

# **Challenging deep learning models in real-world applications**

Learning with few or no data and looking for explainability

Introduction

---

Céline Hudelot

Ecole d'été de Peyresq 2022

# Foreword



Prof. Céline Hudelot, Computer Science  
Head of the MICS Laboratory  
Research on semantic data interpretation

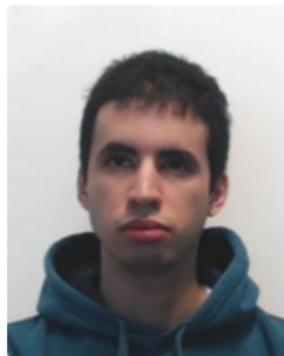
<https://scholar.google.fr/citations?user=gFlAh6MAAAAJ&hl=fr>

# Foreword

Other contributors of this course



Dr. Victor Bouvier  
Research Scientist, Dataiku



Yassine Ouali  
PhD, MICS

## The Deep Learning breakthrough

---

# Some impressive applications of AI

Self-driving cars in the road of Paris



See :<https://www.youtube.com/watch?v=9mBLl6JuvsM>

# Some impressive applications of AI

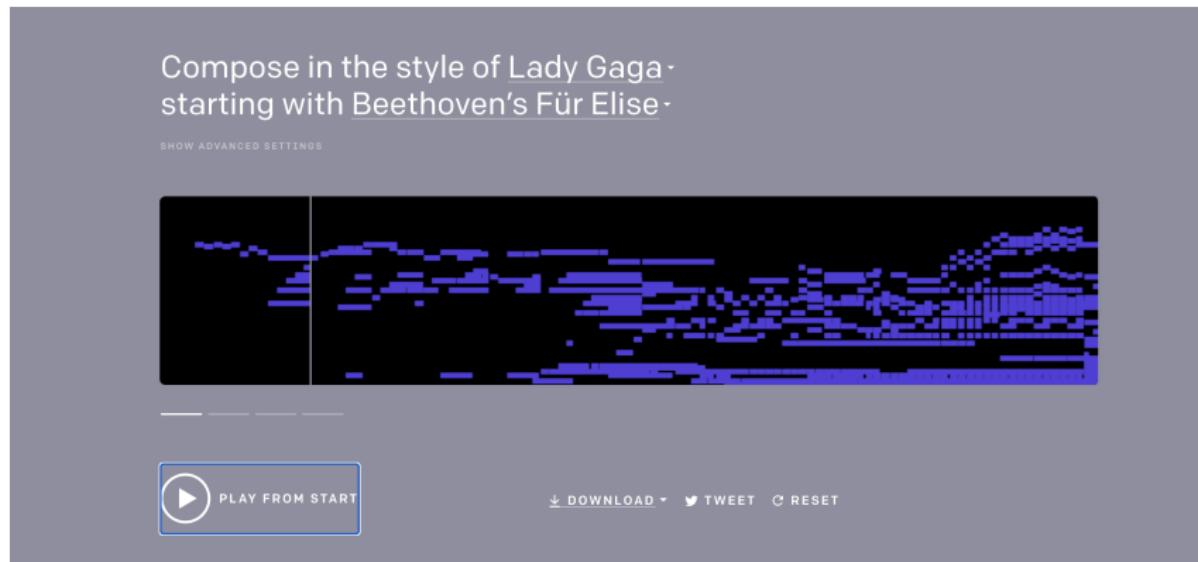
## Realistic data generation



**Figure 1:** Test it here : <https://thispersondoesnotexist.com/>

# Some impressive applications of AI

Able to play music



**Figure 2:** Test it here : <https://openai.com/blog/musenet/>

# Some impressive applications of AI

Able to create art



**Figure 3:** The next Rembrandt: <https://www.nextrembrandt.com/>

This raises problems of intellectual property: who is the author?  
(<https://cacm.acm.org/magazines/2020/7/245693-ai-authorship/fulltext>).

# Some impressive applications of AI

Advancing knowledge: predicting the structure of proteins from their amino acid sequence

## 'It will change everything': DeepMind's AI makes gigantic leap in solving protein structures

Google's deep-learning program for determining the 3D shapes of proteins stands to transform biology, say scientists.

Even Colossal



A protein's function is determined by its 3D shape. Credit: DeepMind

An artificial intelligence (AI) network developed by Google AI offshoot DeepMind has made a gigantic leap in solving one of biology's grandest challenges – determining a protein's 3D shape from its amino acid sequence.

3D viewer ⓘ

Model Confidence:

- Very high ( $\text{pLDDT} > 90$ )
- Confident ( $90 > \text{pLDDT} > 70$ )
- Low ( $70 > \text{pLDDT} > 50$ )
- Very low ( $\text{pLDDT} < 50$ )

AlphaFold produces a per-residue confidence score ( $\text{pLDDT}$ ) between 0 and 100. Some regions below 50  $\text{pLDDT}$  may be unstructured in isolation.

Sequence of AF-Q18Q05-F1 t 1:Free fatty a... t A \*

Sequence of AF-Q18Q05-F1 t 1:Free fatty a... t A \*

Views

Predicted aligned error

Sequence Features (coming soon)

Adjusted residue

# Able to predict the structure of proteins from their amino acid sequence

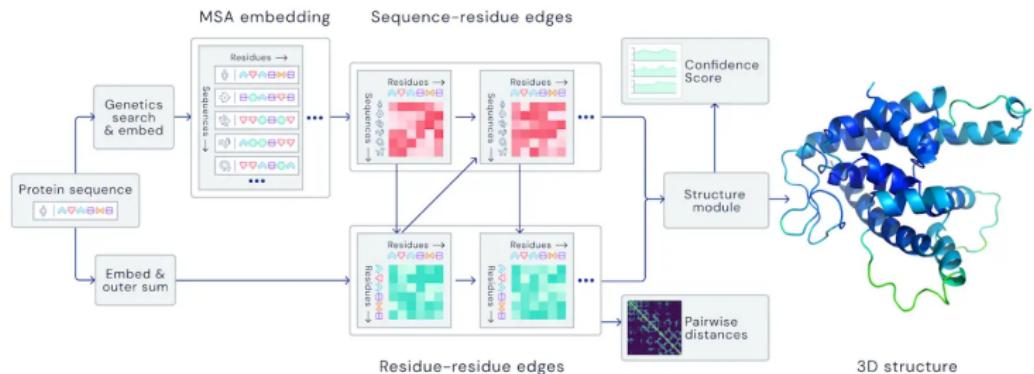


Figure 4: AlphaFold

# Some impressive applications of Deep Learning

Able to solve PDE!

ARTIFICIAL INTELLIGENCE

## AI has cracked a key mathematical puzzle for understanding our world

Partial differential equations can describe everything from planetary motion to plate tectonics, but they're notoriously hard to solve.

By Karen Hao

October 30, 2020

# The Deep Learning breakthrough

---

Which AI ?

# Two major antagonistic approaches<sup>1</sup>

## Two different assumptions

---

<sup>1</sup> D. Cardon et al - La Revanche des neurones - <https://hal.archives-ouvertes.fr/hal-02005537/document>

# Two major antagonistic approaches<sup>1</sup>

## Two different assumptions

- Human reasoning and knowledge are complex: knowledge implicitly in data.

---

<sup>1</sup> D. Cardon et al - La Revanche des neurones - <https://hal.archives-ouvertes.fr/hal-02005537/document>

# Two major antagonistic approaches<sup>1</sup>

## Two different assumptions

- Human reasoning and knowledge are complex: knowledge implicitly in data.
  - Statistic or data-centric AI - Connectionist approaches - Learning from data.

---

<sup>1</sup> D. Cardon et al - La Revanche des neurones - <https://hal.archives-ouvertes.fr/hal-02005537/document>

# Two major antagonistic approaches<sup>1</sup>

## Two different assumptions

- Human reasoning and knowledge are complex: knowledge implicitly in data.
  - Statistic or data-centric AI - Connectionist approaches - Learning from data.
  - Exploitation of the **past experience** represented by annotated data, building calibrated predictive models from it.

---

<sup>1</sup> D. Cardon et al - La Revanche des neurones - <https://hal.archives-ouvertes.fr/hal-02005537/document>

# Two major antagonistic approaches<sup>1</sup>

## Two different assumptions

- Human reasoning and knowledge are complex: knowledge implicitly in data.
  - Statistic or data-centric AI - Connectionist approaches - Learning from data.
  - Exploitation of the **past experience** represented by annotated data, building calibrated predictive models from it.
- Human reasoning can be captured, even if partially incomplete: explicit representation of knowledge (using **symbols** rather than statistics to represent the world).

---

<sup>1</sup> D. Cardon et al - La Revanche des neurones - <https://hal.archives-ouvertes.fr/hal-02005537/document>

# Two major antagonistic approaches<sup>1</sup>

## Two different assumptions

- Human reasoning and knowledge are complex: knowledge implicitly in data.
  - Statistic or data-centric AI - Connectionist approaches - Learning from data.
  - Exploitation of the **past experience** represented by annotated data, building calibrated predictive models from it.
- Human reasoning can be captured, even if partially incomplete: explicit representation of knowledge (using **symbols** rather than statistics to represent the world).
  - Symbolic AI - Based on the modeling of logical reasoning, on **formalisms for knowledge representation and reasoning**.

---

<sup>1</sup> D. Cardon et al - La Revanche des neurones - <https://hal.archives-ouvertes.fr/hal-02005537/document>

# Two major antagonistic approaches<sup>1</sup>

## Two different assumptions

- Human reasoning and knowledge are complex: knowledge implicitly in data.
  - Statistic or data-centric AI - Connectionist approaches - Learning from data.
  - Exploitation of the **past experience** represented by annotated data, building calibrated predictive models from it.
- Human reasoning can be captured, even if partially incomplete: explicit representation of knowledge (using **symbols** rather than statistics to represent the world).
  - Symbolic AI - Based on the modeling of logical reasoning, on **formalisms for knowledge representation and reasoning**.



---

<sup>1</sup> D. Cardon et al - La Revanche des neurones - <https://hal.archives-ouvertes.fr/hal-02005537/document>

# Data-driven Artificial Intelligence

Exploits the past experience represented by labeled data to build calibrated predictive models from **labeled** data.

# Data-driven Artificial Intelligence

Exploits the past experience represented by labeled data to build calibrated predictive models from **labeled** data.

At the origin of the recent AI explosion, thanks to:

- increased availability of **data**, 'big data'.
- improvement of processing methods and **algorithms** (in particular deep neural networks)
- increasing of the **Computing** capacity.

# Data-driven Artificial Intelligence

Exploits the past experience represented by labeled data to build calibrated predictive models from **labeled** data.

At the origin of the recent AI explosion, thanks to:

- increased availability of **data**, 'big data'.
- improvement of processing methods and **algorithms** (in particular deep neural networks)
- increasing of the **Computing** capacity.

## Principle

We want to predict  $Y$  from  $X$ , as for instance:

- $X$ : radiology image,  $Y$ : presence of a tumor?
- $X$ : sensor and monitoring data  $Y$ : rule life prediction of the system ?

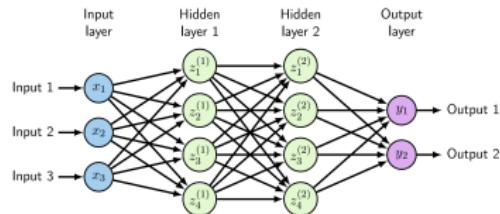
Determination of the function  $\psi$  (model) such that  $Y = \psi(X)$ , and  $\psi$  is estimated from **labeled data**:

$N$  situations in which one knows at the same time  $X$  and  $Y$ :  $(X_i, Y_i)_{1 \leq i \leq N}$

# Data-driven Artificial Intelligence

## Deep neural networks

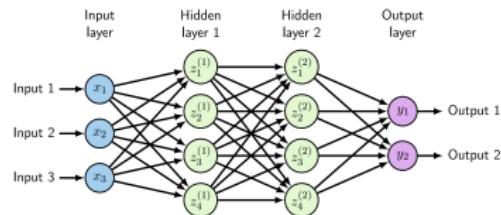
$Y = \psi(X)$ , with  $\psi(X) = h_M \circ g_M \circ \dots \circ h_1 \circ g_1(X)$  where  $h_i$  some non-linear transformations and  $g_i$  some affine transformations.



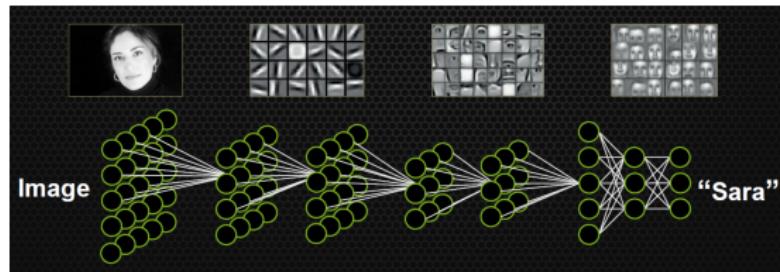
# Data-driven Artificial Intelligence

## Deep neural networks

$Y = \psi(X)$ , with  $\psi(X) = h_M \circ g_M \circ \dots \circ h_1 \circ g_1(X)$  where  $h_i$  some non-linear transformations and  $g_i$  some affine transformations.



## Representation learning



The Deep layers capture complex features in the image to extract the most relevant information for the prediction task [Lee et al., 2009].

# Motivations of Deep Learning

---

# Motivations of Deep Learning

---

Limits of 'traditional' Machine Learning

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

## What do you do?

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

## What do you do?

1. Define the task and collect data,

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

## What do you do?

1. Define the task and collect data,
2. Define an objective (a metric) to maximize  $\triangleright$  or a loss to minimize  $\ell$ ,

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

## What do you do?

1. Define the task and collect data,
2. Define an objective (a metric) to maximize  $\triangleright$  or a loss to minimize  $\ell$ ,
3. Specify a model  $\triangleright (f_\theta)_{\theta \in \Theta}$

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

## What do you do?

1. Define the task and collect data,
2. Define an objective (a metric) to maximize  $\triangleright$  or a loss to minimize  $\ell$ ,
3. Specify a model  $\triangleright (f_\theta)_{\theta \in \Theta}$
4. Split the data into a train and a test set,

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

## What do you do?

1. Define the task and collect data,
2. Define an objective (a metric) to maximize  $\triangleright$  or a loss to minimize  $\ell$ ,
3. Specify a model  $\triangleright (f_\theta)_{\theta \in \Theta}$
4. Split the data into a train and a test set,
5. Train the model  $\triangleright \hat{\theta} := \arg \min_{\theta} \frac{1}{n} \sum_{(x,y) \in \text{train}} \ell(f_\theta(x), y)$

# Road map for building a ML model

## Business case

- You are working in some company where a support team helps users.
- The boss finds that his support team spends a lot of time dealing spam mails rather than dealing with real user requests.
- You are asked to build a model that detect spam mails.

