

# Elements of DeFi

<https://web3.princeton.edu/elements-of-defi/>

**Professor** Pramod Viswanath

Princeton University

# **Lecture 5:**

# **Decentralized Exchanges**

# Last Lecture

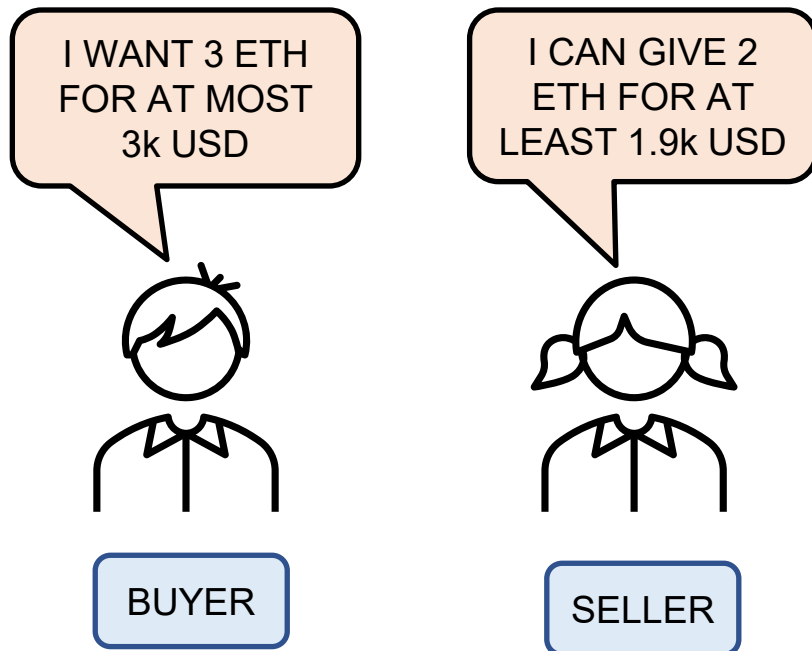
- Pricing smart contracts and computation on a blockchain
  - Gas fees
- Miner incentives
  - Transaction rewards
- Miner Extractable Value (MEV)
  - Basic strategies

# **This lecture: Decentralized Exchanges**

- Most basic element of finance: Market Making
- Traditional Market Makers
  - Limit Order Books
  - Peer-to-peer, centralized
  - hard to decentralize
- Automated Market Makers
  - Peer-to-pool-to-peer, can be decentralized
  - Basic example

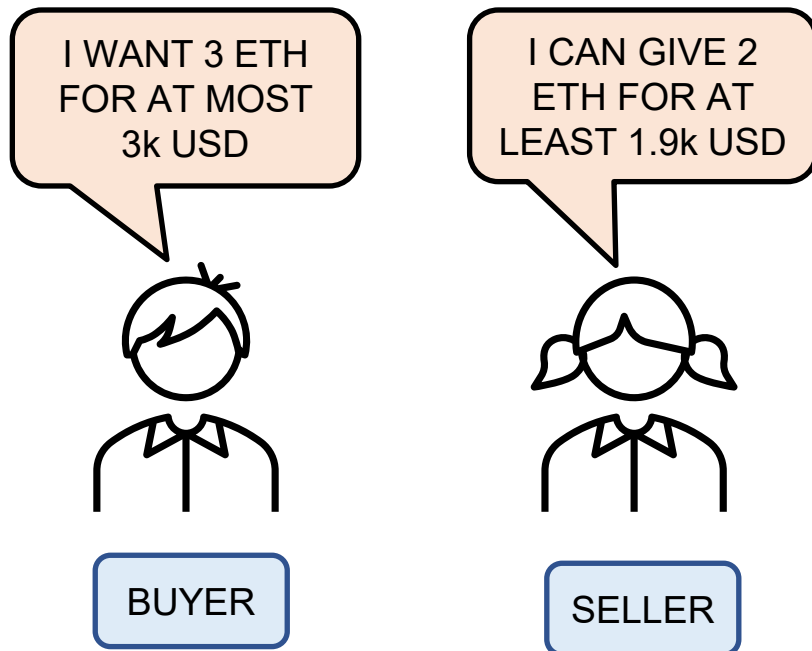
# Exchanges

- As a trader: want to swap token A with token B
- Need a place which lists people willing to do the reverse
- How would you state the intention to trade ?



# Limit Orders

- Intention to trade takes the form of a **limit order**
- What are the conditions for opposite limit orders to match?
- Can these two orders match ?
- Which order remains “on the book” after matching ?



# Limit Order Books

- Collection of such limit orders
- Orders coming in are of two types
  - Market orders (get satisfied)
  - Limit orders (stay on the book)
- Buy order with largest price = **bid**
- Sell order with least price = **ask**
- **Ask > Bid** always



