# fig\_mechanisms

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# 1 Figure 5 - Reaction mechanism determination

Author: Mathieu Baltussen

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Python implementation: CPython Python version : 3.9.5 IPython version : 7.28.0

 seaborn
 : 0.11.1

 pandas
 : 1.2.4

 theano
 : 1.1.2

 numpy
 : 1.20.3

 arviz
 : 0.11.4

 networkx
 : 2.6.3

 scipy
 : 1.6.2

```
matplotlib: 3.4.2
sys : 3.9.5 | packaged by conda-forge | (default, Jun 19 2021, 00:32:32)
[GCC 9.3.0]
pymc3 : 3.11.4
```

# 1.1 Helper functions for plotting

```
[2]: def savefig(name):
    plt.savefig(f"../figures/{name}.svg")
    plt.savefig(f"../figures/{name}.png", dpi=300)

XSIZE = 7.0 #inch
YSIZE = XSIZE/np.sqrt(2) #inch

def generate_figure(n_col, n_row):
    DX = XSIZE/n_col
    DY = YSIZE/n_col
    YLENGTH = n_row*DY

fig = plt.figure(figsize=(XSIZE, YLENGTH), constrained_layout=True)
    gs = fig.add_gridspec(n_row, n_col)
    return fig, gs
```

## 1.2 Data loading

```
[35]: experiments = pd.read_csv(f"../data/kinetic_studies.csv").query(
          f'enzyme == "G6PDH"'
      data = []
      for t in experiments.itertuples():
          df = pd.read_csv(f"../data/{t.data_path}")
          df = df.assign(
              kf=t.flowrate / (60 * t.volume),
              G6PDH=t.enzyme_concentration,
              code=t.experiment_code,
          )
          data.append(df)
      data = pd.concat(data).reset_index(drop=True)
      data = data.assign(
          NAD_obs=data.NAD_in - data.NADH_obs,
          G6P_obs=data.G6P_in - data.NADH_obs,
          G6PdL_obs=data.NADH_obs
      )
```

