LSE Department of Statistics Practitioners' Challenge 2020 Report

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1 Introduction

The purpose of this research project was to establish a robust methodology to calculate the correlations between the risk factors most likely to impact Aviva's balance sheet over a 1-year period. To that end, we identified a total of five risk factors which we sought to model, namely interest rates, the yield on high-quality corporate bonds, the risk-free rate, inflation, and Gross Domestic Product (GDP). In addition, two statistical models were used to forecast rolling correlations among pairs of risk factors: 1) an ARMA model based on a rolling window, and 2) the DCC-GARCH model. Through this project, we found the DCC-GARCH model to be the superior method to forecast these correlations.

This paper is structured as follows: firstly, we list the risk factors we have identified that are mostly like to impact Aviva's balance sheet and provide the necessary justification; secondly, we provide an overview of the theoretical framework of this project; thirdly, we outline the methodology employed to calculate the correlations in practice; lastly, we present the results of our research and provide an evaluation of the methods used.

2 Risk Factors

Credit, longevity, persistency and equity risks remain the highest individual contributors to UK insurers undiversified and diversified Solvency Capital Requirement (SCR). Under Solvency II, firms are required to put money aside so that if (multiple) extreme (the standard is a 1 in 200) events occur, they need not declare bankruptcy. Therefore, the risk factors chosen were those which are likely to have the largest impact on Aviva's balance sheet. In order to identify the most significant factors, an assessment of Aviva's balance sheet alongside their product offerings was undertaken. It was determined that general insurance underwriting risk is one of Aviva's most crucial factors, while Equity Release Mortgages (ERMs) are being increasingly used. Given the subjectivity involved in valuing and managing ERMs and the materiality of this asset class, such assets are receiving increasing regulatory scrutiny, and hence this was taken into consideration when selecting the risk factors.

1. Interest rates: Interest rate risk for insurance companies is a significant factor in determining profitability. Changes in interest rates can affect the assets and the liabilities of an insurance company. Insurance companies have substantial investments in interest-sensitive assets, such as bonds, as well as market interest rate-sensitive products for their customers, such as pension products. As such, an insurer's profitability rises and falls in concert with interest rate increases or decreases. Drops in interest rates can decrease an insurance company's liabilities by decreasing its future obligations to policyholders. The net impact on the company's profitability is determined by whether the decrease in liabilities is greater or less than any reduction in assets that is experienced (Ozdagli & Wang, 2019). While the precise effect of interest rate changes on a specific insurance company may be uncertain, historical analysis shows that the overall trend is for the profitability of the insurance sector to increase in an environment of rising interest rates. Overall price-to-earnings (P/E) ratios for insurance company stocks usually increase in fairly direct proportion to increases in interest rates.

2. Yield on high-quality corporate bonds: Pension scheme risk exposes a firm to demographic risks that are similar to the underwriting risks run by the firm. A particular example of strong correlations would be where a firms insurance business exposes it to longevity risk (James & Normand, 2019). Where pension schemes are valued on the Solvency II balance sheet under IAS19, insurers use the yield on high-quality corporate bonds for the valuation of the liabilities. Unfortunately, we were unable to retrieve data on a yield curve from the set of UK listed high-quality corporate bonds rated AAA, AA, or A that accurately represent the high-quality corporate bond market. Therefore, we used market performance data from the FTSE100 index as we would be able to infer high-quality corporate bond performance from this. The FTSE100 index's relationship with bond prices and yields tends to work as follows:

When the cost of borrowing money (interest) is low (which is usually met with low bond yields) it tends to stimulate business growth. Business growth causes stocks to go up. When the economy is expanding and stock prices are rising, there is an increasing demand for borrowed money. When all of these factors are increasing in value at the same time, it causes inflation. When there is inflation, lenders charge higher rates of interest to offset the effects of inflation. This means that new bonds will have higher interest rates. In order to remain competitive, the prices of existing bonds will drop, which causes their yield to rise.

3. Risk-free rate: The risk-free rate is the rate of return of an investment with no risk of loss. Most often long-term government bond yields are used as the risk-free rate. Non-life (indicating general insurance) underwriting risk continues to be the greatest risk for the insurers with around two-thirds identifying it as such (Drummond, 2019). As such, Aviva's most material risk changed from market to non-life underwriting in 2018. This followed a slight reduction in the level of market risk whilst non-life underwriting risk is broadly unchanged since 2017. On an aggregate basis, it was found that 63% of insurers assets at the 2018 year-end were held in either corporate or government bonds (see Figure 1). Furthermore, general liability insurers, in aggregate, were noted as holding a greater proportion of their investments in government and corporate bonds than property insurers (78% versus 33%). This may reflect the longer-tailed nature of the liabilities of general liability insurers, compared to property insurers, meaning longer duration assets are required (see Figure 2).

Figure 1: Aggregate investment holdings (Source: Drummond, 2019)

