Analysis of Static and Dynamic GHZ Circuits for Scalable Quantum Computing

Abstract

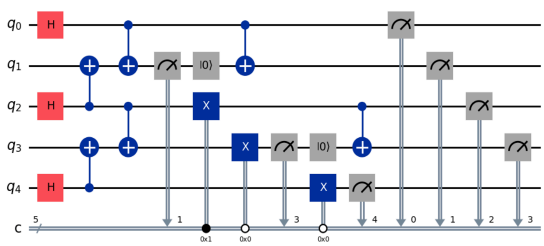
This report examines the efficacy of static and dynamic Greenberger-Horne-Zeilinger (GHZ) circuits in creating entangled quantum states over a system of five qubits. GHZ states are pivotal for quantum computing applications, such as error correction, quantum cryptography, and algorithms requiring entanglement. By comparing the state fidelity, circuit depth, and the number and type of gates between static and dynamic GHZ circuits, we aim to deduce which circuit design offers more promise for scaling to a higher number of qubits in practical quantum computing.

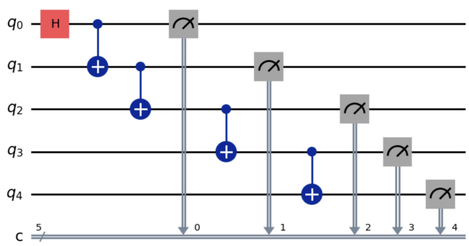
Introduction

Quantum computing leverages the principles of quantum mechanics to process information in ways unattainable by classical computers. A quintessential feature of quantum computing is the entanglement of qubits, with GHZ states being a specific, highly entangled state across multiple qubits. The creation of such states is a litmus test for a quantum computer's ability to maintain coherence and control over its qubits. Static GHZ circuits operate with a fixed sequence of gates, with no feedback or conditional operations. Conversely, dynamic GHZ circuits can adjust their operations in real-time, offering a potentially more flexible and error-resilient pathway for state preparation. With the advent of quantum technologies, it is crucial to determine which of these circuit designs is more suited for scalability – a vital characteristic for practical quantum computation.

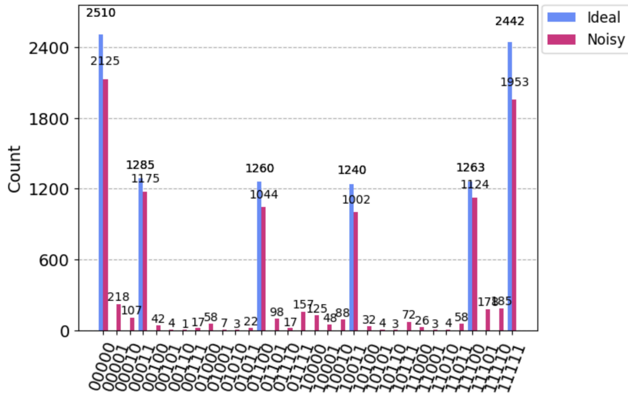
Methodology

Two types of GHZ circuits were designed and implemented: static and dynamic. The static circuit was constructed using a series of Hadamard and CNOT gates, followed by measurement operations. The dynamic circuit included additional gates like Pauli-X and reset operations, providing the ability to react to the state of the qubits during the execution of the circuit. Each circuit was tested under ideal and noisy conditions to simulate real-world quantum environments. The state fidelity metric was used to quantify the accuracy of the GHZ state generated compared to the theoretical ideal. Circuit depth provided insight into the temporal efficiency of the circuit, while the number and type of gates offered a measure of complexity and resource demand. On the left circuit is the model of dynamic GHZ circuit, right circuit is the model of static GHZ circuit.

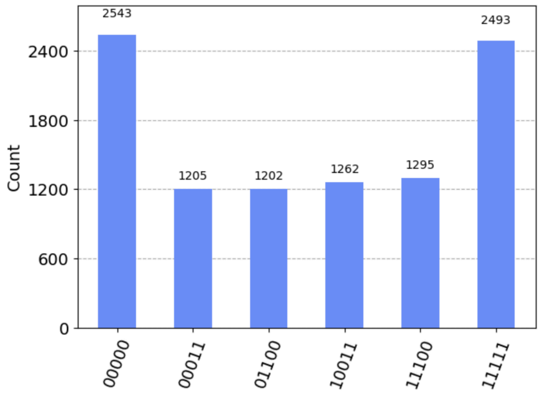




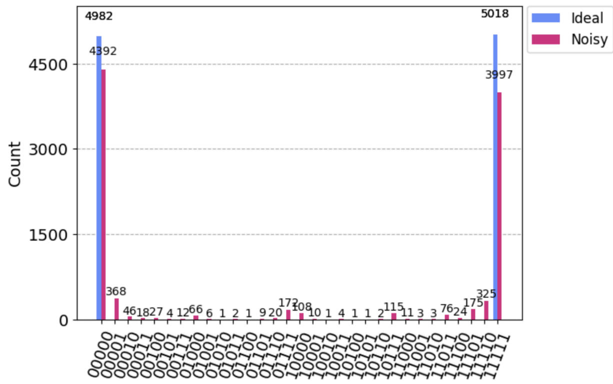
Results



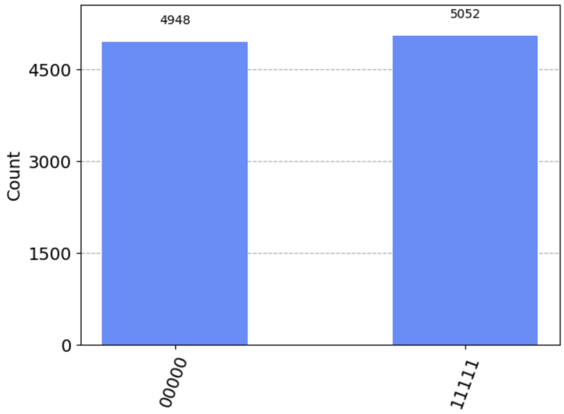
This figure shows the Ideal and Noise count for each state in the Dynamic GHZ circuit



This figure shows the probability for each state in the Dynamic GHZ circuit



This figure shows the Ideal and Noise count for each state in the GHZ circuit



This figure shows the probability for each state in the GHZ circuit

Static GHZ Circuit:
- State Fidelity: 98.76%
- Circuit Depth: 6
- Number of Qubits: 5
- Number of Gates: 10 (5 measures, 4 CNOTs, 1 Hadamard)

Dynamic GHZ Circuit:
- State Fidelity: 96.37%
- Circuit Depth: 10
- Number of Qubits: 5
- Number of Gates: 19 (7 measures, 6 CNOTs, 3 Hadamards, 3 Pauli-X, 2 resets)

The histograms generated from the experiments revealed a higher fidelity in the static circuit compared to the dynamic one. However, the dynamic circuit showed a greater resilience to noise, evident from the less drastic reduction in peak fidelity under noisy conditions. The probability distribution histograms displayed a more pronounced concentration of probability in the correct GHZ states for the static circuit in both ideal and noisy conditions.

Analysis
While the static GHZ circuit boasts higher fidelity, suggesting it is currently better suited for smaller systems where fidelity is paramount, the dynamic circuit's design with conditional operations may provide better error mitigation and adaptability. This adaptability is crucial when scaling up to larger systems, where error rates typically increase, and dynamic behavior can help maintain the integrity of the quantum state. The increased circuit depth and number of gates in the dynamic circuit reflect its higher complexity, which may introduce additional error sources but also allows for more sophisticated error correction techniques.

Conclusion
The static GHZ circuit, with its simplicity and high fidelity, is favorable for near-term applications where the number of qubits is limited, and high fidelity is required. However, for scalability and robustness in larger quantum systems, the dynamic GHZ circuit's flexibility and error resilience make it a promising candidate.

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