### 1.3 Creation of models

```
[36]: exp_idx, exp_coords = data.code.factorize(sort=True)
      obs_idx, obs_coords = data.index.factorize(sort=True)
      coords = {"exp": exp_coords, 'obs': obs_coords}
      with pm.Model(coords=coords) as model_0:
          k_cat = pm.Uniform("k_cat", 0, 500)
          K_G6P = pm.Uniform("K_G6P", 1, 4000)
          K_NAD = pm.Uniform("K_NAD", 1, 2000)
          # KI_NADH = pm.Uniform("KI_NADH", 1, 5000)
          sigma = pm.Exponential("sigma", 0.5, dims='exp')
          G6PDH = data.G6PDH.values
          NADH = data.NADH_obs.values
          NAD = data.NAD_obs.values
          G6P = data.G6P obs.values
          G6PdL = data.G6PdL_obs.values
          G6P_in = data.G6P_in.values
          NAD_in = data.NAD_in.values
          kf = data.kf.values
          NADH_obs = pm.Normal("NADH_obs",
                      mu = k_cat*G6PDH*G6P*NAD/(kf*K_G6P*K_NAD*(1 + G6P/K_G6P)*(1+NAD/K)
       \hookrightarrow K_NAD),
                      sigma=sigma[exp_idx],
                      observed= NADH
      with pm.Model(coords=coords) as model_1:
          k_cat = pm.Uniform("k_cat", 0, 500)
          K_G6P = pm.Uniform("K_G6P", 1, 4000)
          K_NAD = pm.Uniform("K_NAD", 1, 2000)
          KI_NADH = pm.Uniform("KI_NADH", 1, 10000)
          sigma = pm.Exponential("sigma", 0.5, dims='exp')
          G6PDH = data.G6PDH.values
          NADH = data.NADH_obs.values
          NAD = data.NAD_obs.values
          G6P = data.G6P obs.values
          G6PdL = data.G6PdL_obs.values
```

```
G6P_in = data.G6P_in.values
                  NAD_in = data.NAD_in.values
                  kf = data.kf.values
                  NADH_obs = pm.Normal("NADH_obs",
                                                                           mu = k_{cat}*G6PDH*G6P*NAD/(kf*K_G6P*K_NAD*(1 + G6P/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6
    \rightarrowK_NAD)*(1+ NADH/KI_NADH)),
                                                                           sigma=sigma[exp_idx],
                                                                           observed= NADH
                                                                           )
with pm.Model(coords=coords) as model_2:
                  k_cat = pm.Uniform("k_cat", 0, 500)
                  K_G6P = pm.Uniform("K_G6P", 1, 4000)
                  K_NAD = pm.Uniform("K_NAD", 1, 2000)
                  KI_NADH = pm.Uniform("KI_NADH", 1, 10000)
                  sigma = pm.Exponential("sigma", 0.5, dims='exp')
                  G6PDH = data.G6PDH.values
                  NADH = data.NADH_obs.values
                  NAD = data.NAD_obs.values
                  G6P = data.G6P_obs.values
                  G6PdL = data.G6PdL_obs.values
                  G6P_in = data.G6P_in.values
                  NAD_in = data.NAD_in.values
                  kf = data.kf.values
                  NADH_obs = pm.Normal("NADH_obs",
                                                                           mu = k_{cat}*G6PDH*G6P*NAD/(kf*K_G6P*K_NAD*(1 + G6P/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6
    →K_NAD+ NADH/KI_NADH)),
                                                                           sigma=sigma[exp_idx],
                                                                           observed= NADH
with pm.Model(coords=coords) as model_3:
                  k_cat = pm.Uniform("k_cat", 0, 500)
                  K_G6P = pm.Uniform("K_G6P", 1, 4000)
                  K_NAD = pm.Uniform("K_NAD", 1, 2000)
                  KI_NADH = pm.Uniform("KI_NADH", 1, 10000)
                  sigma = pm.Exponential("sigma", 0.5, dims='exp')
```

```
G6PDH = data.G6PDH.values
    NADH = data.NADH_obs.values
    NAD = data.NAD_obs.values
    G6P = data.G6P_obs.values
    G6PdL = data.G6PdL_obs.values
    G6P_in = data.G6P_in.values
    NAD_in = data.NAD_in.values
    kf = data.kf.values
    NADH_obs = pm.Normal("NADH_obs",
                mu = k_cat*G6PDH*G6P*NAD/(kf*K_G6P*K_NAD*(1 + G6P/K_G6P + NADH/
 \hookrightarrow KI_NADH)*(1+NAD/K_NAD)),
                sigma=sigma[exp_idx],
                observed= NADH
with pm.Model(coords=coords) as model_4:
    k_cat = pm.Uniform("k_cat", 0, 500)
    K_G6P = pm.Uniform("K_G6P", 1, 4000)
    K_NAD = pm.Uniform("K_NAD", 1, 2000)
    KI_NADH = pm.Uniform("KI_NADH", 1, 10000)
    sigma = pm.Exponential("sigma", 0.5, dims='exp')
    G6PDH = data.G6PDH.values
    NADH = data.NADH_obs.values
    NAD = data.NAD_obs.values
    G6P = data.G6P_obs.values
    G6PdL = data.G6PdL_obs.values
    G6P_in = data.G6P_in.values
    NAD_in = data.NAD_in.values
    kf = data.kf.values
    NADH_obs = pm.Normal("NADH_obs",
                mu = k_{G6PDH*G6P*NAD/(kf*K_G6P*K_NAD*(1 + G6P/K_G6P + NADH/E))}
→KI_NADH)*(1+NAD/K_NAD+ NADH/KI_NADH)),
                sigma=sigma[exp_idx],
                observed= NADH
with pm.Model(coords=coords) as model_5:
    k_cat = pm.Uniform("k_cat", 0, 500)
```

```
K_G6P = pm.Uniform("K_G6P", 1, 4000)
          K_NAD = pm.Uniform("K_NAD", 1, 2000)
          KI_NADH = pm.Uniform("KI_NADH", 1, 10000)
          sigma = pm.Exponential("sigma", 0.5, dims='exp')
          G6PDH = data.G6PDH.values
          NADH = data.NADH_obs.values
          NAD = data.NAD_obs.values
          G6P = data.G6P obs.values
          G6PdL = data.G6PdL_obs.values
          G6P_in = data.G6P_in.values
          NAD_in = data.NAD_in.values
          kf = data.kf.values
          NADH_obs = pm.Normal("NADH_obs",
                                           mu = k_{cat}*G6PDH*G6P*NAD/(kf*K_G6P*K_NAD*(1 + G6P/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6P)*(1+NAD/K_G6
  \rightarrowK_NAD)*(1+ NADH/KI_NADH)*(1+ NADH/KI_NADH)),
                                           sigma=sigma[exp idx],
                                           observed= NADH
                                           )
with pm.Model(coords=coords) as model_6:
          k_cat = pm.Uniform("k_cat", 0, 500)
          K_G6P = pm.Uniform("K_G6P", 1, 4000)
          K_NAD = pm.Uniform("K_NAD", 1, 2000)
          KI_NADH = pm.Uniform("KI_NADH", 1, 10000)
          sigma = pm.Exponential("sigma", 0.5, dims='exp')
          G6PDH = data.G6PDH.values
          NADH = data.NADH obs.values
          NAD = data.NAD_obs.values
          G6P = data.G6P obs.values
          G6PdL = data.G6PdL_obs.values
          G6P_in = data.G6P_in.values
          NAD_in = data.NAD_in.values
          kf = data.kf.values
          NADH_obs = pm.Normal("NADH_obs",
                                           mu = k_cat*G6PDH*G6P*NAD/(kf*K_G6P*K_NAD*KI_NADH*(1 + G6P/
  \rightarrowK_G6P)*(1+NAD/K_NAD)*(1+ NADH/KI_NADH)),
                                           sigma=sigma[exp_idx],
```

```
observed= NADH
)
```