## What do you do?

1. Define the task and collect data,
2. Data cleaning and feature engineering ▷ 90% of your time! Why..?
3. Define an objective (a metric) to maximize ▷ or a loss to minimize  $\ell$ ,
4. Specify a model ▷  $(f_\theta)_{\theta \in \Theta}$
5. Split the data into a train and a test set,
6. Train the model ▷  $\hat{\theta} := \arg \min_{\theta} \frac{1}{n} \sum_{(x,y) \in \text{train}} \ell(f_\theta(x), y)$
7. Evaluate the model according to a metric.

# Motivations of Deep Learning

---

**Features engineering: How to  
'represent' your data?**

# Features engineering

## Features engineering

Features engineering is the process that consists in transforming the features such that learning the task from the latter is easier (or leads to better generalization) compared to the former.

# Features engineering

## Features engineering

Features engineering is the process that consists in transforming the features such that learning the task from the latter is easier (or leads to better generalization) compared to the former.

- ▷ Examples of features engineering

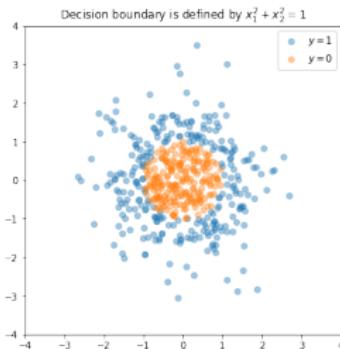
# Features engineering

## Features engineering

Features engineering is the process that consists in transforming the features such that learning the task from the latter is easier (or leads to better generalization) compared to the former.

### ▷ Examples of features engineering

**Unit circle:**  $y := (\|x\|^2 > 1)$  where  $x \sim \mathcal{N}(0, 1)$



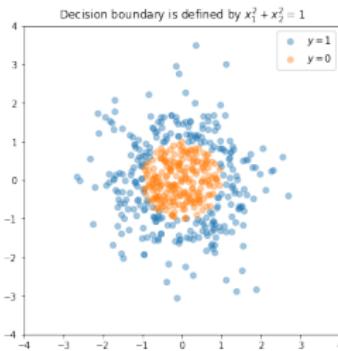
# Features engineering

## Features engineering

Features engineering is the process that consists in transforming the features such that learning the task from the latter is easier (or leads to better generalization) compared to the former.

### ▷ Examples of features engineering

**Unit circle:**  $y := (\|x\|^2 > 1)$  where  $x \sim \mathcal{N}(0, 1)$



▷ How do you address this problem? (Notebook session:  
[colab.research.google.com/drive/1NugMhZ9VEE3Mwt50avgFV1hPWX17N5Wo?usp=sharing](https://colab.research.google.com/drive/1NugMhZ9VEE3Mwt50avgFV1hPWX17N5Wo?usp=sharing))

# Representation

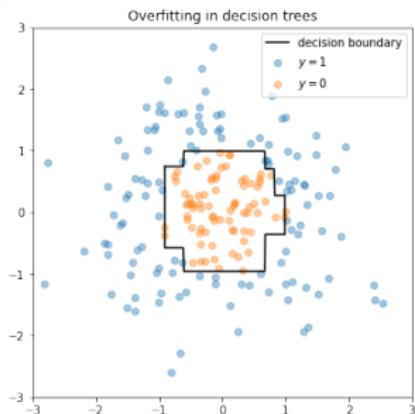
- Features engineering (representation) is mandatory in two cases:

# Representation

- Features engineering (representation) is mandatory in two cases:
  1. Our model has not enough capacity for separating the data ▷  
*Logistic Regression for the unit circle.*

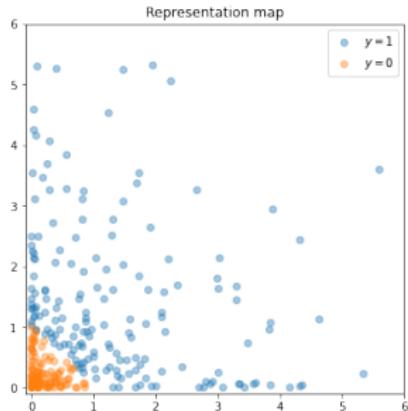
# Representation

- Features engineering (representation) is mandatory in two cases:
  1. Our model has not enough capacity for separating the data ▷ *Logistic Regression for the unit circle.*
  2. Our model has too much capacity and overfit the data ▷ *Decision Tree for the unit circle.*



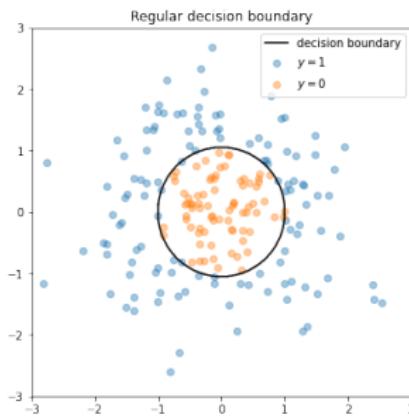
# Representation

- Features engineering (representation) is mandatory in two cases:
  1. Our model has not enough capacity for separating the data ▷ *Logistic Regression for the unit circle.*
  2. Our model has too much capacity and overfit the data ▷ *Decision Tree for the unit circle.*
- If provided with a suitable representation ▷  $\varphi(x) := (x_1^2, x_2^2)$



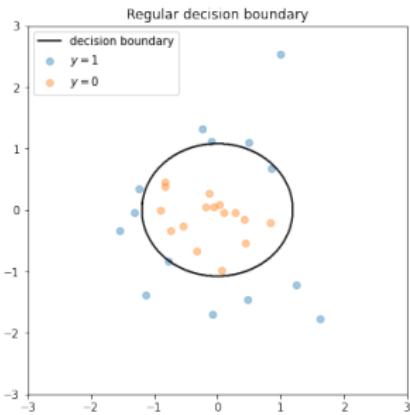
# Representation

- Features engineering (representation) is mandatory in two cases:
  1. Our model has not enough capacity for separating the data ▷ *Logistic Regression for the unit circle.*
  2. Our model has too much capacity and overfit the data ▷ *Decision Tree for the unit circle.*
- If provided with a suitable representation ▷  $\varphi(x) := (x_1^2, x_2^2)$ 
  - model benefits from a strong regularity ▷ *circle shape of the decision boundary for the unit circle.*



# Representation

- Features engineering (representation) is mandatory in two cases:
  1. Our model has not enough capacity for separating the data ▷ *Logistic Regression for the unit circle.*
  2. Our model has too much capacity and overfit the data ▷ *Decision Tree for the unit circle.*
- If provided with a suitable representation ▷  $\varphi(x) := (x_1^2, x_2^2)$ 
  - model benefits from a strong regularity ▷ *circle shape of the decision boundary for the unit circle.*
  - model can generalize with few samples ▷  $\sim 30$  are enough!



# Representation

**Model benefits from a strong regularity...**

A representation encodes our inductive bias ▷

*the hypothesis space is biased to solutions we found 'plausible'.*

# Representation

**Model benefits from a strong regularity...**

A representation encodes our inductive bias ▷

*the hypothesis space is biased to solutions we found 'plausible'.*

**Is it easy to define a good representation?**

- Does the data scientist know *a priori* the shape of the solution?
- Does it exist a representation that can be reasonably hand-crafted?

# Representation

**Model benefits from a strong regularity...**

A representation encodes our inductive bias ▷

*the hypothesis space is biased to solutions we found 'plausible'.*

**Is it easy to define a good representation?**

- Does the data scientist know *a priori* the shape of the solution?
- Does it exist a representation that can be reasonably hand-crafted?

**Maybe, you have already work with data representation**

# Representation

**Model benefits from a strong regularity...**

A representation encodes our inductive bias ▷

*the hypothesis space is biased to solutions we found 'plausible'.*

**Is it easy to define a good representation?**

- Does the data scientist know *a priori* the shape of the solution?
- Does it exists a representation that can be reasonably hand-crafted?

**Maybe, you have already work with data representation** ▷ Kernel trick in SVM!

# Learning representations

**Input image**

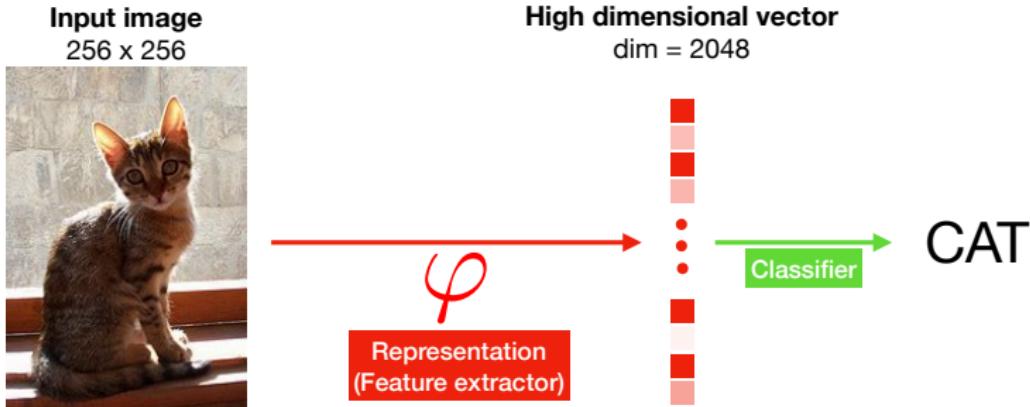
256 x 256



# Learning representations



# Learning representations

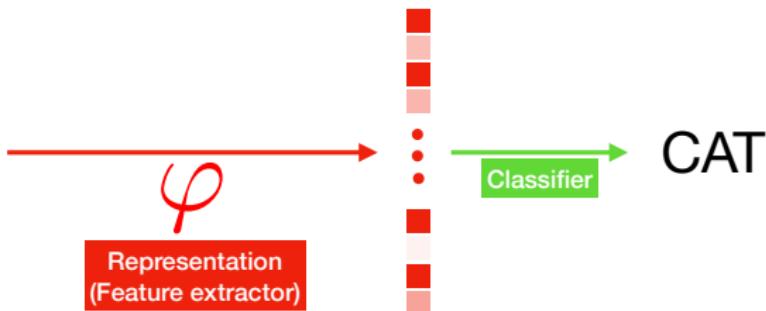


# Learning representations

- Not trainable
- Trainable

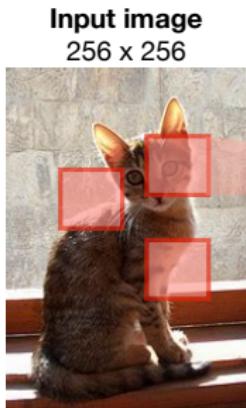


**High dimensional vector**  
dim = 2048

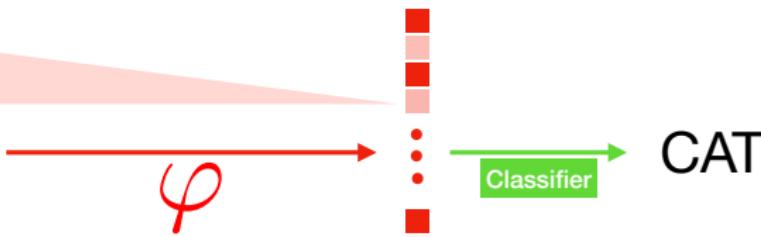


# Learning representations

- Not trainable
- Trainable



High dimensional vector  
dim = 2048



Convolutional filters, SIFT, Visual bag-of words...

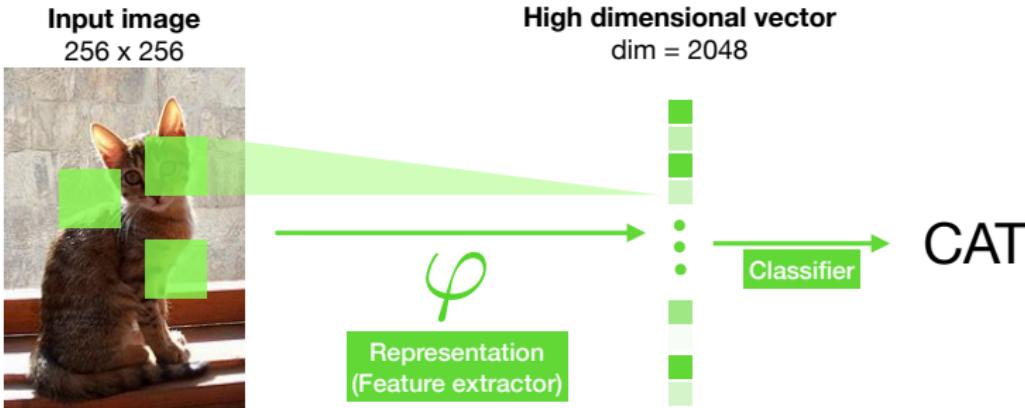
Operation	Kernel $w$	Image result $p(x)$
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
Edge detection	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$	
	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & 0 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	

Source wikipedia

# Learning representations

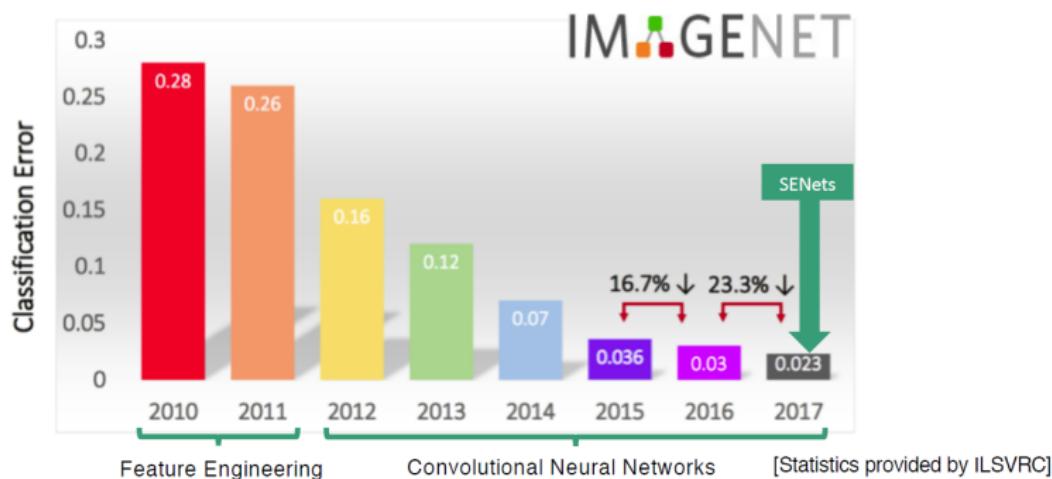
## Learning Representations = Deep Learning

 Not trainable  
 Trainable



- Representation is a function from the input space to the features space
- Defined by a large numbers of parameters
- Deep Learning is finding strong inductive bias for learning a good representation (convolutional neural network, recurrent neural network, transformers...)