# Limit Order Books

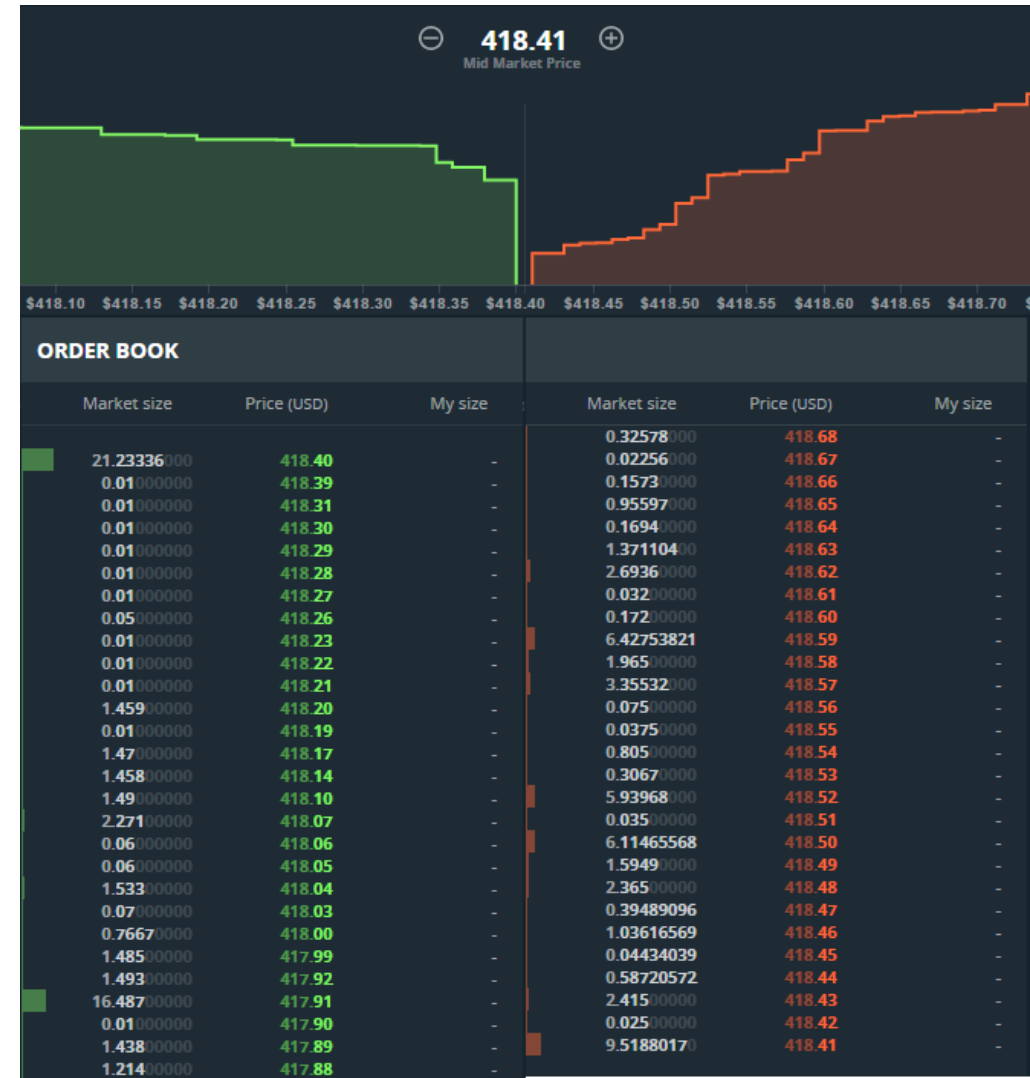
- Volume of limit orders = **liquidity**
- High liquidity markets have a very small **spread** (ask-bid), and vice-versa
- How would LOB look during a crash?
- Why would a trader want a more liquid market? - **slippage**





# Properties of LOB

- Centralized
- Negligible fees
- Fast matching
- No loss due to price fluctuation – infact they are very profitable in periods of volatility
- Incentivizes market makers to provide liquidity