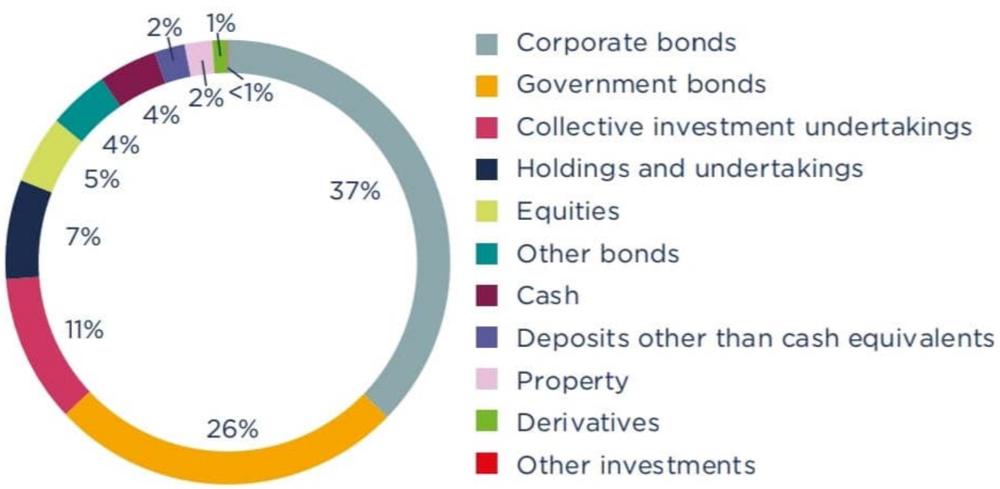
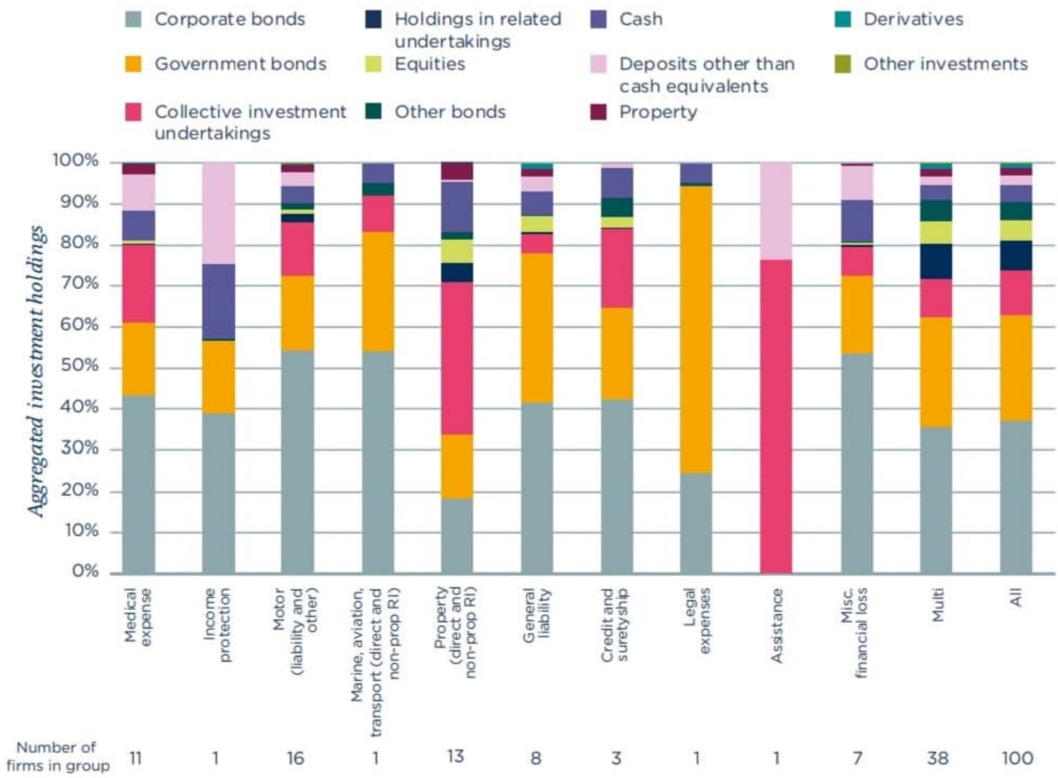


Figure 2: Aggregate investment holdings by type of insurer (Source: Drummond, 2019)



4. Inflation: Inflation - Inflation impacts insurers claims and general expenses, the value of liabilities and, less directly, the value of assets. Inflation affects life and non-life insurers in different ways. For non-life insurers, unanticipated inflation leads to higher claims costs, thereby eroding profitability (Ahlgrim & D'Arcy, 2012). Extended periods of accelerating inflation are especially problematic for long-tail casualty lines of business.

For life insurers, both inflation and deflation are risks. Inflation is often accompanied by rising interest rates, which reduce the value of return guarantees. In the case of deflation, or if very low inflation persists, interest rates tend to fall. This makes it more difficult for life insurers with large portfolios of minimum interest rate guarantee savings products to earn the appropriate asset returns.

5. Gross Domestic Product (GDP): As insurance companies generate income by investing premium payments, the economy can greatly impact an insurance business. Insurance companies invest premiums in dividend-paying stocks, mortgage-backed securities, real estate and financial institutions, such as banks, all of which are vulnerable to economic changes. When the economy is doing well, investment returns will increase and insurance companies may be more likely to accept a claim. With a slow economy, however, the returns will decrease. Insurance companies will need to recover the invested money somehow (Ul Din, Abu-Bakar, & Regupathi, 2017). They will do this by taking a loan themselves, or by challenging their existing operations, especially the significant ones like claims. Settling claims efficiently is the solution.

3 Theoretical Framework

There are many existing methods to estimate conditional correlations in the field of statistics. This section provides a summary of the methodologies to establish 1. rolling window correlations, and 2. models for correlation forecasting.

3.1 Methods of Establishing Rolling Windows

Two popular methods for establishing rolling windows are the rolling window estimator and the exponential smoother. The **rolling correlation estimator** is defined for returns with zero mean and can be computed through the following formula:

$$\hat{\rho}_{12,t} = \frac{\sum_{s=t-n-1}^{t=1} r_{1,s} \cdot r_{2,s}}{\sqrt{(\sum_{s=t-n-1}^{t=1} r_{1,s}^2) \cdot (\sum_{s=t-n-1}^{t=1} r_{2,s}^2)}}$$

This method provides estimates in the range $[-1, 1]$ and gives equal weight to all observations fewer than n periods in the past and zero weight to observations older than that.

An alternative method that does not pick a specific termination point but emphasises current observations through assigning greater weights to more recent observations is the **exponential smoother**, that is given by the formula:

$$\hat{\rho}_{12,t} = \frac{\sum_{s=1}^{t=1} \lambda^{t-s-1} \cdot r_{1,s} \cdot r_{2,s}}{\sqrt{(\sum_{s=1}^{t=1} \lambda^{t-s-1} \cdot r_{1,s}^2) \cdot (\sum_{s=1}^{t=1} \lambda^{t-s-1} \cdot r_{2,s}^2)}}$$

3.2 Methods for Forecasting: The DCC-GARCH model

The Dynamic Conditional Correlation (DCC-GARCH) model (Engle, 2002) decomposes the conditional covariance matrix of a multivariate time series into the product of the conditional standard deviations and the conditional correlations. Whereas in the Constant Conditional Correlation (CCC-GARCH model) (Bollerslev, 1990) the conditional correlation matrix is assumed to be a matrix of *constants*, the DCC-GARCH model enables us to model *time-varying* conditional correlations. The remainder of this section is devoted to providing a brief overview of the theory of the DCC-GARCH model.