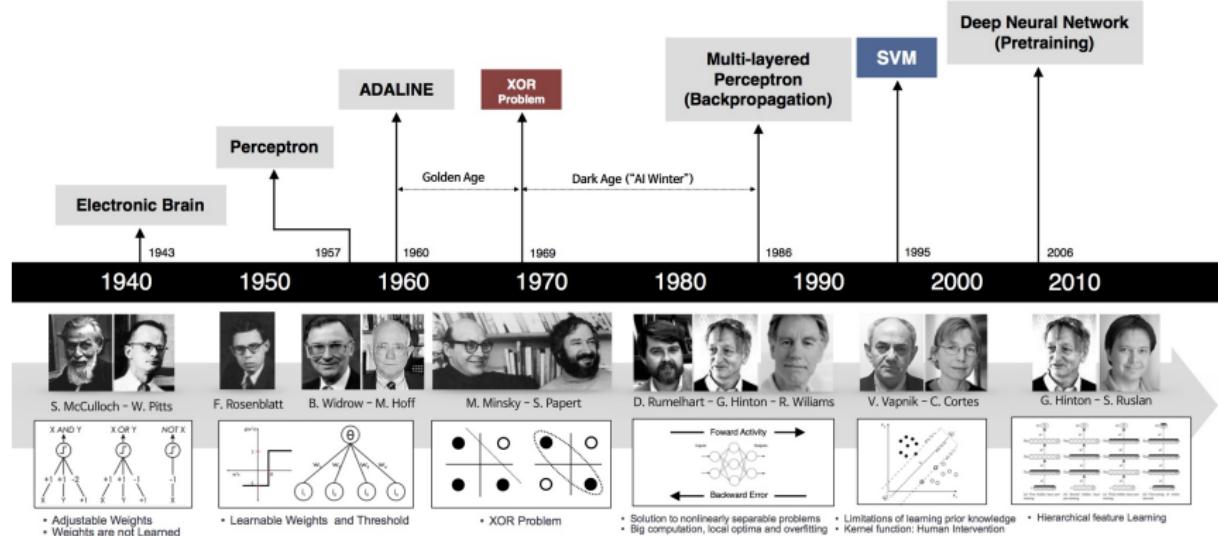
# Learning representations



# The Deep Learning timeline

---

# The Deep Learning timeline



[beamandrew.github.io/deeplearning/2017/02/23/deep\\_learning\\_101\\_part1.html](http://beamandrew.github.io/deeplearning/2017/02/23/deep_learning_101_part1.html)

## What we have learnt so far

- Deep Learning is a new Machine Learning approach where the representation (features extractor) is learned.

## What we have learnt so far

- Deep Learning is a new Machine Learning approach where the representation (features extractor) is learned.
- Representation is (most of the time) an over-parametrized mapping.

## What we have learnt so far

- Deep Learning is a new Machine Learning approach where the representation (features extractor) is learned.
- Representation is (most of the time) an over-parametrized mapping.
- 'Deep' refers to the process of composing (stacking) simple functions in order to build an over-parametrized representation.

## What we have learnt so far

- Deep Learning is a new Machine Learning approach where the representation (features extractor) is learned.
- Representation is (most of the time) an over-parametrized mapping.
- 'Deep' refers to the process of composing (stacking) simple functions in order to build an over-parametrized representation.
- Inductive bias (assumptions made about the shape of the learner) reduces the number of parameters.

## What we have learnt so far

- Deep Learning is a new Machine Learning approach where the representation (features extractor) is learned.
- Representation is (most of the time) an over-parametrized mapping.
- 'Deep' refers to the process of composing (stacking) simple functions in order to build an over-parametrized representation.
- Inductive bias (assumptions made about the shape of the learner) reduces the number of parameters.

### To go deeper in deep learning

1. Linear network (Rosenblatt's perceptron)

# What we have learnt so far

- Deep Learning is a new Machine Learning approach where the representation (features extractor) is learned.
- Representation is (most of the time) an over-parametrized mapping.
- 'Deep' refers to the process of composing (stacking) simple functions in order to build an over-parametrized representation.
- Inductive bias (assumptions made about the shape of the learner) reduces the number of parameters.

## To go deeper in deep learning

1. Linear network (Rosenblatt's perceptron)
2. Multi-layers perceptron.

## What we have learnt so far

- Deep Learning is a new Machine Learning approach where the representation (features extractor) is learned.
- Representation is (most of the time) an over-parametrized mapping.
- 'Deep' refers to the process of composing (stacking) simple functions in order to build an over-parametrized representation.
- Inductive bias (assumptions made about the shape of the learner) reduces the number of parameters.

### To go deeper in deep learning

1. Linear network (Rosenblatt's perceptron)
2. Multi-layers perceptron.
3. Training a neural network.

## Linear network

---

# Linear Neural Network

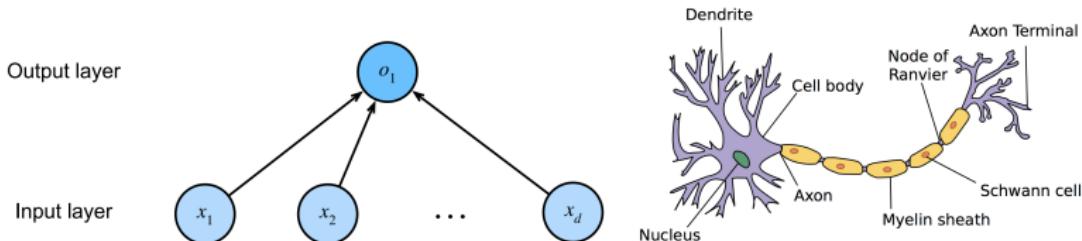
## Two examples

- **Regression:** Linear regression  $\triangleright y = x^\top \theta + \mathcal{N}(0, \sigma^2)$
- **Classification:** Logistic regression  $\triangleright p(y|x) = \sigma(x^\top \theta + \mathcal{N}(0, \sigma^2))$   
where  $\sigma(x) := 1/(1 + \exp(-x))$ .

# Linear Neural Network

## Two examples

- **Regression:** Linear regression  $\triangleright y = x^\top \theta + \mathcal{N}(0, \sigma^2)$
- **Classification:** Logistic regression  $\triangleright p(y|x) = \sigma(x^\top \theta + \mathcal{N}(0, \sigma^2))$   
where  $\sigma(x) := 1/(1 + \exp(-x))$ .



**Figure 5:** (Left): Illustration of a linear neural network from Rosentblatt.  
(Right) Biological inspiration of artifical neurons from Warren McCulloch and Walter Pitts. From d21.ai/d21-en.pdf.

# Linear Neural Network

## Linear Neural Network

- **Regression:** Linear regression  $\triangleright f_\theta(x) := x^\top w + b$
- **Classification:** Logistic regression  $\triangleright f_\theta(x) := \sigma(x^\top w + b)$ .

with  $\theta = (w, b)$ .

# Linear Neural Network

## Linear Neural Network

- **Regression:** Linear regression  $\triangleright f_\theta(x) := x^\top w + b$
- **Classification:** Logistic regression  $\triangleright f_\theta(x) := \sigma(x^\top w + b)$ .

with  $\theta = (w, b)$ . ▷ Learning is finding the optimal  $\theta$ .

## Defining a loss using Maximum Likelihood Estimation

- **Regression:**  $\ell(y, f_\theta(x)) := (y - f_\theta(x))^2$
- **Classification:**  $\ell(y, f_\theta(x)) := -y \log(f_\theta(x)) - (1 - y) \log(1 - f_\theta(x))$

$$\mathcal{L}(\theta) := \frac{1}{n} \sum_{(x,y)} \ell(y, f_\theta(x))$$

# Linear Neural Network

## Linear Neural Network

- **Regression:** Linear regression  $\triangleright f_\theta(x) := x^\top w + b$
- **Classification:** Logistic regression  $\triangleright f_\theta(x) := \sigma(x^\top w + b)$ .

with  $\theta = (w, b)$ .  $\triangleright$  Learning is finding the optimal  $\theta$ .

## Defining a loss using Maximum Likelihood Estimation

- **Regression:**  $\ell(y, f_\theta(x)) := (y - f_\theta(x))^2$
- **Classification:**  $\ell(y, f_\theta(x)) := -y \log(f_\theta(x)) - (1 - y) \log(1 - f_\theta(x))$

$$\mathcal{L}(\theta) := \frac{1}{n} \sum_{(x,y)} \ell(y, f_\theta(x))$$

$\triangleright$  Learning is  $\hat{\theta} := \arg \min_{\theta} \mathcal{L}(\theta)$

# Linear Neural Network

## Linear Neural Network

- **Regression:** Linear regression  $\triangleright f_\theta(x) := x^\top w + b$
- **Classification:** Logistic regression  $\triangleright f_\theta(x) := \sigma(x^\top w + b)$ .

with  $\theta = (w, b)$ .  $\triangleright$  Learning is finding the optimal  $\theta$ .

## Defining a loss using Maximum Likelihood Estimation

- **Regression:**  $\ell(y, f_\theta(x)) := (y - f_\theta(x))^2$
- **Classification:**  $\ell(y, f_\theta(x)) := -y \log(f_\theta(x)) - (1 - y) \log(1 - f_\theta(x))$

$$\mathcal{L}(\theta) := \frac{1}{n} \sum_{(x,y)} \ell(y, f_\theta(x))$$

$\triangleright$  Learning is  $\hat{\theta} := \arg \min_{\theta} \mathcal{L}(\theta)$

We need more flexible learning process (optimization procedure)

## **Linear network**

---

### **Gradient Descent**

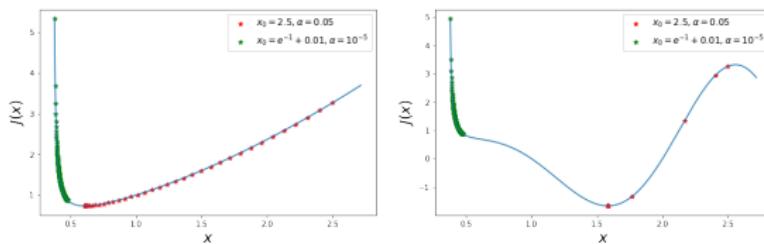
# Gradient Descent

## Gradient Descent (GD)

Let  $J$  a function from  $\mathbb{R}^d \rightarrow \mathbb{R}$ , Gradient Descent consists in localizing a **local minimum** of  $f$  as follows:

- Initialize  $x_0 \in \mathbb{R}^d$ .
- For a given number of iterations:

$$x_{t+1} \leftarrow x_t - \alpha(\nabla_x J)(x_t)$$



**Figure 6:** (Left) GD can minimize arbitrary complicated functions, here  $J(x) = x^2/(1 + \log(x))$ . (Right) Gradient descent with different initialization may lead to different minima, here  $J(x) = x^2 \sin(\pi x)/(1 + \log(x))$

# From Gradient Descent to Learning

## Momentum

$$x_{t+1} \leftarrow x_t - \alpha(t)(\nabla_x J)(x_t)$$

- The choice of momentum  $\alpha(t)$  is often a trade-off between **speed** and **accuracy** when finding a minimum.
- [ruder.io/optimizing-gradient-descent](http://ruder.io/optimizing-gradient-descent)

# From Gradient Descent to Learning

## Momentum

$$x_{t+1} \leftarrow x_t - \alpha(t)(\nabla_x J)(x_t)$$

- The choice of momentum  $\alpha(t)$  is often a trade-off between **speed** and **accuracy** when finding a minimum.
- [ruder.io/optimizing-gradient-descent](http://ruder.io/optimizing-gradient-descent)

## Learning by Gradient Descent

$$\theta_{t+1} \leftarrow \theta_t - \eta \alpha(t)(\nabla_\theta \mathcal{L})(\theta_t)$$

- $\eta$  is the **learning rate**.
- Each iteration needs a **full-scan** of the dataset

# From Gradient Descent to Learning

## Momentum

$$x_{t+1} \leftarrow x_t - \alpha(t)(\nabla_x J)(x_t)$$

- The choice of momentum  $\alpha(t)$  is often a trade-off between **speed** and **accuracy** when finding a minimum.
- [ruder.io/optimizing-gradient-descent](http://ruder.io/optimizing-gradient-descent)

## Learning by Gradient Descent

$$\theta_{t+1} \leftarrow \theta_t - \eta \alpha(t)(\nabla_\theta \mathcal{L})(\theta_t)$$

- $\eta$  is the **learning rate**.
- Each iteration needs a **full-scan** of the dataset ▷ **Stochastic Gradient Descent**.

## **Linear network**

---

**Stochastic Gradient Descent**

# Stochastic Gradient Descent

## Stochastic Gradient Descent

Stochastic Gradient Descent is Gradient Descent where each iteration is performed on a random subset of the dataset (typically of size between 16 and 256 samples).

- **Input:**  $\Theta$  parameters,  $\eta$  learning rate,  $b$  batch size,  $\mathcal{D}$  dataset.
- Initialize  $\theta_0 \in \Theta$ .
- For a given number of iterations:

# Stochastic Gradient Descent

## Stochastic Gradient Descent

Stochastic Gradient Descent is Gradient Descent where each iteration is performed on a random subset of the dataset (typically of size between 16 and 256 samples).

