Appendix

A. Proof of Lemma 1

Proof. Suppose that we add quantum rotation noise to the quantum classifier with quantum test state σ and quantum gates $g = \prod_{x_i}^{n} \cos\theta_{x_i}$. Given distinct n numbers $\theta_{x_1}, \theta_{x_2}, \ldots, \theta_{x_n}$ that satisify $0 < h_1 < \tan\theta_{x_i} < h_2$ for all $i \in \{1, 2, \ldots, n\}$, then the corresponding probability of class K becomes the superposition of other input states, that is,

$$
\begin{aligned}
\widetilde{y_k}(\sigma) &= g\, y_k(\sigma) + g \sum_{x_1=1}^{n} \tan\theta_{x_1} y_k(\sigma_{x_1}) \\
&\quad + g \sum_{x_1, x_2 = 1}^{n} \tan\theta_{x_1} \tan\theta_{x_2} y_k(\sigma_{x_1}) y_k(\sigma_{x_2}) + \cdots \\
&= g y_k(\sigma) + g \sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_k(\sigma_{x_i})\}. \qquad (2)
\end{aligned}
$$

Our objective is to prove that $\underset{k}{argmax}\ \widetilde{y}_k(\sigma) = C$. Here we consider binary case. Without loss of generality, we assume that $\widetilde{y}_1(\sigma) > \widetilde{y}_2(\sigma)$. From equation 2, we formulate the probabilities of label 1 and label 2 respectively, i.e.,

$$
\widetilde{y_1}(\sigma) = g\, y_1(\sigma) + g \sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_1(\sigma_{x_i})\}. \qquad (3)
$$

$$
\widetilde{y_2}(\sigma) = g\, y_2(\sigma) + g \sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_2(\sigma_{x_i})\}. \qquad (4)
$$

Here, the relation, $\widetilde{y_1}(\sigma) > \widetilde{y_2}(\sigma)$, shall be required to prove our objective. With equation 3 and 4, it can be rewritten to be

$$
\begin{aligned}
&g\, y_1(\sigma) + g \sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_1(\sigma_{x_i})\} \\
&\quad > g\, y_2(\sigma) + g \sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_2(\sigma_{x_i})\}.
\end{aligned}
$$

We can further rearrange the inequality to be

$$
\begin{aligned}
y_1(\sigma) - y_2(\sigma) > &\sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_2(\sigma_{x_i})\} \\
&- \sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_1(\sigma_{x_i})\}.
\end{aligned}
$$

Consider worst case : All $y_2(\sigma_{x_i}) = 1$, which leads that

$$
\sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_1(\sigma_{x_i})\} = 0 \qquad (5)
$$

With the rearrangement,

$$
\sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_2(\sigma_{x_i})\} = \prod_{i=1}^{n} (1 + \tan\theta_{x_i} y_2(\sigma_{x_i})) - 1,
$$

we obtain the constrain,

$$
\begin{aligned}
(1 + h_1)^n - 1 \le &\sum_{\ell=1}^{n} \{\prod_{i=1}^{\ell} \tan\theta_{x_i} y_2(\sigma_{x_i})\} \\
&\le (1 + h_2)^n - 1.
\end{aligned}
$$

Equation 5 becomes

$$
y_1(\sigma) - y_2(\sigma) \ge (1 + h_1)^n - 1. \qquad (6)
$$

Combining equation 6 and the property, $1 = y_1(\sigma) + y_2(\sigma)$, we can derive that

$$
2y_1(\sigma) > (1 + h_1)^n \Rightarrow y_1(\sigma) > \frac{(1 + h_1)^n}{2}.
$$

Therefore, if there is a positive number h satisfies

$$
y_C(\sigma) > \frac{(1 + h)^n}{2},
$$

then $C = \underset{k}{argmax}\ \widetilde{y_k}(\sigma)$. $\square$