



# Searching for primordial features from CMB and LSS surveys

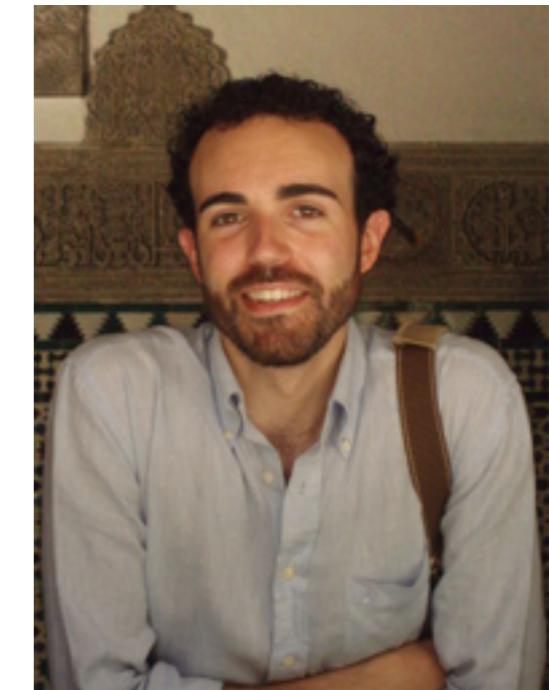
collab. with **A. Achucarro, V. Atal, P. Ortiz, J. Torrado**



- [PRD 89 (2014) 103006]
- [PRD 90 (2014) 023511]
- [PRD 91 (2015) 064039]

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# Outline

1. State-of-the-art of inflation from observational point of view
2. Observational hints of oscillatory features
3. Models with a transient reduction of the speed of sound
4. Search with CMB power spectrum
5. Search with LSS survey
6. Conclusion

# What do we mean by inflation from a phenomenological point of view?

$A_s$  — the size of the primordial scalar power spectrum

$n_s$  — the power index of the primordial scalar power spectrum

$r$  — the primordial tensor-to-scalar ratio

$f_{nl}$  — the size of 3 point function of primordial curvature fluctuations

$$P_s(k) \sim A_s \left( \frac{k}{k_0} \right)^{n_s - 1}$$

$$P_t(k) \sim A_t \left( \frac{k}{k_p} \right)^{n_t}$$

$$r \sim \frac{P_t(k_*)}{P_s(k_*)}$$

$$B_\Phi(k_1, k_2, k_3) = f_{NL} F(k_1, k_2, k_3).$$

[1] Parameter	[2] 2013N(DS)	[6] 2015F(CHM) (Plik)	([2] – [6])/ $\sigma_{[6]}$
$100\theta_{\text{MC}}$	$1.04131 \pm 0.00063$	$1.04086 \pm 0.00048$	0.71
$\Omega_b h^2$	$0.02205 \pm 0.00028$	$0.02222 \pm 0.00023$	-0.61
$\Omega_c h^2$	$0.1199 \pm 0.0027$	$0.1199 \pm 0.0022$	0.00
$H_0$	$67.3 \pm 1.2$	$67.26 \pm 0.98$	0.03
$n_s$	$0.9603 \pm 0.0073$	$0.9652 \pm 0.0062$	-0.67
$\Omega_m$	$0.315 \pm 0.017$	$0.316 \pm 0.014$	-0.06
$\sigma_8$	$0.829 \pm 0.012$	$0.830 \pm 0.015$	-0.08
$\tau$	$0.089 \pm 0.013$	$0.078 \pm 0.019$	0.85 
$10^9 A_s e^{-2\tau}$	$1.836 \pm 0.013$	$1.881 \pm 0.014$	-3.46 

Planck2013 TT  
+WMAP low-ell  
polarization

Planck2015 TT  
+Planck2015 low-ell  
polarization

Tension  
between  
2013 and 2015

$A_s \sim 4E-9$

$r < 0.12$  at 95%  
BKP

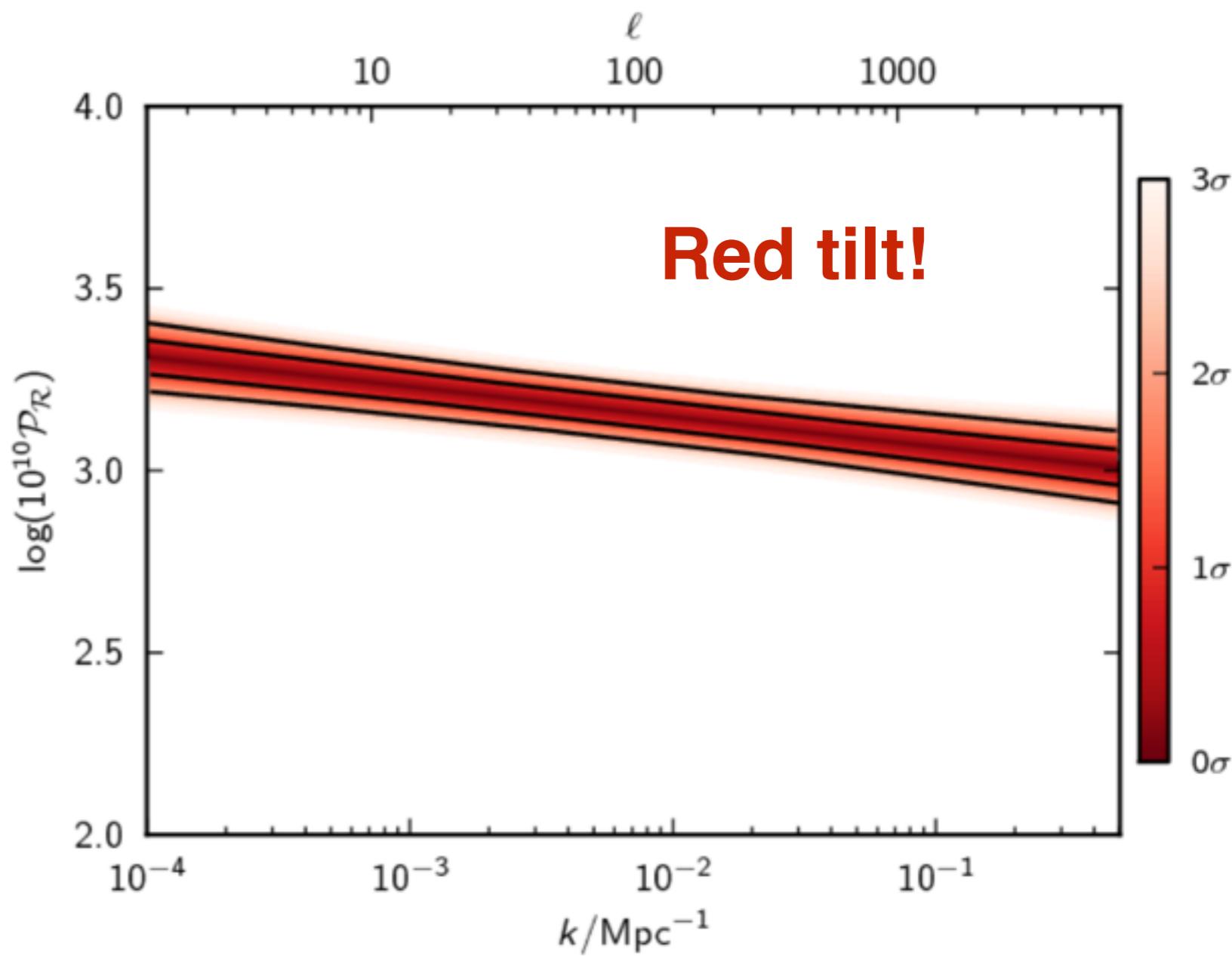
All primordial type  
 $f_{nl}$  are consistent with 0

# n\_s

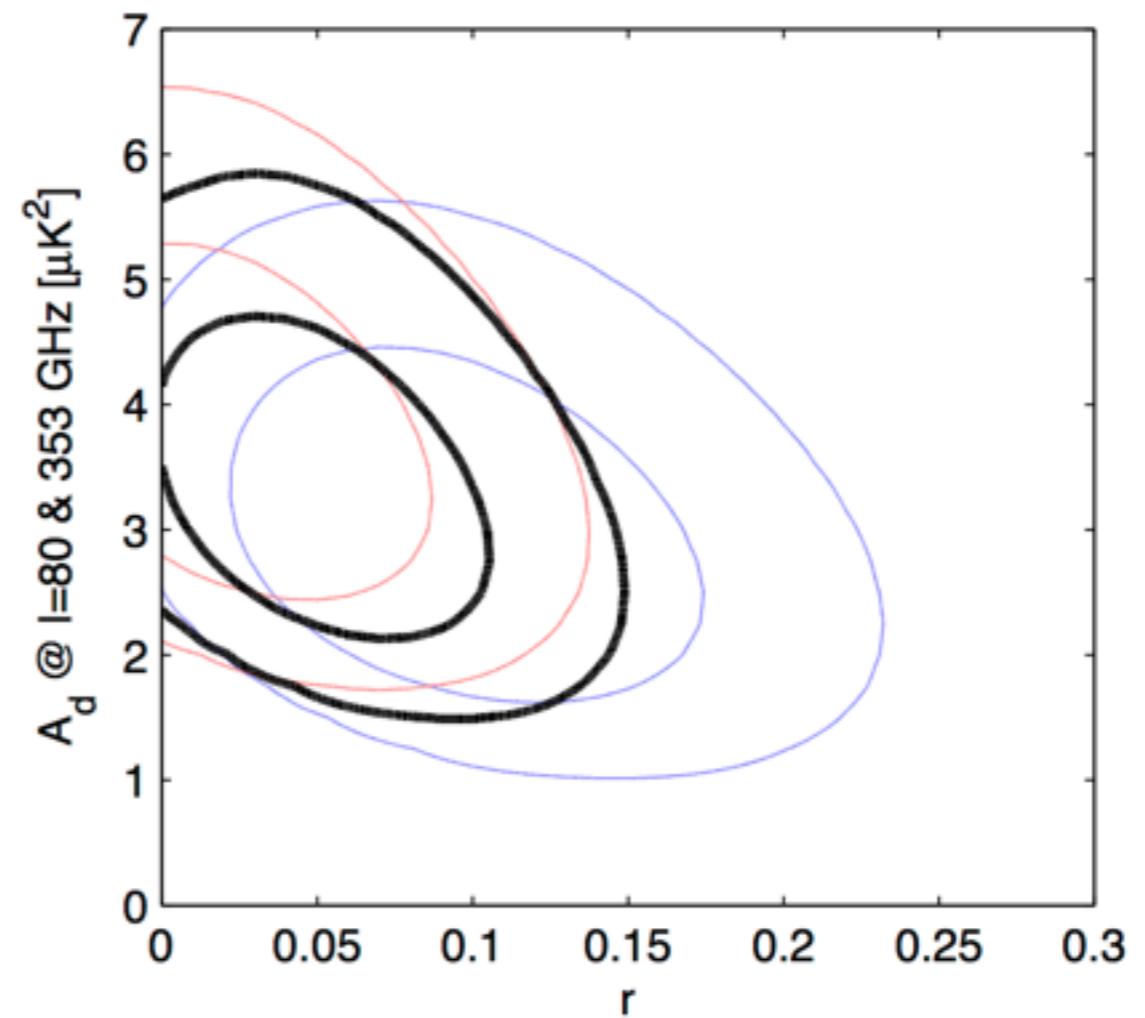
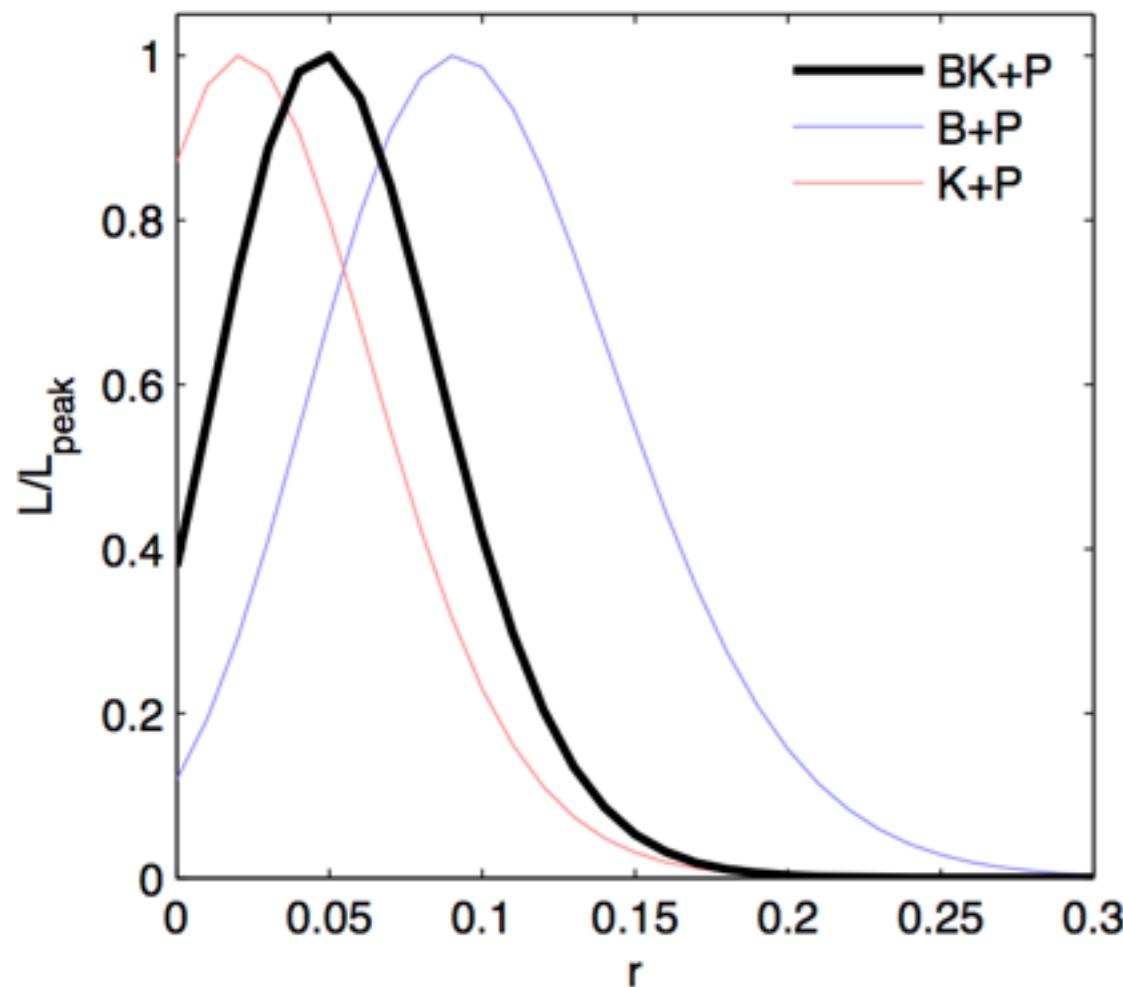
$$P_s(k) \sim A_s \left( \frac{k}{k_0} \right)^{n_s - 1}$$

$$0.9645 \pm 0.0049$$

Power spectrum is  
larger on low-k,  
smaller on high-k

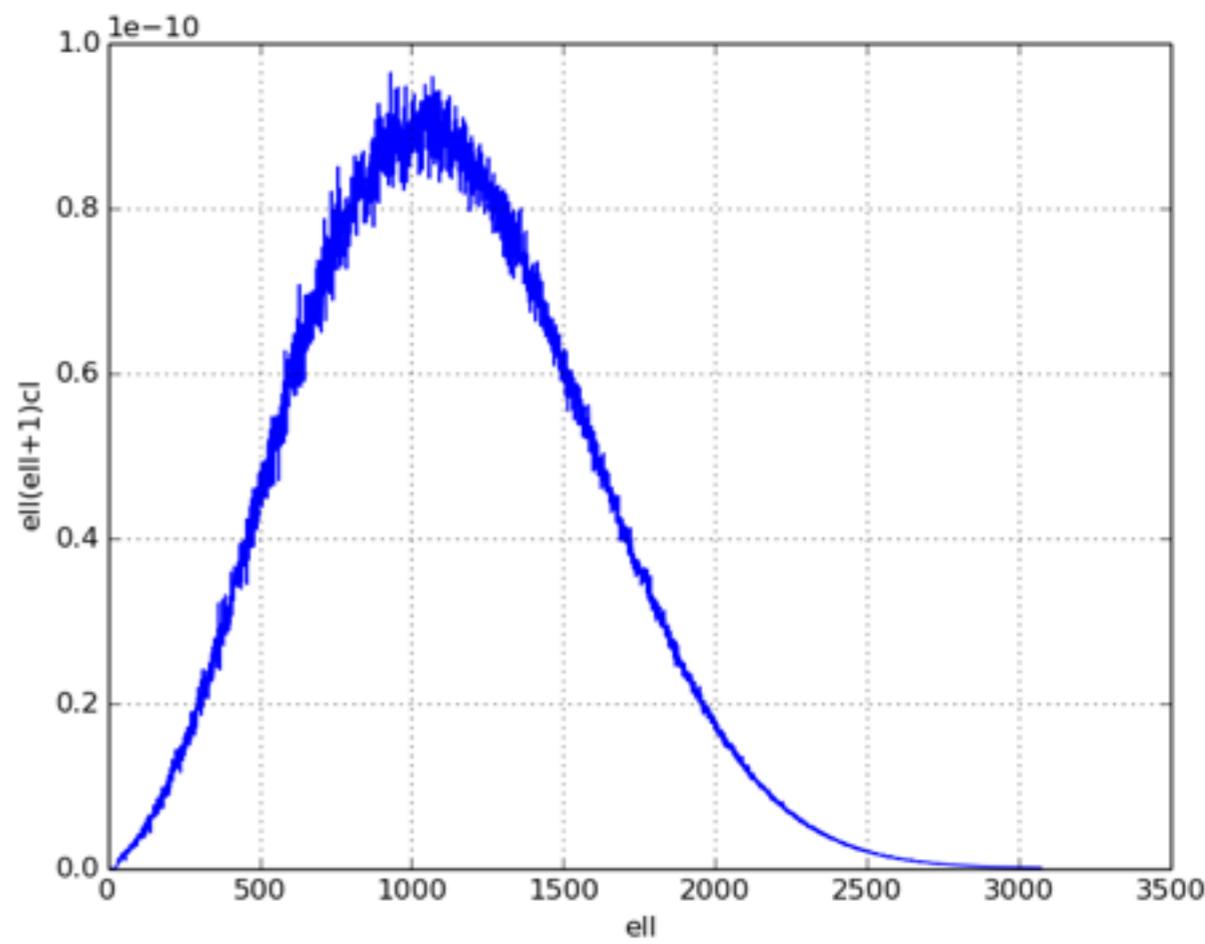
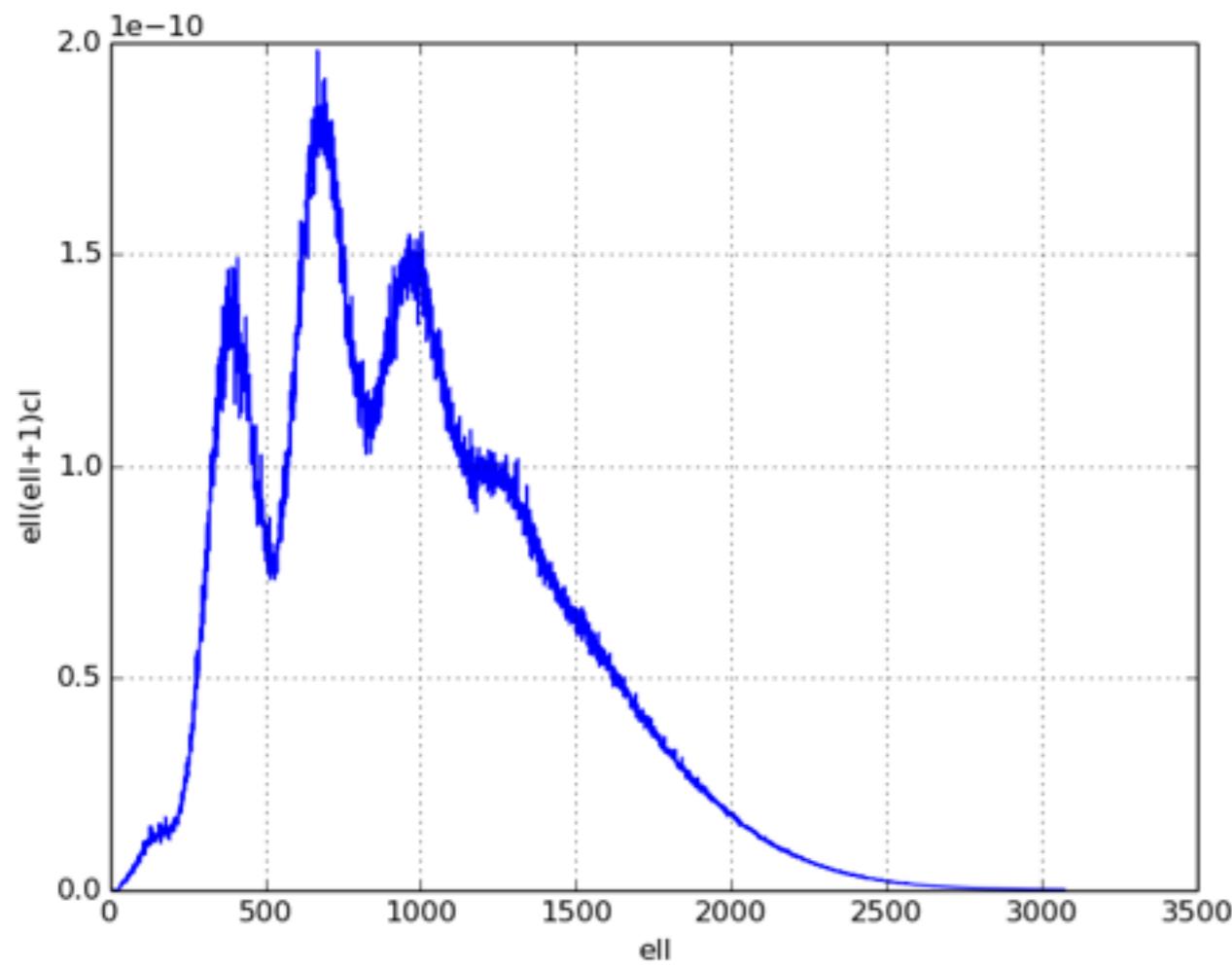


**r**



BICEP-II: NO detection of primordial B-mode!

Dust contamination!



**Say CMB died ONCE MORE!**



$$\ell(\ell+1)C_\ell^{\text{BB}}/(2\pi) (\mu\text{K}^2)$$

# A Instrumental specifications

Experiments	$f_{sky} [\%]$	$\nu [GHz]$	$\theta_{FWHW} [']$	$\delta P [\mu K']$
AdvACT	50	90	2.2	7.8
	50	150	1.3	6.9
	50	230	0.9	25
CLASS	70	38	90	39
	70	93	40	13
	70	148	24	15
	70	217	18	43
Keck/BICEP3	1	95	30	9.0
	1	150	30	2.3
	1	220	30	10
Simons Array	20	90	5.2	15.2
	20	150	3.5	12.3
	20	220	2.7	23.6
SPT-3G	6	95	1	6.0
	6	150	1	3.5
	6	220	1	6.0
EBEX	1	150	8	5.8
	1	250	8	17
	1	410	8	150
Spider	7.5	94	49	17.8
	7.5	150	30	13.6
	7.5	280	17	52.6
CMBPol	70	30	26	19.2
	70	45	17	8.3
	70	70	11	4.2
	70	100	8	3.2
	70	150	5	3.1
	70	220	3.5	4.8
	70	340	2.3	21.6
CoRE	70	45	23	9.1
	70	75	14	4.7
	70	105	10	4.6
	70	135	7.8	4.6
	70	165	6.4	4.6
	70	195	5.4	4.5
	70	225	4.7	4.6
	70	255	4.1	10.5
	70	285	3.7	17.4
	70	315	3.3	46.6
	70	375	2.8	119
LiteBIRD	70	60	32	10.3
	70	78	58	6.5
	70	100	45	4.7
	70	140	32	3.7
	70	195	24	3.1
	70	280	16	3.8

2013

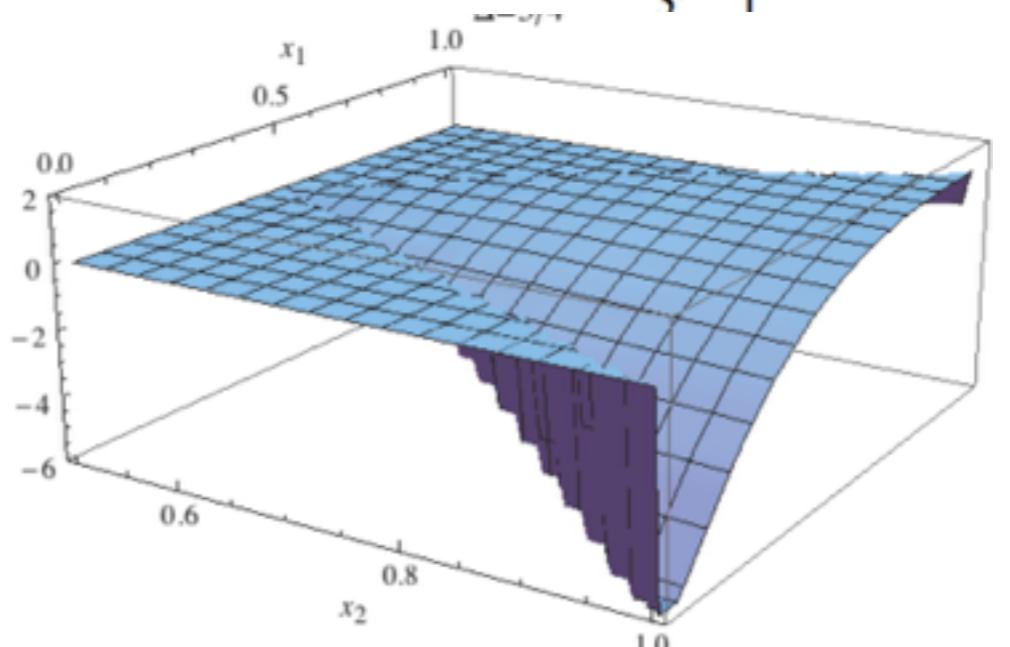
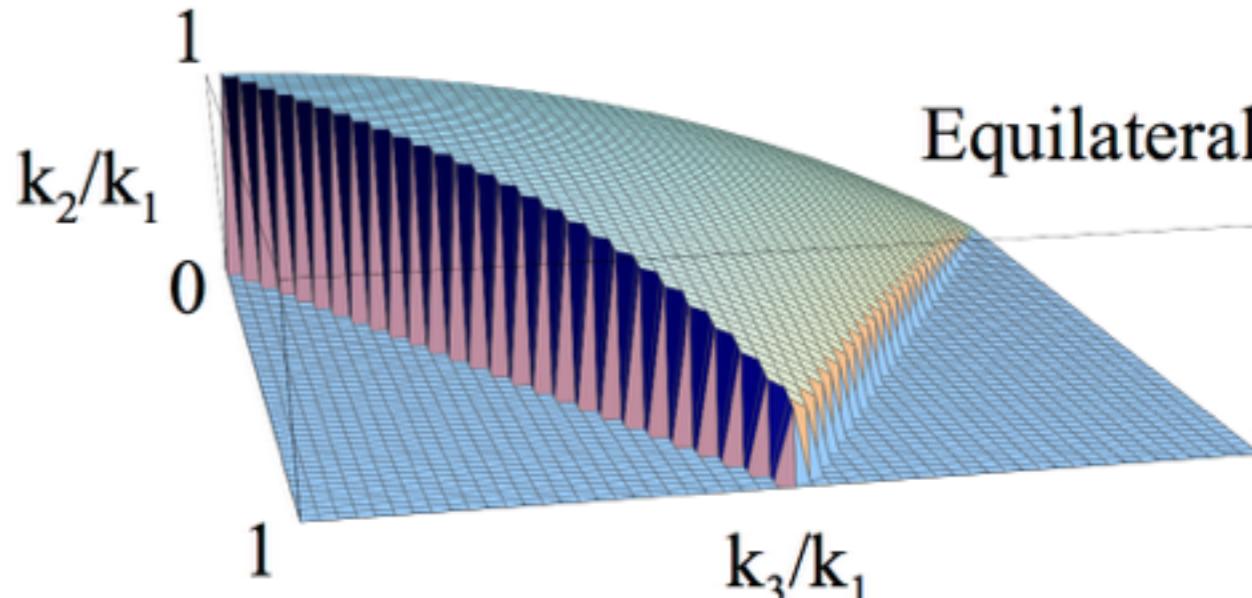
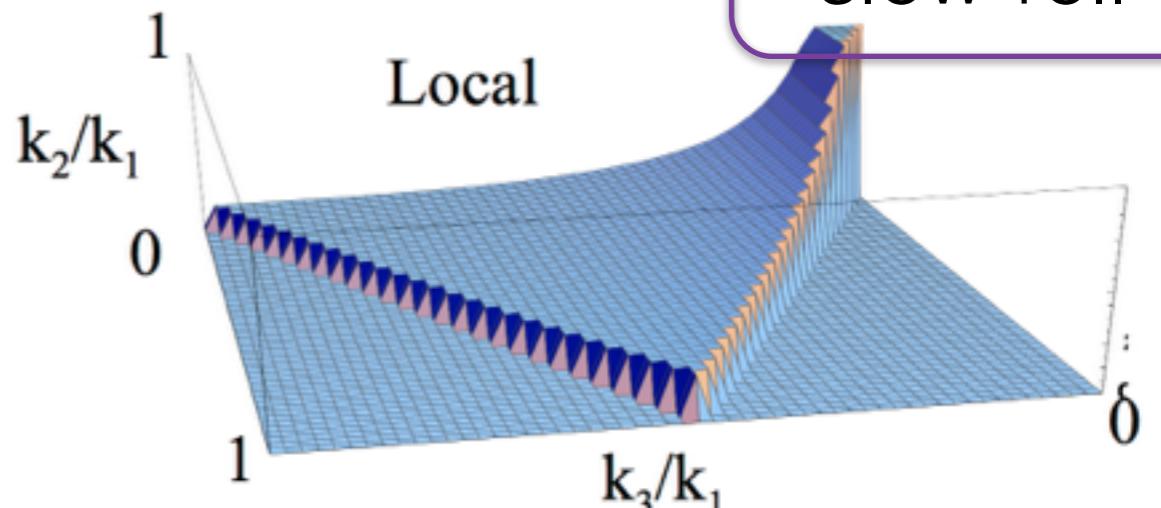
$f_{\text{nl}}$