# Centralized exchanges

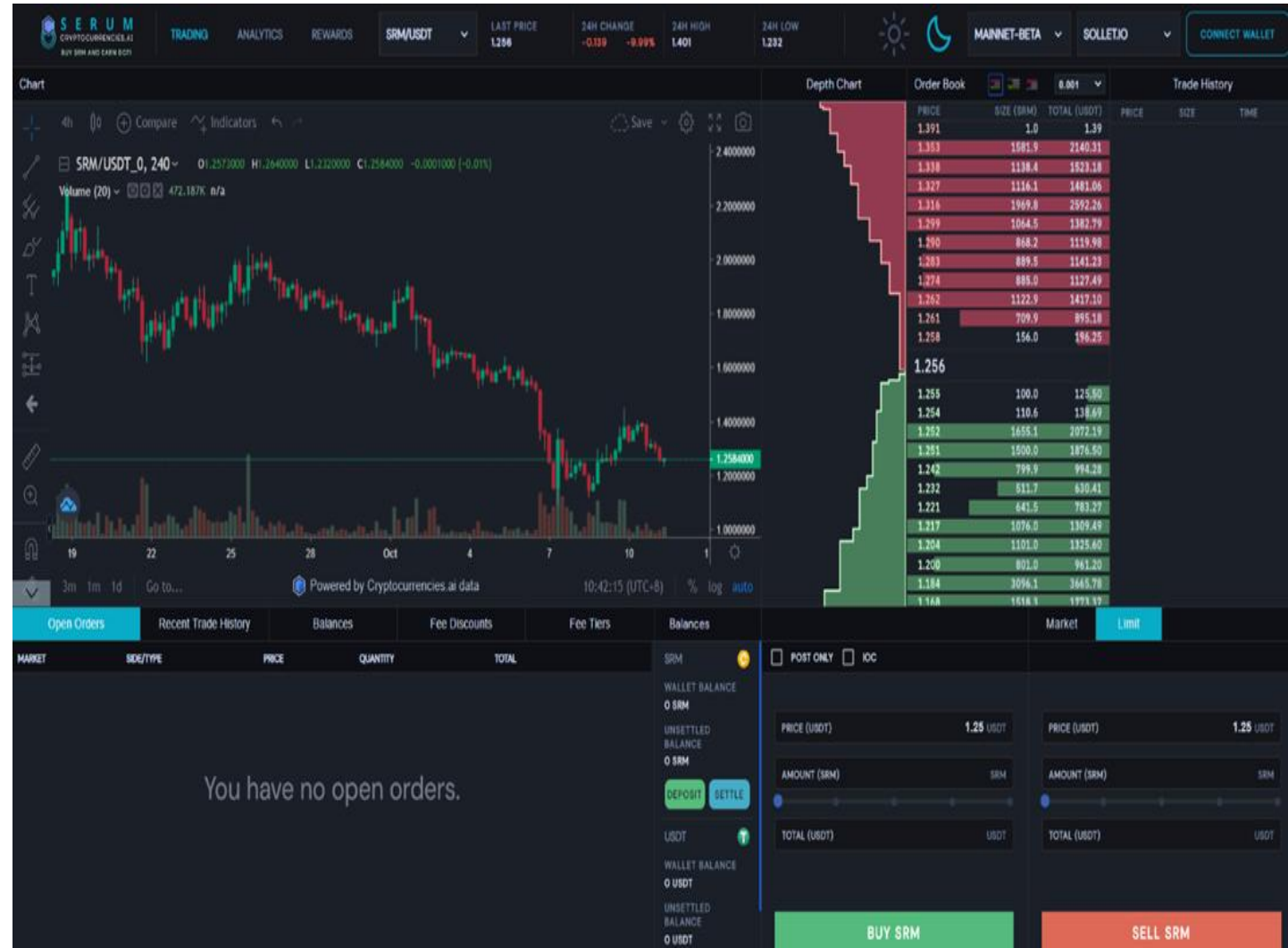
- Traditional exchanges - NYSE, NASDAQ, Shenzhen, ICE
  - Each handles \$1-3 trillion in volume
  - Need to be heavily regulated to avoid insider trading
  - Drives down profit margin
- Blockchain exchanges – Binance, Coinbase, FTX
  - Frequent regulatory issues leading to bans – Binance
  - Safety of customer funds not guaranteed – FTX
  - Need solutions like proof-of-reserves
  - Transfer custodianship of tokens – defeats the point of blockchains

# Decentralize?

- Convenient and transparent for swapping blockchain tokens
- The exchange knows about your order before the rest of the market – **frontrunning**
- Exchange has the power to **cancel** txns
- e.g. Decentralized Exchanges – **Uniswap, Curve, Balancer**

# Decentralized LOB

- Examples : Serum, **Demex**
- Order matching verified by everyone on chain
- Permissionless
- A liquidity provider can cancel and rearrange orders

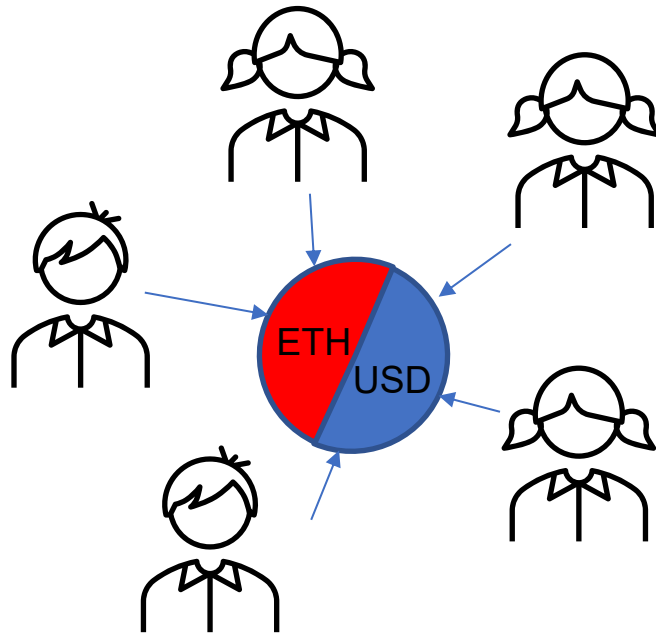


# Shortcomings of Decentralized LOB

- Need high volume markets to work well – otherwise takes long time to satisfy an order
- Need to satisfy orders when traders arrive at non-overlapping times
- Fees to cancel and shift limit orders
- Higher fees than in centralized LOB