Suppose we have returns, $\mathbf{a}_t$, from n assets with expected value 0 and covariance matrix $\mathbf{H}_t$. Then the DCC-GARCH model is defined as:

$$\mathbf{r}_t = \boldsymbol{\mu}_t + \mathbf{a}_t \tag{1}$$

$$\mathbf{a}_t = \mathbf{H}_t^{1/2}\boldsymbol{\epsilon}_t \tag{2}$$

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t \tag{3}$$

where:
$\mathbf{r}_t \in \mathbb{R}^{n\times 1}$ is the log returns of n assets at time t.
$\boldsymbol{\mu}_t \in \mathbb{R}^{n\times 1}$ is the expected value of the conditional log returns, and is assumed to be constant.
$\mathbf{a}_t \in \mathbb{R}^{n\times 1}$ is the mean-corrected returns of n assets at time t, and $\mathbb{E}[\mathbf{a}_t] = 0$.
$\mathbf{H}_t \in \mathbb{R}^{n\times n}$ is the matrix of the conditional variances of $\mathbf{a}_t$ at time t.
$\boldsymbol{\epsilon}_t \in \mathbb{R}^{n\times 1}$ is the vector of i.i.d. errors such that $\mathbb{E}[\boldsymbol{\epsilon}_t] = 0$ and $\mathbb{E}[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{'}] = \mathbf{I}_n$, which is an $n \times n$ identity matrix and $\boldsymbol{\epsilon}_t^{'}$ indicates the transpose of $\boldsymbol{\epsilon}_t$. It is assumed for this project that the errors follow a multivariate Student's t-distribution.
$\mathbf{D}_t \in \mathbb{R}^{n\times n}$ is a diagonal matrix of conditional standard deviations of $\mathbf{a}_t$ at time t.
$\mathbf{R}_t \in \mathbb{R}^{n\times n}$ is the matrix of conditional correlations of $\mathbf{a}_t$ at time t. In the CCC-GARCH model, the elements of $\mathbf{R}_t$ are constants, whilst in the DCC-GARCH model, they are time-varying.

In particular, $\mathbf{D}_t = diag(\sqrt{h_{1t}}, \sqrt{h_{2t}}, ..., \sqrt{h_{nt}})$ where $\sqrt{h_{it}}$ are the conditional standard deviations from the n different univariate GARCH models. Moreover, we have that $\mathbf{R}_t$ takes the following form:

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho_{12,t} & \rho_{13,t} & \cdots & \rho_{1n,t} \\ \rho_{12,t} & 1 & \rho_{23,t} & \cdots & \rho_{2n,t} \\ \rho_{13,t} & \rho_{23,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \rho_{n-1,n,t} \\ \rho_{1n,t} & \rho_{2n,t} & \cdots & \rho_{n-1,n,t} & 1 \end{bmatrix} \tag{4}$$

The elements of $\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$ are:

$$[\mathbf{H}_t]_{ij} = \rho_{ij}\sqrt{h_{it}h_{jt}} \tag{5}$$

where $\rho_{ii} = 1$.

Now, in order for $\mathbf{R}_t$ to be a conditional correlation matrix, two conditions must be met:

1. $\mathbf{H}_t$ has to be positive-definite as it is a covariance matrix. To ensure this, $\mathbf{R}_t$ has to be positive-definite.

2. All the elements of $\mathbf{R}_t$ have to be less than or equal to 1 in magnitude.

In order to resolve these two issues, Engel proposed that $\mathbf{R}_t$ be further decomposed into:

$$\mathbf{R}_t = diag(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t diag(\mathbf{Q}_t)^{-1/2} \tag{6}$$

$$\mathbf{Q}_t = (1-\alpha-\beta)\mathbf{S} + \alpha\epsilon_{t-1}\epsilon_{t-1}^{'} + \beta\mathbf{Q}_{t-1} \tag{7}$$

where $\mathbf{S} = Cov[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{'}] = \mathbb{E}[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{'}]$ is the unconditional covariance matrix of the standardised errors $\boldsymbol{\epsilon}_t$. $\mathbf{S}$ can be estimated as:

$$\hat{\mathbf{S}} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{'} \tag{8}$$

Furthermore, α and β are scalars, and $diag(\mathbf{Q}_t)^{-1/2}$ consists of the square roots of the diagonal elements of $\mathbf{Q}_t$. Pre- and post-multiplication of $\mathbf{Q}_t$ by $diag(\mathbf{Q}_t)^{-1/2}$ thus ensures that each element of $\mathbf{R}_t$ is less than or equal to 1 in magnitude.

In order to guarantee that $\mathbf{R}_t$ is positive-definite (to ensure that the first condition is satisfied), $\mathbf{Q}_t$ has to be positive-definite. By rearranging (7), it can be seen that the process $\mathbf{Q}_t$ is modelled using a GARCH(1,1). Hence, the requirements for $\mathbf{Q}_t$ to positive definite are that $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta < 1$. Moreover, the fact that the conditional correlations are modelled by a GARCH(1,1) implies that the conditional correlations at time t are computed using all the previous correlations from time $t = 1, ..., t-1$ (a form of smoothing).

4 Methodology

4.1 Overview of approaches taken

Having provided an overview of several statistical methods for forecasting conditional correlations, two different approaches were adopted for this project. Firstly, we attempted to directly model the pairwise univariate correlations between each of the five risk factors using an **ARMA(p,q) model with a variable rolling window**. This involved creating a rolling window of 12 observations between two time series based on training data, and then computing the correlation between the two time series for that rolling window of observations *to create a new time series of correlations.* Next, an ARMA(p,q) model was fitted directly to this time series of correlations, whose order was chosen using the Akaike Information Criteria (AIC). Lastly, 12 rolling 1-step ahead forecasts were obtained and compared with the actual rolling correlation in the test data. This process was repeated for each combination of the five univariate time series.

The second approach was to model the overall conditional correlation matrix using the **DCC-GARCH**. Estimation of the DCC-GARCH model was conducted in R using the **rmgarch** package, and was achieved using a two-step procedure. Firstly, univariate GARCH(p,q) models were fitted separately to the five risk factors that were previously identified. The optimal values of p and q were chosen using the AIC and were noted down. Secondly, a DCC-GARCH(1,1) model was fitted to the multivariate time series to obtain estimates for α and β. Finally, 12 1-step ahead rolling forecasts for the conditional correlation matrix were computed.