Shape & Method	$f_{\text{NL}}(\text{KSW})$	
	Independent	ISW-lensing subtracted
SMICA		
Local .....	$9.8 \pm 5.8$	<b><math>2.7 \pm 5.8</math></b>
Equilateral .....	$-37 \pm 75$	<b><math>-42 \pm 75</math></b>
Orthogonal .....	$-46 \pm 39$	<b><math>-25 \pm 39</math></b>

2015

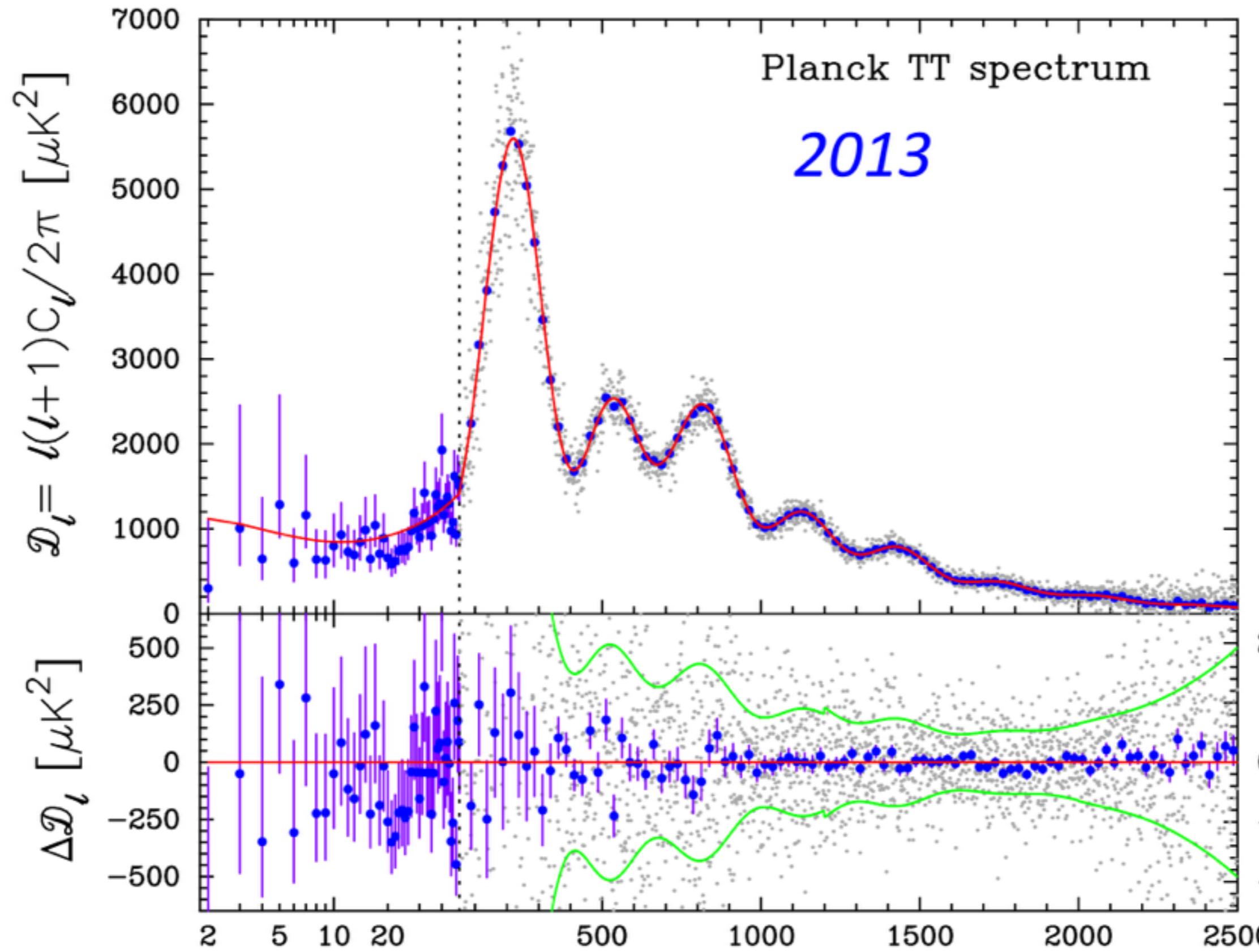
non-canonical  
kinetic, e.g. DBI

Shape and method	$f_{\text{NL}}(\text{KSW})$	
	Independent	ISW-lensing subtracted
SMICA ( $T$ )		
Local .....	$10.2 \pm 5.7$	<b><math>2.5 \pm 5.7</math></b>
Equilateral .....	$-13 \pm 70$	<b><math>-16 \pm 70</math></b>
Orthogonal .....	$-56 \pm 33$	<b><math>-34 \pm 33</math></b>
SMICA ( $T+E$ )		
Local .....	$6.5 \pm 5.0$	<b><math>0.8 \pm 5.0</math></b>
Equilateral .....	$3 \pm 43$	<b><math>-4 \pm 43</math></b>
Orthogonal .....	$-36 \pm 21$	<b><math>-26 \pm 21</math></b>

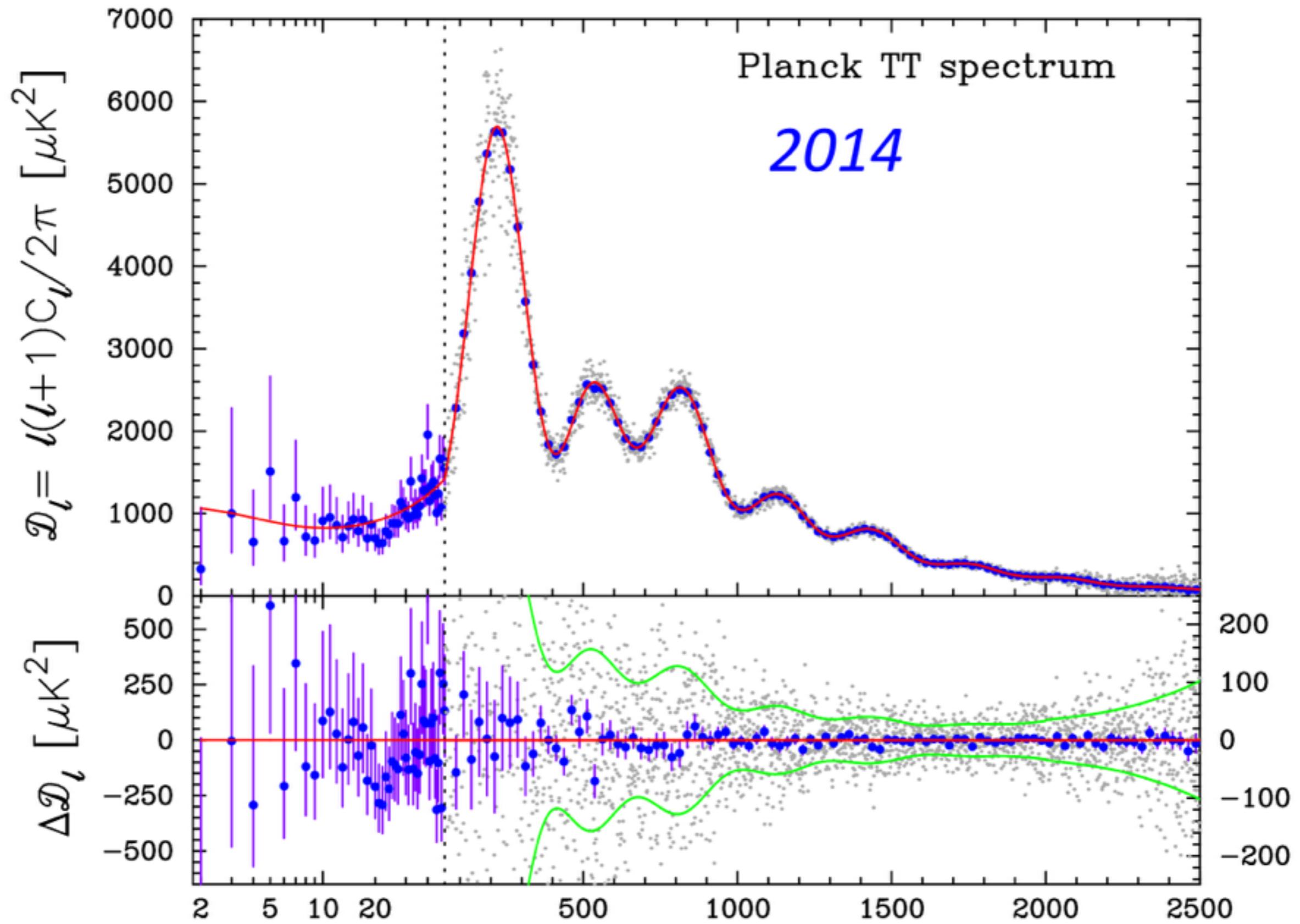


single field  
slow-roll

**A\_s**



Multipole  $l$  Efstathiou @ Ferrara



*preliminary*

Multipole  $l$  Efstathiou @ Ferrara

# 1. As and tau

Absolute calibration:

1. Both 2013 and 2015 TT Planck likelihood code is composed by data from 100, 143, 217 GHz channels.
2. 2013 TT pipeline only calibrate 100 and 217 GHz relatively to 143 GHz channel, but fix the absolute amplitude in 143 GHz channel.
3. In 2015 pipeline, they do the same as 2013 for 100 and 217 GHz channel, but marginalise the absolute amplitude ( $y_p$ ) in 143 GHz. This overall calibration uncertainty is then propagated to  $As^*Exp(-2\tau)$ , effective amplitude of TT spectrum.

$$C_\ell^{TT} \sim \boxed{A_s e^{-2\tau}} \int |\Delta_\ell(k)|^2 k^{n_s - 1} d \log k$$

low tau: seems good news for warm DM. (have  
more time to form the structures before the  
formation of the star) 2013

$$\tau = 0.089 \pm 0.032 \quad (68\%; \textit{Planck+lensing}).$$

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$$\tau = 0.078_{-0.019}^{+0.019}, z_{\text{re}} = 9.9_{-1.6}^{+1.8}, \textit{Planck TT+lowP}; \quad (17a)$$

$$\tau = 0.070_{-0.024}^{+0.024}, z_{\text{re}} = 9.0_{-2.1}^{+2.5}, \textit{Planck TT+lensing}; \quad (17b)$$

$$\tau = 0.066_{-0.016}^{+0.016}, z_{\text{re}} = 8.8_{-1.4}^{+1.7}, \textit{Planck TT+lowP} \quad (17c)$$

+lensing;

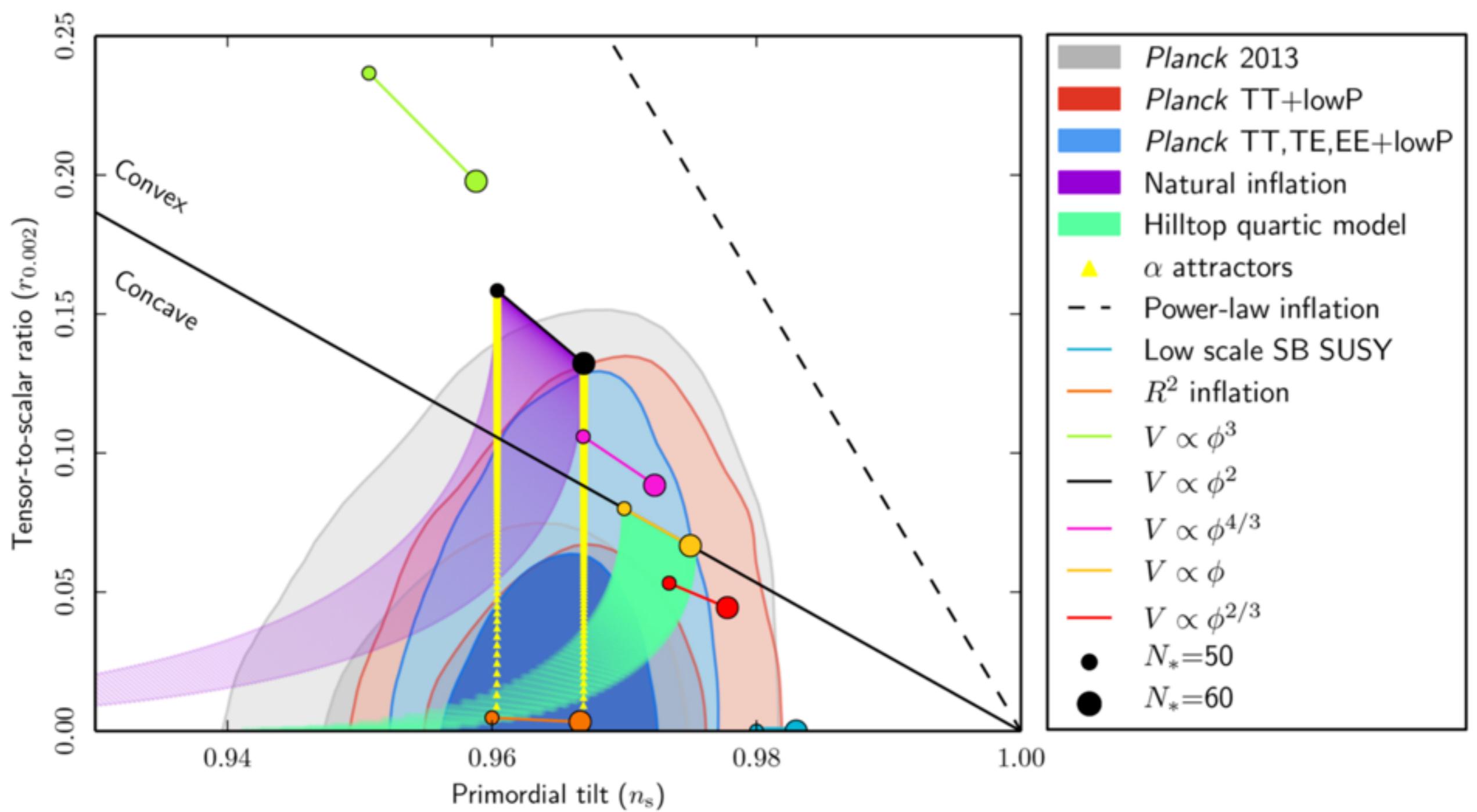
$$\tau = 0.067_{-0.016}^{+0.016}, z_{\text{re}} = 8.9_{-1.4}^{+1.7}, \textit{Planck TT+lensing} \quad (17d)$$

+BAO;

$$\tau = 0.066_{-0.013}^{+0.013}, z_{\text{re}} = 8.8_{-1.2}^{+1.3}, \textit{Planck TT+lowP} \quad (17e)$$

+lensing+BAO.

# Inflationary model



**Fig. 12.** Marginalized joint 68 % and 95 % CL regions for  $n_s$  and  $r_{0.002}$  from *Planck* in combination with other data sets, compared to the theoretical predictions of selected inflationary models.

$r \sim 16/N \sim 0.1$

Two classes of inflation

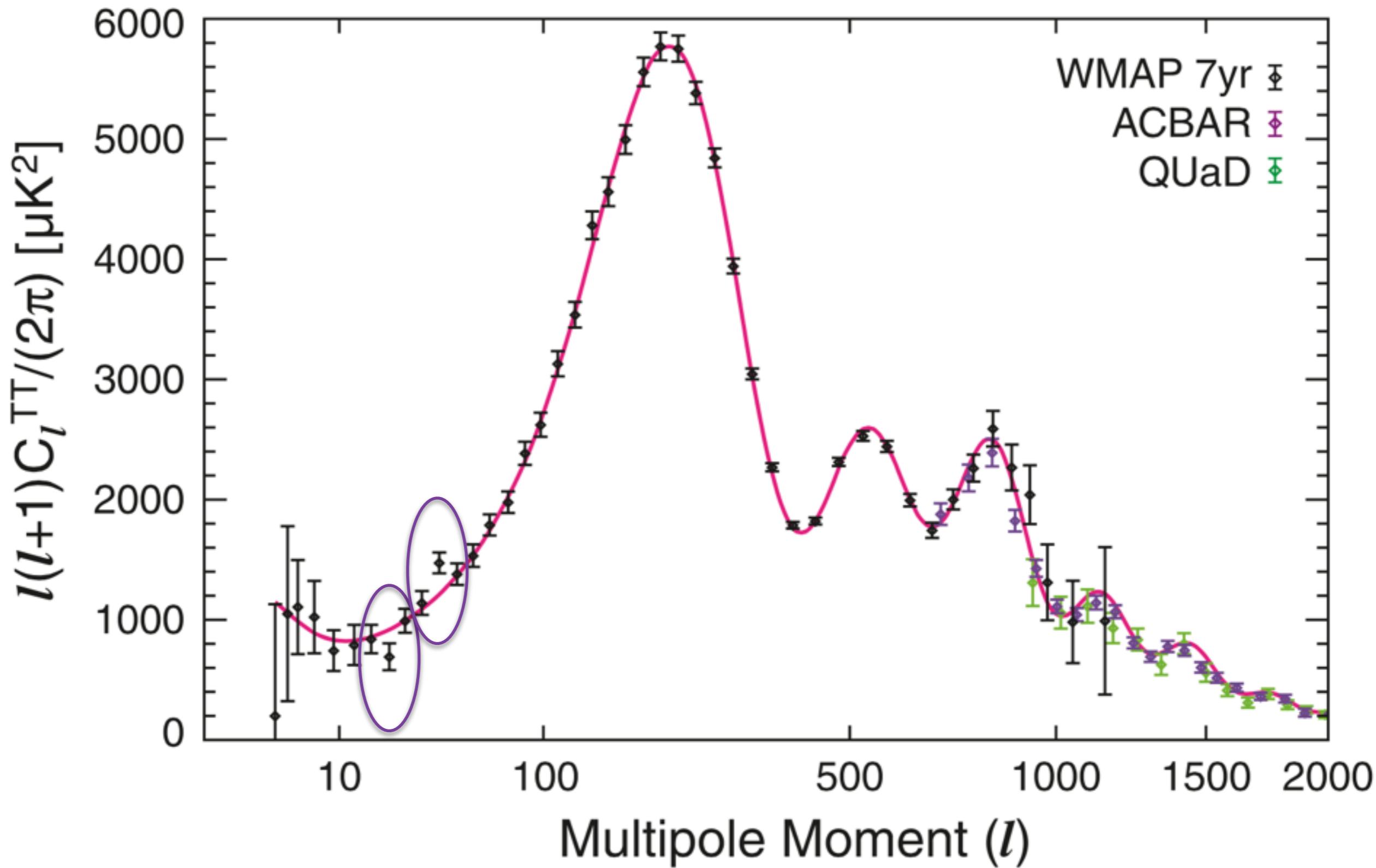
[D.Roest, JCAP 1401 (2014) 007]

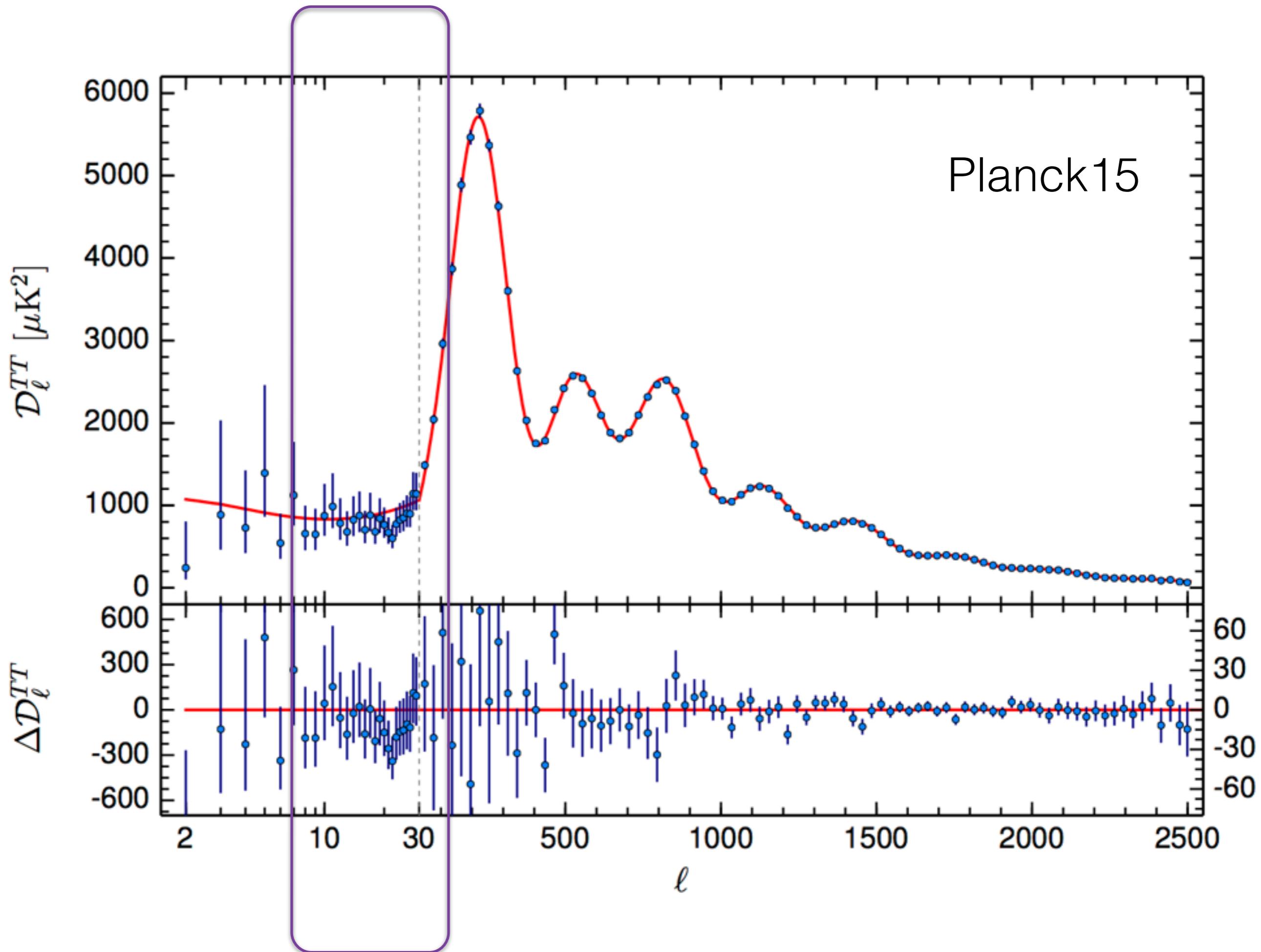
$r \sim 12/N^2 \sim 1E-3$

Is this end of the  
story?