- **Input:**  $\Theta$  parameters,  $\eta$  learning rate,  $b$  batch size,  $\mathcal{D}$  dataset.
- Initialize  $\theta_0 \in \Theta$ .
- For a given number of iterations:
  - Sample  $\mathcal{B} \sim \mathcal{D}$  such that  $|\mathcal{B}| = b$ .

# Stochastic Gradient Descent

## Stochastic Gradient Descent

Stochastic Gradient Descent is Gradient Descent where each iteration is performed on a random subset of the dataset (typically of size between 16 and 256 samples).

- **Input:**  $\Theta$  parameters,  $\eta$  learning rate,  $b$  batch size,  $\mathcal{D}$  dataset.
- Initialize  $\theta_0 \in \Theta$ .
- For a given number of iterations:
  - Sample  $\mathcal{B} \sim \mathcal{D}$  such that  $|\mathcal{B}| = b$ .
  - Compute the batch loss  $\mathcal{L}^{\mathcal{B}}(\theta) := \frac{1}{b} \sum_{(x,y) \in \mathcal{B}} \ell(y, f_{\theta}(x))$ .

# Stochastic Gradient Descent

## Stochastic Gradient Descent

Stochastic Gradient Descent is Gradient Descent where each iteration is performed on a random subset of the dataset (typically of size between 16 and 256 samples).

- **Input:**  $\Theta$  parameters,  $\eta$  learning rate,  $b$  batch size,  $\mathcal{D}$  dataset.
- Initialize  $\theta_0 \in \Theta$ .
- For a given number of iterations:
  - Sample  $\mathcal{B} \sim \mathcal{D}$  such that  $|\mathcal{B}| = b$ .
  - Compute the batch loss  $\mathcal{L}^{\mathcal{B}}(\theta) := \frac{1}{b} \sum_{(x,y) \in \mathcal{B}} \ell(y, f_{\theta}(x))$ .
  - Update parameters according to:

$$\theta_{t+1} \leftarrow \theta_t - \eta \alpha(t) (\nabla_{\theta} \mathcal{L}^{\mathcal{B}})(\theta_t)$$

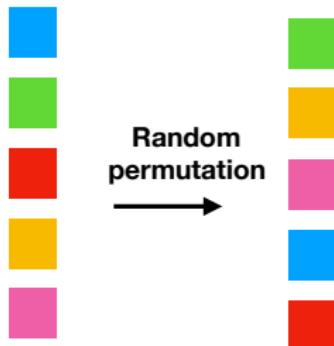
# Stochastic Gradient Descent

**Dataset**

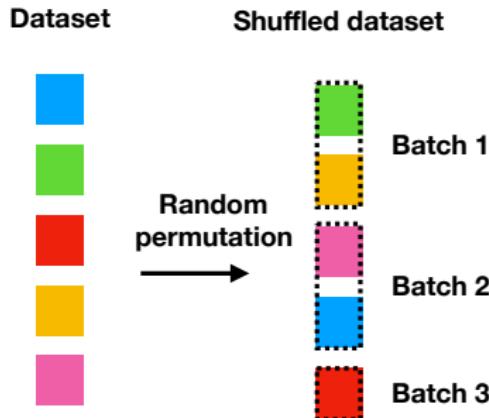


# Stochastic Gradient Descent

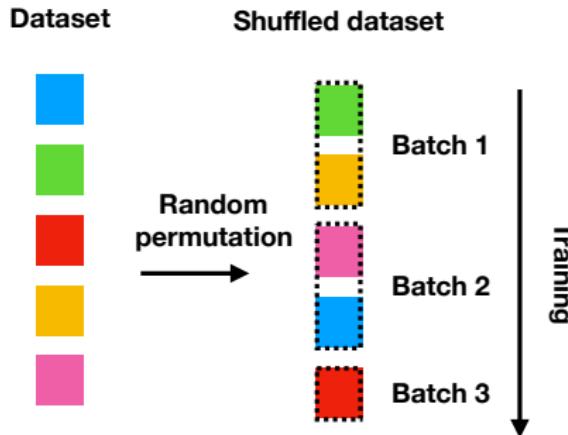
Dataset      Shuffled dataset



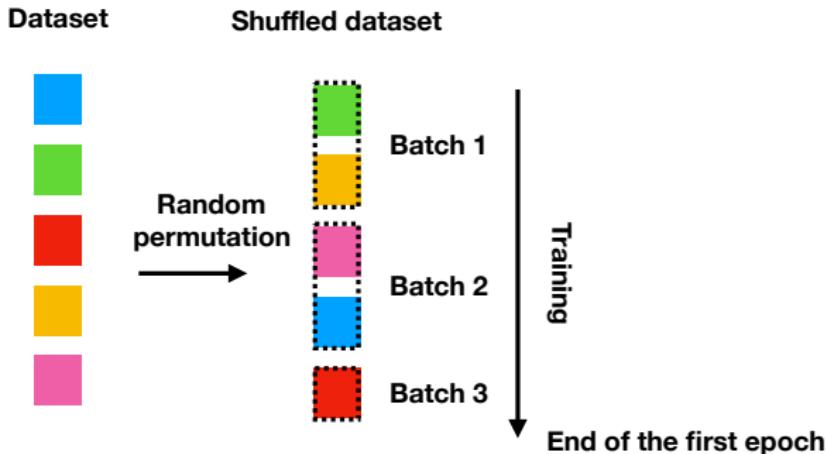
# Stochastic Gradient Descent



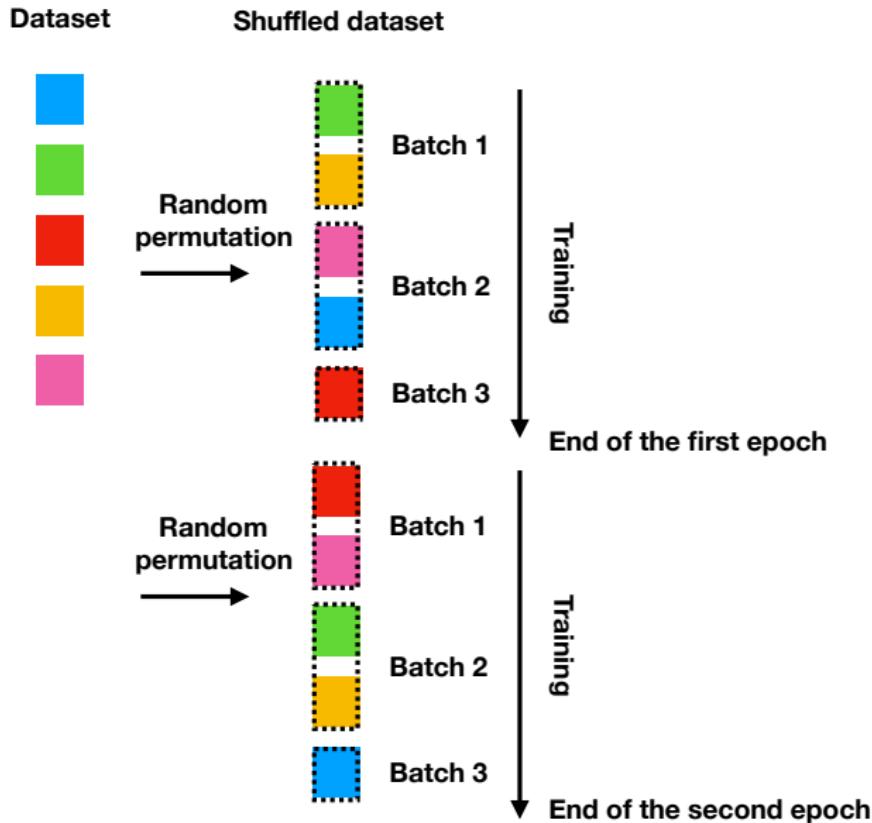
# Stochastic Gradient Descent



# Stochastic Gradient Descent



# Stochastic Gradient Descent



## **Linear network**

---

### **Limitations**

# Limitations of the Linear Neural Network

## Quick summary

- Linear neural network:  $f_\theta(x) := a(x^\top w + b)$

# Limitations of the Linear Neural Network

## Quick summary

- Linear neural network:  $f_\theta(x) := a(x^\top w + b)$
- $w$  are weights, and  $b$  is the bias of the layer,  $a$  is an activation function.

# Limitations of the Linear Neural Network

## Quick summary

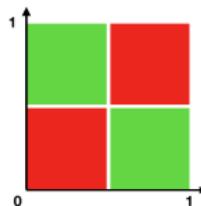
- Linear neural network:  $f_\theta(x) := a(x^\top w + b)$
- $w$  are weights, and  $b$  is the bias of the layer,  $a$  is an activation function.
- SGD allows to learn  $\theta := (w, b)$  for arbitrary loss.

# Limitations of the Linear Neural Network

## Quick summary

- Linear neural network:  $f_{\theta}(x) := a(x^T w + b)$
- $w$  are weights, and  $b$  is the bias of the layer,  $a$  is an activation function.
- SGD allows to learn  $\theta := (w, b)$  for arbitrary loss.

## Limitations: the overkill XOR function



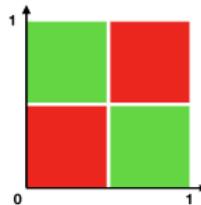
- $(0, 0) \mapsto 0, (1, 1) \mapsto 0, (1, 0) \mapsto 1$  and  $(0, 1) \mapsto 1$ .
- A linear neural network can not learn the XOR function.

# Limitations of the Linear Neural Network

## Quick summary

- Linear neural network:  $f_\theta(x) := a(x^\top w + b)$
- $w$  are weights, and  $b$  is the bias of the layer,  $a$  is an activation function.
- SGD allows to learn  $\theta := (w, b)$  for arbitrary loss.

## Limitations: the overkill XOR function



- $(0, 0) \mapsto 0, (1, 1) \mapsto 0, (1, 0) \mapsto 1$  and  $(0, 1) \mapsto 1$ .
  - A linear neural network can not learn the XOR function.
- ▷ Because we need a representation layer!

# Multi-Layer Perceptron (MLP)

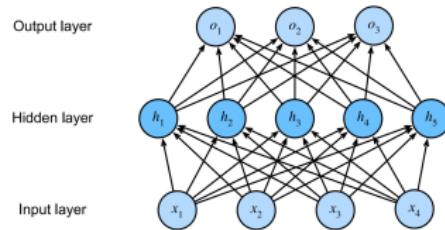
---

# **Multi-Layer Perceptron (MLP)**

---

**One-hidden layer Neural Network**

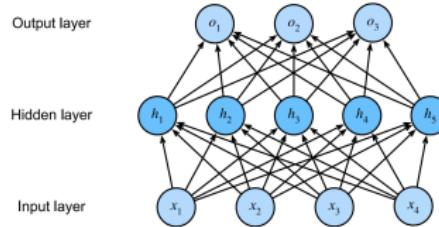
# One-hidden layer Neural Network



**Figure 7:** A MLP with one-hidden layer with five units. From d2l.ai/d2l-en.pdf.

**Forward pass:**  $x \rightarrow h \rightarrow o$

# One-hidden layer Neural Network

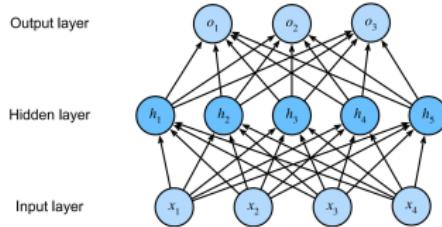


**Figure 7:** A MLP with one-hidden layer with five units. From d2l.ai/d2l-en.pdf.

**Forward pass:**  $x \rightarrow h \rightarrow o$

- 1st layer:  $h = a^{(1)} (\textcolor{orange}{x}^\top w^{(1)} + b^{(1)})$

# One-hidden layer Neural Network

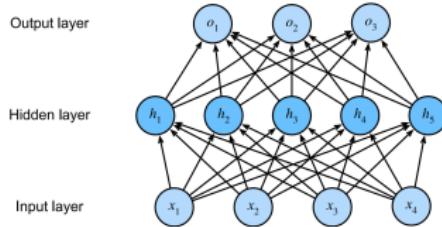


**Figure 7:** A MLP with one-hidden layer with five units. From d2l.ai/d2l-en.pdf.

**Forward pass:**  $x \rightarrow h \rightarrow o$

- 1st layer:  $h = a^{(1)} (\textcolor{orange}{x}^\top w^{(1)} + b^{(1)})$
- 2nd layer:  $\textcolor{green}{o} = a^{(2)} (\textcolor{blue}{h}^\top w^{(2)} + b^{(2)})$

# One-hidden layer Neural Network

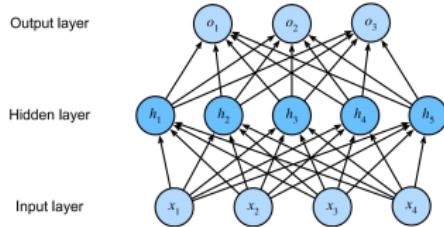


**Figure 7:** A MLP with one-hidden layer with five units. From d2l.ai/d2l-en.pdf.

**Forward pass:**  $\textcolor{orange}{x} \longrightarrow h \longrightarrow o$

- 1st layer:  $\textcolor{blue}{h} = a^{(1)} (\textcolor{orange}{x}^\top w^{(1)} + b^{(1)})$
- 2nd layer:  $\textcolor{green}{o} = a^{(2)} (\textcolor{blue}{h}^\top w^{(2)} + b^{(2)})$
- $\theta := (w^{(1)}, b^{(1)}, w^{(2)}, b^{(2)})$  defines  $\textcolor{green}{o} = f_\theta(\textcolor{orange}{x})$

# One-hidden layer Neural Network



**Figure 7:** A MLP with one-hidden layer with five units. From d2l.ai/d2l-en.pdf.

**Forward pass:**  $\textcolor{orange}{x} \longrightarrow h \longrightarrow o$

- 1st layer:  $\textcolor{blue}{h} = a^{(1)} (\textcolor{orange}{x}^\top w^{(1)} + b^{(1)})$
- 2nd layer:  $\textcolor{green}{o} = a^{(2)} (\textcolor{blue}{h}^\top w^{(2)} + b^{(2)})$
- $\theta := (w^{(1)}, b^{(1)}, w^{(2)}, b^{(2)})$  defines  $\textcolor{green}{o} = f_\theta(\textcolor{orange}{x})$

▷  $h$  is the (hidden) representation of  $\textcolor{orange}{x}$ !

