Elements of DeFi

https://web3.princeton.edu/elements-of-defi/

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Lecture 6:
CFMMs

Last lecture: Decentralized Exchanges

- Most basic element of finance: Market Making

- Traditional Market Makers
  - Limit Order Books
  - Peer-to-peer, centralized
  - hard to decentralize

- Automated Market Makers
  - Peer-to-pool-to-peer, can be decentralized
  - Basic example

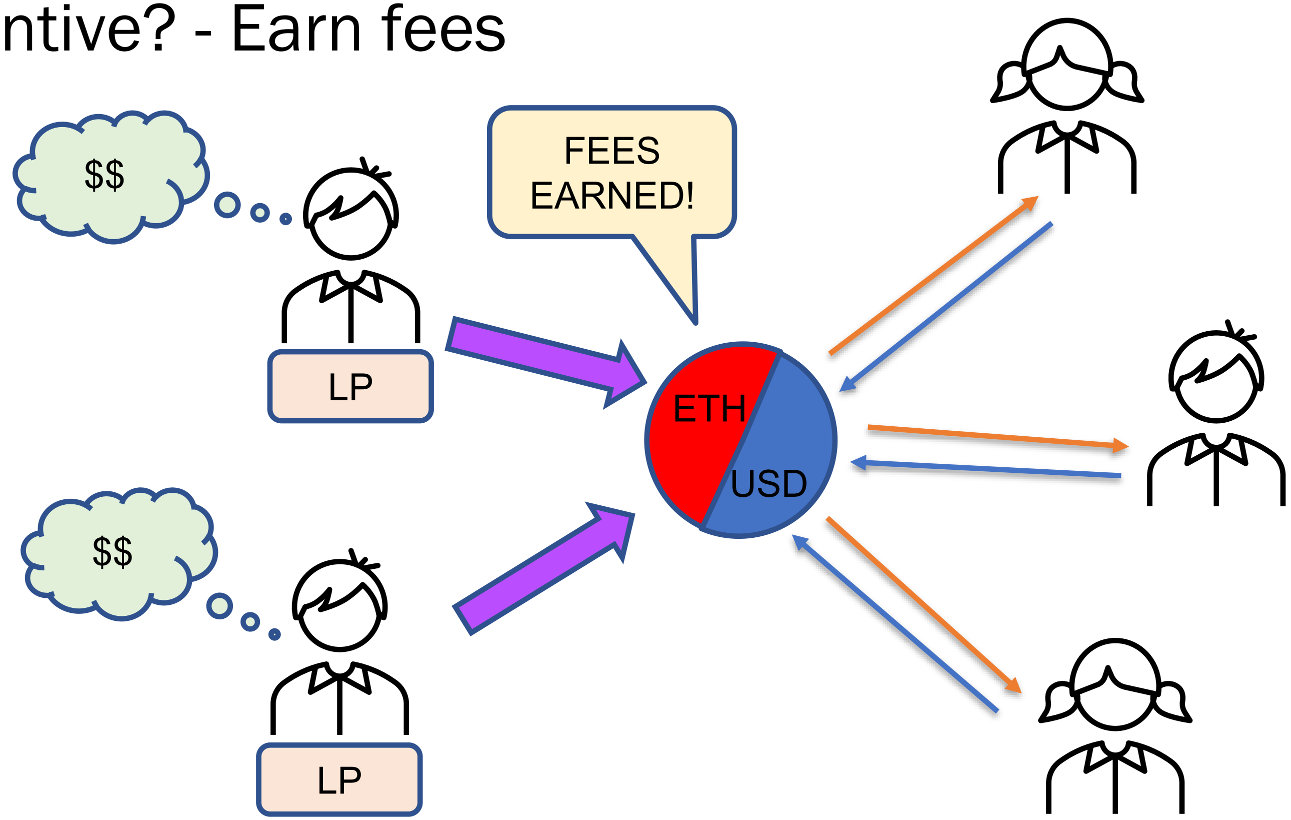
This Lecture : CFMMs and their properties

- Look at CFMMs from trader’s perspective
  - Recall : basics and pricing
  - Slippage
  - Arbitrage
  - Relation with curvature

- Look at CFMMs from liquidity provider’s perspective
  - Impermanent Loss
  - Arbitrage Loss
  - Picking the bonding curve
  - Fees

Recall : Automated Market Makers

- Keep a pool of orders that can satisfy any incoming trade
- Entities with large amount of idle liquidity pitch in to make the pool– liquidity providers (LPs)
- Incentive? - Earn fees