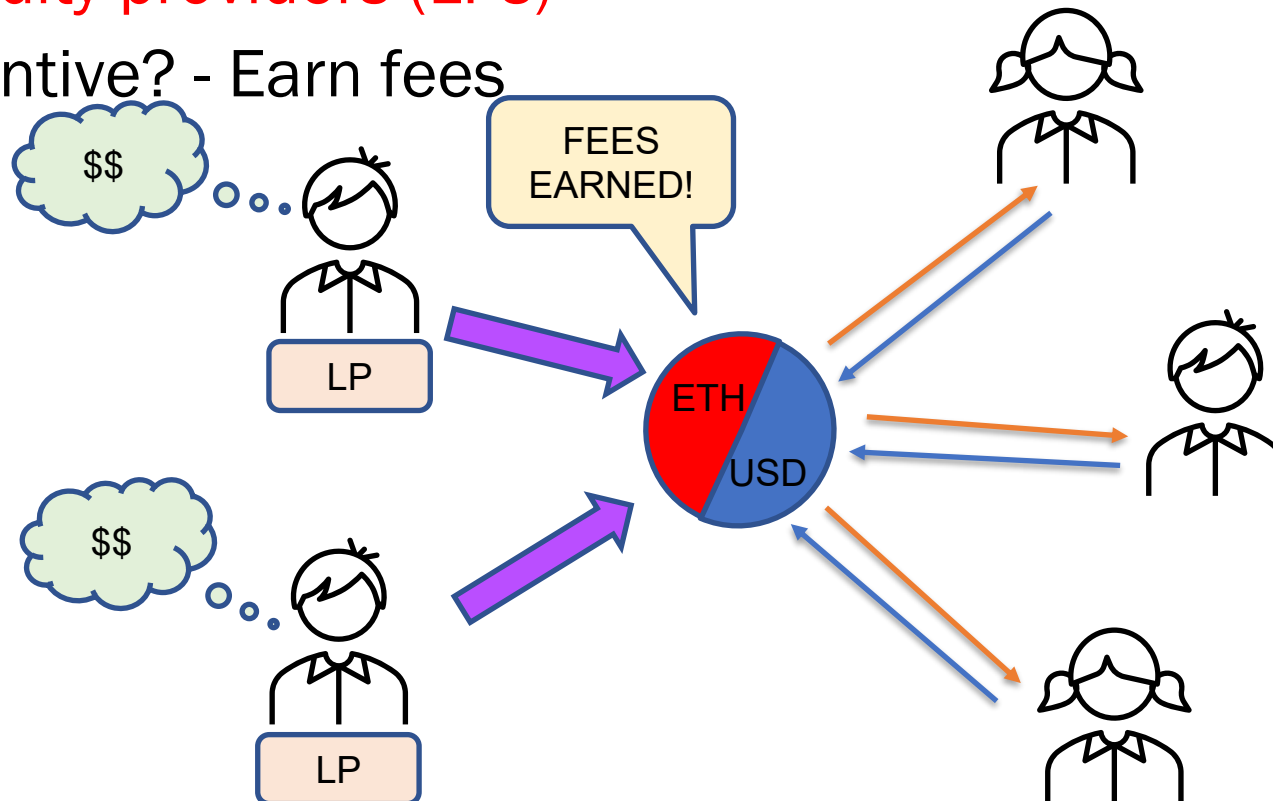
# Automated Market Makers

- Need to guarantee instant exchange when volume is lacking
- Idea : **Peer-Pool-Peer** instead of **Peer-Peer** matching