# Features in Power Spectrum

**low-ell anomaly**

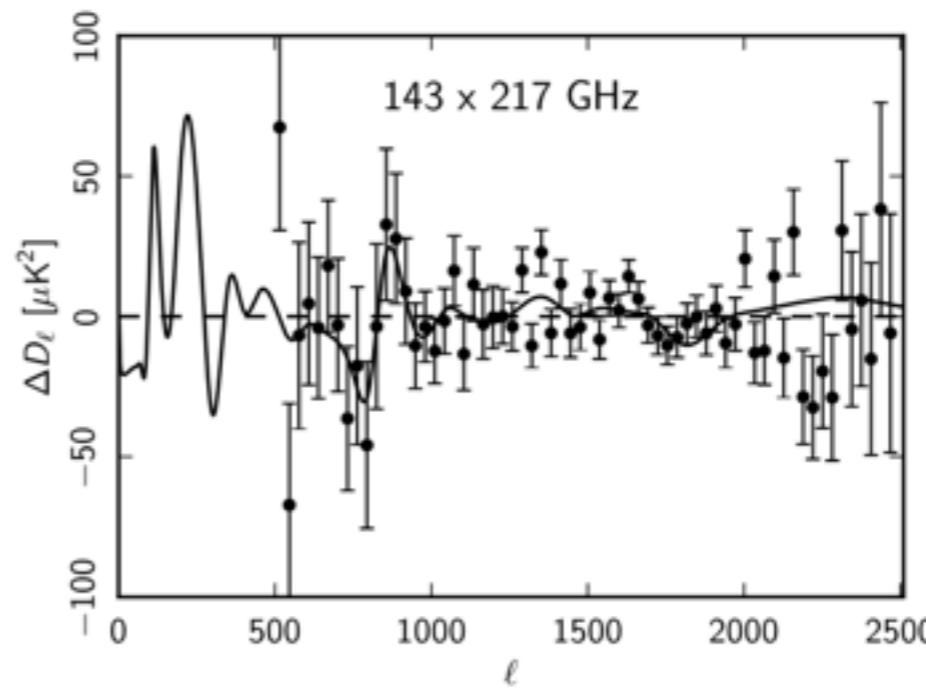
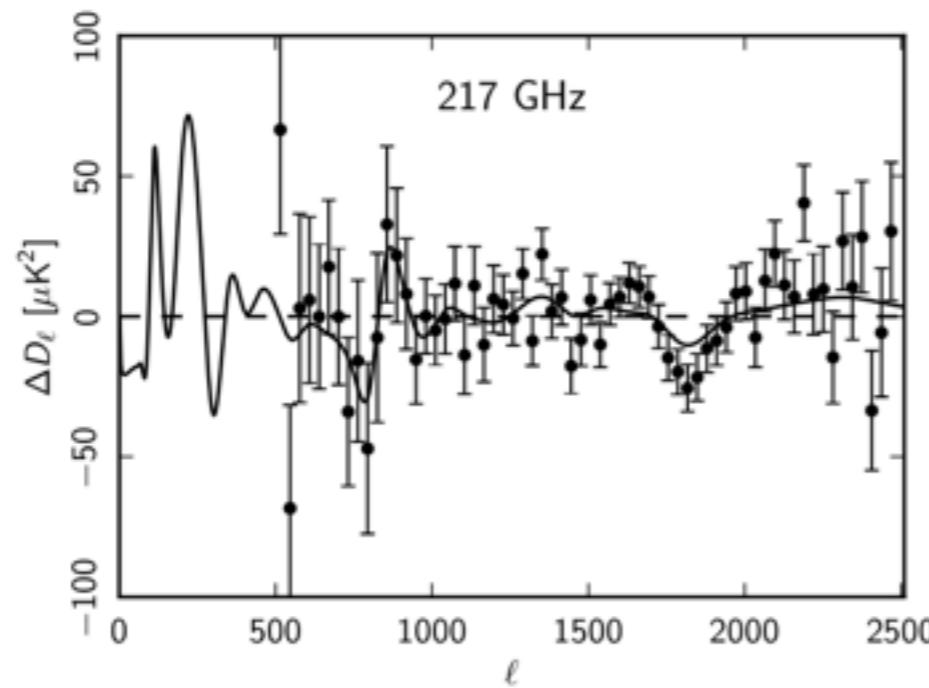
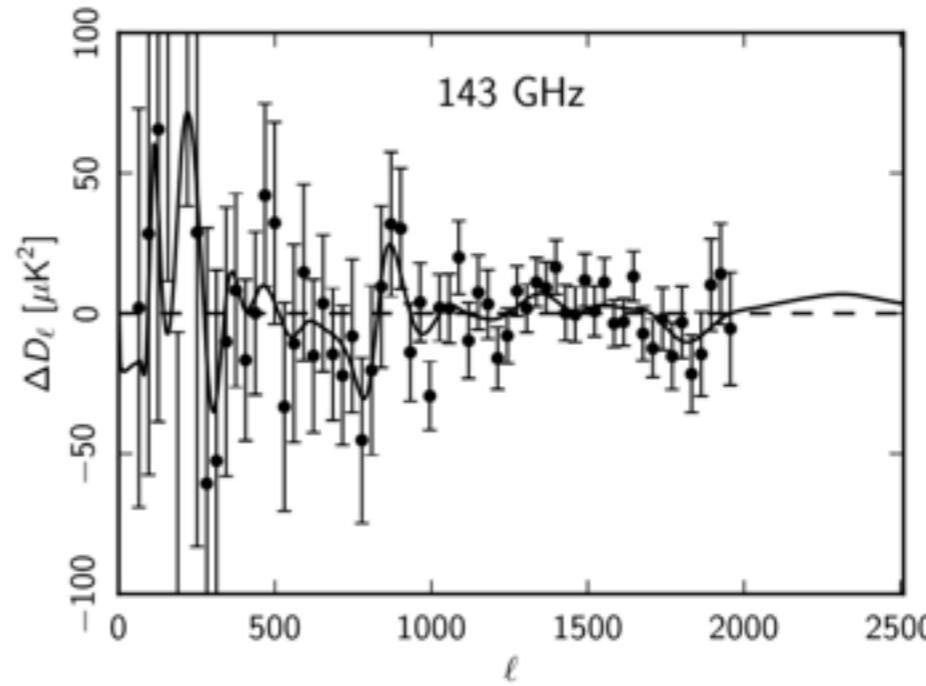
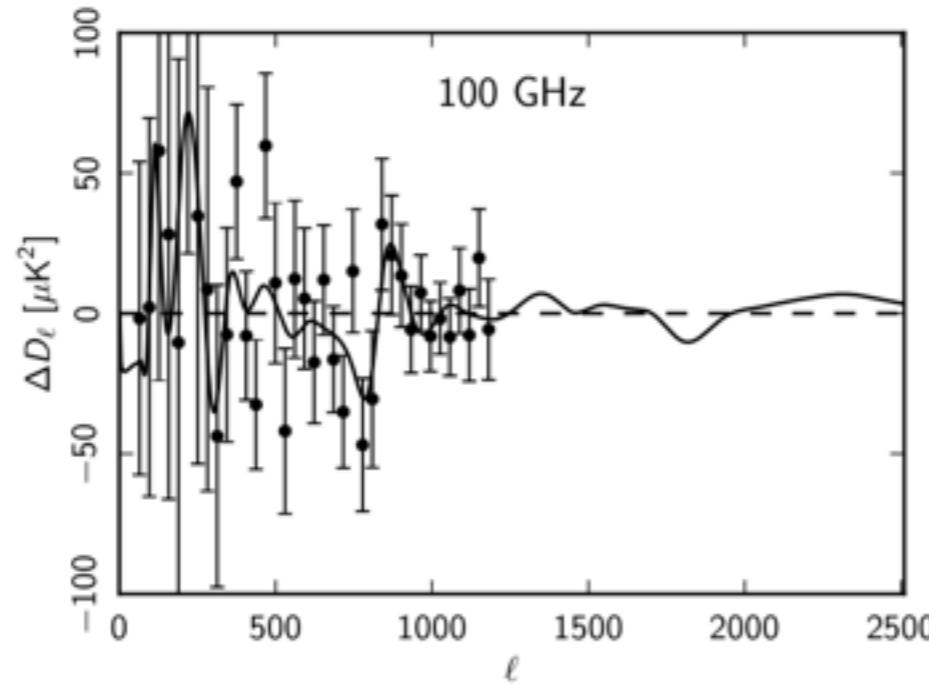




- The low-ell anomaly,  $\ell \sim (20, 40)$ , is in the sky, not due to systematics!
- Are they primordial signal?
- But the significance is not strong enough, due to the cosmic variance.

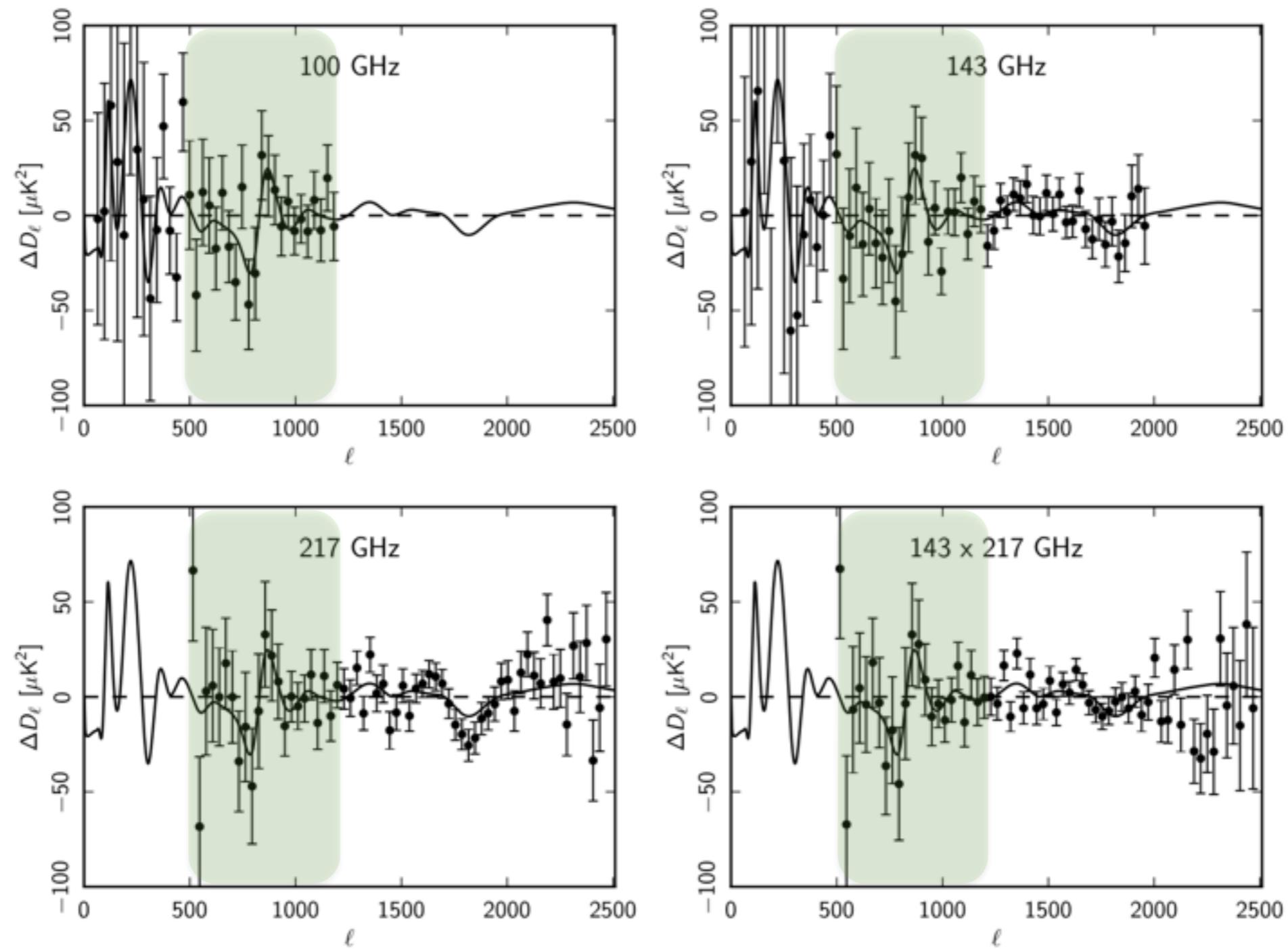
# 1. Observational hints of oscillatory features

TT spectrum residual from best-fit LCDM model



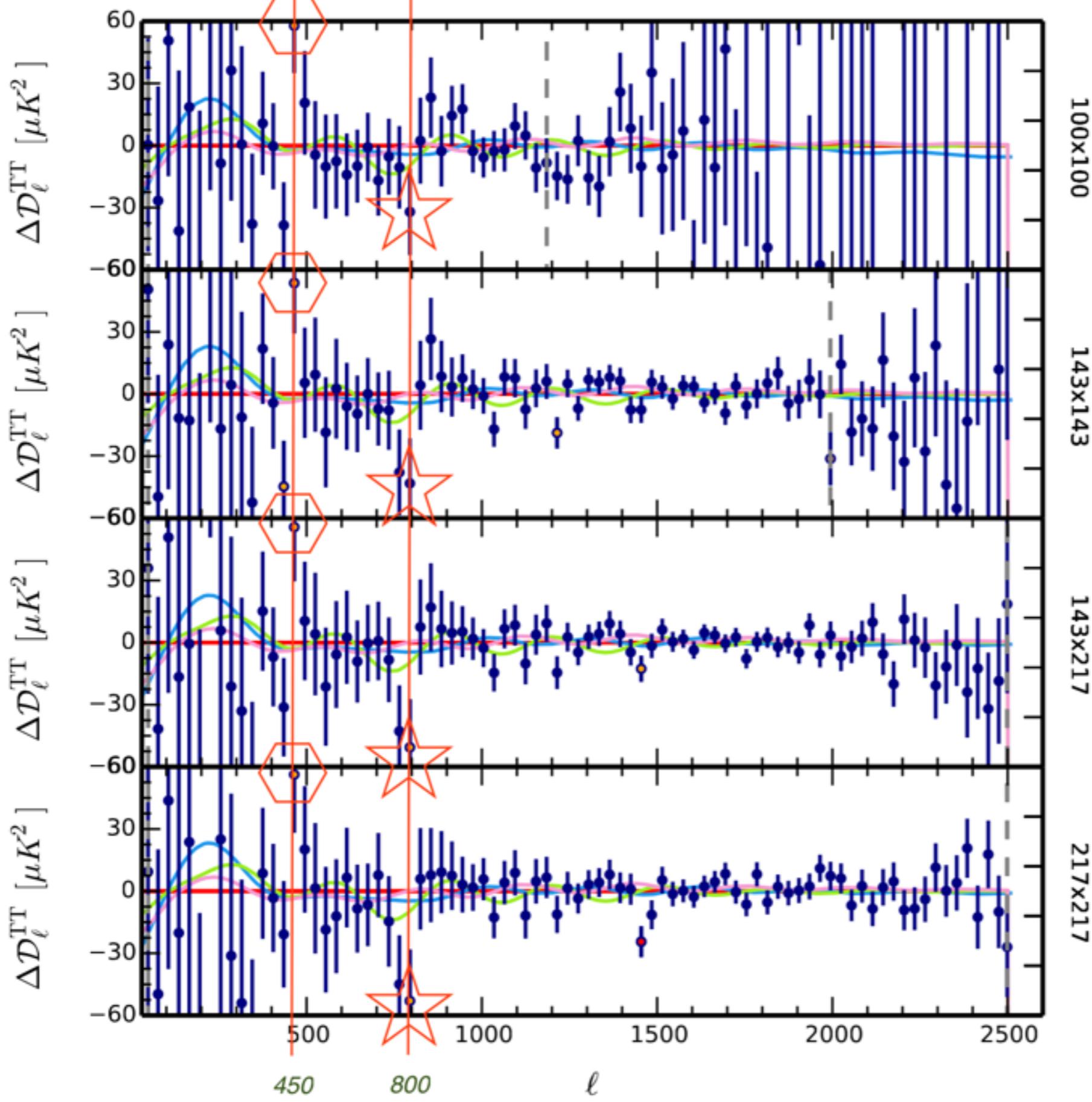
# Spectrum residual from best-fit LCDM model

$$l \in (500, 1200)$$



Appears in all channels

2015

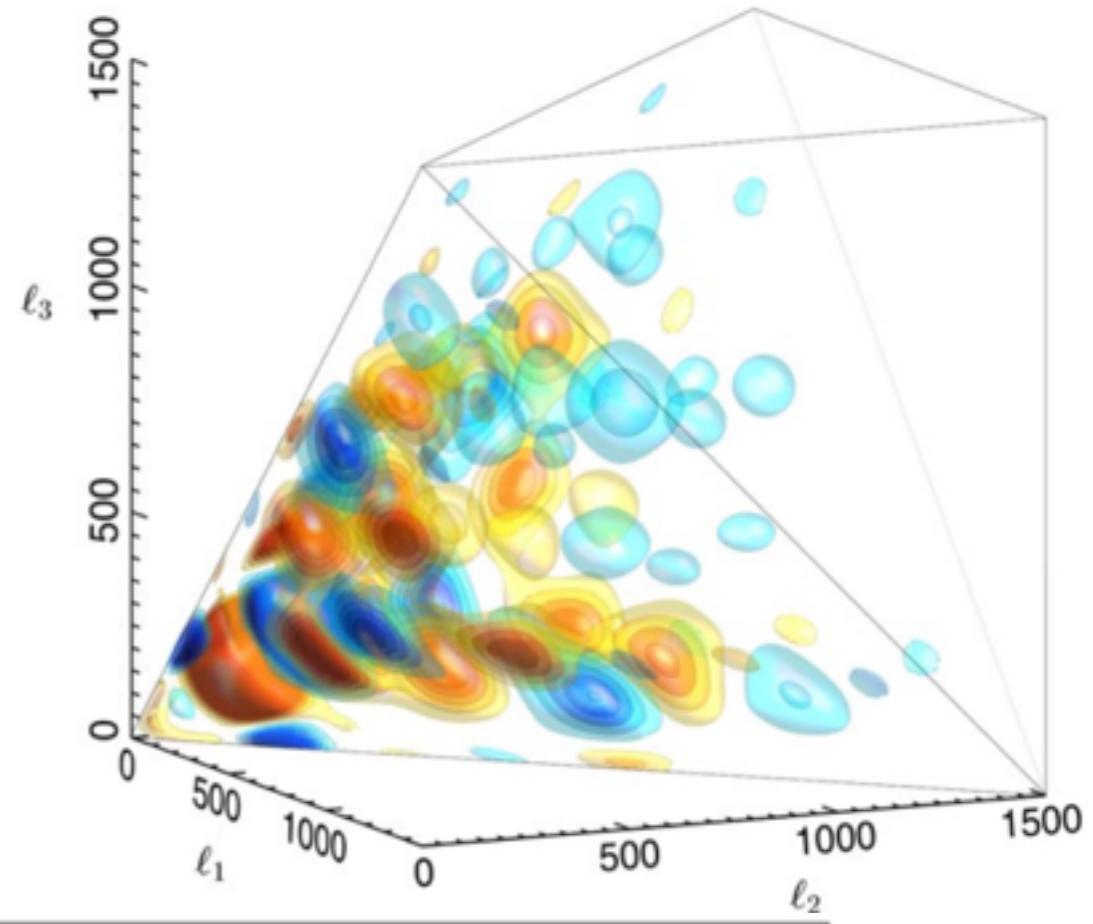


# Observational hints of oscillatory features

## 2. CMB bispectrum

$$B(k_1, k_2, k_3) = \frac{6A^2 f_{\text{NL}}^{\text{feat}}}{(k_1 k_2 k_3)^2} \sin \left( 2\pi \frac{\sum_{i=1}^3 k_i}{3k_c} + \phi \right)$$

The best-fit template to the reconstructed CMB bisp  
 $\sim 3\sigma$  detection



$f_{\text{NL}} \pm \Delta f_{\text{NL}} \ (\sigma)$

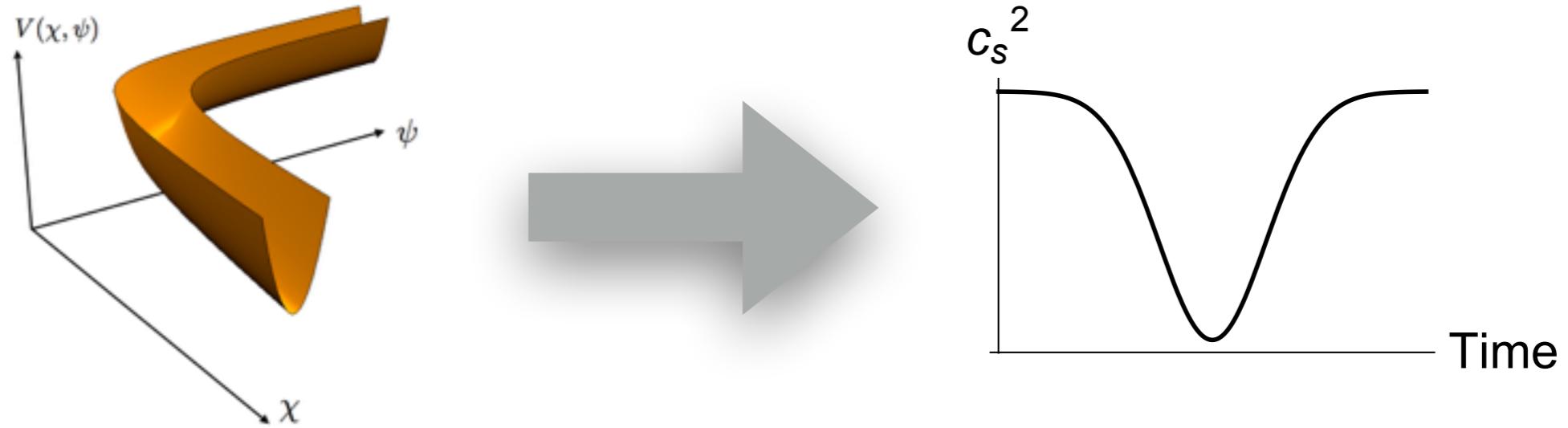
Wavenumber $k_c$ ; phase	$\Delta k = 0.015$	$\Delta k = 0.03$	$\Delta k = 0.045$	Full
0.01125; $\phi = 0$ . . . . .	$765 \pm 275 \ (-2.8)$	$703 \pm 241 \ (-2.9)$	$648 \pm 218 \ (-3.0)$	$434 \pm 170 \ (-2.6)$
0.01750; $\phi = 0$ . . . . .	$-661 \pm 234 \ (-2.8)$	$-494 \pm 192 \ (-2.6)$	$-425 \pm 171 \ (-2.5)$	$-335 \pm 137 \ (-2.4)$
0.01750; $\phi = 3\pi/4$ . . .	$399 \pm 207 \ (-1.9)$	$438 \pm 183 \ (-2.4)$	$442 \pm 165 \ (-2.7)$	$366 \pm 126 \ (-2.9)$
0.01875; $\phi = 0$ . . . . .	$-562 \pm 211 \ (-2.7)$	$-559 \pm 180 \ (-3.1)$	$-515 \pm 159 \ (-3.2)$	$-348 \pm 118 \ (-3.0)$
0.01875; $\phi = \pi/4$ . . . .	$-646 \pm 240 \ (-2.7)$	$-525 \pm 189 \ (-2.8)$	$-468 \pm 164 \ (-2.9)$	$-323 \pm 120 \ (-2.7)$
0.02000; $\phi = \pi/4$ . . . .	$-665 \pm 229 \ (-2.9)$	$-593 \pm 185 \ (-3.2)$	$-500 \pm 160 \ (-3.1)$	$-298 \pm 119 \ (-2.5)$

## 2. Models with a transient reduction of the speed of sound

Two field model:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi) \right]$$

Assumption: 1 light & 1 heavy fields



derivative coupling, e.g.  $\dot{\phi}_1\phi_2 \Rightarrow$  a turn

EFT for inflation:

[C. Cheung et. al. JHEP 0803 (2008) 014]

[S. Weinberg Phys.Rev. D77 (2008) 123541] light adiabatic  
[A. Achucarro et. al. JHEP 1205 (2012) 066] heavy isocurvature

$$\phi^a(t, \mathbf{x}) = \phi_0^a(t + \pi) + N^a(t + \pi)\mathcal{F}$$

## After Integrating out heavy field

effective action  
for light field:

$$S_{\text{eff}} = - \int d^4x \, a^3 M_{\text{pl}}^2 \dot{H} \left\{ \dot{\pi}^2 - \frac{(\nabla \pi)^2}{a^2} + (c_s^{-2} - 1) \dot{\pi}^2 \right. \\ \left. + (c_s^{-2} - 1) \dot{\pi} \left[ \dot{\pi}^2 - \frac{(\nabla \pi)^2}{a^2} \right] + (c_s^{-2} - 1)^2 \frac{\dot{\pi}^3}{2} - 2 \frac{\dot{c}_s}{c_s^3} \pi \dot{\pi}^2 + \dots \right\}$$

Primordial spectrum:  $\mathcal{P}_{\mathcal{R}} \propto \mathcal{O}(\epsilon) + \mathcal{O}(\epsilon(1 - c_s^{-2}))$  sub-leading

Primordial bispectrum:  $\mathcal{B} \propto \mathcal{O}\left(\frac{\dot{c}_s}{H c_s}\right) + \mathcal{O}(\epsilon)$  leading

$$\epsilon \sim \mathcal{O}(0.01)$$

$$1 - c_s^{-2} \sim \mathcal{O}(0.1)$$

$$\frac{\dot{c}_s}{H c_s} \sim \mathcal{O}(0.1)$$

**Do NOT interrupt slow roll condition!**

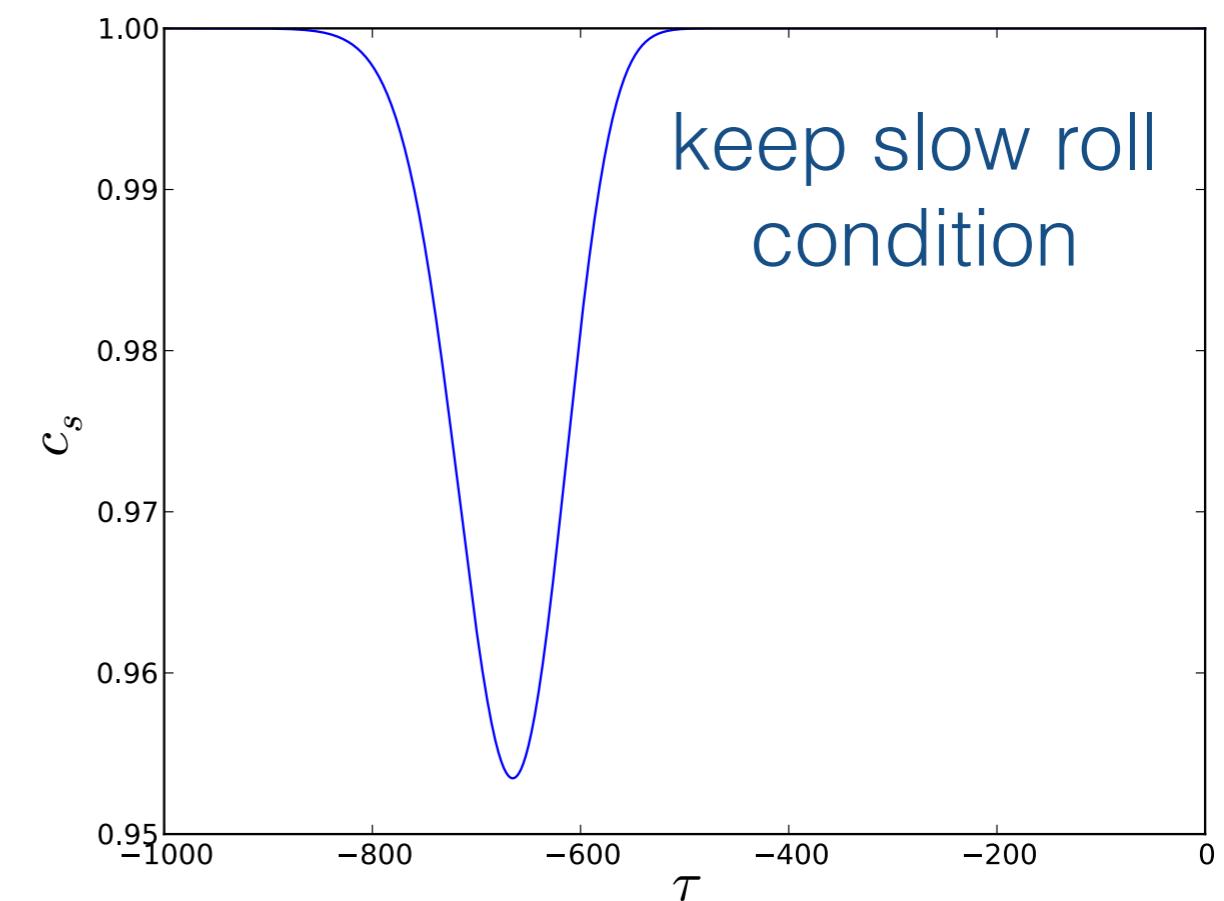
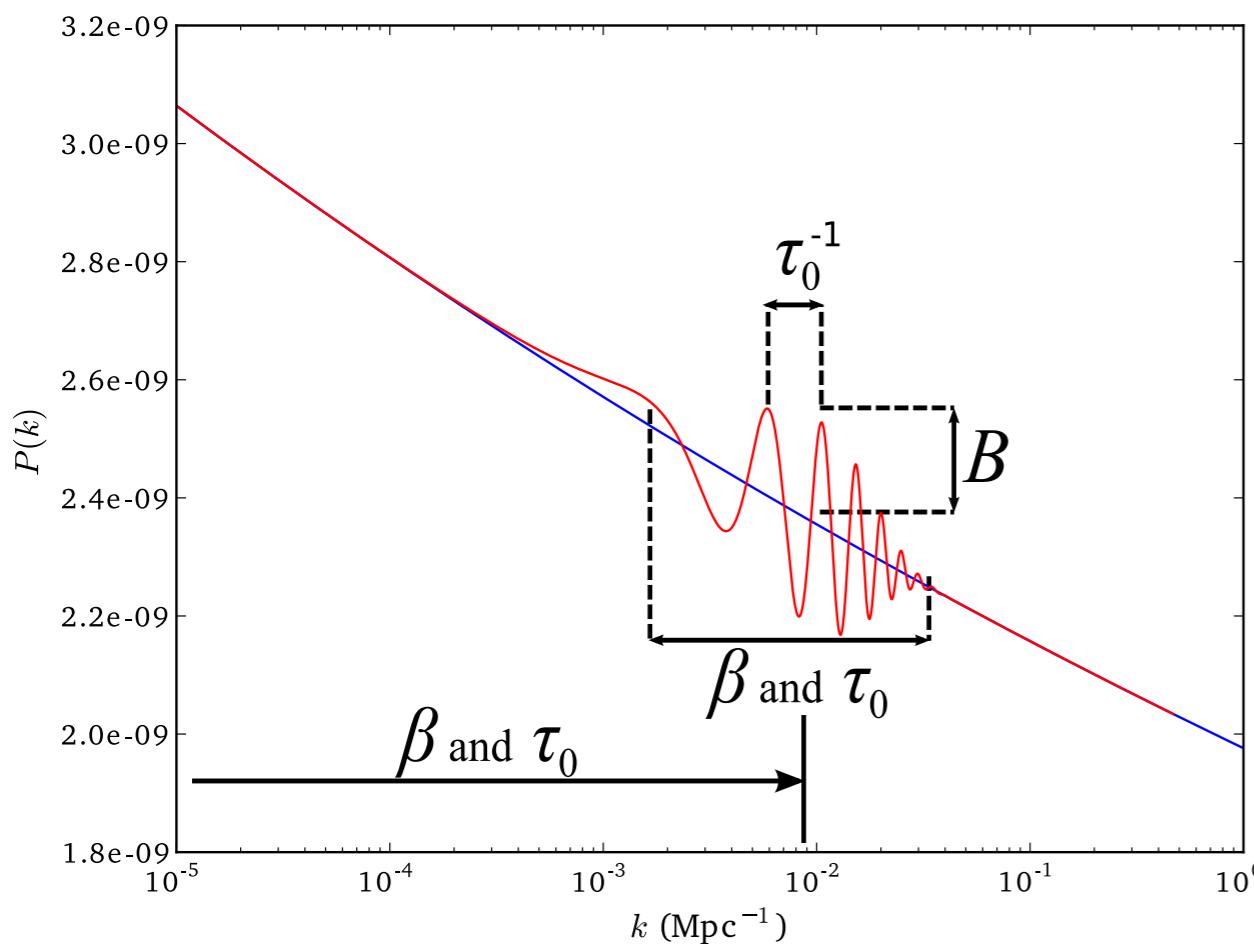
# Oscillatory features in the transient sound speed reduction models— Power spectrum

$$\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}}(k) = k \int_{-\infty}^0 d\tau (1 - c_s^{-2}) \sin(2k\tau)$$