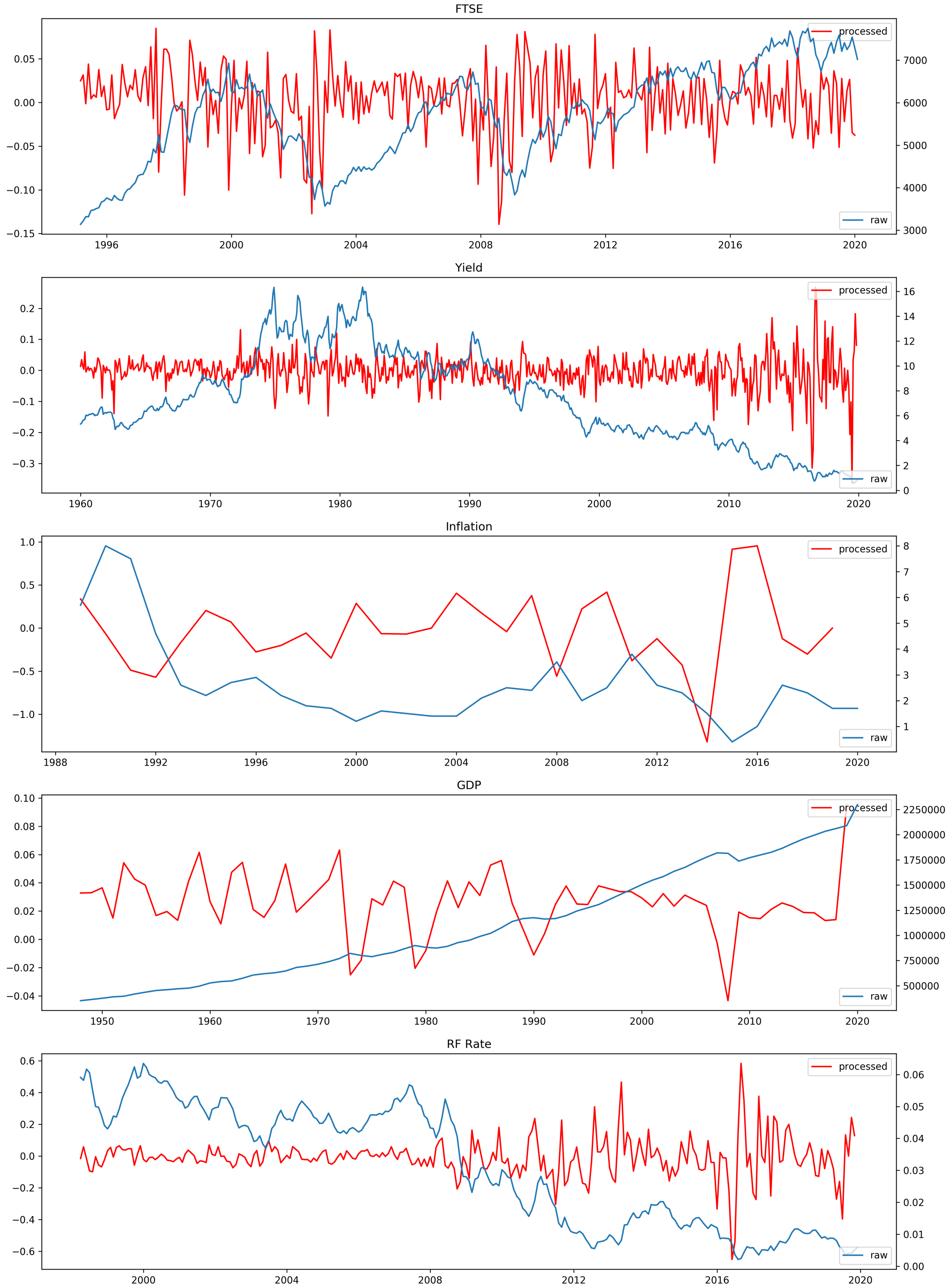
We note that the errors for the DCC-GARCH model were all assumed to follow a Student's t-distribution. In addition, stationarity of all the time series data was confirmed using the Dickey-Fuller test. The univariate GARCH models for each of the five risk factors are as follows:

- FTSE: GARCH(1,1)
- Yield: GARCH(1,2)
- GDP: GARCH(1,1)
- Inflation: GARCH(1,2)
- Risk-free rate: GARCH(1,2)

4.2 Data preprocessing

Before analysing the data, some preprocessing was necessary. The datasets for the risk factors had dates ranging from 1980 to 2020. Hence, all dates that had missing values for any of the time series were removed so as to ensure each factor had consistent representation in the final dataset. We also noted that data for all the risk factors had a monthly frequency apart from GDP and inflation which had a yearly frequency. To remedy this, they were upsampled into monthly frequencies using linear interpolation. Linear interpolation was selected since it was simple to implement and easy to understand. Other interpolation methods attempted were polynomial, spline and piecewise-polynomial. Furthermore, log-returns were calculated over the entirety of this dataset and the Dickey-Fuller test was applied to confirme stationarity for each of the time series. Plots of the raw and log-transformed data are shown in Figure 3 below. Note that the blue lines indicate the unprocessed data, whilst the red lines indicate the log-returns of each risk factor. Indeed, each of the plots suggest that the log-returns of each risk factor is stationary.

Figure 3: Plots of raw and proccessed data for each risk factor



5 Results

5.1 ARMA(p,q) model with a variable rolling window

Firstly, using the AIC as a criteria for choosing ARMA models for a 12-month rolling window yielded the following models:

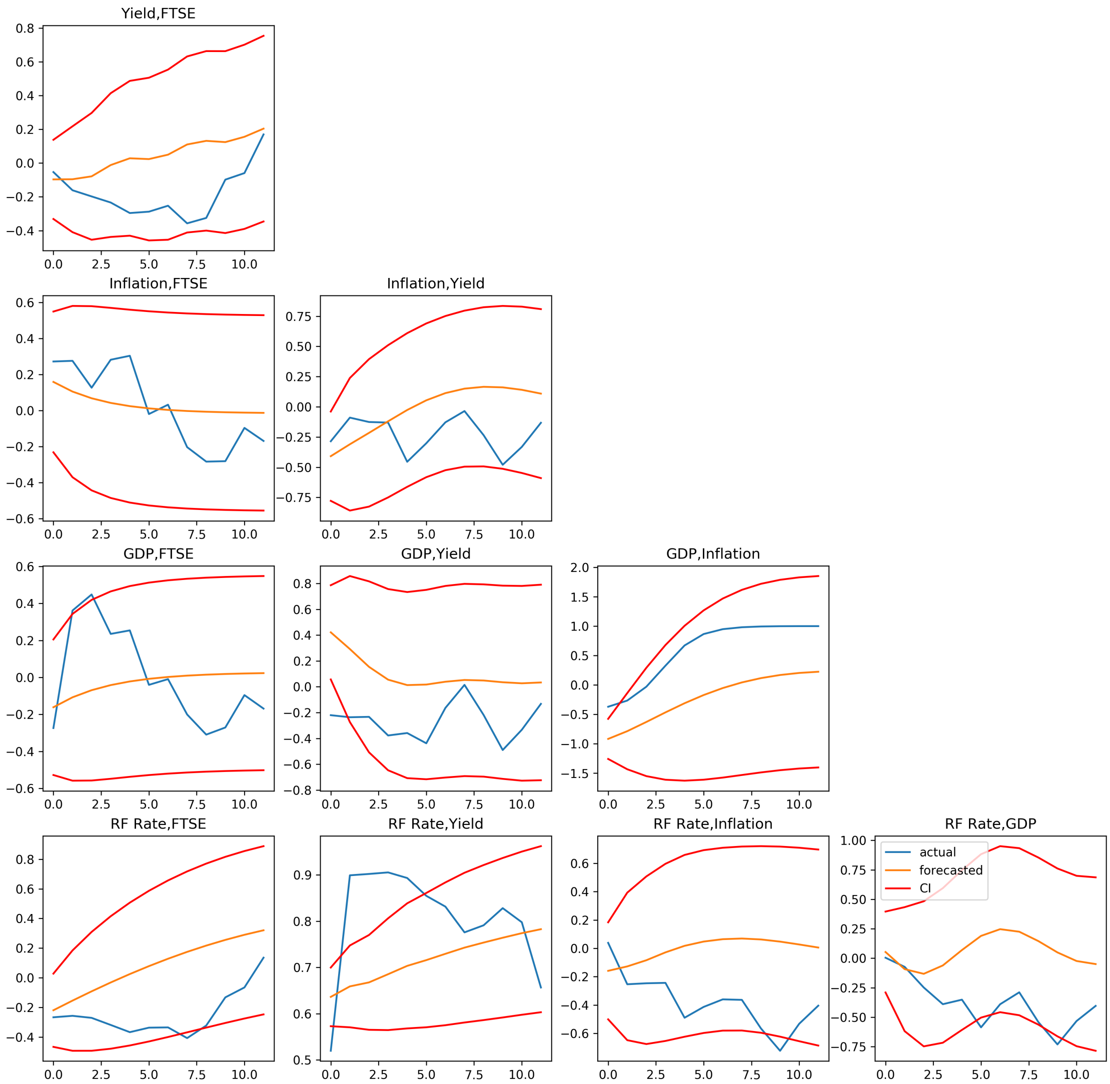
Table 1: ARMA model orders for each pair of risk factors