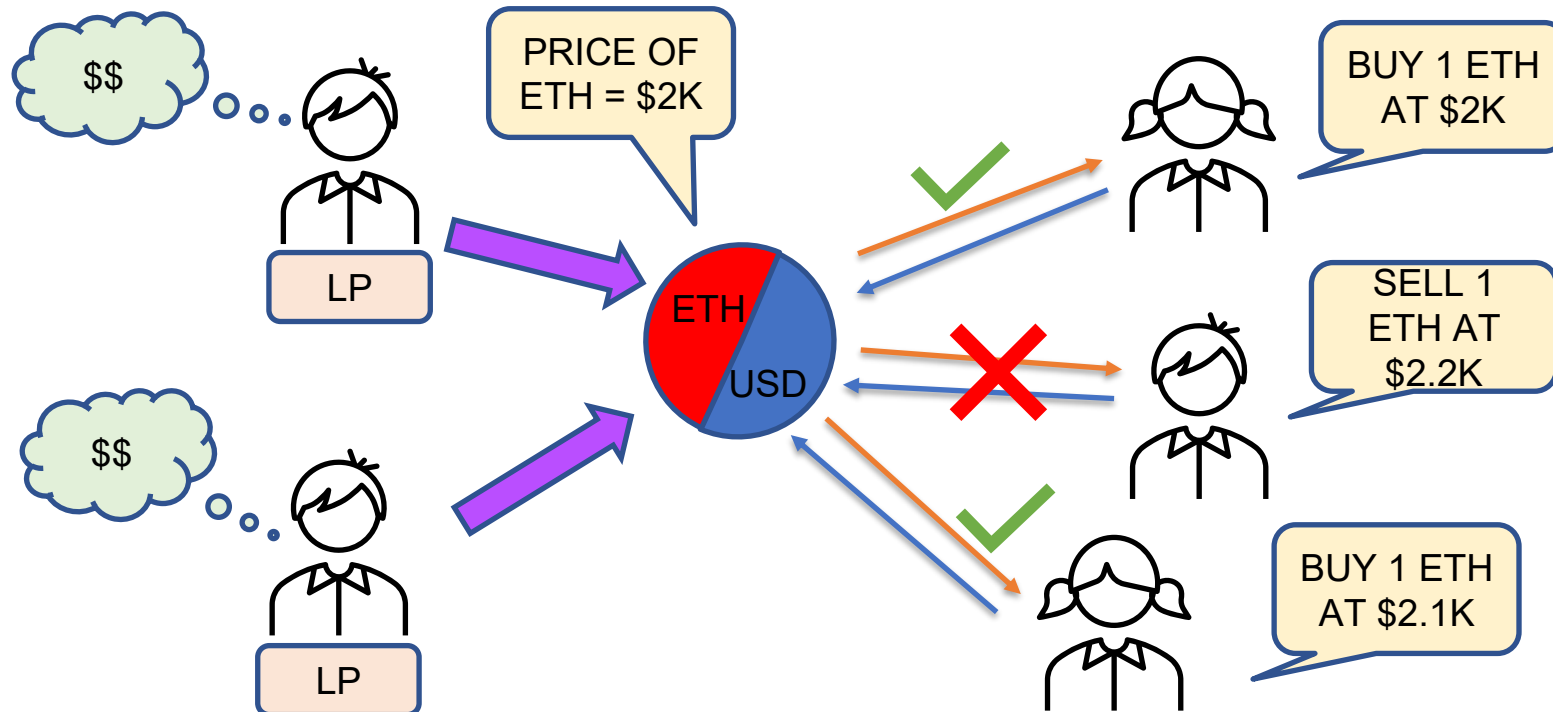
# Automated Market Makers

- Keep a **pool** of orders that can satisfy any incoming trade
- Entities with large amount of idle liquidity pitch in to make the pool—**liquidity providers (LPs)**
- Incentive? - Earn fees



# Automated Market Makers

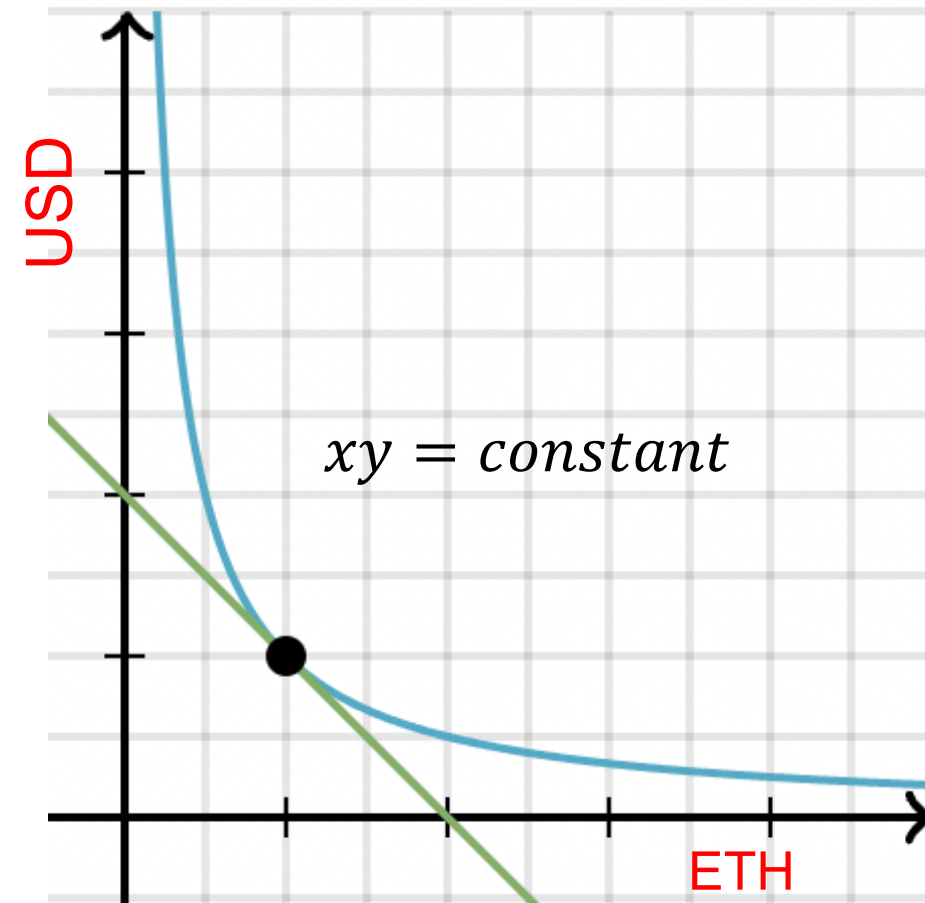
- LPs do not constrain prices like in LOB
- How do you decide prices? Which trades are allowed?
- Make the reserves/inventory of the AMM follow specific rules





# A solution: CFMMs

- CFMM: **Constant Function Market Makers**
- Use **Bonding Curves** to constrain reserves
- Intuition:
  - What happens when AMM has lots of ETH?
  - What happens when AMM has lots of USD?



# CFMMs: general model

- Each CFMM pool between tokens A and B have reserves that satisfy

$$\psi(x, y) = \psi(x + \Delta_x, y - \Delta_y)$$

OR

$$\psi(x, y) = \textit{constant}$$

- Here  $x$  = reserves of token A, and  $y$  = reserves of token B
- Each trade earns **fees** – given to liquidity providers

# Top CFMMs

- Uniswap, Sushiswap:

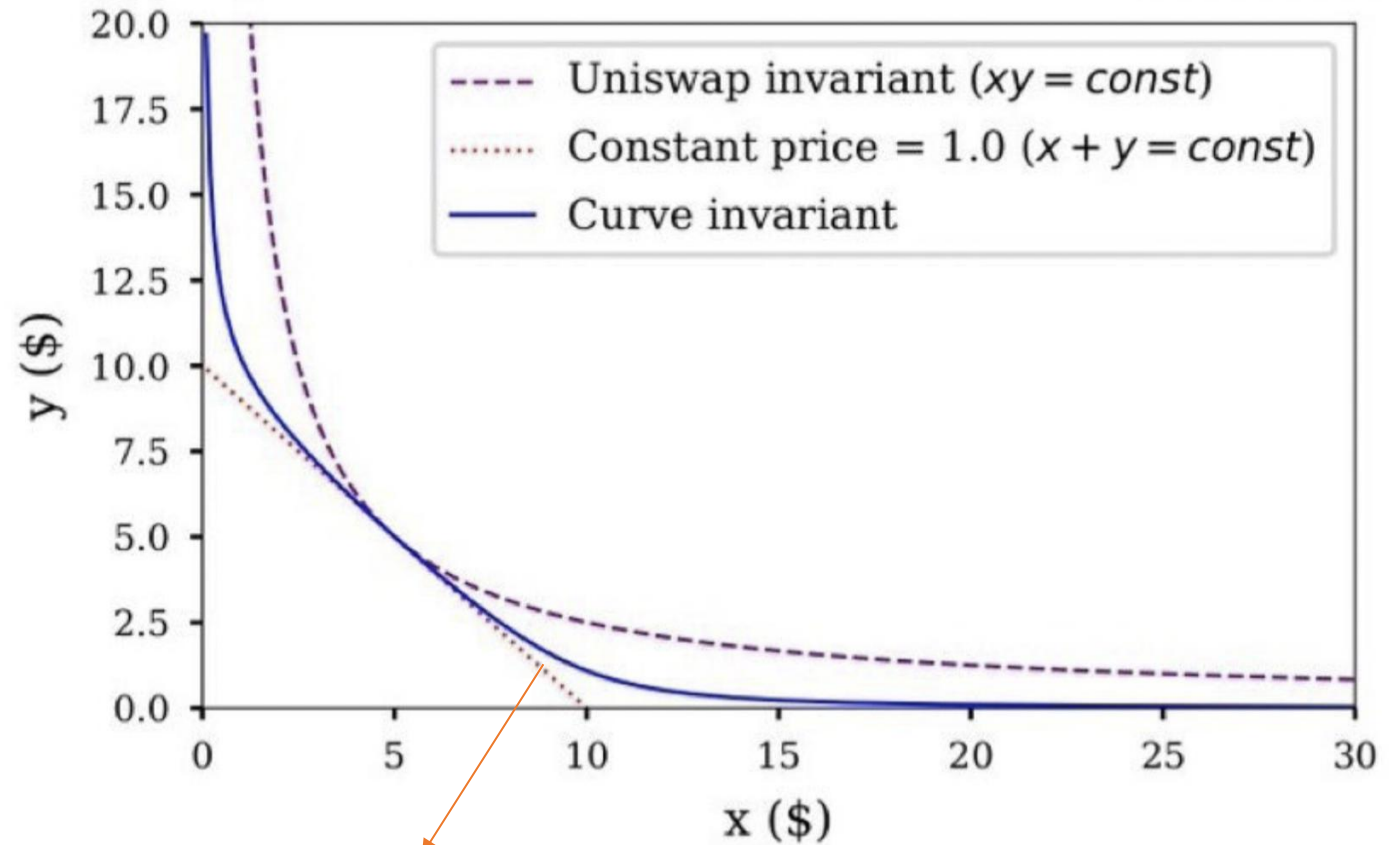
$$xy = \text{constant}$$

- Balancer:

$$x^\theta y^{1-\theta} = \text{constant}$$

- Curve:

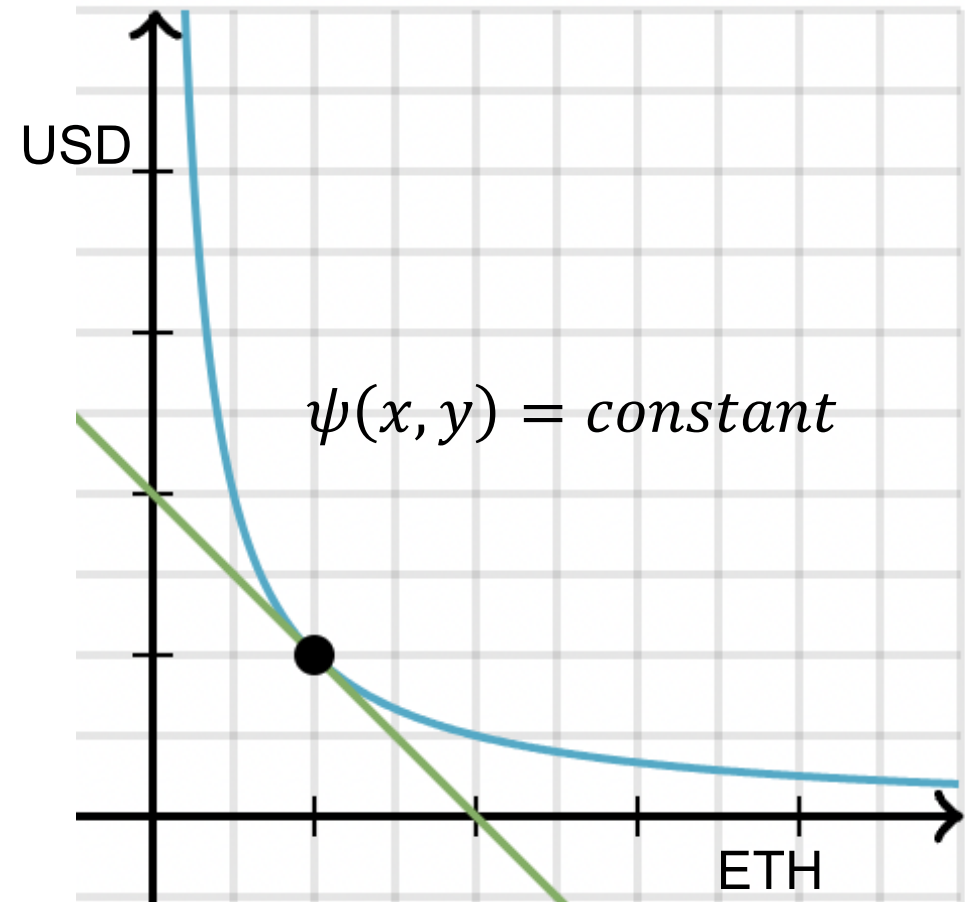
$$x + y + \frac{\alpha}{xy} = \text{constant}$$