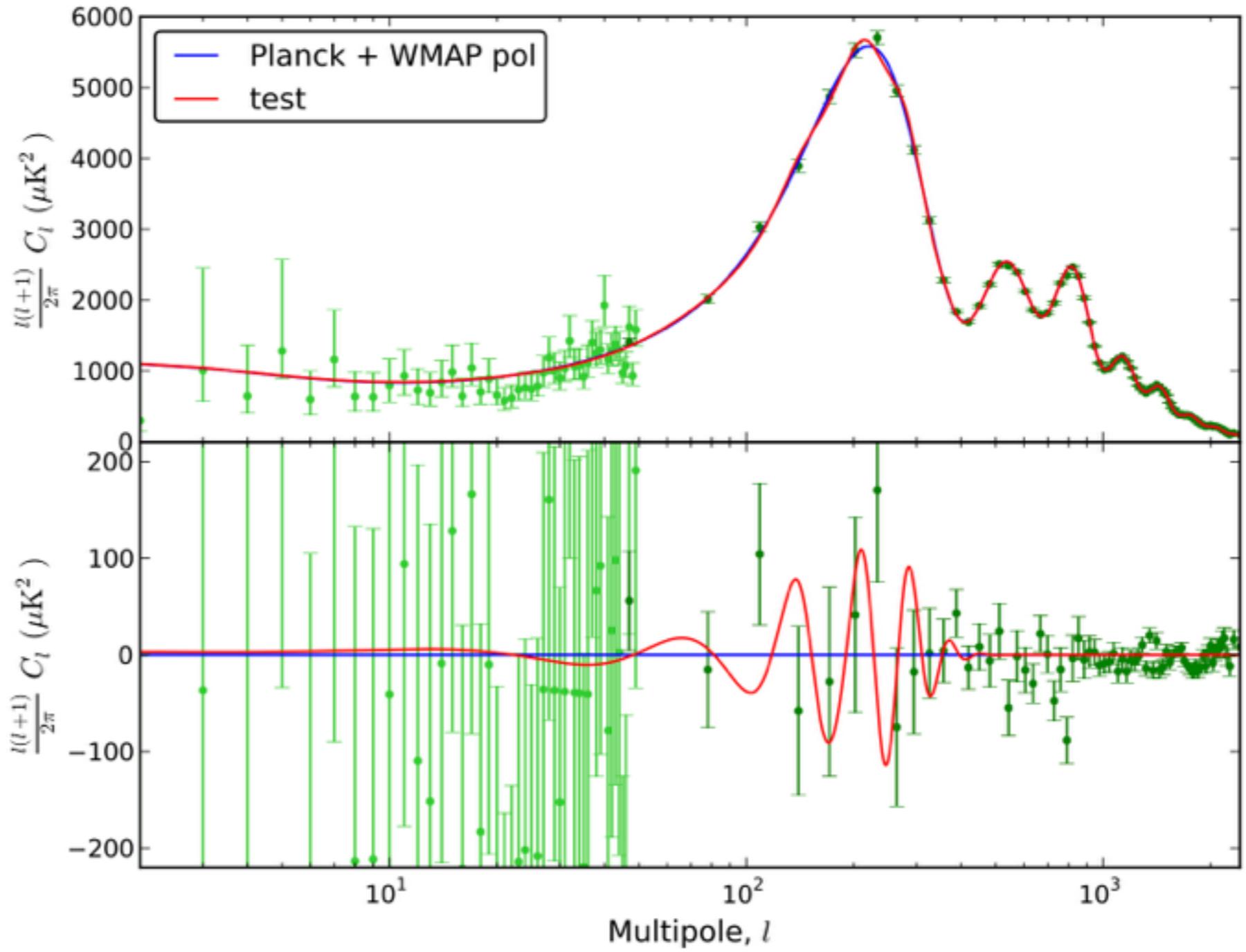
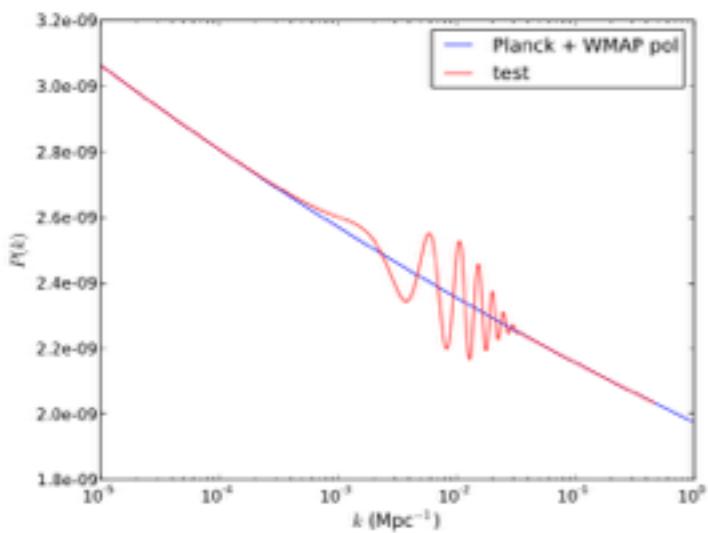
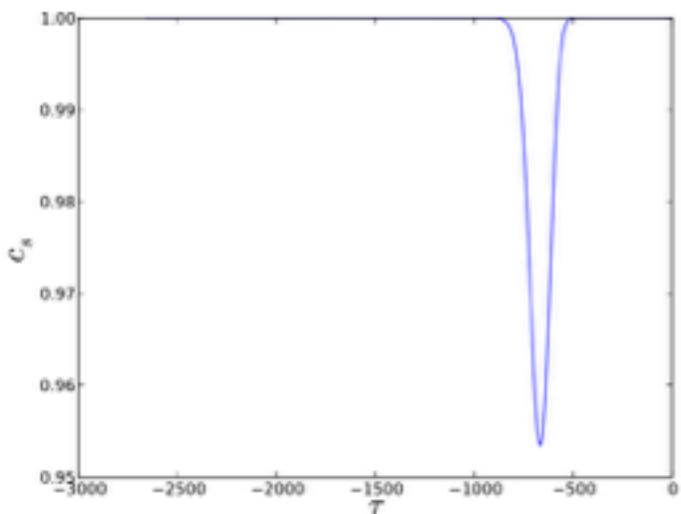
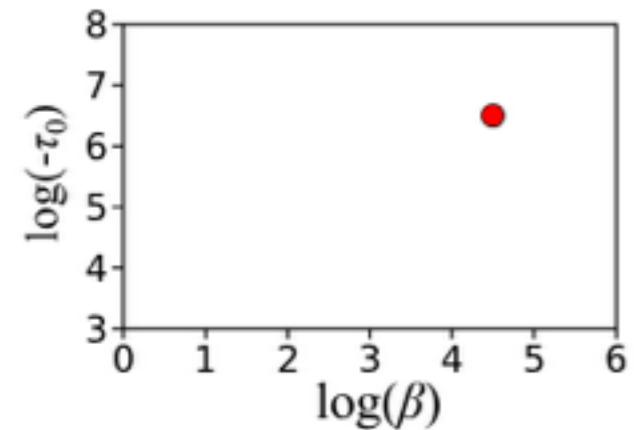
Gaussian reduction in e-folds

[A.Achucarro et. al. PRD 89 (2014) 103006]

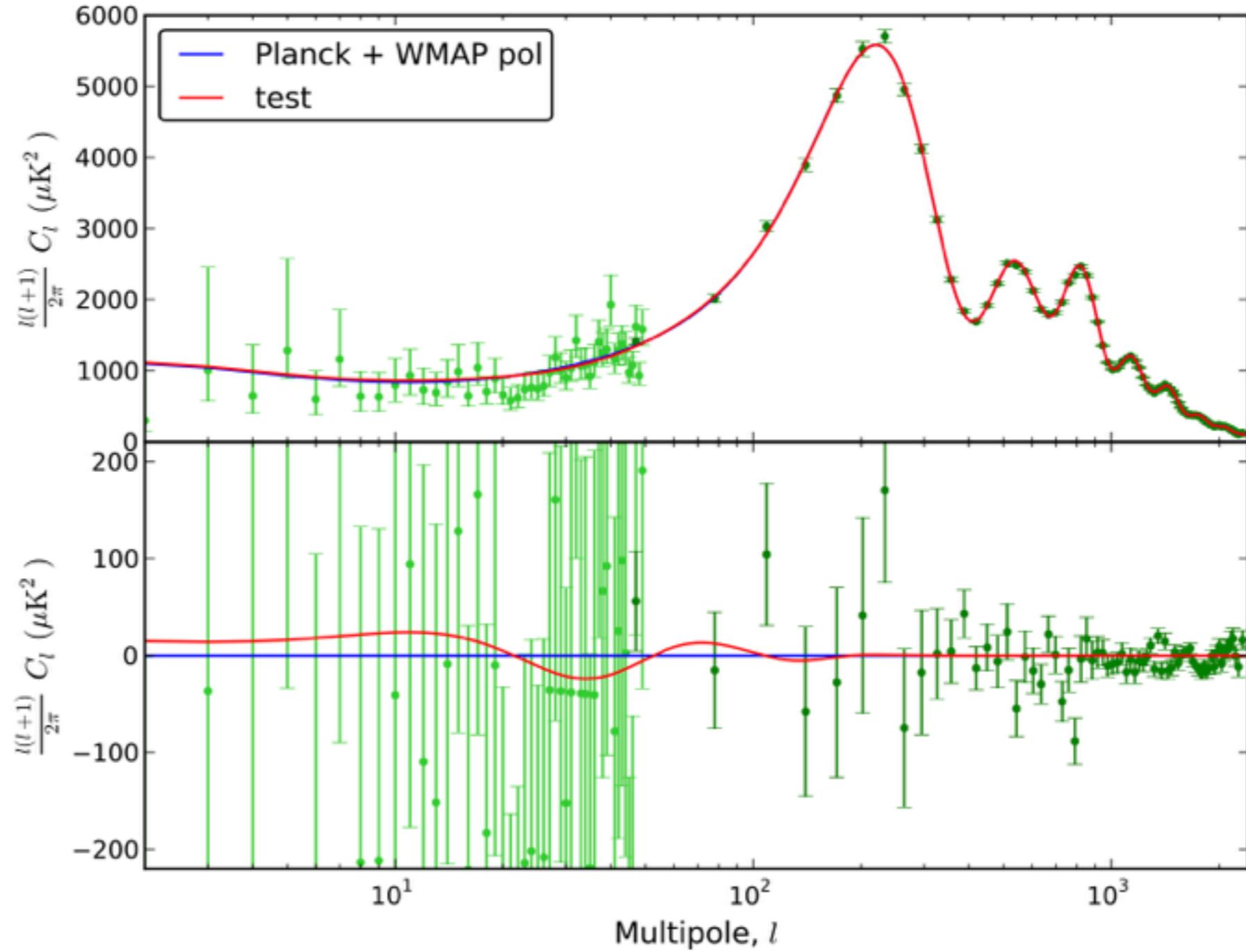
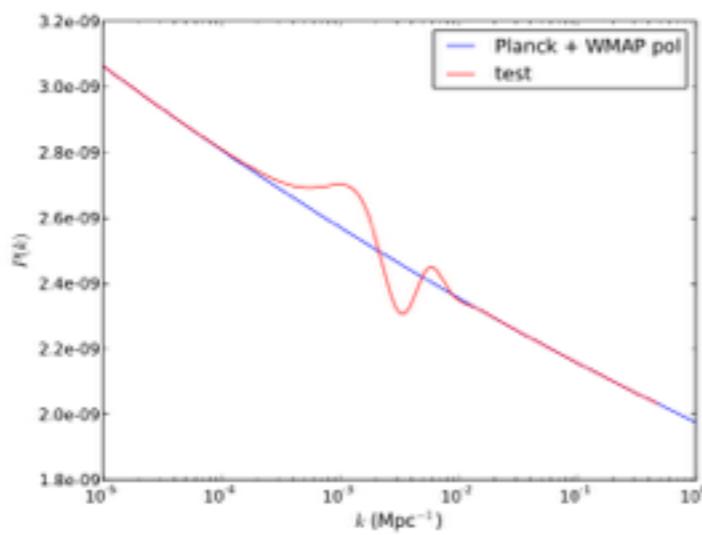
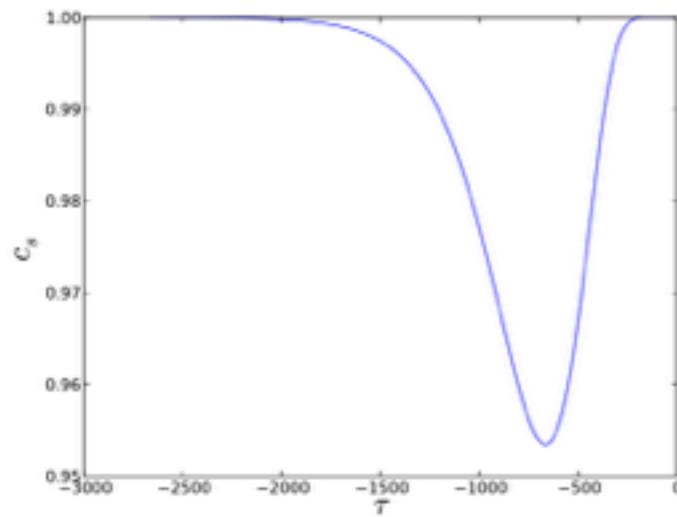
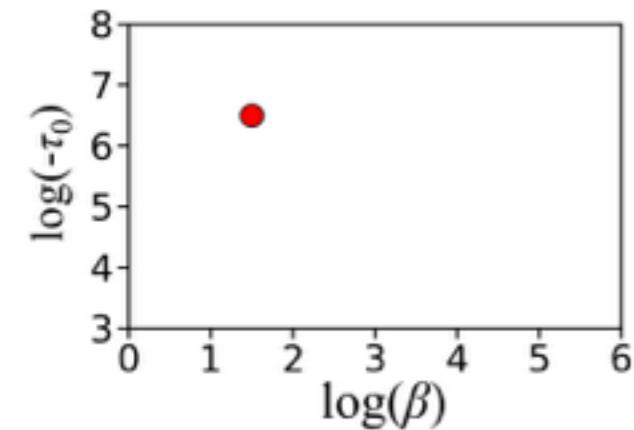
$$1 - c_s^{-2} = B e^{-\beta \left( \log \frac{\tau}{\tau_0} \right)^2}$$



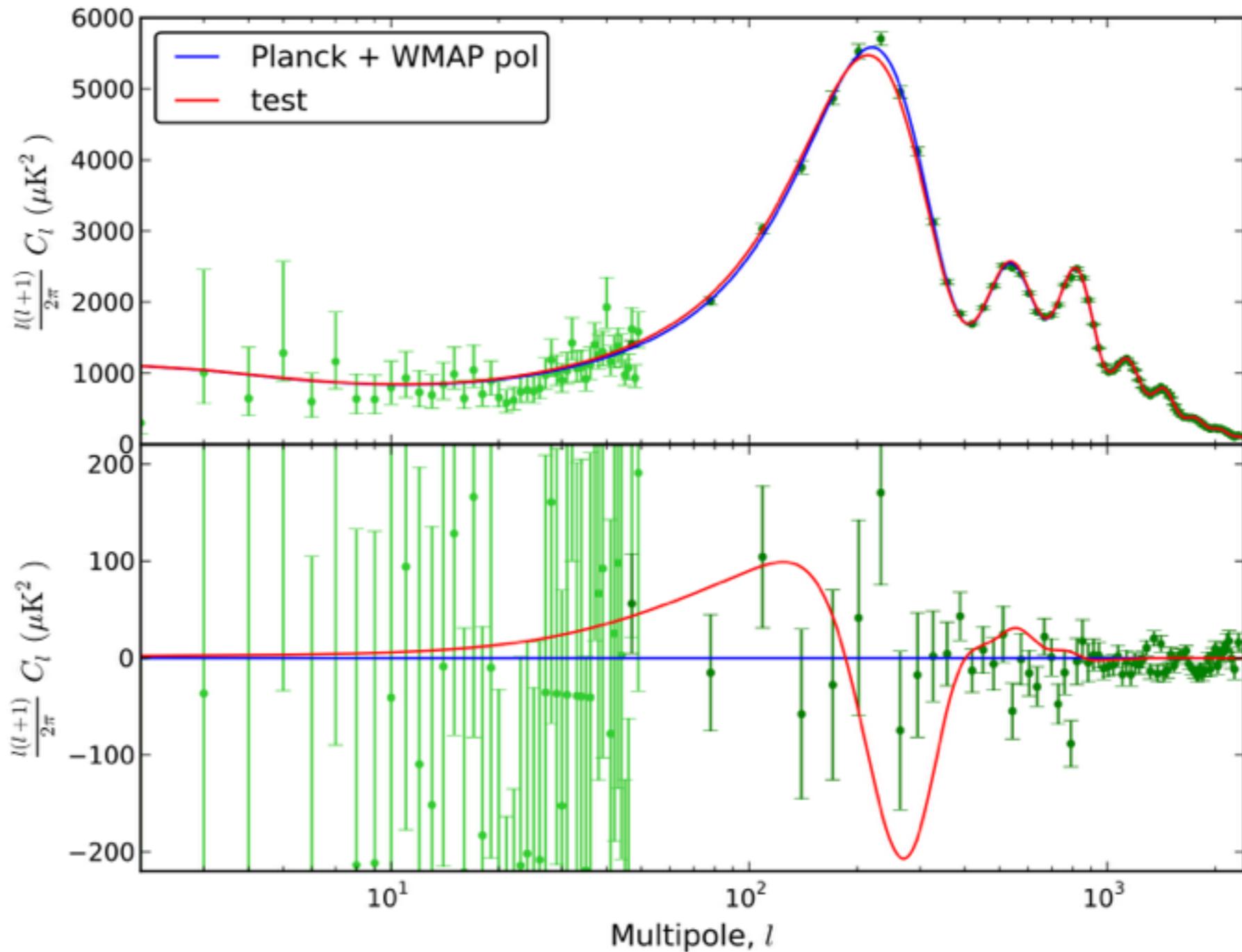
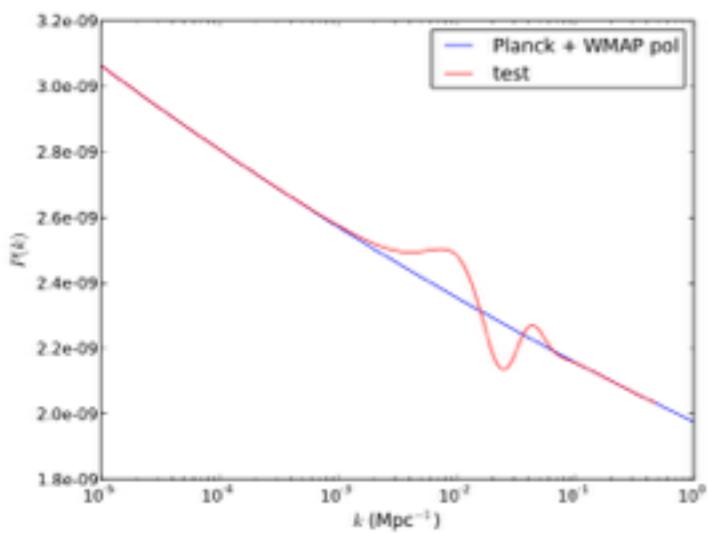
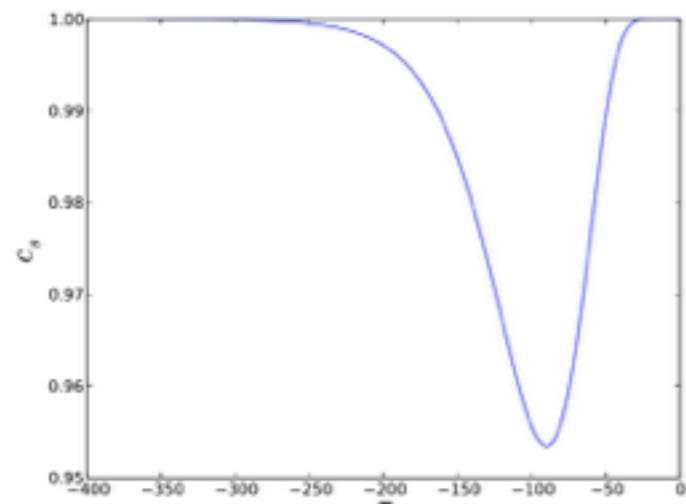
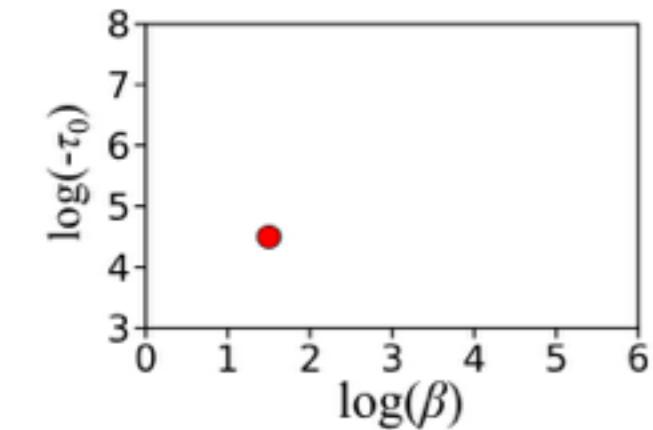
# Some examples ( $B = -0.1$ )



