
Data Structures and Algorithms in Java™

Sixth Edition

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Instructor's Solutions Manual

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Chapter 10 Maps, Hash Tables, and Skip Lists

Hints and Solutions

Reinforcement

R-10.1) Hint The first insertion is $O(1)$, the second is $O(2)$, ...

R-10.2) Hint Use the `PositionalList.remove` method to delete an entry from the map.

R-10.3) Hint Take advantage of the existing `findIndex` method.

R-10.3) Solution

```
public boolean containsKey(K key) {  
    return (findIndex(key) != -1);  
}
```

R-10.4) Hint Think about which of the schemes use the array supporting the hash table exclusively and which of the schemes use additional storage external to the hash table.

R-10.5) Hint There is a lot of symmetry and repetition in this string, so avoid a hash code that would not deal with this.

R-10.5) Solution Either polynomial hash codes or cyclic shift hash codes would be good.

R-10.6) Hint Try to mimic the figure in the book.

R-10.6) Solution

0	1	2	3	4	5	6	7	8	9	10
13	94				44			12	16	20
	39				88			23	5	
					11					

R-10.7) Hint Try to mimic the figure in the book.

R-10.7) Solution

0	1	2	3	4	5	6	7	8	9	10
13	94	39	16	5	44	88	11	12	23	20

R-10.8) Hint The failure occurs because no empty slot is found. For the drawing, try to mimic the figure in the book.

R-10.8) Solution

0	1	2	3	4	5	6	7	8	9	10
13	94	39	11		44	88	16	12	23	30

The probe sequence when inserting value 5 fails to ever find the sole remaining empty slot (at index 4).

R-10.9) Hint Try to mimic the figure in the book.

R-10.9) Solution

0	1	2	3	4	5	6	7	8	9	10
13	94	23	88	39	44	11	5	12	16	20

R-10.10) Hint Think of the worst-case time for inserting every entry in the same cell in the hash table.

R-10.10) Solution The worst-case time is $O(n^2)$. The best case is $O(n)$.

R-10.11) Hint Mimic the way the figure is drawn.

R-10.11) Solution

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		12	18	41	30	36	25		54			38	10			28		

R-10.12) Hint Combine the hash values of the two components to make a hash value for the pair.

R-10.12) Solution

```
public int hashCode() {
    return (a.hashCode() ^ b.hashCode());
}

public boolean equals(Object o) {
    if (o == null) return false;
    if (getClass() != o.getClass()) return false;
    Pair other = (Pair) o;
    return (a.equals(other.a) && b.equals(other.b));
}
```

R-10.13) Hint There is a subtle flaw—but can you find it?

R-10.13) Solution The problem occurs if null is allowed as a value within the map. In that case, there is ambiguity when `bucket.remove(k)` returns null, as that may indicate that nothing was removed, or that a key was removed having null as its value. Therefore, we might need to decrease `n`, even when null is returned.

R-10.14) Hint The load factor can be controlled from within the abstract class, but there must be means for setting the parameter (either through the constructor, or a new method).

R-10.15) Hint It is okay to insert a new entry on “top” of the deactivated entry object.

R-10.16) Hint You will need to keep track of the number of probes in order to apply quadratic probing.

R-10.17) Hint Think of where the entry with minimum key is stored.

R-10.18) Hint Since the map will still contain n entries at the end, you can assume that each `remove()` operation takes the same asymptotic time.

R-10.19) Hint Take advantage of the existing `findIndex` method.

R-10.19) Solution

```
public boolean containsKey(K key) {
    int j = findIndex(key);
    return (j < table.size() && compare(key, table.get(j)) == 0);
}
```

R-10.20) Hint The crucial methods are `get` and `put`.

R-10.21) Hint In the new code, what happens when `key` equals `table.get(mid)`?

R-10.21) Solution This version is certainly wrong. This can be seen by considering for example a call of `findIndex(20, 0, 2)` for a table with contents `{10, 20, 30}`. Such a call should return index 1, yet the newly proposed code will return index 3.

It is worth noting that the alternate version would produce the correct result if only line 4 had been written as

```
if (compare(key, table.get(mid)) <= 0)
```

thereby grouping the equal case with the `if` clause (not the `else` clause).

R-10.22) Hint Assume that a skip list is used to implement the sorted map.

R-10.22) Solution For this problem, we assume we are using a skip list as the implementation for the sorted map. (If a `SortedTableMap` were used, with lower price/performance pairs, each pair is inserted at the beginning of the array list, and thus the insertion time dominates the search time, for overall $O(n^2)$ time.)

With a skip list, the expected running time for maintaining a maxima set if we insert n pairs such that each pair has lower price and performance than one before it is $O(n \log n)$, since each pair is inserted and does not remove any others. Thus, the i^{th} insertion runs in $O(\log i)$ time, and the total expected running time is $\sum_{i=1}^n \log i$, which is $O(n \log n)$.

If, on the other hand, each pair had a lower price and higher performance than the one before it, then the overall running time is $O(n)$, because each new pair causes the removal of the pair that came before it. Therefore the size of the map is constant, and thus all its operations run in expected $O(1)$ time.

R-10.23) Hint Mimic the style of the figures in the book.

R-10.24) Hint You must link out the removed entry's tower from all the lists it belongs to.

R-10.25) Hint Compare to the implementation of the `addAll` method given in Section 10.5.1.

R-10.25) Solution Loop through the second set, while removing each of its elements from the first set (if it exists).

R-10.26) Hint Compare to the implementation of the `addAll` method given in Section 10.5.1.

R-10.26) Solution Loop through the first set, and remove each element that cannot be found in the second set.

R-10.27) Hint Recall that most skip-list operations run in $O(\log k)$ time for a set with size k .

R-10.27) Solution $O(m \log(n + m))$ time.

R-10.28) Hint Recall that most hash-table operations run in $O(1)$ expected time.

R-10.28) Solution $O(m)$

R-10.29) Hint Recall that most hash-table operations run in $O(1)$ expected time.

R-10.29) Solution $O(m)$

R-10.30) Hint Recall that most hash-table operations run in $O(1)$ expected time.

R-10.30) Solution $O(n)$

R-10.31) Hint Something from this chapter should be helpful!

R-10.31) Solution We should use a sorted multiset, as we want to maintain multiple entries with the same birthday and we want to be able to do neighboring queries.

Creativity

C-10.32) Hint Consider a bootstrapping method for finding the primes up to $\sqrt{2M}$.

C-10.33) Hint Your solution should make a single call to `findIndex`.

C-10.33) Solution

```

public V putIfAbsent(K key, V value) {
    int j = findIndex(key);
    if (j == -1) {
        table.add(new MapEntry<>(key, value));    // add new entry
        return null;
    } else {
        return table.get(j).getValue();           // return existing value
    }
}

```

C-10.34) Hint You may assume that such a method is supported by the auxiliary `UnsortedTableMap`

C-10.35) Hint You must only call the `findSlot` utility once.

C-10.36) Hint Model your solution after the existing support for other map methods.

C-10.37) Hint You may assume that such a method is supported by the auxiliary `UnsortedTableMap`

C-10.38) Hint The heart of the process can be performed by the `findSlot` utility.

C-10.39) Hint When might the load factor fall below the threshold, and how can you detect this from within `AbstractHashMap`?

C-10.40) Hint Start by defining an appropriate subclass of `AbstractMap.MapEntry` that includes the new `next` field that is desired.

C-10.41) Hint You need to do some shifting of entries to close up the “gap” just made, but you should only do this for entries that need to move.

C-10.41) Solution When we remove an entry from a hash table that uses linear probing without using deleted element markers, we must ensure that any cluster of consecutively filled cells remains intact, unless no entries in that cluster after a newly introduced gap have a hash value that falls before such a gap. Assume that we wish to delete an entry stored at index j and that the cluster it belongs to goes until an empty cell at index k . For ease of exposition only, we assume $k > j$, thus the cluster does not wrap around the end of the array. To fill the hole in index j , we wish to find the next entry of the cluster that has a hash value $h \leq j$. If no such entry exists, we may leave a hole at j . Otherwise, assume index i holds the first such entry. We move that entry to j and then we repeat the process to fill the hole that results at index i , finding the next subsequent entry of the cluster with hash value $h \leq i$. Note that the entire process takes time linear in the length of the original cluster of filled cells.

C-10.42) Hint For part a, note that the symmetry will halve the range of possible values. For part b, note that such automatic collisions will not occur.

C-10.43) Hint Perhaps borrow techniques from the Progression hierarchy of Chapter 2, or use Java's iterators.

C-10.44) Hint Maintain a secondary PositionalList instance that represents the FIFO order, and store positions within that list with each entry of the hash table.

C-10.45) Hint Each map entry instance should store its current index within the table.

C-10.46) Hint Each map entry instance should store its current index within the table.

C-10.47) Hint Each map entry instance should store its position within the bucket list.

C-10.48) Hint Try out some examples.

C-10.48) Solution The original version of Code Fragment 10.11 does not guarantee that it finds the leftmost occurrence when duplicates exist, because it stops searching as soon as it finds a match. For example, if the entire table was filled with duplicates, it will report the middle index.

If the version in Exercise R-10.21 had been written as intended, with the \leq operator on line 4, then it is a correct implementation and indeed guarantees that it finds the leftmost of all duplicates.

C-10.49) Hint Do a "double" binary search.

C-10.49) Solution In order to find the k^{th} smallest key in the union of the keys from S and T , we can do a "double" binary search. In other words, we will begin by examining the $k/2^{\text{th}}$ element in the array list S . Next, we will find the largest element in T that is less than $S[k/2]$ by binary search. Then, we will add the indices of the elements we were examining in S and T . If the sum equals k , then the max of the two elements is our result. If the sum is greater than k , then we will do a binary search to the right (upwards) in S . If the sum is less than k , then we will do a binary search to the left (downwards) in S . This is followed once again by searching in T for largest element less than the current element in S , etc. This method does a binary search on S which requires $O(\log n)$ "probes." However, for each probe of the search, it does a binary search on T which takes $O(\log n)$ time. Thus, the entire method takes $O(\log^2 n)$ time.

C-10.50) Hint Dovetail two binary searches.

C-10.51) Hint Do a lazy iteration through indices of the underlying array list.

C-10.52) Hint Manage a primary iteration through all nonempty buckets, and then a secondary iteration through each element of a bucket.

C-10.53) Hint Do a lazy iteration through the nonempty cells of the table.

C-10.54) Hint Think about first sorting the pairs by cost.

C-10.54) Solution Sort the pairs by cost. Then scan this list looking at the performance values. Remove any that have performance values worse than the (unremoved) pair that came before.

C-10.55) Hint In the insertion algorithm, first repeatedly flip the coin to determine the level where you should start inserting the new entry.

C-10.56) Hint Consider augmenting each node v in a higher level with the number of missing entries in the gap from v to the next node over.

C-10.57) Hint Consider augmenting each node v in a higher level with the number of missing entries in the gap from v to the next node over.

C-10.58) Hint Think first about how you can determine the number of 1's in any row in $O(\log n)$ time.

C-10.58) Solution To count the number of 1's in A , we can do a binary search on each row of A to determine the position of the last 1 in that row. Then we can simply sum up these values to obtain the total number of 1's in A . This takes $O(\log n)$ time to find the last 1 in each row. Done for each of the n rows, then this takes $O(n \log n)$ time.

C-10.59) Hint Consider a two-pass solution, with use of an auxiliary structure.

C-10.59) Solution

```
public void retainAll(Set<E> other) {
    // let's first build a list of elements to remove
    ArrayList<E> leaving = new ArrayList<E>();
    for (E element : this)
        if (!other.contains(element))
            leaving.add(element);
    // now let's remove those elements from the set
    for (E element : leaving)
        remove(element);
}
```

C-10.60) Hint Think of describing your algorithms in terms of boolean operations on the bit vectors.

C-10.61) Hint Think of how you could transform D into L .

C-10.62) Hint Consider the way subMap was implemented for SortedTableMap.

C-10.62) Solution Simply do a binary search to find an element equal to k . Then step back through the array until you reach the first element equal

to k . Finally, step forward through the area reporting entries until you reach the first key that is not equal to k . This takes $O(\log n)$ time for the search and then at most s time to search back to the beginning of the run of k 's and s time return all of the elements with key k . Therefore, we have a solution running in at most $O(s + \log n)$ time.

C-10.63) Hint You need some way of grouping together entries with the same key.

C-10.63) Solution Use a skip list map, with only r nodes in the bottom level, and with each of these having a secondary linked list of elements with the same key. Thus, all the query and update operations will run in $O(\log r)$ expected time, and the `getAll` method will run in $O(\log r + s)$ expected time.

C-10.64) Hint You need some way of grouping together elements with the same key.

C-10.64) Solution The solution is actually quite simple—just use something like the `HashMultimap` class given in the book, but make sure that the capacity of the underlying hash table is always $O(n)$. Another possibility is to use a linked hash table, which keeps its entries in a linked list in addition to the hash table. Either way we would get that the `entries()` method would run in $O(n)$ time.

Projects

P-10.65) Hint The biggest challenge is detecting the case of an infinite loop.

P-10.66) Hint When searching for an existing key, make sure to consider both of the possible buckets.

P-10.67) Hint Maintain a secondary `PositionalList` instance that represents the FIFO order, and store positions within that list with each entry of the hash table.

P-10.68) Hint We've already implemented them for you; all that's left is the experiments.

P-10.69) Hint In a Unix/Linux system, a good place to start is `/usr/dict`.

P-10.70) Hint It is okay to generate these phone numbers more-or-less at random.

P-10.71) Hint Sentinels can be used in place of the theoretical $-\infty$ and $+\infty$.

P-10.72) Hint Try to make your screen images mimic the skip list figures in the book.

P-10.73) Hint You need to find some way to let the intermediate nodes in the skip list keep track of the number of elements that have been skipped over.

P-10.74) Hint For each word t that results from a minor change to s , you can test if t is in W in $O(1)$ time.