(w, x, y, z)$ be the statement that “Player one” wins. Let $\not W(w, x, y, z)$ be the statement that “Player two” wins. Write out with quantifiers, connectives and the statement $W(w, x, y, z)$ the statement that “Player one” has a winning strategy and that “Player two” has a winning strategy. Show that these statements are negations. Bonus: By defining the notion of a strategy, can you reduce Problem 7 to Problem 6?