# Universal approximation theorem

## Forward pass

- 1st layer:  $h = a^{(1)} (\mathbf{x}^\top \mathbf{w}^{(1)} + b^{(1)})$
- 2nd layer:  $o = a^{(2)} (\mathbf{h}^\top \mathbf{w}^{(2)} + b^{(2)})$

# Universal approximation theorem

## Forward pass

- 1st layer:  $h = a^{(1)}(x^\top w^{(1)} + b^{(1)})$
- 2nd layer:  $o = a^{(2)}(h^\top w^{(2)} + b^{(2)})$

## The role of non-linearity

If  $a^{(1)}$  is the identity function  $\Rightarrow$  the linear network. Typical  $a^{(1)}$ :

- Sigmoid:  $\sigma(x) := \frac{1}{1+\exp(-x)}$
- Tangent-hyperbolic:  $\text{Tanh}(x) := \frac{e^{-x} - e^x}{e^{-x} + e^x}$
- Rectified Linear Unit:  $\text{ReLU}(x) := \max(0, x)$

# Universal approximation theorem

## Forward pass

- 1st layer:  $h = a^{(1)}(x^\top w^{(1)} + b^{(1)})$
- 2nd layer:  $o = a^{(2)}(h^\top w^{(2)} + b^{(2)})$

## The role of non-linearity

If  $a^{(1)}$  is the identity function  $\Rightarrow$  the linear network. Typical  $a^{(1)}$ :

- Sigmoid:  $\sigma(x) := \frac{1}{1+\exp(-x)}$
- Tangente-hyperbolic:  $\text{Tanh}(x) := \frac{e^{-x}-e^x}{e^{-x}+e^x}$
- Rectified Linear Unit:  $\text{ReLU}(x) := \max(0, x)$

## Universal approximation theorem

For any continuous function on a compact, it exists a One-hidden layer network with continuous, bounded, non-constant activation, which achieves uniformly an arbitrary small error on the compact.

# Universal approximation theorem

## Forward pass

- 1st layer:  $h = a^{(1)}(x^\top w^{(1)} + b^{(1)})$
- 2nd layer:  $o = a^{(2)}(h^\top w^{(2)} + b^{(2)})$

## The role of non-linearity

If  $a^{(1)}$  is the identity function  $\Rightarrow$  the linear network. Typical  $a^{(1)}$ :

- Sigmoid:  $\sigma(x) := \frac{1}{1+\exp(-x)}$
- Tangente-hyperbolic:  $\text{Tanh}(x) := \frac{e^{-x}-e^x}{e^{-x}+e^x}$
- Rectified Linear Unit:  $\text{ReLU}(x) := \max(0, x)$

## Universal approximation theorem

For any continuous function on a compact, it exists a One-hidden layer network with continuous, bounded, non-constant activation, which achieves uniformly an arbitrary small error on the compact. ▷ Increases the number of units (dimension) of  $h$ !

# Multi-Layer Perceptron (MLP)

---

Training a MLP by Back-propagation

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.
- It is based on the **chain rule** for computing derivatives.

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.
- It is based on the **chain rule** for computing derivatives.
- Roughly, for computing the gradient of a layer, we use gradient of upper layers

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.
- It is based on the **chain rule** for computing derivatives.
- Roughly, for computing the gradient of a layer, we use gradient of upper layers ▷ That's why we "*Back-propagate errors*".

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.
- It is based on the **chain rule** for computing derivatives.
- Roughly, for computing the gradient of a layer, we use gradient of upper layers ▷ That's why we "*Back-propagate errors*".
- Usually we distinguish two passes:

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.
- It is based on the **chain rule** for computing derivatives.
- Roughly, for computing the gradient of a layer, we use gradient of upper layers ▷ That's why we "*Back-propagate errors*".
- Usually we distinguish two passes:
  - **forward pass:** From inputs to outputs (network as a function)

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.
- It is based on the **chain rule** for computing derivatives.
- Roughly, for computing the gradient of a layer, we use gradient of upper layers ▷ That's why we "*Back-propagate errors*".
- Usually we distinguish two passes:
  - **forward pass:** From inputs to outputs (network as a function)
  - **backward pass:** Gradient from outputs to inputs (chain rule)

# Back-propagation

## Backpropagation demystified

- Backprop is a memory efficient algorithm for computing gradients of MLP's parameters.
- It is based on the **chain rule** for computing derivatives.
- Roughly, for computing the gradient of a layer, we use gradient of upper layers ▷ That's why we "*Back-propagate errors*".
- Usually we distinguish two passes:
  - **forward pass:** From inputs to outputs (network as a function)
  - **backward pass:** Gradient from outputs to inputs (chain rule)

## Chain rule

Let the computational graph  $x \rightarrow y \rightarrow z$

$$\frac{\partial z}{\partial x} = \text{prod} \left( \frac{\partial z}{\partial y}, \frac{\partial y}{\partial x} \right)$$

where prod is the multiplication if variables are real, matrix product if vectors, ...

# Backprop in a One-hidden layer MLP

## (Bias free) Forward pass

- 1st layer:  $h = a^{(1)} (\mathbf{x}^\top \mathbf{w}^{(1)})$
- 2nd layer:  $o = a^{(2)} (\mathbf{h}^\top \mathbf{w}^{(2)})$
- Loss:  $\ell(o)$

# Backprop in a One-hidden layer MLP

## (Bias free) Forward pass

- 1st layer:  $h = a^{(1)} (x^\top w^{(1)})$
- 2nd layer:  $o = a^{(2)} (h^\top w^{(2)})$
- Loss:  $\ell(o)$
- **Gradient wrt  $w^{(2)}$ :**  $\frac{\partial \ell}{\partial w^{(2)}}$   
$$\frac{\partial \ell}{\partial w^{(2)}} = \frac{\partial \ell}{\partial o} \underbrace{\frac{\partial o}{\partial w^{(2)}}}_{G_o} = \underbrace{\frac{\partial \ell}{\partial o}}_{G_o} h^\top \underbrace{a^{(2)'}(h^\top w^{(2)} + b)}_{G_2} = G_o(h^\top G_2)$$

# Backprop in a One-hidden layer MLP

## (Bias free) Forward pass

- 1st layer:  $h = \color{red}{a^{(1)}} (\color{orange}{x}^\top w^{(1)})$
- 2nd layer:  $\color{green}{o} = \color{red}{a^{(2)}} (\color{blue}{h}^\top w^{(2)})$
- Loss:  $\ell(\color{green}{o})$
- **Gradient wrt  $w^{(2)}$ :**  $\frac{\partial \ell}{\partial w^{(2)}}$   
$$\frac{\partial \ell}{\partial w^{(2)}} = \frac{\partial \ell}{\partial o} \frac{\partial o}{\partial w^{(2)}} = \underbrace{\frac{\partial \ell}{\partial o}}_{G_o} h^\top \underbrace{a^{(2)'}(h^\top w^{(2)} + b)}_{G_2} = G_o(h^\top G_2)$$
- **Gradient wrt  $w^{(1)}$ :**  $\frac{\partial \ell}{\partial w^{(1)}}$

# Backprop in a One-hidden layer MLP

## (Bias free) Forward pass

- 1st layer:  $h = a^{(1)}(x^\top w^{(1)})$
- 2nd layer:  $o = a^{(2)}(h^\top w^{(2)})$
- Loss:  $\ell(o)$

- Gradient wrt  $w^{(2)}$ :  $\frac{\partial \ell}{\partial w^{(2)}}$

$$\frac{\partial \ell}{\partial w^{(2)}} = \frac{\partial \ell}{\partial o} \frac{\partial o}{\partial w^{(2)}} = \underbrace{\frac{\partial \ell}{\partial o}}_{G_o} h^\top \underbrace{a^{(2)'}(h^\top w^{(2)} + b)}_{G_2} = G_o(h^\top G_2)$$

- Gradient wrt  $w^{(1)}$ :  $\frac{\partial \ell}{\partial w^{(1)}}$

$$\frac{\partial \ell}{\partial w^{(1)}} = \frac{\partial \ell}{\partial o} \frac{\partial o}{\partial h} \frac{\partial h}{\partial w^{(1)}} = G_o \left( a^{(2)'}(h^\top w^{(2)} + b)^\top w^{(2)} \right) \frac{\partial h}{\partial w^{(1)}}$$

# Backprop in a One-hidden layer MLP

## (Bias free) Forward pass

- 1st layer:  $h = a^{(1)}(x^\top w^{(1)})$
- 2nd layer:  $o = a^{(2)}(h^\top w^{(2)})$
- Loss:  $\ell(o)$

- Gradient wrt  $w^{(2)}$ :  $\frac{\partial \ell}{\partial w^{(2)}}$

$$\frac{\partial \ell}{\partial w^{(2)}} = \frac{\partial \ell}{\partial o} \frac{\partial o}{\partial w^{(2)}} = \underbrace{\frac{\partial \ell}{\partial o}}_{G_o} h^\top \underbrace{a^{(2)'}(h^\top w^{(2)} + b)}_{G_2} = G_o(h^\top G_2)$$

- Gradient wrt  $w^{(1)}$ :  $\frac{\partial \ell}{\partial w^{(1)}}$

$$\frac{\partial \ell}{\partial w^{(1)}} = \frac{\partial \ell}{\partial o} \frac{\partial o}{\partial h} \frac{\partial h}{\partial w^{(1)}} = G_o \left( a^{(2)'}(h^\top w^{(2)} + b)^\top w^{(2)} \right) \frac{\partial h}{\partial w^{(1)}}$$

$$\frac{\partial \ell}{\partial w^{(1)}} = G_o(G_2^\top w^{(2)})(x^\top G_1)$$

# Backprop in a One-hidden layer MLP

## (Bias free) Forward pass

- 1st layer:  $h = a^{(1)}(x^\top w^{(1)})$
- 2nd layer:  $o = a^{(2)}(h^\top w^{(2)})$
- Loss:  $\ell(o)$

- Gradient wrt  $w^{(2)}$ :  $\frac{\partial \ell}{\partial w^{(2)}}$

$$\frac{\partial \ell}{\partial w^{(2)}} = \frac{\partial \ell}{\partial o} \frac{\partial o}{\partial w^{(2)}} = \underbrace{\frac{\partial \ell}{\partial o}}_{G_o} h^\top \underbrace{a^{(2)'}(h^\top w^{(2)} + b)}_{G_2} = G_o(h^\top G_2)$$

- Gradient wrt  $w^{(1)}$ :  $\frac{\partial \ell}{\partial w^{(1)}}$

$$\frac{\partial \ell}{\partial w^{(1)}} = \frac{\partial \ell}{\partial o} \frac{\partial o}{\partial h} \frac{\partial h}{\partial w^{(1)}} = G_o \left( a^{(2)'}(h^\top w^{(2)} + b)^\top w^{(2)} \right) \frac{\partial h}{\partial w^{(1)}}$$

$$\frac{\partial \ell}{\partial w^{(1)}} = G_o(G_2^\top w^{(2)})(x^\top G_1)$$

▷ Just one additional gradient to compute  $G_1 := \frac{\partial h}{\partial w^{(1)}}$

## **Training a neural network**

---

# Training a neural network

---

## Overview

## Overview of usual steps for training a neural network

1. Define your neural network  $\triangleright (f_\theta)_{\theta \in \Theta}$

## Overview of usual steps for training a neural network

1. Define your neural network  $\triangleright (f_\theta)_{\theta \in \Theta}$
2. Define your dataset  $\triangleright \text{Normalization, Augmentation, ...}$

## Overview of usual steps for training a neural network

1. Define your neural network  $\triangleright (f_\theta)_{\theta \in \Theta}$
2. Define your dataset  $\triangleright$  *Normalization, Augmentation, ...*
3. Define your loss

# Overview of usual steps for training a neural network

1. Define your neural network  $\triangleright (f_\theta)_{\theta \in \Theta}$
2. Define your dataset  $\triangleright$  *Normalization, Augmentation, ...*
3. Define your loss
4. Regularize your network

## Overview of usual steps for training a neural network

1. Define your neural network  $\triangleright (f_\theta)_{\theta \in \Theta}$
2. Define your dataset  $\triangleright \text{Normalization, Augmentation, ...}$
3. Define your loss
4. Regularize your network
5. Define your optimizer  $\triangleright \text{What kind of momentum you want to use}$

# Overview of usual steps for training a neural network

1. Define your neural network  $\triangleright (f_\theta)_{\theta \in \Theta}$
2. Define your dataset  $\triangleright$  *Normalization, Augmentation, ...*
3. Define your loss
4. Regularize your network
5. Define your optimizer  $\triangleright$  *What kind of momentum you want to use*
6. Perform SGD until you have reached a stopping criterion (**Callback**)

# Training a neural network

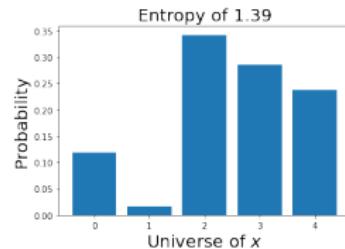
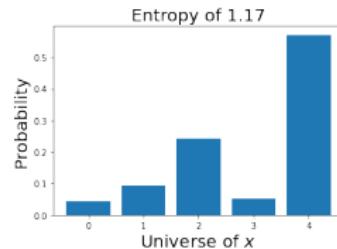
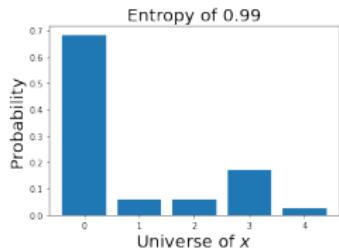
---

Defining a loss

# Basics of Information theory

Let  $p$  and  $q$  two distributions.

