第九次作业解答:

1. 解: (5) $\mathcal{L}\left\{\frac{1}{b^2-a^2}(\cos at-\sin bt)\right\} = \frac{1}{b^2-a^2}\cdot\left(\frac{p}{p^2+a^2}-\frac{b}{p^2+b^2}\right)$

(9) $\mathcal{L}\{te^{5t}\} = -\left(\frac{1}{p-5}\right)' = \frac{1}{(p-5)^2}$

(12) $\mathcal{L}\left\{\frac{d^2}{dt^2}(e^{-at}\sin wt)\right\} = p^2\mathcal{L}[e^{-at}\sin wt] - p\cdot f(+0) - f'(+0)$

$$= p^2\cdot\frac{w}{(p+a)^2+w^2} - w$$

(14) $\mathcal{L}\left\{\int_0^t te^{2t}dt\right\} = \frac{\mathcal{L}\{te^{2t}\}}{p} = \frac{-\left(\frac{1}{p-2}\right)'}{p} = \frac{1}{p(p-2)^2}$

(16) $\mathcal{L}\left\{\int_0^t (t-\tau)^n e^{-a\tau}\cos w\tau\, d\tau\right\} = \mathcal{L}\{t^n * e^{-at}\cos wt\}$

$$= \mathcal{L}\{t^n\}\mathcal{L}\{e^{-at}\cos wt\}$$

$$= \frac{n!}{p^{n+1}}\cdot\frac{p+a}{(p+a)^2+w^2} = \frac{n!\,(p+a)}{p^{n+1}\cdot((p+a)^2+w^2)}$$

(18) $\mathcal{L}\{\cos w(t-\varphi)\,h(t-2\varphi)\}$

$= \mathcal{L}\{\cos w((t-2\varphi)+\varphi)\,h(t-2\varphi)\}$

$= e^{-2p\varphi}\mathcal{L}\{\cos w(t+\varphi)\}$

$= e^{-2p\varphi}\cdot\mathcal{L}\{\cos wt\cdot\cos w\varphi - \sin wt\cdot\sin w\varphi\}$

$= e^{-2p\varphi}\cdot\left[\frac{p\cos w\varphi}{p^2+w^2} - \frac{w\sin w\varphi}{p^2+w^2}\right]$

6. 解: (2) $\mathcal{L}^{-1}\left\{\frac{1-p}{p^3+p^2+p+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{p+1}-\frac{p}{p^2+1}\right\} = e^{-t}-\cos t$

(4) $\mathcal{L}^{-1}\left\{\frac{1}{p(p+a)}\right\} = \mathcal{L}^{-1}\left[\frac{1}{a}\left(\frac{1}{p}-\frac{1}{p+a}\right)\right] = \frac{1}{a}(1-e^{-at})$

(6) $\mathcal{L}^{-1}\left\{\frac{1}{(p^2+1)(p^2+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{2}\left(\frac{1}{p^2+1}-\frac{1}{p^2+3}\right)\right\} = \frac{1}{2}\left(\sin t - \frac{1}{\sqrt{3}}\sin\sqrt{3}t\right)$

(8) $\mathcal{L}^{-1}\left\{\frac{1}{p(p-2)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{4p}-\frac{1}{4(p-2)}+\frac{1}{2(p-2)^2}\right\} = \frac{1}{4}(1-e^{2t}+2te^{2t})$

(12) $\mathcal{L}^{-1}\left\{\frac{a^2 p}{p^4+a^4}\right\} = \mathcal{L}^{-1}\left\{\frac{p}{p^4+4\left(\frac{a}{\sqrt{2}}\right)^4}\cdot a^2\right\} = a^2\cdot\frac{1}{2\cdot\left(\frac{a}{\sqrt{2}}\right)^2}\left(\sin\frac{a}{\sqrt{2}}t\right)\left(\operatorname{sh}\frac{a}{\sqrt{2}}t\right) = \left(\sin\frac{a}{\sqrt{2}}t\right)\cdot\left(\operatorname{sh}\frac{a}{\sqrt{2}}t\right)$

(P19页表格中，$\frac{p}{p^4+4w^4} \sim \frac{1}{2w^2}\sin wt\,\operatorname{sh} wt$)

(18) $\mathcal{L}^{-1}\left\{\frac{1-p}{(p+1)(p^2+1)}e^{-10p}\right\} = \mathcal{L}^{-1}\left\{e^{-10p}\left(\frac{1}{p+1}-\frac{p}{p^2+1}\right)\right\} = \left(e^{-(t-10)}-\cos(t-10)\right)h(t-10)$

7、解：(3) 设 $\mathcal{L}\{y(t)\}=Y(p)$，则 $\mathcal{L}\{y''(t)\} = p^2Y - py(0) - y'(0) = p^2Y-1$

$\mathcal{L}\{y'(t)\} = pY - y(0) = pY$.

∴对原方程两边作拉氏变换：

$(p^2Y-1)-(a+b)pY+abY=0. \Rightarrow Y=\frac{1}{(p-a)(p-b)}$

$\therefore y(t)=\mathcal{L}^{-1}\left\{\frac{1}{(p-a)(p-b)}\right\}=\frac{1}{b-a}\left(e^{at}-e^{bt}\right)$

(5) 设 $\mathcal{L}\{y(t)\}=Y(p)$，则 $\mathcal{L}\{y''(t)\}=p^2Y-py(0)-y'(0)=p^2Y+p+2$

$\mathcal{L}\{4\sin t\}=\frac{4}{p^2+1}$. $\mathcal{L}\{5\cos 2t\}=\frac{5p}{p^2+4}$

∴对原方程两边作拉氏变换：

$(p^2Y+p+2)-Y=\frac{4}{p^2+1}+\frac{5p}{p^2+4} \Rightarrow Y=-\frac{p^3+2p^2+p+8}{(p^2+1)(p^2+4)}=-\left(\frac{2}{p^2+1}+\frac{p}{p^2+4}\right)$

$\therefore y(t)=\mathcal{L}^{-1}\left\{-\left(\frac{2}{p^2+1}+\frac{p}{p^2+4}\right)\right\}=-\cos 2t-2\sin t$

(17) 设 $\mathcal{L}\{x(t)\}=X(p)$，$\Rightarrow Y(p)$. $Z(p)$ 同理，

则 $\mathcal{L}\{x'(t)\}=pX$. $\mathcal{L}\{y'(t)\}=pY$. $\mathcal{L}\{z'(t)\}=pZ$. $\mathcal{L}\{t\}=\frac{1}{p^2}$

$$\therefore \begin{cases} pX-pY=0 \\ pY+pZ=\frac{1}{p} \\ pX-pZ=\frac{1}{p^2} \end{cases} \Rightarrow \begin{cases} X=\frac{1}{2p^2}+\frac{1}{2p^3} \\ Y=\frac{1}{2p^2}+\frac{1}{2p^3} \\ Z=\frac{1}{2p^2}-\frac{1}{2p^3} \end{cases} \Rightarrow \begin{cases} x(t)=\frac{1}{2}t+\frac{t^2}{4} \\ y(t)=\frac{1}{2}t+\frac{t^2}{4} \\ z(t)=\frac{1}{2}t-\frac{t^2}{4} \end{cases}$$

(18). 同上设.

$$\begin{cases} pY-a+pX-b=4Y+\frac{1}{p} \\ pY-a+X=3Y+\frac{2}{p^3} \end{cases} \Rightarrow \begin{cases} X=\frac{2}{p^3}+\frac{3}{2}\frac{1}{p^2}+\frac{1}{4p}+\frac{\left(b-\frac{1}{4}\right)}{p-2}+\frac{a-b}{(p-2)^2} \\ Y=\frac{1}{2p^2}+\frac{1}{4p}+\frac{a-\frac{1}{4}}{p-2}+\frac{(a-b)}{(p-2)^2} \end{cases} \Rightarrow \begin{cases} x(t)=t^2+\frac{3}{2}t+\frac{1}{4}+\left(b-\frac{1}{4}\right)e^{2t}+(a-b)te^{2t} \\ y(t)=\frac{1}{2}t+\frac{1}{4}+\left(a-\frac{1}{4}\right)e^{2t}+(a-b)te^{2t} \end{cases}$$