# 1.4 Model sampling

```
[37]: with model 0:
          idata_0 = pm.sample(1000, step=pm.NUTS(target_accept=0.95), tune=1000,__
       →return_inferencedata=True)
      with model_1:
          idata_1 = pm.sample(1000, step=pm.NUTS(target_accept=0.95), tune=1000, __
       →return_inferencedata=True)
      with model 2:
          idata_2 = pm.sample(1000, step=pm.NUTS(target_accept=0.95), tune=1000, __
       →return inferencedata=True)
      with model 3:
          idata_3 = pm.sample(1000, step=pm.NUTS(target_accept=0.95), tune=1000, __
       →return_inferencedata=True)
      with model 4:
          idata_4 = pm.sample(1000, step=pm.NUTS(target_accept=0.95), tune=1000, __
       →return inferencedata=True)
      with model_5:
          idata_5 = pm.sample(1000, step=pm.NUTS(target_accept=0.95), tune=1000, __
       →return_inferencedata=True)
      with model 6:
          idata_6 = pm.sample(1000, step=pm.NUTS(target_accept=0.95), tune=1000,__
       →return inferencedata=True)
     Multiprocess sampling (4 chains in 4 jobs)
     NUTS: [sigma, K_NAD, K_G6P, k_cat]
     <IPython.core.display.HTML object>
     Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000) draws
     total) took 11 seconds.
     Multiprocess sampling (4 chains in 4 jobs)
     NUTS: [sigma, KI_NADH, K_NAD, K_G6P, k_cat]
     <IPython.core.display.HTML object>
     Sampling 4 chains for 1 000 tune and 1 000 draw iterations (4 000 + 4 000 draws
     total) took 15 seconds.
     The number of effective samples is smaller than 25% for some parameters.
     Multiprocess sampling (4 chains in 4 jobs)
     NUTS: [sigma, KI_NADH, K_NAD, K_G6P, k_cat]
     <IPython.core.display.HTML object>
     Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws
     total) took 28 seconds.
```

There were 57 divergences after tuning. Increase `target\_accept` or reparameterize.

The acceptance probability does not match the target. It is 0.8336170236522807, but should be close to 0.95. Try to increase the number of tuning steps.

There was 1 divergence after tuning. Increase `target\_accept` or reparameterize.

There were 2 divergences after tuning. Increase `target\_accept` or reparameterize.

The rhat statistic is larger than 1.05 for some parameters. This indicates slight problems during sampling.

The estimated number of effective samples is smaller than 200 for some parameters.

Multiprocess sampling (4 chains in 4 jobs)
NUTS: [sigma, KI\_NADH, K\_NAD, K\_G6P, k\_cat]

<IPython.core.display.HTML object>

Sampling 4 chains for  $1_000$  tune and  $1_000$  draw iterations  $(4_000 + 4_000)$  draws total) took 18 seconds.

The number of effective samples is smaller than 25% for some parameters.

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [sigma, KI\_NADH, K\_NAD, K\_G6P, k\_cat]

<IPython.core.display.HTML object>

Sampling 4 chains for  $1_000$  tune and  $1_000$  draw iterations  $(4_000 + 4_000)$  draws total) took 12 seconds.

The number of effective samples is smaller than 25% for some parameters.

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [sigma, KI\_NADH, K\_NAD, K\_G6P, k\_cat]

<IPython.core.display.HTML object>

Sampling 4 chains for  $1_000$  tune and  $1_000$  draw iterations  $(4_000 + 4_000)$  draws total) took 10 seconds.

Multiprocess sampling (4 chains in 4 jobs)

NUTS: [sigma, KI\_NADH, K\_NAD, K\_G6P, k\_cat]

<IPython.core.display.HTML object>

Sampling 4 chains for 1\_000 tune and 1\_000 draw iterations (4\_000 + 4\_000 draws total) took 8 seconds.

The number of effective samples is smaller than 25% for some parameters.