Recall : CFMMs

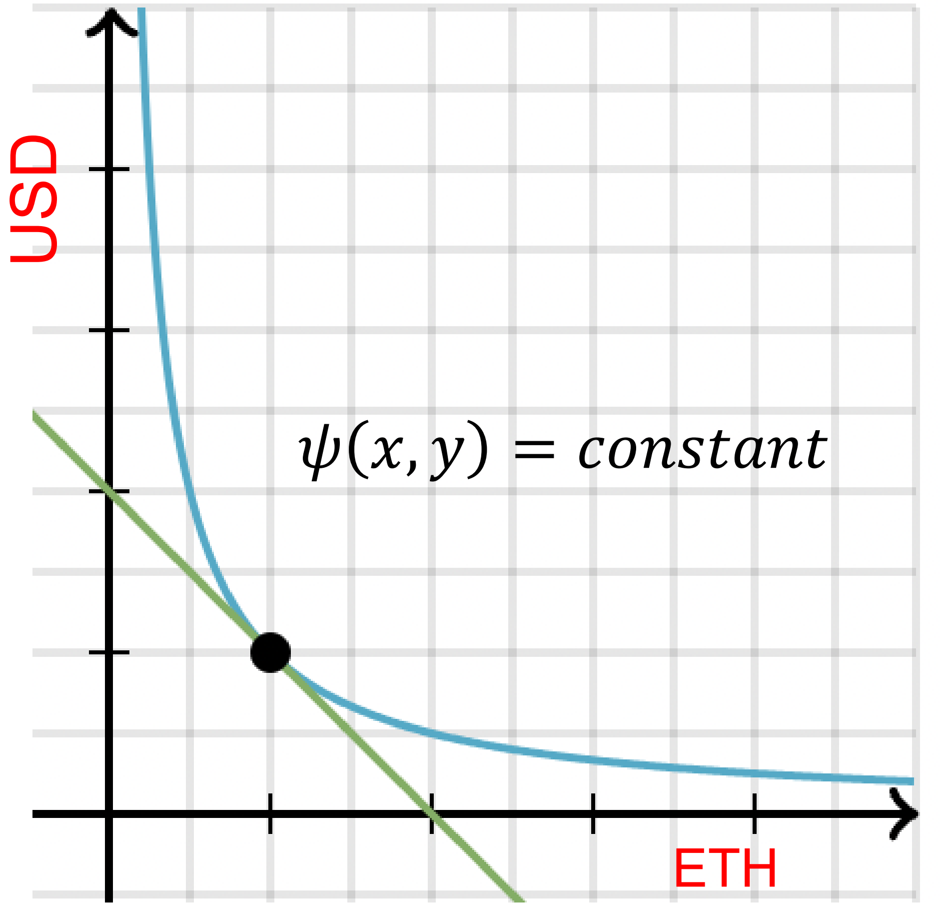
- CFMM: Constant Function Market Makers

- Use Bonding Curves to constrain reserves

$$\psi(x, y) = \psi(x + \Delta_x, y - \Delta_y)$$

OR

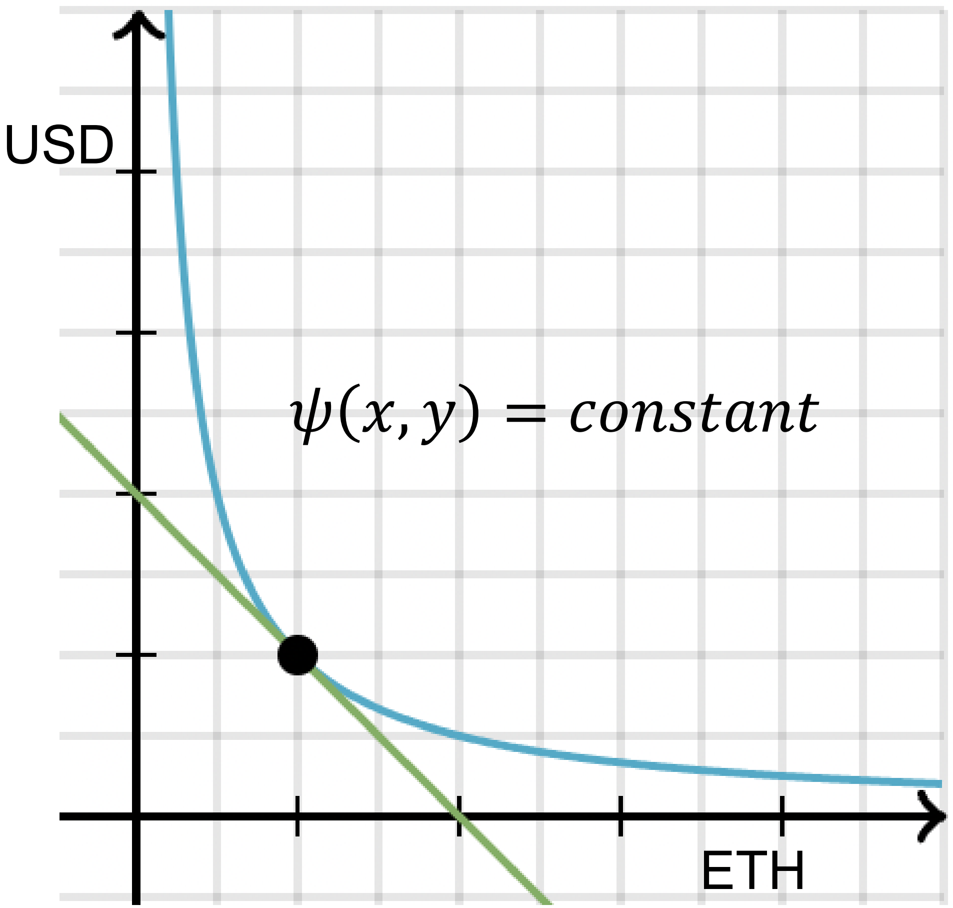
$$\psi(x, y) = constant$$



Recall : Pricing in CFMMs

- For a general curve what is the **price** at any point?
- Price = To buy a small amount of ETH, how much USD should I pay?
- **Slope of the tangent**
- Formula :

$$Price = P_x = \frac{dy}{dx} = -\frac{\partial_x \psi}{\partial_y \psi}$$



Recall : Pricing in CFMM

- Uniswap, Sushiswap:

$$xy = constant \Rightarrow P_x = \frac{y}{x}$$

- Balancer:

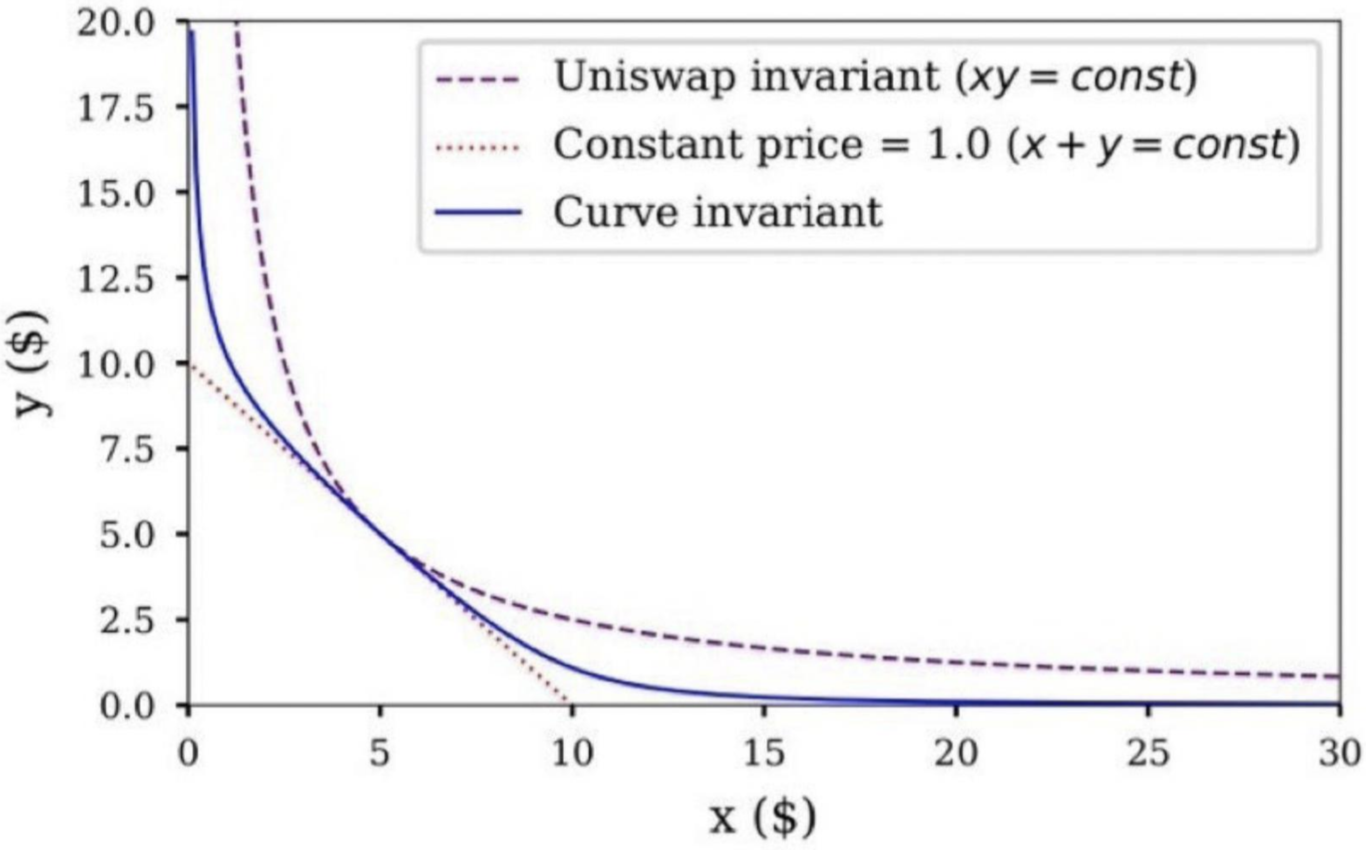
$$x^{\theta} y^{1-\theta} = constant \Rightarrow P_x = \frac{\theta y}{(1-\theta)x}$$