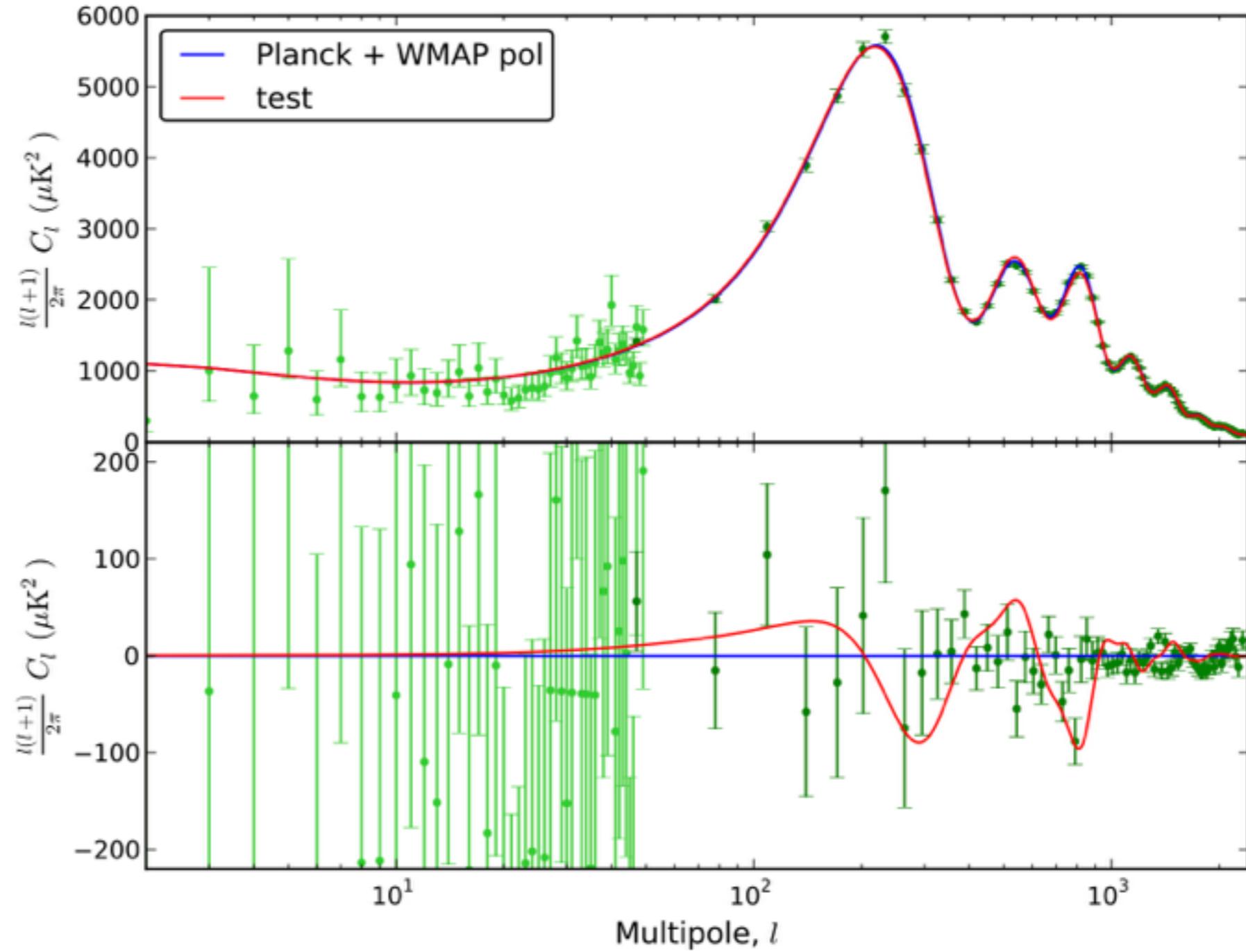
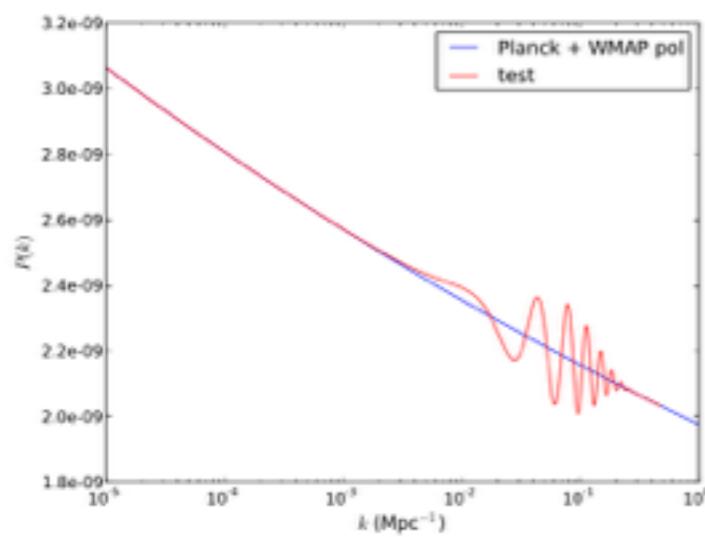
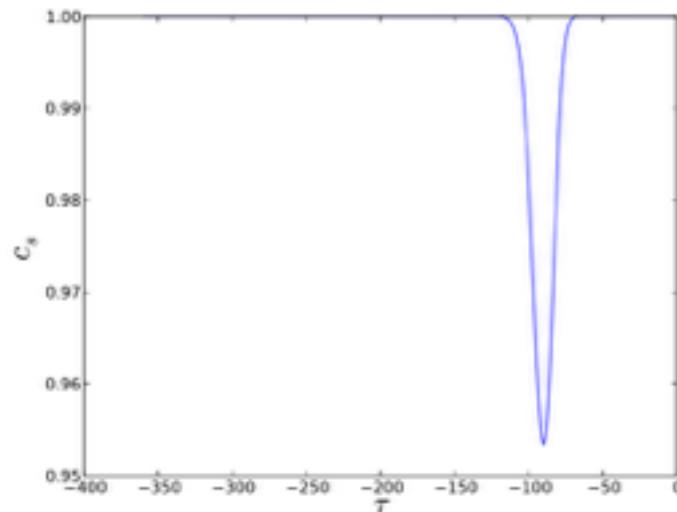
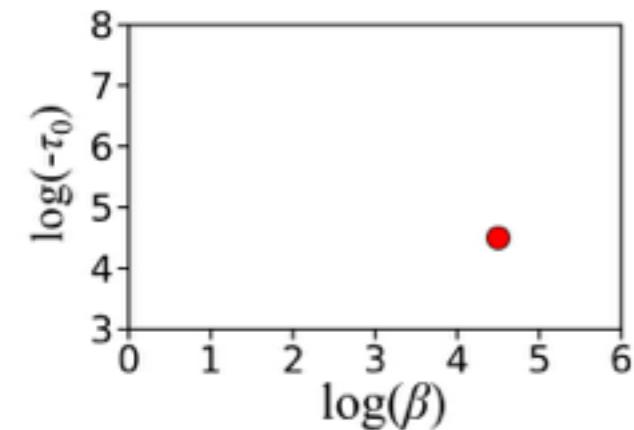
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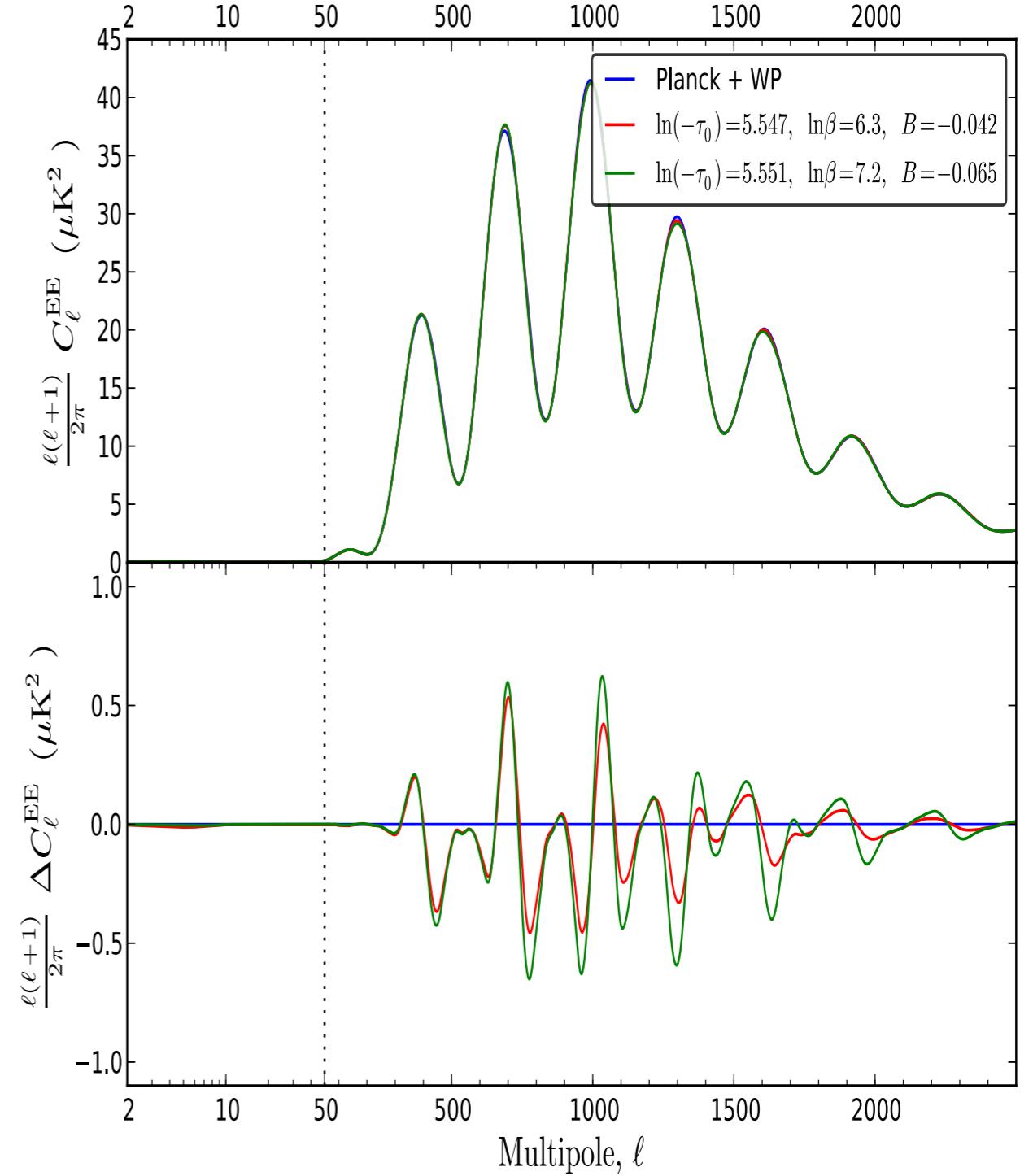
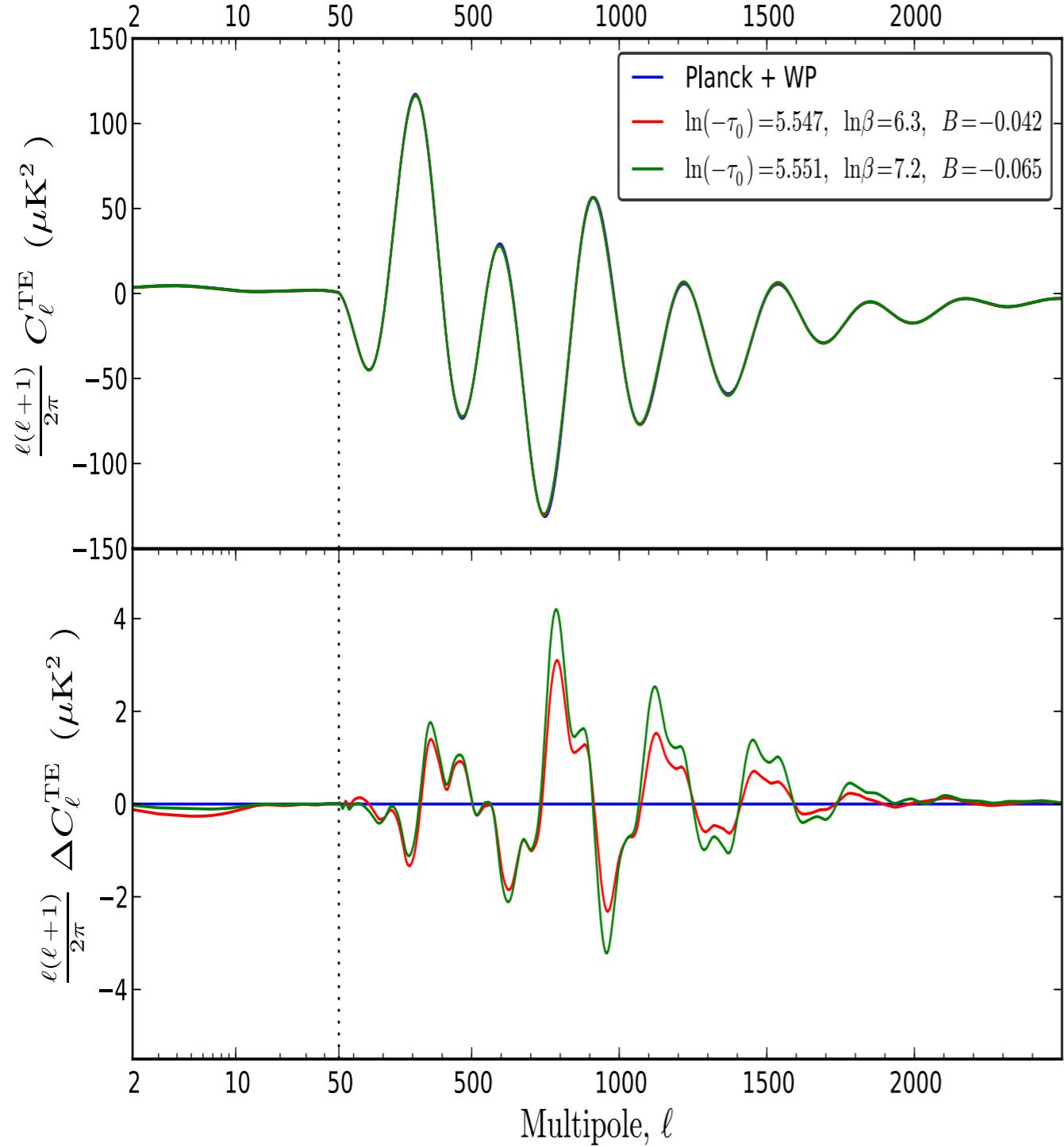


# Some examples ( $B = -0.1$ )



# Some examples ( $B = -0.1$ )





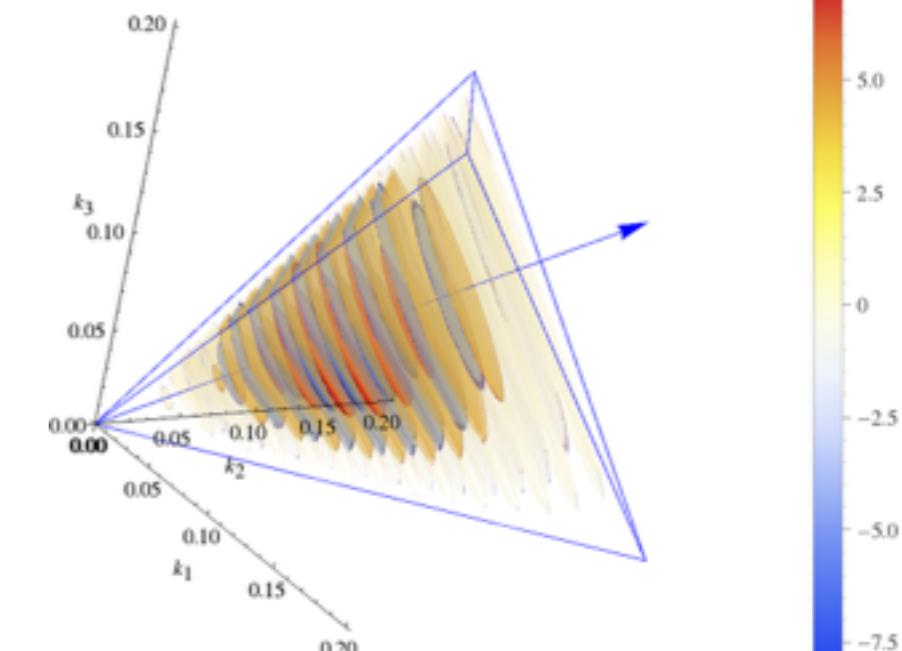
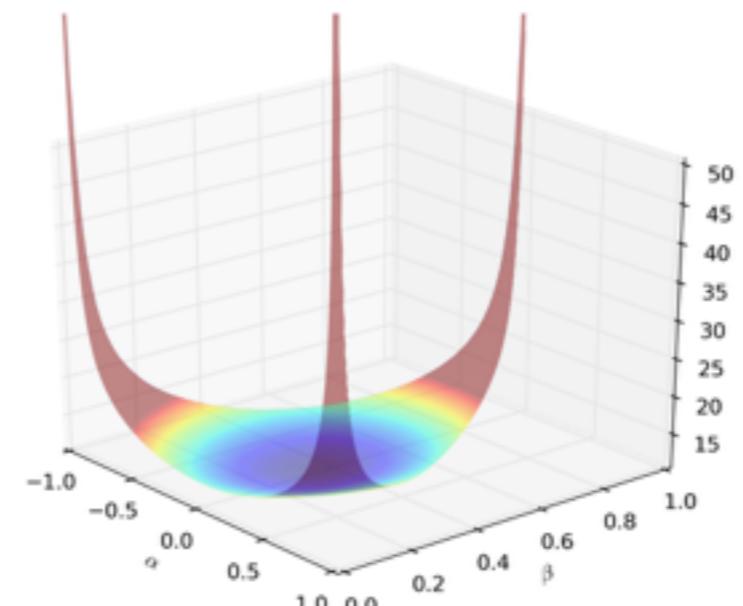
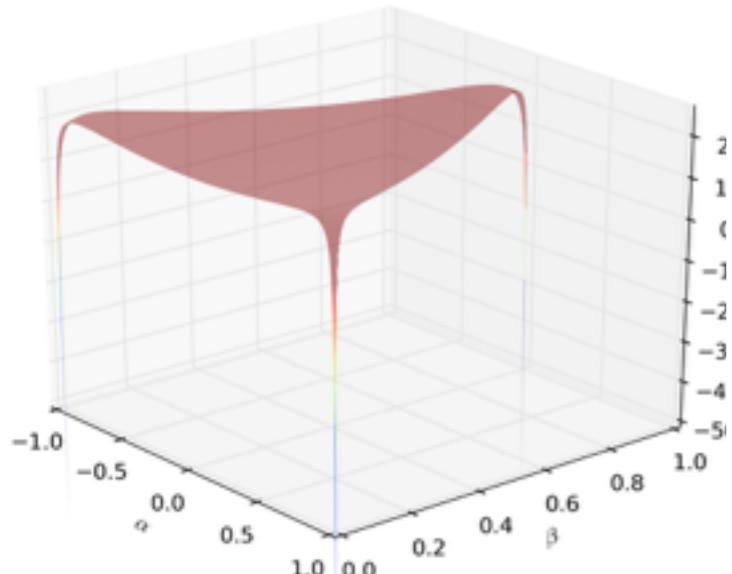
$\sim 10\%$  effect

## 2. Primordial Bispectrum (leading order)

$$B(k_1, k_2, k_3) = \frac{6\Delta_\phi^2}{k_1^2 k_2^2 k_3^2} \frac{(2\pi)^4}{96} \frac{|B|\mathcal{D}_s(K)k_1^2}{k_1 k_2 k_3}$$

$$\left\{ \tau_0 \cos(\tau_0 K) \left[ k_2(k_1 - k_3) + \frac{\tau_0^2}{2\beta} K k_2 k_3 \left( \frac{3}{2}k_1 - k_2 \right) - \frac{1}{2\beta} \left( \frac{1}{2}k_1^2 - k_2^2 \right) \right] \right. \\ \left. + \sin(\tau_0 K) \left[ \frac{1}{2}\tau_0^2 k_1 k_2 k_3 - \frac{1}{K} \left( \frac{1}{2}k_1^2 - 2k_2^2 \right) - \frac{\tau_0^2}{2\beta} k_2 (2k_1^2 - k_2 k_3) \right] \right\} + 5 \text{ perm.}$$

$$\mathcal{D}_s(K) = -\sqrt{\frac{\pi}{\beta}} 2K\tau_0 \exp\left(-\frac{K^2\tau_0^2}{4\beta}\right), \quad K = k_1 + k_2 + k_3$$



$K = 0.19$

$K = 0.21$

removing  $\frac{1}{k_1^3 k_2^3} + \frac{1}{k_1^3 k_3^3} + \frac{1}{k_2^3 k_3^3}$

Step in sound speed:

[Adshead et al. PhysRevD.84.043519], [Bartolo et al. JCAP 1310 (2013) 038]

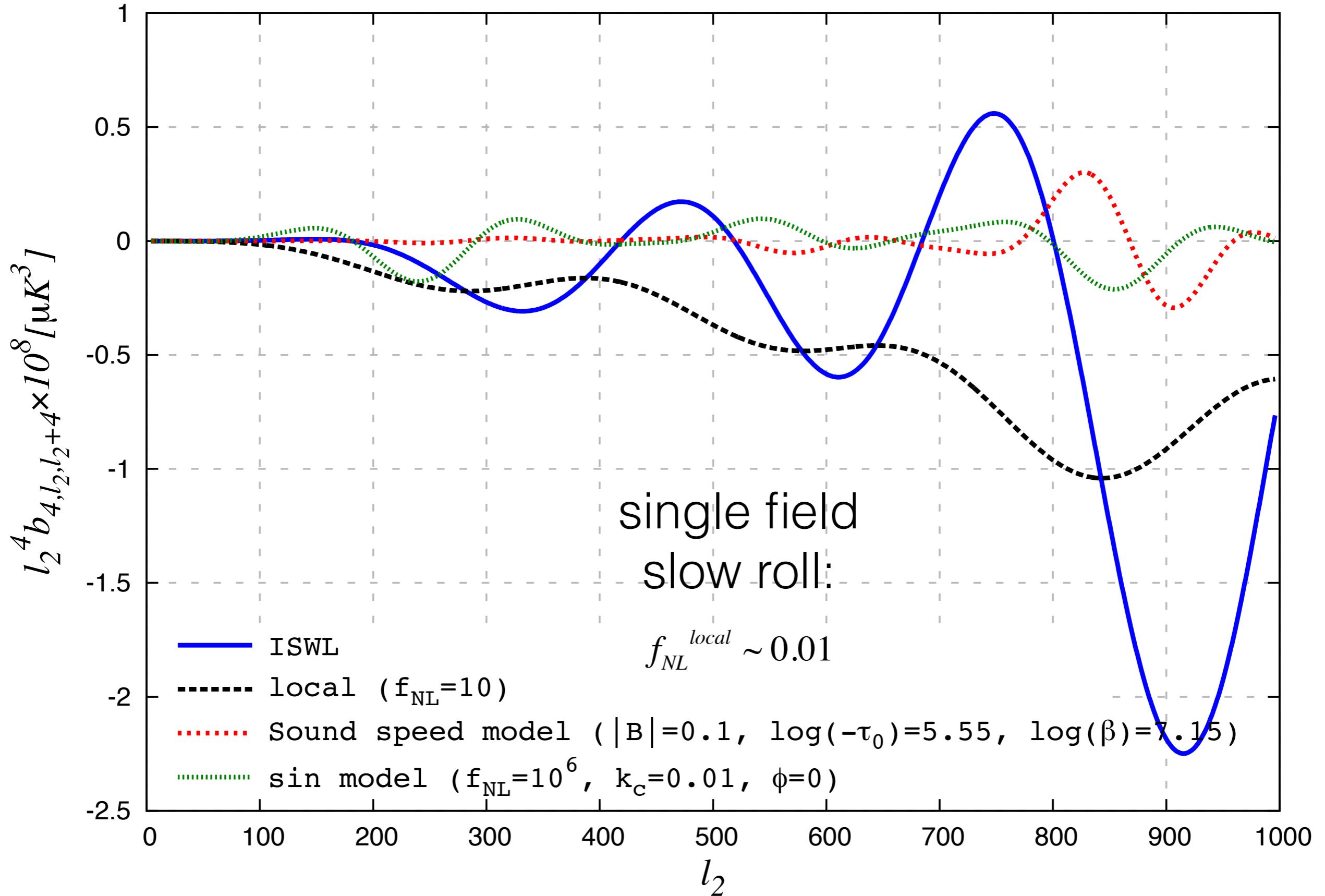
[Miranda et al. Phys.Rev. D86 (2012)], [Park et al. Phys.Rev. D85 (2012)]

[Adshead et al. PhysRevD.84.043519], [Nakashima et al. Prog.Theor.Phys. 125 (2011)]

[Bean et al. JCAP 0803 (2008) 026], [Cannone et al. Phys.Rev. D89 (2014)]

$l_1 = 4, l_3 = l_2 + 4$

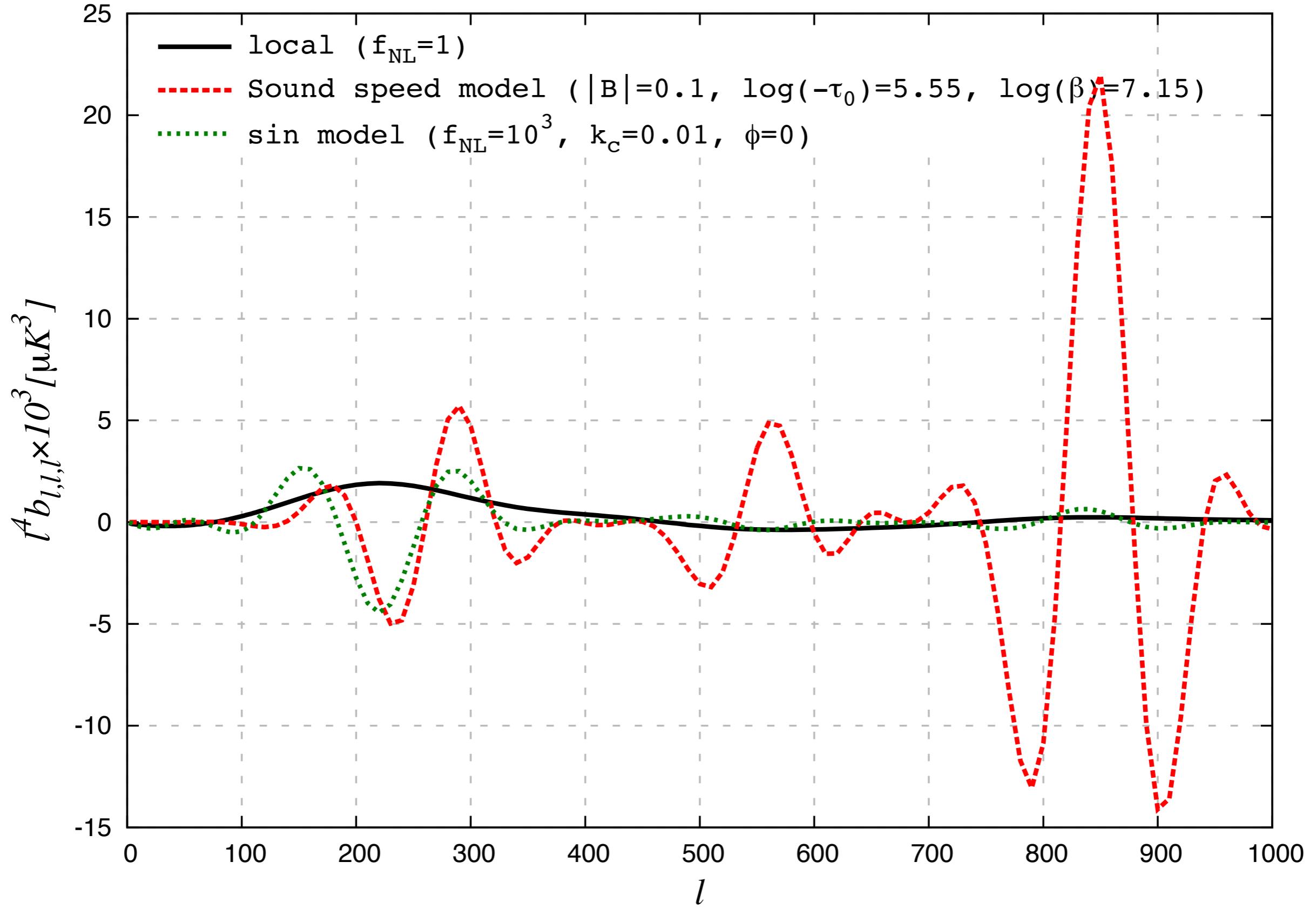
squeezed



$l_1 = l_2 = l_3$

equilateral

Also see Munchmeyer's  
& Van Tent's talks



# Other studies and searches for features in the CMB Power spectrum and bispectrum

Linear oscillation (e.g. step-like features in V)

Adshead, Hu, Miranda (2013), Benetti (2013), Miranda, Hu (2013) Fergusson et al. 1410.5114

Log-spaced oscillation (e.g. monodromy inflation)

Meerburg, Spergel, Wandelt (2013a, 2013b, 2014) (incl. also linear) Peiris, Easter, Flauger (2013), Münchmeyer, Meerburg, Wandelt (2014)

Others sources of features  
(e.g. multi-field dynamics, non-Bunch-Davis vacuum)

Danielsson (2002), Greene, Schalm, Shiu, v.d. Schaar (2004) Meerburg, v.d. Schaar, Corasaniti (2009), Jackson, Schaalm (2010), Gao, Langlois, Mizuno (2012, 2013), Saito, Takamizu (2013), Noumi, Yamaguchi (2013), Miranda, Hu, Dvorkin (2014), Cai, Chen, Ferreira, Quintin (2014) ...

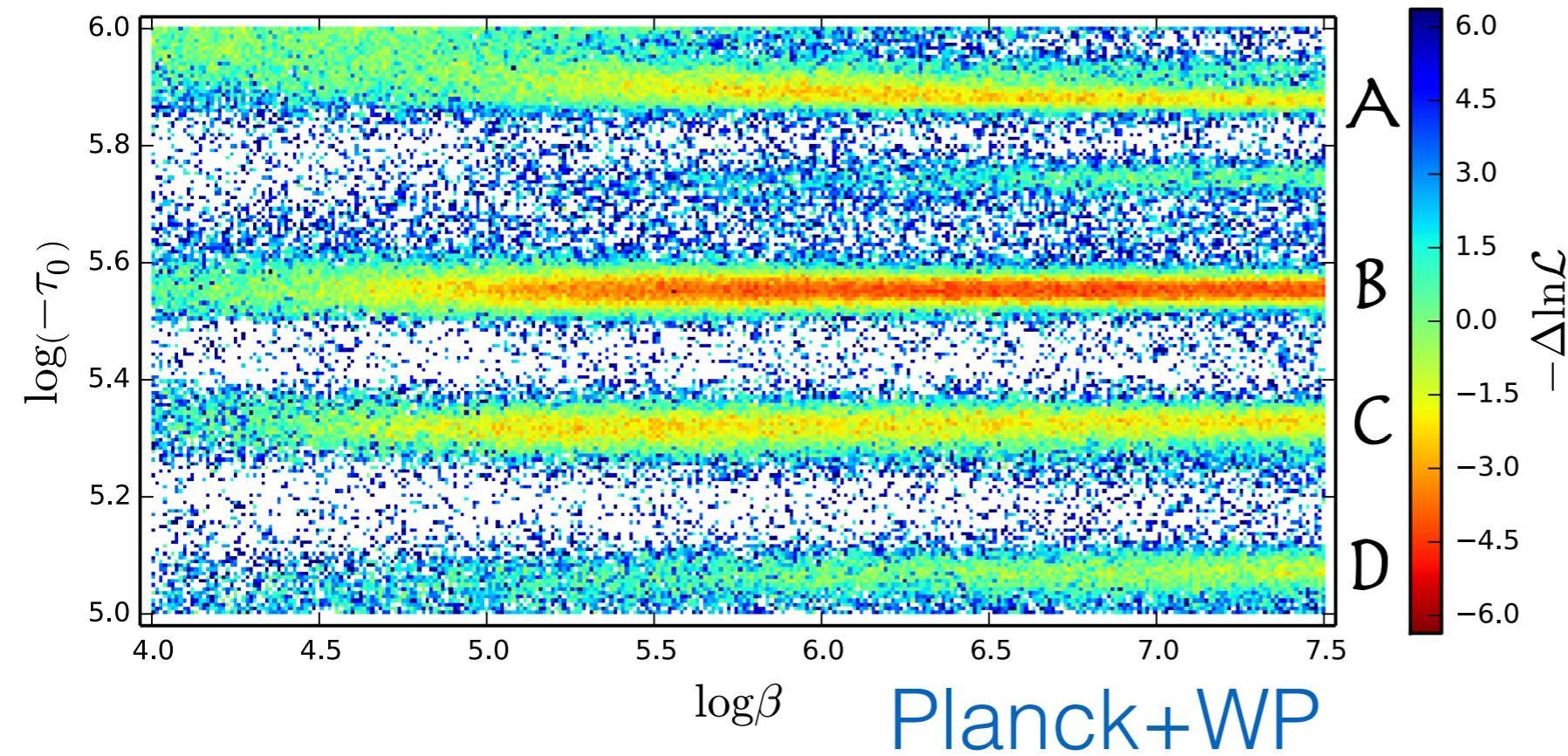
And, of course, Planck's team search for features:

Ade et al. (2013) "Constraints on Inflation"