### 1.5 Posterior predictive sampling

```
with model 2:
          post_pred_2 = pm.sample_posterior_predictive(idata_2,__
       →var_names=['NADH_obs'] )
      with model 3:
          post_pred_3 = pm.sample_posterior_predictive(idata_3,__
       →var names=['NADH obs'] )
      with model_4:
          post_pred_4 = pm.sample_posterior_predictive(idata_4,__
       →var_names=['NADH_obs'] )
      with model 5:
          post_pred_5 = pm.sample_posterior_predictive(idata_5,__

¬var_names=['NADH_obs'] )
     <IPython.core.display.HTML object>
     <IPython.core.display.HTML object>
     <IPython.core.display.HTML object>
     <IPython.core.display.HTML object>
     <IPython.core.display.HTML object>
     <IPython.core.display.HTML object>
     1.6 Creation of figure
[49]: posterior_0 = idata_0.to_dataframe(['posterior'], include_coords=False)
      posterior_1 = idata_1.to_dataframe(['posterior'], include_coords=False)
      posterior_2 = idata_2.to_dataframe(['posterior'], include_coords=False)
      posterior 3 = idata 3.to dataframe(['posterior'], include_coords=False)
      posterior 4 = idata 4.to_dataframe(['posterior'], include_coords=False)
      posterior_5 = idata_5.to_dataframe(['posterior'], include_coords=False)
      posteriors = [posterior_0,posterior_2,posterior_3, posterior_1,posterior_5]
[55]: | fig = plt.figure(figsize=(XSIZE, YSIZE), constrained_layout=True)
      gs0 = fig.add gridspec(2, 1)
      gs = gs0[0].subgridspec(1,2)
      ax_1 = fig.add_subplot(gs[0])
      sns.lineplot(ax=ax_1, data=data, x='G6P_in', y='NADH_obs', hue="NAD_in", u
      ⇒palette='crest', style='code', markers=True)
      ax_1.set_xlim(0, 3200)
      ax_1.set_ylim(0, 1500)
      ax_1.set_xlabel(r"$[$G6P$]_{in}^(\mu M)$")
```

 $ax_1.set_ylabel(r"$[$NADH$]_{obs}^(\mu M)$")$ 

```
ax_1.legend(loc='upper center', fontsize='x-small', ncol=2)
ax_3 = fig.add_subplot(gs[1])
ax_3.axis('off')
ax_3.text(x=-0.15, y=0.9, va='center', ha='left', fontsize=8,_
⇒bbox=dict(boxstyle='round', fc='white', ec='CO', pad=0.5),
            s=r"$H_0$: $v = \frac{k_{cat}[E][G6P][NAD]}{K_{G6P}K_{NAD}(1+[G6P]/Ser}
\hookrightarrowK_{G6P})(1+[NAD]/K_{NAD})}$")
ax_3.text(x=-0.15, y=0.7, va='center', ha='left', fontsize=8,__
⇒bbox=dict(boxstyle='round', fc='white', ec='C1', pad=0.5),
            s=r"$H 1$: $v = \frac{k {cat}[E][G6P][NAD]}{K {G6P}K {NAD}(1+[G6P]/}
\hookrightarrowK_{G6P})(1+[NAD]/K_{NAD}+[NADH]/KI_{NADH})}$")
ax 3.text(x=-0.15, y=0.5, va='center', ha='left', fontsize=8,
⇒bbox=dict(boxstyle='round', fc='white', ec='C2', pad=0.5),
            s=r"$H 2$: $v = \frac{k {cat}[E][G6P][NAD]}{K {G6P}K {NAD}(1+[G6P]/}
\hookrightarrowK_{G6P}+[NADH]/KI_{NADH})(1+[NAD]/K_{NAD})}$")
ax_3.text(x=-0.15, y=0.3, va='center', ha='left', fontsize=8,_\( \sigma \)
⇒bbox=dict(boxstyle='round', fc='white', ec='C3', pad=0.5),
            s=r"$H 3$: $v = \frac{k {cat}[E][G6P][NAD]}{K {G6P}K {NAD}(1+[G6P]/}
\hookrightarrowK {G6P})(1+[NAD]/K {NAD})(1+ [NADH]/KI {NADH}))$")
ax_3.text(x=-0.15, y=0.1, va='center', ha='left', fontsize=8,__
⇒bbox=dict(boxstyle='round', fc='white', ec='C4', pad=0.5),
            s=r"$H_4$: $v = \frac{k_{cat}[E][G6P][NAD]}{K_{G6P}K_{NAD}(1+[G6P]/E]}
\hookrightarrowK {G6P})(1+[NAD]/K {NAD})(1+ [NADH]/KI {NADH})^2}$")
ax_1.text(-0.25, 0.95, 'A', transform=ax_1.transAxes, weight="bold", size=10)
ax_3.text(-0.3, 0.95, 'B', transform=ax_3.transAxes, weight="bold", size=10)
gs = gs0[1].subgridspec(2,3)
ax_4 = fig.add_subplot(gs[0,0])
ax 5 = fig.add subplot(gs[1,0])
ax_6 = fig.add_subplot(gs[0,1])
ax_7 = fig.add_subplot(gs[1,1])
ax 4.ticklabel format(style='sci', scilimits=(-1,1), axis='y')
ax_5.ticklabel_format(style='sci', scilimits=(-1,1), axis='y')
ax_6.ticklabel_format(style='sci', scilimits=(-1,1), axis='y')
ax_7.ticklabel_format(style='sci', scilimits=(-1,1), axis='y')
for i, post in enumerate(posteriors):
    sns.kdeplot(post['k_cat'], ax=ax_4, fill=True, color=f'C{i}',_
\rightarrowlabel=r"$H_{}$".format(i))
ax_4.set_xlabel(r"$k_{cat}^{min^{-1}})$")
ax_4.set_ylabel(r"$P(k_{cat})$")
ax_4.set_xlim(0, 300)
```

```
ax_4.legend(loc='upper right', ncol=2, markerscale=0.5, fontsize='x-small', u
→title fontsize='small')
for i, post in enumerate(posteriors):
    sns.kdeplot(post['K_G6P'], ax=ax_5, fill=True, color=f'C{i}')
ax 5.set xlabel(r"$K {G6P}~(\mu M)$")
ax_5.set_ylabel(r"$P(K_{G6P})$")
ax_5.set_xlim(0, 4000)
for i, post in enumerate(posteriors):
    sns.kdeplot(post['K_NAD'], ax=ax_6, fill=True, color=f'C{i}')
ax_6.set_xlabel(r"$K_{NAD}^(\underline{Mu M})$")
ax_6.set_ylabel(r"$P(K_{NAD})$")
ax_6.set_xlim(0, 2000)
for i, post in enumerate(posteriors):
    try:
        sns.kdeplot(post['KI_NADH'], ax=ax_7, fill=True, color=f'C{i}')
    except KeyError as e:
        print(e)
ax 7.set xlabel(r"$KI {NADH}~(\mu M)$")
ax_7.set_ylabel(r"$P(KI_{NADH})$")
ax_7.set_xlim(0, 8000)
sns.despine()
ax_8 = fig.add_subplot(gs[:, 2])
model_comparison = az.compare({
   r'$H_0$': idata_0,
   r'$H_1$': idata_2,
   r'$H_2$': idata_3,
   r'$H_3$': idata_1,
   r'$H 4$': idata 5
}, ic='loo', method='BB-pseudo-BMA')
print(model_comparison)
ax_8 = az.plot_compare(model_comparison, ax=ax_8, insample_dev=False,_
→plot_standard_error=False)
ax_8.scatter(model_comparison["loo"], ax_8.get_yticks()[::2], c=[ "C4", "C3", _
→"C0","C2","C1"], s=64, ec='black', zorder=10)
ax_8.set_xlabel("Log-score")
ax 8.set xlim(-290, -255)
ax_4.text(-0.25, 1.0, 'C', transform=ax_4.transAxes, weight="bold", size=10)
ax_8.text(-0.15, 1.0, 'D', transform=ax_8.transAxes, weight="bold", size=10)
savefig('fig_mechanisms')
plt.show()
```