Correlation	FTSE	Yield	Inflation	GDP	Risk-free rate
FTSE	-				
Yield	4,3	-			
Inflation	1,0	4,1	-		
GDP	1,0	4,2	2,0	-	
Risk-free rate	2,1	4,0	4,1	4,2	-

The table above describes the models used for each pair of correlation. The data is presented in the format (p,q) where p and q correspond to the parameters in the ARMA(p,q) model. With these models, the 12 step ahead forecasts were then obtained and plotted in Figure 4 below. Note that the pairwise correlations of the 5 factors are presented in a lower diagonal matrix format, with red lines corresponding to the prediction intervals around the ARMA forecasts (yellow lines). The true rolling correlation is seen in blue.

It can be seen that the forecasts are often constant or with a low drift. They hence do not capture the volatility of the correlations. Moreover, the prediction intervals are extremely wide and in the case of GDP and inflation, exceed the range of $[-1, 1]$. Note that this pair of risk factors have interpolated values and hence this may have affected the forecasts. It can thus be concluded that the ARMA models are less effective at forecasting the correlations between the risk factors.

Figure 4: Plots of the forecasted correlations made using the ARMA models and the true correlations computed using a 12-month rolling window for each pair of risk factors

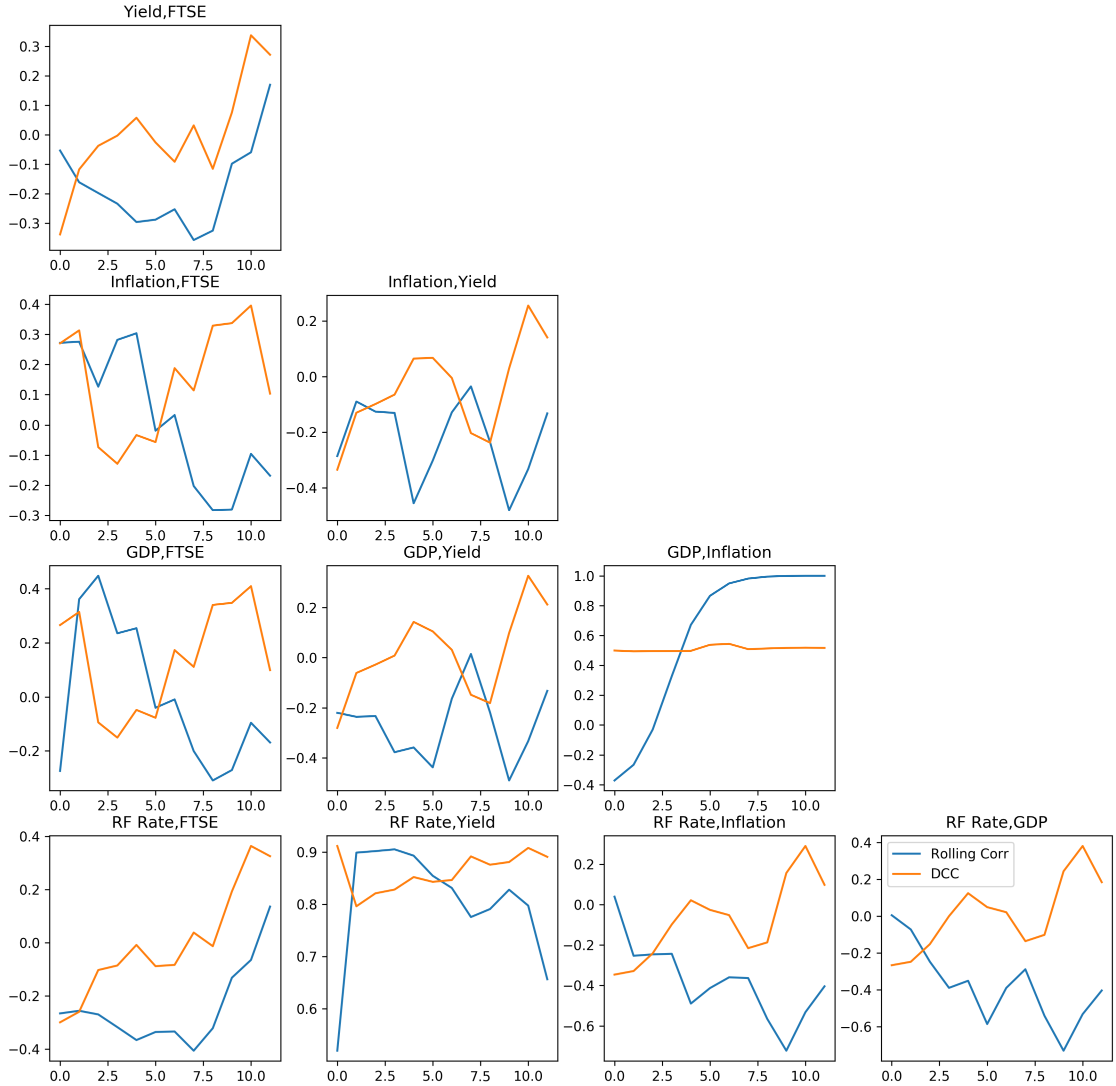


5.2 DCC-GARCH

The estimates of the DCC-GARCH joint parameters α and β were 0.186 and 0.562 respectively, and as $0.186 + 0.562 = 0.748 < 1$, the condition for the GARCH model was satisfied. Both parameters were statistically highly significant, implying that the conditional correlations are indeed time-varying and that a DCC-GARCH model was appropriate.