- Constant Price :

$$x + y = constant \Rightarrow P_x = 1$$

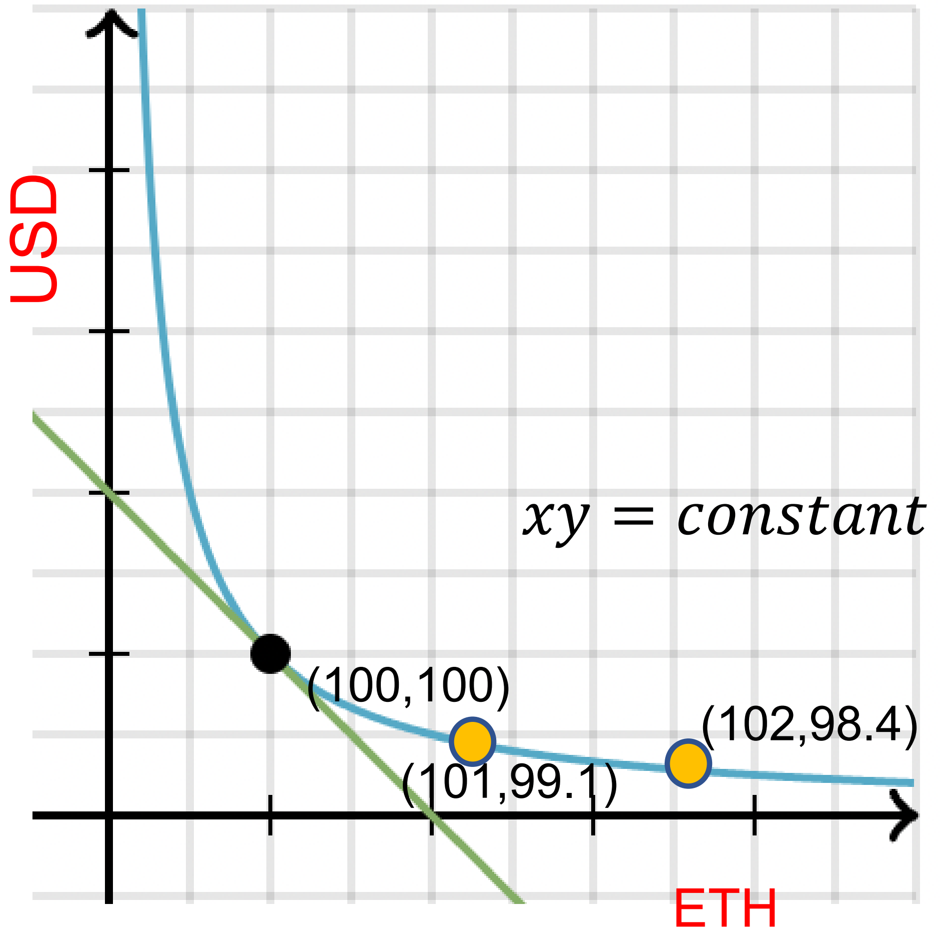
- Curve:

$$x + y + \frac{\alpha}{xy} = constant$$



Slippage

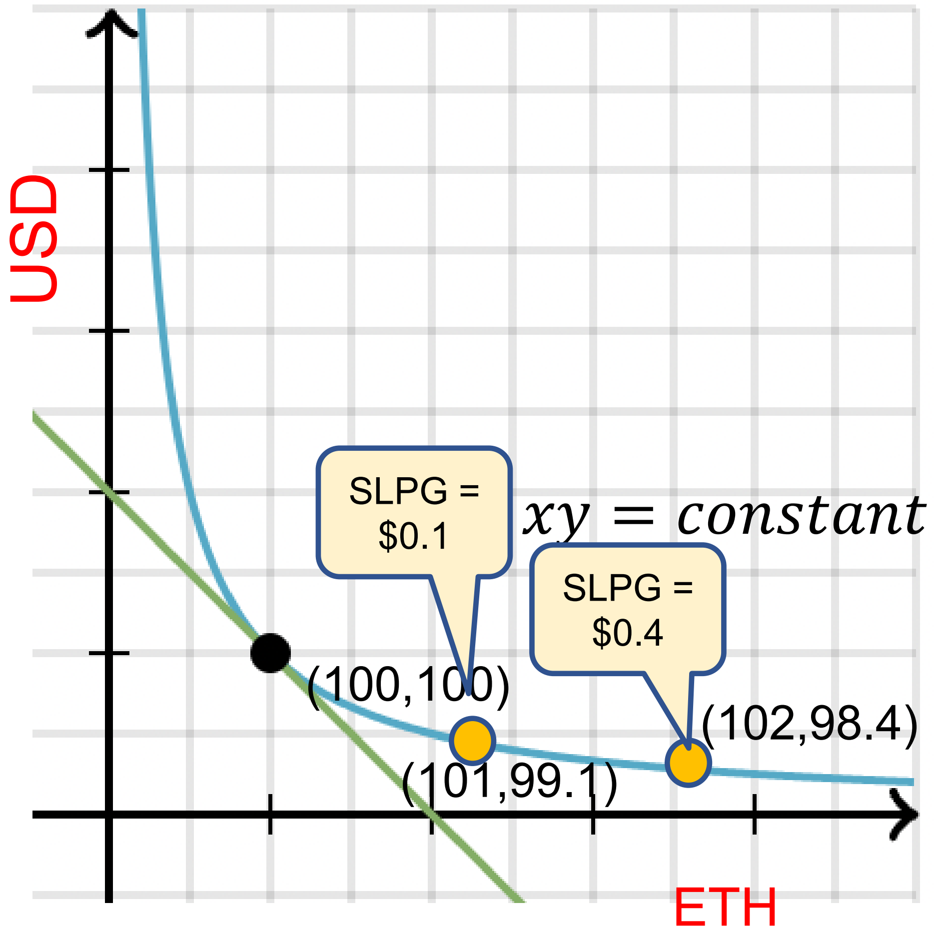
- The price is defined for a small trade
- What if you want to do a larger trade? Does the price you pay stay constant with size?
- Slippage is the amount of asset you lose out on because of the price change
- E.G :
  - You want to sell 1 ETH
  - You want to sell 2 ETH



Slippage and Curvature

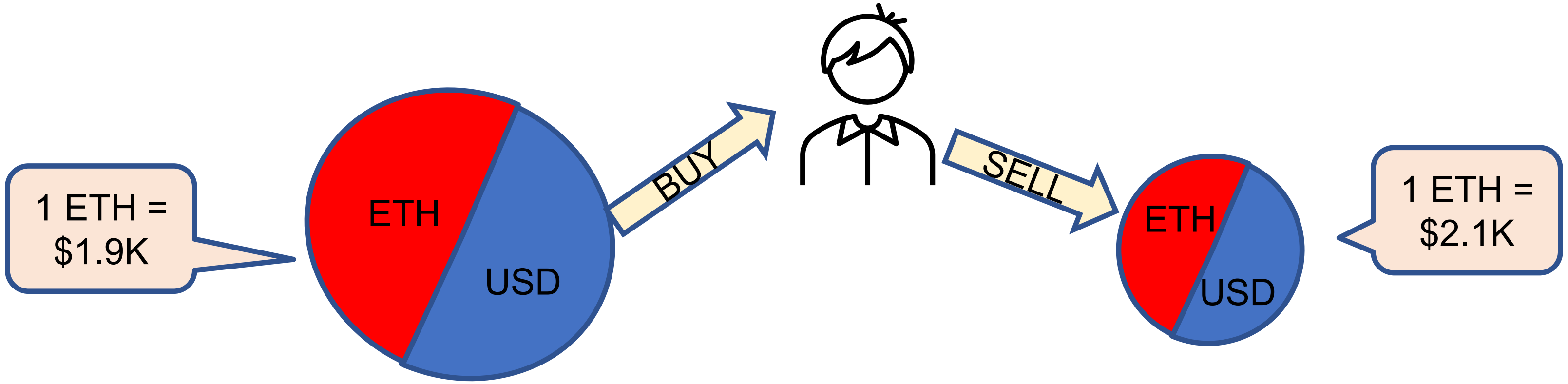
- E.G :
  - You want to sell 1 ETH
  - You want to sell 2 ETH