Why is this curve called constant price?

# Pricing in CFMMs

- Intuition from last slide :  
constant price = constant slope
- For a general curve what is the **price** at any point?
- **Slope of the tangent**
- Formula for price of ETH in terms of USD?



# Pricing in CFMMs

- Formula for price of ETH in terms of USD?

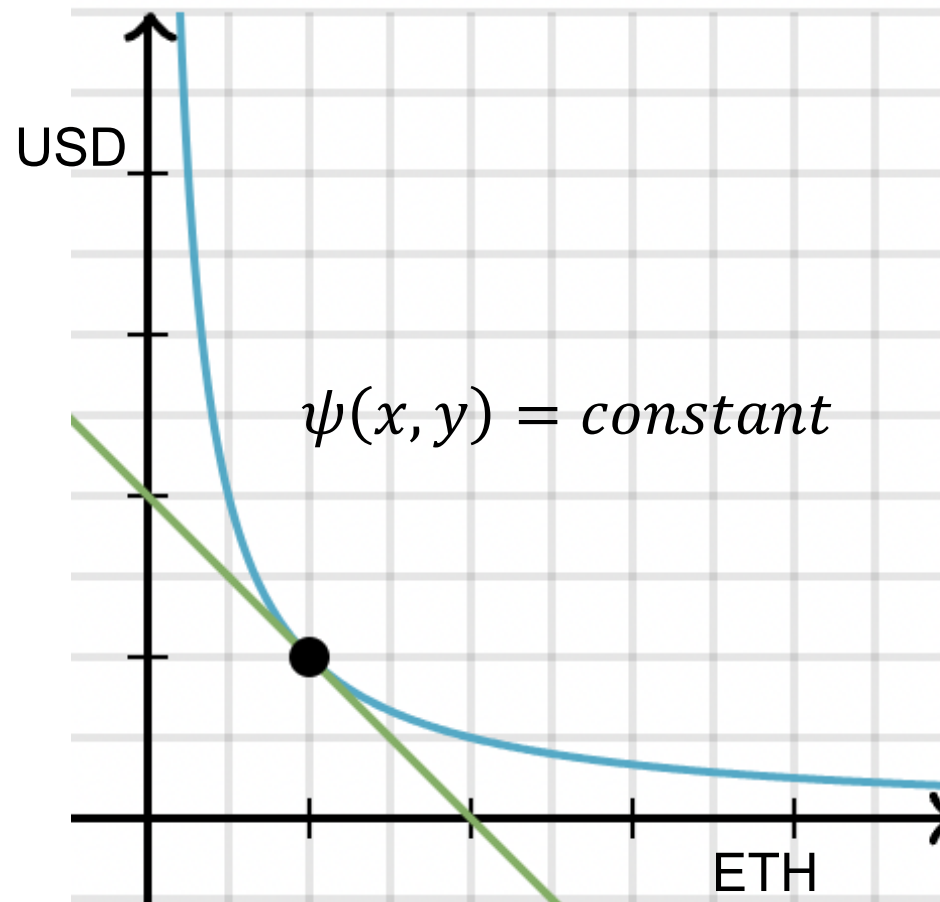
$$\text{Price} = P_x = \frac{dy}{dx} = - \frac{\partial_x \psi}{\partial_y \psi}$$

WHY?

$$d\psi(x, y) = 0$$

$$\Rightarrow \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0$$

- Example :
  - $xy = \text{constant}$  – what is the price?



# Popular CFMM and Pricing

- Uniswap, Sushiswap:

$$xy = \text{constant} \quad P_x = \frac{y}{x}$$

- Balancer:

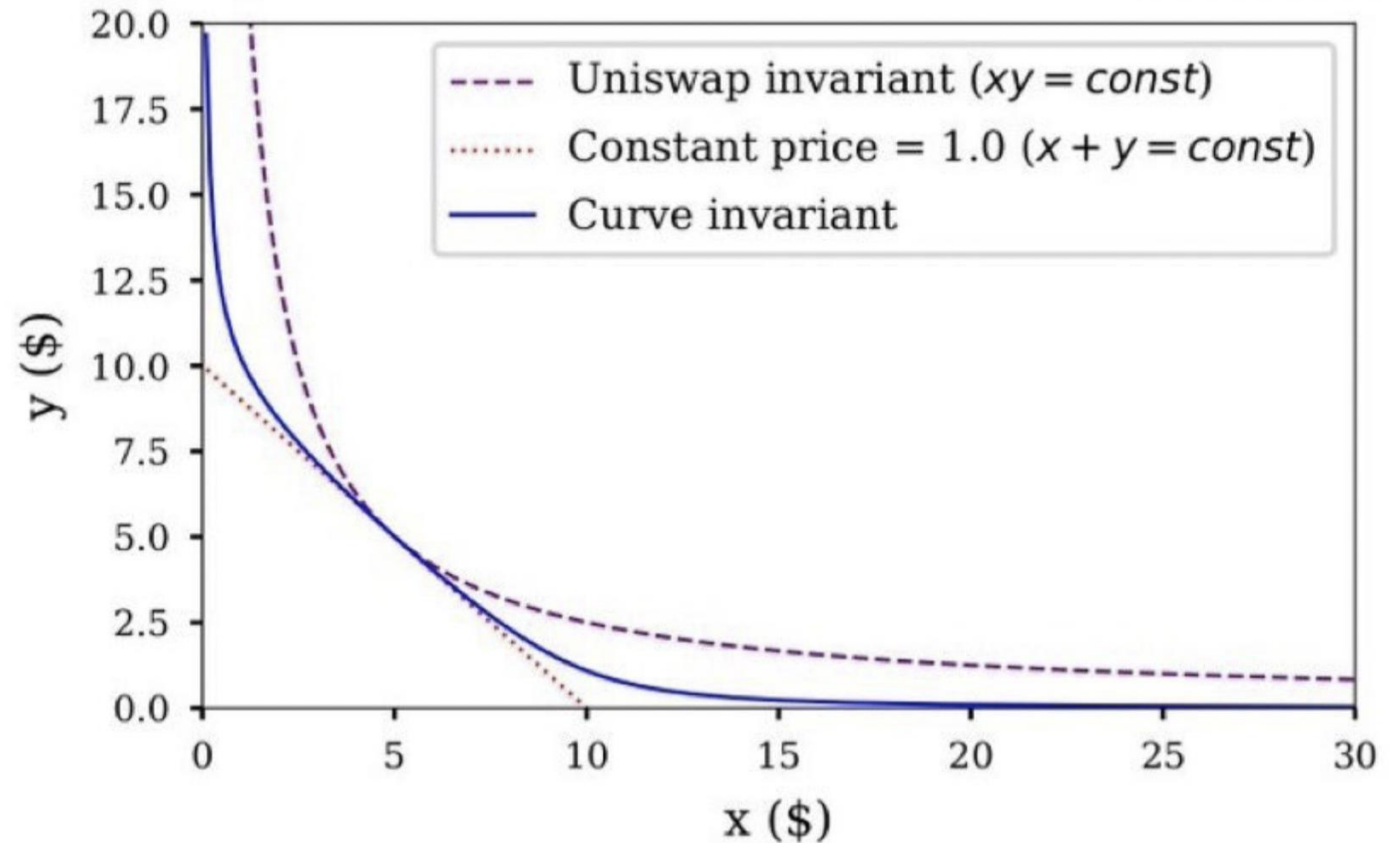
$$x^\theta y^{1-\theta} = \text{constant} \quad P_x = \frac{\theta y}{(1-\theta)x}$$

- Constant Price :

$$x + y = \text{constant} \quad P_x = 1$$

- Curve:

$$x + y + \frac{\alpha}{xy} = \text{constant}$$



# Conclusion

- Ways of exchanging assets in DeFi
  - CLOB
  - AMM
- Focus on AMMs
  - Inspired from betting markets
  - CFMMs and their pricing

# Next Lecture

- What curve to pick?
- Trader strategies
  - Routing
- Liquidity Provider performance
  - Arbitrage, exposure to informed traders
- Efficient CFMM:
  - Fees set such that both liquidity providers and traders are satisfied



LECTURE ENDS

# CFMM origins : betting markets

- Suppose you want the forecast of an uncertain event (weather, elections, sport) from a group of experts
- Event has N outcomes, each expert gives pmf p as their prediction
- If outcome 'i' happens, then each expert rewarded  $S(p,i)$
- What we want : every expert to speak the truth
- How should S be to incentivize each expert to be truthful?

$$q = \operatorname{argmax}_p \sum_{i=1 \dots N} q_i S(p, i)$$

# CFMM origins : betting markets

$$q = \operatorname{argmax}_p \sum_{i=1 \dots N} q_i S(p, i)$$

- What happens if  $S(p, i) = p_i$  ?
- Is there a truthful scoring rule ?
- **Yes** – there are many!
- Examples :

$$S(p, i) = p_i - \sum_{j=1 \dots N} \frac{p_j^2}{2}$$

$$S(p, i) = \ln(p_i)$$

# CFMM origins : betting markets

- So far, each bet was only interacting with one “expert”
- What if we want aggregate opinion by large group of people?
- Use the scoring rule sequentially
  - Start at  $t=0$ , with beliefs set at some  $q_0$
  - At time  $t$ , a trader reports their belief  $q_t$  if it is different from  $q_{(t-1)}$
  - At time  $T$ , check outcome and reward trader  $t$  with  $S(q_{t,i}) - S(q_{(t-1),i})$
- Truthful  $S$  stay the same! Why?

# CFMM origins : betting markets

- Problem : Reporting pmfs is unintuitive – how can we make this more like a real market?