Once the forecasts using the DCC-GARCH were computed, it was necessary to evalute the accuracy of these forecasts. To that end, we used the rolling correlation of the data spanning both the training and test data as a proxy for the true conditional correlation. As an intial test, the 12-month rolling correlation was calculated for the test set and compared with the DCC-GARCH forecasts. The results are seen in Figure 5 below. In the plots, the blue line refers to the rolling correlation values while the yellow line refers to the DCC-GARCH forecast.

Figure 5: Plots of the forecasted correlations made using the DCC-GARCH and the true correlations computed using a 12-month rolling window for each pair of risk factors



Further rolling correlations were subsequently calculated, with window lengths ranging from 4 to 60 months. For each window length, we calculated the Mean Squared Error (MSE) and Mean Absolute Error (MAE) between the rolling correlation and DCC forecast. The tables below (Figures 6 and 7) show which lengths give the optimal error for each pair of correlation by presenting the top 10 best rolling correlation lengths in terms of lowest error value, in decreasing order. As an example, for risk factors Yield and FTSE, the 22 month rolling correlation gives the lowest error in terms of both MSE and MAE, followed by 23 months (MSE) and 21 months (MAE).

Figure 6: Top 10 rolling window lengths for each pair of risk factors that minimise the Mean Squared Error between the rolling correlations and DCC forecasts

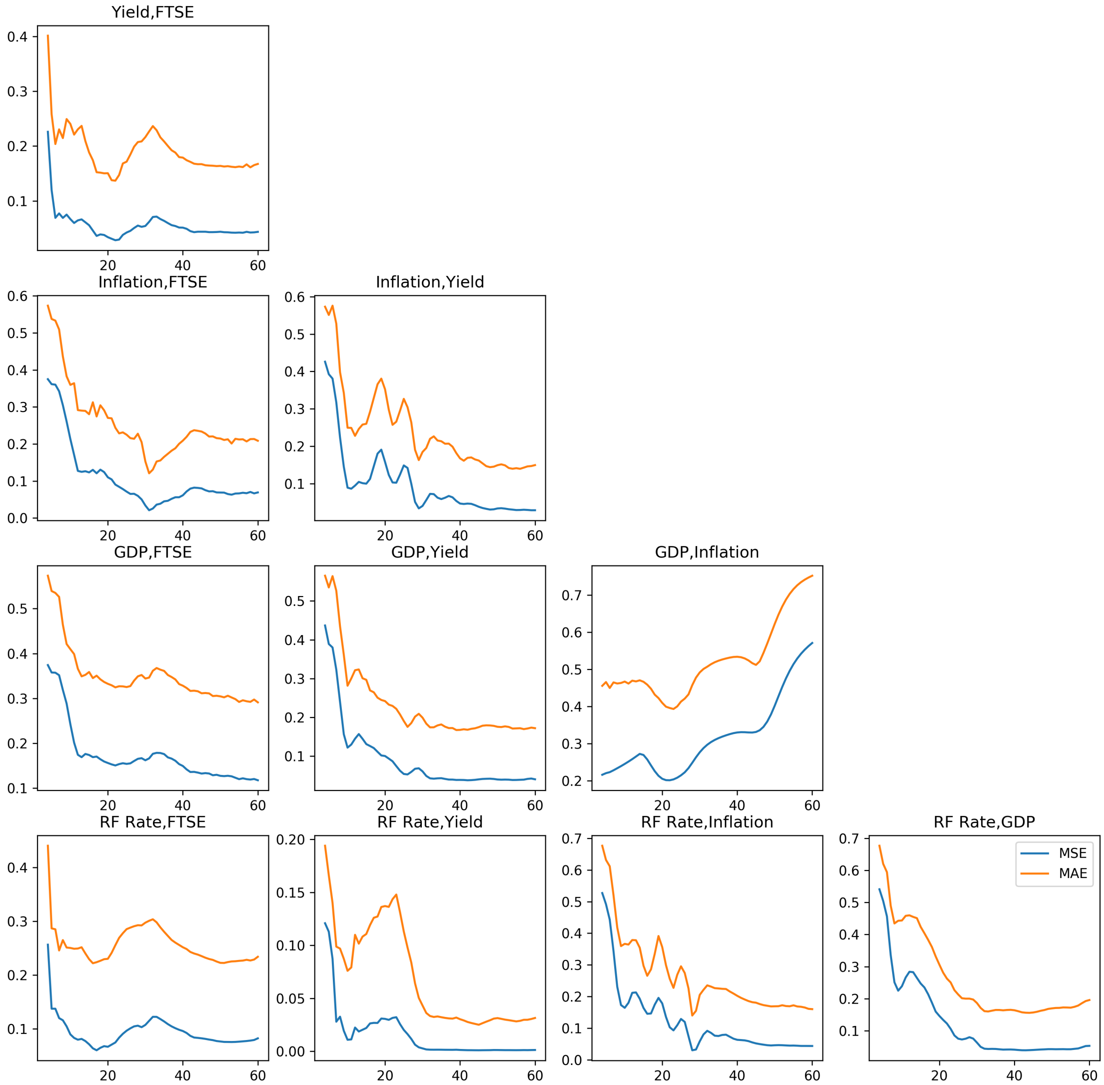
	Yield,FTSE	Inflation,FTSE	Inflation,Yield	GDP,FTSE	GDP,Yield	GDP,Inflation	RF Rate,FTSE	RF Rate,Yield	RF Rate,Inflation	RF Rate,GDP
0	22	31	59	60	42	22	17	45	28	43
1	23	32	60	58	43	21	16	44	29	42
2	21	30	55	55	54	23	18	46	60	44
3	20	33	56	57	41	20	20	43	59	41
4	17	34	58	59	55	24	19	55	58	45
5	19	35	57	56	39	19	21	42	57	46
6	24	36	54	54	40	25	15	47	56	40
7	18	29	48	53	44	4	22	54	54	37
8	56	37	49	51	56	5	53	56	55	38
9	54	39	53	50	51	26	52	48	49	47

Figure 7: Top 10 rolling window lengths for each pair of risk factors that minimise the Mean Absolute Error between the rolling correlations and DCC forecasts