- What is the slippage in both cases?
- If trade size doubles, then slippage ….?
- Reason ?
  - Price depends on tangent, slippage depends on curvature!
- As a trader, you would like to have low slippage



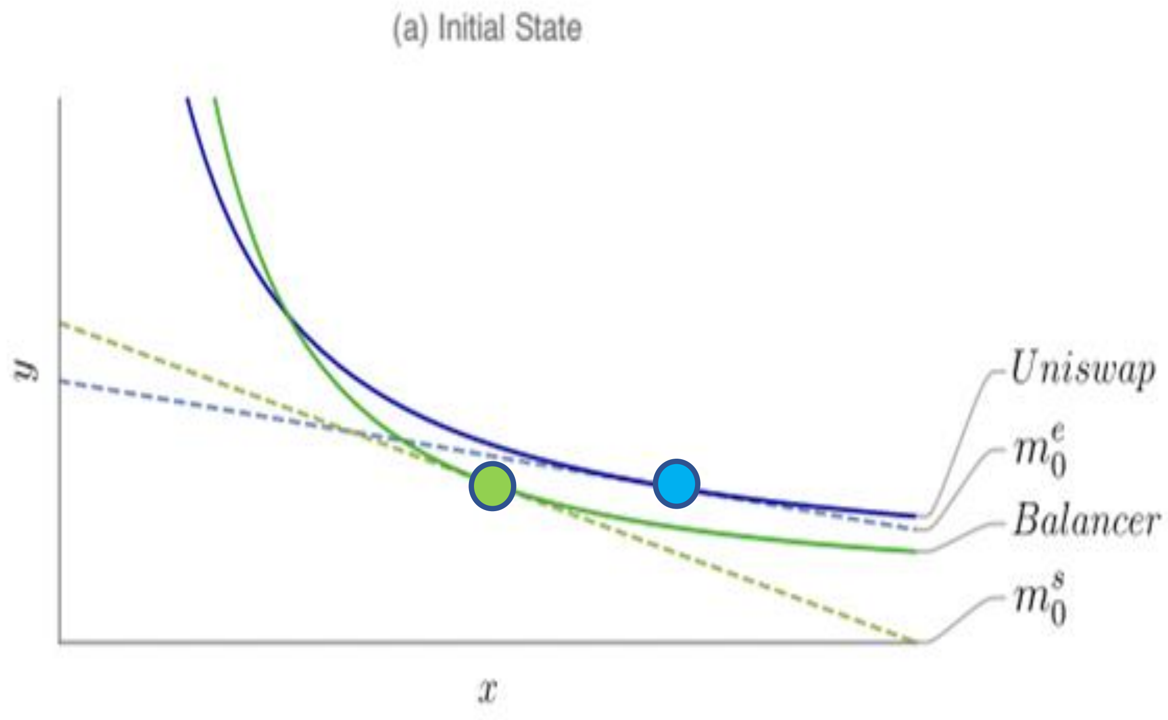
Arbitrage

- What is arbitrage?
- Riskless profit obtained by exploiting price differences
- What happens after arbitrage?
- Both pools reach same price – arbitrage no longer profitable after this



Arbitrage and Curvature

- Arbitrage between two CFMMs – what is the trade?
- Which one undergoes a larger change in price ? Why ?



Arbitrage and Curvature

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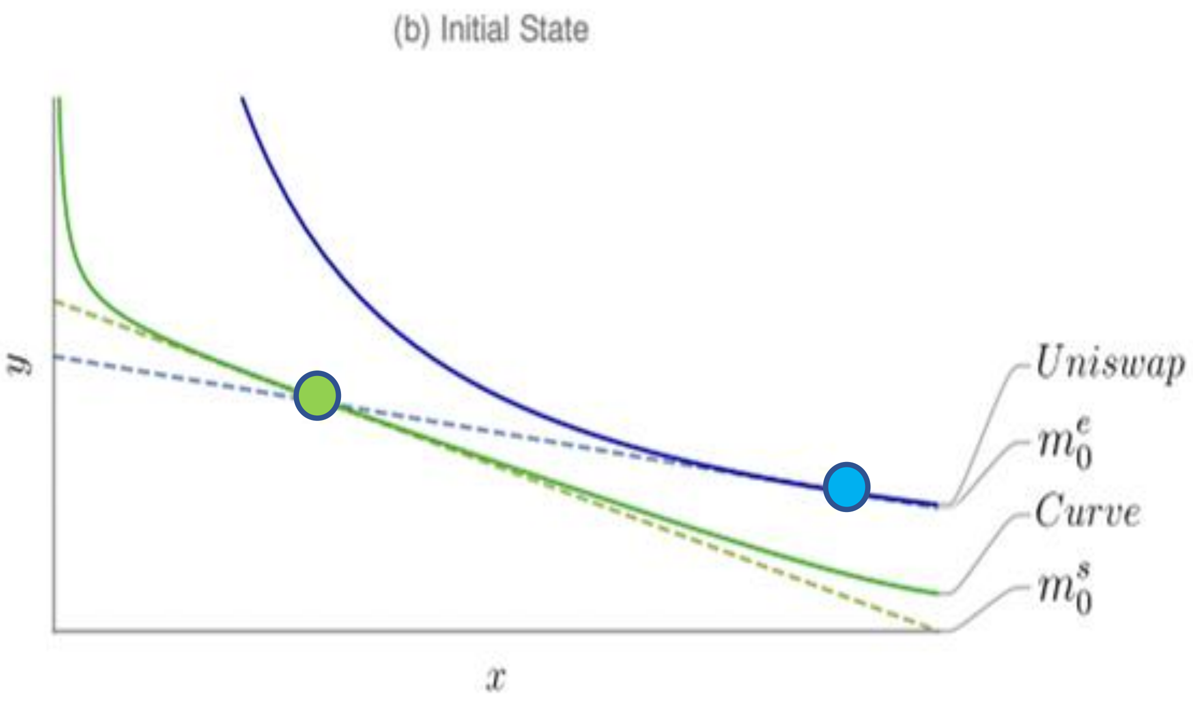
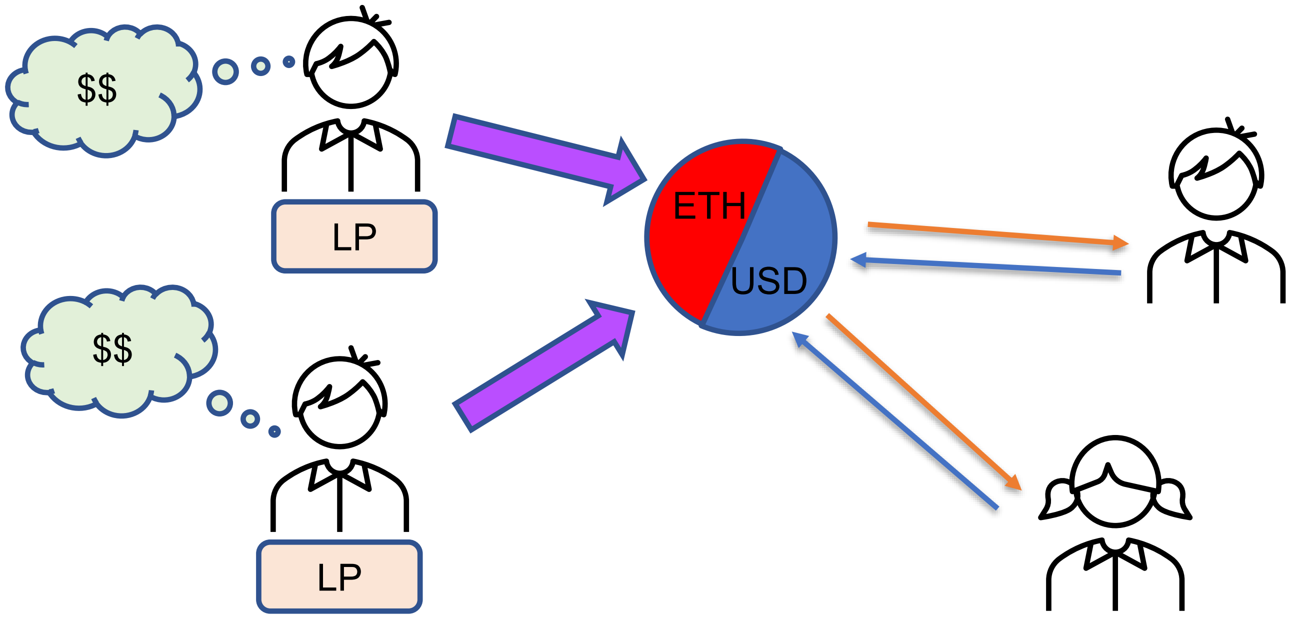