### 3. Search with CMB map–TT spectrum

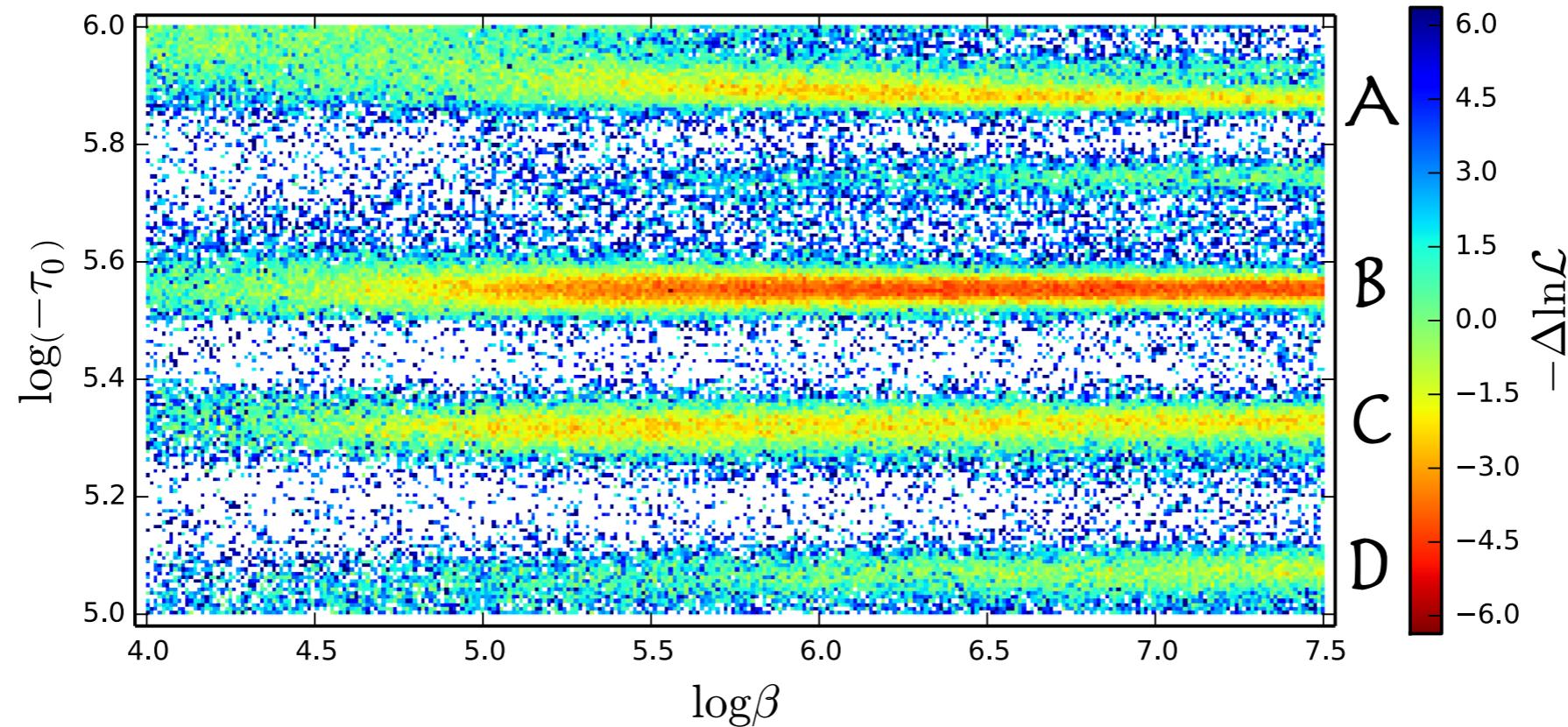
2013

profile likelihood

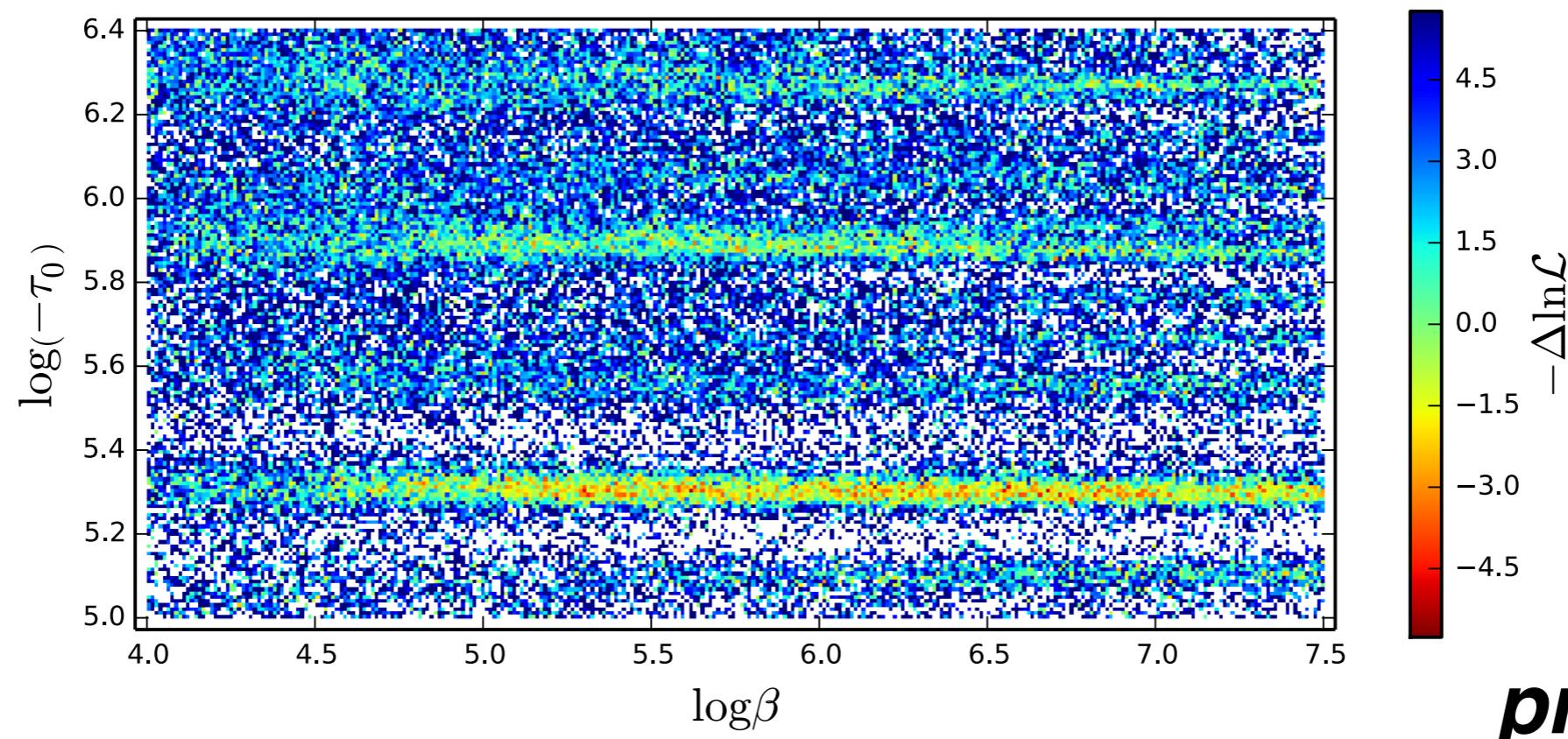


#	$-B \times 10^2$	$\ln \beta$	$\ln(-\tau_0)$	$\Delta\chi^2$
A	(4.5) $3.7^{+1.6}_{-3.0}$	(5.7) $5.7^{+0.9}_{-1.0}$	(5.895) $5.910^{+0.027}_{-0.035}$	-4.3
B	(4.2) $4.3 \pm 2.0$	(6.3) $6.3^{+1.2}_{-0.4}$	(5.547) $5.550^{+0.016}_{-0.015}$	-8.3
C	(3.6) $3.1^{+1.6}_{-1.9}$	(6.5) $5.6^{+1.9}_{-0.7}$	(5.331) $5.327^{+0.026}_{-0.034}$	-6.2
D	(4.4)	(6.5)	(5.06)	-3.3

degeneracy  
of featured and  
vanilla  
parameters is  
negligible

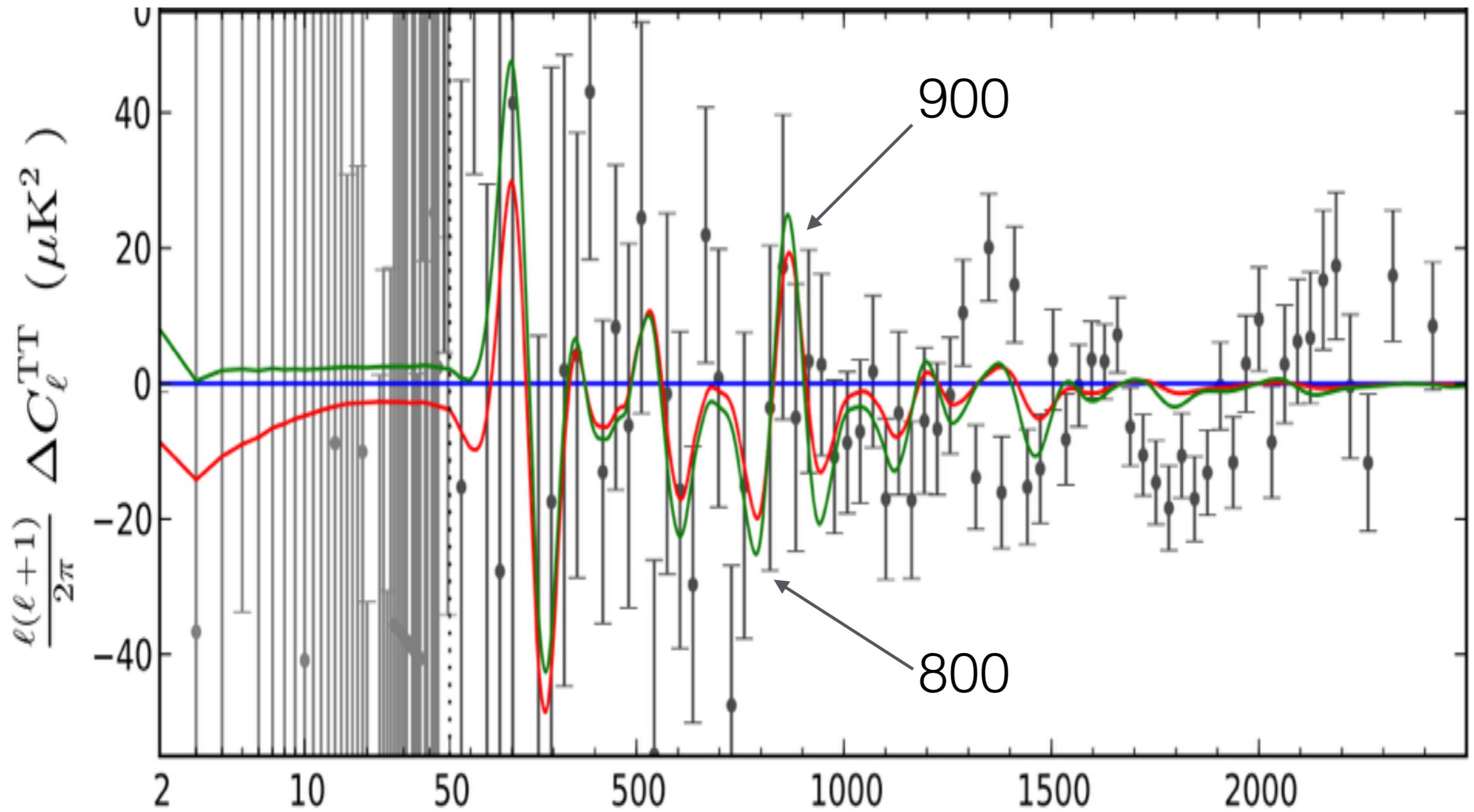


2013

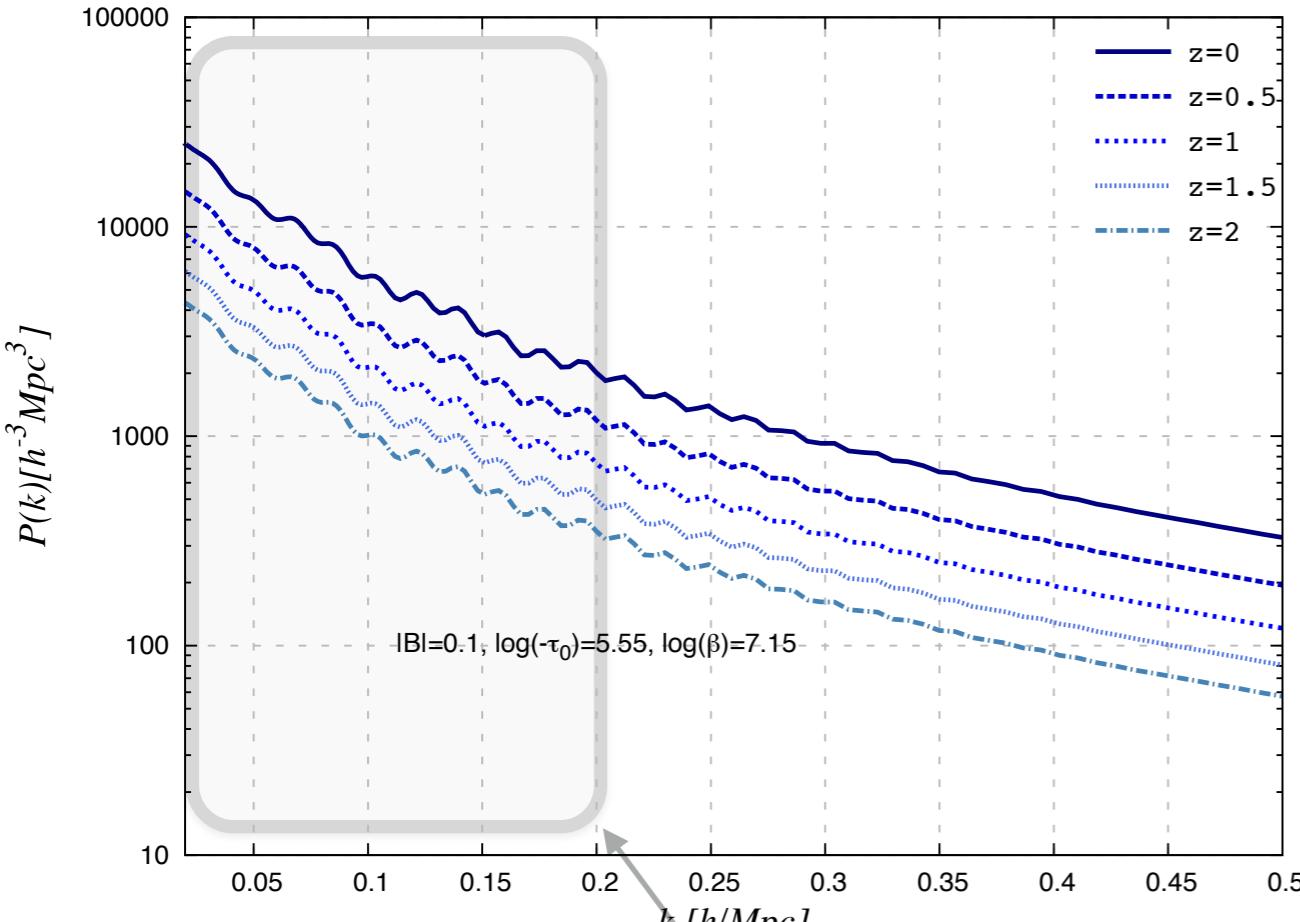


2015

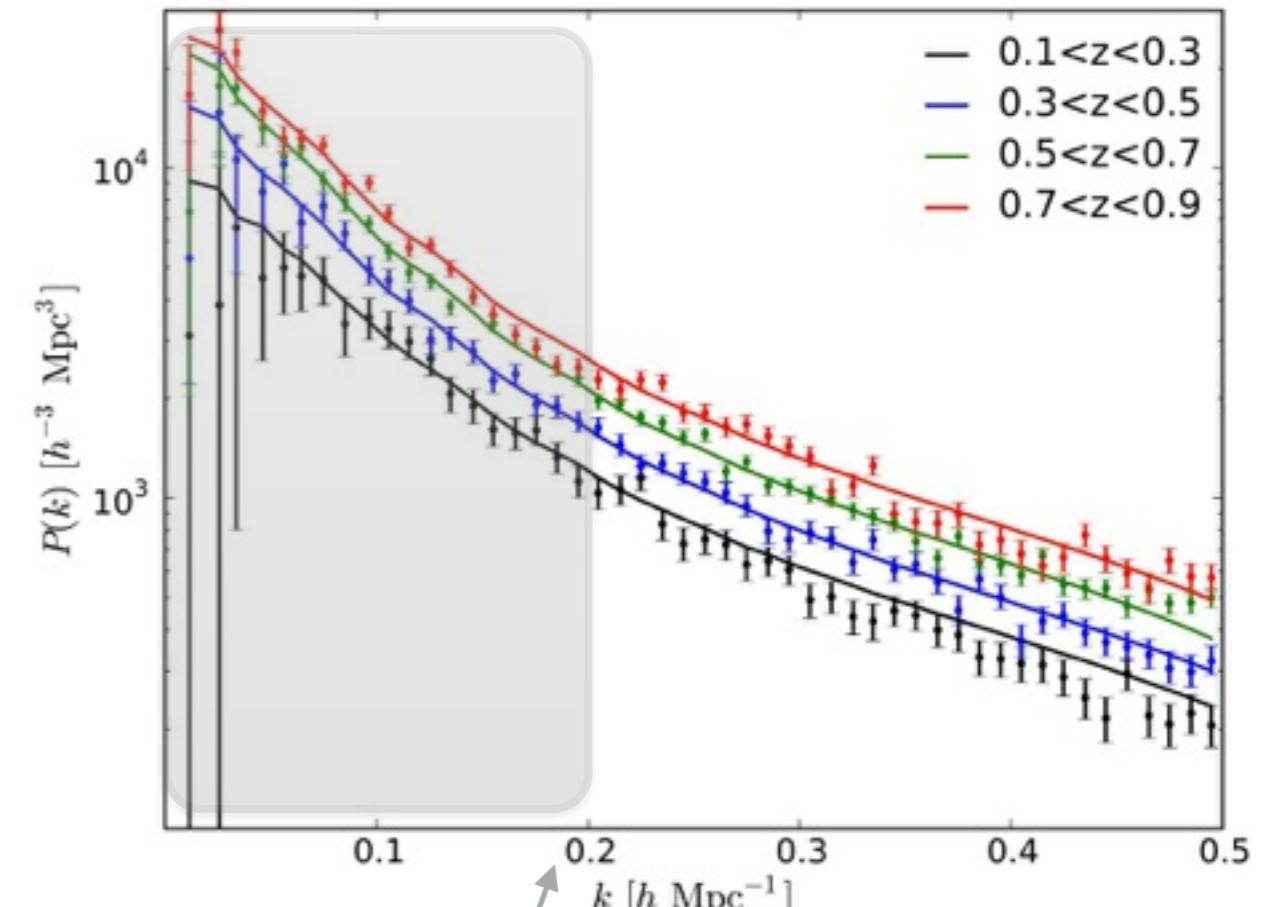
*preliminary*



## 4. Search with LSS survey—WiggleZ

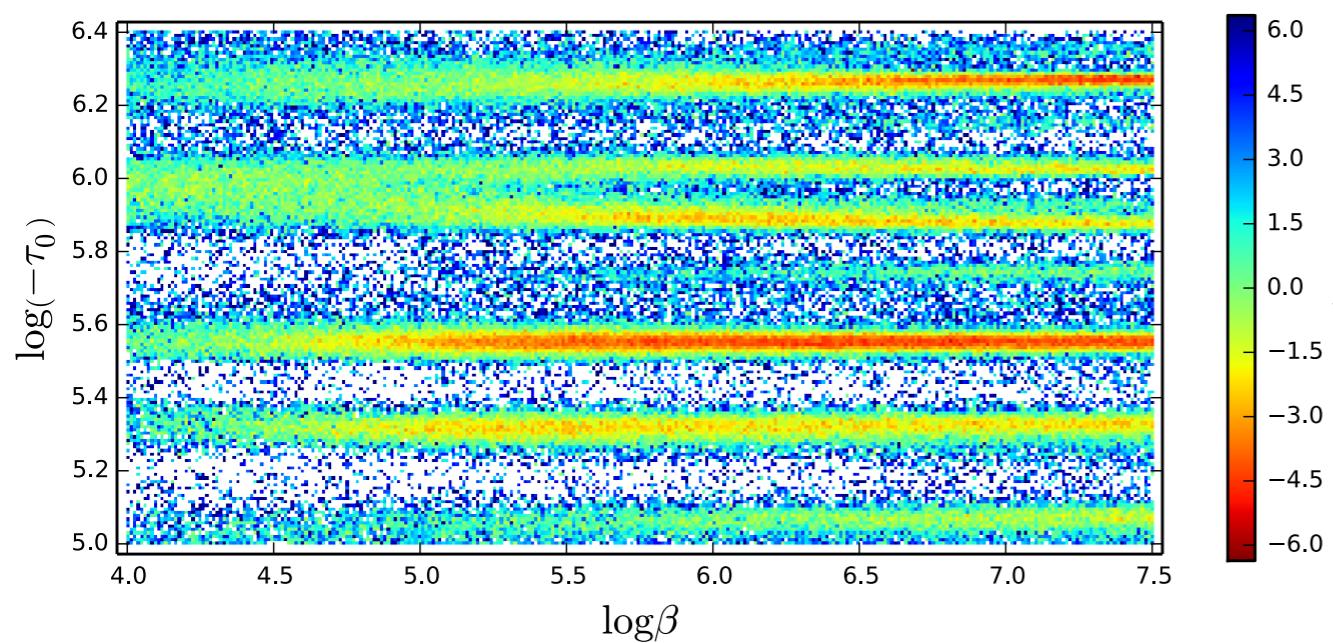


features shows  
around  $k \sim (0.1, 0.2)$

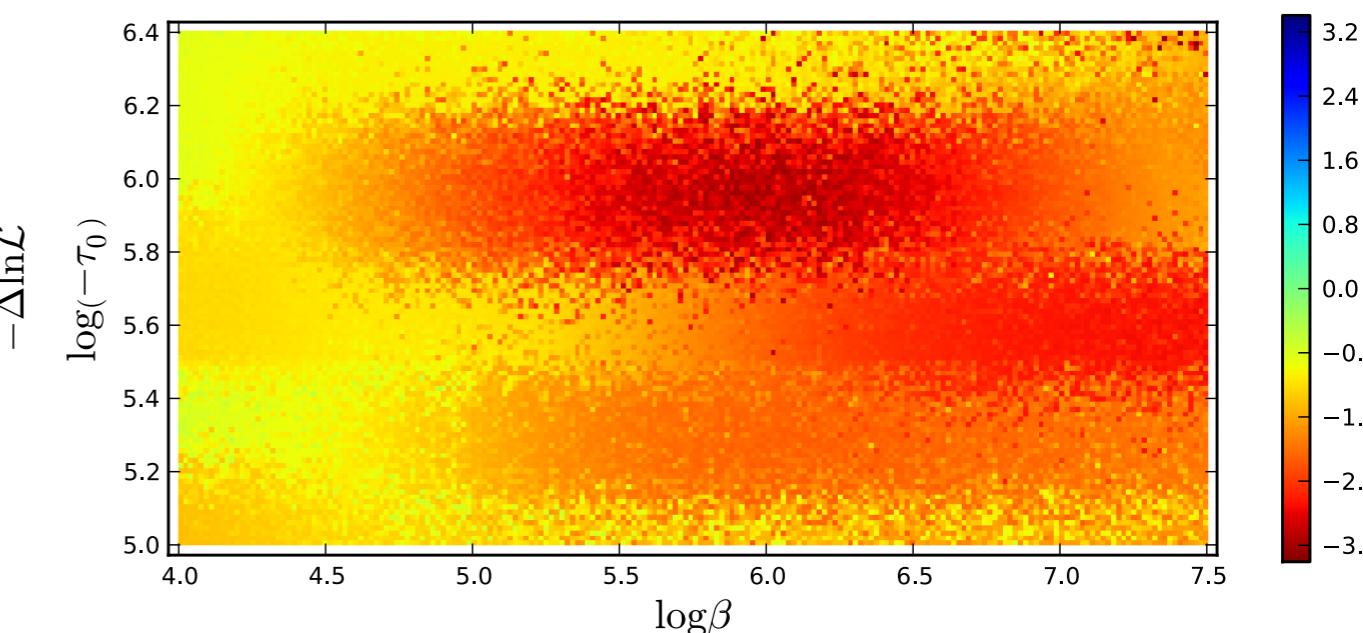


Search up to  
 $k = 0.2$

# Independent search with different data

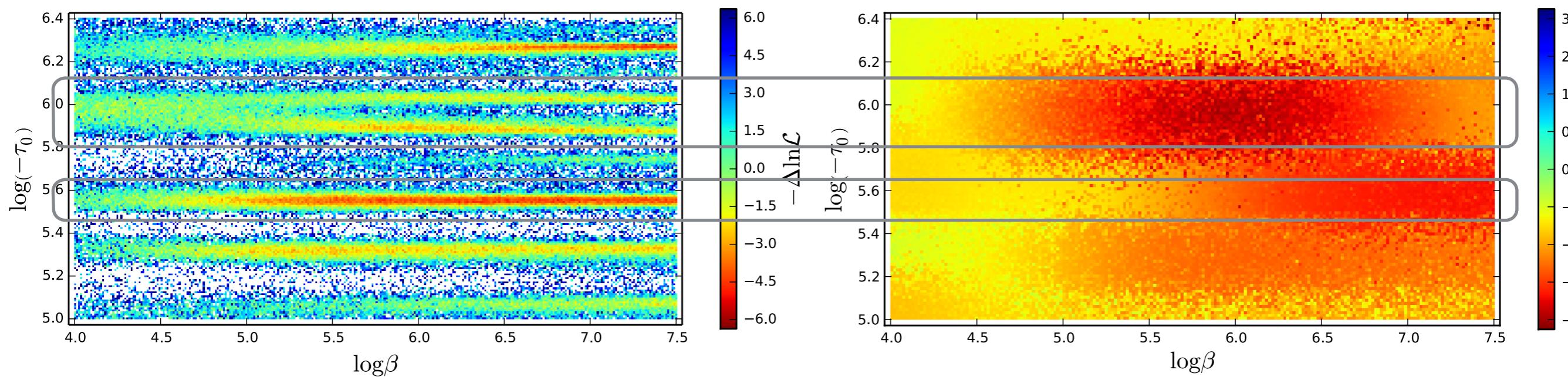


Planck+WP



WiggleZ

# Independent search with different data

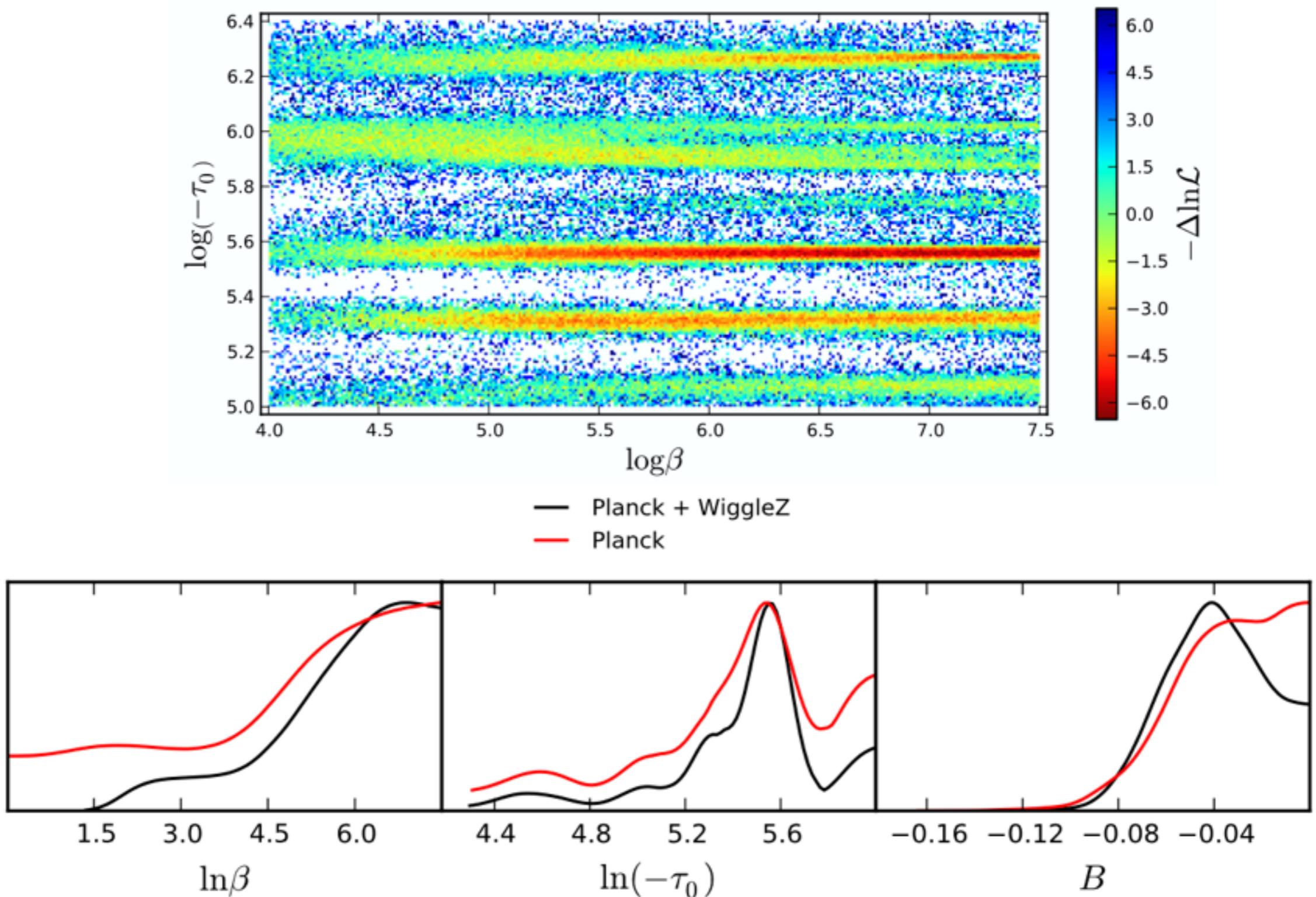


Planck+WP

WiggleZ

Two coincident modes  
including the best-fit mode

# Combine Planck and WiggleZ



get better constrained in Planck+WiggleZ

# Bayesian Evidence

Evidence:  $\mathcal{Z} = \int \mathcal{L}(\mathbf{D}|M(\boldsymbol{\theta})) \pi(\boldsymbol{\theta}) d^D \boldsymbol{\theta}$