	Yield,FTSE	Inflation,FTSE	Inflation,Yield	GDP,FTSE	GDP,Yield	GDP,Inflation	RF Rate,FTSE	RF Rate,Yield	RF Rate,Inflation	RF Rate,GDP
0	22	31	56	60	39	23	16	45	28	44
1	21	32	54	55	40	22	51	44	29	43
2	23	33	55	58	42	21	50	46	60	45
3	19	30	53	57	41	24	17	43	59	42
4	20	34	57	56	57	20	52	42	58	46
5	18	35	48	59	43	25	49	47	57	33
6	17	36	49	54	54	26	53	55	56	41
7	58	37	58	53	58	19	54	56	49	32
8	54	38	47	51	55	18	55	54	50	47
9	56	39	59	50	44	27	18	41	54	34

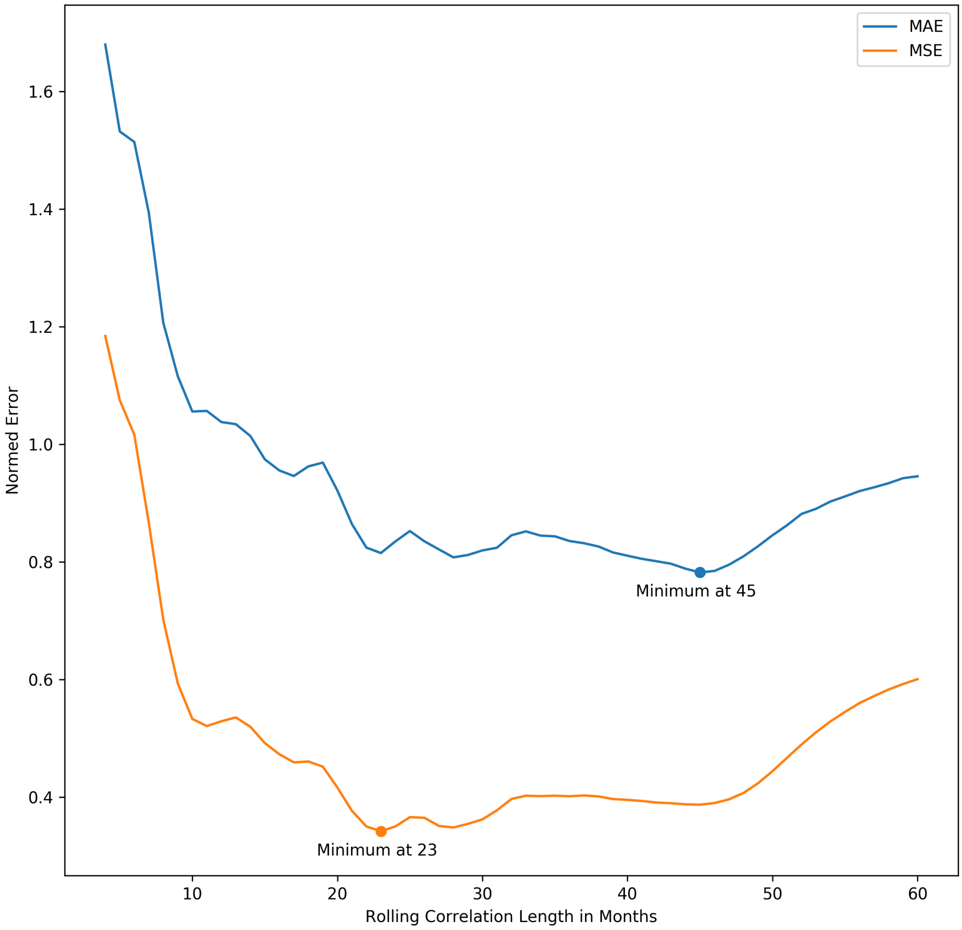
Figure 8 below presents another plot for the errors and the rolling window correlation length for each pair of correlations, which is shown in a lower diagonal matrix form. For example in the top left plot, for Yield and FTSE pairwise correlations, we see the MSE and MAE decreasing with increasing rolling window length.

Figure 8: Plot of MSE/MAE vs the rolling window lengths for each risk factor



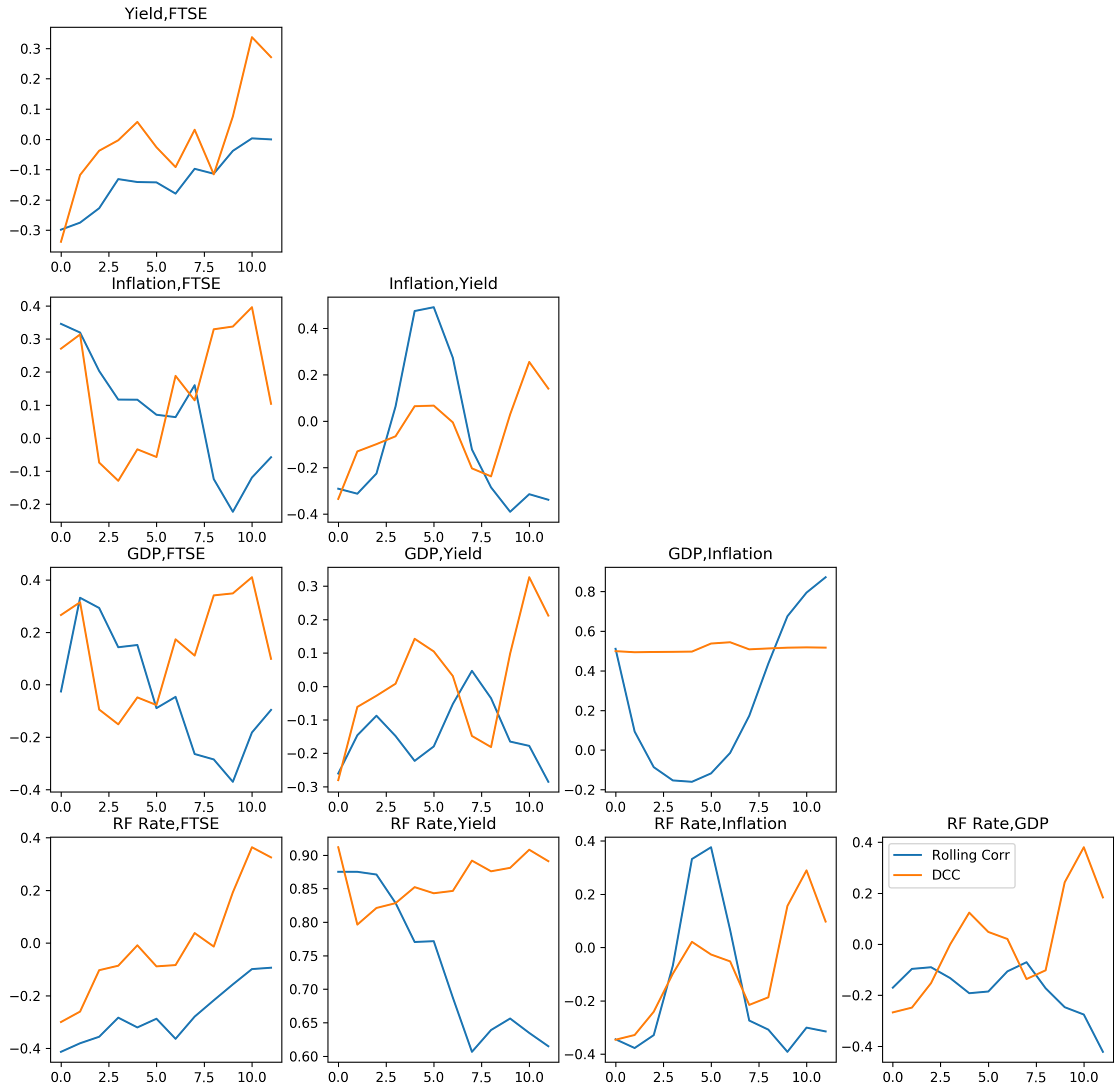
We note that each rolling correlation had an error value at each window length ranging from length 4 to 60 months. Across 10 rolling correlations, we then obtained a vector of dimension 10 of the error values at each specific rolling window length. By taking the norm of this vector, we obtain a normalised error value for each rolling window length. In this case, the L_2 norm was chosen. We then plotted the normalised error value against the rolling window length as shown in Figure 9 below. Based on MSE, it can be seen that the minimum error occurs at a window length of 23 months. We thus concluded that this is the ideal length for rolling window correlations. Finally, we plotted the rolling correlation of length 23 months against the DCC-GARCH forecasts, as shown in Figure 10 below. Visually, it can be seen that the DCC-GARCH forecasts are more accurate at capturing the volatility of the rolling correlations compared to the ARMA models seen earlier. The DCC-GARCH also guarantees that the output values are in $[-1, 1]$.