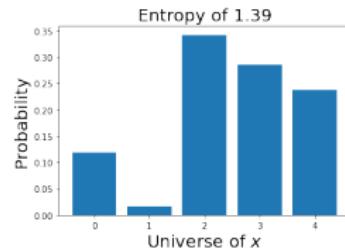
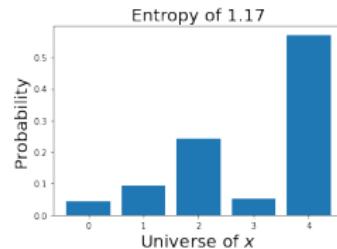
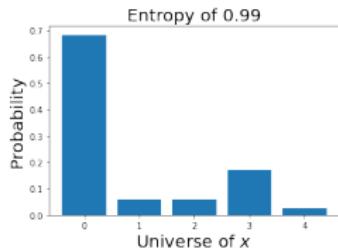
- **Entropy:**  $H(p) := \mathbb{E}_{x \sim p}[-\log p(x)]$



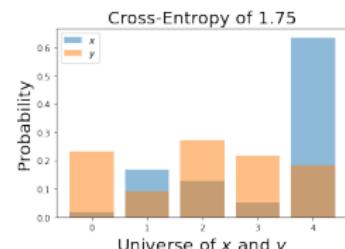
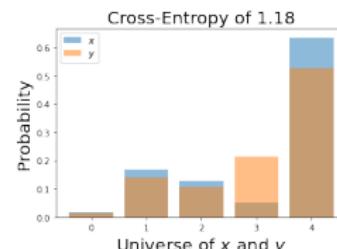
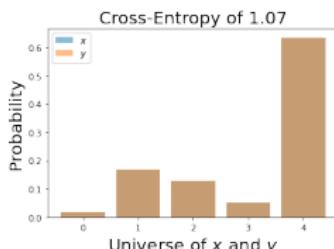
# Basics of Information theory

Let  $p$  and  $q$  two distributions.

- **Entropy:**  $H(p) := \mathbb{E}_{x \sim p}[-\log p(x)]$



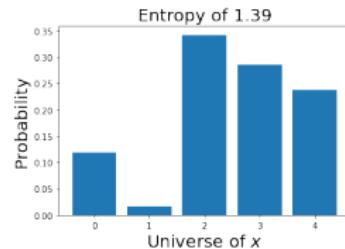
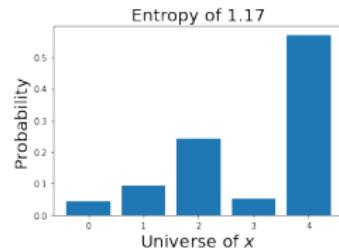
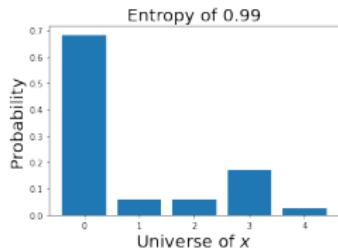
- **Cross-Entropy:**  $H(p, q) := \mathbb{E}_{x \sim p}[-\log q(x)]$



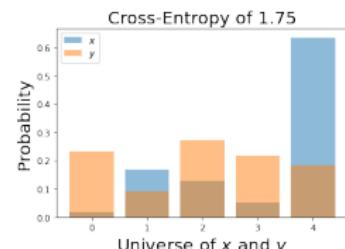
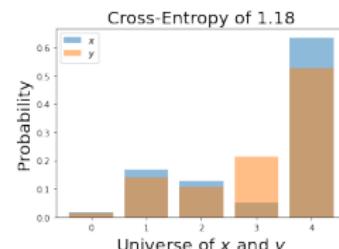
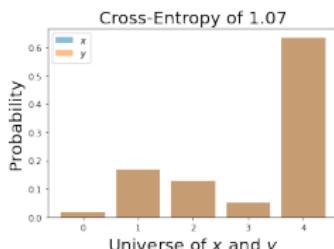
# Basics of Information theory

Let  $p$  and  $q$  two distributions.

- **Entropy:**  $H(p) := \mathbb{E}_{x \sim p}[-\log p(x)]$



- **Cross-Entropy:**  $H(p, q) := \mathbb{E}_{x \sim p}[-\log q(x)]$



▷ Given  $p$ , the cross-entropy is minimal when  $q = p$

# Binary Cross-Entropy

## Binary cross-entropy

- Particular case when the universe is binary (positive and negative)

$$p := p(X = 1), q := q(X = 1)$$

# Binary Cross-Entropy

## Binary cross-entropy

- Particular case when the universe is binary (positive and negative)

$$p := p(X = 1), q := q(X = 1)$$

- Given  $p$  and  $q$ , the (binary-)cross-entropy is:

$$H(p, q) = -p \log q - (1 - p) \log(1 - q)$$

# Binary Cross-Entropy

## Binary cross-entropy

- Particular case when the universe is binary (positive and negative)

$$p := p(X = 1), q := q(X = 1)$$

- Given  $p$  and  $q$ , the (binary-)cross-entropy is:

$$H(p, q) = -p \log q - (1 - p) \log(1 - q)$$

- $f_\theta$  is a neural net such that:
  - $f_\theta(x) \in [0, 1] \triangleright$  uses  $\sigma(\cdot)$  for outputs.

# Binary Cross-Entropy

## Binary cross-entropy

- Particular case when the universe is binary (positive and negative)

$$p := p(X = 1), q := q(X = 1)$$

- Given  $p$  and  $q$ , the (binary-)cross-entropy is:

$$H(p, q) = -p \log q - (1 - p) \log(1 - q)$$

- $f_\theta$  is a neural net such that:
  - $f_\theta(x) \in [0, 1] \triangleright$  uses  $\sigma(\cdot)$  for outputs.
  - $f_\theta(x)$  is designed to fit  $p(Y = 1|X = x)$  by minimizing:

$$\ell(y, f_\theta(x)) := -y \log f_\theta(x) - (1 - y) \log(1 - f_\theta(x))$$

# (Categorical) Cross-Entropy

## Categorical cross-entropy

- Let  $(x, y)$  a training data point.
- $y$  is a categorical variable  $\triangleright \text{cat, dog, cow, plane, ...}$

# (Categorical) Cross-Entropy

## Categorical cross-entropy

- Let  $(x, y)$  a training data point.
- $y$  is a categorical variable  $\triangleright \text{cat, dog, cow, plane, ...}$
- Let  $\mathcal{C}$  the number of categories and  $c = \underbrace{\{0, \dots, 0, 1, 0, \dots, 0\}}_{c \text{ dimensions}}$  is a **one-hot vectorization** of  $y$ .

# (Categorical) Cross-Entropy

## Categorical cross-entropy

- Let  $(x, y)$  a training data point.
- $y$  is a categorical variable  $\triangleright \text{cat, dog, cow, plane, ...}$
- Let  $\mathcal{C}$  the number of categories and  $c = \underbrace{\{0, \dots, 0, 1, 0, \dots, 0\}}_{c \text{ dimensions}}$  is a **one-hot vectorization** of  $y$ .
- $f_\theta(x) \rightarrow [0, 1]^\mathcal{C}$  maps  $x$  to  $\mathcal{C}$ -dimensional space.

# (Categorical) Cross-Entropy

## Categorical cross-entropy

- Let  $(x, y)$  a training data point.
- $y$  is a categorical variable  $\triangleright \text{cat, dog, cow, plane, ...}$
- Let  $\mathcal{C}$  the number of categories and  $c = \underbrace{\{0, \dots, 0, 1, 0, \dots, 0\}}_{c \text{ dimensions}}$  is a **one-hot vectorization** of  $y$ .
- $f_\theta(x) \rightarrow [0, 1]^\mathcal{C}$  maps  $x$  to  $\mathcal{C}$ -dimensional space.
- $f_\theta(x)_c$  is designed to fit  $p(c|X=x)$  i.e.  $x$  belongs to the  $c$ -th class by minimizing:

$$\ell(y, f_\theta(x)) := -\log f_\theta(x)_{c(y)}$$

## Softmax-layer

$$\ell(y, f_\theta(x)) := -\log f_\theta(x)_{c(y)}$$

### Softmax-layer

- Maps an arbitrary dimensional vector  $z \in \mathbb{R}^C$  into a probability distribution over the dimensions,

## Softmax-layer

$$\ell(y, f_\theta(x)) := -\log f_\theta(x)_{c(y)}$$

### Softmax-layer

- Maps an arbitrary dimensional vector  $z \in \mathbb{R}^C$  into a probability distribution over the dimensions,
- with higher probabilities to higher values of  $z$ ,

## Softmax-layer

$$\ell(y, f_\theta(x)) := -\log f_\theta(x)_{c(y)}$$

### Softmax-layer

- Maps an arbitrary dimensional vector  $z \in \mathbb{R}^C$  into a probability distribution over the dimensions,
- with higher probabilities to higher values of  $z$ ,
- ultimately, the higher coordinate has a probability close to 1 and others are pushed to 0,

## Softmax-layer

$$\ell(y, f_\theta(x)) := -\log f_\theta(x)_{c(y)}$$

### Softmax-layer

- Maps an arbitrary dimensional vector  $z \in \mathbb{R}^C$  into a probability distribution over the dimensions,
- with higher probabilities to higher values of  $z$ ,
- ultimately, the higher coordinate has a probability close to 1 and others are pushed to 0,
- it is differentiable approximation of the arg max:

## Softmax-layer

$$\ell(y, f_\theta(x)) := -\log f_\theta(x)_{c(y)}$$

### Softmax-layer

- Maps an arbitrary dimensional vector  $z \in \mathbb{R}^C$  into a probability distribution over the dimensions,
- with higher probabilities to higher values of  $z$ ,
- ultimately, the higher coordinate has a probability close to 1 and others are pushed to 0,
- it is differentiable approximation of the arg max:

$$\text{Softmax}(z) := \frac{1}{\sum_c e^{z_c}} (e^{z_1}, \dots, e^{z_c})$$

# Training a neural network

---

Regularizing neural networks

## $L^2$ = Ridge = Weight decay = Tikhonov

The  $L^2$  penalty is the most common regularization of models:

$$\mathcal{L}_{\text{reg}}(\theta) = \mathcal{L}(\theta) + \lambda ||\theta||^2$$

### Linear Regression

- $X \in \mathbb{R}^{n \times p}$ : (samples, features),  $Y \in \mathbb{R}^{n \times 1}$ : (samples, value),
- **No-regularization:**  $||Y - X^\top \theta||^2$

## $L^2$ = Ridge = Weight decay = Tikhonov

The  $L^2$  penalty is the most common regularization of models:

$$\mathcal{L}_{\text{reg}}(\theta) = \mathcal{L}(\theta) + \lambda ||\theta||^2$$

### Linear Regression

- $X \in \mathbb{R}^{n \times p}$ : (samples, features),  $Y \in \mathbb{R}^{n \times 1}$ : (samples, value),
- **No-regularization:**  $||Y - X^\top \theta||^2$

$$\hat{\theta} = (X^\top X)^{-1}(X^\top Y)$$

## $L^2$ = Ridge = Weight decay = Tikhonov

The  $L^2$  penalty is the most common regularization of models:

$$\mathcal{L}_{\text{reg}}(\theta) = \mathcal{L}(\theta) + \lambda ||\theta||^2$$

### Linear Regression

- $X \in \mathbb{R}^{n \times p}$ : (samples, features),  $Y \in \mathbb{R}^{n \times 1}$ : (samples, value),
- **No-regularization:**  $||Y - X^\top \theta||^2$

$$\hat{\theta} = (X^\top X)^{-1}(X^\top Y)$$

- **$L^2$ -regularization:**  $||Y - X^\top \theta||^2 + \lambda ||\theta||^2$

## $L^2$ = Ridge = Weight decay = Tikhonov

The  $L^2$  penalty is the most common regularization of models:

$$\mathcal{L}_{\text{reg}}(\theta) = \mathcal{L}(\theta) + \lambda ||\theta||^2$$

### Linear Regression

- $X \in \mathbb{R}^{n \times p}$ : (samples, features),  $Y \in \mathbb{R}^{n \times 1}$ : (samples, value),
- **No-regularization:**  $||Y - X^\top \theta||^2$

$$\hat{\theta} = (X^\top X)^{-1}(X^\top Y)$$

- **$L^2$ -regularization:**  $||Y - X^\top \theta||^2 + \lambda ||\theta||^2$

$$\hat{\theta} = (X^\top X + \lambda I)^{-1}(X^\top Y)$$

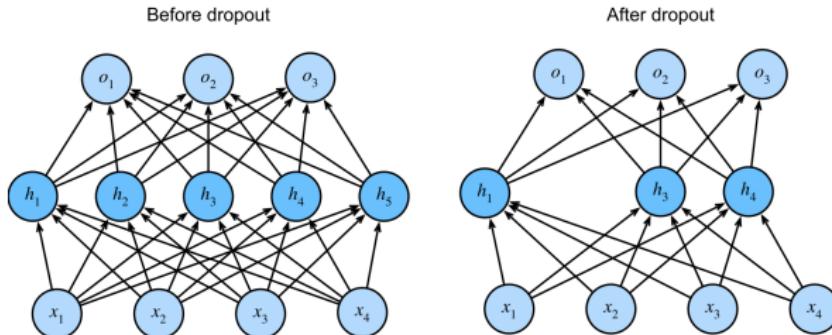
## Training a neural network

---

Dropout regularization

# Dropout: Ensembling of neural networks

Dropout consists in randomly deleting some units during training.

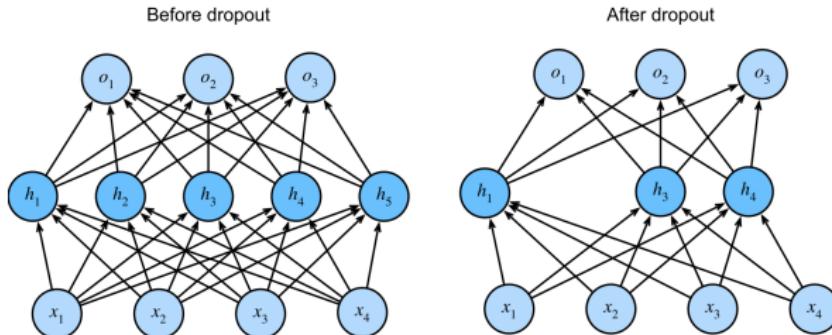


**Figure 8:** MLP before and after dropout. From d21.ai/d21-en.pdf.

$$\text{Dropout}_p(h)_i = \begin{cases} 0 & \text{with probability } p \\ \frac{h_i}{1-p} & \text{with probability } 1 - p \end{cases}$$

# Dropout: Ensembling of neural networks

Dropout consists in randomly deleting some units during training.



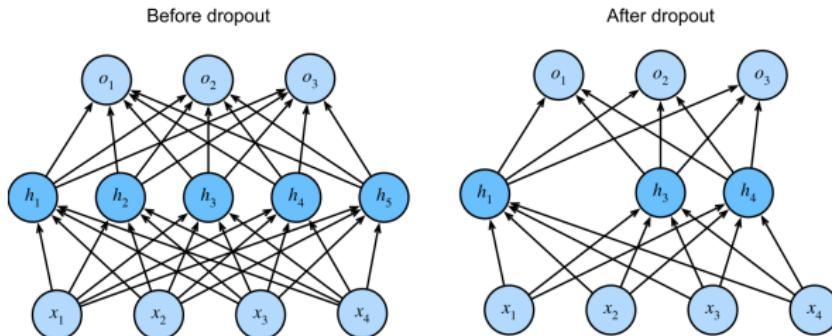
**Figure 8:** MLP before and after dropout. From d21.ai/d21-en.pdf.

$$\text{Dropout}_p(h)_i = \begin{cases} 0 & \text{with probability } p \\ \frac{h_i}{1-p} & \text{with probability } 1 - p \end{cases}$$

- Ensures an unbiased layer i.e.,  $h = \mathbb{E}_p[\text{Dropout}_p(h)]$

# Dropout: Ensembling of neural networks

Dropout consists in randomly deleting some units during training.



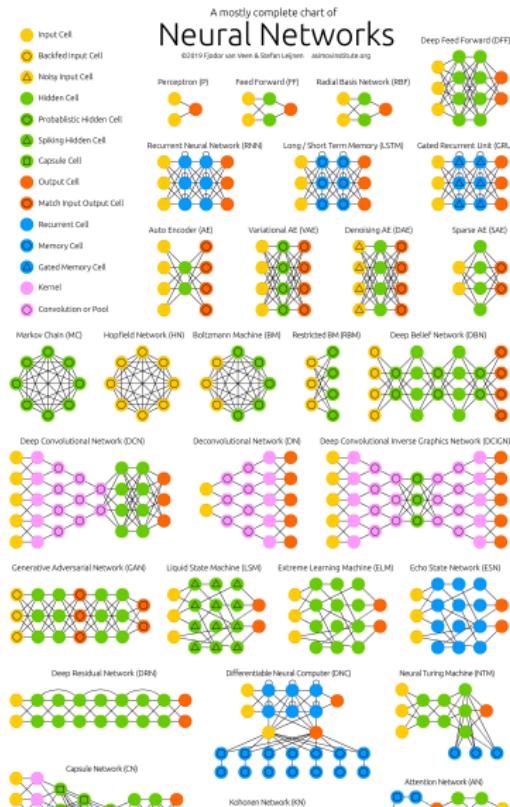
**Figure 8:** MLP before and after dropout. From d21.ai/d21-en.pdf.

$$\text{Dropout}_p(h)_i = \begin{cases} 0 & \text{with probability } p \\ \frac{h_i}{1-p} & \text{with probability } 1 - p \end{cases}$$

- Ensures an unbiased layer i.e.,  $h = \mathbb{E}_p[\text{Dropout}_p(h)]$
- **At test-time,  $p = 0$ .**

# Deep neural model zoo

<https://www.asimovinstitute.org/neural-network-zoo/>



# Training a neural network

---

## The Needs

# Deep Learning : Needed resources

Huge annotated data



---

<sup>2</sup>Unbiased Look at Dataset Bias :

<http://people.csail.mit.edu/torralba/research/bias/>

# Deep Learning : Needed resources

Huge annotated data



But annotation not prevent from bias in data <sup>2</sup>,

---

<sup>2</sup>Unbiased Look at Dataset Bias :

<http://people.csail.mit.edu/torralba/research/bias/>

# Deep Learning : Needed ressources

Computing and storage ressources

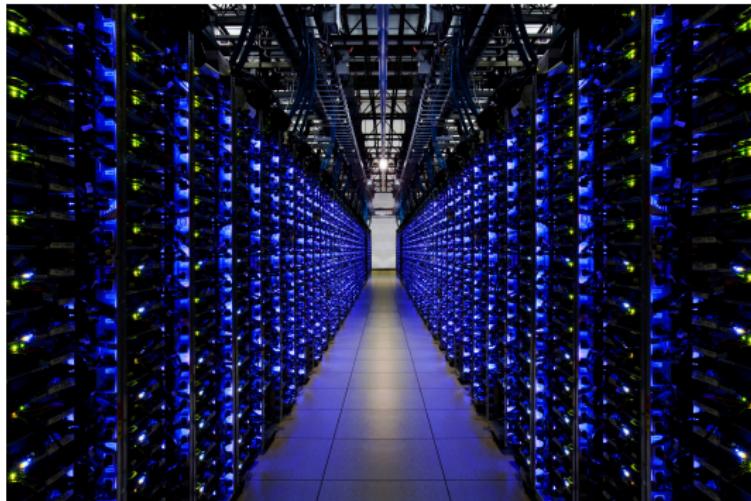


---

<sup>3</sup>[https://www.technologyreview.com/f/614056/  
ai-research-has-an-environment-climate-toll/?utm\\_campaign=site\\_visitor.  
unpaid.engagement&utm\\_source=twitter&utm\\_medium=tr\\_social](https://www.technologyreview.com/f/614056/ai-research-has-an-environment-climate-toll/?utm_campaign=site_visitor.unpaid.engagement&utm_source=twitter&utm_medium=tr_social)

# Deep Learning : Needed ressources

Computing and storage ressources



To a green AI<sup>3</sup>

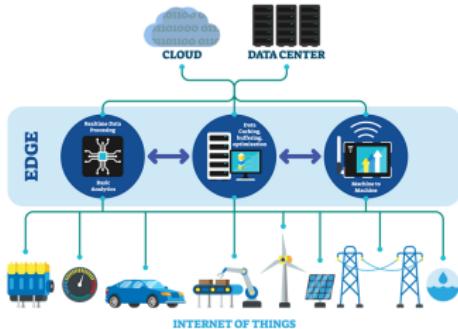
<sup>3</sup>[https://www.technologyreview.com/f/614056/ai-research-has-an-environment-climate-toll/?utm\\_campaign=site\\_visitor.unpaid.engagement&utm\\_source=twitter&utm\\_medium=tr\\_social](https://www.technologyreview.com/f/614056/ai-research-has-an-environment-climate-toll/?utm_campaign=site_visitor.unpaid.engagement&utm_source=twitter&utm_medium=tr_social)

## Deep Learning : Needed ressources

Computer science and IT.



# Edge Computing



## **Deep Learning : where are we ?**

---

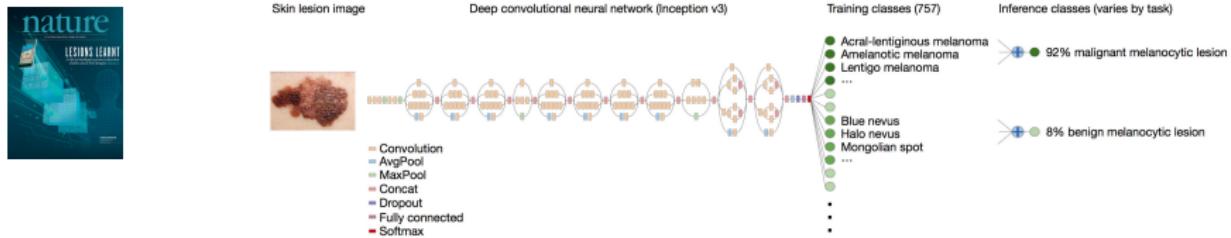
## **Deep Learning : where are we ?**

---

**Highly performant deep Learning :  
More performant than humans?**

# Let's look at a model that claims it!

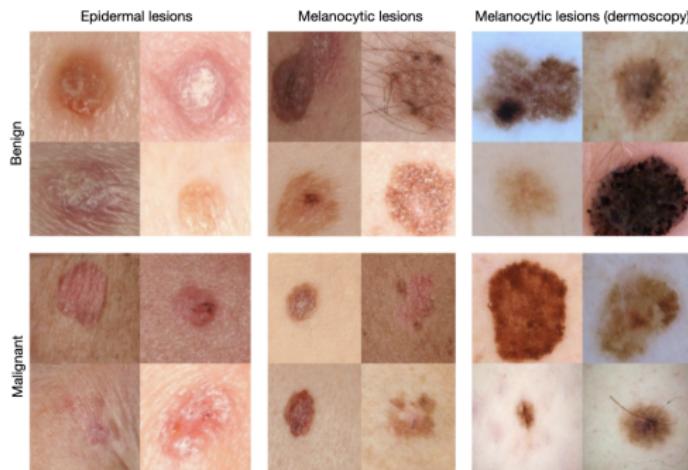
Dermatologist-level classification of skin cancer with deep neural networks [Esteva et al, 17]



# Let's look at a model that claims it!

## The task

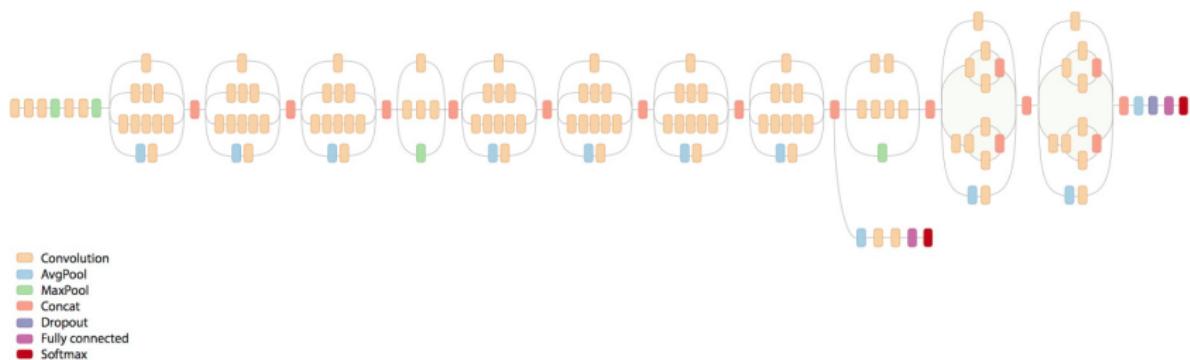
Skin cancer detection : a fine-grained visual recognition task ( 2,032 different diseases, fine-grained variability) but evaluation is done on two binary classification tasks (*keratinocyte carcinomas versus benign seborrheic keratoses; and malignant melanomas versus benign nevi*).



# Let's look at a model that claims it!

## The model

Inception v3 CNN architecture, pre-trained on ImageNet and fine-tuned on the target dataset<sup>4</sup>



---

<sup>4</sup><https://ai.googleblog.com/2016/03/train-your-own-image-classifier-with.html>

## Dermatologist-level classification of skin cancer with deep neural networks [Esteva et al, 17]

## The data

- 129,450 clinical images, including 3,374 dermoscopy images, **annotated by dermatologists**.
  - Images are organized in a tree-structured taxonomy of 2,032 diseases, derived by dermatologists using a bottom-up procedure: individual diseases, initialized as leaf nodes, were merged based on clinical and visual similarity, until the entire structure was connected.
  - Training dataset ; 127,463 training and validation images and testing dataset : 1,942 biopsy-labelled test images with no-overlap (same lesion, multiple viewpoints)



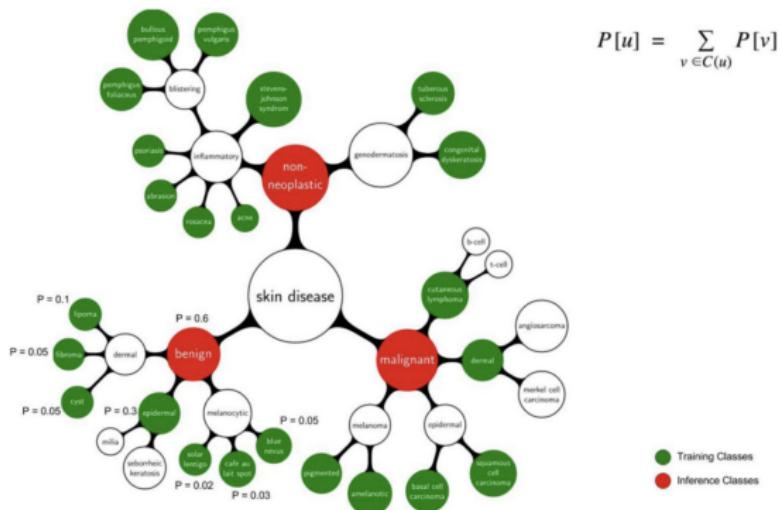
## Dermatologist-level classification of skin cancer with deep neural networks [Esteva et al, 17]

### **From Disease to training classes**

Disease partitioning algorithm : partitions individual diseases into training classes whose individual diseases are clinically and visually similar and with constraints on the size of the class ( $\text{maxClassSize} = 1,000$ ) : **disease partition of 757 classes.**

# Dermatologist-level classification of skin cancer with deep neural networks [Esteva et al, 17]

From training classes to inference classes.

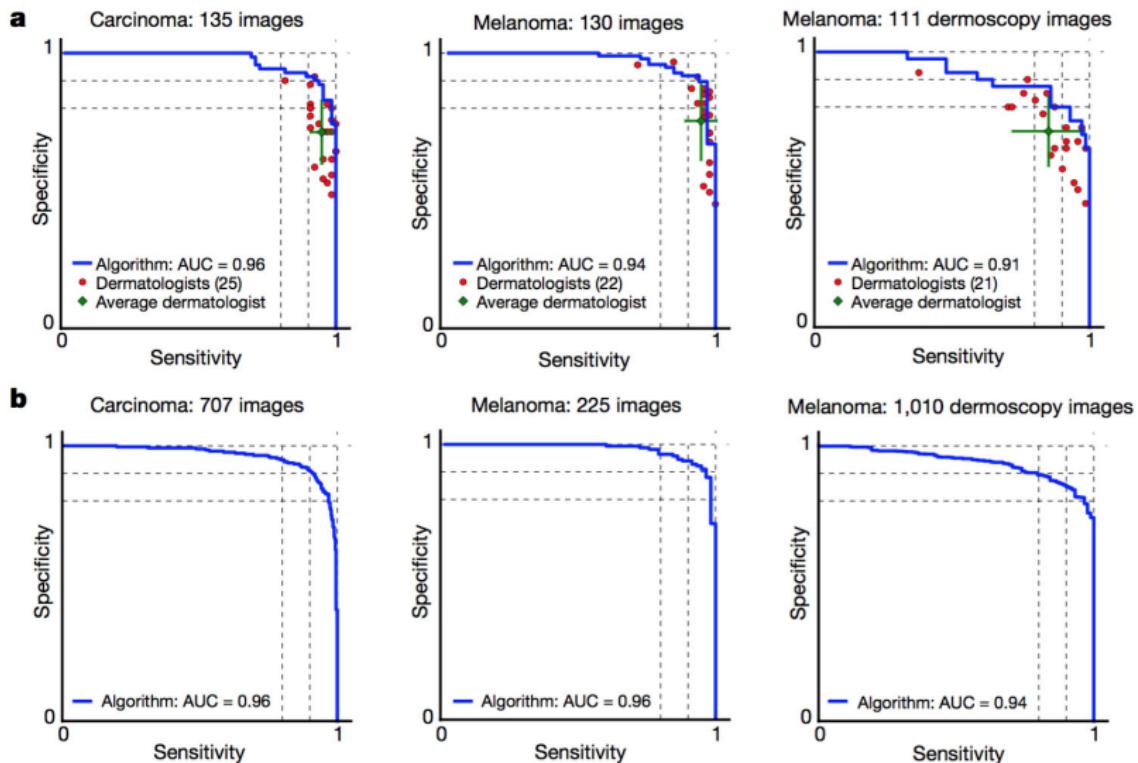


# Dermatologist-level classification of skin cancer with deep neural networks [Esteva et al, 17]

## Experimental protocol

- Test against 21 board-certified dermatologists on biopsy-proven clinical images.
- Two critical binary classification use cases:
  - malignant carcinomas versus benign seborrheic keratoses: identification of the most common cancers
  - malignant melanomas versus benign nevi : identification of the deadliest skin cancer

# Dermatologist-level classification of skin cancer with deep neural networks [Esteva et al, 17]



## Results

- The richness of the approach is in the building of the database and the ontology-based annotation.
- The classification task is simple.

# Performant but what about the confiance on the decision ?

## Skin images

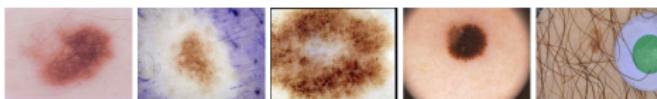
- The ISIC database is a database of annotated dermoscopic images.
- Use in different challenges.
- Deep neural networks (AUC=71%) have better results than dermatologists (AUC=67%)

(Bissoto el al, 2019) (De)Constructing Bias on Skin Lesion Datasets

([https://openaccess.thecvf.com/content\\_CVPRW\\_2019/papers/ISIC/Bissoto\\_DeConstructing\\_Bias\\_on\\_Skin\\_Lesion\\_Datasets\\_CVPRW\\_2019\\_paper.pdf](https://openaccess.thecvf.com/content_CVPRW_2019/papers/ISIC/Bissoto_DeConstructing_Bias_on_Skin_Lesion_Datasets_CVPRW_2019_paper.pdf))

# Performant but what about the confiance on the decision ?

Black boxes pose the problem of bias in the data.



(a) Traditional images



(b) Only Skin images



(c) Bbox images



(d) Bbox70 images

# Performant but what about the confiance on the decision ?

Black boxes pose the problem of bias in the data.

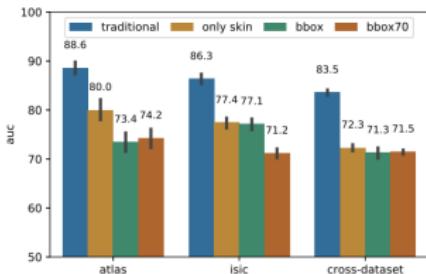


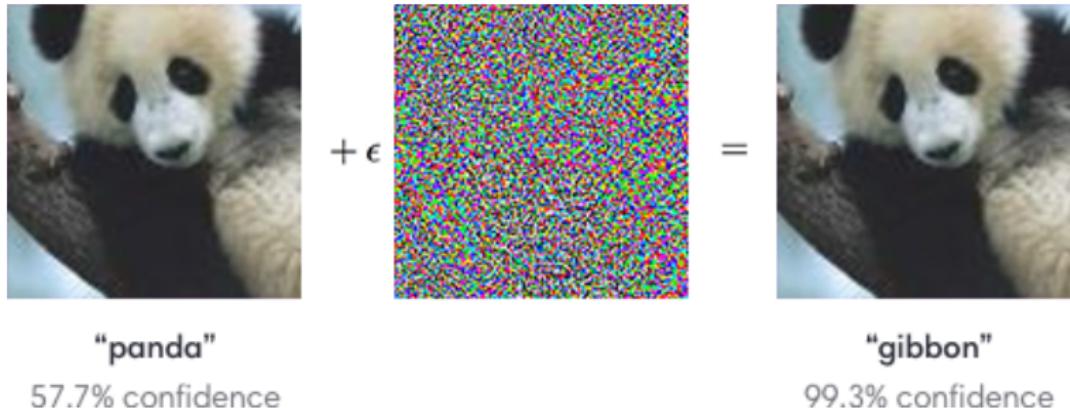
Figure 3: Models' performance over the disturbed datasets. We first remove all the pixel colors inside the lesion (*only skin*), proceeding to remove border information (*bbox*), and finally, removing the size (diameter) of the lesion (*bbox70*). Surprisingly, even when we destruct all clinical-meaningful information, the network finds a way to learn to classify skin lesion images much better than chance.

# Deep Learning : where are we ?

---

Trustworthy?

# Not trustworthy !



**Figure 9:** Goodfellow et al, Explaining and Harnessing adversarial examples. See  
<https://arxiv.org/pdf/1412.6572.pdf>

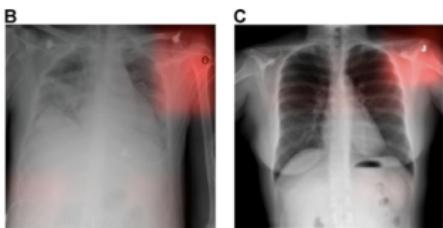
Performant AI systems are often **black box models**, i.e. models, whose internals are either unknown to the observer or they are known but uninterpretable by humans<sup>5</sup>.  
And they can be fooled.

<sup>5</sup> Guidotti et al - A Survey of Methods for Explaining Black Box Models  
<https://dl.acm.org/doi/pdf/10.1145/3236009>

# Not trustworthy !

*CNNs learn to predict pneumonia by detecting hospital which took the image*

- Study on detecting pneumonia using 158,323 chest radiographs
- CNNs robustly identified hospital system and department within a hospital
- CNN has learned to detect a metal token that radiology technicians place on the patient in the corner of the image



Variable generalization performance of a deep learning model to detect pneumonia in chest radiographs: A cross-sectional study.  
[Zech JR1](#), [Badgeley MA2](#), [Liu M2](#), [Costa AB3](#), [Titano JJ4](#), [Germann EK3](#). <https://www.ncbi.nlm.nih.gov/pubmed/30399157>

Slide credit : K. Saenko

# Not trustworthy !

Many accidents in AI systems !

The screenshot shows the homepage of the AI Incident Database (AID). At the top, there's a navigation bar with the AID logo and a search bar. Below the header, there's a section for 'Analysis' featuring an article titled 'AI Accidents: An Emerging Threat' by Zachary Arnold and Helen Toner from July 2021. To the right of this article is a grid of four news cards, each with a thumbnail image and a brief description:

- Researchers find evidence of racial, gender, and socioeconomic bias in chest X-ray classifiers** (Technology Review - 2018)
- Google Photos Still Has a Problem with Gorillas** (Technology Review - 2018)
- A Closer Inspection of Tesla's Autopilot Safety Statistics** (Medium - 2018)
- Google Photos Mistakenly Labels Black People 'Gorillas'** (Bloomberg Technology - 2018)

At the bottom right of the grid, there's a note: "ago — which uses a combination of".

<https://cset.georgetown.edu/publication/ai-accidents-an-emerging-threat/>  
<https://incidentdatabase.ai/?lang=en>

# Not trustworthy !

## Concrete problems for AI safety !

- **Avoiding negative side effects** : *Can we transform an RL agent's reward function to avoid undesired effects on the environment?* E.g, build a robot that move an object while avoiding knocking or breaking anything over, without programming a penalty for each possible bad behavior?
- **Safe exploration** : *Can reinforcement learning (RL) agents learn about their environment without executing catastrophic actions?* E.g, RL agent learn to navigate an environment without ever falling off a ledge?
- **Robustness to distributional shift** : *Can machine learning systems be robust to changes in the data distribution, or at least fail gracefully?* E.g, build image classifiers that indicate appropriate uncertainty when shown new kinds of images, instead using inapplicable learned model?
- **Avoiding “reward hacking” and “wireheading”** : *Can we prevent agents from “gaming” their reward functions, such as by distorting their observations?* E.g, train an RL agent to minimize the number of dirty surfaces, without looking for dirty surfaces or creating new dirty surfaces to clean up?
- **Scalable oversight.** : *Can RL agents efficiently achieve goals for which feedback is very expensive?* E.g, build agents that try to clean a room in the way the user would be happiest with, even though feedback is very rare and cheap approximations during training?

# Ethics and dilemma

## Autonomous cars and moral decisions<sup>6</sup>

An autonomous car is an intelligent agent capable of perceiving and acting on its environment while moving with little or no human intervention. For the vehicle to move safely and understand its environment, a huge amount of data must be captured by a multitude of different sensors in the car at any given time. This data is then processed by the vehicle's autonomous driving system.

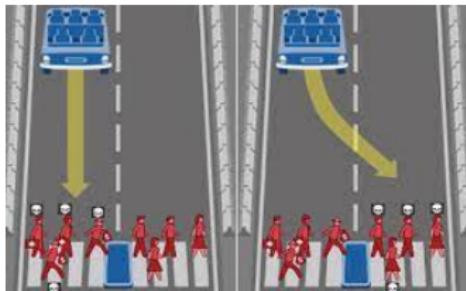


**Figure 10:** Source :Shutterstock.com/Senha

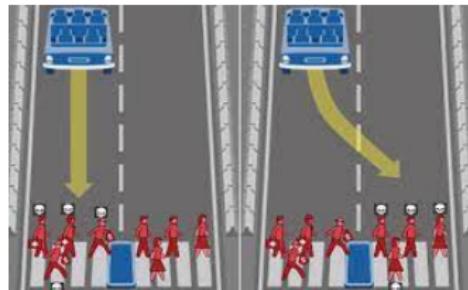
---

<sup>6</sup><https://www.youtube.com/watch?v=HzYG56HLxbI&feature=youtu.be>

# Autonomous cars and moral decisions

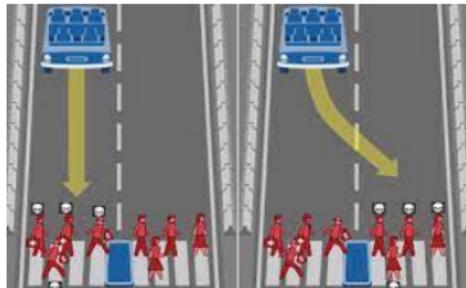


# Autonomous cars and moral decisions



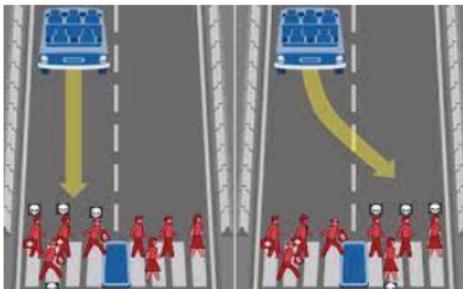
- From a **ethics dilemma** : what should a car do or not do in a specific scenario?

# Autonomous cars and moral decisions



- From a **ethics dilemma** : what should a car do or not do in a specific scenario?
- to a **social dilemma**: how to make the company accept and apply the compromises that suit it?

# Autonomous cars and moral decisions



- From a **ethics dilemma** : what should a car do or not do in a specific scenario?
- to a **social dilemma**: how to make the company accept and apply the compromises that suit it?
- AI is not just a technical, economical or legislative problem, it is also a problem of society's **cooperation**. <https://www.moralmachine.net/>

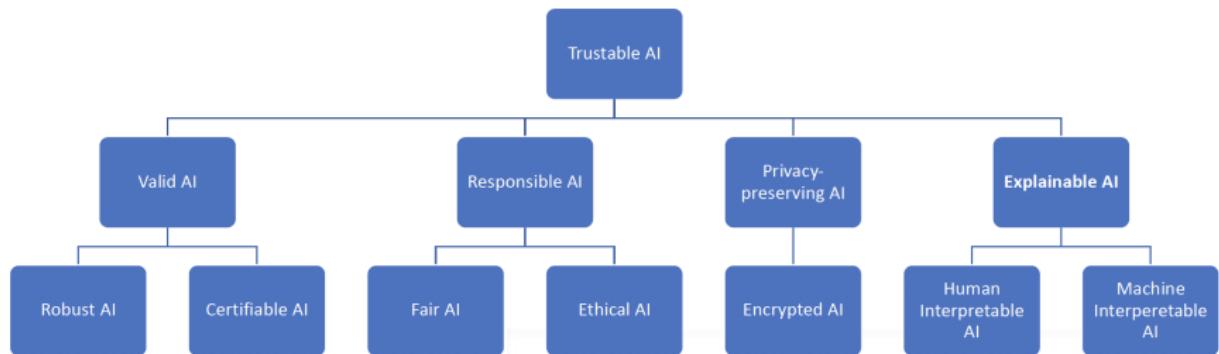
# Quelles décisions morales les voitures sans conducteur devraient-elles prendre ?

Take the time to look at the TED conference of Iyad Rahwan



[https://www.ted.com/talks/iyad\\_rahwan\\_what\\_moral\\_decisions\\_should\\_driverless\\_cars\\_make/transcript](https://www.ted.com/talks/iyad_rahwan_what_moral_decisions_should_driverless_cars_make/transcript)

# Requirements for AI adoption



**Figure 11:** Source : <https://xaitutorial2019.github.io/>

## Conclusion

---

# Conclusion

- Deep learning is a key element of the recent success of AI.
- Performance of deep learning models is highly correlated to the availability of huge high-quality annotated datasets.
  - But, this availability assumption is not realistic: **deep learning in low data regime**
- Deep learning and more generally AI is face to methodological issues to tackle :
  - Robustness (to shifts)
  - Safety
  - Privacy
  - Ethics
  - **Explainability**