#### 'KI\_NADH'

/home/mathieu/anaconda3/envs/phd/lib/python3.9/site-

packages/arviz/stats/stats.py:694: UserWarning: Estimated shape parameter of Pareto distribution is greater than 0.7 for one or more samples. You should consider using a more robust model, this is because importance sampling is less likely to work well if the marginal posterior and LOO posterior are very different. This is more likely to happen with a non-robust model and highly influential observations.

#### warnings.warn(

/home/mathieu/anaconda3/envs/phd/lib/python3.9/site-

packages/arviz/stats/stats.py:927: RuntimeWarning: overflow encountered in exp
weights = 1 / np.exp(len\_scale - len\_scale[:, None]).sum(axis=1)

/home/mathieu/anaconda3/envs/phd/lib/python3.9/site-

packages/arviz/stats/stats.py:694: UserWarning: Estimated shape parameter of Pareto distribution is greater than 0.7 for one or more samples. You should consider using a more robust model, this is because importance sampling is less likely to work well if the marginal posterior and LOO posterior are very different. This is more likely to happen with a non-robust model and highly influential observations.

#### warnings.warn(

/home/mathieu/anaconda3/envs/phd/lib/python3.9/site-

packages/arviz/stats/stats.py:694: UserWarning: Estimated shape parameter of Pareto distribution is greater than 0.7 for one or more samples. You should consider using a more robust model, this is because importance sampling is less likely to work well if the marginal posterior and LOO posterior are very different. This is more likely to happen with a non-robust model and highly influential observations.

#### warnings.warn(

/home/mathieu/anaconda3/envs/phd/lib/python3.9/site-

packages/arviz/stats/stats.py:694: UserWarning: Estimated shape parameter of Pareto distribution is greater than 0.7 for one or more samples. You should consider using a more robust model, this is because importance sampling is less likely to work well if the marginal posterior and LOO posterior are very different. This is more likely to happen with a non-robust model and highly influential observations.