Liquidity Providers : Loss and risks

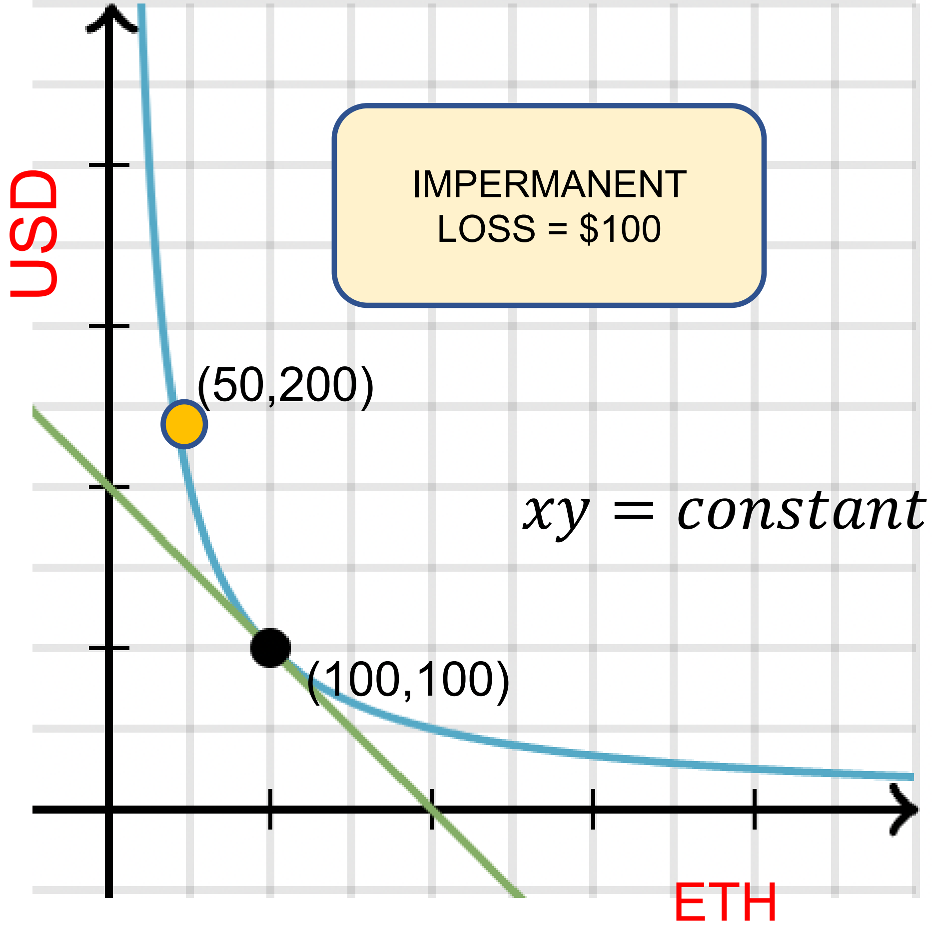
- We now look at CFMMs from the perspective of LPs (Liquidity Provider)
- They facilitate the market via their assets
- What risks/losses can they face? – two types
- How should they recover these losses? - fees



Impermanent Loss (Loss-Vs-Holding)

Looking from the LP's perspective :

- What happens when price goes from 1 USD to 4 USD?
- Compare with the old reserves held static
- Value of old reserves = 100 + 4x100 USD
- Value of new reserves = 200 + 4x50 USD
- What happens when price goes back to 1 USD?

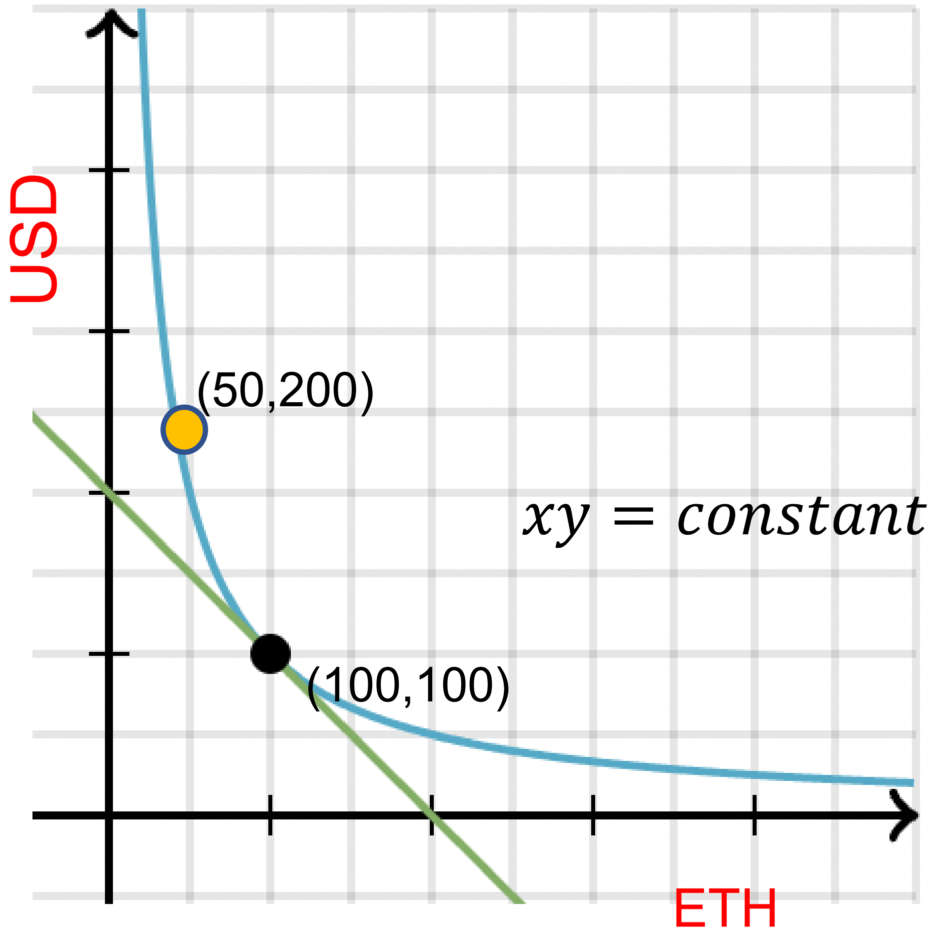


Impermanent Loss (Loss-Vs-Holding)

Impermanent Loss :

- Value of reserves if held static – Value of reserves under CFMM
- Always positive! Why?
- CFMM always sells off the token appreciating in value

- Loss disappears when price moves back to the reference point – impermanent
- What is the bet that LPs are making ?