$M_0$  : Base-LCDM model

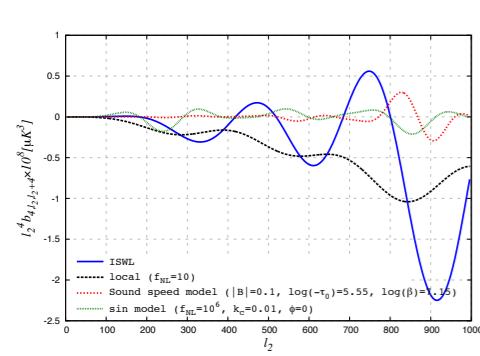
$M_1$  : Sound speed model

R<1: data favers M0

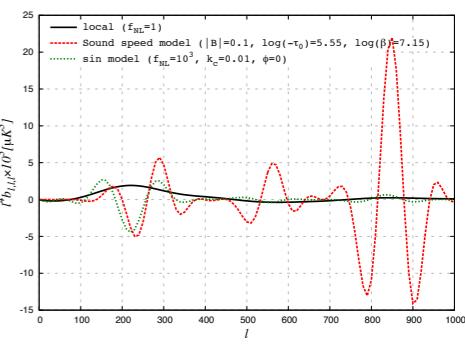
R>1: data favers M1

Model	Data set	posterior	evidence	Bayesian ratio
$M_1$	Planck	9801.918 (9796.27)	$-4955.61 \pm 0.31$	$\exp(0.46) \simeq 1.6$
$M_0$	Planck	9807.154 (9805.90)	$-4956.07 \pm 0.31$	
$M_1$	Planck+ WiggleZ	10253.570 (10249.20)	$-5183.05 \pm 0.32$	$\exp(0.62) \simeq 1.9$
$M_0$	Planck+ WiggleZ	10262.042 (10258.80)	$-5183.67 \pm 0.31$	

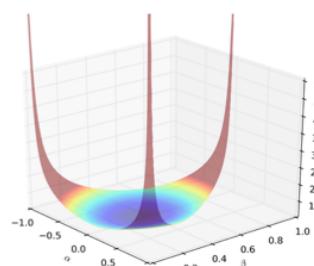
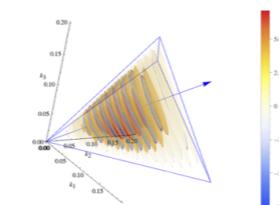
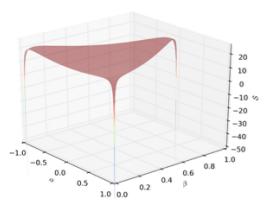
Jeffreys's criterion ( $1 < R < 3$ ): *Barely worth mentioning!*



# Conclusion

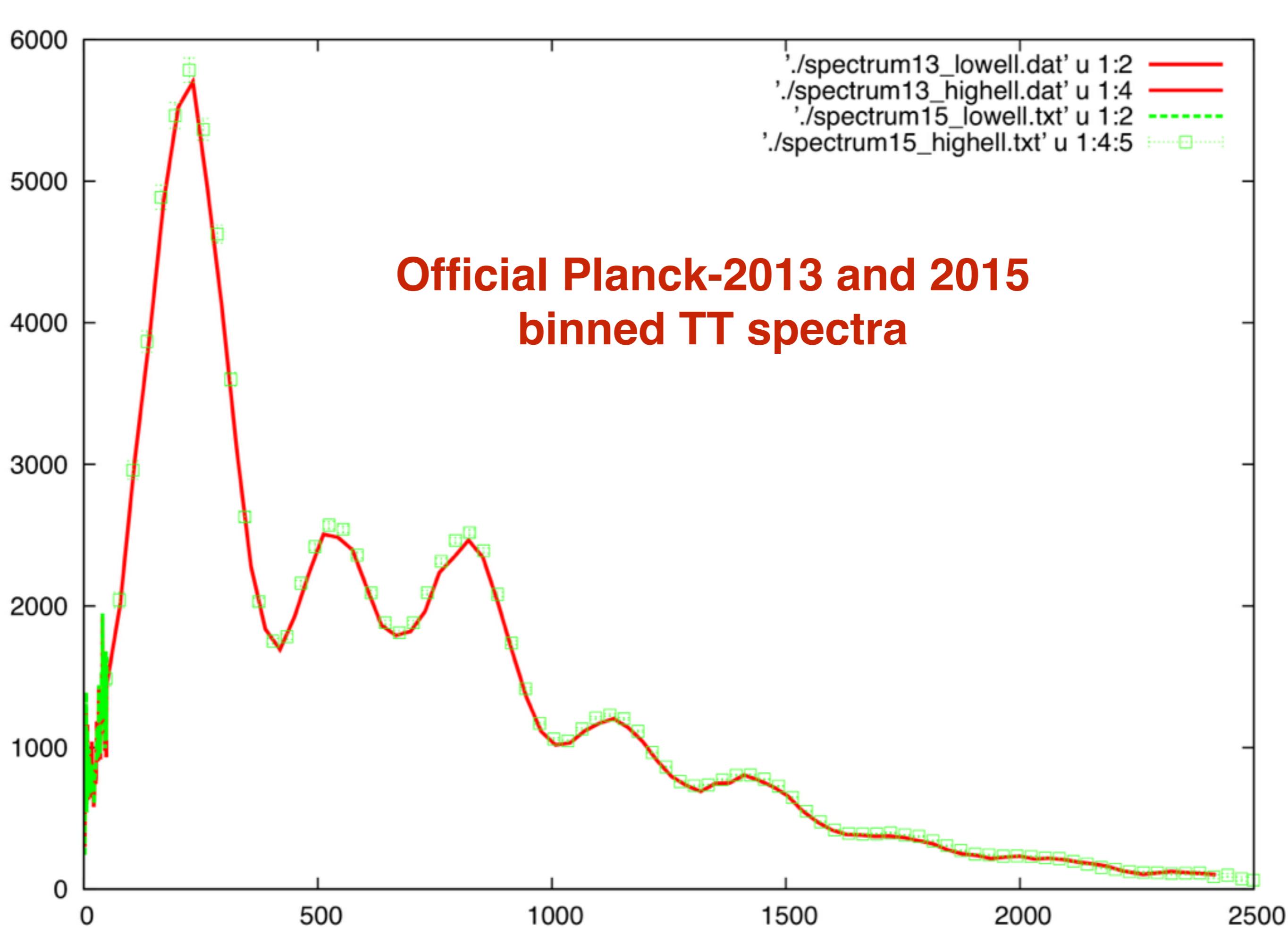


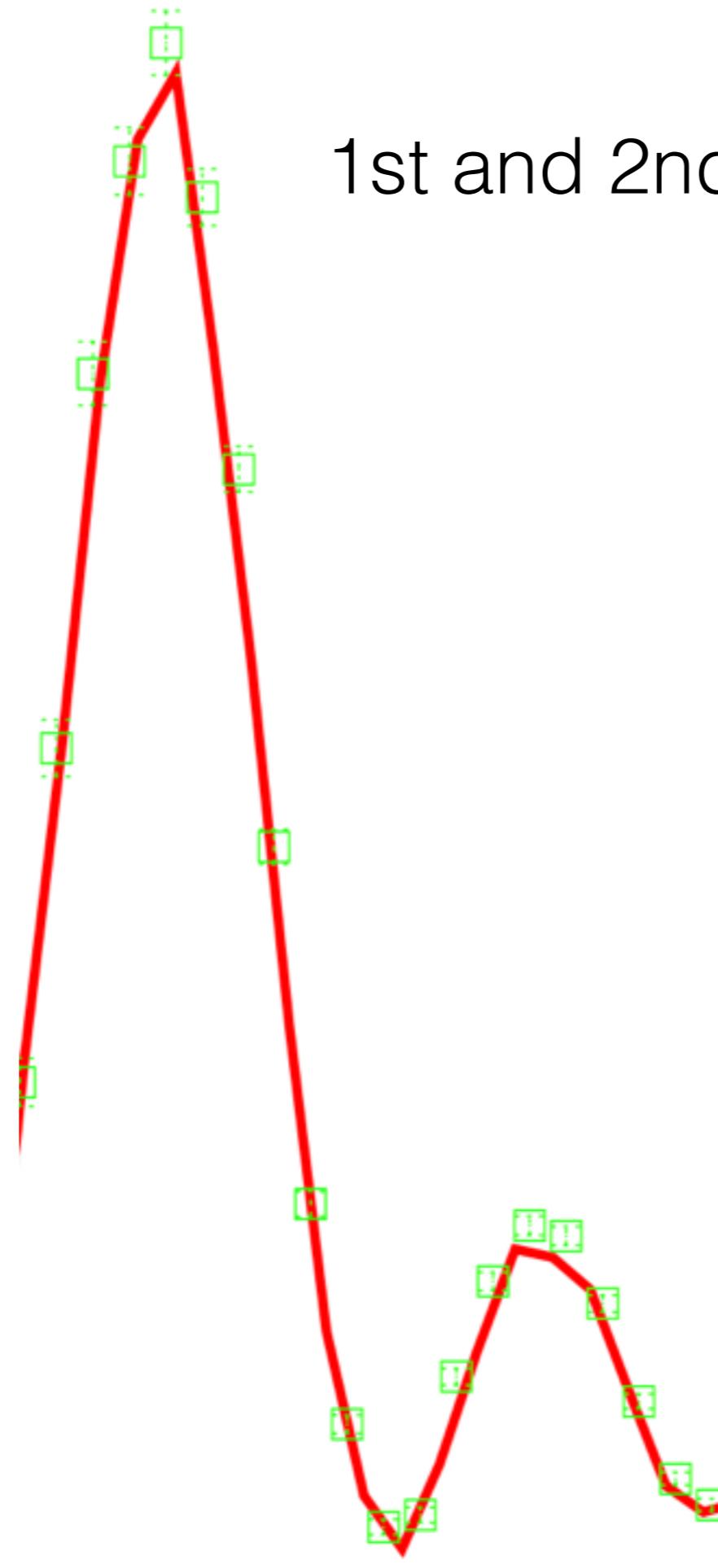
1. A transient reduction of the speed of sound generically gives primordial oscillatory features.
2. It could produce sizeable and distinguishable features in CMB spectrum, bispectrum and matter spectrum.
3. Planck-2013 and WiggleZ data shows a coincidence in the best-fit mode.
4. The statistical significance is not big enough to claim a detection.



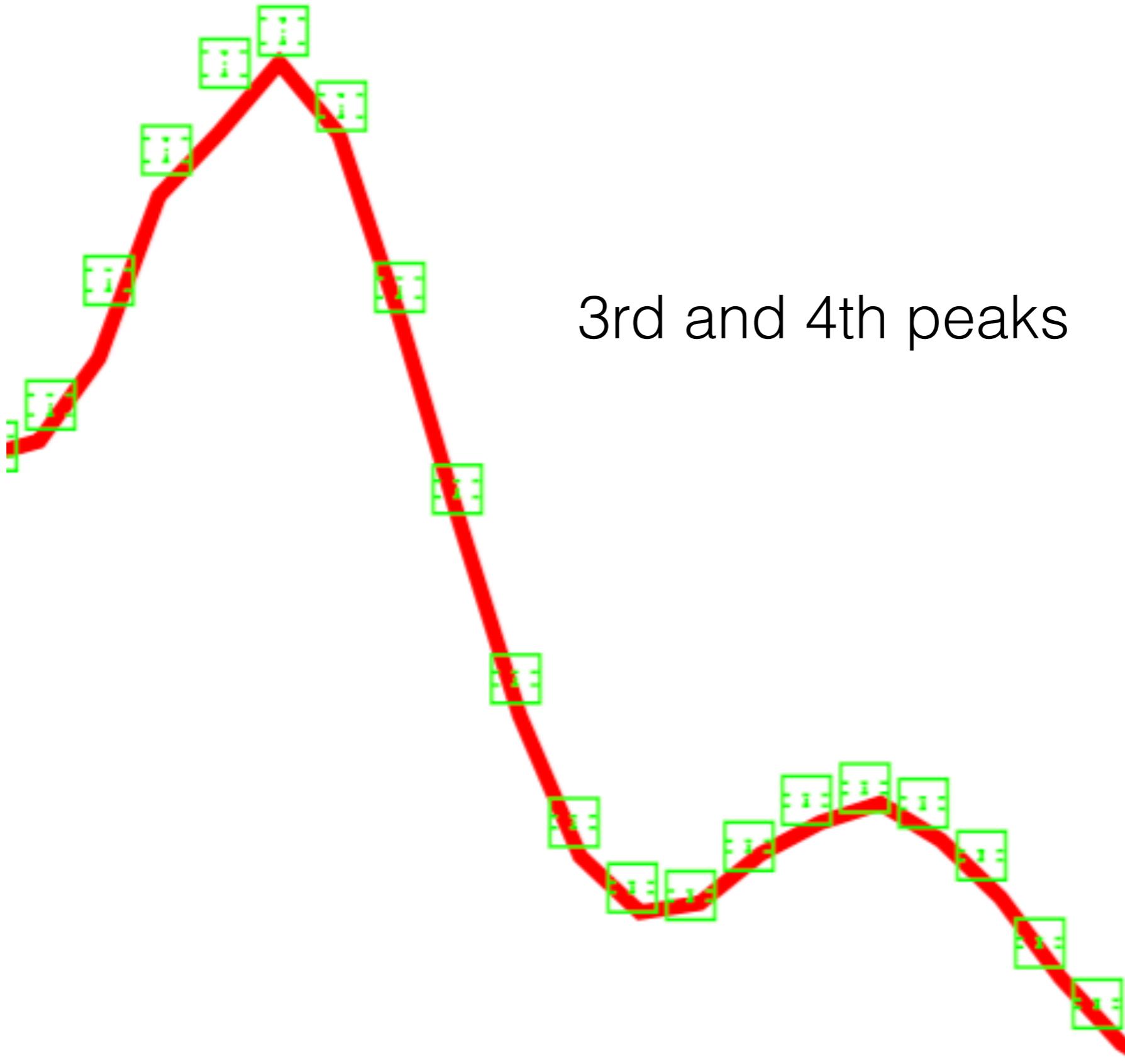
**Thank you!**

bonus slide



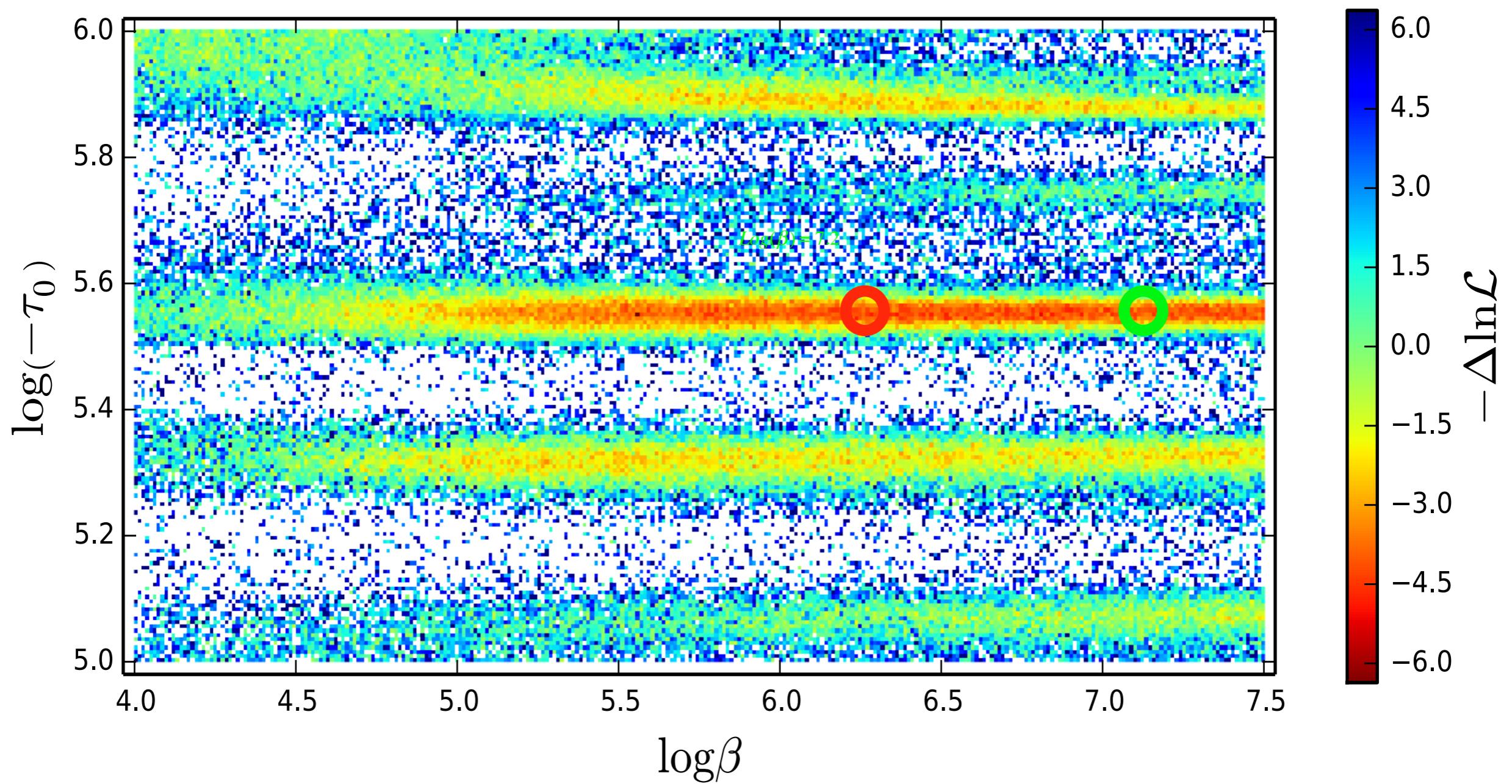


1st and 2nd peak

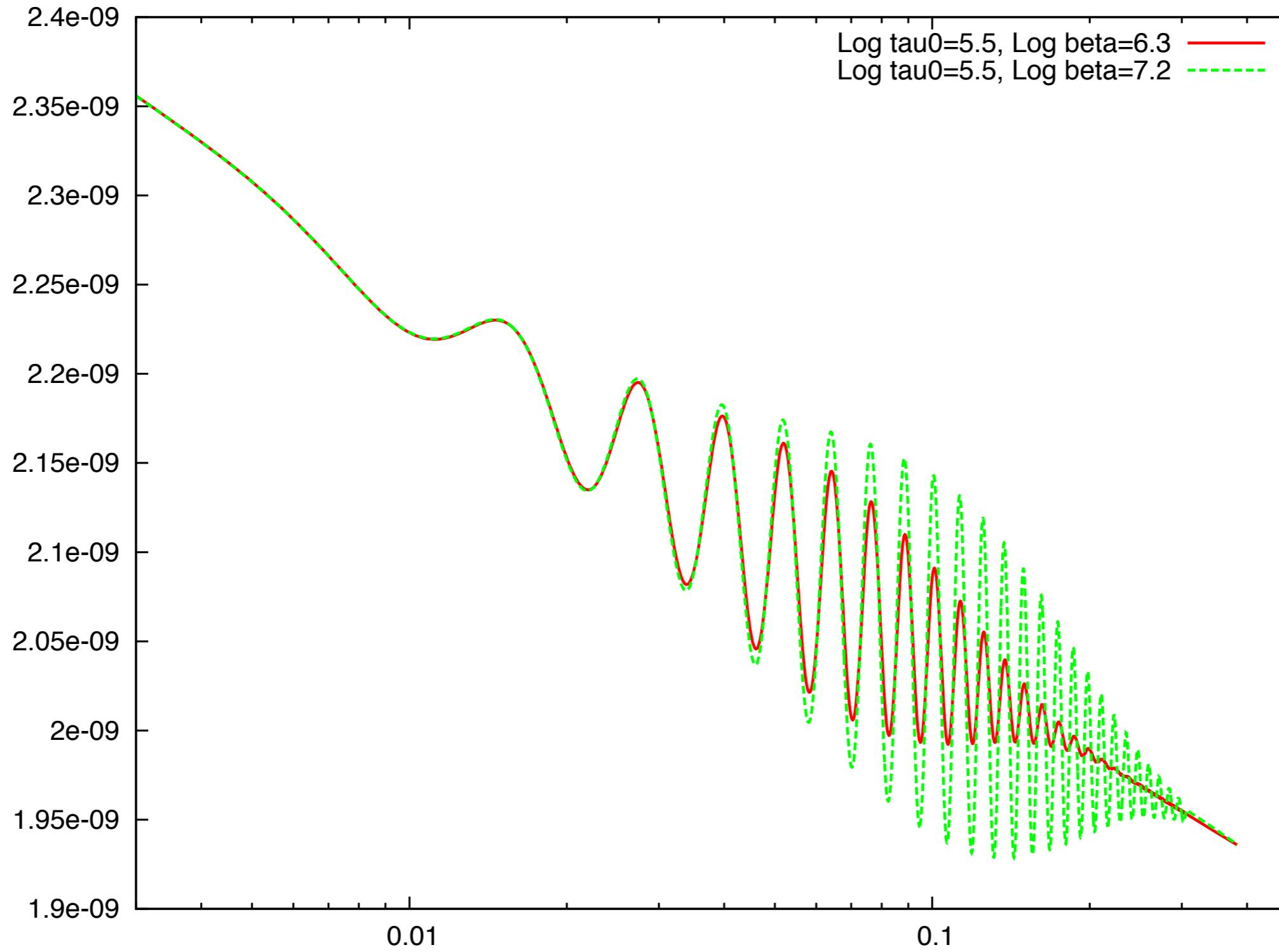


3rd and 4th peaks

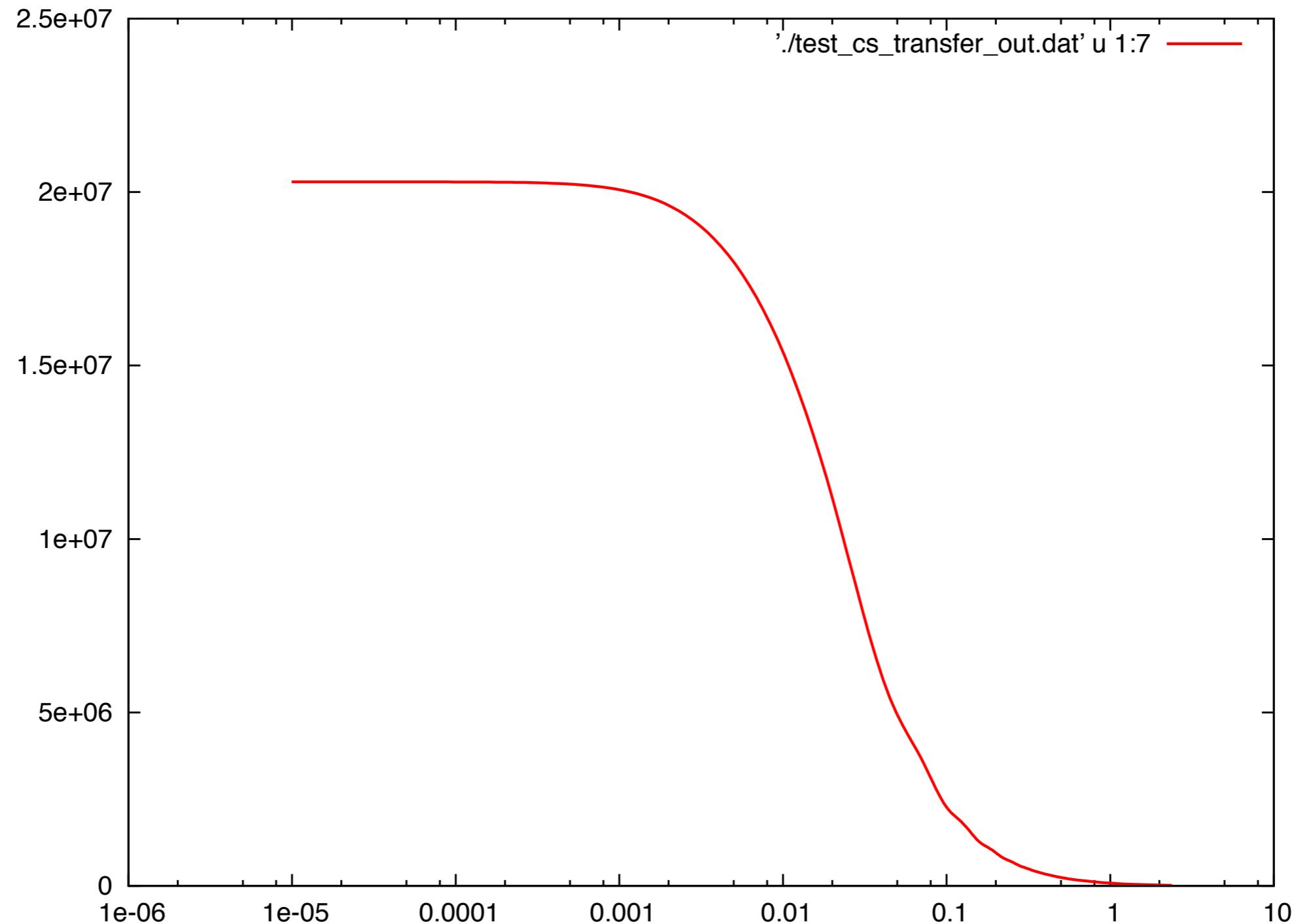
Two mode with the same frequency  $\log(-\tau_0) = 5.5$   
but with different location  $\log(\beta) = 6.3$  (red)  $\log(\beta) = 7.2$  (green)