# warnings.warn(

/home/mathieu/anaconda3/envs/phd/lib/python3.9/site-

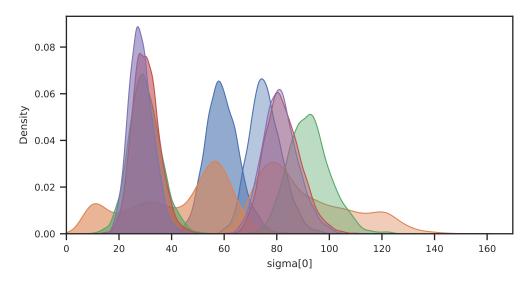
packages/arviz/stats/stats.py:694: UserWarning: Estimated shape parameter of Pareto distribution is greater than 0.7 for one or more samples. You should consider using a more robust model, this is because importance sampling is less likely to work well if the marginal posterior and LOO posterior are very different. This is more likely to happen with a non-robust model and highly influential observations.

#### warnings.warn(

	rank	100	p_loo	$d_loo$	weight	se	\
\$H_4\$	0 -259	9.811332	15.229934	0.000000	7.283544e-01	17.038761	
\$H 3\$	1 -262	2.461983	16.666034	2.650651	9.164697e-02	11.439590	

```
$H_0$
                   2 -269.170218
                                                 14.250073
                                                                         9.358886
                                                                                            1.504520e-01 10.765705
$H_2$
                   3 -269.383489
                                                 18.365049
                                                                         9.572156
                                                                                            2.954612e-02
                                                                                                                        16.173247
$H_1$
                                                 43.565673
                                                                                            5.392143e-07
                   4 -285.289822
                                                                       25.478490
                                                                                                                        11.641045
                         dse
                                   warning loo_scale
$H_4$
               0.000000
                                         True
                                                               log
$H_3$
                2.268839
                                         True
                                                               log
$H_0$
              10.492043
                                         True
                                                               log
$H_2$
                5.540026
                                         True
                                                               log
$H_1$
                9.413428
                                         True
                                                               log
                                                                                 В
                1400
                                            NAD in
                                                                                         H_0 \colon V = \frac{k_{Cat}[E][G6P][NAD]}{K_{G6P}K_{NAD}(1 + [G6P]/K_{G6P})(1 + [NAD]/K_{NAD})}
                                            1000
                1200
                                                                                                                k_{cat}[E][G6P][NAD]
(K_{G6P})(1 + [NAD]/K_{NAD})
            [NADH]<sub>obs</sub> (µM)
                1000
                                                                                                               k_{cat}[E][G6P][NAD]
P]/K<sub>G6P</sub> + [NADH]/KI<sub>NADH</sub>)(1
                 800
                 600
                                                                                         H_3 \colon v = \frac{k_{Cat}[F][G6P][NAD]}{K_{G6P}K_{NAD}(1 + [G6P]/K_{G6P})(1 + [NAD]/K_{NAD})(1 + [NADH]/KI_{NADH})}
                 400
                                                                                         H_4: V = \frac{k_{cat}[E][G6P][NAD]}{K_{G6P}K_{NAD}(1 + [G6P]/K_{G6P})(1 + [NAD]/K_{NAD})(1 + [NADH]/K_{NADH})}
                 200
                                              1500
                                                       2000
                                                                2500
                                                                        3000
                              500
                                      1000
                                            [G6P]_{in} (\mu M)
          C
                   1e-1
                                                                                                              D
               2
                                                           P(K<sub>NAD</sub>)
0.5
            P(k_{cat})
                                                                                                              H_4
                                                                                                              H_3
               0
                                                               0.0
                             100 150 200 250 300
                                                                            500
                                                                                    1000
                  0
                       50
                                                                                             1500
                                                                                                      2000
                               k_{cat} \ (min^{-1})
                                                                                 K_{NAD} (\mu M)
                                                                                                              H_0
                                                                     1e-3
                   1e-3
                                                                                                              H_2
          6.5 J. 5.0 (K<sup>26b</sup>)
                                                                                                              H_1
             0.0 -
                                                                 Ω
                         1000
                                   2000
                                            3000
                                                                           2000
                                                                                    4000
                                                                                             6000
                                                                                                      8000
                                                     4000
                                                                                                                -290
                                                                                                                           -280
                                                                                                                                     -270
                                                                                                                                                -260
                                K_{G6P} (\mu M)
                                                                                 KI_{NADH} (\mu M)
                                                                                                                               Log-score
```

```
print(e)
ax.set_xlim(0)
plt.show()
```