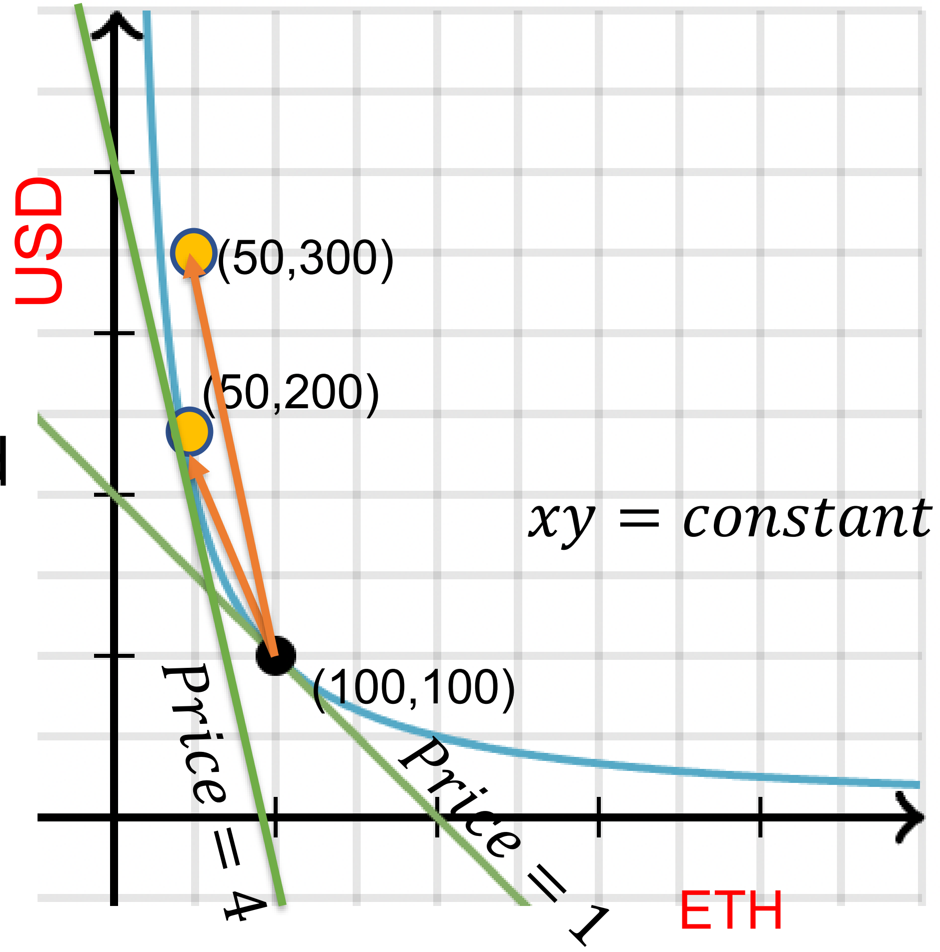
Arbitrage Loss (Loss-Vs-Rebalancing)

- Same situation as before – price jumps from 1 USD to 4 USD

- What is the trade that CFMM does?

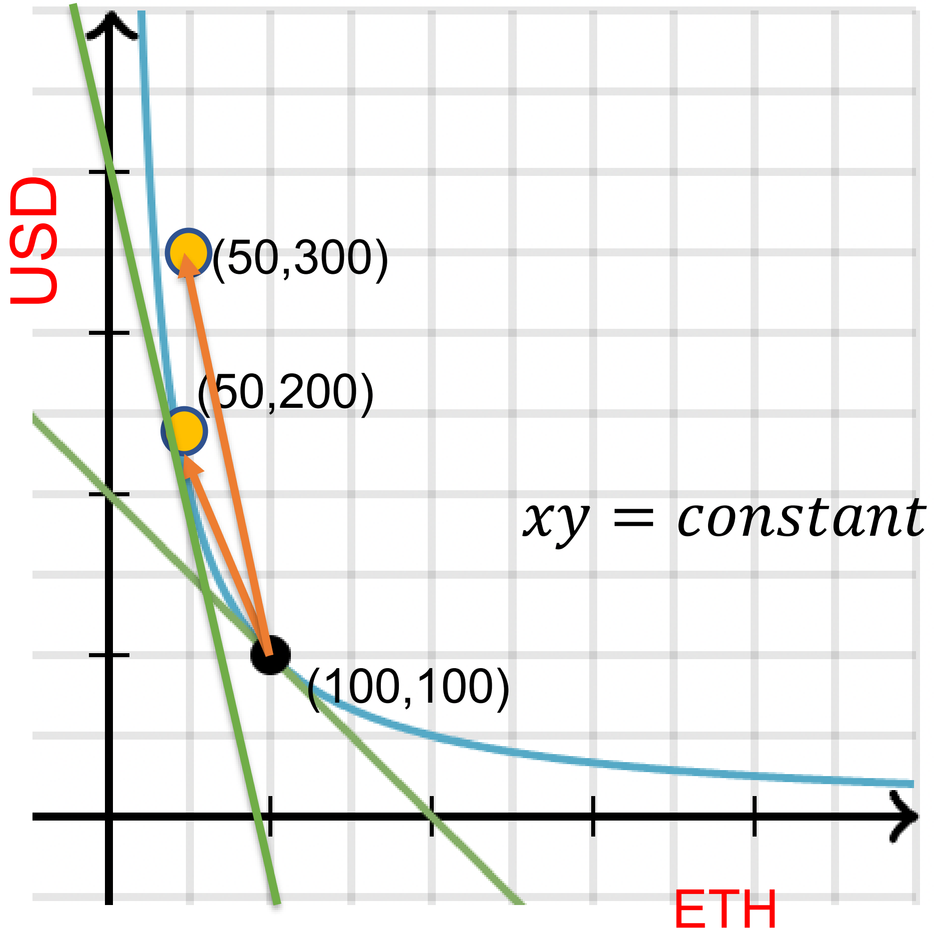
- If CFMM "knew" the real price, what would the trade have looked like?