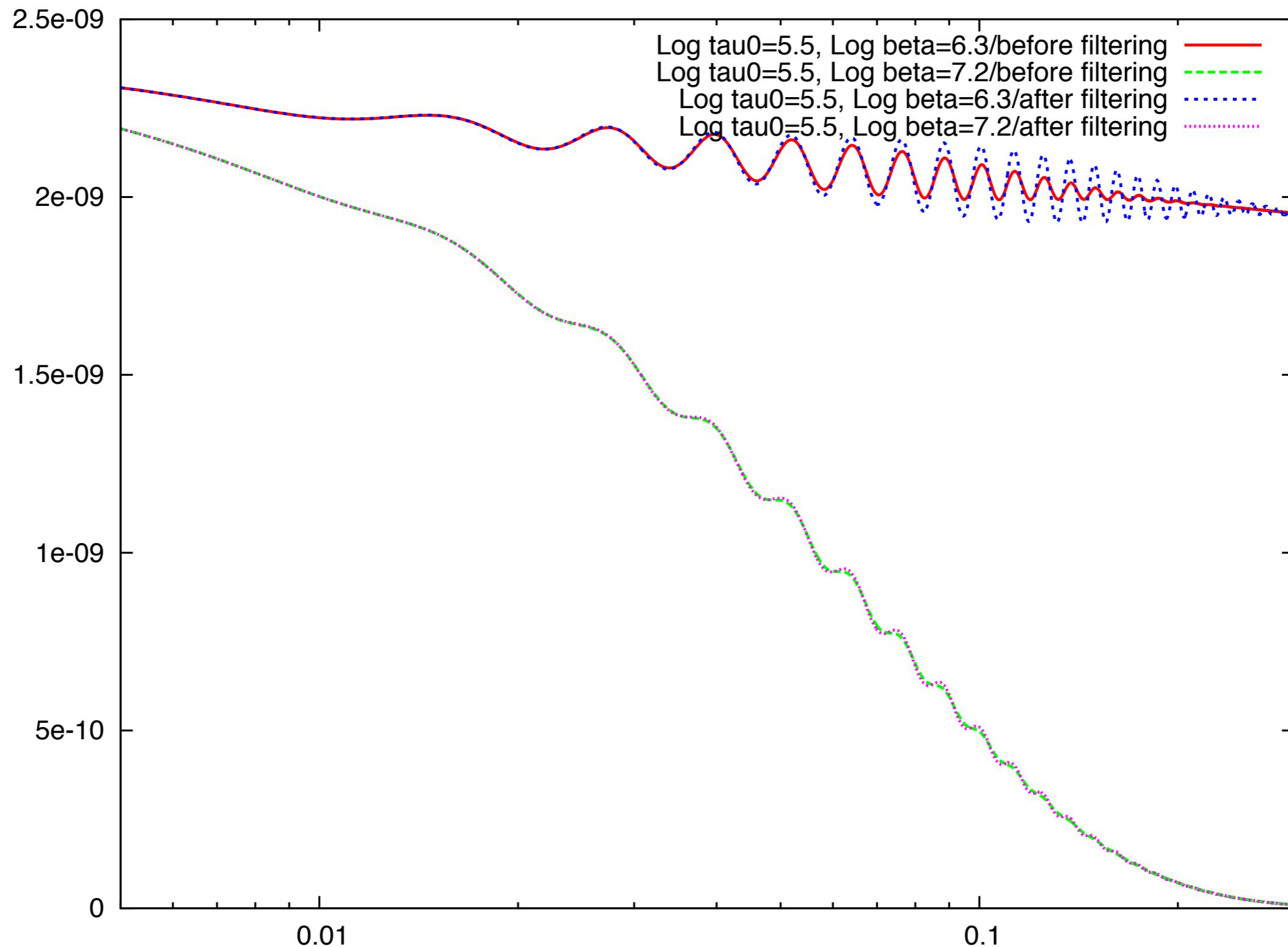
# Primordial power spectrum



# Transfer function



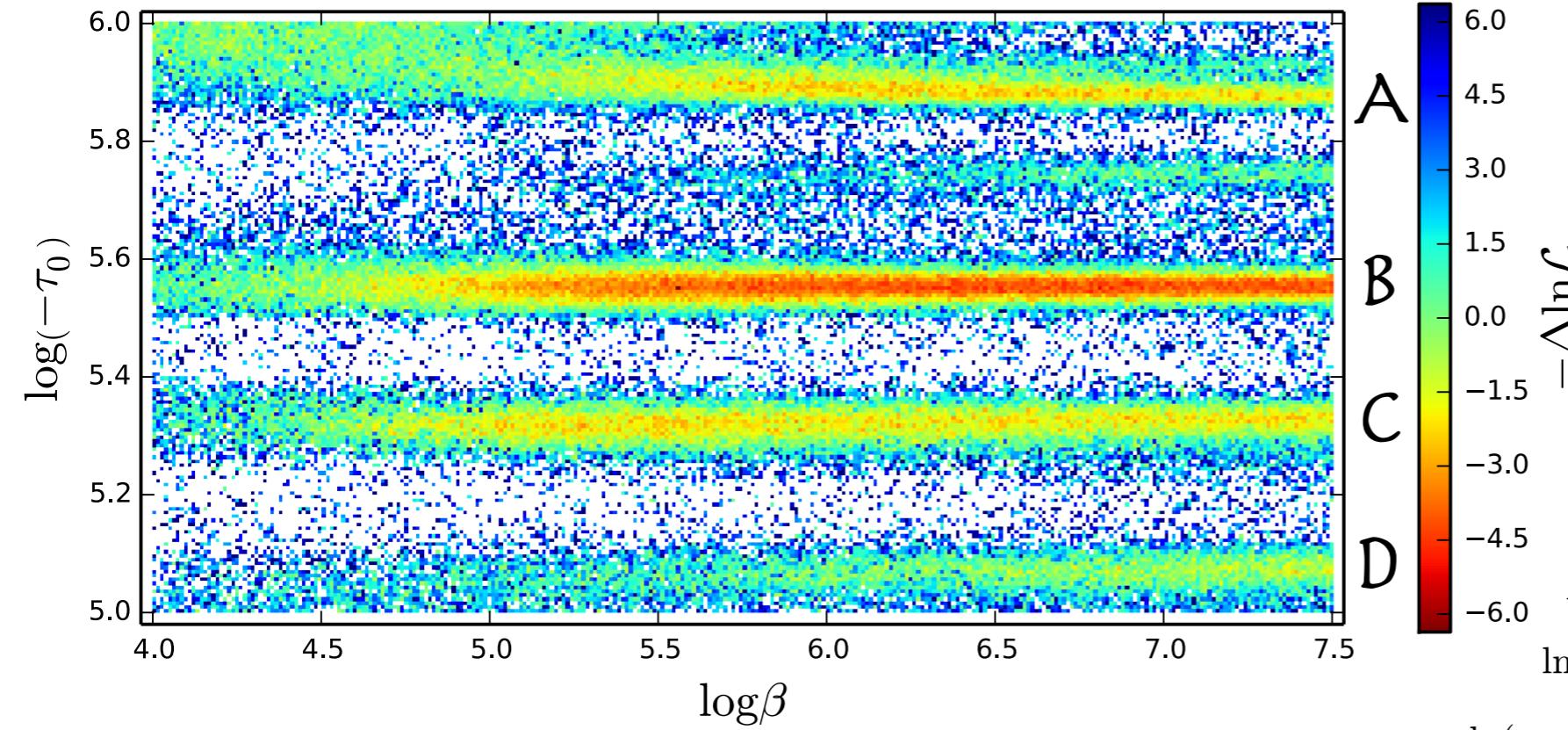
After convolving with transfer function  
they looks similar, due to the damping effect on small scale



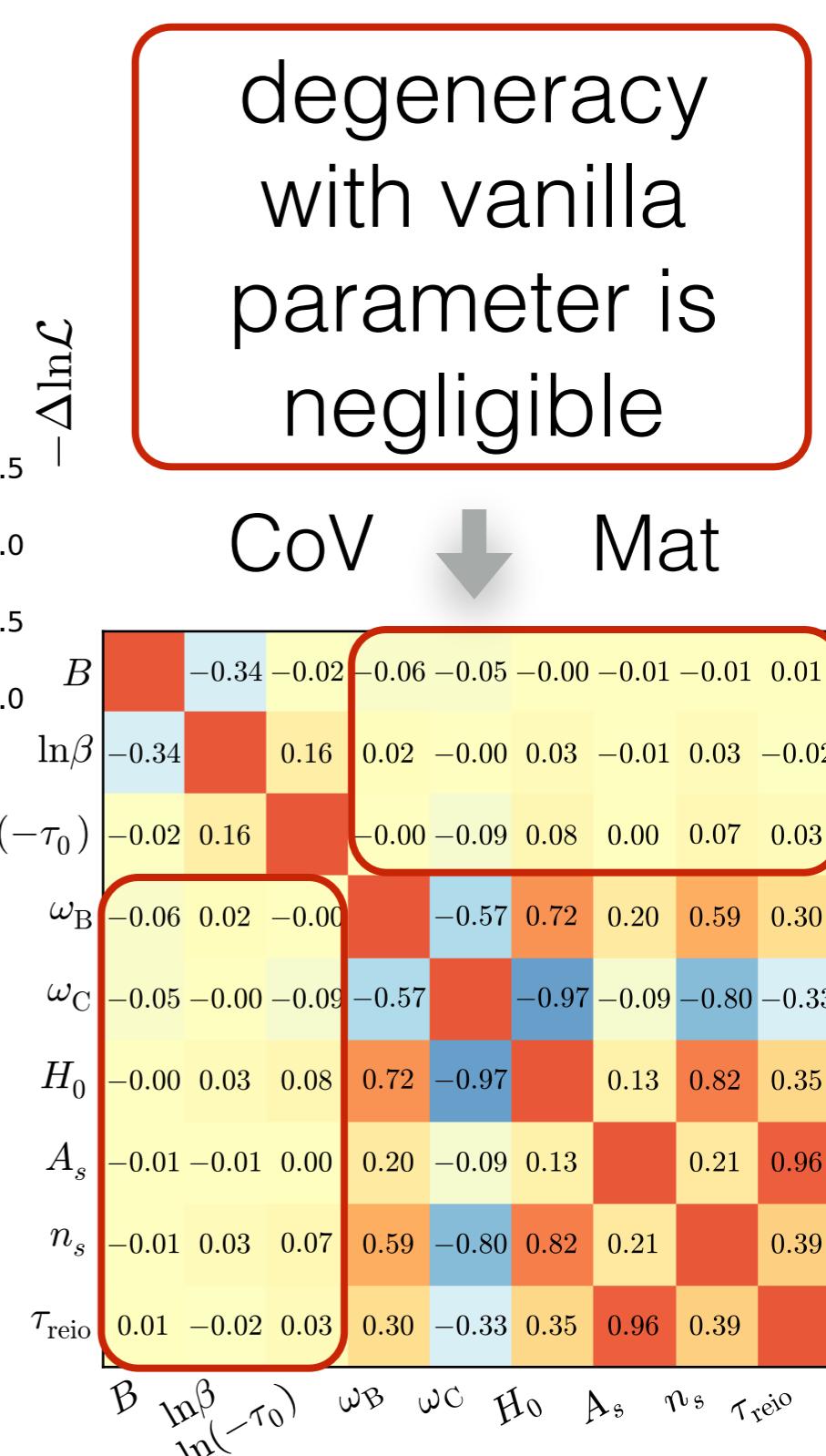
### 3. Search with CMB map—TT spectrum

profile likelihood

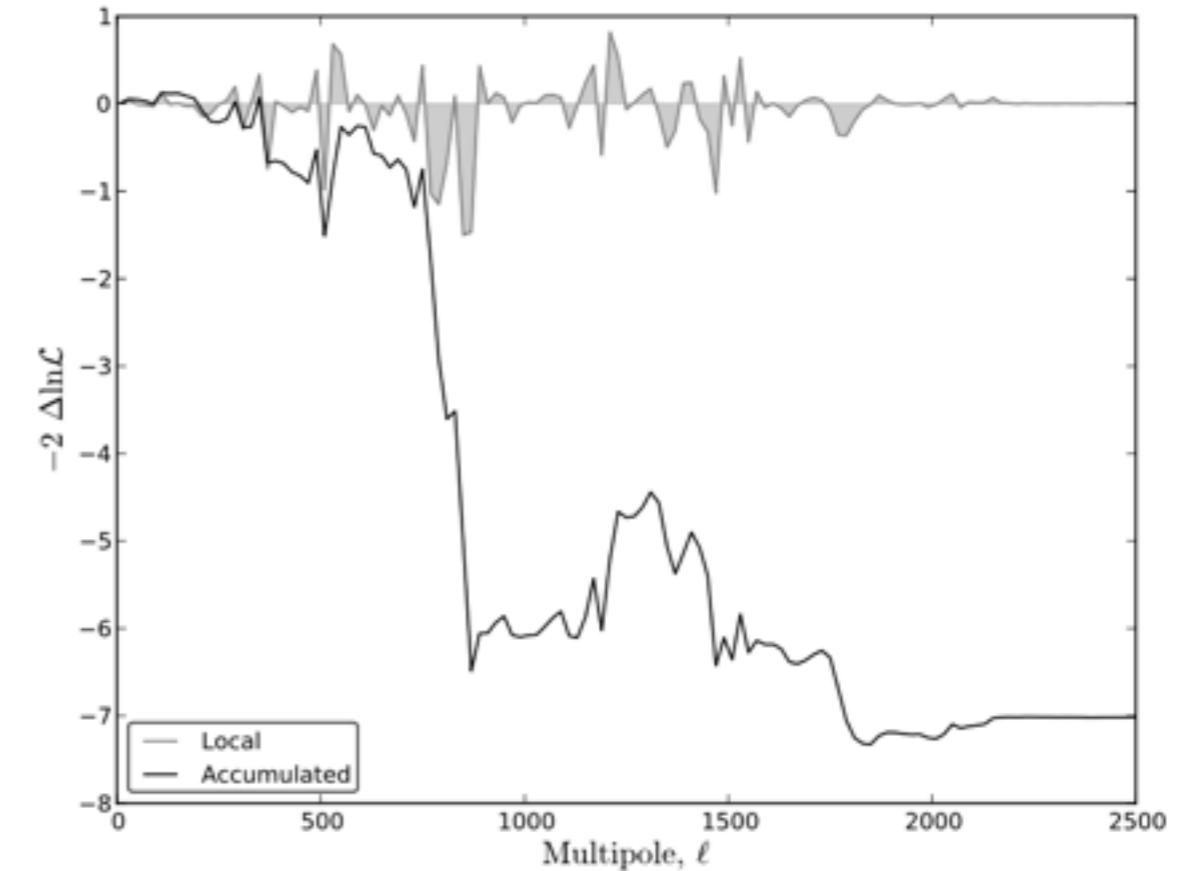
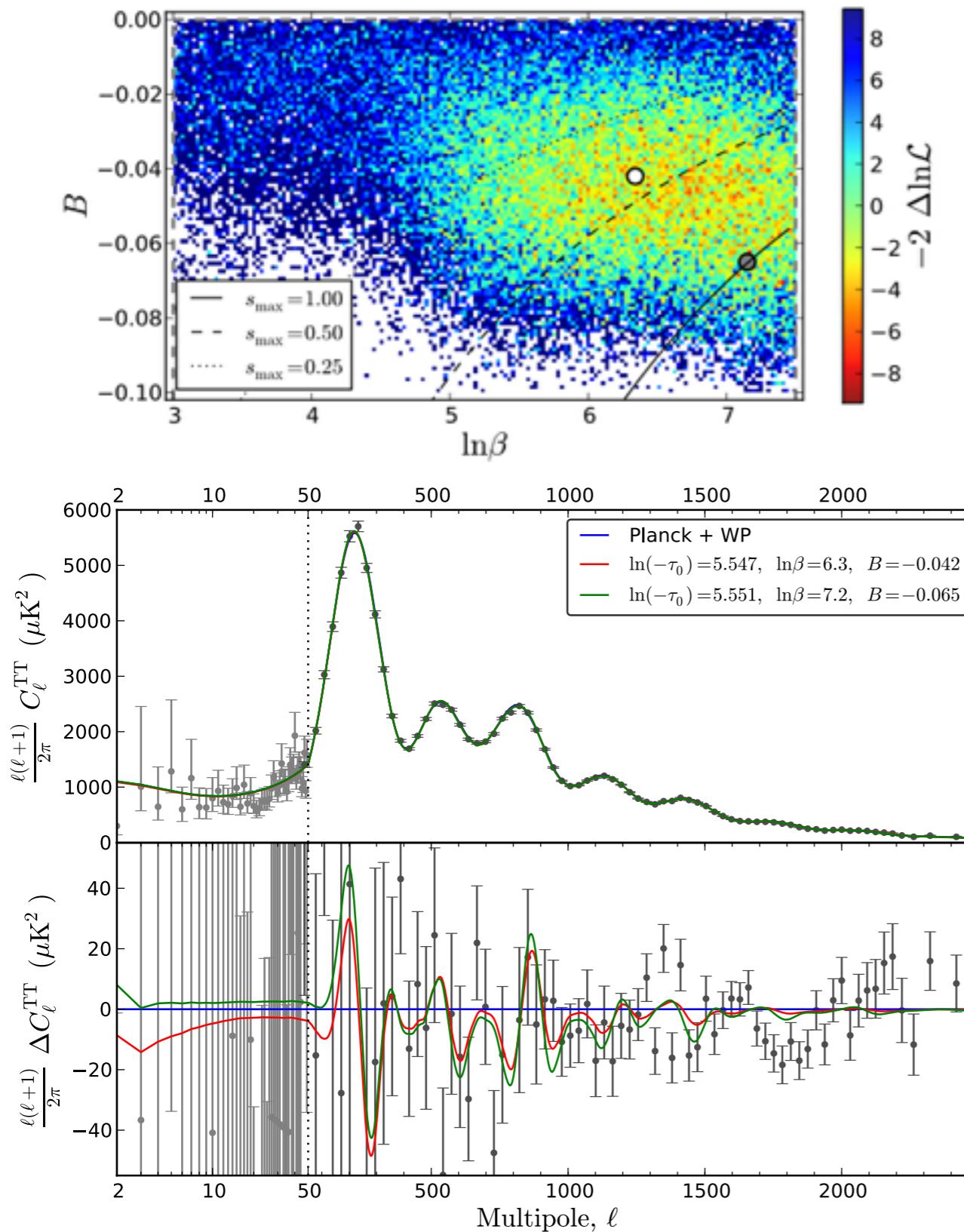
Planck+WP



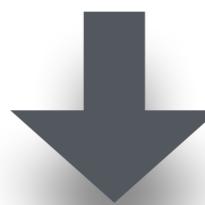
#	$-B \times 10^2$	$\ln \beta$	$\ln(-\tau_0)$	$\Delta\chi^2$
A	$(4.5) 3.7^{+1.6}_{-3.0}$	$(5.7) 5.7^{+0.9}_{-1.0}$	$(5.895) 5.910^{+0.027}_{-0.035}$	-4.3
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C	$(3.6) 3.1^{+1.6}_{-1.9}$	$(6.5) 5.6^{+1.9}_{-0.7}$	$(5.331) 5.327^{+0.026}_{-0.034}$	-6.2
D	(4.4)	(6.5)	(5.06)	-3.3



# Search with CMB map—Zoom in best-fit



Need to consider  
**look-elsewhere effect!**



Enlarge the  
parameter space

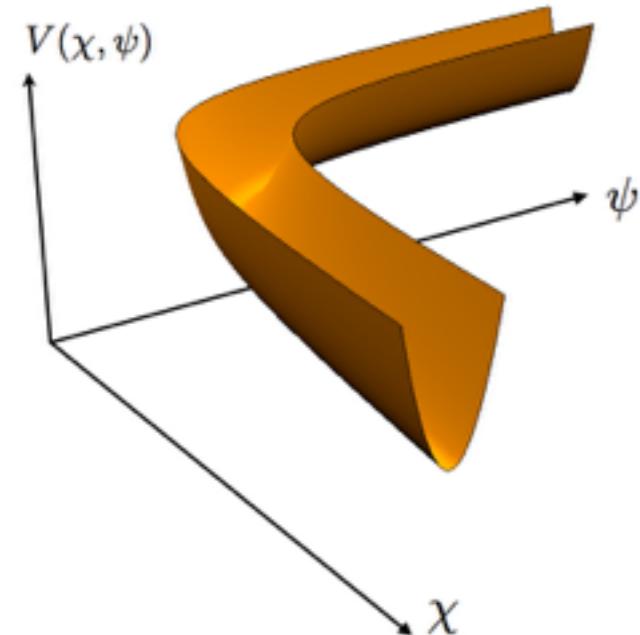
## 2. Models with a transient reduction of the speed of sound

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi) \right]$$

A.Achucarro et. al.  
JHEP 1205  
(2012) 066

effective action:

$$\begin{aligned} S = & \frac{1}{2} \int d^4x \dot{\phi}_0^2 \left\{ c_s^{-2} \dot{\pi}^2 - (\nabla\pi)^2 + \left( \frac{1}{c_s^2} - 1 \right) \dot{\pi} [\dot{\pi}^2 - (\nabla\pi)^2] + \left( \frac{1}{c_s^2} - 1 \right)^2 \frac{\dot{\pi}^3}{2} \right. \\ & \left. + 2 \frac{\ddot{\phi}_0}{\dot{\phi}_0} \left[ \frac{\dot{\pi}^2}{c_s^2} - (\nabla\pi)^2 \right] \pi - 2 \frac{\dot{c}_s}{c_s^3} \dot{\pi}^2 \pi \right\}, \end{aligned}$$

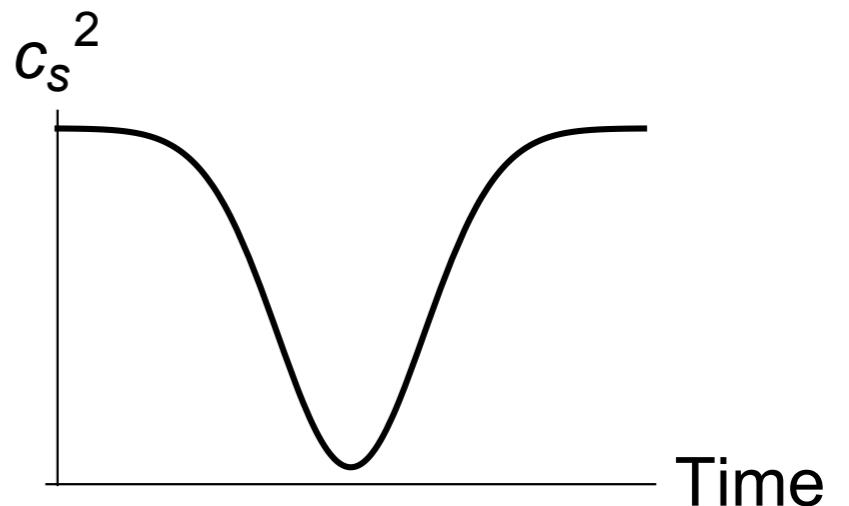


$\phi^a(t, \mathbf{x}) = \phi_0^a(t + \pi) + N^a(t + \pi)\mathcal{F}$

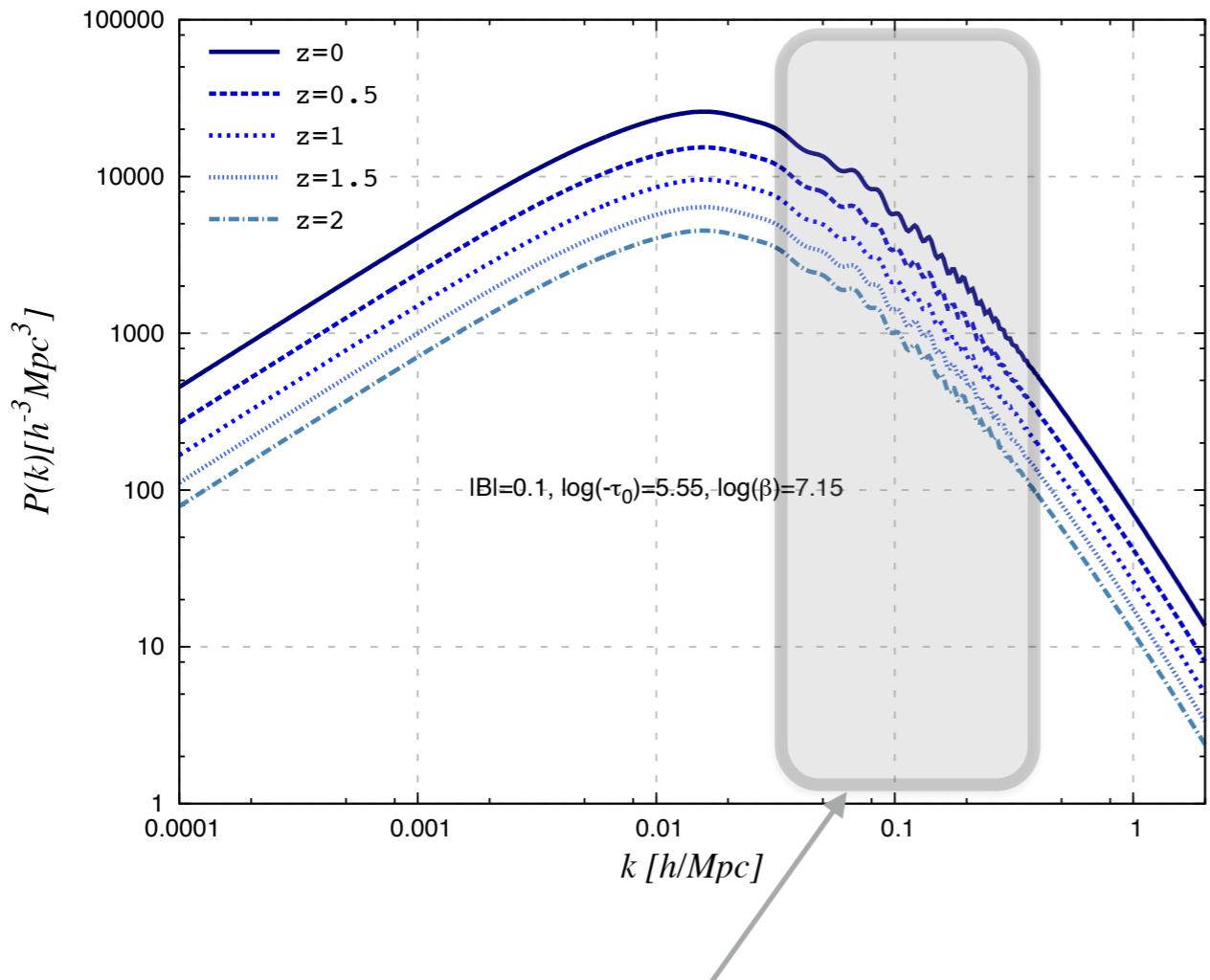
light adiabatic      ↓      heavy isocurvature

integrating out      ↓      heavy field

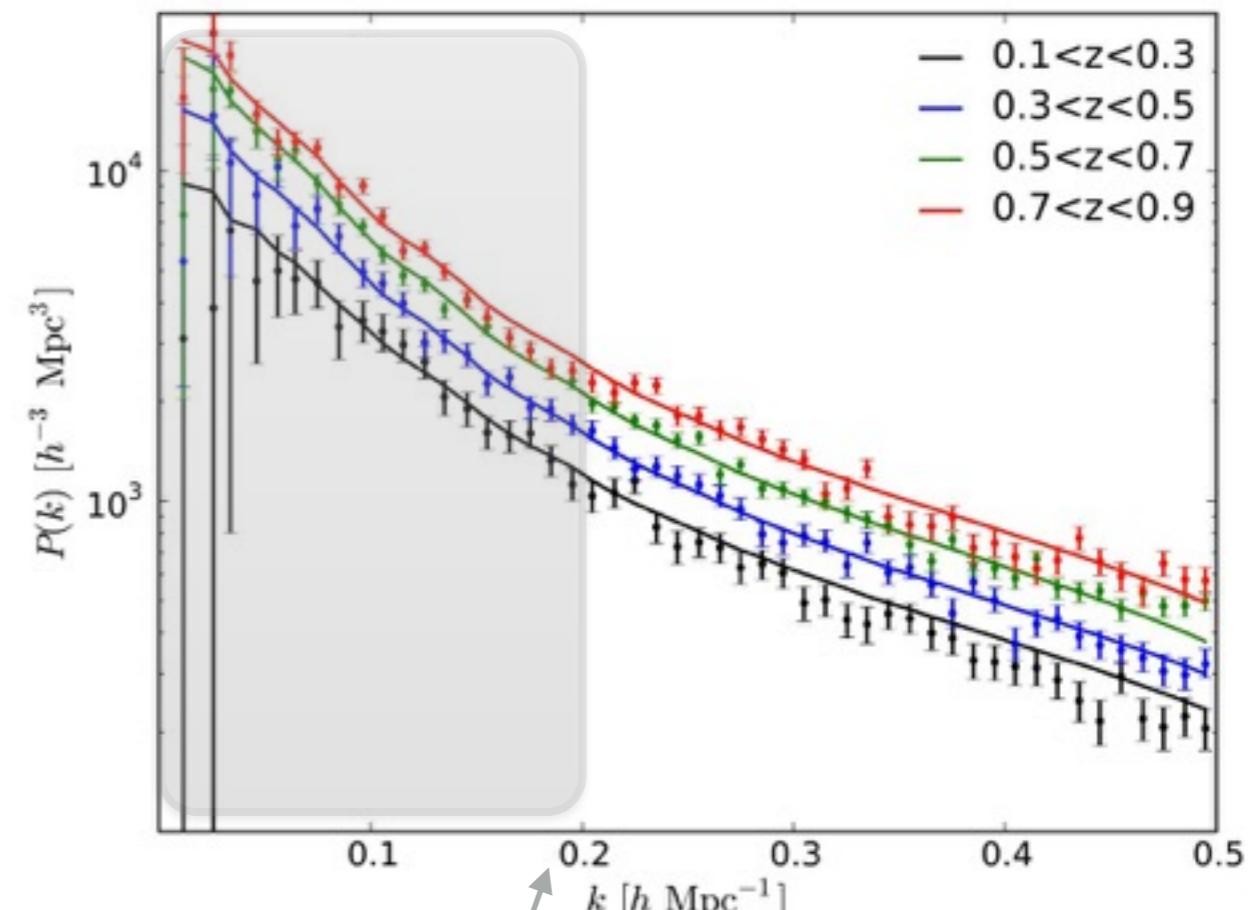
turn  
sound speed  
reduced



## 4. Search with LSS survey—WiggleZ



features shows  
around  $k \sim (0.1, 0.2)$



Search up to  
 $k=0.2$