### 1.7 Old notebook cells

```
NameError Traceback (most recent call last)
```

```
[118]: import scipy.integrate as integrate
       from numba import njit
       control_inputs = pd.DataFrame(
           {
               "kf": 0.125, "E": 2.0,
               # "G6P in": [2000, 3000, 4000, 5000, 2000, 3000, 4000, 5000],
               # "NAD_in": [3000, 3000, 3000, 3000, 3000, 3000, 3000],
               "G6P in": [3000, 3000, 3000, 3000, 3000, 3000, 3000, 3000],
               "NAD_in": [3000, 3000, 3000, 3000, 2000, 3000, 4000, 5000],
               "NADH_in": [2000, 3000, 4000, 5000, 3000, 3000, 3000, 3000]
           }
       )
       def reactor_2(controls, parameters):
           def reactor(t,c, k, controls):
               G6P, NAD, NADH = c
               v = k.k_{cat} * controls.E * G6P * NAD / (k.K_G6P*k.K_NAD*(1+G6P/k.
        \rightarrowK_G6P)*(1+NAD/k.K_NAD + NADH/k.KI_NADH))
               return [
                   -v + controls.kf * (controls.G6P_in - G6P),
                   -v + controls.kf * (controls.NAD_in - NAD),
                   v + controls.kf * (controls.NADH_in - NADH),
               1
           sol = [[integrate.solve ivp(reactor, y0=[control.G6P in, control.NAD in,
        →control.NADH_in], t_span=(0, 60), args=(param, control), vectorized=False).
        \rightarrowy[2,-1] for i, control in controls.iterrows()] for j, param in parameters.
        →iterrows()]
           return sol
       def reactor_5(controls, parameters):
           def reactor(t,c, k, controls):
               G6P, NAD, NADH = c
               v = k.k_{cat} * controls.E * G6P * NAD / (k.K_G6P*k.K_NAD*(1+G6P/k.
        \hookrightarrowK G6P)*(1+NAD/k.K NAD)*(1+ NADH/k.KI NADH)**2)
```

```
return [
                 -v + controls.kf * (controls.G6P_in - G6P),
                 -v + controls.kf * (controls.NAD_in - NAD),
                 v + controls.kf * (controls.NADH_in - NADH),
             ]
         sol = [[integrate.solve_ivp(reactor, y0=[control.G6P_in, control.NAD_in,_
      →control.NADH_in], t_span=(0, 60), args=(param, control), vectorized=False).
      \rightarrowy[2,-1] for i, control in controls.iterrows()] for j, param in parameters.
      →iterrows()]
         return sol
     pred_2 = reactor_2(control_inputs, posterior_2.sample(200))
    pred_5 = reactor_5(control_inputs, posterior_5.sample(200))
[]: with sns.color_palette('plasma', n_colors=6):
         fig, axes = plt.subplots(2,2, figsize=(8,6), constrained layout=True)
         for i, post in enumerate(posteriors):
             sns.kdeplot(post['k_cat'], ax=axes[0][0], fill=True, color=f'C{i}',_
      →label=f"Model {i}")
         axes[0][0].set xlabel(r"$k {cat}~(min^{-1})$")
         axes[0][0].set_ylabel(r"$P(k_{cat})$")
         axes[0][0].set xlim(0, 250)
         axes[0][0].legend(loc='upper left')
         for i, post in enumerate(posteriors):
             sns.kdeplot(post['K_G6P'], ax=axes[0][1], fill=True, color=f'C{i}')
         axes[0][1].set_xlabel(r"$K_{G6P}^(\underline{Mu M})$")
         axes[0][1].set_ylabel(r"$P(K_{G6P})$")
         axes[0][1].set_xlim(0, 4000)
         for i, post in enumerate(posteriors):
             sns.kdeplot(post['K_NAD'], ax=axes[1][0], fill=True, color=f'C{i}')
         axes[1][0].set xlabel(r"$K {NAD}~(\mu M)$")
```

sns.kdeplot(post['KI\_NADH'], ax=axes[1][1], fill=True,\_\_

axes[1][0].set\_ylabel(r"\$P(K\_{NAD})\$")

for i, post in enumerate(posteriors):

axes[1][0].set xlim(0, 2000)

except KeyError as e:

print(e)

try:

```
axes[1][1].set_xlabel(r"$KI_{NADH}~(\mu M)$")
axes[1][1].set_ylabel(r"$P(KI_{NADH})$")
axes[1][1].set_xlim(0, 8000)

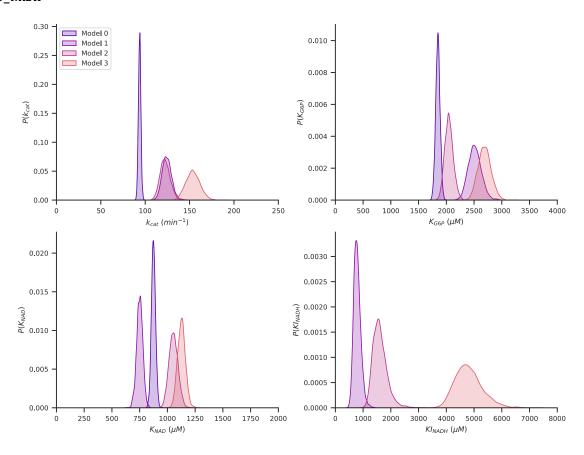
sns.despine()
plt.show()

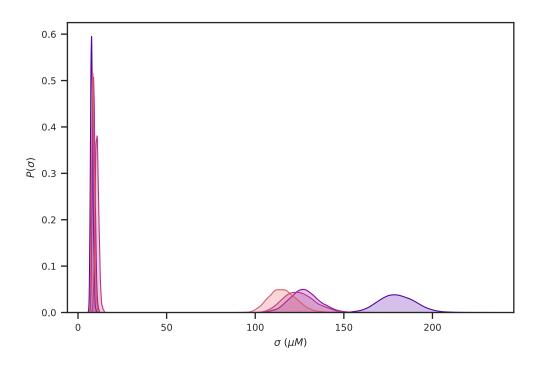
fig, ax = plt.subplots(figsize=(6,4))
for i, post in enumerate(posteriors):
    sns.kdeplot(post['sigma[0]'], ax=ax, fill=True, color=f'C{i}')
    sns.kdeplot(post['sigma[1]'], ax=ax, fill=True, color=f'C{i}')
    # sns.kdeplot(post['sigma[0]'], ax=ax, fill=True, color=f'C{i}')

ax.set_xlabel(r"$\sigma^(\mu M)$")
ax.set_ylabel(r"$P(\sigma)$")

# ax.set_xlim(0, 2000)
```