- Loss due to arbitrage = 100 USD



Arbitrage Loss (Loss-Vs-Rebalancing)

- Value of trade assuming external price – Value of trade assuming AMM price
- Is also equal to the profit made by the arbitrageur
- Is non-recoverable and cumulative
- Loss proportional to price volatility (Millionis et al, 2022)



Difference between the two losses

Impermanent Loss	Arbitrage Loss
• Is path-independent	• Is path-dependent
• Recovered when price reaches initial point again	• Cannot be recovered
• Can be hedged against	• Need fees to make up for loss
• Depends only on final and initial prices	• Depends external price volatility

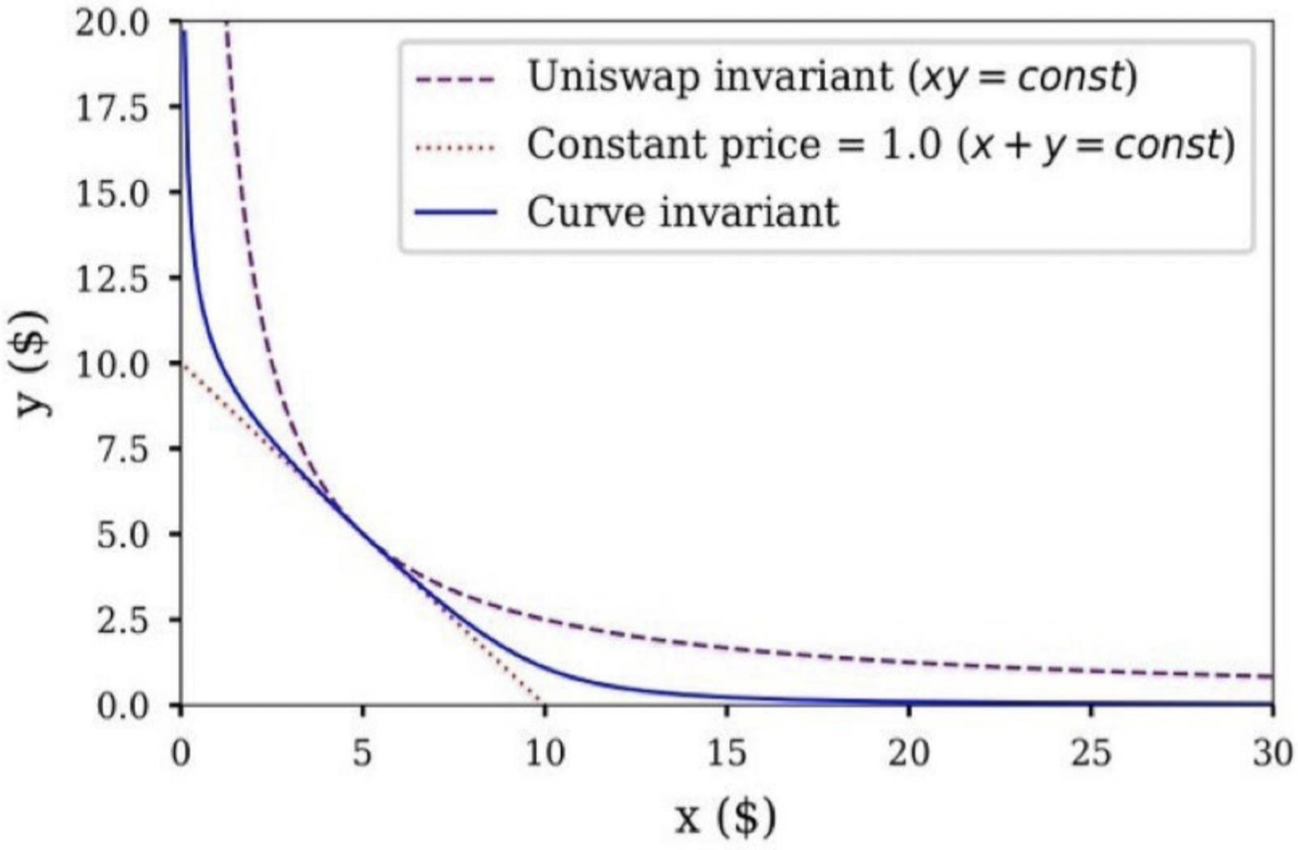
Choosing the right curve

Lesson for the trader :

- Lesser the curvature, better trading quality because of less slippage

- Lesser the curvature, larger profit through arbitrage

- Uniswap vs Curve



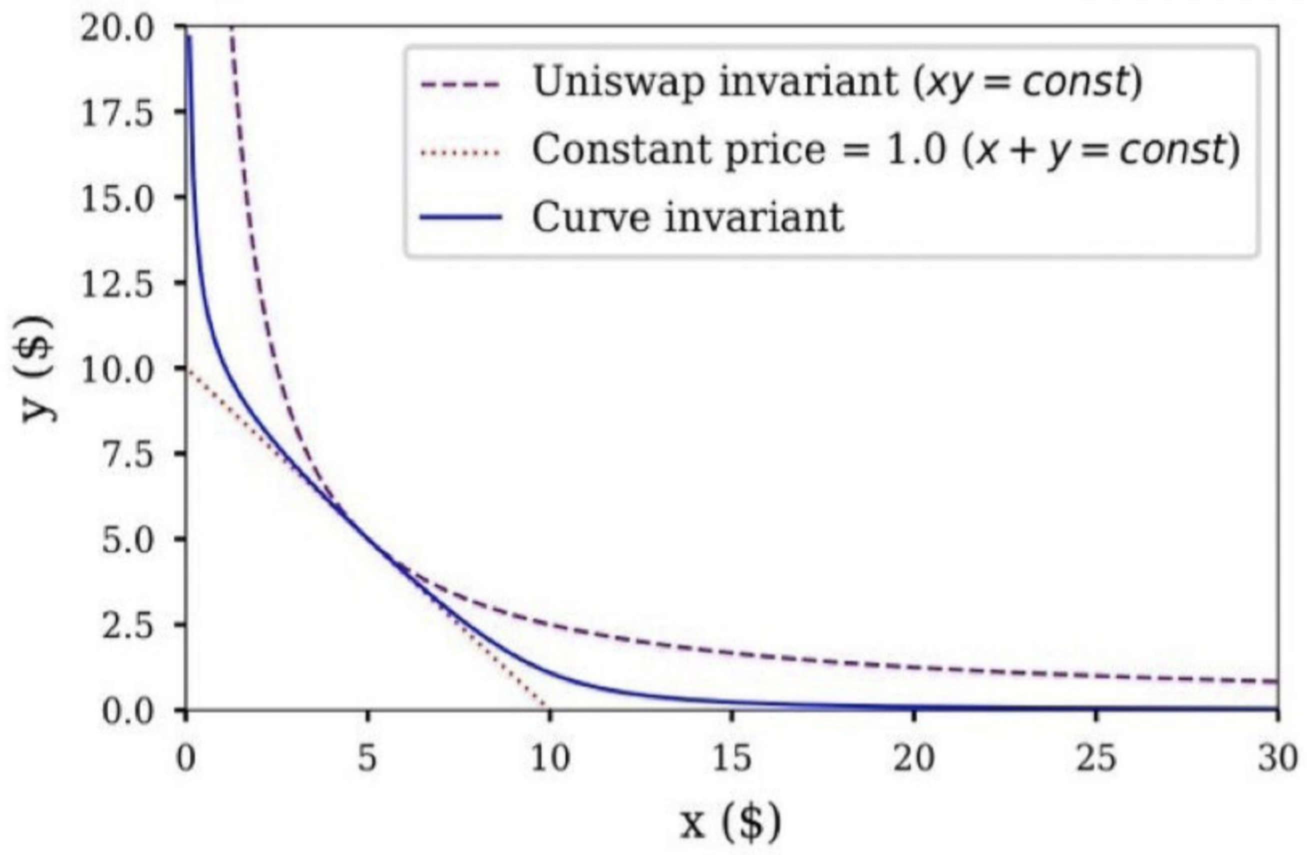
Choosing the right curve

Lesson for the LP :

- If prices expected to be correlated/stable : use less curvature

- If prices expected to be uncorrelated/independent : use more curvature

- Uniswap vs Curve



Fees

- In exchange of allocating liquidity for trades, LPs take fee from asset coming in

$$\psi(x,y) = \psi\big(x + \gamma\Delta_x, y - \Delta_y\big)$$

$$Fee = ((1-\gamma)\Delta_x)$$

USE THIS CONDITION FOR SMALL TRADES

- Typically, $\gamma \approx 0.99$

- Induces an effective ask and a bid price

$$P^{ask} = P_x/\gamma$$

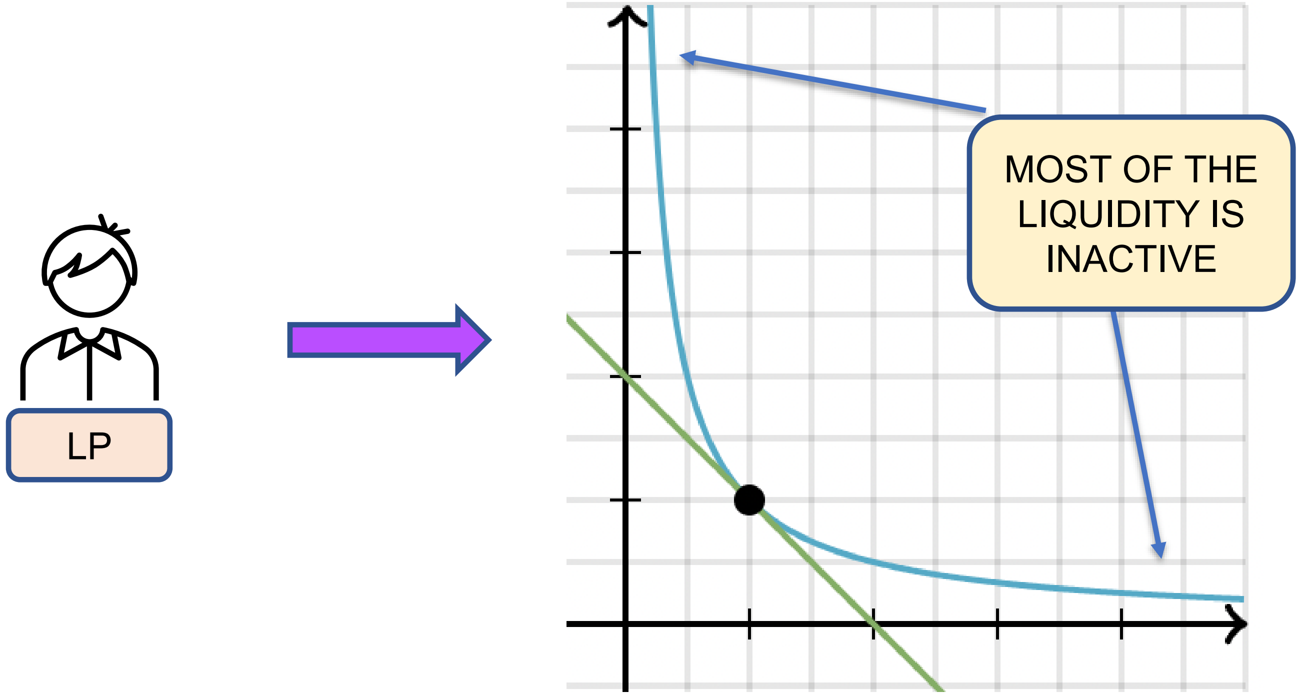
$$P^{bid} = \gamma P_x$$

EXERCISE: PROVE THIS

Problems with CFMMs: Capital Inefficiency

Capital inefficiency : Less capital efficiency than LOBs - why?

- LPs cannot move liquidity around
- Was possible in LOBs



Problems with CFMMs: Arbitrage Loss

Arbitrage loss :

- *increases* with volatility (recall in the tradfi case, more volatility gave more profit) -> Fees have to give a return *and* cover these losses

- LPs are sitting ducks – easily fleeced by arbitrageurs

- "If I see a Uniswap LP in the wild, I go up to them, shake their hand and thank them for their service"

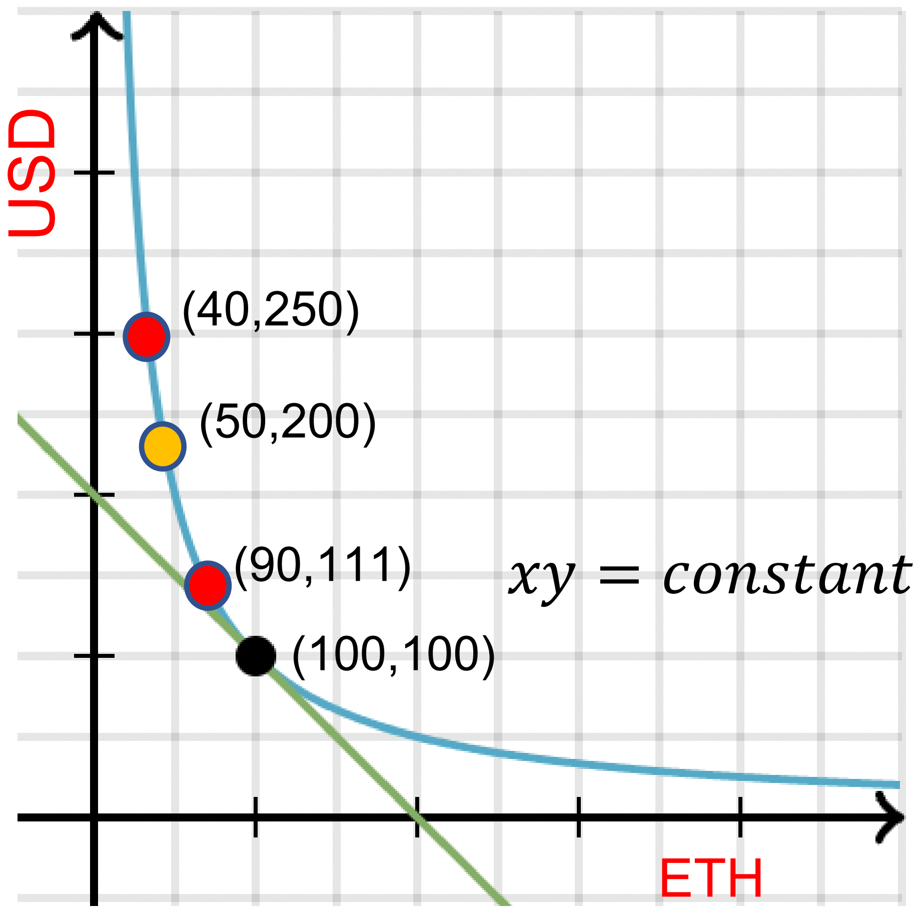
  - Mark Twain (probably)

Problems with CFMMs: Front Running

<u>MEV</u> : Sandwich Attack

- User wants to do a normal trade :
  - Buy 50 ETH, (has to pay 100 USD)

- If miner sees a large buy txn,
  - introduce a buy txn just before it : buy 10 ETH
  - Put the txn
  - introduce a sell txn just after it : sell 10 ETH
- Miner gets profit with no risk : 39 USD
- User gets a worse price : 139 USD



Next Lecture

- Make CFMMs more capital efficient
  - LP’s POV : Concentrated liquidity – move liquidity around
  - Trader’s POV : DEX aggregators – Batching + Routing – avoid arbitrage losses

- Private CFMMs – to avoid MEV

- CFMMs as derivatives

LECTURE ENDS