Figure 9: Plot of normed error (MSE/MAE) vs the rolling correlation window length in months



From Figure 9, it can be seen that the MSE/MAE curves are roughly convex, first decreasing and then increasing with increasing rolling window length. Subsequently research could further investigate the optimal window length through adding a penalty term for larger window lengths (regularisation), hence favouring shorter window lengths.

Figure 10: Plots of the forecasted correlations made using the DCC-GARCH and the true correlations computed using a 23-month rolling window for each pair of risk factors



6 Evaluation

6.1 Limitations of the DCC-GARCH for forecasting

Although the DCC-GARCH model is capable of computing the matrix of conditional correlations for a set of time series data, there are several known issues regarding the statistical properties of the model. These issues are concisely summarised by Caporin & McAleer (2013). Two of them include:

- The matrix $\mathbf{R}_t$ from the DCC-GARCH *does not necessarily represent the dynamic conditional correlation matrix of the process* due to the way it is constructed. Namely, Caporin and McAleer argue that the way $\mathbf{R}_t$ is standardised simply using the formula $\mathbf{R}_t = diag(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t diag(\mathbf{Q}_t)^{-1/2}$ does not adequately qualify $\mathbf{R}_t$ to be proper dynamic conditional correlation matrix. This is due to the fact that $\mathbf{Q}_t$ is **not** the conditional covariance of $\boldsymbol{\epsilon}_t$, which is in fact $\mathbb{E}[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{'}|\mathcal{F}_{t-1}] = \mathbf{R}_t$. Hence, $\mathbf{Q}_t$

has no proper interpretation as either a dynamic conditional covariance or dynamic conditional correlation matrix (Caporin & McAleer, 2013).

- There is no derivation of the DCC-GARCH and its mathematical properties, and a lack of any demonstration of the asymptotic properties of the estimated parameters. It also does not have testable regularity conditions. Hence, there is no guarantee that the model is consistent, nor that the DCC-GARCH estimates have any connection to the definition of dynamic conditional correlations (Caporin & McAleer, 2013).

Owing to these issues among others, it has thus been suggested that the DCC-GARCH be used as a diagnostic check rather than a proper model for correlations. That being said, it has been noted that the DCC-GARCH does perform well empirically, due to reasons such as ease of estimation (Caporin & McAleer, 2013). Caporin and McAleer suggest that it can play a useful role in forecasting out-of-sample dynamic conditional covariances and correlations (which was indeed the purpose of this project).

6.2 Further extensions

In this study, we only looked at the Pearson correlation coefficient to compare the relationship between two variables. In long-run time series analysis, an alternative that could be looked at is the cointegration of two time series. Furthermore, change point detection was an interesting topic that we could not explore owing to a lack of time. By utilising change point detection, we could identify multiple regimes in the time series and apply different models to each of them. We could then further model these regime changes with markov chains. Lastly, Principal Component GARCH offers an interesting alternative to volatility modelling. By applying Principal Components Analysis (PCA) to the high dimensional dataset, we can reduce it to a lower dimensional space of orthogonal (and hence independent) principal components. We could then apply a GARCH model to these principal components.

7 Conclusion

Our research revealed that the DCC-GARCH model was the superior method for robustly calculating the correlations between the chosen risk factors. The factors were chosen on the preface of their proposed relationship between one another and more importantly, the influential relationship they held with Aviva's products, capital model and assets and liabilities. Validation of these factors comes in the form of our model successfully forecasting the expected link between the factors. For example, we observed that equity and interest rate level dependencies range from low negative to medium positive, while dependencies between the risk free rate and other market risks are typically medium positive, which were all as expected.

Furthermore, we determined that 23 months was the optimal length to minimise the MSE between the true rolling correlations of the observed data and DCC-GARCH forecasts. We believe that this is the ideal tool for forecasting and analysing volatility of time series for several reasons:

- The model helps in the detection of possible changes in conditional correlations over time, which allows for us to detect dynamic investor behaviour in response to news and innovations.
- The DCC-GARCH measure is appropriate to investigate possible markets during crisis periods.
- The DCC-GARCH model estimates correlation coefficients of the standardised residuals and so accounts for heteroskedasticity directly.
- Since the volatility is adjusted by the procedure, the time-varying correlation does not have bias from volatility. Unlike the volatility-adjusted cross-market correlations, DCC-GARCH continuously adjusts the correlation for time-varying volatility. This will certainly improve the accuracy of a volatility adjustment, which is important as the Prudential Regulation Authority (PRA) has proposed a new supervisory statement related to the application of a dynamic volatility adjustment (VA) in the modelling of Solvency II (SII) market risk stresses, whereby the VA is a stabilising measure intended to avoid excessive short term volatility of own funds under SII.

This report also notes that the DCC-GARCH model suffers from several drawbacks as notably stated in Caporin & McAleer (2013), and thus should be used with caution. Moreover, further research on models such as the Principle Component GARCH could be conducted to assess their viability as an alternative to the DCC-GARCH.

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