### 'KI\_NADH'





```
[]: model_comparison = az.compare({
    'model_0': idata_0,
    'model_1': idata_1,
    'model_2': idata_2,
    'model_3': idata_3,
    'model_4': idata_4,
    'model_5': idata_5,
}, ic='loo')
print(model_comparison)
az.plot_compare(model_comparison)
```

/home/mathieu/anaconda3/envs/phd/lib/python3.9/sitepackages/arviz/stats/stats.py:145: UserWarning: The default method used to estimate the weights for each model, has changed from BB-pseudo-BMA to stacking warnings.warn(

/home/mathieu/anaconda3/envs/phd/lib/python3.9/sitepackages/arviz/stats/stats.py:655: UserWarning: Estimated shape parameter of Pareto distribution is greater than 0.7 for one or more samples. You should consider using a more robust model, this is because importance sampling is less likely to work well if the marginal posterior and LOO posterior are very different. This is more likely to happen with a non-robust model and highly

```
warnings.warn(
```

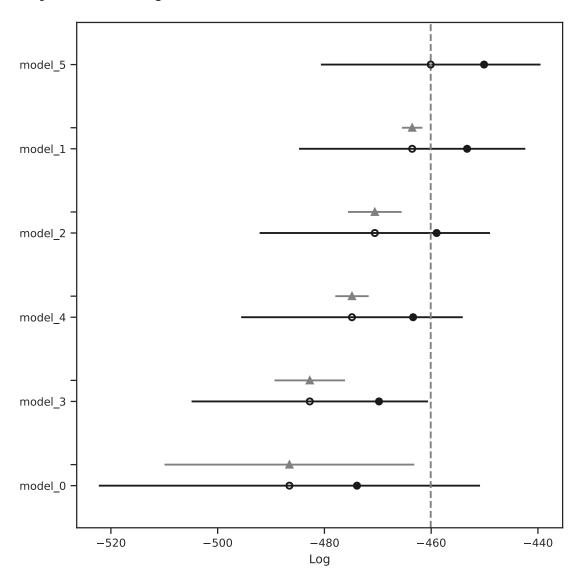
influential observations.

rank loo p\_loo d\_loo weight se \
model\_5 0 -460.086011 9.995511 0.000000 4.308970e-01 20.572814

model_1	1 -463.559869	10.288714	3.473858	0.000000e+00	21.193274
model_2	2 -470.557057	11.579056	10.471046	0.000000e+00	21.580386
${\tt model\_4}$	3 -474.838169	11.453683	14.752158	2.548756e-13	20.754832
model_3	4 -482.743593	12.970784	22.657582	4.585695e-13	22.142735
model_0	5 -486.556216	12.652510	26.470204	5.691030e-01	35.711813

	dse	warning	loo_scale
model_5	0.000000	False	log
$model_1$	1.915862	False	log
$model_2$	5.030913	False	log
${\tt model\_4}$	3.128812	False	log
model_3	6.605258	False	log
model_0	23.389344	True	log

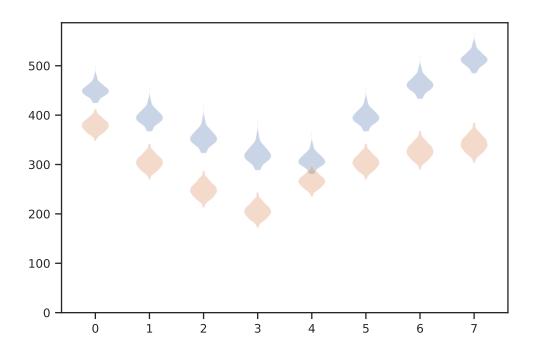
<AxesSubplot:xlabel='Log'>



[120]: (0.0, 587.1013063884353)

plt.ylim(0)

 $\hookrightarrow$ showextrema=False)



[]: